Ch27 Electromagnetic Induction

So far: $\begin{cases} \text{Gauss Law} \\ \text{Ampere Law} \end{cases}$

Faraday's Law: $\mathcal{E} = -\dfrac{d\phi_B}{dt}$

ϕ_B (Phi sub B) : magnetic flux : flux of the magnetic field $\vec{B}$ through a surface that is enclosed by a loop :

Notes: 1) $\phi_B = \int \vec{B} \odot d\vec{A}$

scalar or dot product : $\underbrace{B \cdot dA \cdot \cos\theta} = B_\perp \, dA$

$B_\perp$: component of $\vec{B}$ that is perpendicular to the surface area

(Note $B_\parallel$ doesn't cross the surface area!)

2) $\dfrac{d\phi_B}{dt}$ = change of the magnetic flux per unit time
- → change of $\vec{B}$
  - → change of strength B
  - → change of direction
- → change of surface area A

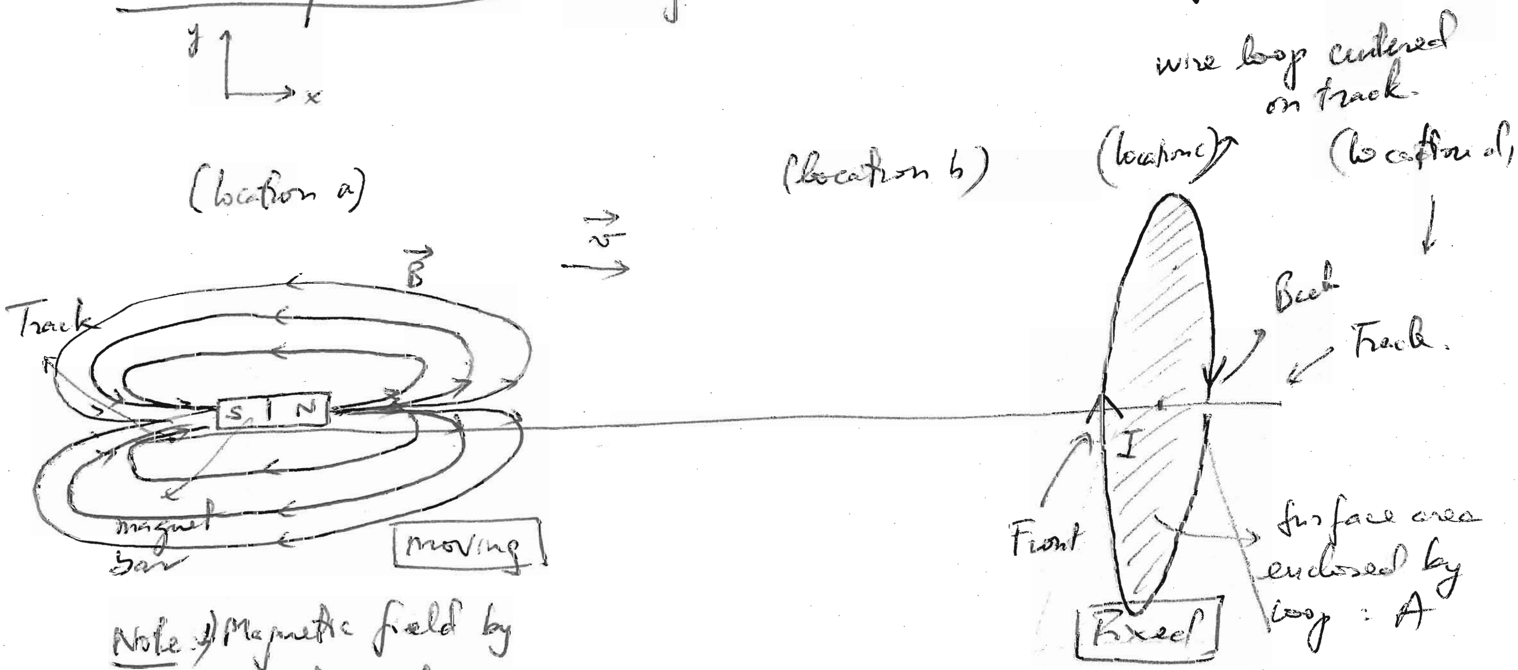
3) $\mathcal{E}$ = induced voltage in the loop that encloses the surface area that defines the magnetic flux

→ Faraday's law: if the magnetic flux through a surface area defined by a loop changes over time → there is an induced voltage in that loop. (can be used to light a bulb, or run a motor, etc.)

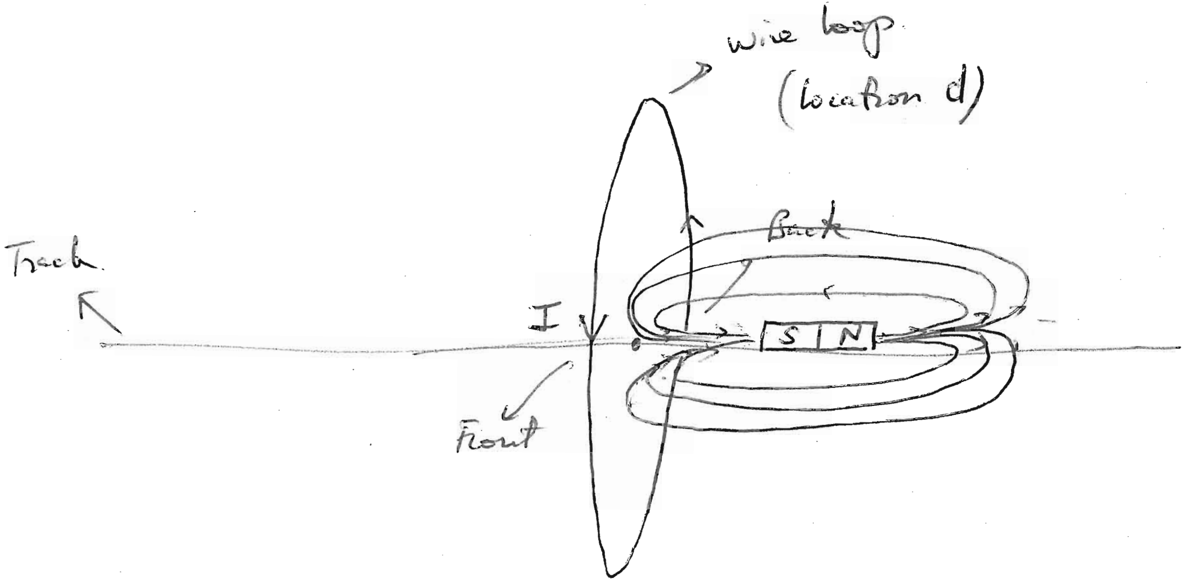
4) The minus sign is Lenz's law : determines the polarity or direction of the induced voltage $\mathcal{E}$:

Lenz's law: the induced voltage will be such that it will oppose the change of the magnetic flux (inertia law). How? By creating an induced magnetic (voltage → current → magnetic field) field
1) Pointing in the opposite direction of the original magnetic field if Φ_B was increasing
2) Pointing in the same direction of the original magnetic field if Φ_B was decreasing.

Visual experiment: a magnet bar sliding on a frictionless track



Note: 1) Magnetic field by magnet is stronger closer to the poles, pointing in $+\hat{i}$ along track.
2) As we slide magnet toward wire loop: there will be an increased magnetic flux through the wire loop → induced voltage $\mathcal{E}$ will create an induced current in the wire loop such that it will create an induced magnetic field pointing in $(-\hat{i})$ to oppose the original magnetic field by the magnet (so to neutralize the increase of the magnetic flux). So current is up on the front of loop (or CW if viewed from the right of loop)



3) As magnet continues sliding beyond loop: as magnet is leaving the loop: there is a decreased magnetic flux through the surface area enclosed by wire loop → induced voltage will create an induced current such that it will create an induced magnetic field pointing in $+\hat{i}$ (same as original field by magnet) So induced current now is downward in the front of loop or CCW if viewed from right of loop.

4) Speed of the magnet @ different locations: a, b, c, d?
along the frictionless track if we give it a push and let it go
a = some distance to the left of loop
b = closer to loop
c = as it crosses center of loop
d = to the right of loop.

$v_b \begin{cases} > v_a \\ = v_a \\ < v_a \checkmark \end{cases}$

There is some interaction b/w magnet & loop via the magnetic field: the induced current in the loop draws energy from the moving magnet (kinetic energy)
(The faster the magnet is approaching the stronger the induced voltage)

$v_d \begin{cases} > v_c \checkmark \\ = v_c \\ < v_c \end{cases}$

→ Conservation of energy: energy transferred into loop as magnet was entering is now being returned. Also as loop exercise the law of inertia it would hold the magnet back as it enters and give it a push as it leaves.

Inductance & Magnetic Energy

Electric

Capacitor: storage device for electrostatic energy

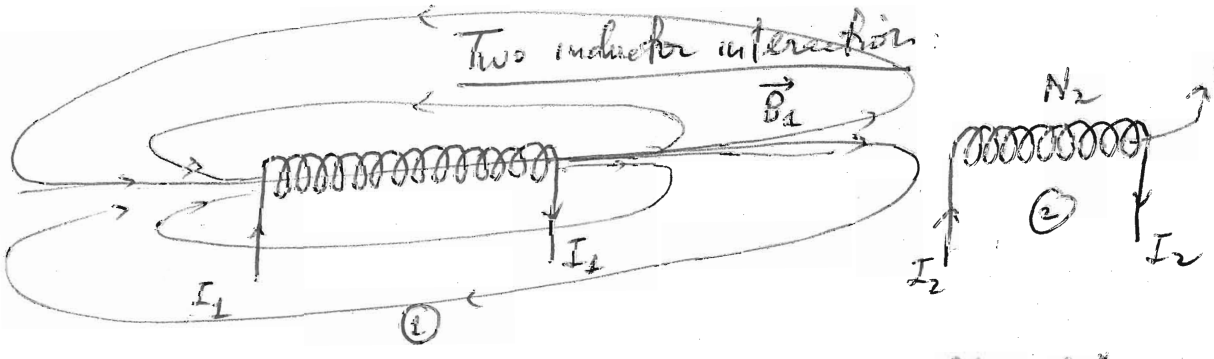
Capacitance $C = \frac{Q}{V}$

Magnetic

Inductor: storage device for magnetic energy

Inductance:
- Self inductance $L = \frac{\Phi}{I}$
- Mutual inductance $M = \frac{\Phi_2}{I_1}$ (There are 2 inductors involved)

Two inductor interaction:



A_2: cross-sectional area of second inductor or solenoid

1) Using Ampere's law: $B_1 = \mu_0 n_1 I_1$
   n_1 = number of turns per unit length

2) Second inductor in proximity of first one

3) Solenoid #2 receives a magnetic flux $\Phi_2 = B_1 \cdot A_2$ $\left\{ \begin{array}{l} \text{a) } \vec{B}_1 \text{ is } \perp A_2 \\ \text{b) } A_2 \text{ sufficiently small so } B_1 \text{ is uniform.} \end{array} \right.$
   ($\Phi = \int \vec{B} \cdot d\vec{A}$)

4) Φ_2 would change over time if
   $\Phi_2 = B_1 \cdot A_2$
   - Magnitude of $\vec{B}_1$ changes over time
   - Direction of $\vec{B}_1$ changes over time
   - A_2 changes over time.

If I_1 changes over time $\rightarrow$ B_1 changes over time $(B_1 = \mu_0 n_1 I_1)$
$\rightarrow$ Φ_2 changes over time $\rightarrow$ induced voltage in the second solenoid

$$\underbrace{\mathcal{E}_2}_{\substack{\text{voltage in}\\ \text{solenoid \#2}}} = -\frac{d\Phi_2}{dt} = -\frac{d}{dt}(B_1 N_2 A_2) = -N_2 A_2 \frac{dB_1}{dt} = \underbrace{-N_2 A_2 \mu_0 n_1}_{M \text{ (Mutual inductance)}} \underbrace{\frac{dI_1}{dt}}_{\substack{\text{current in}\\ \text{solenoid \#1}}}$$

Faraday's & Lenz laws

mutual interaction.

$\rightarrow$ Mutual inductance M relates the induced voltage in inductor #2 due to a changing current in inductor #1, and viceversa!

$$\underbrace{\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}}$$

$$\Phi_2 = M I_1 \rightarrow \boxed{M = \frac{\Phi_2}{I_1}}$$

SI unit: easier to look at $M = -\dfrac{\mathcal{E}_2}{\frac{dI_1}{dt}} \rightarrow \dfrac{V}{\frac{A}{s}} = \dfrac{V.s}{A} \equiv H$ (Henry)

Two questions:

1) What about the effect of $\vec{B}_2$ of the second solenoid on the first solenoid? $\mathcal{E}_1 = -\dfrac{d\Phi_1}{dt} = -\boxed{A_1 N_1 \mu_0 n_2}\dfrac{dI_2}{dt}$

M (same mutual inductance!)

2) What about the effect of solenoid #1 on itself? or what is the self magnetic flux: $\Phi = B_1 \cdot A_1$: If I_1 change over time $\rightarrow$ B_1 changes over time $\rightarrow$ self flux changes over time
$\rightarrow$ there is a self induced voltage as well : $\mathcal{E} = -\dfrac{d\Phi}{dt} = -\mu_0 n_1 \dfrac{dI_1}{dt} \cdot N_1 A_1$

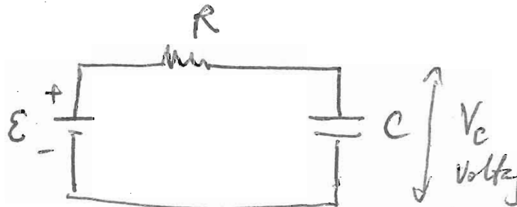
$$\mathcal{E} = -\underbrace{\mu_0 n_1 N_1 A_1}_{L \text{ self inductance}} \frac{dI_1}{dt}$$

L: self inductance: relate the self-induced voltage with the changing current in the same inductor or solenoid.

SI unit same as for M: H (Henry)

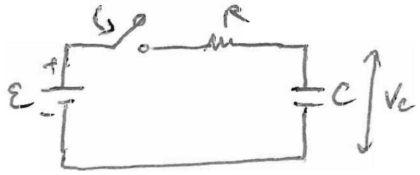
$$\phi_{self} = L \cdot I \quad \text{or} \quad \boxed{L = \frac{\phi_{self}}{I}}$$

RC Circuits



Voltage across capacitor

switch



Behavior over time:

$t=0$ switch is closed, C acts like a short-circuit (a wire) or $V_c = 0$ $\left(I(t=0) = \frac{\mathcal{E}}{R}, \text{max}\right)$

$t=\infty$ long after switch is closed, C acts like an open circuit ($I=0$) $V_c = \mathcal{E}$ (max.)

$0 < t < \infty$ $\boxed{I_c(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}}$

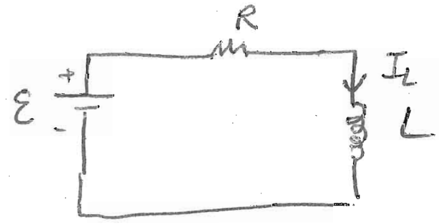
$\tau_{RC} = \frac{1}{RC}$: time constant.

I_c : starts out @ max

V_c : starts out @ min

↳ $\boxed{V_c \text{ doesn't change instantaneously}}$

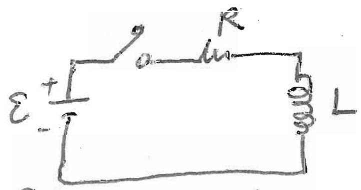
RL Circuits



current thru solenoid or inductor

↓

current is induced by a change of magnetic flux thru solenoid



Behavior over time:

$t=0$ switch is closed, L acts like an open circuit $I_L = 0$ $V_L = \mathcal{E}$.

$t=\infty$ sufficiently long after switch is closed current reaches max. value, L acts like a short-circuit (a wire): $V_L = 0$

$0 < t < \infty \rightarrow$ $\boxed{V_L = \mathcal{E} e^{-\frac{t}{\left(\frac{L}{R}\right)}}}$

$\tau_{RL} = \frac{1}{\frac{L}{R}} = \frac{R}{L}$: time constant

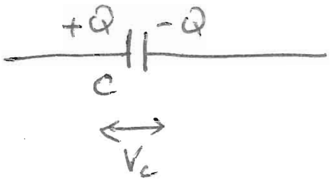
V_L starts out @ max

I_L starts out @ min

↳ $\boxed{I_L \text{ doesn't change instantaneously}}$

Energy Storage

Electrostatic Energy in Capacitors



$$\boxed{U_C = \frac{1}{2} C V_c^2} \quad (J)$$

F (Farad) V (Volt)

Remember: V_c doesn't change instantaneously

Object of mass m: when given a push its velocity doesn't change instantaneously
mass is inertia to velocity:

$\hookrightarrow U_{kinetic} = \frac{1}{2} m v^2$

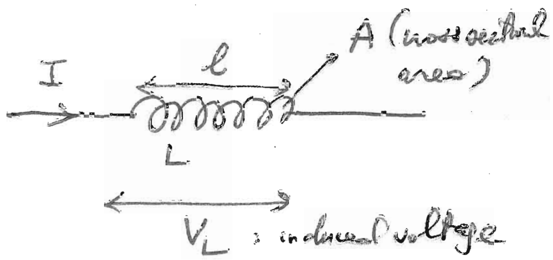
$\hookrightarrow$ C is inertia to electric potential

Energy density $\left[u_C = \frac{U_C}{Vol} = \frac{1}{2}\varepsilon_0 E^2\right] \quad \left(\frac{J}{m^3}\right)$

Capacitor: Vol = A · d

A: cross-sectional area of plates; d: spacing b/w plates

Magnetic Energy in inductors



V_L: induced voltage

Faraday's law

$$\boxed{V_L = -L\frac{dI}{dt}}$$

$\phi = LI \rightarrow V_L = -\frac{d\phi}{dt}$

$$U_L = \int_0^t P_L\, dt = \int_0^t I|V_L|\, dt$$

P_L: energy per unit time $(P = I|V_L|)$

$$= L\int_0^t I\frac{dI}{dt}\, dt = \frac{1}{2}L\left[I^2(t) - \cancel{I^2(0)}\right]$$

$$\boxed{U_L = \frac{1}{2} L I^2}$$

Remember: I_L doesn't change instantaneously

$\hookrightarrow$ L is the inertia to electric current

Energy density $\left[u_L = \frac{U_L}{A\cdot l} = \frac{\frac{1}{2}\cancel{A\mu_0 n^2 l}\frac{B^2}{\mu_0^2 n^2}}{\cancel{A\cdot l}} = \frac{B^2}{2\mu_0}\right]$

Inductor or solenoid:

Vol = A · l

A: cross-sectional area; l: length of solenoid

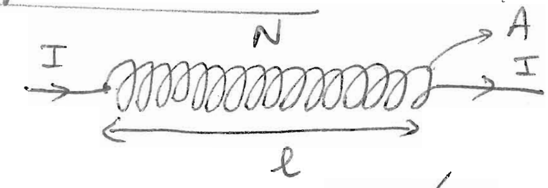
$$\boxed{L = A\mu_0\frac{N^2}{l} = A\mu_0 n^2 l}$$

$$B = \mu_0 n I \rightarrow \boxed{I = \frac{B}{\mu_0 n}}$$

$$n = \frac{N}{l}$$

Self inductance for a solenoid $L = A\mu_0 n^2 l$:

Proof:

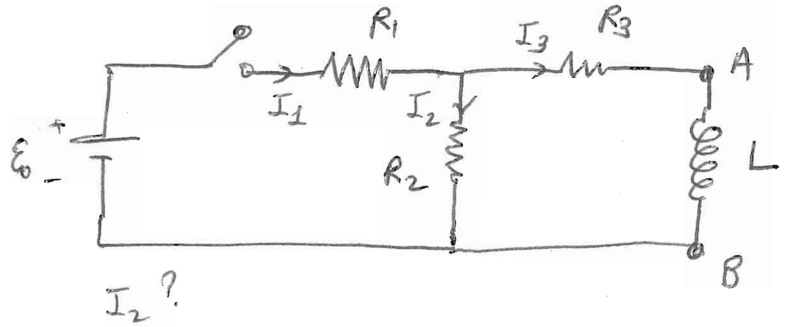


$\begin{cases} B_{solenoid} = \mu_0 n I \\ n = \dfrac{N}{l} \end{cases}$

$$L = \frac{\phi_{self}}{I} = \frac{N \cdot B A}{I} = \frac{N \cdot \mu_0 n \cancel{I} \cdot A}{\cancel{I}}$$

$$= \mu_0 A N n \cdot \frac{l}{l} = \boxed{\mu_0 A n^2 l}$$

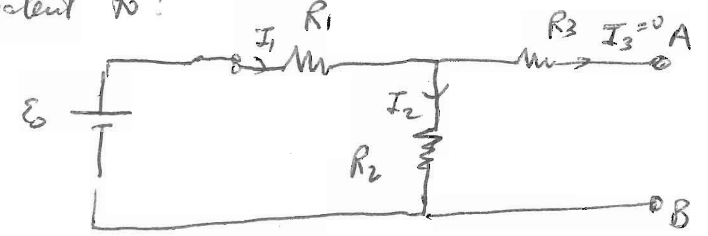
27.62



Given information:
$\mathcal{E}_0 = 12V$
$R_1 = 4\Omega$; $R_2 = 8\Omega$; $R_3 = 2\Omega$
$L = 2H$

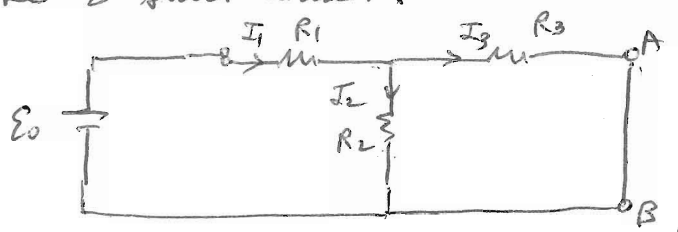
I_2 ?

1) Soon after switch is closed $(t=0)$: (inductor is an inertia to current)
I_3 was 0 before switch is closed, still 0 right after switch is closed (inductor acts like an open circuit b/w A & B). The circuit is equivalent to:



$$\boxed{I_2 = \frac{\mathcal{E}_0}{R_1 + R_2} = \frac{12}{4+8} = 1A}$$

2) Long after switch is closed $(t=\infty)$: I_L reaches max value, inductor acts like a short-circuit. The circuit is equivalent to



Solve for this circuit
1) Using parallel & series combination
2) Using loop analysis
3) Using node analysis

Use parallel & series combination:

a) Find $I_1 = \dfrac{\mathcal{E}_0}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}} = \dfrac{12}{4 + \dfrac{8 \cdot 2}{8+2}} = 2.14A$

b) Find I_2: I_1 is dividing into $\left\{ \begin{array}{l} I_2 = I_1 \cdot \dfrac{R_3}{R_2 + R_3} \\ I_3 = I_1 \cdot \dfrac{R_2}{R_2 + R_3} \end{array} \right\} I_2 + I_3 = I_1$

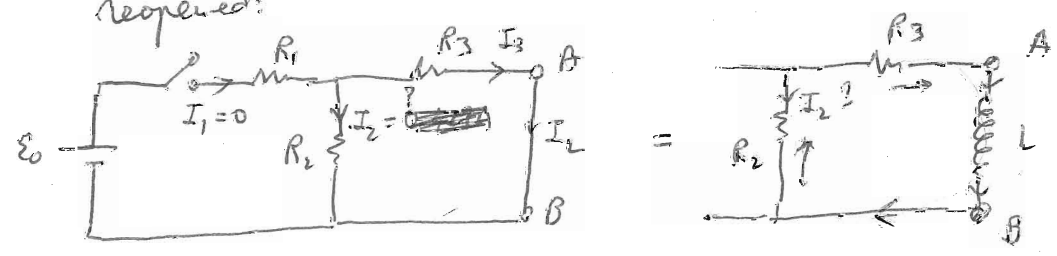
$\boxed{I_2 = 2.14 \dfrac{2}{8+2} = 0.429A}$ $\quad I_3 = I_L = 2.14 \dfrac{8}{10} = 1.71A$

(See next page for solution of same circuit using loop analysis)

3) <u>Switch is reopened long after it was closed</u>:

At the inductor: (inductance is an inertia to current) →

@ $t = \infty$: I_L was max (L acts like a short-circuit) : b/c of the inertia it will stay at max for a moment if the switch is reopened:



$I_L = I_3 = \cancel{0.429}A = 1.71 A$

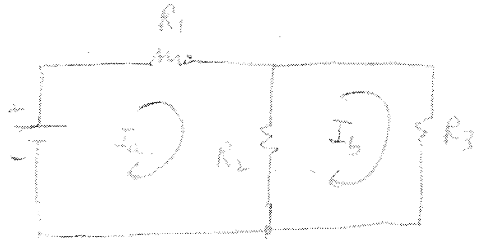
$I_2 = \cancel{0.429A} = -1.71A$

(I_2 was defined as downward)

This came out from the energy stored in the inductor

Current will decrease to zero when all energy stored in the inductor is dissipated @ $(R_2 + R_3)$.

Curiosity



→ We used parallel & series connection.

Loop analysis: 2 loops: I_a & I_b

$$1)\quad +\mathcal{E}_0 - R_1 I_a - (I_a - I_b) R_2 = 0$$

$$+\quad 2)\quad -(I_b - I_a) R_2 - R_3 I_b = 0$$

$$\mathcal{E}_0 - R_1 I_a - R_3 I_b = 0 \rightarrow \boxed{I_a = \frac{\mathcal{E}_0 - R_3 I_b}{R_1}}$$

$$2)\quad -I_b(R_2+R_3) + I_a R_2 = 0$$

$$I_b(R_2+R_3) - \frac{R_2}{R_1}(\mathcal{E}_0 - R_3 I_b) = 0$$

$$I_b\left(R_2+R_3+\frac{R_2R_3}{R_1}\right) = \frac{R_2}{R_1}\mathcal{E}_0$$

$$I_b = \frac{\frac{R_2}{R_1}\mathcal{E}_0}{R_2+R_3+\frac{R_2R_3}{R_1}}$$

$$= \frac{\frac{8}{4}\,12}{8+2+\frac{8\times 2}{4}}$$

$$\boxed{I_b = \frac{12}{7}\ A}$$

Current across R_2 = $(I_a - I_b)$

$$I_a = \frac{12 - 2\times\frac{12}{7}}{4} = 3\left(1-\frac{2}{7}\right) = \frac{15}{7}\ A$$

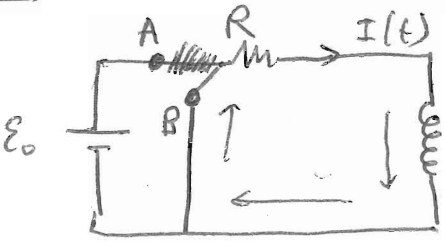
$$\frac{15}{7} - \frac{12}{7} = \frac{3}{7} = \boxed{0.429 A}$$

Conclusion: If there is only one battery, shortest solution is to do parallel & series reductions.

(143)

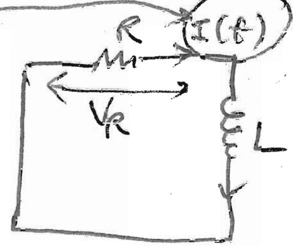
(27.69)

Eq. 27-8: in an RL circuit: { when $\mathcal{E}_0$ is disconnected: $I(t) = I_0 e^{-\frac{t}{L/R}}$



(switch @ B)

=



Find power dissipation in R: $P(t) = I \cdot V_R$

a) $P(t) = I(t)\, V_R(t) = I(t) \cdot I(t) \cdot R = I^2(t) \cdot R$

$= I_0^2 R\, e^{-\frac{2t}{\left(\frac{L}{R}\right)}}$ → time constant

b) Total energy dissipated in R b/w $t=0$ & $t=\infty$:

$$E_R = \int_{t=0}^{t=\infty} P(t)\, dt = I_0^2 R \int_0^{\infty} e^{-\frac{2t}{L/R}}\, dt = I_0^2 \cancel{R} \left[\frac{e^{-\frac{2t}{L/R}}}{-\frac{2\cancel{R}}{L}} \right]_0^{\infty}$$

$$= -\frac{1}{2} I_0^2 L \left[\underset{\downarrow \atop 0}{e^{-\infty}} - 1 \right] = \boxed{\frac{1}{2} L I_0^2}$$

→ Conclusion: current provided by the inductor after the battery is disconnected will end when all magnetic energy stored in the inductor is dissipated in the resistor.

(27.70) An electric and a magnetic field have the same energy density $\boxed{u_C = u_L}$ → Find the ratio $\frac{E}{B}$?

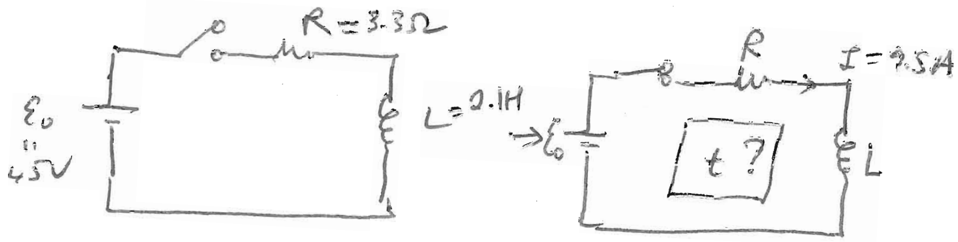
$$\frac{1}{2}\varepsilon_0 E^2 = \frac{\cancel{1} B^2}{2\mu_0} \to \frac{E^2}{B^2} = \frac{1}{\varepsilon_0 \mu_0} \to \frac{E}{B} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

epsilon mu

$$\frac{E}{B} = \frac{1}{\sqrt{8.85 \times 10^{-12} \times 4\pi \times 10^{-7}}} = 2.99 \times 10^8 \frac{m}{s} = c$$ (speed of light)

27.56

In a RL circuit with a known current $I = 9.5A$ find out how long the switch has been closed?



L is an inertia to current I : when switch is closed the current is still 0 : current will build up from $\boxed{0 \to max.}$

Notes
1) Now V_L starts @ max $\to$ 0 : $V_L(t) = \mathcal{E}_0 e^{-\frac{t}{(L/R)}}$
2) In an inductor: $V_L = L\frac{dI}{dt} \to dI = \frac{1}{L}V_L\,dt$

To obtain an expression for current @ time t:

$$\int_0^t dI = \int_0^t \frac{1}{L}V_L\,dt$$

$$I(t) - \cancel{I(0)}^{\,0} = \frac{1}{L}\int_0^t \mathcal{E}_0 e^{-\frac{t}{L/R}}\,dt = \frac{\mathcal{E}_0}{L}\left[\frac{e^{-\frac{t}{L/R}}}{\left(-\frac{R}{L}\right)}\right]_0^t$$

$$I(t) = -\frac{\mathcal{E}_0}{R}\left[e^{-\frac{t}{L/R}} - 1\right]$$

$$\boxed{I(t) = \frac{\mathcal{E}_0}{R}\left[1 - e^{-\frac{t}{L/R}}\right]}$$

(Check: $I(0) = 0$!
$I(\infty) = \frac{\mathcal{E}_0}{R}$!)

Now use $I(t) = 9.5A$ and solve for t:

$$9.5 = \frac{45}{3.3}\left[1 - e^{-\frac{t}{2.1/3.3}}\right] \to e^{-\frac{t}{2.1/3.3}} = \left(1 - \frac{9.5}{45}3.3\right)$$

$$-\frac{t}{\frac{2.1}{3.3}} = \ln\left(1 - \frac{9.5}{45}3.3\right) \to t = -\frac{2.1}{3.3}\ln\left(1 - \frac{9.5}{45}3.3\right)$$

$$\boxed{t = 0.76\,s}$$

Ch 29 Maxwell's Equations & Electromagnetic Waves

→ So far we have seen some connection b/w electric and magnetic field:

Ampere's law. $\underset{\text{Amperian Loop}}{\oint} \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{A}$ $(\vec{E} \to \vec{J} \to \vec{B})$

Faraday's law: $\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{induced voltage}} = -\int \underbrace{\frac{\partial \vec{B}}{\partial t}}_{\text{time derivative of the magnetic field}} \cdot d\vec{A}$ $\left(\frac{\partial \vec{B}}{\partial t} \to \vec{E}\right)$

Note: if $\frac{\partial \vec{B}}{\partial t} = 0$ the cycle of $\vec{E} \to \vec{B} \to \frac{\partial \vec{B}}{\partial t} \to \vec{E}$ is broken

→ The ultimate connection was discovered by Maxwell who saw the connections b/w all previous laws: Gauss, Ampere, Faraday's.

Maxwell's equations:

1) Gauss' Law: $\underbrace{\oint \vec{E} \cdot d\vec{A}}_{\text{Electric flux through the Gaussian surface}} = \frac{q_{enclosed}}{\varepsilon_0}$

2) "Gauss' Law for B": $\oint \vec{B} \cdot d\vec{A} = 0$ ← no magnetic monopole has been discovered.

3) Ampere's Law: (Modified) $\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{A} + \underbrace{\mu_0 I_{displacement}}_{\text{Maxwell's modification!}}$

4) Faraday's law: $\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{induced voltage}} = -\frac{d\Phi_B}{dt} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

Maxwell's contribution: the displacement current:

$$\boxed{I_{displacement} = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}}$$

Modified Ampere's Law (by Maxwell):

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

Observations:

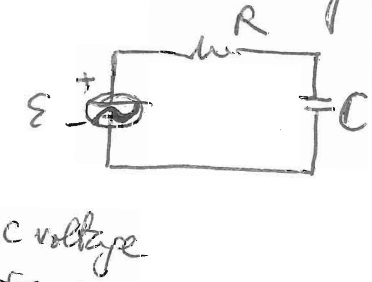
1) Now we can say:
$$\begin{cases} \dfrac{\partial \vec{E}}{\partial t} \xrightarrow{\text{creates}} \vec{B} & \text{(Modified Ampere's law)} \\ \dfrac{\partial \vec{B}}{\partial t} \xrightarrow{\text{creates}} \vec{E} & \text{(Faraday's)} \end{cases}$$

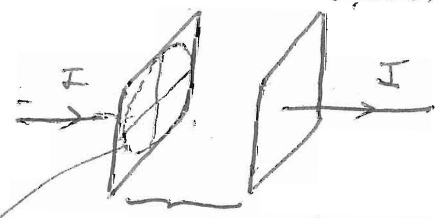
Cycle: $\frac{\partial \vec{E}}{\partial t} \rightarrow \vec{B} \rightarrow \frac{\partial \vec{B}}{\partial t} \rightarrow \vec{E}$

Since the fields create themselves they don't need a medium (with materials: air, molecules, water molecules, etc.) to propagate
(e.g.: light from the Sun)
(no longer need the postulate of an "ether"!)

Other EM waves: cell phone signals, radio signals, signals from space probe
(still c is not ∞!)

2) Maxwell's displacement current also explains a technicality about a measured magnetic field around a capacitor in an RC circuit, with an AC voltage source





No physical current b/w plate

AC: Alternating current

→ Amperian loop parallel to left plate; $I_{enclosed} = 0$ (does not go through loop)

→ Ampere's Law (before Maxwell) → $B = 0$

However a B could be measured around the left plate & in gap b/w the plates. :

With Maxwell's modification:

$$\oint \vec{B}\cdot d\vec{l} = \underbrace{\mu_0 I_{enclosed}}_{\substack{\text{didn't provide}\\ \text{a B with an}\\ \text{Amperian parallel}\\ \text{to left plate.}}} + \underbrace{\mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}}_{\substack{\text{this explains the}\\ \text{measured magnetic}\\ \text{field.}}}$$

$B \neq 0$!

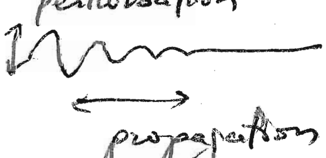
3) Starting from Maxwell's equations → EM wave equations:

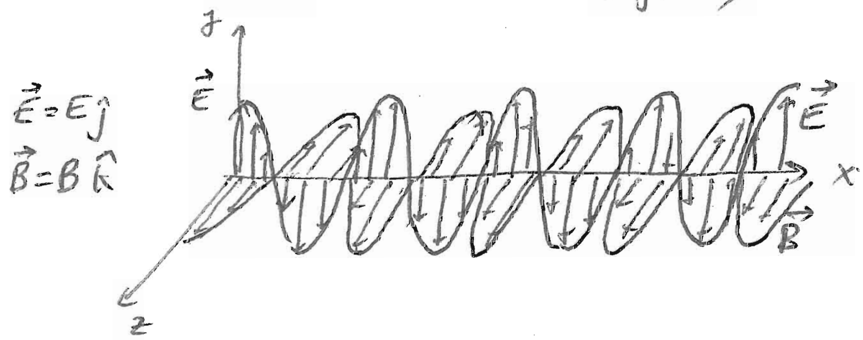
$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \\ \frac{\partial^2 B}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \end{aligned}\right\}$$ Relating 2nd derivatives in space & time

$$\frac{\partial^2 y}{\partial x^2} = \underbrace{\mu_0 \varepsilon_0}_{\omega^2} \frac{\partial^2 y}{\partial t^2}$$

(Transverse waves along a string)

→ EM waves are transverse wave: propagation is ⊥ perturbation (field)





Propagation is along $\hat{j} \times \hat{k} = \hat{i}$ (RHR).

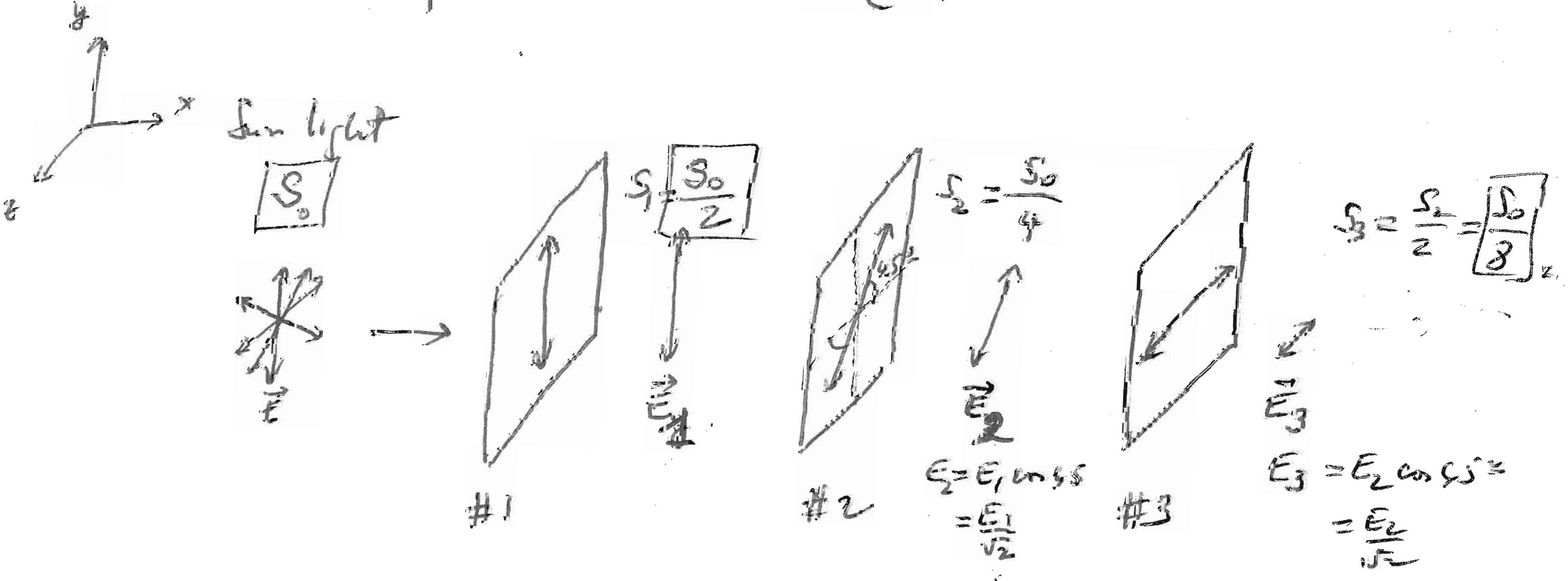
4) $\vec{E}$ & $\vec{B}$ are vectors → their directions are important
→ Polarization of EM waves
↳ Sun glasses: reduce intensity of sun light by allowing

only $\vec{E}$ along certain direction to pass through using "polarized material" (has an axis of polarization and only allows $\vec{E}$ along that direction to pass through)

29.44

Unpolarized light (sun light) passing through a sandwich of 3 polarizers

- Unpolarized light: has its electric fields pointing along all possible directions
- Polarizer: has a polarization axis, and it allows only $\vec{E} \parallel$ polarization axis to pass through → after a polarizer: $\vec{E}$ points along its axis only
- Intensity $S \propto E^2$ $\left(\vec{S} = \frac{\vec{E} \times \vec{B}}{c} \right)$



- Any field $\vec{E}$ (pointing along any direction) will have 2 components, one along the vertical, one along horizontal direction. → $\vec{E}$ in sun light $\begin{cases} \frac{1}{2} & \text{vertical} \\ \frac{1}{2} & \text{horizontal} \end{cases}$

29.50 Radio waves are EM waves:

Radio station : transmitter sends out waves in all directions

@ 1.5 km $\vec{E}$ has amplitude of $350 \frac{mV}{m}$

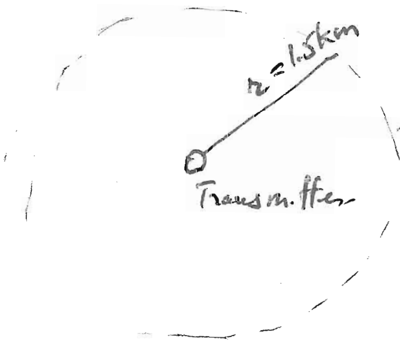
a) What was the power at transmitter

Calculate power from intensity of radiation S

$$S = \frac{\text{Power}}{\text{Area}}$$

$$S = \frac{\text{Energy}}{\text{Time}} \frac{\text{length}}{\underbrace{\text{Area} \times \text{length}}_{\text{Vol.}}} = \text{energy density} \times c$$

$$S = \left(\frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0} \right) c = \varepsilon_0 E^2 c$$



Area of sphere = $4\pi r^2$

$$\left\{ \begin{array}{l} \frac{E}{B} = c \\ c^2 = \frac{1}{\mu_0 \varepsilon_0} \rightarrow \varepsilon_0 = \frac{1}{\mu_0 c^2} \end{array} \right.$$

$$\text{Power} = S \cdot \text{Area} = \varepsilon_0 E^2 c \cdot \text{Area}.$$

$$\text{Power} = \varepsilon_0 E^2 c \cdot 4\pi r^2$$

Electric field is a sinusoid

$$E^2 \sim \sin^2(kx - \omega t)$$

$$\overline{E^2} \sim \overline{\sin^2(kx-\omega t)} = \frac{1}{2}$$

(sin goes b/w -1 & 1
$\sin^2$ goes b/w 0 & 1)

$$\overline{\text{Power}} = \frac{1}{2} \varepsilon_0 E_{max}^2 \, c \, 4\pi r^2$$

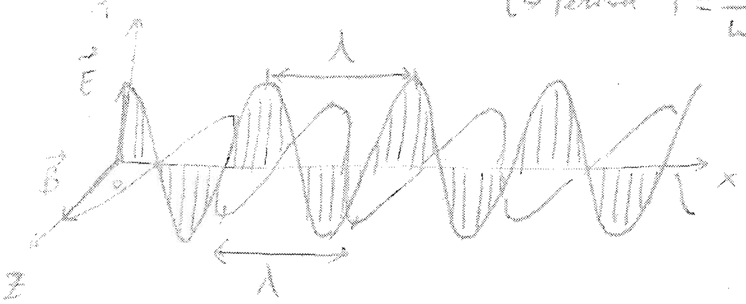
$$= \frac{1}{2} \times 8.85 \times 10^{-12} \times (350 \times 10^{-3})^2 \times 3 \times 10^8 \times 4\pi \times (1.5 \times 10^3)^2 = \boxed{4.59 \text{ kW}}$$

Electromagnetic waves. (Cont.)

1) $\boxed{\text{Vector nature of } \vec{E} \text{ \& } \vec{B}}$ $\boxed{\text{(Wave properties)}}$

$$\vec{E} = \underbrace{E}_{\text{Max. Magnitude}} \sin(kx - \omega t)\,\hat{j}$$

- Field points along $+y$
- Propagation is along x-axis
- Minus sign b/w kx & ωt → propagation is in $+x$ direction
- Wavelength $\lambda = \frac{2\pi}{k}$: sep. b/w consecutive peaks or troughs in space
- Period $T = \frac{2\pi}{\omega}$: sep. b/w consecutive peaks or troughs in time.

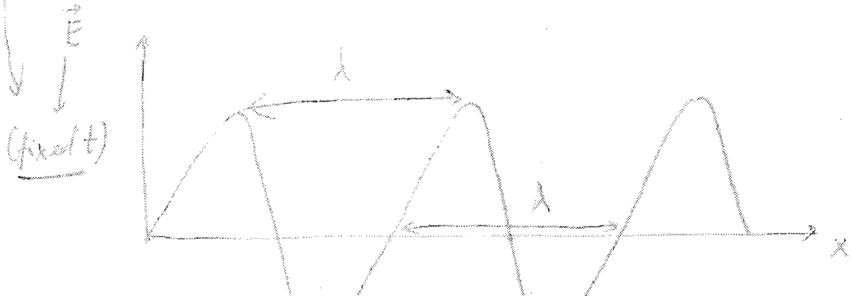


$\vec{B}$ has to be perpendicular to $\vec{E}$

EM wave ~ transverse wave in a string
↳ perturbation perpendicular to direction of propagation

$$\vec{B} = B \sin(kx - \omega t)\,\hat{k}$$

→ Also: direction of propagation is given by $\underbrace{\vec{v}}_{\text{direction of}}$ $\vec{E} \underbrace{\times}_{\text{cross product}} \vec{B}$ using RHR



Two consecutive same zeroes (both increasing or decreasing) are also separated by λ