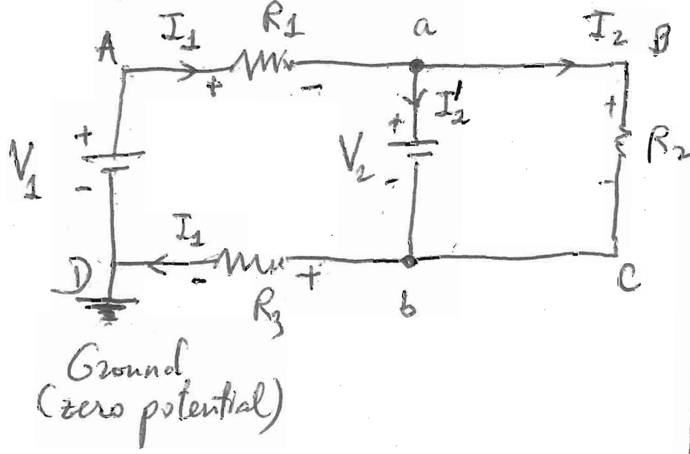
15) Circuits with resistors only but can't be solved using simple series & parallel combinations:



Note:

1) R_1 & R_2 in series? No, since I_1 flows through R_1 but I_2 flows through R_2!

2) R_1 & R_2 in parallel?

Potential or Voltage difference

across R_2 is $\boxed{V_2}$

across R_1 is $I_1R_1 = V_1 - V_a$

$$= V_1 - (V_2 + I_1R_3)$$

$$= \boxed{V_1 - V_2 - I_1R_3}$$

Must use $\boxed{\text{loop or node analysis.}}$

Loop Analysis

→ Total voltage difference across elements in a closed loop is zero. (Conservation of energy)

Loop: in our example we have two independent loops: AabD, aBCb. There is also ABCD, but it is not independent of the other two.

→ Assign directions for currents in the loops (the equations will tell the correct direction via signs in numeric results)

→ Sign convention for voltage:

1) Voltage @ battery will have + sign if the current goes from − to + through that battery
2) Voltage @ battery will have − sign if the current you assigned goes from + to −
3) Voltage @ any resistor will be negative

$\boxed{\sum V_i = 0}$

Node Analysis

→ Total current @ any node is zero (conservation of charge).

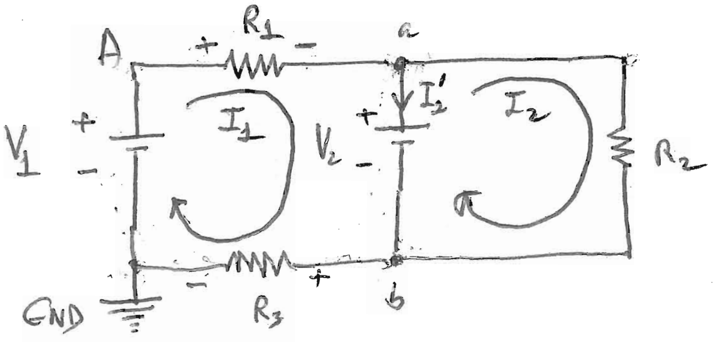
Node: where more than 2 branches converge. (In our example: 2 nodes: a & b)

→ Assign directions for currents at nodes (equations will tell the correct direction)

→ Sign convention for currents

1) Current into node will have a + sign
2) Current leaving node will have − sign

$\boxed{\sum_i I_i = 0}$



(Ground or zero potential)

$\boxed{\text{Assign}}$ directions for currents $\rightarrow$ $\begin{cases} I_1 : \text{clockwise (CW)} \\ I_2 : \text{CW} \\ I_2' \text{ (b/w a \& b)} : \text{downward} \end{cases}$

Loop Analysis

For each loop : $\sum_i V_i = 0$

Loop 1 : $+V_1 - I_1R_1 - V_2 - I_1R_3 = 0$ (1)

Loop 2 : $+V_2 - I_2R_2 = 0$ (2)

Solve for I_1 & I_2 (V_1, V_2, R_1, R_2, R_3 are given)

(2) $\boxed{I_2 = \frac{V_2}{R_2}}$

(1) $V_1 - V_2 = I_1(R_1 + R_3)$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

Node Analysis

For each node : $\sum_i I_i = 0$

Node a : $\boxed{I_1 - I_2' - I_2 = 0}$

Node b : $-I_1 + I_2' + I_2 = 0$

Only one independent node!

Solve for I_1 & I_2 (V_1, V_2, R_1, R_2, R_3 are given)

Ohm's Law $\begin{cases} I_1 = \frac{V_1 - V_a}{R_1} \\ I_2 = \frac{V_a - V_b}{R_2} = \frac{V_2}{R_2} \end{cases}$

Node eq: $\frac{V_1 - V_a}{R_1} - I_2' - \frac{\overset{\nearrow V_2}{\boxed{V_a - V_b}}}{R_2} = 0$

Notes $\begin{cases} V_a = \overset{a \to b \to GND}{\boxed{V_2 + I_1R_3}} = \overset{a \to A \to GND}{\boxed{-I_1R_1 + V_1}} \\ I_2' = I_1 - I_2 \end{cases}$

$\rightarrow V_2 + I_1R_3 = -I_1R_1 + V_1$

$I_1(R_3 + R_1) = V_1 - V_2$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

$\rightarrow \frac{V_1 - (V_2 + I_1R_3)}{R_1} - (I_1 - I_2) - \frac{V_2}{R_2} = 0$

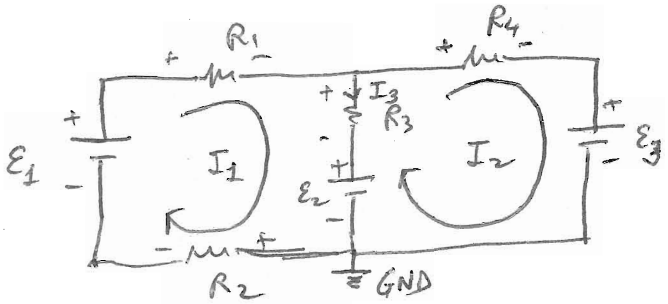
$$\frac{V_1 - V_2}{R_1} - I_1\underbrace{\left(\frac{R_3}{R_1} + 1\right)}_{\frac{R_3 + R_1}{R_1}} + \underbrace{I_2 - \frac{V_2}{R_2}}_{0} = 0$$

$\rightarrow \frac{V_1 - V_2}{\cancel{R_1}} - I_1 \cdot \frac{R_3 + R_1}{\cancel{R_1}} = 0$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

25.53

100



$\varepsilon_1 = 6V$; $\varepsilon_2 = 1.5V$; $\varepsilon_3 = 4.5V$

$R_1 = 270\Omega$; $R_2 = 150\Omega$;

$R_3 = 560\Omega$; $R_4 = 820\Omega$.

→ 2 loops → Assign directions for currents: I_1 (cw) & I_2 (cw)

Question: I_3? or $\boxed{I_1 - I_2 = I_3}$

Loop Analysis:

loop 1: $+\varepsilon_1 - I_1 R_1 - I_3 R_3 - \varepsilon_2 - I_1 R_2 = 0$ (1)

+

loop 2: $+\varepsilon_2 - (I_2 - I_1) R_3 - I_2 R_4 - \varepsilon_3 = 0$ (2)

2 eqs. with 2 unknowns I_1 & I_2

$$\varepsilon_1 + \cancel{\varepsilon_2} - I_1(R_1+R_2) - I_2 R_4 - \cancel{\varepsilon_2} - \varepsilon_3 = 0$$

Goal: solve for I_1 & I_2

$$I_1 = \frac{\varepsilon_1 - \varepsilon_3 - I_2 R_4}{R_1 + R_2}$$

$$\boxed{I_1 = \frac{1.5 - 820 I_2}{420}} \quad (3)$$

(2) Plug (3) into (2):

$$\underbrace{\varepsilon_2 - \varepsilon_3}_{} - I_2 \underbrace{(R_3 + R_4)}_{} + \frac{1.5 - 820 I_2}{420} \cdot R_3 = 0$$

$$-3 - I_2\, 1380 + 1.5 \cdot \frac{560}{420} - \frac{820 \times 560}{420} I_2 = 0$$

$$-1 - 2473.2\, I_2 = 0 \rightarrow \boxed{I_2 = -0.4\, mA}$$

m → 10^{-3}

I_2 is actually in the CCW direction in loop #2.

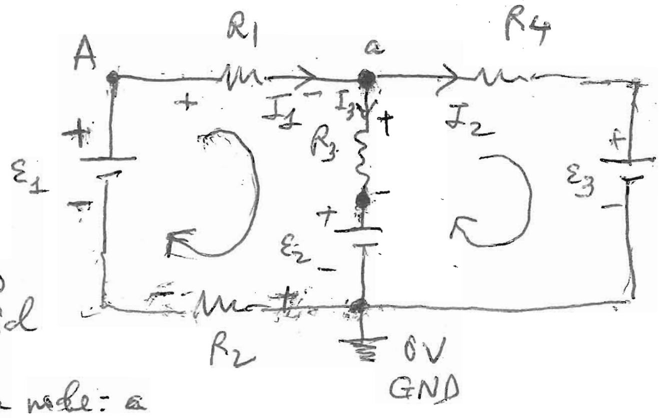
Back in (3) $\boxed{I_1 = \frac{1.5 + 820 \times 0.4 \times 10^{-3}}{420} = +4.36\, mA}$

I_1 is actually in the CW direction in loop #1.

Finally: $\boxed{I_3 = I_1 - I_2 = 4.36\, mA - (-0.4\, mA) = +4.76\, mA}$

downward through R_3 as assumed.

Node Analysis:



1) Set the GND or 0 potential

2) Defines the node: a

3) Assign directions for currents: I_1 (into a), I_2 (leaving a); I_3 (leaving a) down R_3
CW in loop #1 (under I_1); CW in loop #2 (under I_2)

4) Write node equation: $I_1 - I_2 - I_3 = 0$

5) Write currents in terms of the voltages: V_a is voltage at node a wrt. GND. V_A is voltage @ A wrt. GND.

Ohm's law

$$I_1 = \frac{V_A - V_a}{R_1} = \frac{\overbrace{(\mathcal{E}_1 - I_1 R_2)}^{V_A} - V_a}{R_1}$$

→ here we need to solve for I_1

$$I_1 R_1 = \mathcal{E}_1 - I_1 R_2 - V_a$$

$$I_1 = \frac{\mathcal{E}_1 - V_a}{R_1 + R_2}$$

$$I_2 = \frac{V_a - \mathcal{E}_3}{R_4}$$

$$I_3 = \frac{V_a - \mathcal{E}_2}{R_3}$$

Plug these currents I_1, I_2, I_3 (all in terms of V_a or potential at our node!) In to our only node equation: $I_1 - I_2 - I_3 = 0$

$$\frac{\mathcal{E}_1 - V_a}{R_1 + R_2} - \frac{V_a - \mathcal{E}_3}{R_4} - \frac{V_a - \mathcal{E}_2}{R_3} = 0$$

Solve for V_a, then obtain the currents!

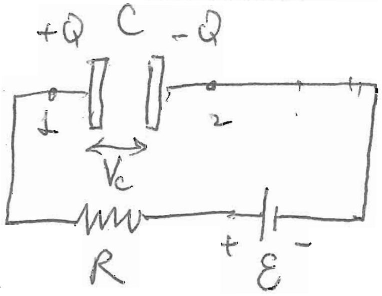
$$\frac{6 - V_a}{420} - \frac{V_a - 4.5}{820} - \frac{V_a - 1.5}{560} = 0 \rightarrow \boxed{V_a = 4.17V}$$

$$I_3 = \frac{4.17 - 1.5}{560} = \frac{2.67}{560} = +4.76mA\ !$$

$$I_1 = \frac{6 - 4.17}{420} = 0.004357 = +4.36\ mA$$

$$I_2 = \frac{4.17 - 4.5}{820} = -0.4\ mA$$

Circuits involving resistors & capacitors:

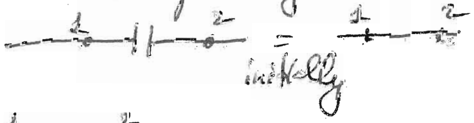


Time evolution: a) @ time $t=0$ the uncharged capacitor C is connected to a circuit with a resistor R and a battery $\mathcal{E}$

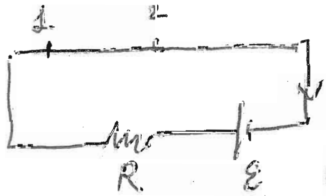
Initial charge: $Q=0$

Initial potential @ capacitor $V_c = 0$

↳ Zero potential across terminals 1 & 2: the capacitor acts like a piece of wire initially



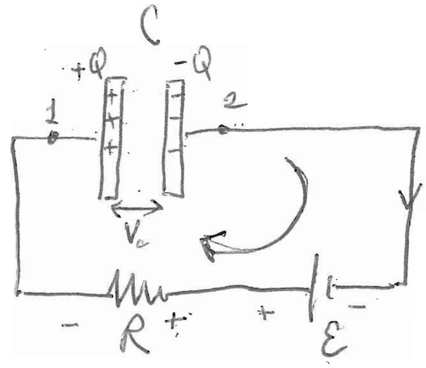
@ $t=0$:



$I_0 = \frac{\mathcal{E}}{R}$ (Ohm's Law)

b) @ time $t>0$ some positive charges are transferred from right plate to left plate through the circuit → the capacitor is being charged. Current is I.

$\boxed{t>0}$



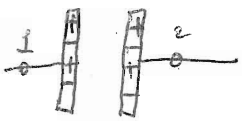
loop equation: $\boxed{\varepsilon - IR - V_c = 0}$

$$I(t>0) = \frac{\varepsilon - V_c}{R}$$

$\left(I_0 = \frac{\varepsilon}{R} \rightarrow I(t>0) \text{ is smaller}\right)$

→ Qualitative description for Current in RC circuit as capacitor is getting charged:

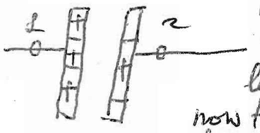
initially: $\boxed{t=0}$



$I = I_{max} = I_0$

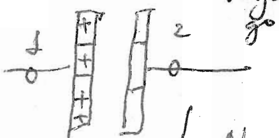
$V_c = 0$

then: $\boxed{t>0}$



Moved one + charge to left plate: now to move more charges we need to go against $\vec{E}$

then:



$I < I_0$

$V_c > 0$

$\boxed{t\to\infty}$ (long enough)

↳ No more + charges to be moved → $\boxed{I(t\to\infty) = 0}$

$V_c = V_{c\,max} = \varepsilon$

→ Capacitor behaves like an open circuit across 1 & 2:

$V_c = \varepsilon$

→ also when Capacitor is fully charged: the electric field b/w plates is max.

→ $V_c = E \cdot d$ → $V_c = max = \varepsilon$

(d = separation b/w plates)

Summary: Qualitative description:

	Current I	Voltage V_c
$t=0$	$I(t) = I_0 = \frac{\varepsilon}{R}$	$V_c(t) = 0$
$t>0$	$I(t) < I_0$	$V_c(t) > 0$
$t\to\infty$	$I(t) = 0$	$V_c(t) = V_{c_{max}} = \varepsilon$

→ Quantitative description for $t>0$:

$$\frac{d}{dt}\left[\varepsilon - IR - V_c = 0\right]$$

$$\frac{d}{dt}\varepsilon = 0 \rightarrow -R\frac{dI}{dt} - \frac{d}{dt}\left(\frac{Q}{C}\right) = 0 \rightarrow -R\frac{dI}{dt} - \frac{1}{C}\underbrace{\frac{dQ}{dt}}_{I} = 0 \rightarrow \boxed{-R\frac{dI}{dt} - \frac{1}{C}I = 0}$$

(ε is supposed to constant over time)

Differential equation of 1st order in I.

$$I = -RC\frac{dI}{dt} \quad \text{or} \quad \underbrace{\frac{dI}{I}}_{\text{integral of this is } \ln I} = \underbrace{-\frac{1}{RC}\,dt}_{\text{integral of this is } -\frac{1}{RC}t \,+\, \text{constant}}$$

$$\left[\ln I = -\frac{1}{RC}t + \text{const.}\right]$$

e

$$e^{\ln I} = e^{-\frac{t}{RC}} \cdot e^{\text{const}}$$

$$\boxed{I(t) = \text{constant} \times e^{-\frac{t}{RC}}}$$

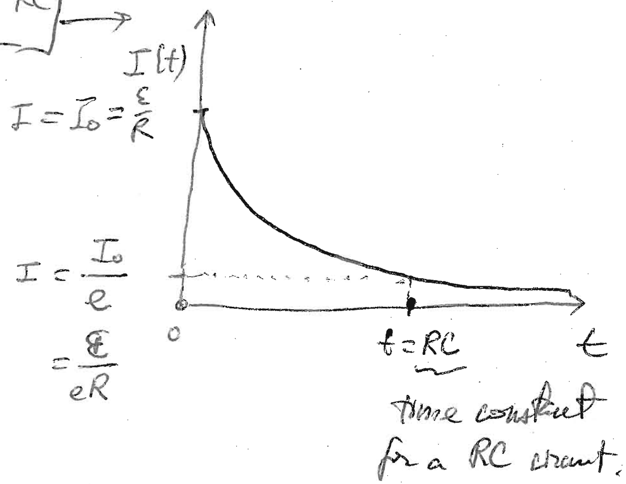
↓

can be found by setting $t=0$:

$\underline{I(t=0)} = \text{constant}$

$I_0 = \frac{\varepsilon}{R}$

$$\boxed{I(t) = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}} \rightarrow$$



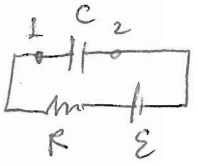
$e = 2.71\ldots$

$$\left[V_c = \frac{Q}{C} = \frac{\int I\,dt}{C} = \frac{\varepsilon}{RC}\underbrace{\int_0^t e^{-\frac{t}{RC}}dt}_{\left(-RC\,e^{-\frac{t}{RC}}\right)_0^t} = -\varepsilon\left[e^{-\frac{t}{RC}} - 1\right] = \boxed{\varepsilon\left(1-e^{-\frac{t}{RC}}\right)}\right.$$

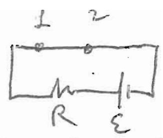
$$I = \frac{dQ}{dt} \rightarrow Q = \int I\,dt$$

$$\begin{cases} t=0 & V_c = 0 \\ t>0 & V_c = \varepsilon\left(1-e^{-\frac{t}{RC}}\right) \\ t\to\infty & V_c = \varepsilon \end{cases}$$

RC Circuit



t = 0 :

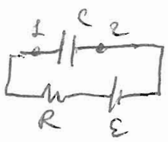


Capacitor acts like a short-circuit

$I = I_0 = \frac{\varepsilon}{R}$ (max)

$V_c = 0$

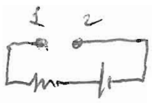
t > 0



$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$

$V_c = \varepsilon(1 - e^{-\frac{t}{RC}})$

t → ∞ (sufficiently long)

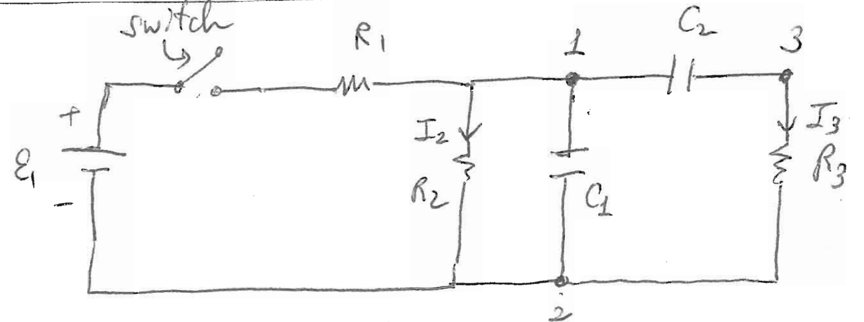


Capacitor acts like an open-circuit

$I(t \to \infty) = 0$

$V_c = \varepsilon$ (max.

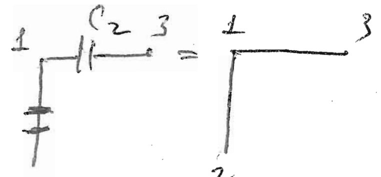
25.64



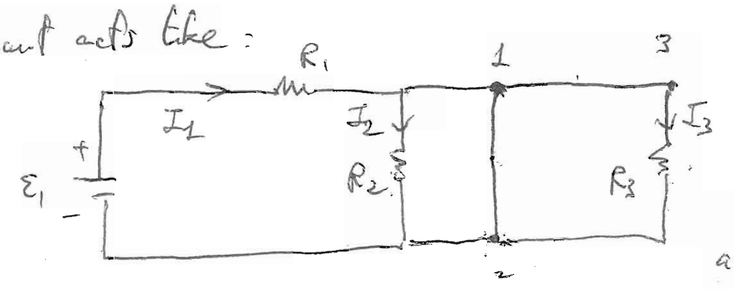
Given data:
- → Switch initially open
- → C_1 & C_2 initially uncharged (act like short-circuit: no potential difference)
- → $R_1 = R_2 = R_3 \equiv R$

Find: current in R_2: I_2
- a) @ t=0 (just after switch is closed or circuit starts running)
- b) @ t → ∞
- c) Describe current I_3

a) @ t=0 capacitors act like short-circuits



Circuit acts like:



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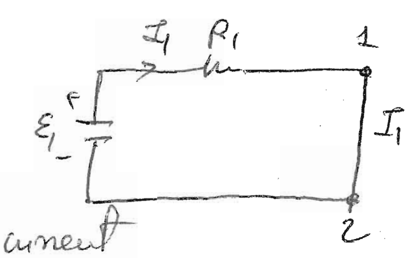
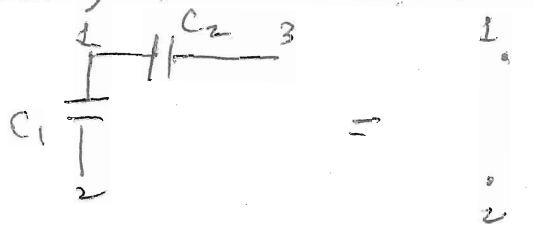


$I_1 = \frac{\varepsilon_1}{R_1}$

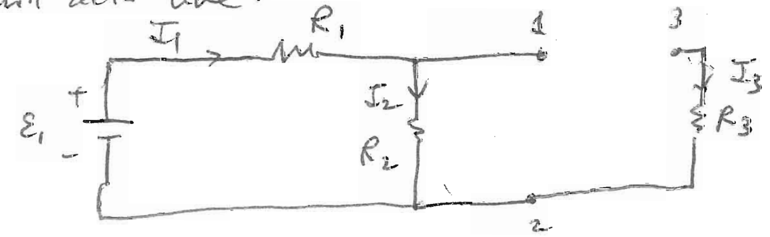
↓

any current would choose the least resistance path. → $I_3 = 0$ ($I_2 = 0$ as well)

b) @ $t \to \infty$: Capacitors are fully charged (no more charges to move) → no current → act like open circuit!

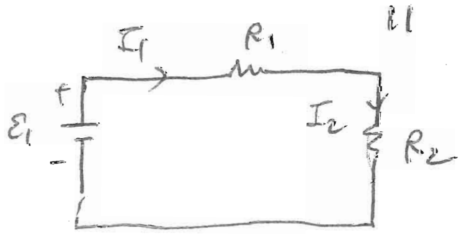


Circuit acts like:



$I_3 = 0$ (dead end street)

||



$$\boxed{I_2 = I_1 = \frac{\varepsilon}{R_1 + R_2}}$$

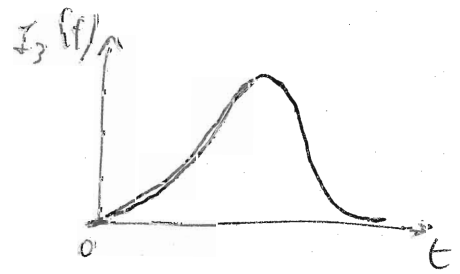
↑ Ohm's Law

c) Describe I_3 qualitatively:

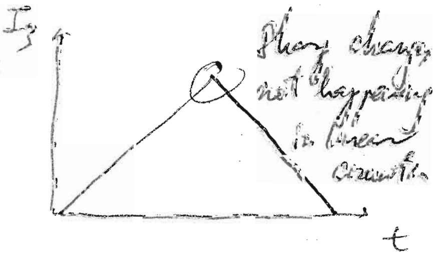
$t = 0$: $I_3 = 0$

$t > 0$: increase then decrease.

$t = \infty$: $I_3 = 0$

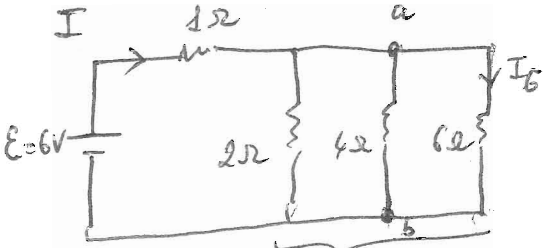


Also:



Sharp change not happening in linear circuits

25.48

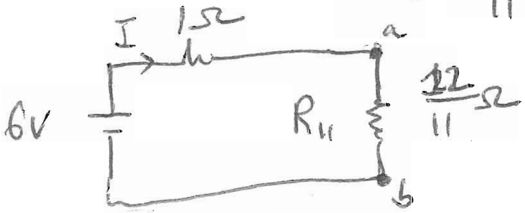


Can be analyzed : parallel & series combination :

parallel : $\frac{1}{R_{\parallel}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12}$

$\rightarrow R_{\parallel} = \frac{12}{11}\,\Omega$

a)



$I = \frac{6}{1+\frac{12}{11}} = 2.86$ A.

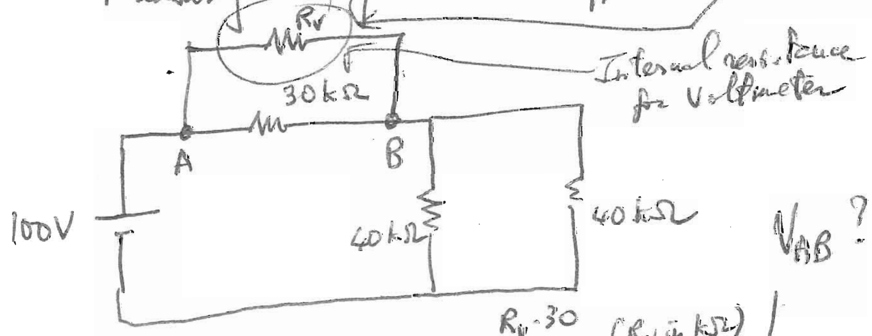
$V_{ab} = I \cdot R_{\parallel} = 2.86 \times \frac{12}{11}$ V

b) I_6 ? → Find V_{ab} :
→ Ohm's law ⇒ $I_6 = \frac{V_{ab}}{6} = \frac{2.86 \times 12}{66}$ A

$= 0.522$ A

25.55

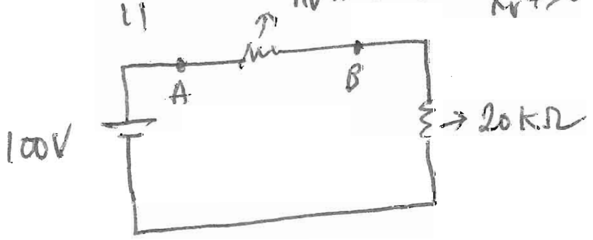
Measuring voltages with different voltmeters (connecting in parallel with the element to measure its voltage)

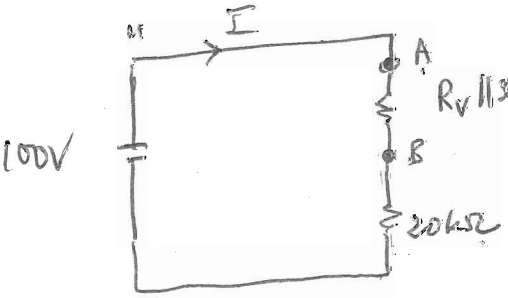


V_{AB} ? $\begin{cases} R_v = 50\,k\Omega \\ R_v = 250\,k\Omega \\ R_v = 10\,M\Omega \text{ (digital)} \end{cases}$

→ Find V_{AB} in terms of R_v

$R_v \parallel 30\,k\Omega = \frac{R_v \cdot 30}{R_v + 20}$ (R_v in kΩ)





→ Voltage division : $V_{AB} = 100 \cdot \frac{\frac{R_v \cdot 30}{R_v+30}}{\frac{R_v \cdot 30}{R_v+30} + 20}$

I

Ohm's law

$V_{AB} = 100 \cdot \frac{R_v \cdot 30}{R_v \cdot 30 + R_v \cdot 20 + 600}$

$$V_{AB} = \frac{3000\,R_V}{50R_V + 600} \quad (R_V \text{ in } k\Omega)$$

Voltage across A & B
as measured by voltmeters

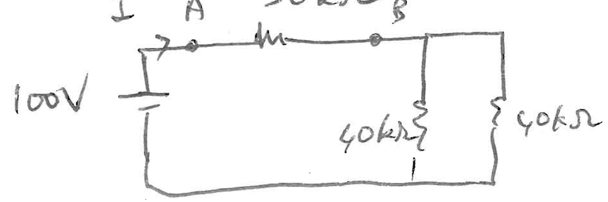
a) $R_V = 50k\Omega$
$\rightarrow V_{AB} = 48.39V$

b) $R_V = 250k\Omega \rightarrow V_{AB} = 57.25V$

c) $R_V = 10M\Omega = 10{,}000k\Omega$

$V_{AB} = 59.93V$
(closest value of V_{AB} in the original circuit w/o R_V)

__Note:__ what was the original V_{AB} (w/o voltmeter → $R_V = \infty$)



$$\Rightarrow V_{AB} = I \cdot 30k\Omega$$

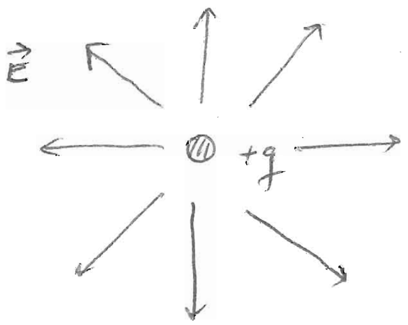
$$= \frac{100}{50\,\cancel{k\Omega}} \cdot 30\cancel{k\Omega}$$

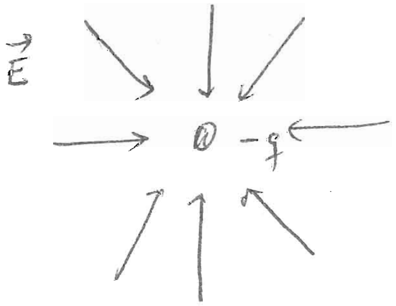
$$\boxed{V_{AB} = 60V} \rightarrow \text{Digital is best.}$$

Ch 26 Magnetic Field

Electric → Magnetic → Electromagnetic (two sides of a same phenomenon)

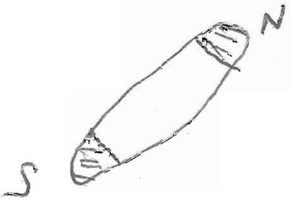
Electric





- Two types of charge: +, -
- like charge repel
  opposite charges attract
- Electric field lines are open

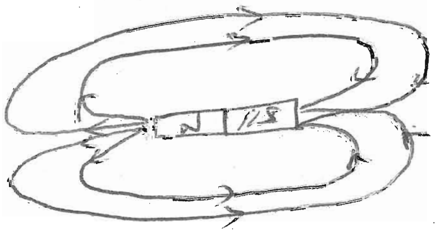
Magnetic



magnetic stones

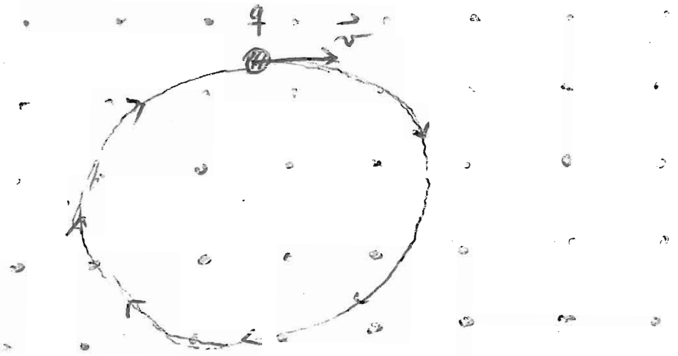
compass
↓
navigation
(Earth has a magnetic field from its "big compass")

- Two types of magnetic monopoles
  N, S
- They are always attached!
  (magnetic monopoles are not found yet!)
- Like pole repel
  Opposite poles attract
  (N & S attract)
- Magnetic field line are closed (N & S are attach)



Effects of the Magnetic Field on Moving charges:

On a moving charge of value q going at velocity $\vec{v}$ on the plane of this page, a uniform magnetic field $\vec{B}$ pointing out of the page (perpendicular to the plane of charge motion) would bend the trajectory of that charge into circles:



Uniform (equal spacing) magnetic field $\vec{B}$ out of page (dots)

Applications:
1) Particles confinement or trap
2) Cyclotron → synchrotron (medical applications, particle physics research)

Closer look:

1) If q instead moves in the same direction as the field $\vec{B}$: the field has no effect on the charge. (angle θ between $\vec{v}$ & $\vec{B}$ is $0°$ or $180°$)
2) If the angle b/w $\vec{v}$ & $\vec{B}$ is $0 < \theta < 90°$: Then the effect of $\vec{B}$ on $\vec{v}$ is intermediate:

$$\vec{F}_{magnetic} = q\vec{v} \underbrace{\times}_{\text{cross product b/w } \vec{v} \,\&\, \vec{B}} \vec{B}$$

cross product b/w $\vec{v}$ & $\vec{B}$ (same as in torque, angular momentum, etc)

$$\vec{v}\times\vec{B} \begin{cases} vB\sin\theta \\ \text{direction is } \perp \text{ both } \vec{v} \,\&\, \vec{B} \end{cases}$$

$$\underset{\substack{\uparrow \\ \text{Force}}}{\vec{F}_{electric}} = q \underset{\substack{\uparrow \\ \text{field}}}{\vec{E}}$$

(velocity is not involved)

3) When $\theta = 90°$, magnetic force is maximum on the moving charge

Cross-product:

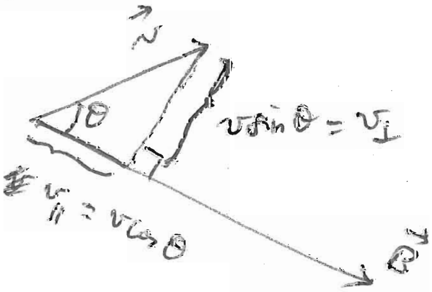
a) $\vec{v}\times\vec{B}$ is another vector that is perpendicular to both $\vec{v}$ & $\vec{B}$. Its magnitude is $vB\sin\theta$ (θ is the angle b/w $\vec{v}$ & $\vec{B}$). Its direction is given by the Right Hand Rule (RHR): when you close your right hand fingers from the 1st first vector ($\vec{v}$) toward the 2nd vector ($\vec{B}$), your thumb indicates the direction of $\boxed{\vec{v}\times\vec{B}}$ since $\vec{F}_{magnetic} = q\vec{v}\times\vec{B}$

- If $q>0$, thumb also points in the direction of $\vec{F}_{magnetic}$
- If $q<0$, $\vec{F}_{magnetic}$ is in the opposite direction

b) A note on $v_\perp$: is the component of $\vec{v}$ that is perpendicular to the magnetic field.

$$|\vec{v}\times\vec{B}| = vB\sin\theta = \underbrace{v\sin\theta}_{v_\perp} B$$

magnitude of $\vec{v}\times\vec{B}$

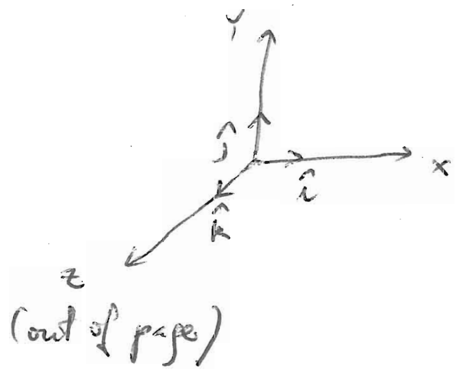


- The parallel component of $\vec{v}$ to the magnetic field is not affected by the field
- The perpendicular component of the velocity ($v_\perp$) to the field will feel the magnetic force: bending it into circular trajectory

$$F_{magnetic} = qv\sin\theta B = q\underset{\uparrow}{v_\perp} B$$

$$\begin{cases} \text{If particle goes} \parallel \vec{B} \rightarrow v_\perp = 0 \\ \qquad \rightarrow F_{magnetic} = 0 \\ \text{If particle goes} \perp \vec{B} \rightarrow v = v_\perp \\ \qquad \rightarrow F_{magnetic} = \text{max.} \end{cases}$$

c)



Standard 3D coord. system.

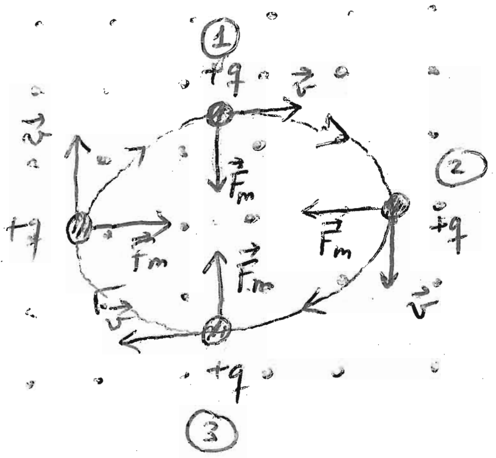
Uniform $\vec{B}$ (equal spacing dots) pointing out of page $\vec{B} = B\hat{k}$

① A charge $+q$ going in $+x$:

$\vec{v} = v\hat{i}$:

$\vec{F}_m = q v B \underbrace{(\hat{i} \times \hat{k})}$

RHR : $-\hat{j}$



CW

② Charge $+q$ going in $-y$:

$\vec{v} = v(-\hat{j})$

$\vec{F}_m = q v B \underbrace{(-\hat{j} \times \hat{k})}$

RHR: $(-\hat{i})$

③ Charge $+q$ in $-x$ direction

$\vec{v} = v(-\hat{i})$

$\vec{F}_m = q v B \underbrace{(-\hat{i} \times \hat{k})}$

RHR : $\hat{j}$

④ Charge $+q$ in $+y$ direction

$\vec{v} = v\hat{j}$

$\vec{F}_m = q v B \underbrace{(\hat{j} \times \hat{k})}$

RHR : $\hat{i}$

(a)

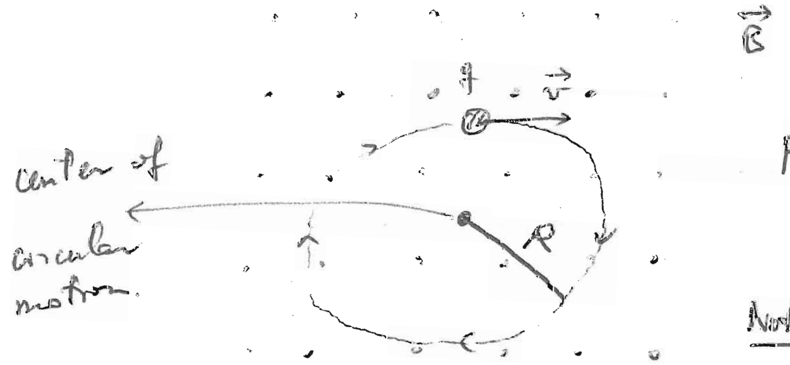
q	$\vec{B}$	direction of motion
$+$	out of page	CW
$-q$	out of page	CCW
$+q$	into page	CCW
$-q$	into page	CW

(ii) F_m provides the radial acceleration for uniform circular motion of the moving charge.

2nd Newton's Law: (radial direction) $\underline{qvB = ma_r}$

$a_r = \frac{v^2}{R} \longrightarrow q\cancel{v}B = m\frac{v^{\cancel{2}}}{R} \rightarrow \boxed{R = \frac{mv}{qB}}$

Magnetic Field and uniform circular motion:



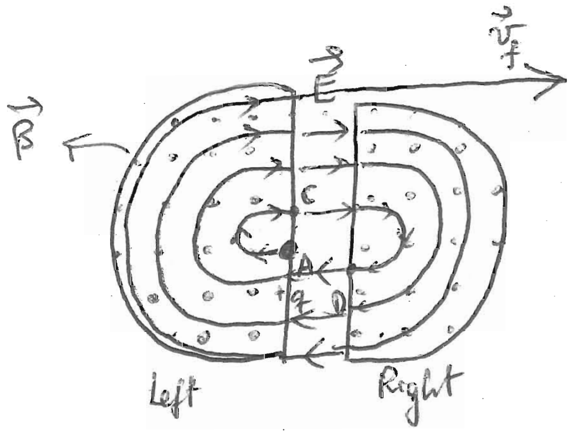
$$R = \frac{mv}{qB}$$

Note: larger B → smaller R
→ A technical difficulty for magnetic fusion (bring particles sufficiently closer together) is to achieve a sufficiently high B. (right now $B_{max} \sim 10T$)

Orbital period: how long for charge particle to complete one orbit: $T = \frac{2\pi R}{v} = \frac{2\pi \cancel{R}}{\frac{qB\cancel{R}}{m}} = \frac{2\pi m}{qB}$

Applications of effect of a magnetic field on a moving charge:

1) Cyclotron (modern version: synchrotron)
- → Goal: to accelerate charged particles to a very high speed using the magnetic & electric fields;
- → Application:
  - → study of subatomic particles and structures (CERN)
  - → lower energy range: medical applications:



- 2 D-shaped chambers filled with a uniform magnetic field (out of page)
- Alternating electric field $\vec{E}$ in the gap b/w chambers: direction of $\vec{E}$ flipping b/w left & right direction in-sync with the circular motion of charge particle.

a) Charge +q starts @ A going into left chamber: F_m is up, it is bent

into circular trajectory, exiting left chamber @ C

2) No $\vec{B}$ in gap, $\vec{E}$ will push charge into right chamber giving it an acceleration during the gap → going into right chamber @ higher speed: $R = \frac{mv}{qB}$ → R is also larger.

4) When particle comes out @ D, $\vec{E}$ is reversed to give particle another push into left chamber.

5) Process is repeated until particle leaving left chamber @ high v_f

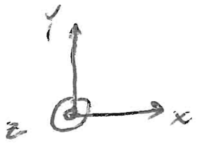
6) $$\frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m}$$

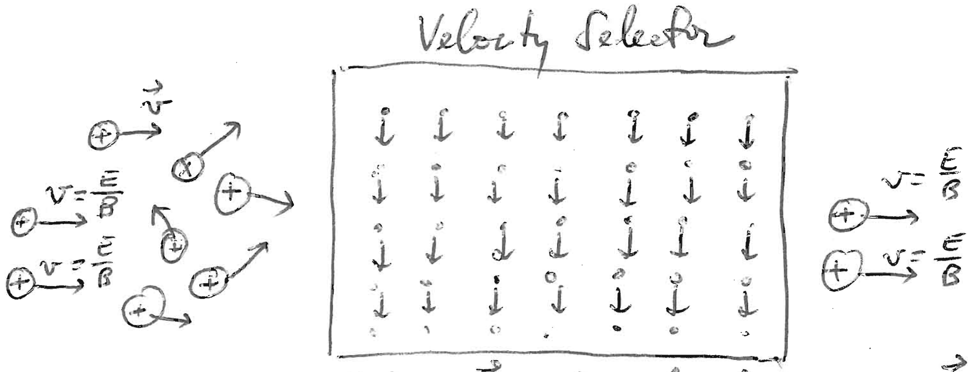
Max KE, { 1) Acceleration from $\vec{E}$
2) Radius of last outer orbit: size of cyclotron

7) When v_f gets closer to $c = 3\times10^8 \frac{m}{s}$ → relativistic corrections → Synchrotron

2) Velocity selector:

↳ Goal: to pick out among a bunch of ions (positively charged particles) at different velocities those with a desired velocity by using a combination of electric & magnetic fields:





This selects those ions with $\vec v = \frac{E}{B}\hat i$

Dots: $\vec E$: uniform & out of page $\vec E = E\hat k$

Arrows: $\vec B$ uniform & downward $\vec B = B(-\hat j)$

Why? A ion coming in with $\vec v = v\hat i$ will feel two forces:

$$\vec F_E = q\vec E = qE\hat k$$

$$\vec F_m = qvB\,(\underbrace{\hat i \times (-\hat j)}_{-\hat k}) = -qvB\hat k$$

} When these forces exactly cancel each other → particle will pass through w/o deflection.

$$\rightarrow (qE - qvB)\hat k = 0 \rightarrow \boxed{v = \frac{E}{B}}$$

Note: a) those ions with $\vec v = v\hat i$ but $v \neq \frac{E}{B}$ will be deflected.

b) Those with velocity components in $\hat j$ and/or $\hat k$ directions will also be deflected.

Calculation of $\vec{B}$:

Electric field

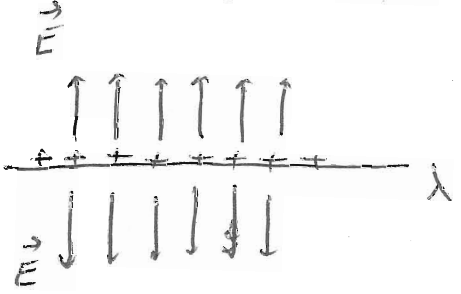
→ Sources: charge:

$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$

$k = 9\times10^9$ (SI)

inverse-square law
or Coulomb's law

→ line of charge of linear charge density λ



$$E = \frac{2k\lambda}{r}$$

r: separation from line of charge.

Magnetic field

→ Sources: moving charge or currents.

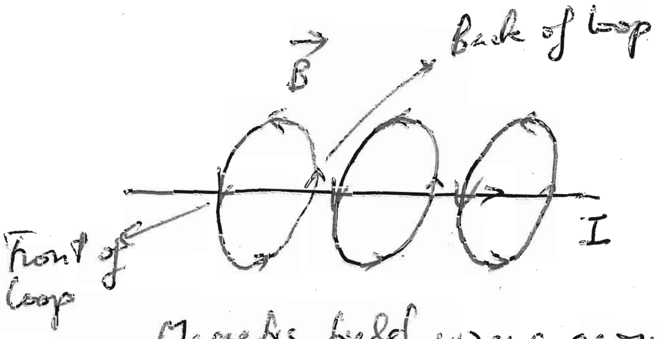
$$d\vec{B} = \frac{\mu_0}{2\pi}\,\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

μ_0: magnetic permeability in vacuum

$$\mu_0 = 4\pi\times10^{-7}\ \frac{N}{A^2}$$

inverse-square law
or Biot-Savart's law

→ line of current



→ Magnetic field wraps around the current I:

→ Right hand thumb in direction of I → wrapping fingers indicate the direction of B

→ Magnetic field lines are closed loop.

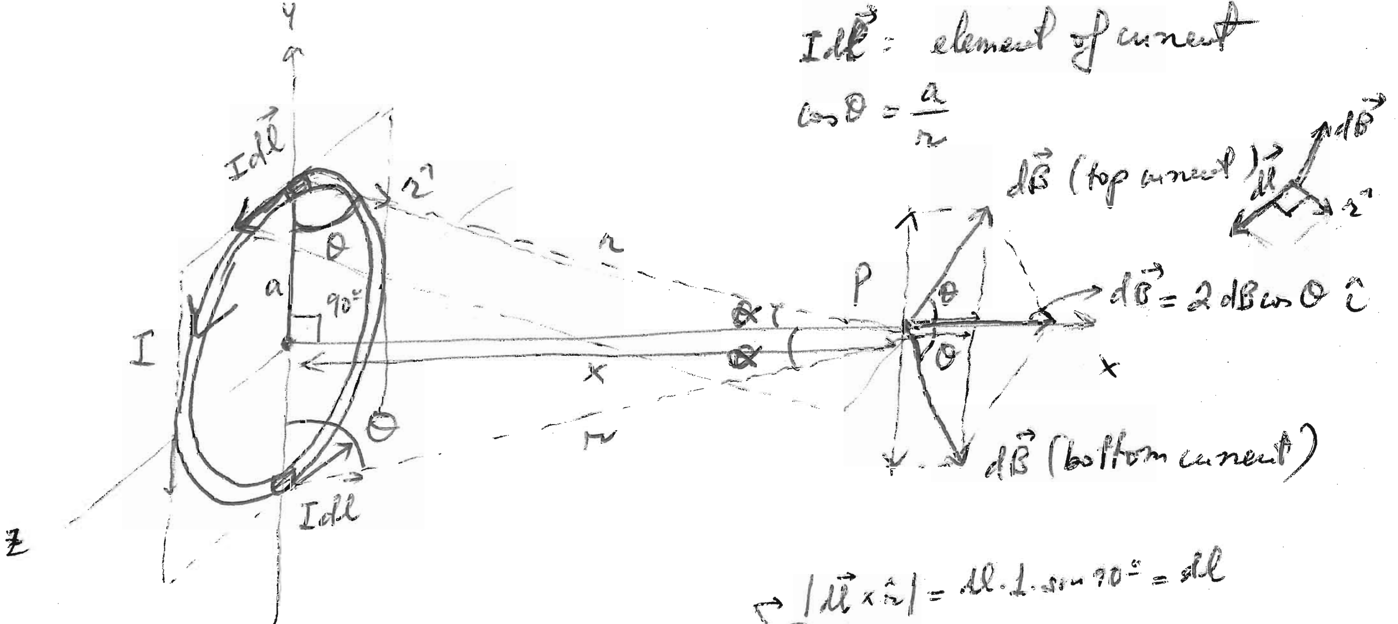
Magnetic Field created by a circular loop of current:

along its axis of symmetry (x-axis)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l}\times\hat{r}}{r^2} \quad \Big\{ \hat{r} : \text{current to point } P.$$

$I d\vec{l}$: element of current

$\cos\theta = \frac{a}{r}$



$$|d\vec{l}\times\hat{r}| = dl\cdot 1\cdot\sin 90° = dl$$

$$\underset{\text{Total (top \& bottom elements of current)}}{d\vec{B}} = 2dB\cos\theta\,\hat{i} = 2\frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\underset{\downarrow \frac{a}{r}}{\cos\theta}\,\hat{i} = \frac{2\mu_0}{4\pi}\frac{I\,dl\,a}{r^3}\hat{i}$$

$$\vec{B}_{\text{at } P} = \int_{\text{Half loop}} d\vec{B}_{\text{Total}} = \frac{2\mu_0}{4\pi}\frac{Ia}{r^3}\hat{i}\underbrace{\int_{\text{Half loop}} dl}_{\pi a} = \frac{\cancel{2}\mu_0 I a^2 \cancel{\pi}}{\cancel{4}_{2}\cancel{\pi}\, r^3}\hat{i}$$

$$\boxed{\vec{B} = \frac{\mu_0 I a^2}{2r^3}\hat{i} = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}\hat{i}}$$

Magnetic field at P along the axis of symmetry due to a circular loop of current I of radius a (P is at separation x from center of loop)

Notes: 1) Very far away from loop: x >> a $\quad (x^2+a^2)^{3/2} \simeq x^3$

$$\rightarrow \vec{B} \simeq \frac{\mu_0 I a^2}{2x^3}\hat{i} \quad \text{(inverse-cube law!)}$$

2) $\vec{E}$ due to a dipole @ x >> d (d = separation b/w the two charges of dipole) was also inverse-cube law.
→ loop of current is the "magnetic dipole"

Calculation of Fields

Electric

1) Vector superposition (direct method)

2) Gauss Law

$$\underbrace{\oint_{\text{Gaussian surface}} \vec{E}\cdot d\vec{A}}_{\phi \text{ (electric flux)}} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Use highly symmetric Gaussian surfaces: $E_\perp A = \phi$

3) Using electric potential V

$$\vec{E} = -\underset{\substack{\downarrow \\ \text{derivative} \\ \text{vector or} \\ \text{gradient}}}{\vec{\nabla}} V$$

Magnetic

1) Vector superposition using Biot-Savart Law

2) Ampere's Law:

$$\oint_{\text{Amperian loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

(line integral)

Highly symmetric Amperian Loop: B.l

3) Using the vector potential $\vec{A}$

$$\vec{B} = \underbrace{\vec{\nabla}\times\vec{A}}_{\text{rotational or curl of } \vec{A}}$$

(derivative vector & a cross product)

Calculation of the magnetic field using Ampere's Law:

1) Determine the Amperian loop (closed loop) taking advantage of the symmetry in the problem:

$$\oint \vec{B}\cdot d\vec{l} = \vec{B}\cdot \oint d\vec{l} = B\cdot L$$

↓ $\vec{B}$ constant along loop; $\vec{B}$ tangential at all points along loop.

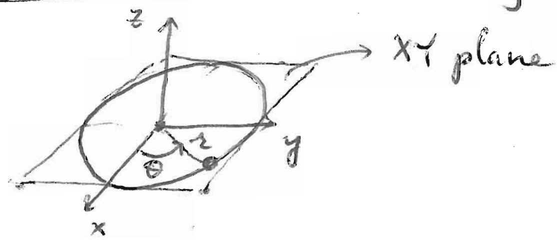
↓ Length of Loop.

2) Find the current enclosed by Amperian loop.

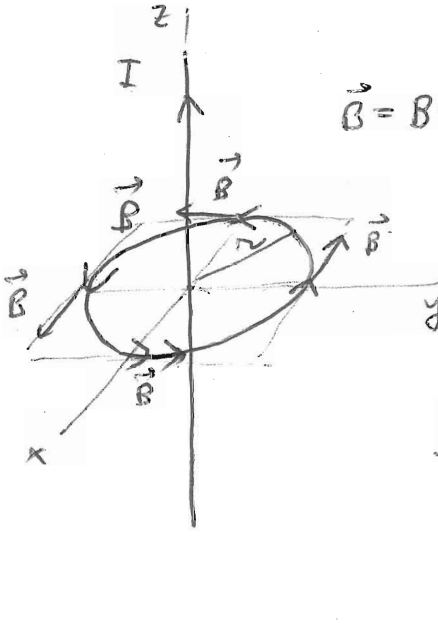
3) Apply Ampere's Law: $BL = \mu_0 I_{enclosed}$

$$B = \frac{\mu_0 I_{enclosed}}{L}$$

Magnetic field due to a long line of current:



Current I along z, $\vec{B}$ wraps around I as given by RHR. (CCW as seen from above). Thumb in direction of current I, the RH fingers show direction of $\vec{B}$ wrapping around it..



$\vec{B} = B\hat{\theta}$

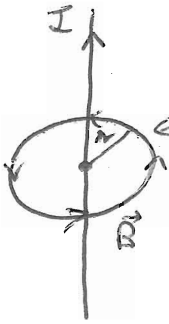
↳ unit vector in direction of angle θ, which is the direction of $\vec{B}$

Note: 1) $\vec{B}$ is tangential at all points along a circle in the XY plane centered at the current I

2) $\vec{B}$ has same strength at all points along this circle.

Application of Ampere's law:

1) Amperian loop: circle of radius r centered at the current.



Amperian loop: if we want to find B at separation r from the current: imagine an Amperian loop of radius r.

2) Current enclosed by the Amperian loop: $I_{enclosed} = I$

3) $\underset{\underset{2\pi r}{\downarrow}}{B.L} = \mu_0 I \rightarrow B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$

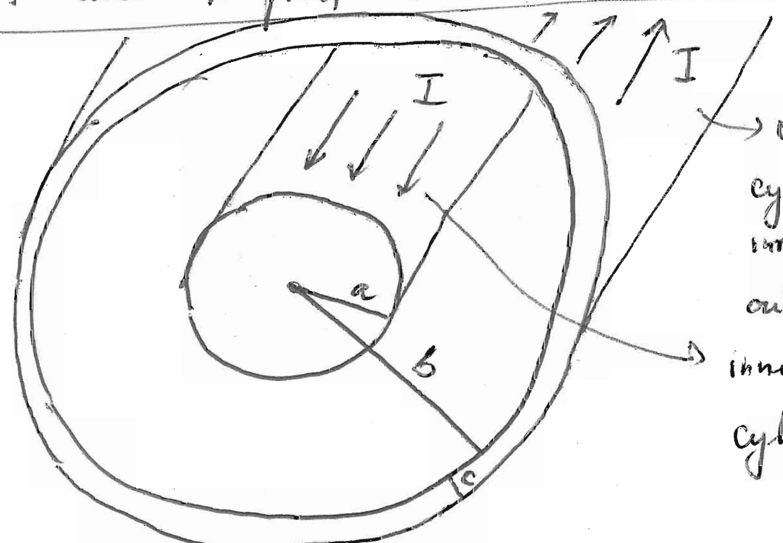
26.68 Use Ampere's law to find $\vec{B}$ in a coaxial cable:

Find $\vec{B}$:

a) $r < a$

b) $a < r < b$

c) $r > (b+c)$



outer conductor: cylindrical shell inner radius b, outer radius b+c

inner conductor: cylinder of radius a

Cylindrical symmetry → field $\vec{B}$ wraps around the current.

a)

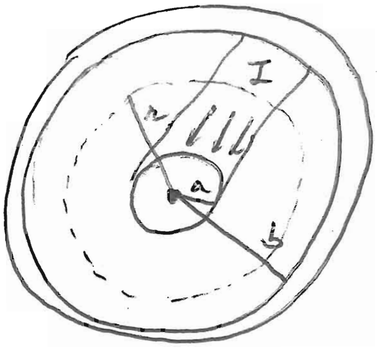
1) Amperian loop of radius r centered @ axis of cable. → length $= 2\pi r$

2) $I_{enclosed} = I \frac{\cancel{\pi} r^2}{\cancel{\pi} a^2}$

3) $B \cdot 2\pi \cancel{r} = \mu_0 I \frac{r^{\cancel{2}}}{a^2} \rightarrow \boxed{B = \frac{\mu_0 r}{2\pi a^2} I}$

within inner conductor.

b)



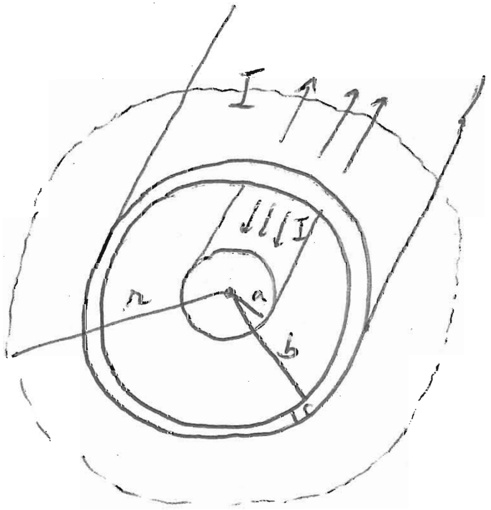
1) Amperian loop is a circle of radius r (a<r<b), centered @ axis of cable → length = 2πr.

2) $I_{enclosed} = I$

3) $B \cdot 2\pi r = \mu_0 \cdot I \rightarrow B = \frac{\mu_0}{2\pi} \frac{I}{r}$

$a < r < b$
b/w inner & outer conductor.

c)



1) Amperian loop is a circle of radius r   r > b+c (outside coaxial cable), centered @ axis of cable, length is 2πr

2) $I_{enclosed} = I - I = 0$!

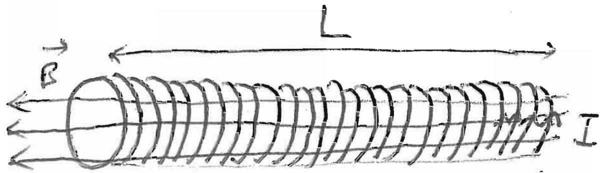
3) $B \cdot 2\pi r = \mu_0 \cdot 0 \rightarrow \boxed{B = 0}$!

outer conductor is the shield.

One wire wrapping around cylinder many turns?

(26.44) Superconducting solenoid (higher B)

$n = 3300$ (turns per unit length) : $n = \frac{N}{L}$



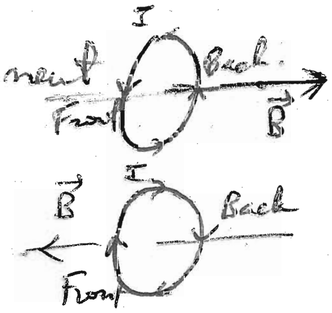
N turns → $n = \frac{N}{L}$

Each turn carries current I

$I = 4100\,A$ (large!, superconducting wire)

Find B in solenoid?

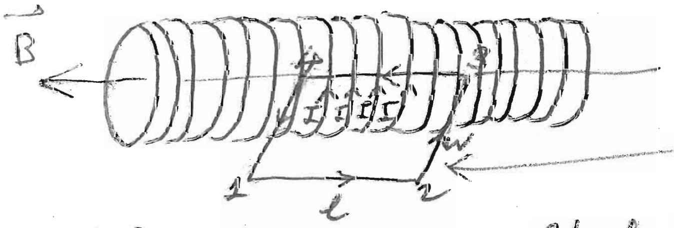
Recall: magnetic due to one circular loop of current



Note: { B is mostly inside solenoid
negligible outside

Apply Ampere's Law:

1) Amperian loop: { $\vec{B}$ tangential to loop
$\vec{B}$ uniform along loop



rectangular amperian loop 1234 of length l, width w
{ 12 is outside solenoid
34 is parallel to axis ~~axis~~ of solenoid and is inside solenoid.

$$\oint \vec{B}\cdot d\vec{l} = \underbrace{B\cdot l}_{34} + \underbrace{0\cdot l}_{12} + \underbrace{\overset{B \perp \text{side}}{0}}_{14} + \underbrace{\overset{B \perp \text{side}}{0}}_{23}$$

2) current enclosed by Amperian loop:

$$I_{enclosed} = N_l\, I = l \cdot \frac{N}{L} I = \underbrace{l \cdot n}\, I$$

turns enclosed by Amperian loop.

3) $B \cdot \cancel{l} = \mu_0 \cancel{l} \cdot n \cdot I \rightarrow \boxed{B = \mu_0 n I}$

$B = 4\pi \times 10^{-7} \times 3300 \times 4100 = 17\,T$

(26.50) $\begin{cases} B = 2T \\ \text{Cyclotron to accelerate deuterium nuclei } (1p + 1n) \end{cases}$

a) Frequency of alternating voltage ($\vec{E}$) in the gap b/w the two D-chambers? → This has to synchronize with the cyclotron frequency of the deuterons:

$f = \frac{1}{T}$, T: orbital period for deuterons

$$T = \frac{2\pi r}{v} = \frac{2\pi \cancel{r}}{\frac{qB\cancel{r}}{m}} = \frac{2\pi m}{qB}$$

2nd Newton's Law & $a_r = \frac{v^2}{r}$ $\Big\}$ $q\cancel{v}B = m\frac{v^{\cancel{2}}}{r}$

↓

Magnetic force → a_r : $qvB = ma_r$ $\Big|$ $v = \frac{qBr}{m}$

$$f = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times 2}{2\pi\times 2\times 1.67\times10^{-27}} = 15.2\ \underset{\substack{\downarrow\\10^{6}}}{\mathrm{M}}\underset{\substack{\downarrow\\s^{-1}}}{\mathrm{Hz}}$$

Proton charge: $q = 1.6\times10^{-19}$ C

Deuteron mass = $2m_p$

b) diameter for D chamber of 0.9m → $KE_{max} = \frac{(qBR)^2}{2m_d}$

$$= \frac{(1.6\times10^{-19}\times 2\times 0.45)^2}{2\times 2\times 1.67\times10^{-27}}\ \text{J}$$

c) ΔV in the gap is 1500V → how many orbits ~~deutons~~ deuterons complete?

Each orbit: 2 pushes of $q\Delta V = 1.6\times10^{-19}\times 1500\ \text{J}\times 2$

→ # orbits = $\frac{KE_{max}}{q\Delta V}$ =

26.76 → long hollow pipe with current I around the pipe

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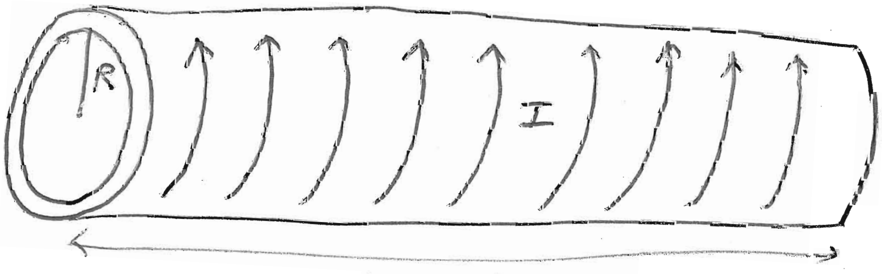
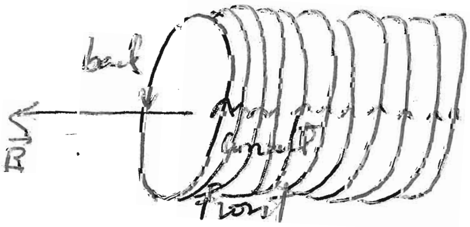
B? $\begin{cases} r<R \\ r>R \end{cases}$



Length of pipe L

→ Help: this pipe could be seen as a stack of many circular loops of current



This problem

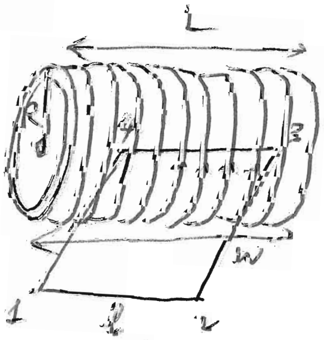
RHR $\begin{cases} 1) \text{ Current} \leftrightarrow \text{fingers then } \vec{B} \leftrightarrow \text{thumb} \\ 2) \text{ Current} \leftrightarrow \text{thumb then } \vec{B} \leftrightarrow \text{fingers.} \end{cases}$

a) → Apply Ampere's Law to find $\vec{B}$:

1) Amperian Loop: $\begin{cases} \vec{B}: \text{tangential to all or part of loop.} \\ \vec{B}: \text{uniform along all or part of loop.} \end{cases}$

↓

rectangular

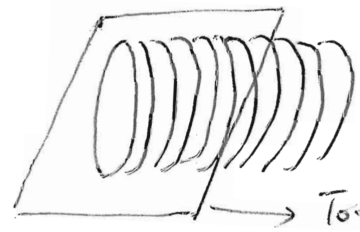


$\oint \vec{B} \cdot \vec{dl} = \underset{34}{B \cdot l} + \underset{\substack{12 \\ \downarrow \\ B=0 \\ \text{outside} \\ \text{pipe}}}{0} + \underbrace{\overset{23}{0} + \overset{14}{0}}_{B \perp l}$

2) $I_{enclosed} = I \cdot \frac{l}{L}$

3) $B \cdot \cancel{l} = \mu_0 I \frac{\cancel{l}}{L} \rightarrow \boxed{B = \mu_0 \frac{I}{L}}$ inside hollow pipe.

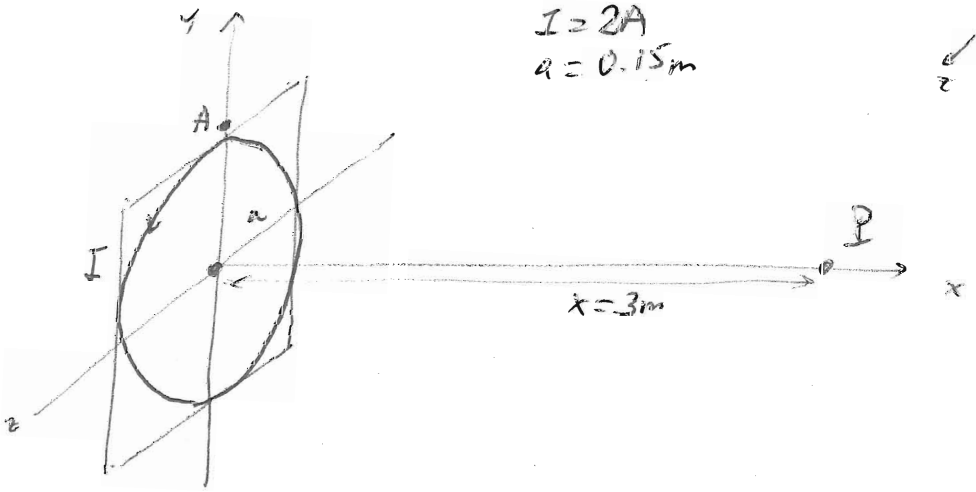
b) Outside hollow pipe:



→ Total current enclosed is 0 (up then down)

$B\cdot l = 0 \rightarrow B = 0$

(26.74)



I = 2A
a = 0.15m

1) Find $\vec{B}$ @ A 1mm outside loop in the loop plane:

$\downarrow$
$\frac{1}{300}$ of the loop diameter → loop curvature is negligible (as we don't notice the Earth's curvature) → Approx.: field due to long straight wire @ A.

$$B = \frac{\mu_0 I}{2\pi y} = 2\times10^{-7} \times 2 \times \frac{1}{10^{-3}}\ T = 4\times10^{-4}\ T = \underset{\substack{\downarrow\\ \text{Gauss.}}}{4G}$$

2) $\vec{B}$ along axis of loop @ 3m from its center

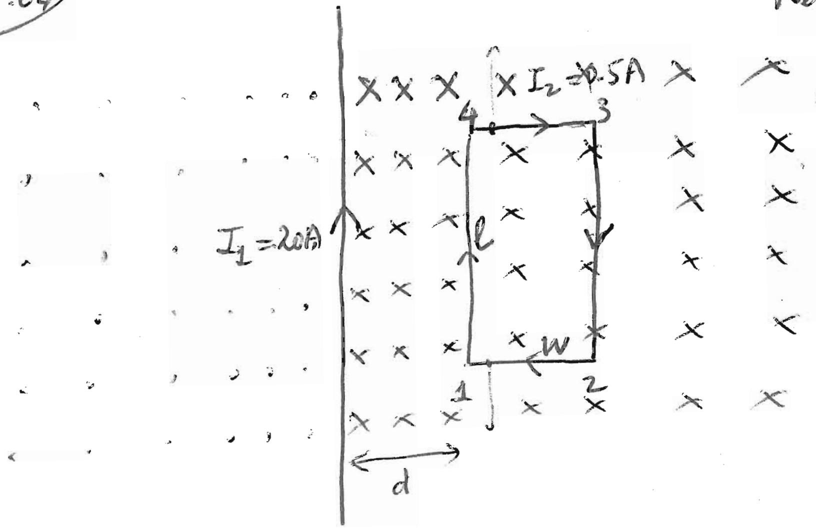
$$\vec{B} = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}\hat{i} \underset{\substack{\downarrow\\ x>>a\\ 3m >> 0.15m}}{\simeq} \underset{\substack{\downarrow\\ \text{dipole}\\ \text{behavior}}}{\frac{\mu_0 I a^2}{2x^3}\hat{i}} = \frac{4\pi\times10^{-7}\times2\times0.15^2}{2\cdot3^3}\hat{i}$$

$$= 1.05\times10^{-9}\,T$$

$$= 1.05\times10^{-5}\,G$$

21.64

Net $F_{magnetic}$ on loop ?



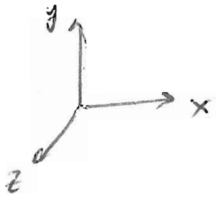
1) I_1 creates a B_1 around it

(RHR: into page right of I_1; out of page left of I_1)

Strength: stronger closer to I_1

in : ×

out : •



2) B_1 applies a magnetic force on any test current I_2 : $\vec{F} = I_2 \underbrace{\vec{l} \times \vec{B}_1}$

{ $l B_1 \sin\theta$
direction by RHR

$\vec{F}_{14} = I_2 l B_1 (-\hat{i})$ @d

$\vec{F}_{23} = I_2 l B_1 (\hat{i})$ @ d+w

$B_1 = \frac{\mu_0 I_1}{2\pi r}$

$$\vec{F}_{net} = \vec{F}_{14} + \vec{F}_{23} = I_2 l \left[-\frac{\mu_0 I_1}{2\pi d} + \frac{\mu_0 I_1}{2\pi (d+w)} \right] \hat{i}$$

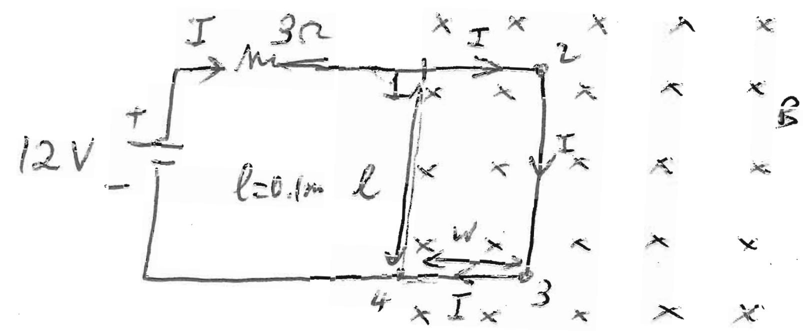
$$= \frac{I_2 l \mu_0 I_1}{2\pi} \left[-\frac{1}{d} + \frac{1}{d+w} \right] \hat{i} = -7.14 \times 10^{-6} N \hat{i}$$

(toward long current I_1)

$\vec{F}_{43}$ & $\vec{F}_{21}$ } cancel by pairs ↳ No net force in the y-direction.

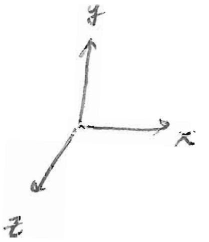
$I_2 w B_1(r) \hat{j}$ $I_2 w B_1(r)(-\hat{j})$

26.52



B uniform & into page

$\vec{B} = B\hat{k}$



Current I will feel a magnetic fr by $\vec{B}$:

$\vec{F} = I\vec{l} \times \vec{B}$

$1 \to 2$: $\vec{l} = w\,\hat{i} \to \vec{F}_{12} = IwB\,\hat{j}$

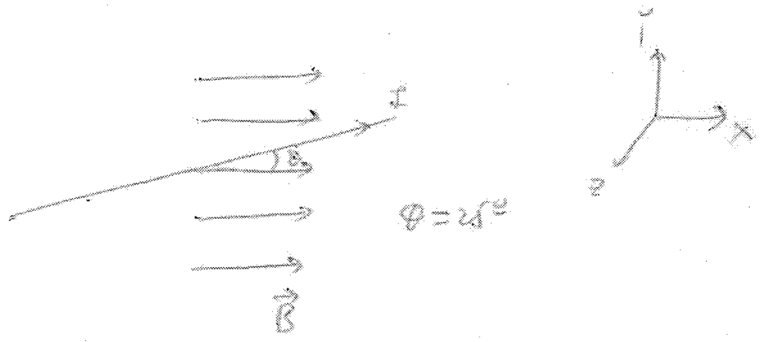
$3 \to 4$: $\vec{l} = w(-\hat{i}) \to \vec{F}_{34} = IwB\,(-\hat{j})$ } cance

$2 \to 3$: $\vec{l} = l(-\hat{j}) \to \vec{F}_{23} = IlB\,\hat{i}$ ← net force on loop or circuit.

$\vec{F}_{net} = IlB\,\hat{i} = \frac{12}{3} \times 0.1 \times 38 \times 10^{-3}\,\hat{i} = 15.2 \text{ mN } \hat{i}$

(toward the right)

(26.29) Wire w/ $I = 15A$ @ 25° with a uniform magnetic field. $\frac{F_m}{L} = 0.31 \frac{N}{m}$ B?



a) → Current consists of moving charges : → B will apply a force on each charge → force on current or wire.

$$\boxed{\vec{F} = q\vec{v} \times \vec{B} = q \underbrace{\frac{\vec{l}}{t}}_{\vec{v}} \times \vec{B} = \underbrace{\frac{q}{t}}_{I} \vec{l} \times \vec{B} = I\vec{l} \times \vec{B}}$$

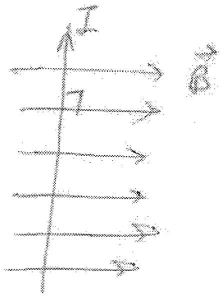
$F = I l B \sin\theta$ (θ angle b/w wire & field)

$\frac{F}{l} = I B \sin\theta \rightarrow B = \frac{\frac{F}{l}}{I \sin\theta} = \frac{0.31}{15 \times \sin 25^{\circ}}$

$B = 48.9\, mT$

b) Max $\frac{F}{l}$ when $\sin\theta = 1$ or $\theta = 90^{\circ}$

$\frac{F}{l} = I B = 15 \times 48.9 \times 10^{-3} = 0.734 \frac{N}{m}$



26.47 Data: $v = 185\,m/s$; $q = 1.4\,\mu C$

$\vec{F}_B = (2.5\,\hat{i} + 7\,\hat{j})\,\mu N$

$\vec{B} = (42\,\hat{i} - 15\,\hat{j})\,mT$

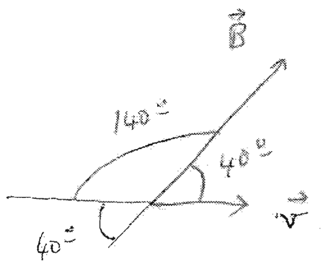
Find θ b/w $\vec{v}$ & $\vec{B}$

$\vec{F} = q\vec{v} \times \vec{B}$ → Magnitude :

$$\begin{cases} F = qvB\sin\theta \\ \sin\theta = \dfrac{F}{qvB} \\ F = |\vec{F}| = \sqrt{F_x^2 + F_y^2} \\ \quad = \sqrt{2.5^2 + 7^2}\ 10^{-6}\,N \\ B = |\vec{B}| = \sqrt{42^2 + 15^2}\ 10^{-3}\,T \end{cases}$$

$$\sin\theta = \frac{\sqrt{2.5^2 + 7^2}\ 10^{-6}}{1.4\times10^{-6} \times 185 \times \sqrt{42^2 + 15^2}\ 10^{-3}} = 0.644 \rightarrow \theta = \sin^{-1} 0.644$$

$$\boxed{\theta = 40.1^\circ}$$



Ch 27 Electromagnetic Induction

Faraday's Law: $\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$

$\vec{B}(t)$ { B const but direction changes

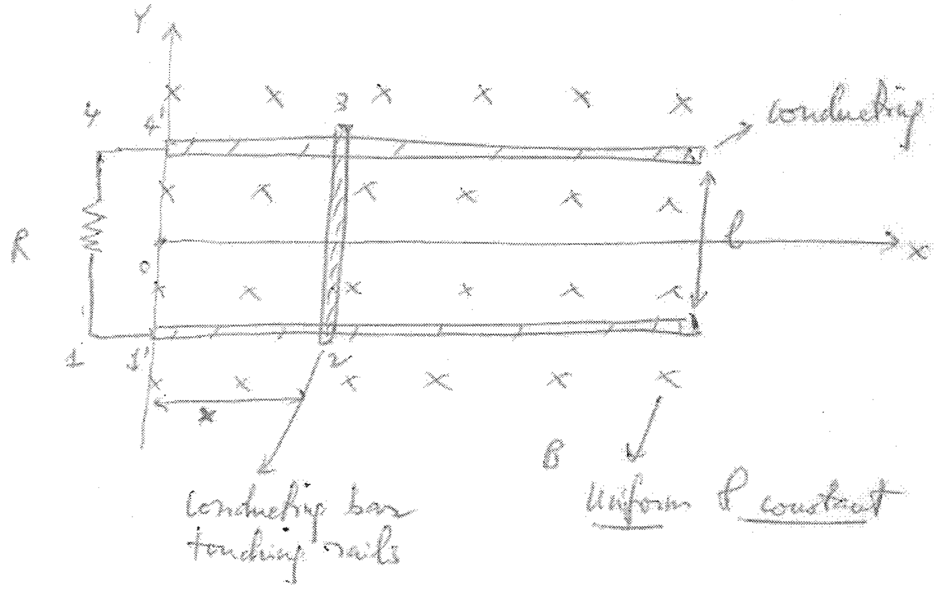
{ $\vec{B}$ change with time

{ A change with time

$\phi_B = \int \vec{B}\cdot d\vec{A}$ = magnetic flux → change

$\mathcal{E}$ = induced e.m.f or induced voltage.

(27.47)



Closed loop 1234 : with a magnetic flux Φ_B through

$$\phi_B = \oint \vec{B}\cdot d\vec{A} = \underbrace{BA}_{\text{area of loop with magnetic field} \rightarrow 1'234'} = B(xl)$$

B & l are constant, but if we move the conducting bar 23 or changing x → then $\frac{d\phi_B}{dt} \neq 0$ → there is an induced $\mathcal{E}$ in the loop (acts like a battery) → a current $I = \frac{\mathcal{E}}{R}$ will show up in the loop.

a) Direction of current in resistor ?

$$\mathcal{E} = \underset{\uparrow}{\bigcirc\!\!\!\!-}\ \frac{d\phi_B}{dt}$$

the induced $\mathcal{E}$ will oppose the change in ϕ_B

1) $\phi_B \uparrow \rightarrow \mathcal{E}$ will be such that it reduce ϕ_B

2) $\phi_B \downarrow \rightarrow \mathcal{E}$ will be such that it increase ϕ_B

If conducting bar 23 goes right $\rightarrow \phi_B \uparrow \rightarrow \mathcal{E}$ will tend to reduce ϕ_B by creating a current in the loop that produces a induced magnetic field out of page to reduce the original field and so to reduce the ϕ_B despite an increase in A due to the conducting bar moving to the right.

$\rightarrow$ I induced will go $4 \rightarrow 1$ across the resistor (downward).

b) What power (work per unit time) is need to pull the bar 23 ?

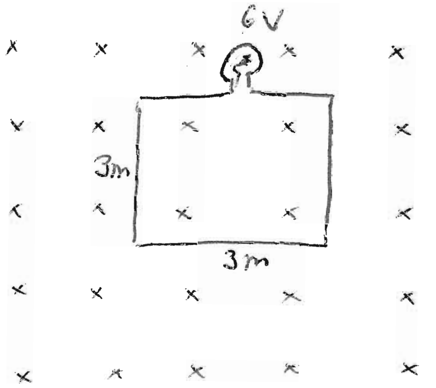
$$P = \underset{\substack{\downarrow \\ \text{induced} \\ \text{current}}}{I.V} \underset{\substack{\downarrow \\ \text{Ohm's Law}}}{=} I^2 R = \left(\frac{\mathcal{E}}{R}\right)^2 R = \frac{\mathcal{E}^2}{R} = \frac{\left(\frac{d\phi_B}{dt}\right)^2}{R}$$

$$\phi_B = B x l \rightarrow \frac{d\phi_B}{dt} = Bl\frac{dx}{dt} = Bl\underset{\substack{\downarrow \\ \text{speed of bar 23}}}{v}$$

$$\boxed{P = \frac{(Blv)^2}{R}}$$

27.40

Electromagnetic voltage induced in a square loop when magnetic flux changes in time due to a changing magnetic field



$\vec{B}(t)$ { uniform magnetic field that reduces: 2T → 0T over time Δt

a) Find Δt for full brightness

b) Direction of induced current

Faraday's Law: $\mathcal{E}_{induced} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B.A)$:

"A changing magnetic flux induces a voltage $\mathcal{E}_{induced}$ in a loop"

a) In this problem $A = 9m^2$ is fixed but B is decreasing 2T→0T in Δt

So $\mathcal{E}_{induced} = -A\frac{\Delta B}{\Delta t}$. Full brightness if Δt is such that $\mathcal{E}_{induced} = 6V$ needed for light bulb:

$$\Delta t = \frac{-A \cdot \Delta B}{\mathcal{E}_{induced}} = \frac{-9\times(0-2)}{6} = 3s$$

b) Induced voltage aims at neutralizing the decrease of magnetic flux into the page. So induced current will be in CW to create a magnetic field into page to compensate for the decrease of the original field into page.