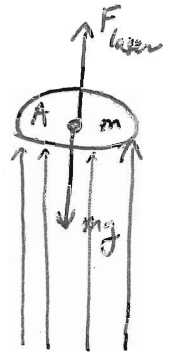
29.56 Use laser beam to hold a light piece of Al foil : $m = 30\,\mu g$

$$F_{laser} - mg = 0$$



<u>Radiation pressure</u>: laser ⟷ EM wave ⟷ pressure P

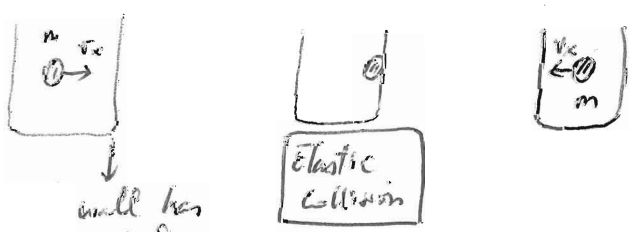
<u>Gas Pressure</u>

→ Gas molecule transfers $\Delta p = 2mv_x$ to wall :

force

$$\text{Pressure } P = \frac{F}{A} = \frac{\frac{\Delta p}{\Delta t}}{A} = \frac{2mv_x}{A\,\Delta t}$$

→ <u>Elastic collision</u>



<u>Total initial momentum</u> = <u>Total final momentum</u>

$$mv_x + 0 = -mv_x + 2mv_x$$

acquired or transferred to wall

<u>Radiation Pressure</u>

→ EM wave ⟷ no mass but still carries a momentum

Force

$$\text{Pressure } P = 2\frac{\text{Power}}{c \cdot A}$$

→ <u>Foil reflects all light</u>

$$\left(\text{Power} = \frac{\text{Work}}{\Delta t} = \frac{F \cdot d}{\Delta t} = F \cdot \text{speed}\right)$$

→ Foil is equiv. to wall in gas situation.

$$F_{laser} = mg$$

$$P \cdot A = mg$$

$$2\frac{\text{Power}}{c \cdot \cancel{A}} \cdot \cancel{A} = mg$$

$$\text{Power} = \frac{mgc}{2}$$

$$= \frac{30 \times 10^{-9} \times 9.81 \times 3 \times 10^{8}}{2}$$

$$= 44.1\,W$$

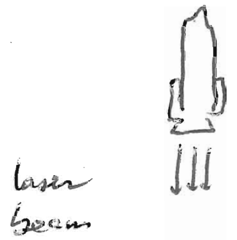
<u>Note</u> : ⟹ Pressure (Rad.) $P = 2\frac{\text{Power}}{c \cdot A} = 2\frac{\bar{S}}{c}$

$\bar{S}$: average radiation intensity (power per unit area)

→ This happens (factor 2) when there is reflection (thin foil reflects all the light)

→ If there is no reflection : Rad. Pressure $P = \frac{\overline{S}}{c}$

29.59 Power needed for a "photon rocket" (using laser beam instead of hot gas)



↳ Needs to provide an upward thrust force of $35 \times 10^6 N$

Here there is no light reflection as in 29.56 → Rad. Pressure : (Force per unit area)

$$P = \frac{\overline{S}}{c}$$

$$F = P \cdot A = \frac{\overline{S}}{c} \cdot A = \frac{\overline{Power}}{\cancel{A} \cdot c} \cdot \cancel{A}$$

$$\overline{Power} = c \cdot F = 3 \times 10^8 \times 35 \times 10^6 = 10^{16} W$$

All of our power generating capability is only $10^{12} W$

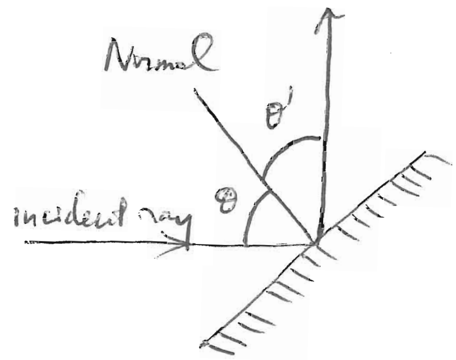
Ch 30: Reflection & Refraction

Optics

- → **Geometrical Optics** (Ch 30, 31): study propagation of light rays in straight line using geometry
- → **Physical Optics** (Ch 32): study propagation of lights, looking @ wave properties of light (superposition: interference & diffraction) in addition to the geometry

Reflection: light ray incident upon a mirror is reflected with $\theta' = \theta$

Mirror: light can only propagate in one side of it, being the other inaccessible.



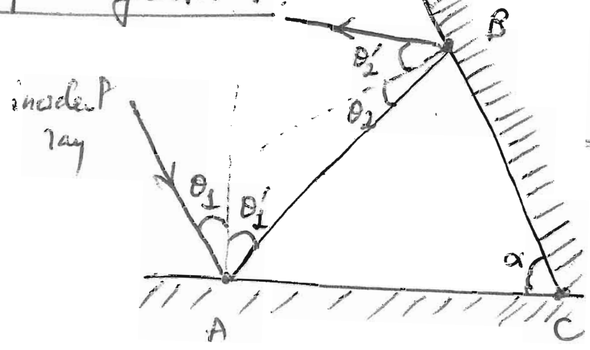
→ **Normal**: is the (imagined) line that is perpendicular to the flat mirror

→ **Incident angle θ**: is the angle b/w the incident & the normal to the surface of the mirror

→ **Reflected angle θ'**: angle b/w reflected ray & normal.

→ **Law of reflection**: $\theta' = \theta$

→ **Multiple reflections**:



→ We know: $\begin{cases} \theta_1' = \theta_1 \\ \theta_2' = \theta_2 \end{cases}$

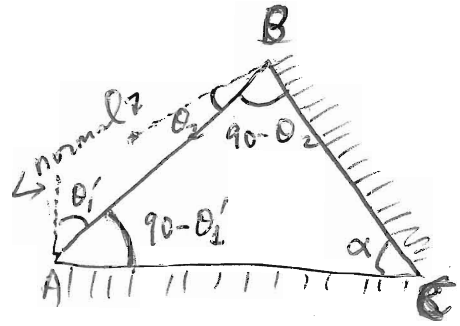
→ Need to relate θ_1 to θ_2' or θ_2 to θ_1' → using the geometry of the set of mirrors

By studying the triangle ABC :

$$90 - \theta_2 + 90 - \theta_1' + \alpha = 180$$

$$\alpha - \theta_1' - \theta_2 = 0$$

$$\theta_2 = \alpha - \theta_1'$$



$$\boxed{\theta_2' = \theta_2 = \alpha - \theta_1' = \alpha - \theta_1}$$

θ_2': 2nd reflected angle

θ_1: 1st incident angle

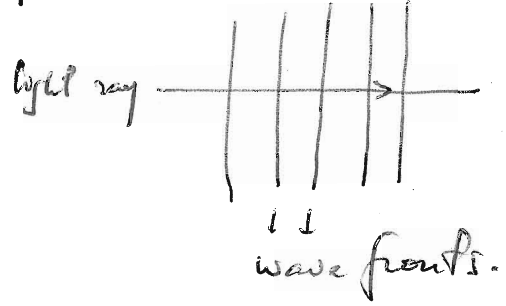
Refraction: when light travels from one medium to another with a different density of matter

- in vacuum : $c = 3 \times 10^8$ m/s
- in water : $v = \frac{c}{n}$ ($n = 1.333$ for water index of refraction)
- $\boxed{\text{refraction} \leftrightarrow \text{change of speed}}$.
- commonly observed:
  - rainbow (water droplets have different indices of refraction for different colors or different wavelengths
  - broken straw :

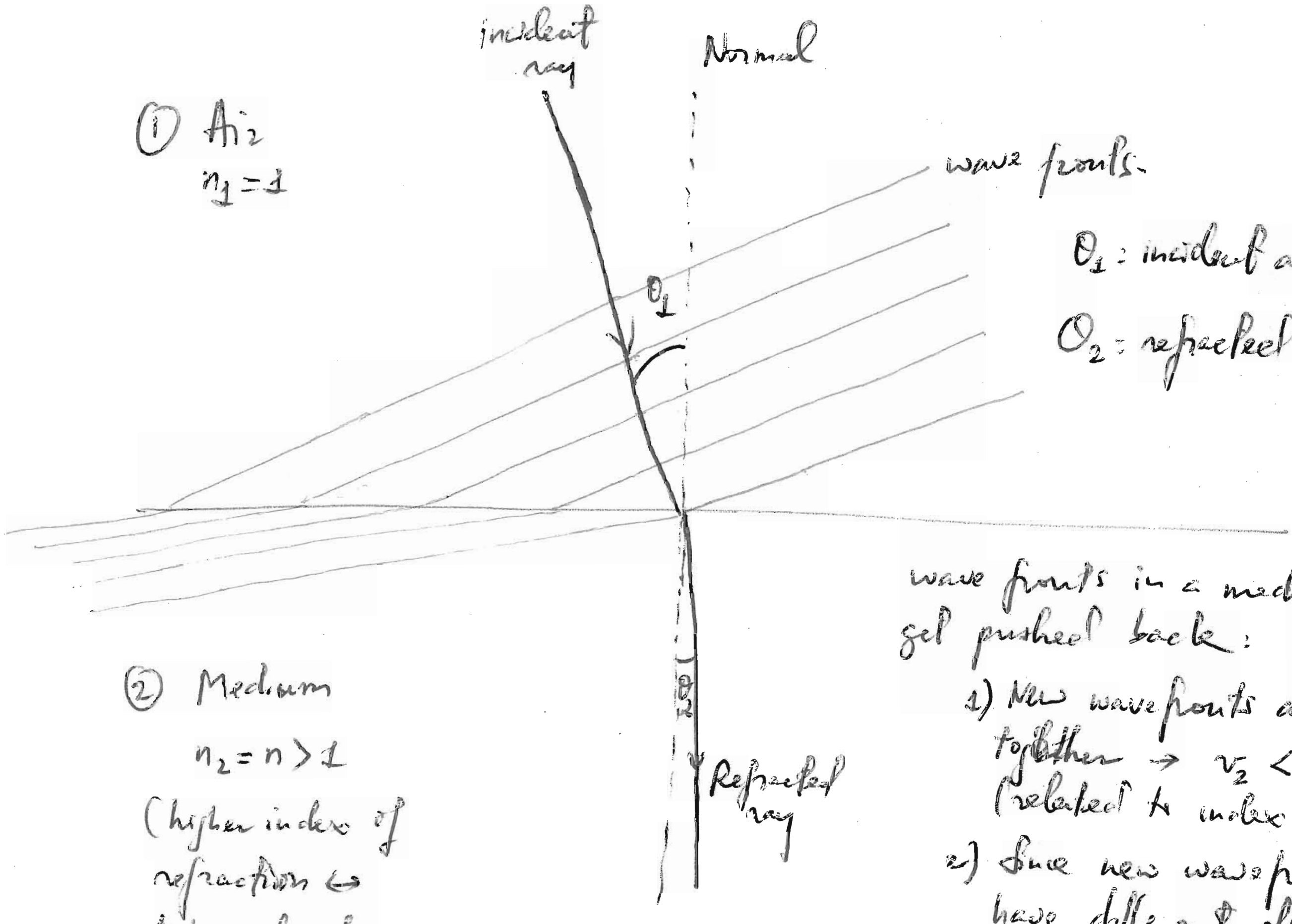
empty glass

half filled with water

Wave fronts: for a light ray wave fronts are straight lines that are perpendicular to the direction of propagation.



Air to a medium.



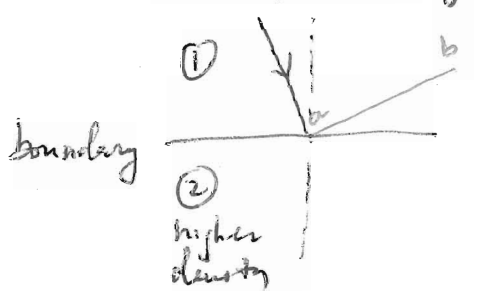
θ_1 : incident angle

θ_2 : refracted angle

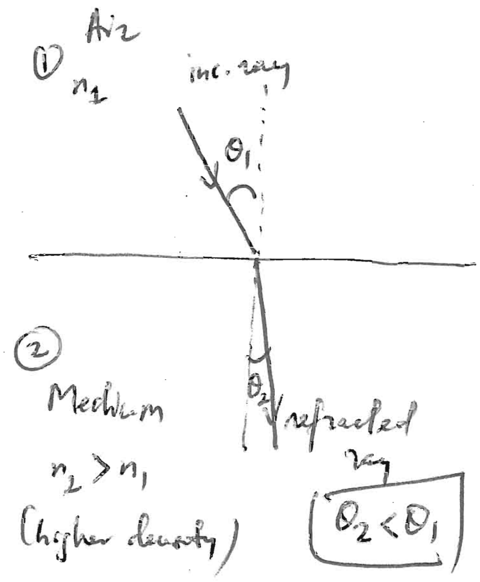
wave fronts in a medium ② get pushed back:

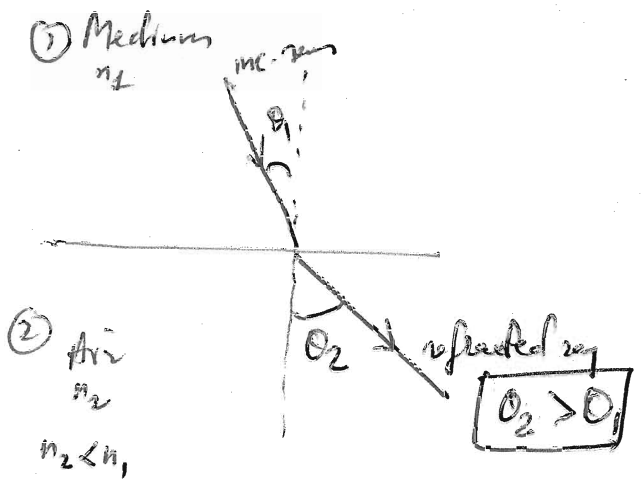
1) New wave fronts are closer together → $v_2 < v_1$ (related to index of refraction)
2) Since new wave fronts have different directions → refracted ray has $\theta_2 < \theta_1$

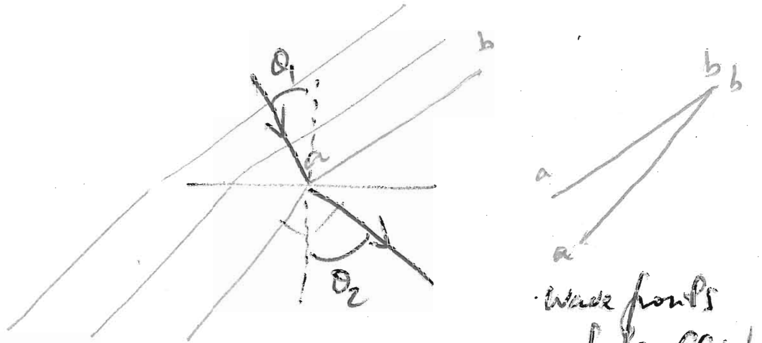
3) Why it gets pushed back?



a hits boundary first while b is still in 1st medium → a gets slowed down while b still travels @ initial speed: wave front will rotate around a in CW : b/c of this $\theta_2 < \theta_1$







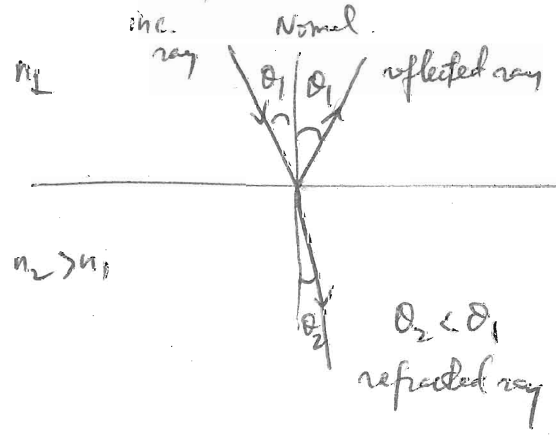
Wave fronts rotate CCW (a gets going faster than b as it crosses the boundary before b)

Law of refraction: Snell's Law:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

- n_1: index of refraction medium #1
- θ_1: incident angle
- n_2: index of refraction medium #2
- θ_2: refracted angle

Note: Normally ($\theta_1 < \theta_c$: critical angle): { Reflection & Refraction



@ $\theta_1 \geq \theta_c$

→ There is NO refraction

→ $\theta_2 = 90°$ (outgoing ray is ∥ boundary)

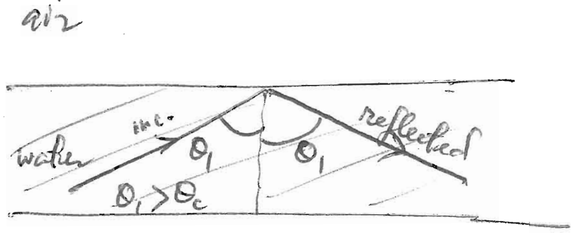
$n_1 \sin\theta_{1c} = n_2$

$\sin\theta_{1c} = \frac{n_2}{n_1}$

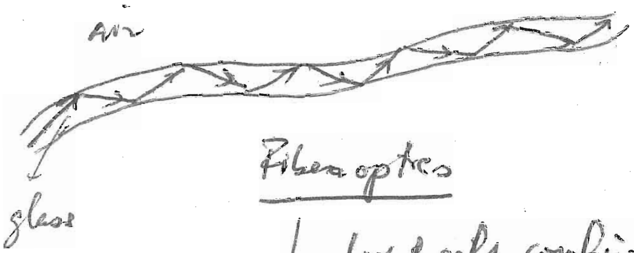
$\theta_{1c} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

→ (Only when $n_1 > n_2$

Critical angle $\leftrightarrow$ Total internal reflection ($n_2 < n_1$): going from higher density (or index of refraction) to lower density



Total internal reflection
(no refraction to air)

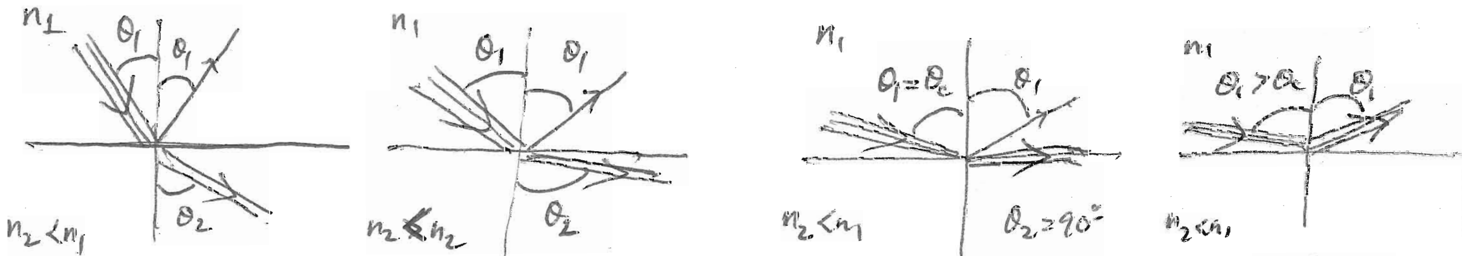


Fiber optics

↳ light gets confined

↳ Can carry lots of data quickly (internet, phone, etc...)

Critical angle: higher index to lower index:



$\theta_2 > \theta_1$

larger θ_1 (still $\theta_1 < \theta_c$)
larger θ_2 (refracted rays are closer to the boundary)

$n_1 \sin\theta_c = n_2$

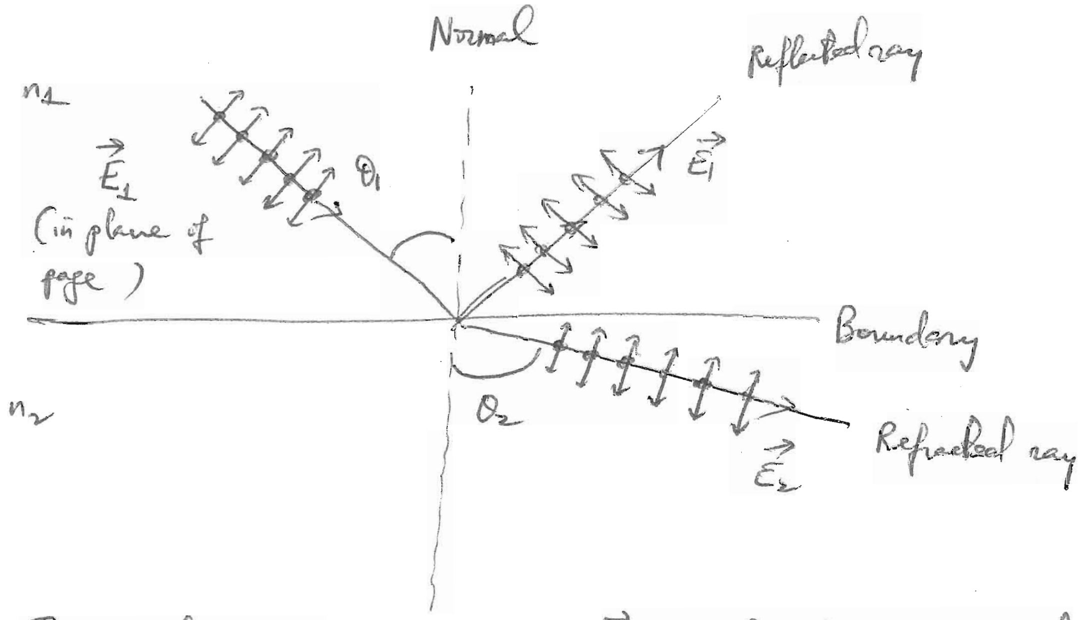
$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

@ $\theta_1 > \theta_c$
→ no further refraction
→ all rays stay in medium ① due to reflection!

→ When $\theta_1 > \theta_c$: no refraction, all reflection

↳ Is there an angle @ which there is no reflection, all refraction?

$\boxed{\text{The polarizing angle or Brewster angle}}$



Note: → For in-plane-of-page rays, $\vec{E}$ could polarize as shown or also in and out of page: ⊙

→ If $\theta_1 = \theta_p$ then only the in & out of page direction for $\vec{E}$ is allowed for reflected ray.

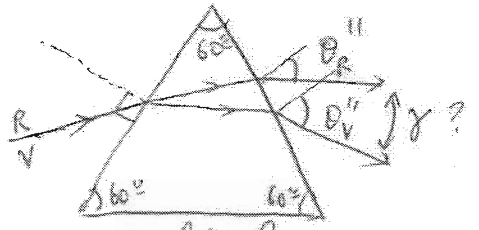
↳ Implication: 1) if $\boxed{\text{at } \theta_1 = \theta_p \text{ or } \theta_B}$ there is no in-and-out component of $\vec{E}$ in the incident ray → since that is the only polarization allowed for reflected ray → there will be no reflection. $\theta_p = \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

2) If $\theta_1 = \theta_p = \theta_B$ and there is both in-and-out and in-plane-of-page polarizations for $\vec{E}$ in the incident ray: there will still be a reflection for the in-plane-of-page polarization component of $\vec{E}$

3) If $\theta_1 = \theta_p = \theta_B$ and there is only the ~~in-plane-of-page~~ in-and-out polarization of $\vec{E}$: there is reflection, The Brewster angle has no meaning.

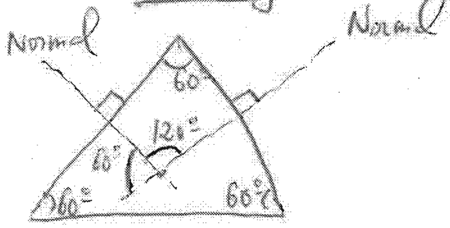
30.28
30.57

30.28



Equilateral.

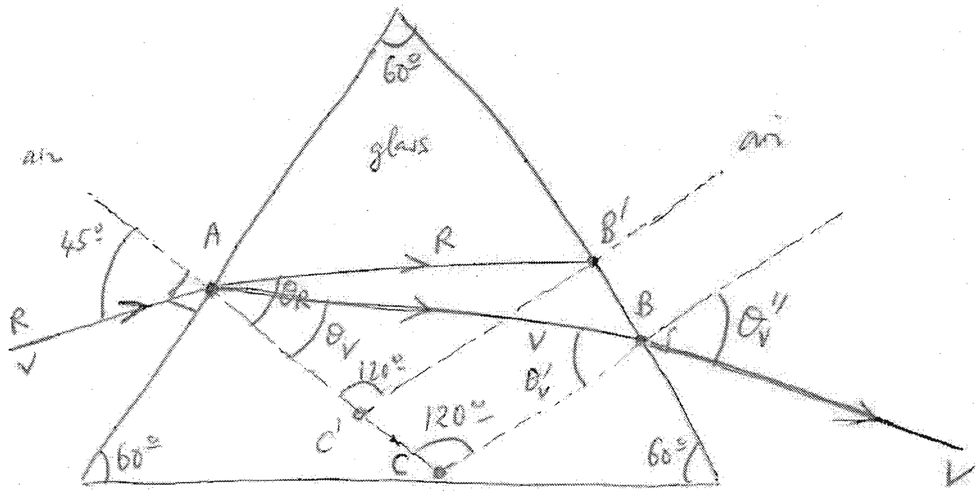
Geometry:



Prism $\left\{ \begin{array}{l} n_R = 1.582 \\ n_V = 1.633 \end{array} \right.$ $\succ\rightarrow$ Dispersion: different colors come out @ different directions the other side of prism.

(Different indices of refraction for different wavelengths)

$\gamma \equiv \theta_V'' - \theta_R''$?



Violet ray: $\left\{ \begin{array}{l} \text{Incident on left boundary @ A : angle } 45° \\ \text{Refracted angle on left boundary @ A : } \theta_V \\ \text{Incident on right boundary @ B : } \theta_V' \\ \text{Refracted angle @ B : } \theta_V'' \end{array} \right.$

- Snell's Law @ A: $\underbrace{1 \sin 45°}_{\frac{1}{\sqrt{2}}} = 1.633 \sin \theta_V \rightarrow \theta_V = \sin^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{1.633}\right) = 25.5°$

- Snell's Law @ B: $1.633 \sin \theta_V' = 1 \sin \theta_V''$

Need one more eq. → from geometry: triangle ABC → $\theta_V + \theta_V' + 120° = 180°$

$\rightarrow \theta_V + \theta_V' = 60° \rightarrow \theta_V' = 60° - \theta_V = 60° - 25.5° = 34.5°$

$$\boxed{\theta_V'' = \sin^{-1}(1.633 \sin 34.5°) = 67.7°}$$

$\boxed{\text{Red ray:}}$ same calculations except $n_R = 1.582$

Snell's law @ A: $1 \sin 45° = 1.582 \sin \theta_R \rightarrow \theta_R = \sin^{-1} \frac{\frac{1}{\sqrt{2}}}{1.582} = 26.5°$

Snell's Law @ B': $1.582 \sin \theta_R' = 1 \sin \theta_R''$

From triangle AB'C': $\theta_R + \theta_R' + 120° = 180°$

$\theta_R' = 60 - \theta_R = 60 - 26.5°$

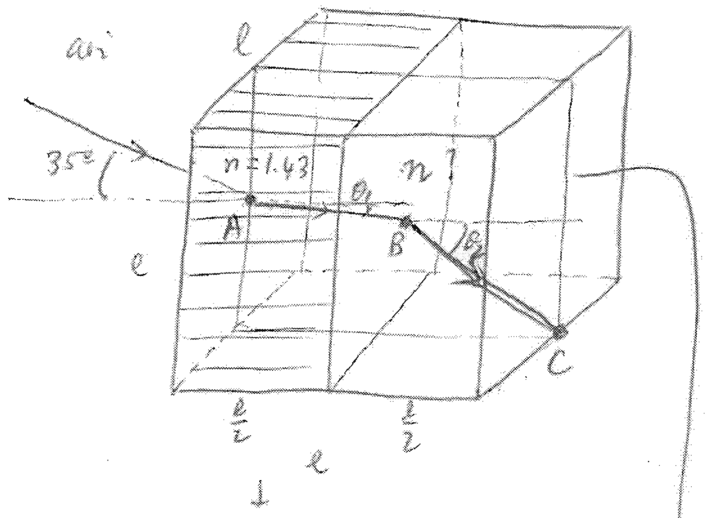
$\theta_R' = 33.5°$

$$\boxed{\theta_R'' = \sin^{-1}(1.582 \sin 33.5°) = 60.8°}$$

$\rightarrow$ Angular dispersion $\delta = \theta_V'' - \theta_R'' = 67.7° - 60.8° = 6.85°$.

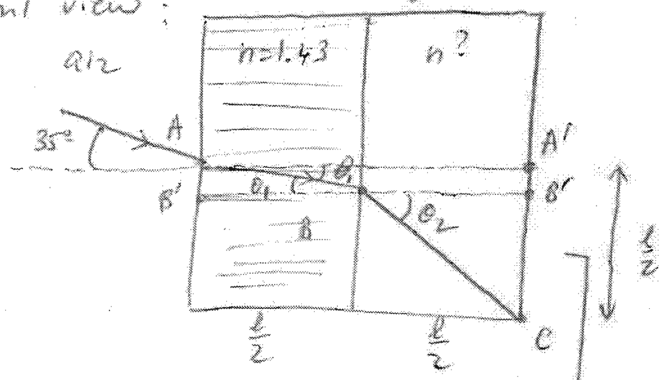
For the beam: Red & Violet are the outer wavelengths of the visible spectrum : other colors ~~below~~ are between these : $1.582 < n < 1.633$.

30.57



1st refraction @ A:

Front view:



Snell's Law:

@ A: $1 \sin 35^\circ = 1.43 \sin \theta_1$

$$\theta_1 = \sin^{-1}\left(\frac{\sin 35^\circ}{1.43}\right)$$

$$\theta_1 = 23.6^\circ$$

@ B: $1.43 \sin 23.6^\circ = n \sin \theta_2$ → 2 unknowns: need one more equation from the geometry of the problem:

$$\tan \theta_2 = \frac{B'C}{\frac{l}{2}}$$

$A'C = \frac{l}{2}$ → $B'C = A'C - A'B'$

Now find $A'B'$: $\frac{A'B'}{\frac{l}{2}} = \tan \theta_1$

$$A'B' = \frac{l}{2} \tan \theta_1$$

$$\boxed{\tan \theta_2 = \frac{\frac{l}{2} - \frac{l}{2}\tan\theta_1}{\frac{l}{2}} = 1 - \tan \theta_1}$$

$$\theta_2 = \tan^{-1}(1 - \tan 23.6^\circ)$$

$$\boxed{\theta_2 = 29.3^\circ}$$

$$\boxed{n = \frac{1.43 \sin(23.6^\circ)}{\sin(29.3^\circ)} = 1.17}^{*}$$

* From Figure given as data $\theta_2 > \theta_1$ → we expect $n < 1.43$ → checked

30.44

$n = 1.52$

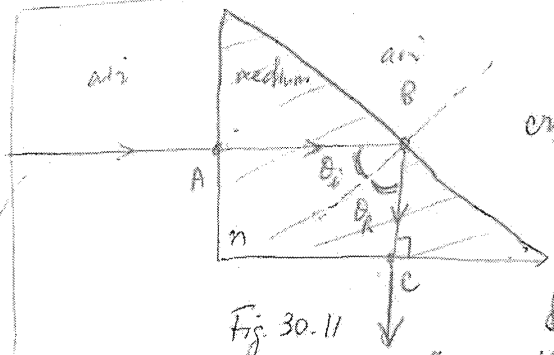


Fig. 30.11

critical angle: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ @ B

$\hookrightarrow \begin{cases} n_2 = 1 \\ n_1 = n = 1.52 \end{cases}$ $\theta_c = \sin^{-1}\left(\frac{1}{1.52}\right) = 41°$

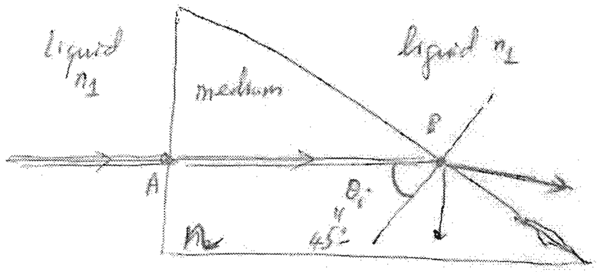
there is Total internal reflection (TIR)

$\theta_R = \theta_i \rightarrow 2\theta_i = 90° \longrightarrow 45° = \boxed{\theta_i \geq 41°}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_1 = 0 \Rightarrow \theta_2 = 0$

Now immersed in liquid: no more Total reflection @ B



$n = 1.52$

What minimum n_1 so there is some refracted light out to liquid?

since w/ or w/o liquid $\theta_i = 45°$ → no longer T.I.R. if

θ_i now is $\leq \theta_c^{liq}$ → $\sin\theta_i = \sin 45° = \frac{1}{\sqrt{2}} \leq \sin\theta_c^{liq}$.

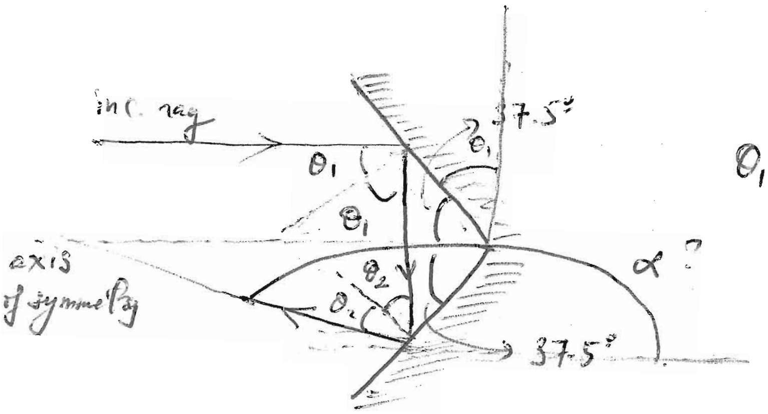
θ_c^{liq}: new critical angle w/ liquid outside (@ B)

$$\theta_c^{liq} = \sin^{-1}\left(\frac{n_1}{n}\right) = \sin^{-1}\left(\frac{n_1}{1.52}\right)$$

$$\sin\theta_c^{liq} = \frac{n_1}{1.52}$$

$$\frac{1}{\sqrt{2}} \leq \frac{n_1}{1.52} \rightarrow \boxed{n_1 \geq \frac{1.52}{\sqrt{2}} = 1.07}$$

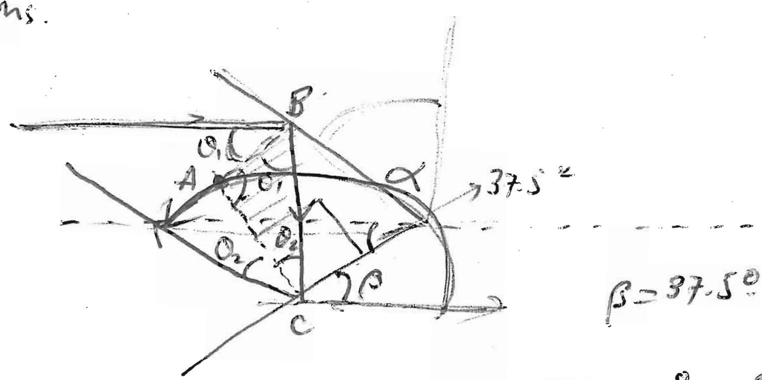
30.29



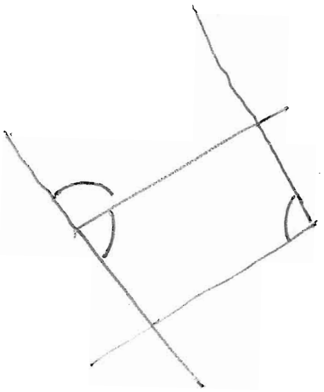
$\theta_1 = 90 - 37.5° = 52.5°$ ✓

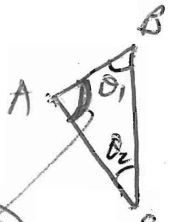
a) 2 reflections.

b) α?



$\beta = 37.5°$





105°

(AB ⊥ mirror #1
AC ⊥ mirror #2)

$\alpha = \beta + 90° + \theta_2$

$\theta_2 = 180 - 105° - \theta_1$

$= 75 - \theta_1$.

$\alpha = 37.5° + 90° + 75 - \theta_1$

$= 37.5° + 165° - 52.5°$

$= 150°$ CCW

(or 210° CW)

Ch 31 Images & Optical Instruments

↳ mirrors & lenses.

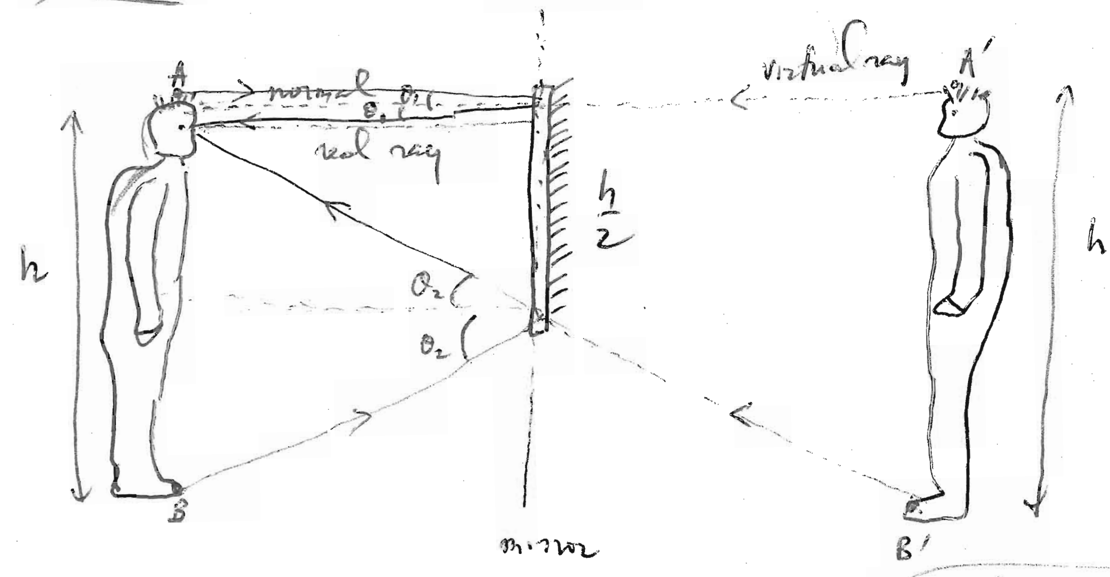
↳ How to form the image of an object for a mirror or a lens (or lenses) using geometrical optics (ray tracing)

Image formation by a mirror (flat)

How tall a mirror to see one's whole body: (height h)

1) $h_m = h$ 2) $h_m = \frac{h}{2}$ 3) $h_m = \frac{h}{3}$

↳ Answer: see hair & toe

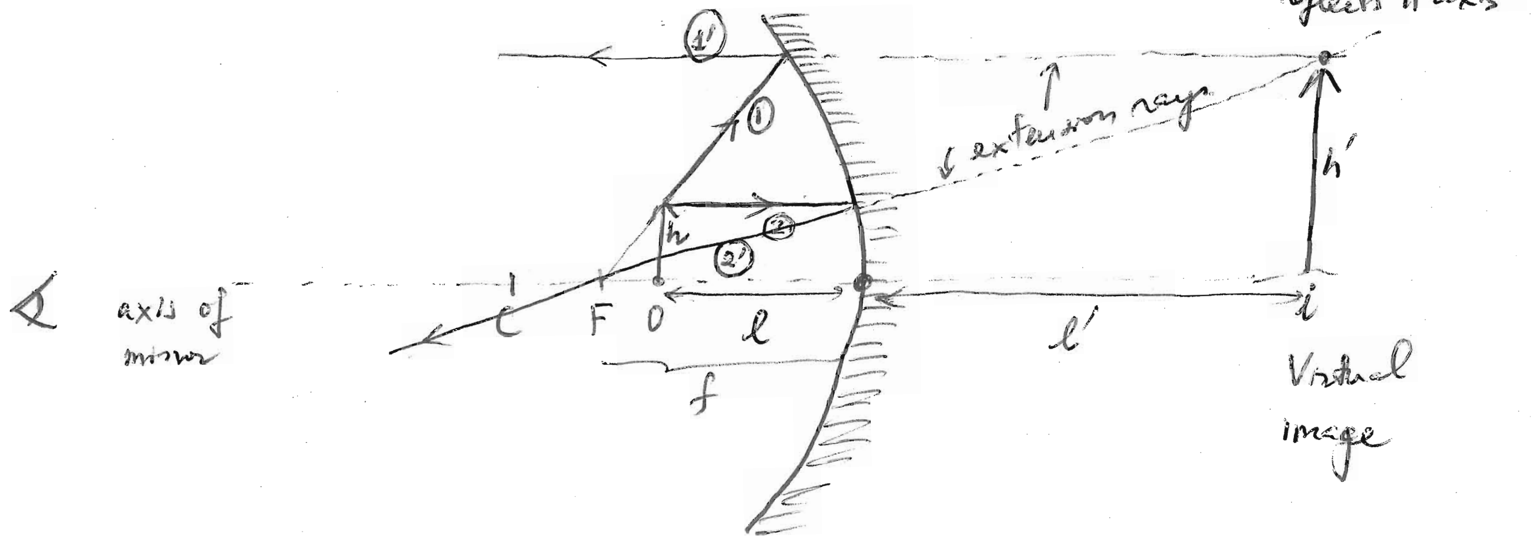


Object

Virtual image
↳ light can't travel through mirror
↳ formed by extension rays or virtual rays.
↳ If these rays are physically blocked, image is unaffected.

Virtual image by a curve mirror:

- C: center of curvature of mirror
- F: focal point
  - 1) Inc. ray ∥ axis reflects thru F
  - 2) Inc. ray thru F reflects ∥ axis
- O: object



Note: 1) in this example F is b/w C & midpoint of mirror

2) Trace 2 rays ① & ② to form the image i:
image is where ①' & ②' converge via their extension rays.

3) location of object and virtual image wrt midpoint of mirror are l, l', respectively. Heights of object and image are h, h', respectively

4) Mirror equation: (derived from the geometry of rays)

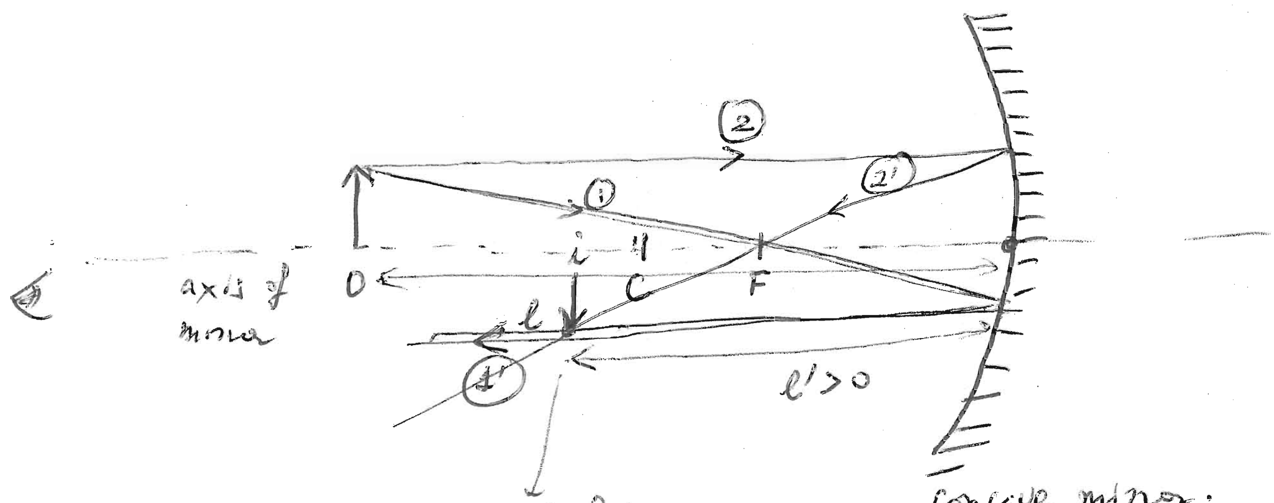
$$\frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \quad (f: \text{focal length})$$

Sign convention: (so same eq. for mirrors & lenses)

- f
  - $+$ concave mirror
  - $-$ convex mirror
- l'
  - $+$ image on same side of mirror as object (real image)
  - $-$ image on the other side of mirror (virtual image)

5) Magnification factor: $M \equiv \dfrac{h'}{h} \underset{\text{geometry}}{=} -\dfrac{l'}{l}$

When will image be real for this mirror ($l' > 0$)?

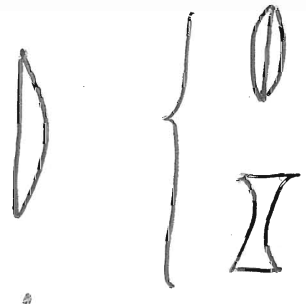


Real image

↳ Formed with real rays.

Concave mirror.
$f > 0$

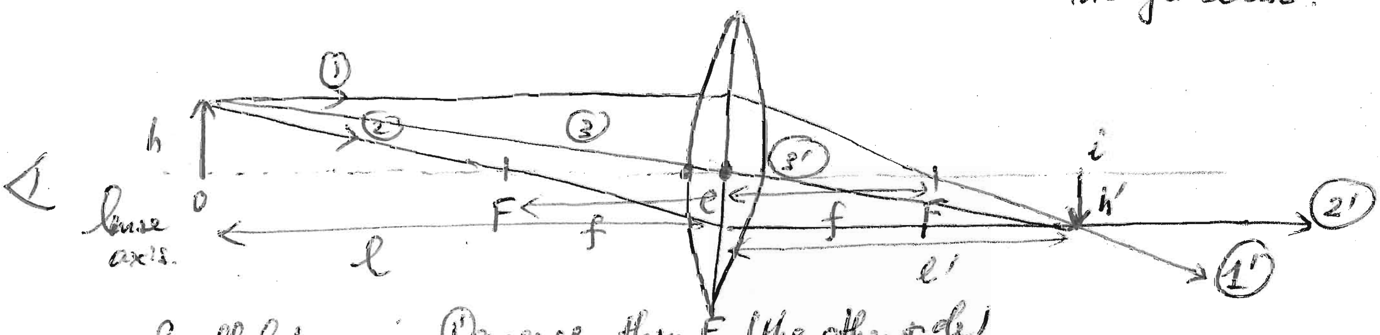
Image formation with lenses:



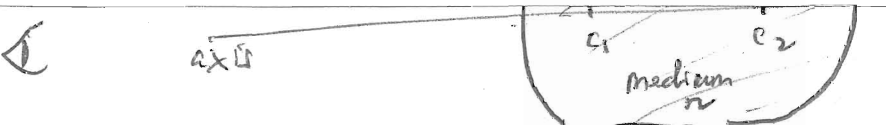
converging lense or convex lense

diverging lense or concave lense

{ C: center
F: focal points (2 symmetric points) : same definition as with mirrors but rays will go through lense.



(1): Parallel to axis (1') emerge thru F (the other side)
(2) Thru F (left of lense), (2') emerge ∥ axis
(3) Thru C and straight to the other side (3')



focal length
R_1 & R_2 sign { + Convex ($R_2 > 0$)

→ lense equation : $\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}$
(same as mirror eq.)

→ Magnification factor $M = \frac{h'}{h} = -\frac{\ell'}{\ell}$
(same as mirrors)

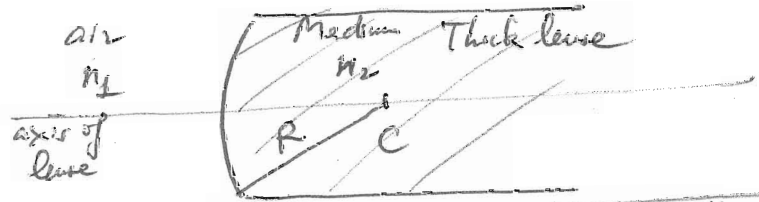
→ Sign convention for lenses $\begin{cases} f \begin{cases} - & \text{concave lenses (diverging)} \\ + & \text{convex lenses (converging)} \end{cases} \\ \ell' \begin{cases} + & \text{image on the other side of lense} \\ - & \text{image on same side of lense as object.} \end{cases} \end{cases}$

Types of lenses :

1) Air - Glass - Air :

Thin lenses :
$\frac{1}{\ell} + \frac{1}{\ell'} + \frac{1}{f}$

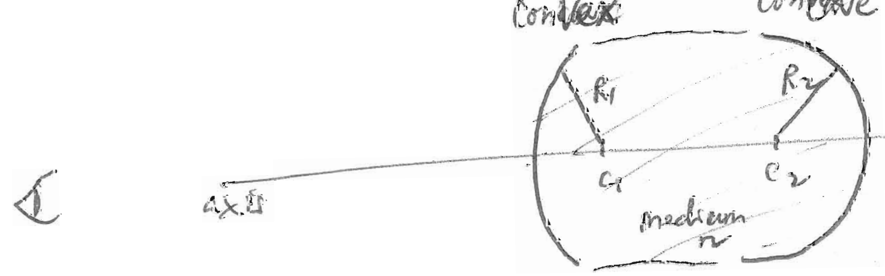
2) Air - Glass (Medium)



Thick lense equation : $\frac{n_1}{\ell} + \frac{n_2}{\ell'} = \frac{n_2 - n_1}{R}$

Sign convention for R $\begin{cases} + & \text{convex} \\ - & \text{concave} \end{cases}$

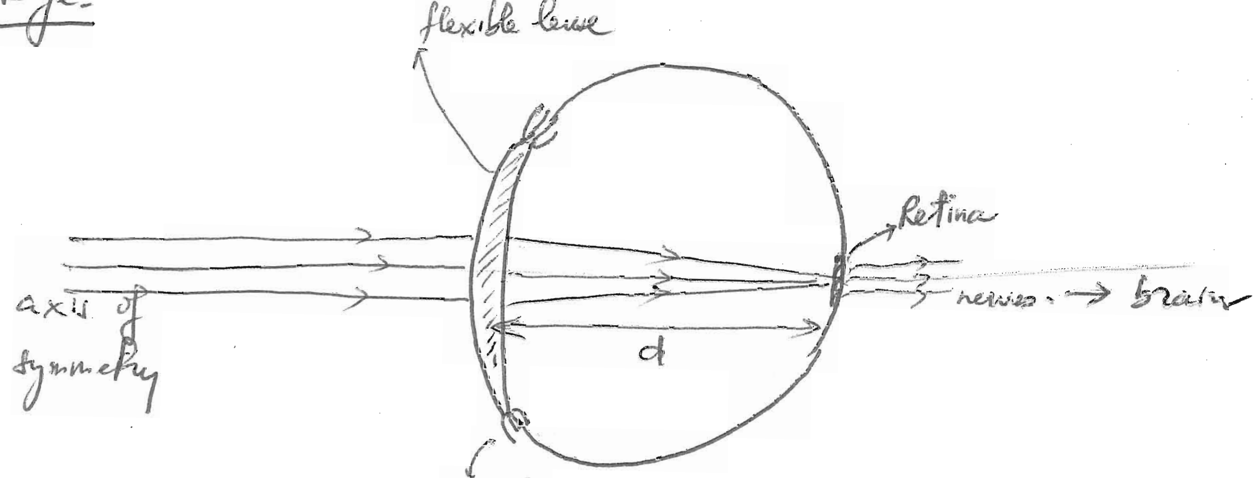
3) Thick air - medium - air : different radii of curvature for left & right lense :



"lense Maker's Eq." :
$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
↓
focal length

R_1 & R_2 sign convention $\begin{cases} + & \text{convex} (R_1 > 0) \\ - & \text{concave} (R_2 < 0) \end{cases}$

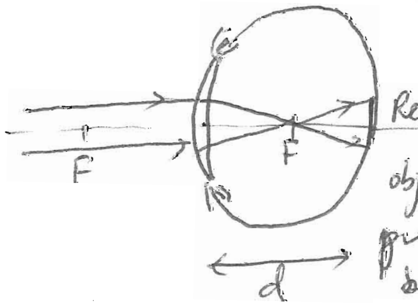
Eye:



muscles
help control focal
length of our lense :
trying to set $f = d$ (to see far away objects
clear & focused)

Near sighted (myopic)

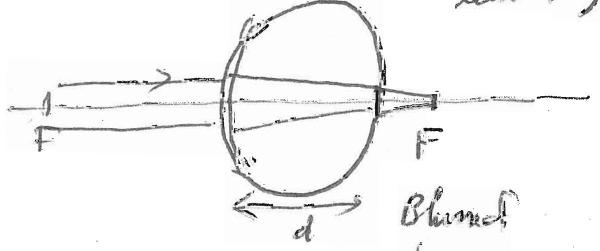
$f < d$ (can see closer objects w/o corrective lenses)



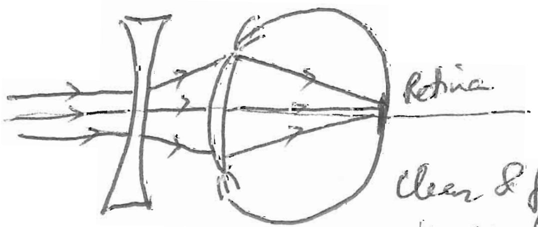
objects (far away) produce a blurred image.

Far sighted (hyperopic)

$f > d$ (can see further objects w/o corrective lenses)



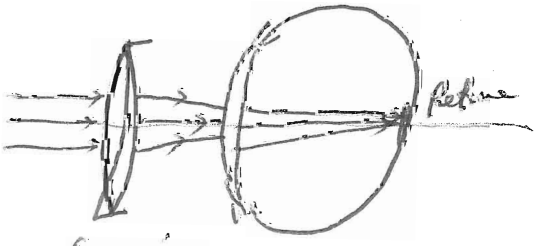
Blurred image.



Diverging corrective lense $f < 0$

Clear & focused image for faraway objects.

Diopter $= \frac{1}{f_{(\text{in m.})}}$ (negative)



Converging corrective lense $f > 0$

Diopter $= \frac{1}{f}$ (positive) → in meter

Ch 32 Interference & Diffraction.

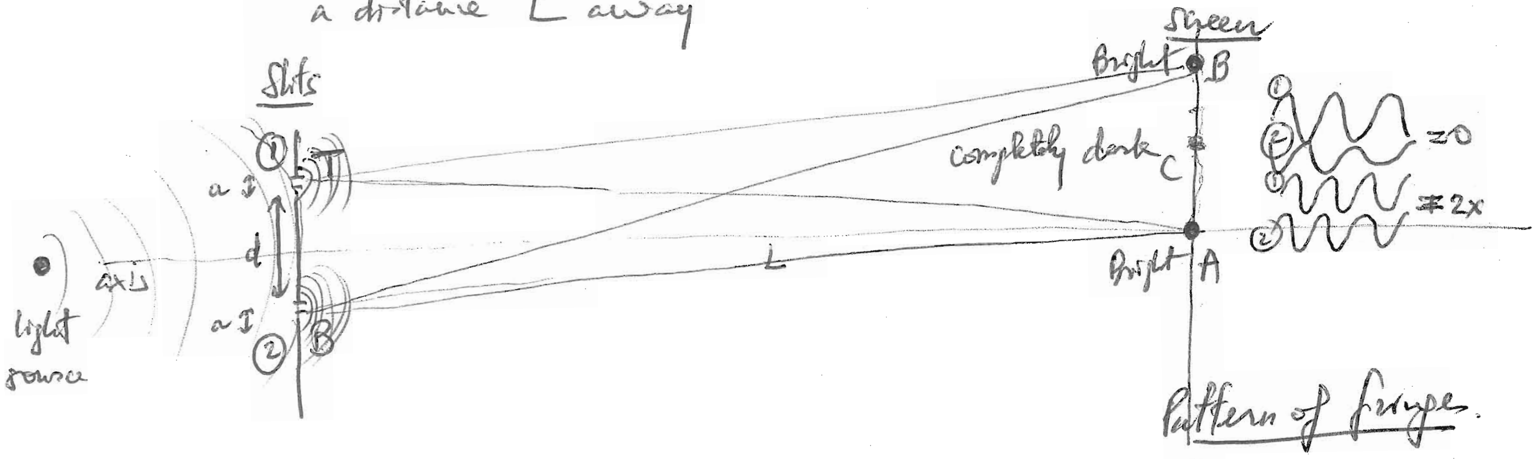
→ Physical optics : uses $\boxed{\text{wave properties}}$ of light in addition to geometry of the problem

→ superposition of waves:
- constructive (in phase)
- destructive (out of phase 180°)

Double-slit Interference:

→ One source of light → 2 identical light waves {in phase, coherent} to start with

→ look at their superposition at different spots on a screen a distance L away



→ Sep. b/w two slits : d

→ Slit width : a

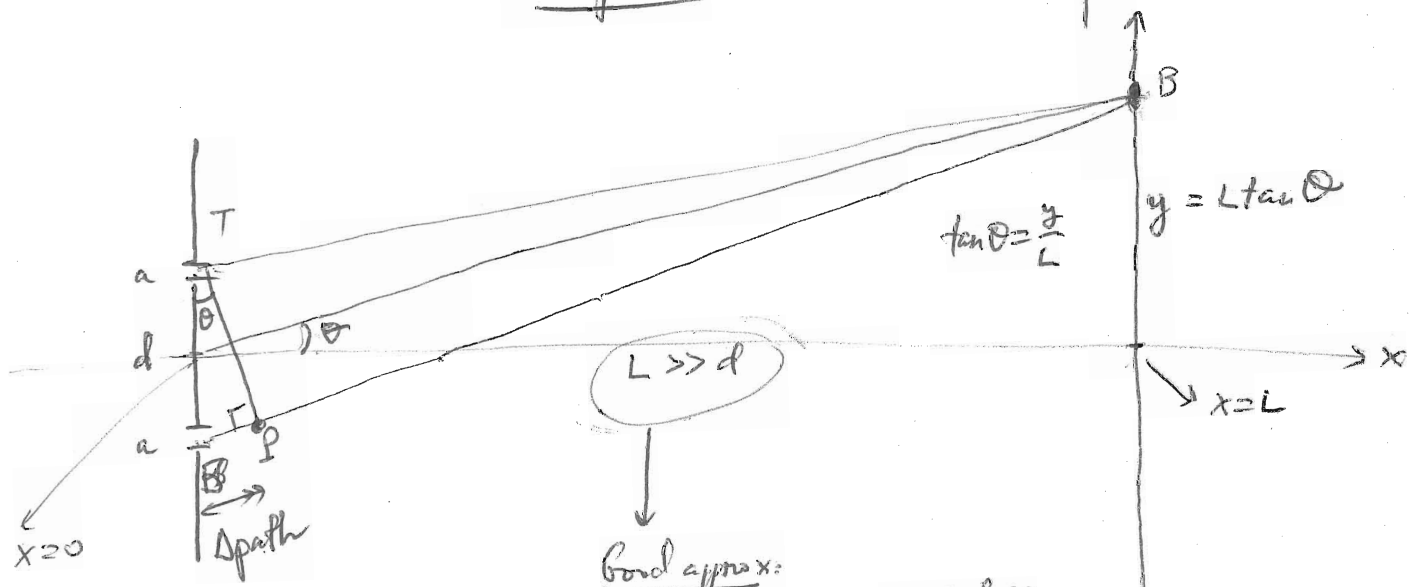
→ Screen far away : $L \gg d$

1) TA = BA : waves ① & ② arrive in phase @ A → constructive interference → bright spot

2) TB < BB such that BB − TB = λ → ① & ② in phase @ B → constructive interference → next bright spot
(λ: wavelength)

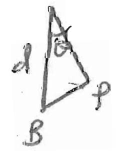
3) TC < BC → BC − TC = $\frac{\lambda}{2}$ → ① & ② out of phase or 180° off @ C → destructive interference → dark spot!

Interference



Good approx:
rays or paths TB & BB
are parallel

↳ $\Delta path = BP = d\sin\theta$



Constructive @ B

$d\sin\theta_n = n\lambda \quad (n = 0, 1, 2, 3, \text{etc.})$

$$\boxed{y_n = L\tan\theta_n = \cancel{\sin^{-1}\left(\frac{n\lambda}{d}\right)} = L\tan\left[\sin^{-1}\frac{n\lambda}{d}\right]}$$

if $\lambda \ll d$: $\sin\theta_n \simeq \tan\theta_n$

↳ $\boxed{y_n \cong \frac{n\lambda}{d}L}$

Destructive @ B

$d\sin\theta_n = (2n+1)\frac{\lambda}{2} \quad (n = 0, 1, 2, \text{etc.})$

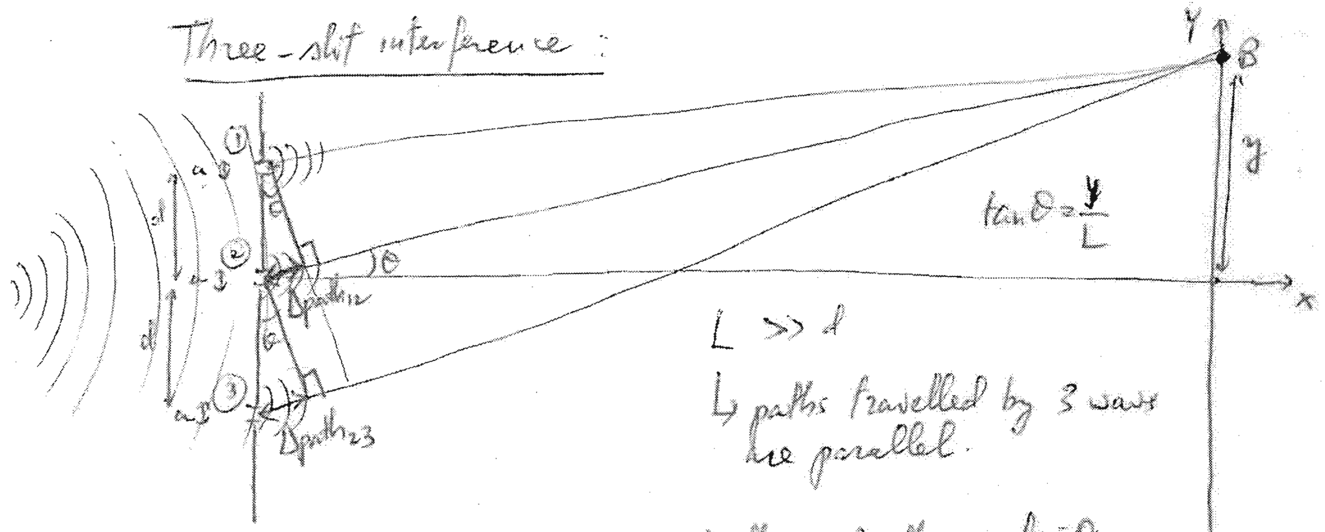
$y_n = L\tan\theta_n$

$$\boxed{y_n = L\tan\left[\sin^{-1}\left[(2n+1)\frac{\lambda}{2d}\right]\right]}$$

if $\lambda \ll d$: $\sin\theta_n \simeq \tan\theta_n$

↳ $\boxed{y_n = \frac{(2n+1)\lambda}{2d}L}$

Three-slit interference :



$\tan\theta = \dfrac{y}{L}$

$L \gg d$

↳ paths travelled by 3 waves are parallel.

one source → 3 identical waves.

$$\begin{cases} \Delta path_{12} = \Delta path_{23} = d\sin\theta \\ \Delta path_{13} = 2d\sin\theta \end{cases}$$

Constructive interference @ B :

→ For our 3 waves :

$$\left.\begin{array}{lll} ① \,\&\, ② : & d\sin\theta_m & = m\lambda \\ ② \,\&\, ③ : & d\sin\theta_m & = m\lambda \\ ① \,\&\, ③ : & \cancel{2}d\sin\theta_m & = \cancel{2}m\lambda \end{array}\right\}$$

$$\boxed{d\sin\theta_m = m\lambda \quad (m = 0, 1, 2, 3, \ldots)}$$

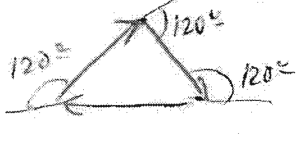
Destructive interference @ B

↳ 2 slits: $\Delta path = (2n+1)\dfrac{\lambda}{2} = (n + \dfrac{1}{2})\lambda \quad (n = 0, 1, 2, 3, \ldots)$

~~3 slits~~ Two waves are 180° out of phase ↳ $\dfrac{360°}{2}$ ↑↓ = 0

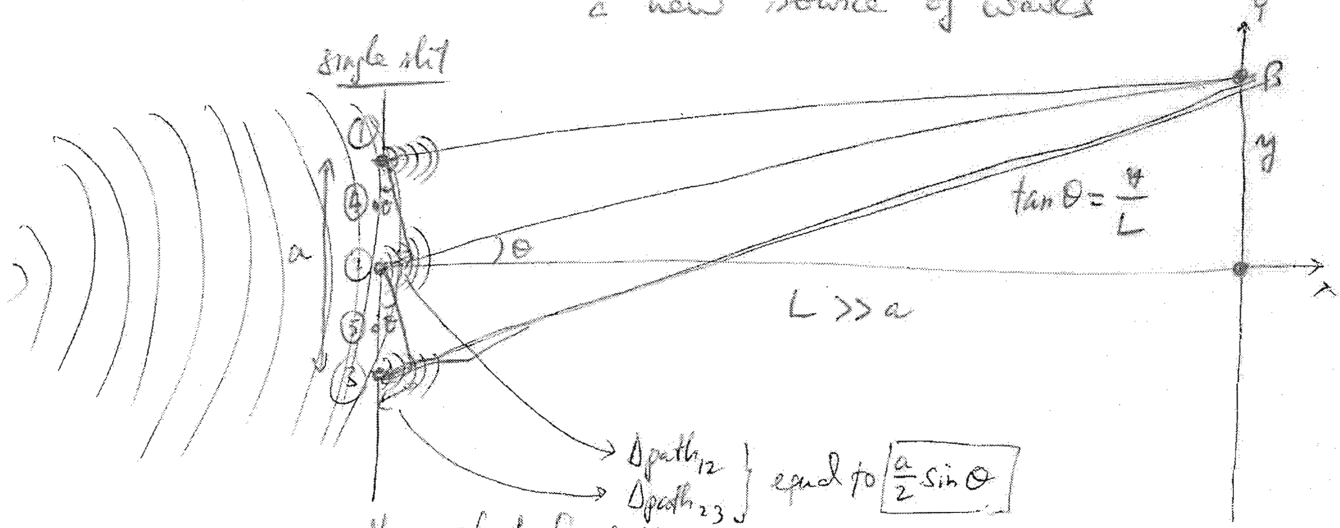
3 slits: $\Delta path = (n + \dfrac{1}{3})\lambda \quad (n = 0, 1, 2, 3, \ldots)$ $\dfrac{360°}{3}$

Three waves ~~should~~ be 120° out of phase :



Diffraction in a single slit: superposition of waves from different points along the slit:

Huyghens' principle: each point on a wavefront can become a new source of waves



e.g. three identical waves come out of points along slit

Destructive interference @ B:

waves 1 & 2, 2 & 3:

$$\Delta path = (2n+1)\frac{\lambda}{2} \quad (n=0,1,2,\dots)$$

$$\frac{a}{\cancel{2}}\sin\theta_n = (2n+1)\frac{\lambda}{\cancel{2}}$$

$$\boxed{a\sin\theta_n = (2n+1)\lambda \quad (n=0,1,2,\dots)}$$

waves 1 & 3, $\Delta path$ 1 & 3 $= a\sin\theta$:

$$a\sin\theta_n = \cancel{2}(2n+1)\frac{\lambda}{\cancel{2}} \quad (n=0,1,2,\dots)$$

$$\boxed{a\sin\theta_n = (2n+1)\lambda \quad (n=0,1,2,\dots)}$$

For all 3 waves:

$$\boxed{a\sin\theta_n = (2n+1)\lambda \quad (n=0,1,2,3,\dots)}$$

$\lambda; 3\lambda; 5\lambda; 7\lambda \dots$

Dark spots for ∞ waves: $a\sin\theta_n = n\lambda \quad (n=1,2,3,\dots)$; $n=0$ is a bright spot

Now waves 1 & 4: $\frac{a}{4}\sin\theta_n = (2n+1)\frac{\lambda}{2} \rightarrow a\sin\theta_2 = 2(2n+1)\lambda = 2\lambda; 6\lambda; 10\lambda; 14\lambda \dots$

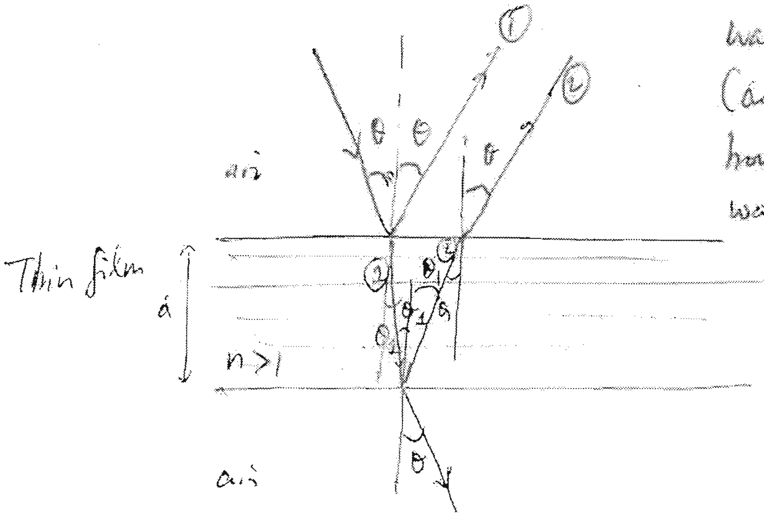
waves 1 & 5: $\frac{3}{4}a\sin\theta_n = 2(2n+1)\frac{\lambda}{2} \rightarrow a\sin\theta_n = \frac{4}{3}(2n+1)\lambda = 4\lambda\ (n=1); 12\lambda\ (n=3)$; etc.

Diffraction limit in optical instruments:

$$\underbrace{\theta_{min}}_{\text{minimum angle b/w objects we can distinguish through a lens}} = \frac{1.22\lambda}{D}$$

D is diameter of slit or lense.

Thin-film interference: (rainbow on thin layer of oil on water)



waves ① & ② come out parallel (as in the double slit experiment), however wave ① stayed in air while wave ② has travelled approximately 2d in the medium

Wave ①: because of a reflection off a higher index medium (like a wave reaching the fixed end of a string → gets inverted): gets inverted → gets a phase shift of 180° or $\frac{\lambda}{2}$

wave ②: has travelled an additional 2d.

→ b/w ① & ②

constructive interference: $2d = m\lambda + \frac{\lambda}{2}$

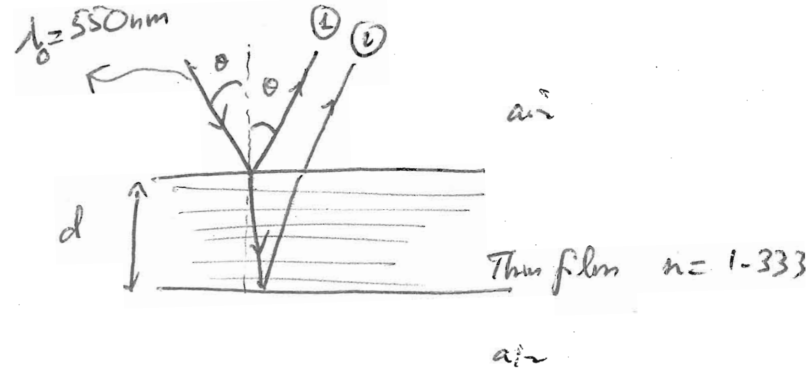
$= (m+\frac{1}{2})\lambda$ $(m=0,1,2\ldots)$ wavelength in film

destructive interference: $2d = (2m+1)\frac{\lambda}{2} + \frac{\lambda}{2}$

$= 2m\frac{\lambda}{2} + \lambda$

$= (m+1)\lambda$ $(m=0,1,2,\ldots)$ wavelength in film

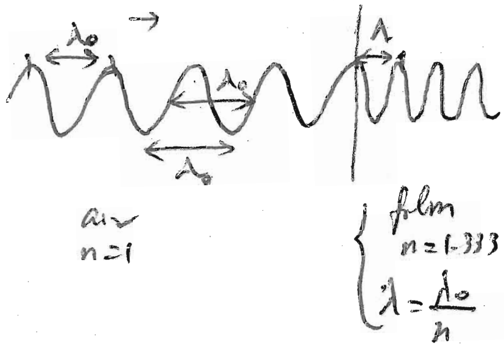
32.21



Notes: ① & ② combine: constructive interference what is d?
Light has a wavelength in air $\lambda_0 = 550nm$.

Constructive interference: $2d = (m + \frac{1}{2})\lambda$ — m: integer = 0, 1, 2, etc.; λ: wavelength in the film → n = 1.333

$\rightarrow 2d = (m + \frac{1}{2})\frac{\lambda_0}{1.333}$



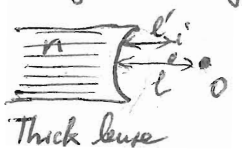
m could be 0, 1, 2, etc.
but for minimum thickness: m=0

$$\boxed{2d_{min} = \frac{1}{2}\frac{\lambda_0}{1.333} \rightarrow d_{min} = \frac{\lambda_0}{4 \times 1.333} = \frac{550nm}{4 \times 1.333} = 103nm}$$

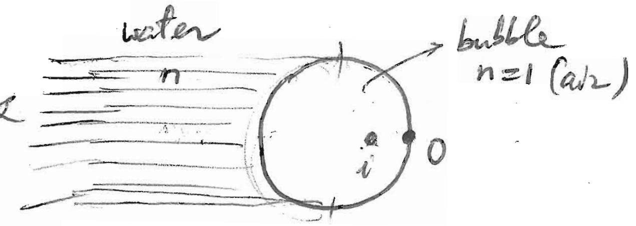
(31.32) Actual and apparent size of a bubble under water.

→ Object: the far side of bubble (o)

→ lense:



Thick lense



↳ $\frac{n_1}{l} + \frac{n_2}{l'} = \frac{n_2 - n_1}{R}$

in this problem:

$n_1 = 1$ (where object is located)

$n_2 = n = 1.333$ (water)

l: object position wrt midpoint of lense also $2R$ (R is the actual radius of bubble) (l always +)

l' = image position wrt midpoint of lense (+ if image on the other side of lense, − if image on same side as object)

↳ $l' = -1.5$ cm

R : radius of curvature of lense {+ convex, − concave} from object (light travels from object)

↳ $\frac{1}{2R} + \frac{1.333}{-1.5} = \frac{0.333}{-R}$ → $\frac{1}{2R} + \frac{0.333}{R} = \frac{1.333}{1.5}$

$$\frac{1}{R}\left(\frac{1}{2} + 0.333\right) = \frac{1.333}{1.5}$$

$$R = \left(\frac{1.333}{1.5}\right)^{-1}\left(\frac{1}{2} + 0.333\right)$$

$$= \frac{0.833 \times 1.5}{1.333} = 0.938\text{cm}$$

→ Actual diameter of bubble $= 2R = 1.87$ cm

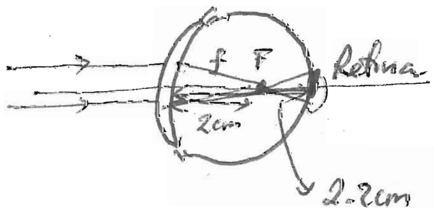
Note: can also look at size of bubble by the side-to-side diameter. Front view of bubble:  : relating l & l' to h & h' via magnification factor: $M = \frac{h'}{h} = -\frac{l'}{l}$

31.36

Eye with a shorter focal length than needed.

↓ parallel rays (faraway objects) are converged short of retina: the extension rays make a blurred image → Nearsighted eye

$$\begin{cases} f = 2.0\text{ cm} \\ f_{required} = 2.2\text{ cm} \end{cases}$$



a) Faraway objects are seen blurred @ retina → nearsighted eye

b) Power (diopter ↔ f in meter!) for the corrective lense.

$$\begin{cases} \text{This eye} = \dfrac{1}{f} = \dfrac{1}{0.02\text{m}} = 50 \text{ diopters} \\ \text{Good eye} = \dfrac{1}{f_{req}} = \dfrac{1}{0.022} = 45.5 \text{ diopters.} \end{cases}$$

↳ Corrective lense $= 45.5 - 50 = -4.5$ diopters.

↳ $f_{lense} = \dfrac{1}{-4.5}\text{ m} < 0$ negative:

→ concave as expected for nearsighted eye (to spread the rays out to bring F to retina)

(32.38) 2-slit interference → find separation b/w bright fringes on a screen → Note: $\begin{cases} \to \text{Not using } L \gg d \ \& \ \lambda \ll d \ ? \\ \to \text{Using } L \gg d \ \& \ \lambda \ll d \ ? \end{cases}$

Data: $\begin{cases} \lambda = 633\,nm \\ d = 6.5\,\mu m \quad \text{(slit spacing)} \\ L = 1.7\,m \quad \text{(slit - screen sep.)} \end{cases}$

Constructive interference $\begin{cases} y_n = L \tan\left[\sin^{-1} \frac{n\lambda}{d}\right] \quad \text{w/o approx.} \\ y_n = \frac{n\lambda}{d} L \quad \text{w/ approx. } \lambda \ll d \end{cases}$

a) $y_2 - y_1 =$ (b/w second & first bright fringe)

exact:

$$L\left\{\tan\left[\sin^{-1}\frac{2\lambda}{d}\right] - \tan\left[\sin^{-1}\frac{\lambda}{d}\right]\right\}$$

$$= 1.7\left\{\tan\left[\sin^{-1}\frac{2\cdot 633\times10^{-9}}{6.5\times10^{-6}}\right] - \tan\left[\sin^{-1}\frac{633\times10^{-9}}{6.5\times10^{-6}}\right]\right\}$$

$$= 17.17\,cm$$

approx: ($\lambda \ll d$)

$$L\frac{\lambda}{d}\left\{2-1\right\} = 1.7 \times \frac{633\times10^{-9}}{6.5\times10^{-6}} = 17\,cm$$ (good)

b) $y_4 - y_3 =$ (b/w fourth & third fringe)

exact:

$$L\left\{\tan\left[\sin^{-1}\frac{4\lambda}{d}\right] - \tan\left[\sin^{-1}\frac{3\lambda}{d}\right]\right\}$$

$$= 20\,cm$$

approx: ($\lambda \ll d$)

$$L\frac{\lambda}{d}\left\{4-3\right\} = 17\,cm$$

(not so good away from the central bright fringe)

32.42 Visible light: $400\,\text{nm} < \lambda < 700\,\text{nm}$

$\underbrace{\lambda_v}$ (violet) $\quad$ $\underbrace{\lambda_r}$ (red)

↓

dispersed by a grating ⟷ diffraction by a single slit:

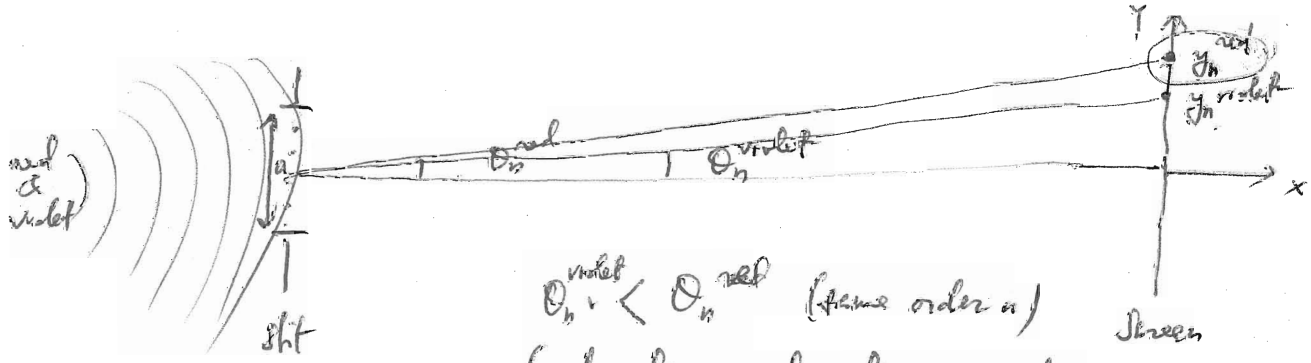
↳ Dark spot (easier to calculate):

↳ $a \sin\theta_n = n\lambda$ $\quad \begin{cases} a: \text{slit width} \\ \theta: \text{angle of spot on screen} \\ n = 1, 2, 3, \text{etc.} \end{cases}$

(location on screen: $y_n = L \tan\theta_n$)

$$\left.\begin{aligned} \theta_n^{red} &= \sin^{-1}\left(\frac{n\lambda_{red}}{a}\right) \\ \theta_n^{violet} &= \sin^{-1}\left(\frac{n\lambda_{violet}}{a}\right) \end{aligned}\right\} \text{ colors get dispersed by a slit.}$$

For a given order n



$\theta_n^{violet} < \theta_n^{red}$ (same order n)

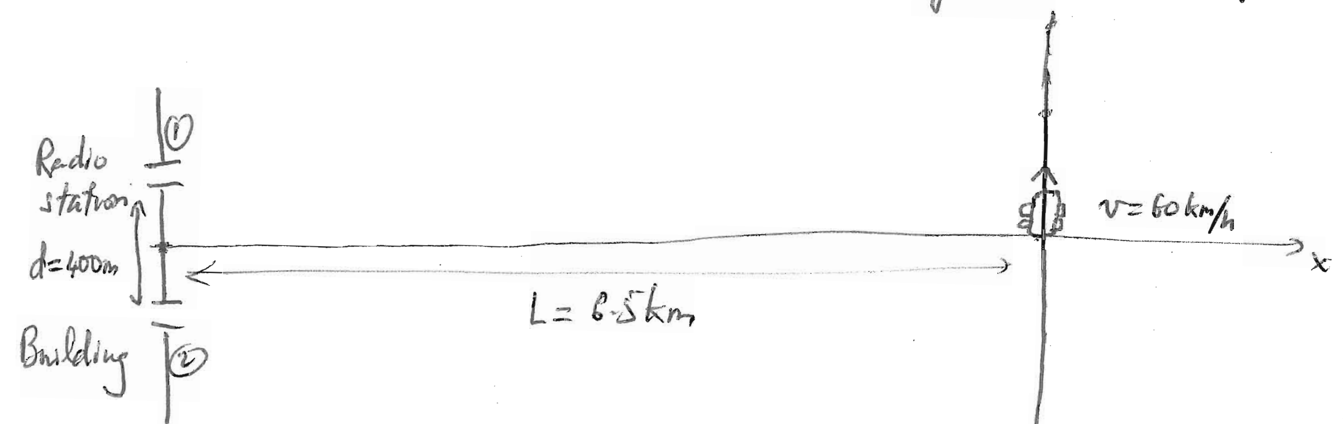
(red gets spread out more due to its longer wavelength)

Question: lowest pair of consecutive orders for an overlap of visible spectra: overlap when $y_n^{red} = y_{n+1}^{violet}$ or $\boxed{\theta_n^{red} = \theta_{n+1}^{violet}}$

$\rightarrow \dfrac{n\lambda_{red}}{\cancel{a}} = \dfrac{(n+1)\lambda_{violet}}{\cancel{a}} \rightarrow n = \dfrac{\lambda_{violet}}{\lambda_{red} - \lambda_{violet}} = \dfrac{400}{700-400} = \dfrac{4}{3} = 1.333$

Note
- → From calculation $n = 1.333$ (min for overlap !)
- → n has to be an integer → pair of consecutive orders for overlap
  - $\boxed{n = 2 \;\&\; n+1 = 3}$ → $\theta_2^{red} \geq \theta_3^{violet}$
  - $n = 1 \;\&\; n+1 = 2$ is not sufficient for overlap since $n > 1.333$.

(32.70) $d = 400m$ { Signal from radio station; Its reflection from a building } Two sources of the same signal } 2-slit interference



↑
line connecting station & reflecting building

→ Road will find "dark" spots (fading radio signal) & "bright" spots (clean radio signal)
→ knowing the speed of vehicle → find how often radio signal will fade.

Radio signal wavelength: λ?

$f = 103.9 \times 10^6 \,Hz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{103.9\times10^6} = 2.89m \ll 400m = d$$

2-slit interference: destructive interference:

approx $\lambda \ll d \Rightarrow y_n = \frac{(2n+1)}{2}\lambda L$ $(n = 0, 1, 2, etc.)$

Separation b/w 2 consecutive fadings:
$$y_{n+1} - y_n = \frac{(2(n+1)+1)}{2d}\lambda L - \frac{(2n+1)}{2d}\lambda L$$
$$= \frac{\lambda L}{2d}\left[\cancel{2n} + 3 - \cancel{2n} - 1\right] = \boxed{\frac{\lambda L}{d}}$$

$$y_{n+1} - y_n = \frac{\lambda L}{d} = \frac{2.89 \times 6.5 \times 10^3}{400} \text{ m}$$ (b/w 2 consecutive fadings)

At $v = 60 \frac{km}{h} = \frac{60}{3.6} \frac{m}{s}$ how often do you hear a fading)?

$\hookrightarrow$ time b/w consecutive fadings $= \frac{y_{n+1} - y_n}{v} = \frac{\lambda L}{d \cdot v} = 2.82s$

How often: how many times per second $= \frac{1}{2.82s}$

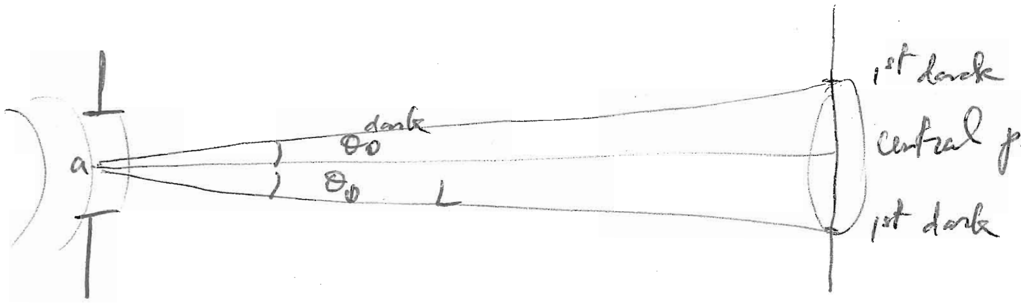
Note: same b/w two consecutive clear signals!

32.27 Single-slit diffraction:

$\lambda = 633nm$

$a = 2.5\mu m$

angular width of central peak $= 2\theta_0^{dark}$



Dark spot for diffraction: $a \sin\theta_n = (2n+1)\lambda \quad (n=0,1,2,\text{etc})$

$$\theta_n = \sin^{-1}\left(\frac{(2n+1)\lambda}{a}\right)$$

$$\theta_0 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{633\times10^{-9}}{25\times10^{-6}}\right)$$

$$= \pm 14.7°$$

$\rightarrow$ Angular width of central peak is $2\theta_0 = 29.4°$

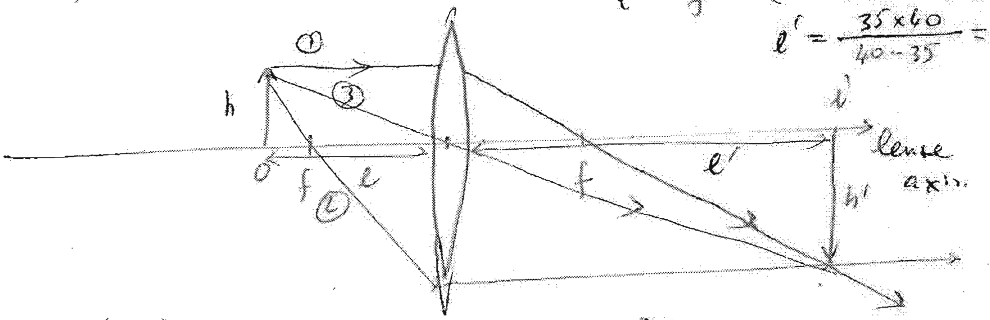
31.50

Converging lense: $\rightarrow$ thin lense equation : $\frac{1}{\ell}+\frac{1}{\ell'}=\frac{1}{f}$

$f=+35\text{cm}$

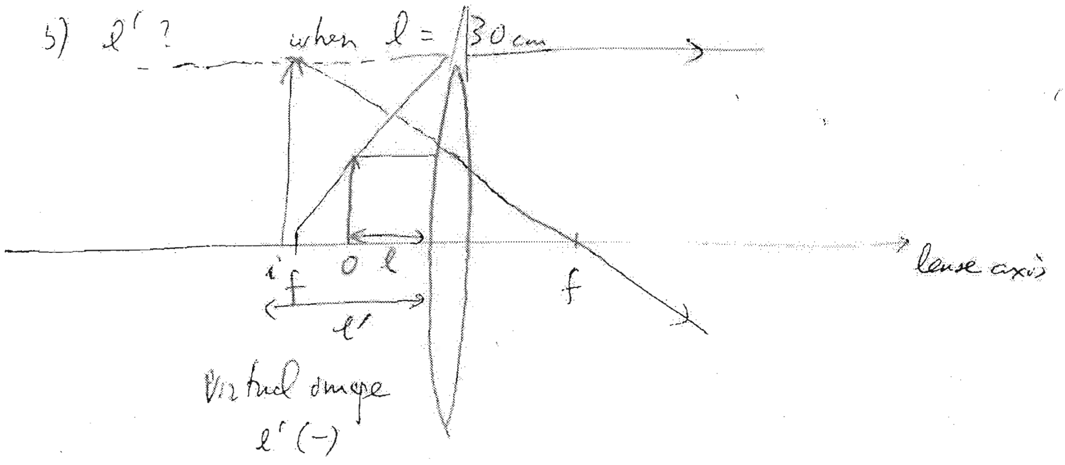
a) ℓ'? when $\ell = 40\text{cm}$ $\rightarrow$ $\frac{1}{\ell'}=\frac{1}{f}-\frac{1}{\ell}=\frac{1}{35\text{cm}}-\frac{1}{40\text{cm}}$

$\ell'=\frac{35\times 40}{40-35}=280\text{cm}$



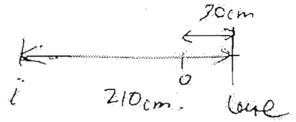
object to image = 40cm + 280cm = 320cm.

b) ℓ'? when $\ell = 30\text{cm}$



$\frac{1}{\ell'}=\frac{1}{f}-\frac{1}{\ell}=\frac{1}{35}-\frac{1}{30}$ $\rightarrow \ell'=\frac{35\times 30}{-5}=-210\text{ cm}$

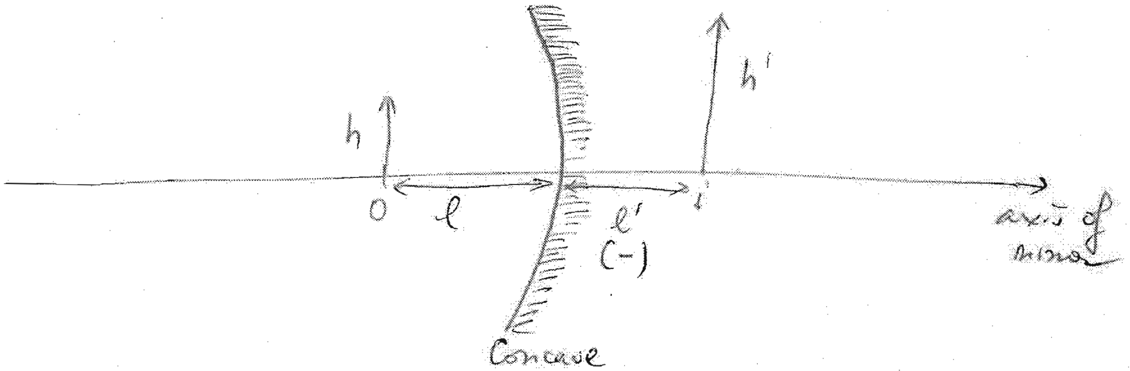
object to image : 180cm.



31.32 31.42 31.50

31.42 concave mirror R?

$h' = 9.5\,cm$ (virtual image)
$h = 5.7\,cm$ (object)
$\ell = 22\,cm$ (from mirror).



Minor equation: $\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}$ $\boxed{R = 2f}$ → need f:

Magnification factor $M = \frac{h'}{h} = -\frac{\ell'}{\ell}$ → $\boxed{\ell' = -\ell\frac{h'}{h}}$

$$\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}$$

$$\frac{1}{\ell} - \frac{1}{\ell\frac{h'}{h}} = \frac{1}{\ell}\left[1 - \frac{1}{\frac{h'}{h}}\right] = \frac{1}{\ell}\left[1 - \frac{h}{h'}\right] = \frac{1}{f}$$

$$R = 2f = \frac{2\ell}{1 - \frac{h}{h'}} = \frac{2 \times 22\,cm}{1 - \frac{5.7}{9.5}} = 110\,cm.$$