# Ch 23 Electrostatic Energy & Capacitors

Like water damp allows storage of gravitational potential energy, capacitors allows storage of electrostatic potential energy:

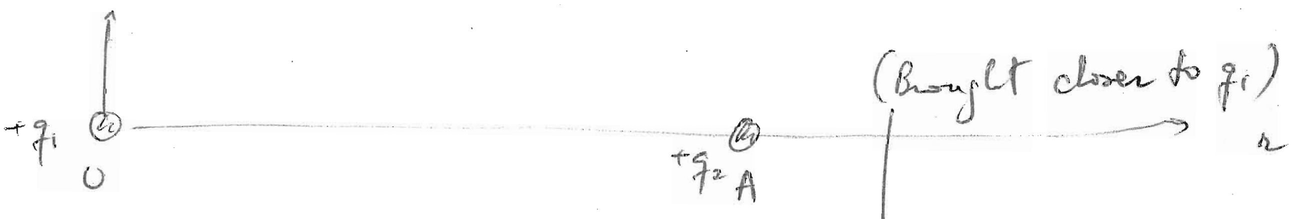
Bringing water up

- Bringing similar charges together
- Separating opposite types of charges.

$$\Delta U_{AB} = -W_{AB} = -\int_A^B \underbrace{\vec{F}}_{\text{force applied}} \overset{\text{scalar or dot product}}{\cdot} \underbrace{d\vec{l}}_{\text{infinitesimal displacement}}$$

Gravitational

Electrostatic: $\frac{kq_1q_2}{r^2}$


(Brought closer to $q_1$)

$$\boxed{\Delta U_{\infty A} = q_2 \Delta V_{\infty A} = q_2 k q_1 \left(\frac{1}{r} - \underbrace{\frac{1}{\infty}}_{0}\right) = \underbrace{\boxed{\frac{kq_1q_2}{r}}}_{\text{energy stored.}}}$$

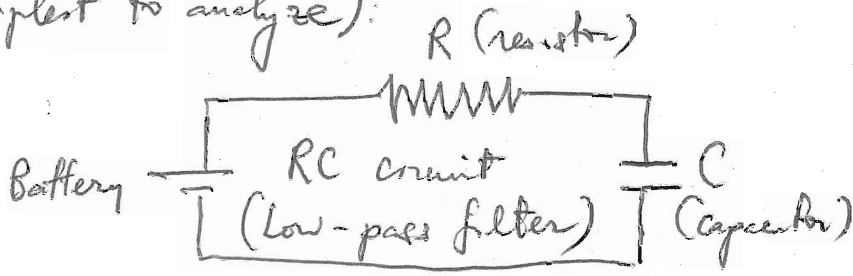
$\left(V = \frac{kq_1}{r}\right)$

Bigger $\begin{cases} r \text{ smaller} \\ q_2 \text{ bigger} \end{cases}$

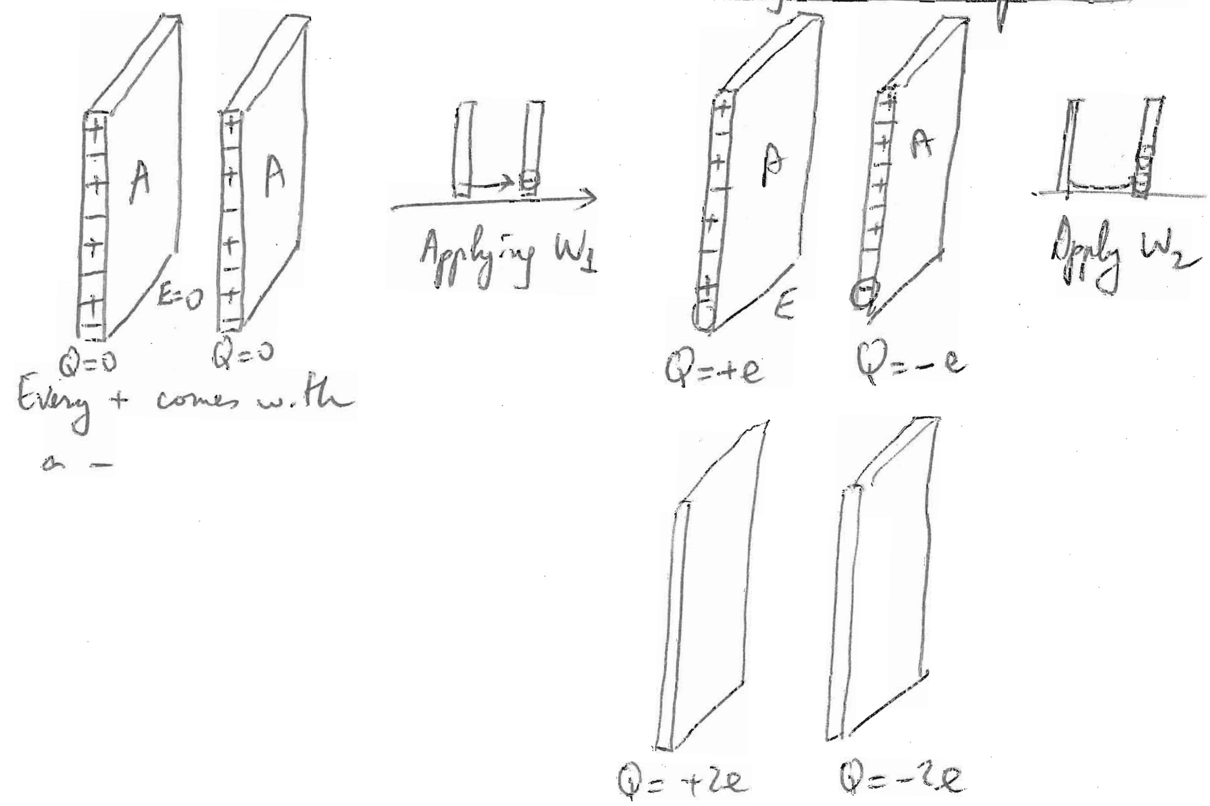
# Capacitors : electrostatic energy storage device

Parallel Plates (simplest to analyze):

↳ Symbol: ||




RC Circuit is used to charge the capacitor:

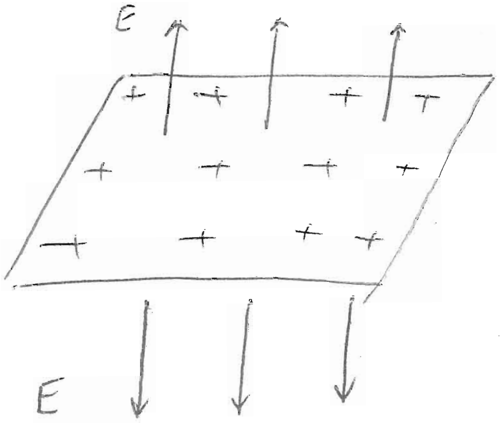



Every + comes with a −

$\boxed{W_2 > W_1}$ to move the $2^{nd}$ $e^-$ : we work against the field created by the $1^{st}$ $e^-$!

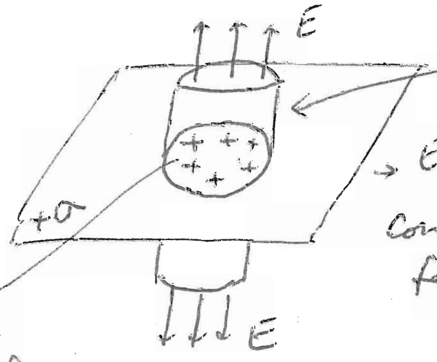
It is harder to move subsequent charge! But you store more energy!

## Electric field by a plate of charge:




Electric field points perpendicular & away from plate (+ charged)

Gaussian surface $\begin{cases} \text{cylindrical} \checkmark \\ \text{rectangular.} \end{cases}$




→ Electric flux: with contribution from both top & bottom faces.

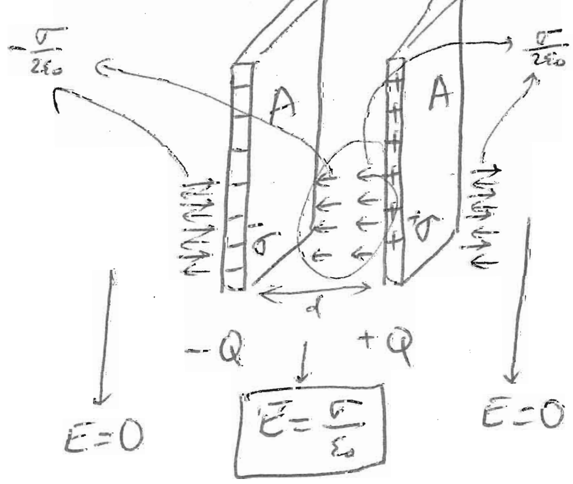
$2EA$

→ Charge enclosed = $\sigma A$

Charge enclosed by cylindrical Gaussian surface: in a cross section $A$

Gauss Law: → $2EA = \dfrac{\sigma A}{\varepsilon_0}$ → $\boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

## Electric field b/w 2 plates of a capacitor:




d: separation b/w plates.

$$\sigma = \frac{Q}{A}$$

Capacitance: $C \equiv \dfrac{Q}{V}$ (charge each plate can hold over the electric potential b/w plate)

## Capacitance of a Parallel Plate Capacitor:

$$\left[ C = \frac{Q}{E\cdot d} = \frac{Q}{\frac{\sigma}{\varepsilon_0}\cdot d} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0}\cdot d} = \frac{A}{d}\varepsilon_0 \right]$$

→ Unit: F for Farad } $\varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{Nm^2}$

→ $C\uparrow$ { larger surface A; smaller separation b/w plates d

→ Importance: total energy stored in a capacitor depends on C.

Total energy stored in a fully charged parallel-plates capacitor

$$dU = -dW = -dq\,V \overset{\text{parallel plate}}{=} +dq\,Ed = +dq\,\frac{\sigma}{\varepsilon_0}d \qquad (V=-Ed) \leftrightarrow \boxed{V=-\int \vec{E}\cdot d\vec{l}}$$

$$= +dq\,\frac{q}{A\varepsilon_0}d$$

$$dU = q\,dq\,\frac{d}{A\varepsilon_0} \rightarrow U = \int dU = \frac{d}{A\varepsilon_0}\underbrace{\int_0^{Q} q\,dq}_{\frac{Q^2}{2}}$$

$Q$ → Fully charged

$$\left[ U = \frac{1}{2}\underbrace{\frac{d}{A\varepsilon_0}}_{\frac{1}{C}}Q^2 = \boxed{\frac{1}{2}\frac{Q^2}{C}} \right] \text{ or } = \frac{1}{2}C\left(\frac{Q}{C}\right)^2 = \frac{1}{2}CV^2 \Big]$$

if we use $C \equiv \frac{Q}{V} \rightarrow \boxed{V = \frac{Q}{C}}$

Alternative expression for U

$$U = \frac{1}{2}\frac{d}{A\varepsilon_0}Q^2 = \frac{1}{2}\frac{(A\varepsilon_0)\,d}{(A\varepsilon_0)^2}Q^2 = \frac{1}{2}A\varepsilon_0 d\underbrace{\boxed{\frac{\sigma}{\varepsilon_0}}^2}_{E^2}$$

$$= \frac{1}{2}\varepsilon_0 E^2 (A\cdot d)$$

$(A\cdot d)$ → volume b/w plates. $A$: surface of a plate; $d$ → sep. b/w plates.

where energy is stored! → volume V

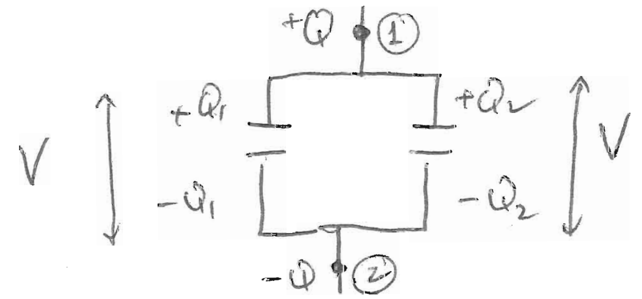
$$\rightarrow \frac{U}{Vol} = \frac{1}{2}\varepsilon_0 E^2$$

Electrostatic (Energy stored per unit volume is $\underbrace{\frac{1}{2}\varepsilon_0 E^2}$ )

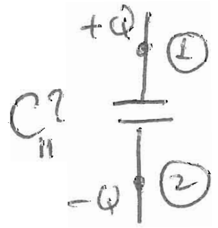
( $\rightarrow \frac{1}{2}mv^2$ ; $\rightarrow \frac{1}{2}I\omega^2$ ) kinetic energies!

## Two different ways to connect two capacitors together

### Parallel Connection




$$\begin{cases} \rightarrow Q = Q_1 + Q_2 \\ V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = V \end{cases}$$

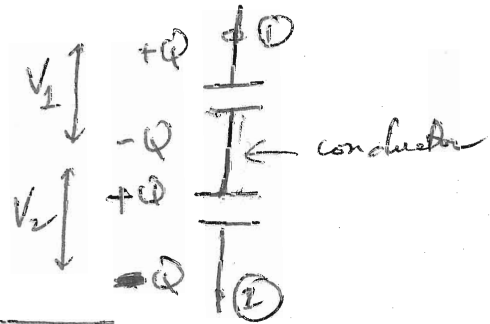



$$C_\parallel = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$$

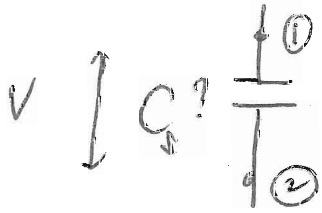
$$\boxed{C_\parallel = C_1 + C_2}$$

One way to increase capacitance!

### Series Connection




$$\boxed{V = V_1 + V_2}$$

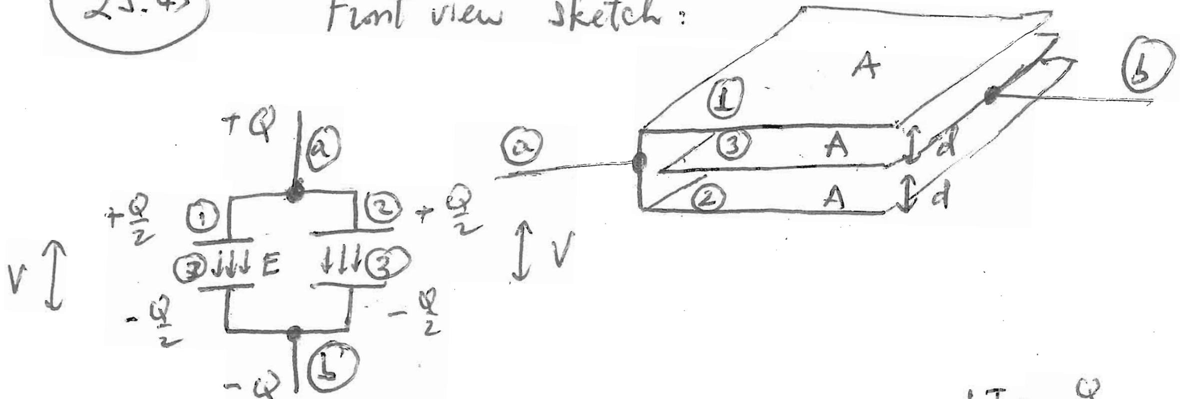



$$C_s = \frac{Q}{V} = \frac{Q}{V_1 + V_2}$$

$$= \frac{\cancel{Q}}{\frac{\cancel{Q}}{C_1} + \frac{\cancel{Q}}{C_2}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}} \text{ or } \boxed{C_s = \frac{C_1 C_2}{C_1 + C_2}}$$

23.43 Front view sketch:




- Top ① & bottom ② plates are connected.
- Middle plate ③ at equal separation from top & bottom. ③ is not touching ① & ②

$$C = \frac{Q}{V} \qquad \begin{cases} \text{Top \& bottom} : +Q \begin{cases} \text{Top} : \frac{Q}{2} \\ \text{Bottom} : \frac{Q}{2} \end{cases} \\ \text{Middle} : -Q \\ E = \frac{\sigma}{\varepsilon_0} = \frac{\frac{Q}{2A}}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0} \end{cases}$$

$$V = E \cdot d = \frac{Q}{2A\varepsilon_0} \cdot d$$

$$C_{Total} = \frac{\cancel{Q}}{\frac{\cancel{Q} \cdot d}{2A\varepsilon_0}} = \frac{2A\varepsilon_0}{d} \quad ✓$$

- $\sigma$: surface charge density: $\frac{\text{charge}}{\text{Area}}$
- $d$: surface – surface separation (not middle of plate to middle of plate)

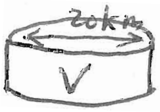
Observation:


23.68

Example 23.4: $E = 10^5 \frac{V}{m}$ (thunder storms) $\rightarrow u = \frac{1}{2}\varepsilon_0 E^2$

$u = \frac{1}{2} 8.85 \times 10^{-12} \times (10^5)^2 = 4.4 \times 10^{-2} \frac{J}{m^3}$ (energy density)

In a cylindrical cloud:




10km $V = \pi \times (10^4)^2 \times 10^4 = \pi 10^{12} m^3$

$\rightarrow$ Total electric energy in cloud = $U = u \cdot V = 4.4 \times 10^{-2} \times \pi \times 10^{12}$ J

$$\boxed{U = 140 GJ}$$

Lightning flashes $\begin{cases} \text{Every } 5s \\ Q = 30C \ \& \ V = 30MV \quad (\text{Mega or } M = 10^6) \end{cases}$

↳ With $U = 140 GJ$ how long will lightning last?
(cloud is not replenishing its electric energy)

Answering steps:

1) How much energy is transferred per lightning flash?

$U_{Flash}$ = energy to move a test charge $q$ across a potential $V$

$U_{Flash} = q \cdot V = 30 \cdot 30 \cdot 10^6 = 9 \times 10^8\ J$ (per flash)
$= 0.9 \times 10^9\ J = 0.9\ GJ$

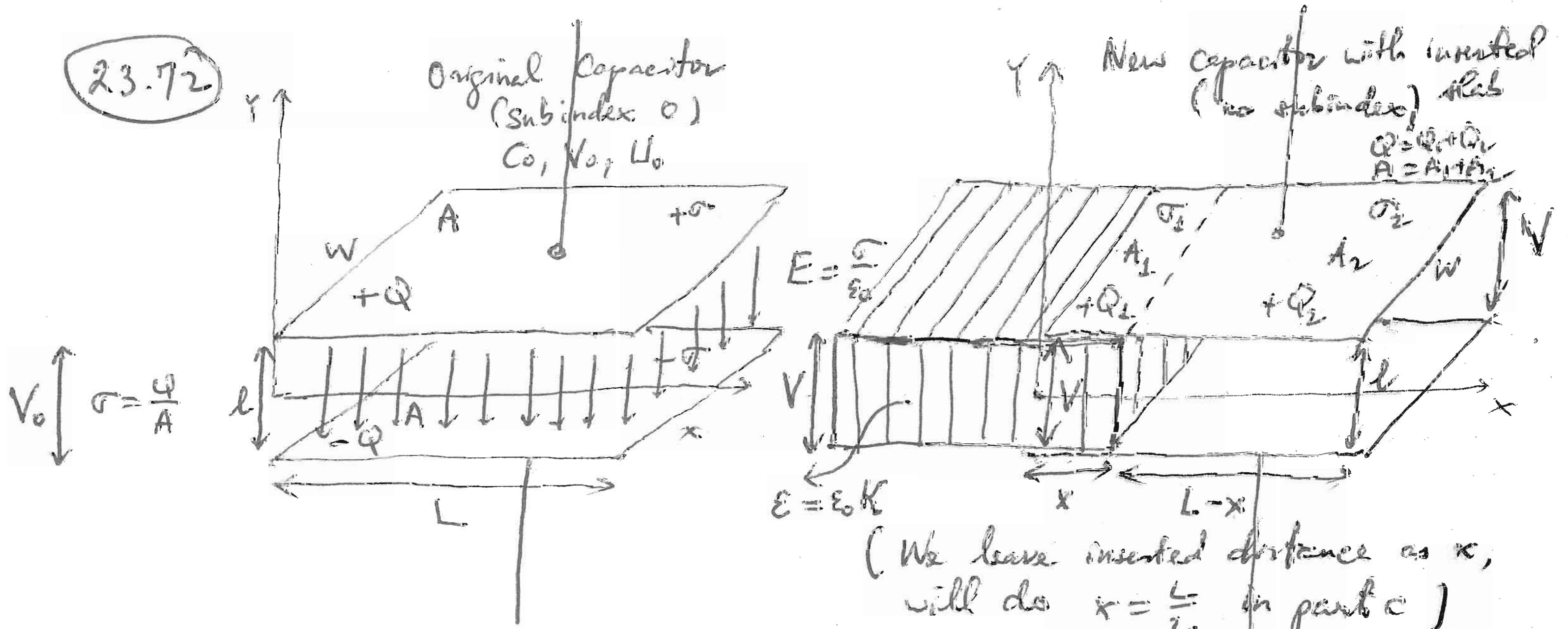
2) How many flashes can our cloud release? $N$

$$N = \frac{\text{Total energy}}{\text{Energy per flash}} = \frac{U}{U_{Flash}} = \frac{140\ GJ}{0.9\ \frac{GJ}{Flash}} \approx 156\ \text{Flashes}$$

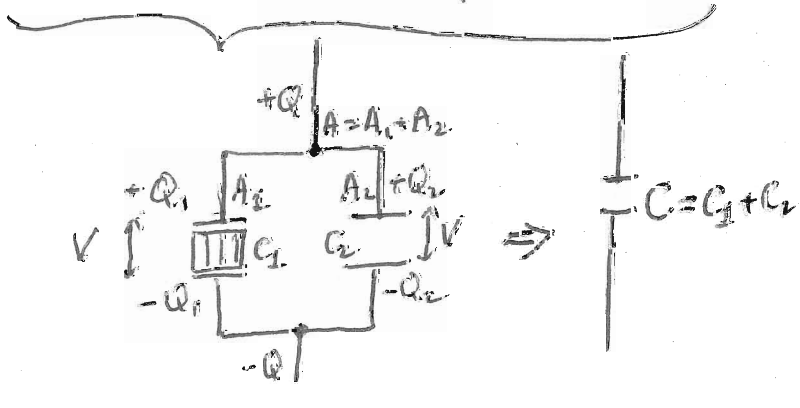
3) How long will lightning last?

$$t = N \cdot \Delta t_{Flash} = 156 \cdot 5\,s \times \frac{1\,min}{60\,s} = 13\ min.$$

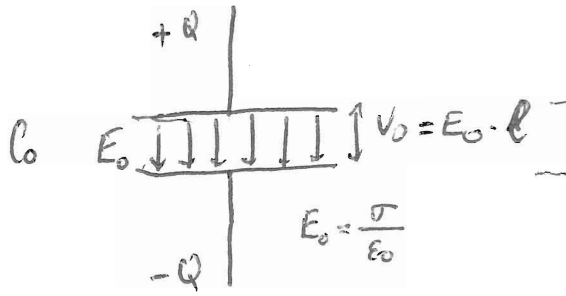
23.72



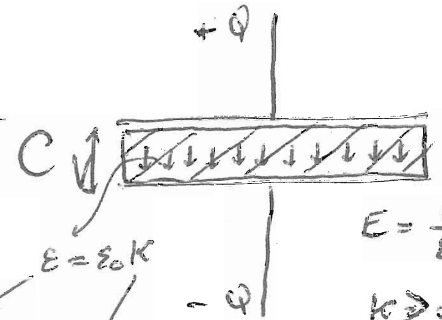

(We leave inserted distance as $x$, will do $x = \frac{L}{2}$ in part c)



# Isolated capacitors: (charges are not changing over time)




$E_0 = \frac{\sigma}{\varepsilon_0}$




$V = E \cdot d = \frac{V_0}{K}$

$E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0 K}$

$K \geq 1$

$\varepsilon = \varepsilon_0 K$

Dielectric coefficient — dielectric constant

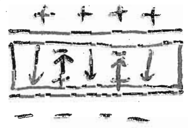
$C_0 = \frac{Q}{V_0}$

$C_0 = \frac{A\varepsilon_0}{l} = \frac{wL\varepsilon_0}{l}$

$C = \frac{Q}{V} = \frac{Q}{\frac{V_0}{K}} = KC_0$

Field is reduced by the dielectric constant $K$

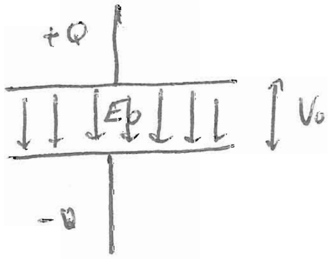
(How?:



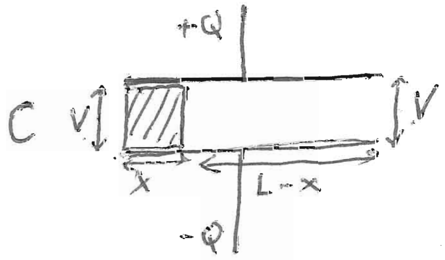
Some rearrangement (not as complete as in conductors) inside dielectric creates an opposing electric field)

In this problem: we need to calculate total new capacitance:

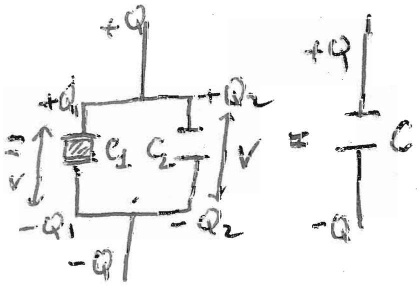
$C_0$




Original



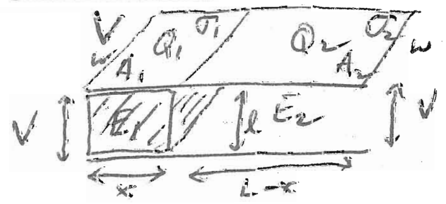

New




$C = C_1 + C_2$

Total new capacitance with dielectric slab inserted a distance x:

$$C = C_1 + C_2 = \frac{Q_1}{V} + \frac{Q_2}{V} = \frac{Q_1 + Q_2}{V}$$




Find $Q_1$ in term of $Q_2$:

$$E_1 = \frac{\sigma_1}{K\varepsilon_0} = \frac{\frac{Q_1}{A_1}}{K\varepsilon_0} = \frac{Q_1}{xwK\varepsilon_0}$$

$$E_2 = \frac{\sigma_2}{\varepsilon_0} = \frac{\frac{Q_2}{A_2}}{\varepsilon_0} = \frac{Q_2}{(L-x)w\,\varepsilon_0}$$

$\Big\}\rightarrow$ Since potential difference is V left & right $E_1 = E_2$, also there is no abrupt change in electric field in nature.

The electric fields in both regions are equal thanks to a proper distribution of charges $Q_1$ & $Q_2$

$$\frac{Q_1}{x w K \varepsilon_0} = \frac{Q_2}{(L-x) w \varepsilon_0} \rightarrow \quad Q_1 = Q_2 \frac{x \cancel{w} K \cancel{\varepsilon_0}}{(L-x)\cancel{w}\cancel{\varepsilon_0}}$$

$$\rightarrow \boxed{Q_1 = Q_2 \frac{xK}{L-x}}$$

$$C = \frac{Q_1 + Q_2}{V} = \frac{Q_2\left(\frac{xK}{L-x} + 1\right)}{V}$$

Write $\frac{Q_2}{V}$ in terms of known information: $L, x, w, l$:

$$\frac{Q_2}{V} = \frac{Q_2}{E_2 \cdot l} = \frac{Q_2}{\frac{\sigma_2}{\varepsilon_0} \cdot l} = \frac{\cancel{Q_2}}{\frac{\cancel{Q_2}}{A_2 \varepsilon_0} \cdot l} = \frac{A_2 \varepsilon_0}{l} = \frac{(L-x) w \cdot \varepsilon_0}{l}$$

(Region 2 has no dielectric insert)

$$\Rightarrow C(x) = \frac{(L-x) w \cdot \varepsilon_0}{l}\left(\frac{xK}{(L-x)} + 1\right) = \frac{w\varepsilon_0}{l}\left[xK + L - x\right]$$

$$\boxed{C(x) = \frac{w\varepsilon_0}{l}\Big[x\underbrace{(K-1)}_{\substack{K>1 \\ \text{positive}}} + L\Big]}$$

Total new capacitance with dielectric slab inserted a distance x in the original spacing.

a) $$\boxed{C\left(x=\frac{L}{2}\right) = \frac{w\varepsilon_0}{l}\left[\frac{L}{2}(K-1) + L\right] = \frac{w\varepsilon_0 L}{l}\Big[\underbrace{\frac{K-1}{2} + 1}_{\frac{K}{2}+\frac{1}{2}}\Big] = \frac{w\varepsilon_0 L}{2l}(K+1)}$$

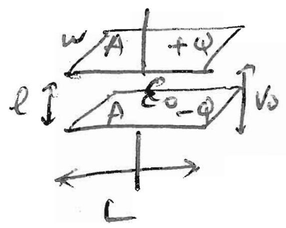
(<u>Note</u>: $C(x=L) = \frac{w\varepsilon_0}{l} LK = \frac{wL\varepsilon_0}{l} K = KC_0$)

b) $$U(x) = \frac{1}{2} C(x) V^2 = \frac{1}{2}\frac{Q^2}{C(x)} \rightarrow U(x) = \frac{Q^2 \quad L}{\frac{2w\varepsilon_0 L}{l}\left[x(K-1) + L\right]}$$

$C = \frac{Q}{V}$ or $V = \frac{Q}{C}$

$U_0$ (see next page)

Original capacitor's energy storage:



$$\boxed{C_0 = \frac{A\varepsilon_0}{d}}$$

$$\left[ U_0 = \frac{1}{2} C_0 V_0^2 = \frac{Q^2}{2C_0} = \boxed{\frac{Q^2}{2 \cdot \frac{wL\varepsilon_0}{\ell}}} \right.$$

$$C_0 = \frac{A\varepsilon_0}{\ell} = \frac{wL\,\varepsilon_0}{\ell}$$

$$\rightarrow U(x) = U_0 \frac{L}{[x(K-1)+L]}$$

$$\rightarrow U\left(x=\frac{L}{2}\right) = U_0 \frac{\cancel{L}}{\left[\frac{\cancel{L}}{2}(K-1)+\cancel{L}\right]} = U_0 \frac{2}{K+1} \text{ or } \frac{C_0 V_0^2}{K+1}$$

c) Capacitor will not suck in the dielectric slab: it opposes the insert: we need to apply a force on the slab to push it in:

$$\vec{F} = -\frac{dU}{dx}\hat{\imath} \qquad \left( E = -\frac{dV}{dx} \text{ when both sides are multiplied by } q_{test} \text{ we get the first equation} \right)$$

$$\vec{F} = -\hat{\imath}\, U_0 L \frac{d}{dx} \frac{1}{[x(K-1)+L]} = +\hat{\imath}\, U_0 L \frac{K-1}{[x(K-1)+L]^2}$$

$$\vec{F}\left(x=\frac{L}{2}\right) = \hat{\imath} \frac{U_0 L(K-1)}{\left[\frac{L}{2}(K-1)+L\right]^2} = \hat{\imath} \frac{U_0(K-1)}{L\left[\frac{K+1}{2}\right]^2} = \hat{\imath} \frac{4U_0(K-1)}{L(K+1)^2}$$

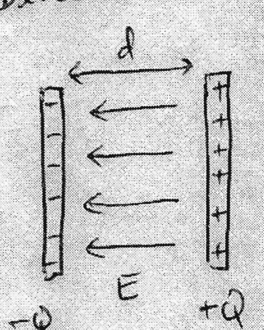
$$\underset{U_0 = \frac{1}{2}C_0V_0^2}{=} \hat{\imath} \frac{2C_0V_0^2(K-1)}{L(K+1)^2}$$

## How to increase the capacitance?

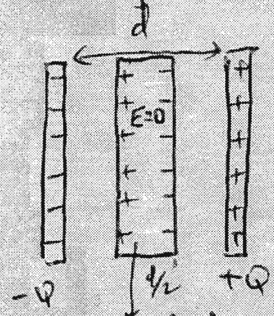
1) Connecting 2 or more capacitors in parallel

2) $C_1 = \frac{A\varepsilon_0}{d}$ (parallel plate capacitors)

2a) Decrease $d$: inserting a conducting slab b/w plates:



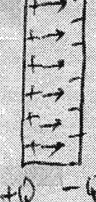

$V = E \cdot d$




conducting slab

free $e^-$ → rearrange

so




$V' = E \cdot \frac{d}{2}$

The field now inside the conducting slab (width $\frac{d}{2}$) is 0

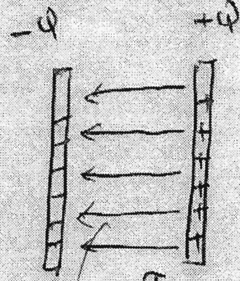
→ we have effectively reduced the separation in half

By inserting a conducting slab of width $\frac{d}{2}$ → the capacitance is doubled $C' = \frac{A\varepsilon_0}{\frac{d}{2}} = 2C$

2b) Notice $\varepsilon_0$ : dielectric constant in vacuum (air)
$\uparrow$ vacuum

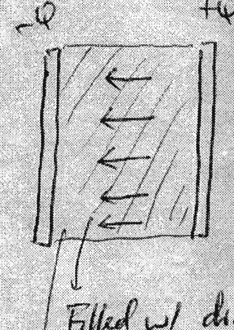
In a medium $\varepsilon = K\varepsilon_0$ ; $K > 1$ → $K$ can help increase the capacitance: if we insert a dielectric of $\varepsilon = K\varepsilon_0$ b/w the plates
$\downarrow$ Kappa

<u>Dielectric</u>: not many free $e^-$ as in a conductor $\rightarrow$
we don't get a perfect rearrangement of charges as in a conductor
So the field within the dielectric insert is only reduced, not 0
(by a factor of K)




$E = \frac{\sigma}{\varepsilon_0}$

air $\rightarrow$ $\varepsilon = \varepsilon_0$




Filled w/ dielectric insert $\varepsilon = K\varepsilon_0$

$$E' = \frac{\sigma}{\varepsilon} = \frac{\sigma}{K\varepsilon_0} = \frac{E}{K}$$

---

$$C = \frac{A\varepsilon_0}{d} \qquad C' = \frac{AK\varepsilon_0}{d} = KC$$

---

$$V = Ed \qquad V' = \frac{E}{K}d = \frac{V}{K}$$

$$C = \frac{Q}{V} \qquad C' = \frac{Q}{V'} = \frac{Q}{\frac{V}{K}} = K\frac{Q}{V} = KC$$

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# Ch-24 Electric Current: I

$I = \frac{dq}{dt}$ (Motion of charges)

{ Macroscopic quantity:
SI unit: $\frac{C}{s} = A$ for Amp

Ohm's Law: $I = \frac{V}{R}$

$V \rightarrow$ Potential difference

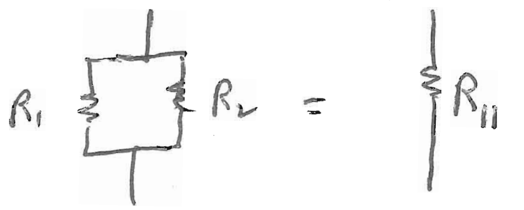
$R \rightarrow$ Resistance ($e^-$ will find some difficulty pushing through atoms in the wire)

$\rightarrow$ Power dissipation in resistors:

$P = I \cdot V$ $\begin{cases} \left(\frac{V}{R}\right) \cdot V = \frac{V^2}{R} \\ \text{or } I \cdot (IR) = I^2 R \end{cases}$

$\left(\frac{J}{s} = W\right)$

## Parallel

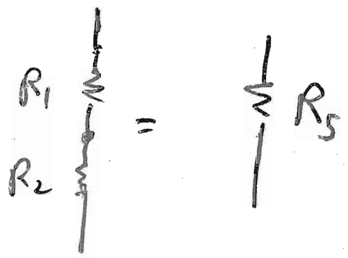



$R_{\parallel} = \frac{R_1 R_2}{R_1 + R_2}$

or $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

(Total resistance is lower than the original)

## Series




$R_S = R_1 + R_2$

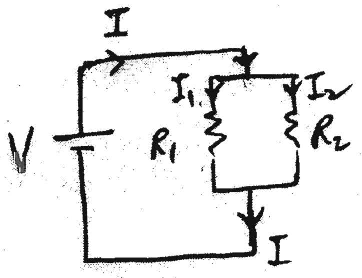
(Total resistance is larger than the original)

# Resistors



## Parallel $\leftrightarrow$ Current division

$I = I_1 + I_2$

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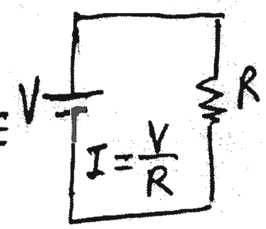



R: is the equivalent resistor for $R_1$ & $R_2$ :

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Ohm's Law:

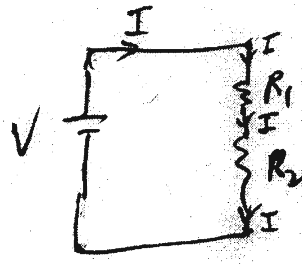
$$I = \frac{V}{R}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

or $\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$

## Series $\leftrightarrow$ Voltage division

$V = V_1 + V_2$

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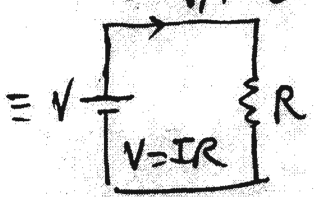



R: is the equivalent resistor for $R_1$ & $R_2$:

$$\begin{cases} V = V_1 + V_2 = IR_1 + IR_2 = I(R_1+R_2) \\ V = IR \end{cases}$$

$$\boxed{R = R_1 + R_2}$$

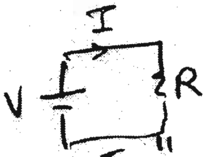
Voltage division:

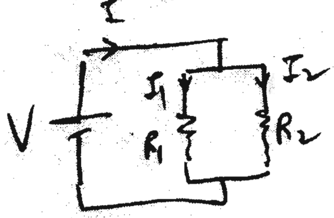
$$V_1 = IR_1 = \underbrace{\frac{V}{R_1+R_2}}_{I} R_1 = \underbrace{\frac{R_1}{R_1+R_2}}_{<1} V$$

$$V_2 = IR_2 = \frac{R_2}{R_1+R_2} V$$

$$V_1 + V_2 = \left(\frac{R_1}{R_1+R_2} + \frac{R_2}{R_1+R_2}\right) V = V$$

## Power Consumption



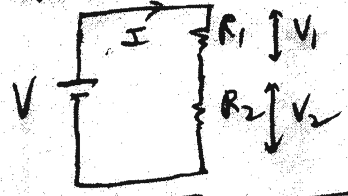

$\boxed{R_1 = R_2}$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1^2}{2R_1}$$

$P_1$ : power consumed @ $R_1$ :

$$P_1 = I_1 V = \frac{I}{2} V = \frac{V^2}{2R}$$

$$\xrightarrow{R = \frac{R_1}{2}} \boxed{P_1 = \frac{V^2}{R_1}}$$

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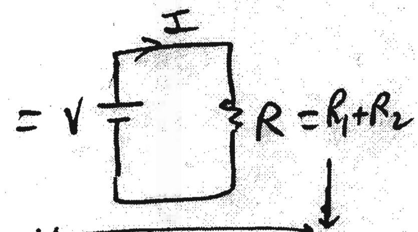



$\boxed{R_1 = R_2} \rightarrow V_1 = \frac{V}{2} = V_2$ $\qquad R = 2R_1$

$P_1$ : power consumed @ $R_1$ :

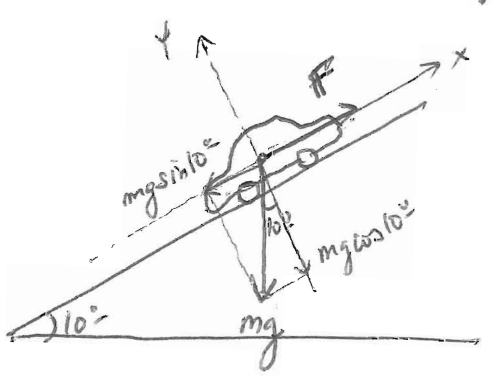
$$P_1 = I V_1 = \frac{V}{R} V_1 = \frac{V}{R}\frac{V}{2} = \frac{V^2}{2R}$$

$$R = 2R_1 \rightarrow \boxed{P_1 = \frac{V^2}{4R_1}}$$

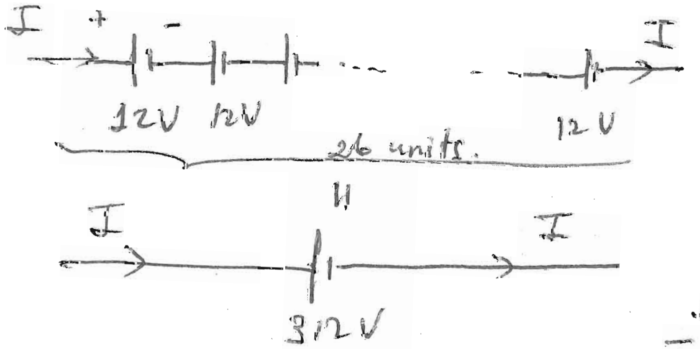
(24-69) EV on a slope: given data

$\{$ 85% efficiency ~~electrical~~ electrical $\rightarrow$ mechanical
Slope of $10°$
$m = 1500$ kg
uphill constant speed $v = 45 \frac{km}{h}$
$= \frac{45}{3.6}$ m/s
$\{$ $26 \times 12V = 312V$ Batteries
$100$ A.h $= Q$ (charge)
Amp×hour $= 3.6\times10^5$ C
$I = \frac{dq}{dt}$ ( Unit : $\frac{C}{s}$ )
Unit: Amp






Batteries in series:




Same current through each
$\rightarrow$ Potential V gets added
$\downarrow$
Total charge delivered is same as that of 1 battery!

Notes: Since there is a downhill force of $mg \sin 10°$, for the car to go uphill @ constant speed $v$ $(a = 0)$, the batteries need to provide enough potential to apply a mechanical force $F = mg \sin 10°$ (if $F < mg \sin 10°$, car goes downhill, if $F > mg \sin 10°$, car accelerates uphill)

Question: how long will batteries last if car goes uphill @ constant speed of $\frac{45}{3.6}$ m/s ?

$t = \frac{\text{total available M.E}}{\text{how fast mechanical energy is consumed}}$

$t = \frac{M.E.}{\text{Power Consumption}}$

→ Total Mechanical Energy available $ME =$ total electrical energy $\times\ e_{\text{electrical} \to \text{mechanical}}$

Total electrical energy : $E = P \cdot \Delta t$

$= I . V . \Delta t$

$= \underbrace{I \Delta t}_{= Q} . V = Q . V = 3.6 \times 10^5 \times 312 \text{ J}$

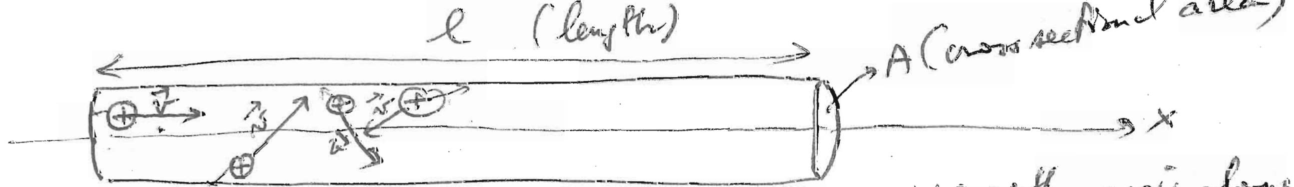
($Q$: Total charge; $V$: Total potential)

→ Power consumption by car or engine: to sustain a constant speed of $\frac{45}{3.6} \frac{m}{s}$ : $P = F . v = \frac{F . \Delta x}{\Delta t}$ (Energy)

$$\to t = \frac{0.85 \times Q \times V}{F . v} = \frac{0.85 \times 3.6 \times 10^6 \times 312}{1500 \times 9.81 \times \sin 10^\circ \times \frac{45}{3.6}} = 2989 s = 49.8 \text{ min}$$

$\boxed{\text{Note:}}$ $mg(v \sin 10^\circ)$ is the power consumed for vertical ascend (rate of change of grav. potential energy on vertical ascend)

**24.61** Drift speed in Al. wire




- → wire with axis along x
- → $e^-$ moving or ⊕ moving in the other direction

$\vec{v}$ for individual charges are very high but random → the average speed along x is small : → drift speed or velocity ($\vec{v}_d$) is what produces the macroscopic current $I$.

$$I = \frac{\Delta q}{\Delta t} = \frac{n \overset{\text{vol.}}{A l} . q}{\frac{l}{v_d}} = n q A v_d \ ?$$

$n$ : # charge per unit volume; $q$: charge of the electron

This problem:
- $I = 20A$
- diameter $= 2.1$ mm
- $\hookrightarrow A = \pi (1.05 \times 10^{-3})^2$
- Al = each atom → 3.5 electrons

Need to find $n$ for Al

We need to find $\boxed{\overset{n}{\text{number of electrons per unit volume}}}$ for Aluminum.

↳ number of atoms per unit volume $\times 3.5$

↳ $\dfrac{\rho_{AL}}{m_{AL}} = \dfrac{2702}{26.96 \times 1.661 \times 10^{-27}} =$

$m_{AL}$: mass of one atom of Aluminum.

↓

Periodic Table: $26.98\ \cancel{a.u.} \times \dfrac{1.661 \times 10^{-27}\ kg}{\cancel{a.u.}}$

$\rho_{AL}$: Al density:

↓

Table: $2702 \dfrac{kg}{m^3}$

$$\rightarrow n = \frac{\rho_{AL}}{m_{AL}} \cdot 3.5 = \frac{2702 \times 3.5}{26.96 \times 1.661 \times 10^{-27}} = 2.1 \times 10^{29} \frac{\text{electrons}}{m^3}$$

$$v_d = \frac{I}{neA} = \frac{20}{2.1 \times 10^{29} \times 1.6 \times 10^{-19} \times \pi \times (1.05 \times 10^{-3})^2} = 0.171 \frac{mm}{s}$$

(very low!)

---

Estimation of instantaneous or random speed of electrons: rough estimate using a gas model:

$$\overline{\cancel{\tfrac{1}{2}} m v^2} = \cancel{\tfrac{3}{2}} kT \quad \text{(monoatomic molecules)}$$

$$v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 298.16^\circ}{9.11 \times 10^{-31}\ kg}}$$

$$= 12 \times 10^4 \frac{m}{s} \quad \text{(very large!)}$$

# Ch 25 Electrical Circuits

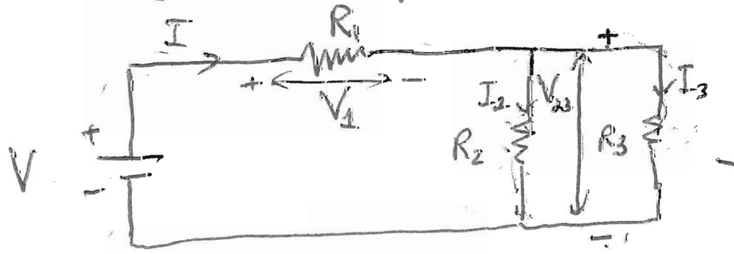
→ Linear: involves elements with a linear relationship b/w voltage V and current I : R, C, L

- R → Resistors (Ω for Ohm)
- C → Capacitors (F for Farad)
- L → Inductors (H for Henry)

→ 2 types in terms of the analysis method:

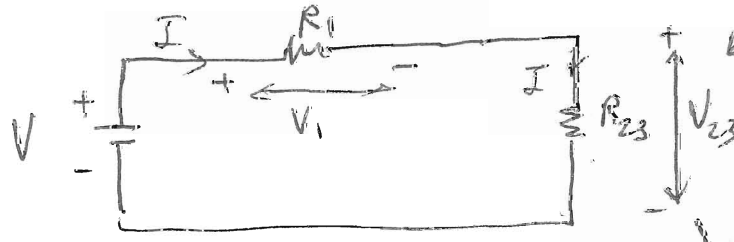
1) Resistors only
   - 1a) All resistors can be replaced by a single equivalent via series & parallel combinations
   - 1b) Need to use Loop or Node Analysis
2) Resistors & Capacitors

1a) Can reduce the circuit to a single battery & a single resistor using series & parallel combinations:



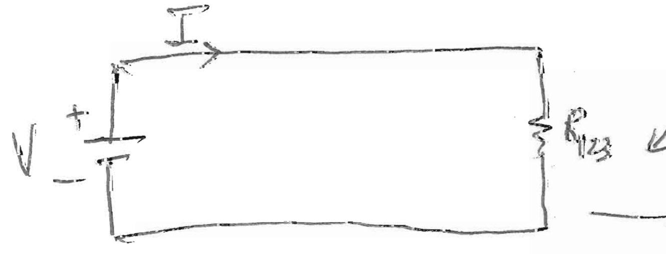

$R_1$ is in series with $(R_2 \parallel R_3)$

$$V = V_1 + V_{23}$$




$$R_{23} = \frac{R_2 R_3}{R_2 + R_3}$$

$$I = \frac{V_{23}}{R_{23}} \Longrightarrow V_{23} = I \cdot R_{23}$$




$$R_{123} = R_1 + R_{23} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

→ Single circuit: (Ohm's law) $\boxed{I = \frac{V}{R_{123}}}$

Now: $I = I_2 + I_3$

$$\begin{cases} I_{2} = \dfrac{V_{23}}{R_2} = I\left(\dfrac{R_{23}}{R_2}\right) = I\dfrac{\cancel{R_2}R_3}{(R_2+R_3)\cancel{R_2}} = I\dfrac{R_3}{R_2+R_3} \\[2ex] I_{3} = \dfrac{V_{23}}{R_3} = I\dfrac{R_{23}}{R_3} = I\dfrac{R_2}{R_2+R_3} \end{cases}$$

Current division: current in branch 2 is proportional to resistance in branch 3, and viceversa!