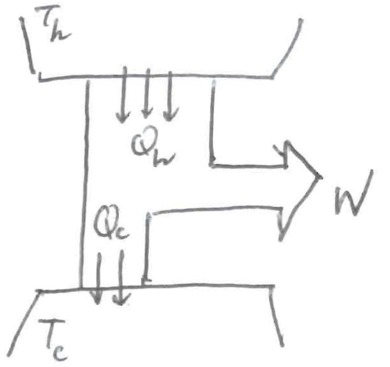
Ch 19 2nd of Thermodynamics (Cont.)

Reversed heat engines · refrigerators

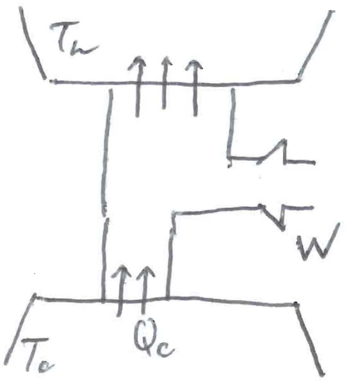
Heat Engine



Q_h : heat absorbed by gas @ high temp. T_h

Q_c : heat loss by gas @ low temp. T_c

Reversed Heat Engine



Q_c : heat absorbed by gas @ low temp T_c (inside of fridge)

Q_h : heat loss @ high temp T_h (back or bottom front of fridge) ↳ kitchen..

W: work received by gas ↓ electrical.

$$e \equiv \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{|Q_c|}{|Q_h|}$$

$\boxed{e < 1}$ ← 2nd Law of Thermodynamics →

C.O.P (Coefficient of performance) $= \dfrac{Q_c}{W}$

$\boxed{\text{C.O.P} < \infty}$

It is impossible to transfer heat from a cold reservoir (inside fridge) to a hot reservoir (kitchen) without requiring any work.

2nd Law of Thermodynamics

Entropy: $\Delta S \equiv \int_1^2 \frac{dQ}{T}$: ΔS: change of entropy b/w states 1 & 2

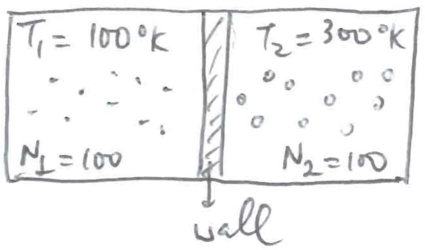
$\Delta S \geq 0$

Entropy of a closed system can never decrease

↳ or disorder

Applying on the universe (a closed system): disorder will just increase

A



V_1 V_2

$V_1 = V_2 = V$

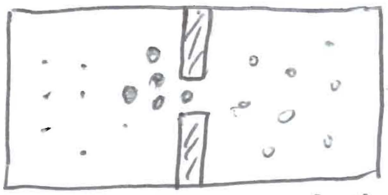
P_1 $P_2 > P_1$

$(PV = nRT)$

(More pressure on hotter gas)

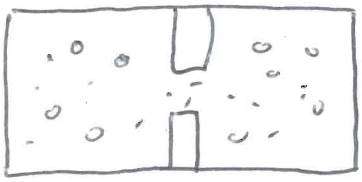
Order: the 2 types of gas are separate.

B



Some of type 2 push their way into the left.

C



P_1 got increased and some of type 1 are pushed ~~into~~ to the right

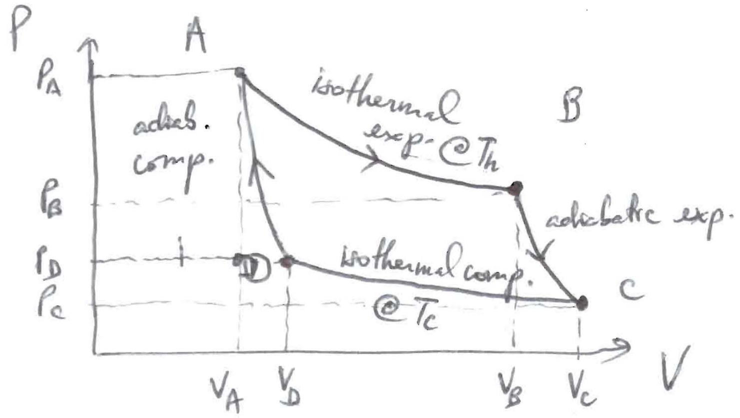
Entropy or disorder will only increase

Heat Engines: More quantitative description:

↳ operate in cycles (the 2^{nd} half of the cycle was to bring engine or system back to original state so to complete and repeat the cycle) → in PV diagram this cycle is a closed loop!

↳ - Carnot engines: 4 reversible processes (2 isothermal, 2 adiabatic)
- Otto cycle: 4 reversible processes (2 adiabatic, 2 isovolumic)

CARNOT ENGINE: efficiency of a Carnot engine is the maximum achievable so far $e_{carnot} = e_{max}$



CARNOT CYCLE
(a closed loop in PV)

Write the e_{max} in terms of the temperatures T_h & T_c:

Isothermal process $A \to B$; $C \to D$

$\Delta U_{CD} = 0$ $(T_c = T_D = T_{cold})$
ideal gas: no interactions → all internal energy from KE → dof $\times \frac{1}{2} kT$
$\Delta U_{AB} = 0$ $(T_A = T_B = T_h)$

$$\rightarrow \begin{cases} Q_{CD} = W_{CD} = nRT_c \ln\left(\frac{V_D}{V_c}\right) \\ Q_{AB} = W_{AB} = nRT_h \ln\left(\frac{V_B}{V_A}\right) \end{cases} \quad e = 1 - \frac{|Q_c|}{|Q_h|}$$

$$e = 1 - \frac{|T_c|}{|T_h|} \underbrace{\frac{\left|\ln\left(\frac{V_D}{V_C}\right)\right|}{\left|\ln\left(\frac{V_B}{V_A}\right)\right|}}$$

Need to work on these ratios of volumes.
using the adiab. process B→C & D→A

B→C : adiab. expansion :

$$T V^{\gamma-1} = \text{constant}$$

$$T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$$

$$\left(\frac{V_B}{V_C}\right)^{\gamma-1} = \frac{T_C}{T_B} = \frac{T_{cold}}{T_h}$$

D→A : adiab. compression

$$T_D V_D^{\gamma-1} = T_A V_A^{\gamma-1}$$

$$\left(\frac{V_D}{V_A}\right)^{\gamma-1} = \frac{T_A}{T_D} = \frac{T_h}{T_{cold}}$$

$$\boxed{\frac{V_B}{V_C} = \frac{V_A}{V_D}}$$

$$\boxed{\frac{V_B}{V_A} = \frac{V_C}{V_D}}$$

$$\boxed{e_{CARNOT} = e_{max}} = 1 - \frac{|T_c|}{|T_h|} \frac{\ln \frac{V_C}{V_D}}{\ln \frac{V_B}{V_A}} = \boxed{1 - \frac{|T_c|}{|T_h|}} = \boxed{1 - \frac{T_c}{T_h}}$$

$\to 1$

if in °K

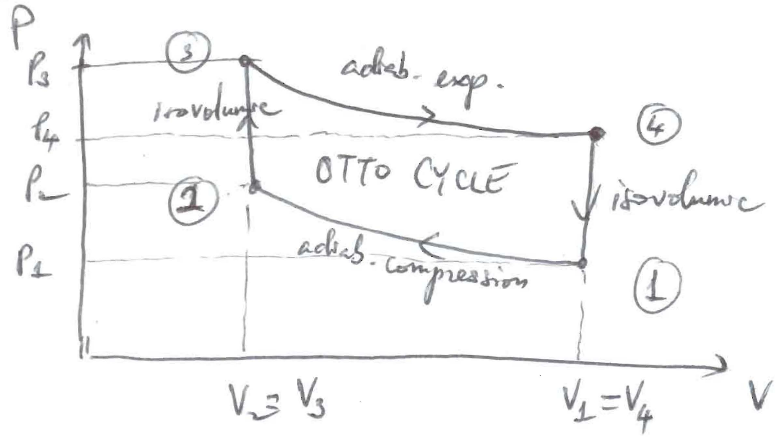
$$\left|\ln\left(\frac{V_B}{V_A}\right)\right| = \ln \frac{V_B}{V_A}$$

A→B exp : $V_B > V_A \to \frac{V_B}{V_A} > 1 \to \ln\left(\frac{V_B}{V_A}\right) > 0$

$$\left|\ln\left(\frac{V_D}{V_C}\right)\right| = -\ln\left(\frac{V_D}{V_C}\right) = \ln \frac{V_C}{V_D}$$

C→D : compression : $V_D < V_C \to \frac{V_D}{V_C} < 1 \to \ln\left(\frac{V_D}{V_C}\right) < 0$

Otto Cycle:



$$e_{OTTO} < e_{CARNOT} = e_{max}.$$

Entropy: $\Delta S_{12} = \int_1^2 \frac{dQ}{T}$

1) Isothermal: $\Delta S_{12} = \frac{1}{T}\int_1^2 dQ = \frac{\Delta Q}{T}$

2) Isovolume: $C_V = \frac{1}{n}\frac{dQ}{dT} \rightarrow dQ = nc_v dT$

$$\Delta S_{12} = nc_v \int_1^2 \frac{dT}{T} = nc_v \ln\left(\frac{T_2}{T_1}\right)$$

Thermodynamics:

Ch 16 : Temp & Heat

Ch 17 : Thermal Behavior of Matter

Absorbs heat Q
1) $T \uparrow$
2) Change of phase
3) Expansion: α, β

Ch 18 : 1^{st} Law of T.D. $\Delta U = Q - W$

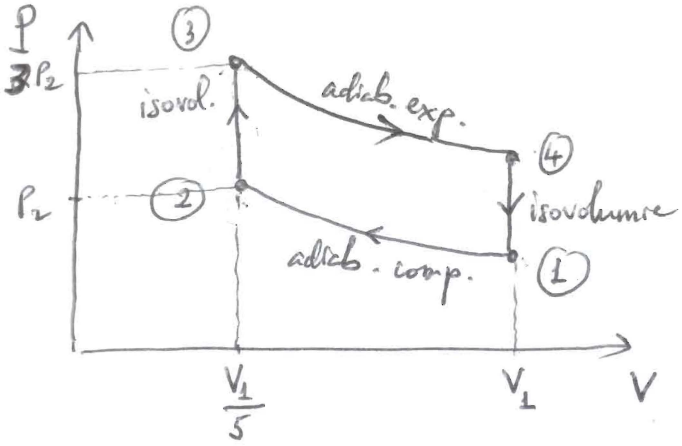
Ch 19 : 2^{nd} & 3^{rd} Law of TD.

$\downarrow$ $e < 1$, C.O.P. $< \infty$

$\downarrow$ $\Delta S = \int_1^2 \frac{dQ}{T} \geq 0$

19.54

Otto Cycle gasoline engine (heat engine)



a) If specific heat ratio is γ → find engine's efficiency e?

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{|Q_c|}{|Q_h|} < 1$$

→ 2nd Law of Thermodynamics

Notes:
1) $W = Q_{net}$ since in one ~~cy~~ cycle $\Delta U = 0$
2) $e = 1 - \frac{T_c}{T_h}$: can't be applied since this the Carnot Engine's efficiency or the max. achievable efficiency.

(i) There are 2 isovolumic processes (② → ③ ; & ④ → ①)

→ use C_V :

$$\left.\begin{aligned} Q_c &= Q_{41} = nC_V(T_1 - T_4) \\ Q_h &= Q_{23} = nC_V(T_3 - T_2) \end{aligned}\right\} e = 1 - \frac{|T_1 - T_4|}{|T_3 - T_2|}$$

(No heat exchange in adiab. processes ① → ② & ③ → ④)

(ii) Next: find relationships b/w the temperatures: using the adiabatic processes:

$T.V^{\gamma-1} = $ const.

$$\begin{cases} ① \to ② : & T_1 . V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \\ ③ \to ④ : & T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \end{cases}$$

$$\left.\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \quad (1) \\ T_4 V_4^{\gamma-1} &= T_3 V_3^{\gamma-1} \quad (2) \end{aligned}\right\} \quad \text{Divide: } \frac{1}{2}$$

$$\left\{\begin{aligned} &\frac{T_1}{T_4}\underbrace{\left(\frac{V_1}{V_4}\right)}_{1}^{\gamma-1} = \frac{T_2}{T_3}\underbrace{\left(\frac{V_2}{V_3}\right)}_{1}^{\gamma-1} \\ &\text{From PV diagram: } \quad V_1 = V_4 \ \& \ V_2 = V_3 \end{aligned}\right\} \quad \boxed{\frac{T_1}{T_4} = \frac{T_2}{T_3}}$$

$$e = 1 - \frac{|T_1 - T_4|}{|T_3 - T_2|} = 1 - \frac{|T_4(\frac{T_1}{T_4} - 1)|}{|T_3(1 - \frac{T_2}{T_3})|} = 1 - \frac{|T_4| \cdot \cancel{|\frac{T_1}{T_4} - 1|}}{|T_3| \cdot \cancel{|1 - \frac{T_2}{T_3}|}}$$

$$\boxed{e = 1 - \frac{|T_4|}{|T_3|}}$$

(iii) Now write $\frac{|T_4|}{|T_3|}$ in term of γ: using adiabatic equation:

$$\left\{\begin{aligned} &③ \to ④: \quad T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \to \frac{T_4}{T_3} = \underbrace{\left(\frac{V_3}{V_4}\right)^{\gamma-1}}_{\left(\frac{1}{5}\right)^{\gamma-1} = 5^{1-\gamma}} \\ &\text{PV diagram: } \left\{\begin{aligned} V_3 &= V_2 = \frac{V_1}{5} \\ V_4 &= V_1 \end{aligned}\right\} \end{aligned}\right.$$

$$\to \boxed{e = 1 - |5^{1-\gamma}| = 1 - 5^{1-\gamma}} \quad \text{For the Otto cycle in this problem}$$

b) Find T_{max} in term of T_{min}: From the PV diagram: $T_{max} = T_3$ (③ highest P & lowest V) ; $T_{min} = T_1$ (①: lowest P, largest V)

Adiab: ③ → ④ : $\frac{T_4}{T_3} = 5^{1-\gamma} \rightarrow T_3 = T_4 \cdot 5^{\gamma-1} = 3 \cdot T_1 \cdot 5^{\gamma-1}$

Adiab: $\left\{ \begin{array}{l} ③ \to ④ \\ ① \to ② \end{array} \right\} \rightarrow \frac{T_1}{T_4} = \frac{T_2}{T_3} \Rightarrow \frac{T_1}{T_4} = \frac{1}{3} \rightarrow \boxed{T_4 = 3 T_1}$

Ideal gas: $PV = nRT$ $\left\{ \begin{array}{l} ②: P_2 V_2 = nRT_2 \\ ③: P_3 V_3 = nRT_3 \end{array} \right\}$ $\frac{P_2 V_2}{P_3 V_3} = \frac{T_2}{T_3} \longrightarrow \boxed{\frac{1}{3} \cdot 1 = \frac{T_2}{T_3}}$

PV diagr.

$\boxed{T_3 = 3 T_2}$

$$\boxed{T_3 = 3 \cdot 5^{\gamma-1} \cdot T_1} \text{ or } \boxed{T_{max} = 3 \cdot 5^{\gamma-1} \cdot T_{min}}$$

c) Compare with efficiency of a Carnot Engine operating b/w

$$\left\{ \begin{array}{l} T_h = T_{max} = T_3 \\ T_c = T_{min} = T_1 \end{array} \right\}$$

$$\left\{ \begin{array}{l} e_{Carnot} = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{|T_c|}{|T_h|} = 1 - \frac{|T_1|}{|T_3|} = 1 - \frac{1}{3 \cdot 5^{\gamma-1}} = 1 - \frac{5^{1-\gamma}}{3} \\ \\ e_{otto} = 1 - 5^{1-\gamma} < e_{Carnot} = e_{max} \quad \text{as expected.} \end{array} \right.$$

19.46

Ideal gas, diatomic $\left\{ \begin{array}{l} \text{d.o.f} = 5 \quad \rightarrow \quad C_v = \frac{5}{2}R \\ \text{(two atoms attached)} \\ n = 5; \quad P = 1\,atm = 1.013 \times 10^5\,Pa; \quad T = 300°K \end{array} \right.$

ΔS_{12} ? $\left\{ \begin{array}{ll} a) & \text{constant vol.} \quad \text{(isovolumic)} \\ b) & \text{constant pressure (isobaric)} \\ c) & \text{adiabatically ?} \quad \rightarrow \text{No heat absorbed} \rightarrow \Delta S_{12} = 0 \end{array} \right.$

$T_2 = 500°K$

$$\Delta S_{12} = \int_1^2 \frac{dQ}{T}$$

a) Isovolumic: $C_v = \frac{1}{n}\frac{dQ}{dT} \rightarrow dQ = nC_v dT$

$$\rightarrow \Delta S_{12}^{isovol.} = nC_v \int_1^2 \frac{dT}{T} = nC_v \ln\left(\frac{T_2}{T_1}\right)$$

$$= 5 \times \frac{5}{2}R \ln \frac{500}{300} = 53.1 \frac{J}{°K}$$

b) Isobaric: $C_p = \frac{1}{n}\frac{dQ}{dT} \rightarrow dQ = nC_p\,dT$

$$\Delta S_{12}^{isobaric} = nC_p \ln \frac{T_2}{T_1} = \Delta S_{12}^{isovol.} \times \frac{7}{5}$$

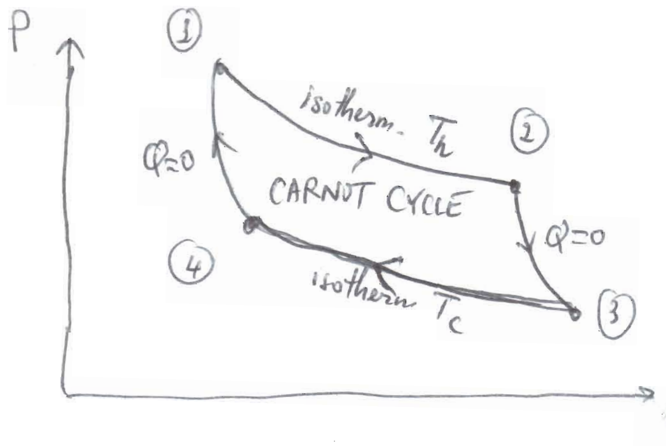
$$= 74.3 \frac{J}{°K}$$

$$\boxed{C_p = C_v + R = \frac{7}{2}R}$$

c) Adiabatic: $\Delta S_{12}^{adiab.} = 0$

$\Delta U = Q - W$, $Q \rightarrow 0$ → gas would receive work to increase its temp. adiabatically.

19.42



$n = 0.2$ (ideal gas)

1: $P_1 = 8\,atm$; $V_1 = 1\,L$

2: $P_2 = 4\,atm$; $V_2 = 2\,L$ ✓ ($PV = nRT$!)

3: $P_3 = 2.05\,atm$; $V_3 = 3.224\,L$

4: $P_4 = 4.1\,atm$; $V_4 = 1.612\,L$

a) $Q_h ? = Q_{12} = W_{12} = nRT \ln\left(\frac{V_2}{V_1}\right) = 0.2 \times 8.314 \times \overset{?}{T} \cdot \ln 2$

Q_h ↓ heat absorbed by gas

isothermal: $\Delta U = 0$

$\Delta U = Q - W$

$= P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = 8 \times 1.013 \times 10^5 \times \frac{1}{10^3} \ln 2 = 561.7\,J$

10^3 ↓ $1\,m^3 = 1000\,L$

b) $Q_c ? = Q_{34} = W_{34} = P_3 V_3 \ln\left(\frac{V_4}{V_3}\right) = 2.05 \times 1.013 \times 10^5 \times \frac{3.224}{10^3} \ln \underbrace{\underbrace{\left(\frac{1.612}{3.224}\right)}_{<1}}_{<0}$

Q_c ↓ heat absorbed by gas.

$= -464.1\,J$

− ↓ heat is loss

c) Work done by gas.

↳ In the whole cycle: $\underbrace{\Delta U}_{\substack{\downarrow \\ 0}} = Q - W \rightarrow \underset{(net)}{Q} = \underset{(net)}{W}$

back to same state

Net work done by gas equals net heat absorbed by gas

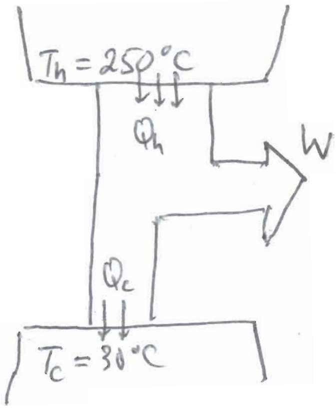
$W = |Q_h| - |Q_c| = 561.7\,J - 464.1 = 97.66\,J$

d) $e = \frac{W}{Q_h} = \frac{\cancel{100.60}\ 97.66}{561.7} = 0.1739 \sim 17.39\ \%$

e) $e = 1 - \frac{T_c}{T_h} = 1 - \frac{\frac{P_3 V_3}{\cancel{nR}}}{\frac{P_2 V_2}{\cancel{nR}}} = 1 - \frac{P_3 V_3}{P_2 V_2} = 1 - \frac{2.05 \times 3.224}{4 \times 2} = 0.17385$

same as d)

19.28



Power plant : steam → heat engine

$$\begin{cases} \frac{W}{t} = \text{Power} = 800\ MW & (1) \\ e = 0.28 \end{cases}$$

a) $e_{max} = e_{Carnot} = 1 - \frac{T_c}{T_h} = 1 - \frac{303.16°K}{523.16°K} = 0.42$ or 42%

b) Rate of waste heat discharge to river $\frac{Q_c}{t}$?

$$e = \frac{W}{Q_h} \rightarrow Q_h = \frac{W}{e} \rightarrow \frac{Q_h}{t} = \frac{\frac{W}{t}}{e} \quad (2)$$

Heat engines operate in cycles: in one cycle $\Delta U = 0$ or

$\Delta U = Q_{net} - W = 0 \rightarrow Q_{net} = W$

$Q_h - Q_c = W$ or $\frac{Q_h}{t} - \frac{Q_c}{t} = \frac{W}{t}$

$$\rightarrow \frac{Q_c}{t} = \frac{Q_h}{t} - \frac{W}{t} \underset{(1)\&(2)}{=} \frac{1}{e}\frac{W}{t} - \frac{W}{t} = \left(\frac{1}{e} - 1\right)\frac{W}{t}$$

Waste Heat loss rate

$$= \left(\frac{1}{0.28} - 1\right) 800\,MW$$

$$= 2057\,MW$$

c) 18 kW heat power per house

How many houses can be heated with the wast heat ?

$$\frac{2057 \times 10^6}{18 \times 10^3} = 114\,000 \text{ house}.$$

Ch. 20: Electric Charge, Force, Field

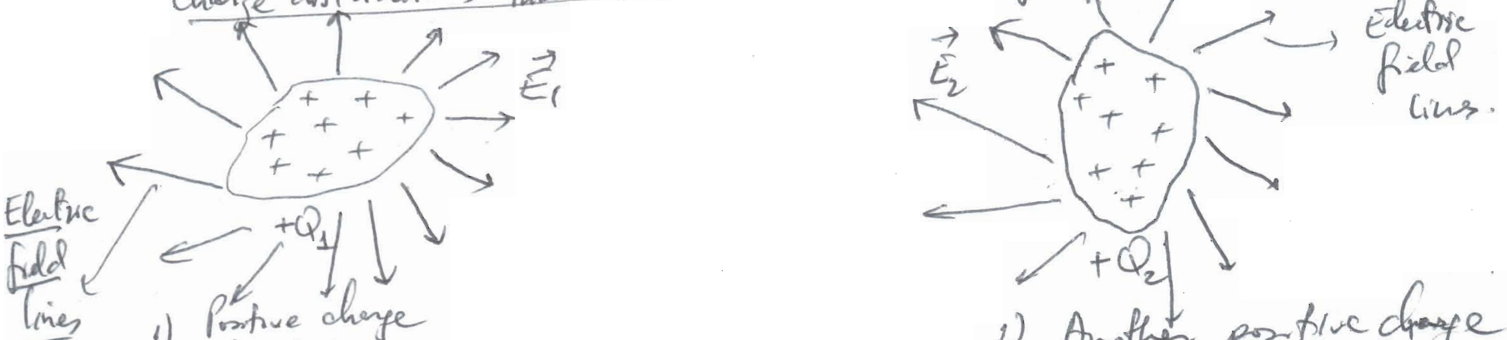
Charge → scalar; Force → vector; Field → vector

- Charge:
  - is a multiple of e^- or e^+ (unit is C for Coulomb)
    $e^- = -1.6 \times 10^{-19}$ C ; $e^+ = +1.6 \times 10^{-19}$ C
  - charge distribution: discrete or a continuous distribution.
    q or Q

- Electric field:
  - charges interact through their electric fields ($\frac{N}{C}$ for $\frac{\text{Newton}}{\text{Coulomb}}$)
  - $\vec{E}$ ($\frac{N}{C}$)

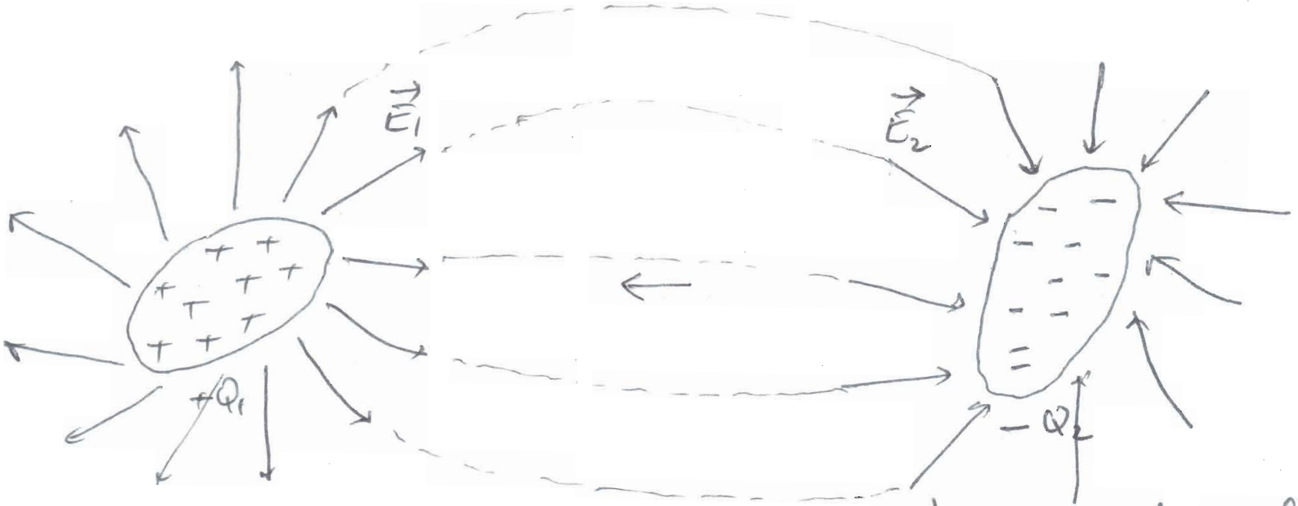
- Electric force:
  1) force felt by a test charge of value q_{test} in the presence of an electric field $\vec{E}$ (this field could be created by another charge or a group of charges)
  2) $\vec{F} = q_{test} \cdot \vec{E}$ (N)

Charge distributions interact via their electric fields.



1) Positive charge distribution: $\vec{E}_1$ pointing away from it
2) Higher density of line closer to the charge distribution: useful feature to describe the electric field (stronger closer to the charge distribution)

1) Another positive charge distribution: $\vec{E}_2$ pointing away from it
2) $+Q_2$ will feel a repulsive force from $+Q_1$ via it its electric field $\vec{E}_1$ over the distance!
Also $+Q_1$ will feel a similar repulsive force from $+Q_2$ via its electric field $\vec{E}_2$. <u>Interaction is mutual</u>



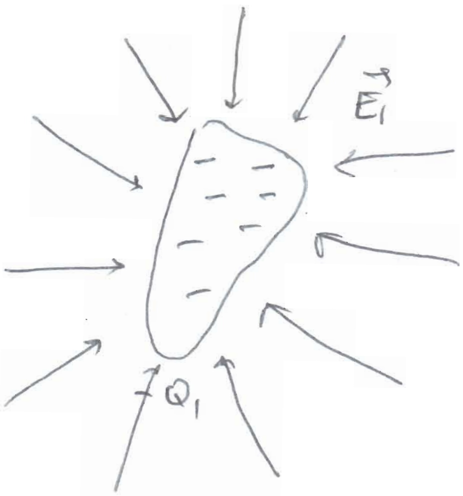
1) Positive charge distribution
$\vec{E}_1$ pointing away from it

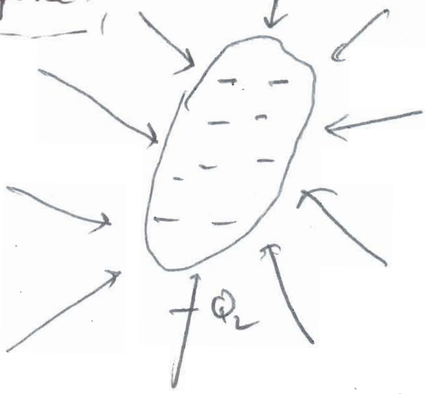
$\boxed{\vec{F} = \vec{E}\, q_{test}}$

$\vec{F}_{21} = -Q_2 \vec{E}_1$
Electric force on 2 by 1 is in the opposite direction as the field $\vec{E}_1$: attractive force.

1) Negative charge distribution,
$\vec{E}_2$ pointing toward it!

2) These electric field lines can connect! $-Q_2$ will feel an attractive force from $+Q_1$ via its electric field $\vec{E}_1$. And vice versa!





1) Neg. charge distribution
$\vec{E}_1$ pointing toward it

1) Neg charge distribution,
$\vec{E}_2$ pointing toward it.

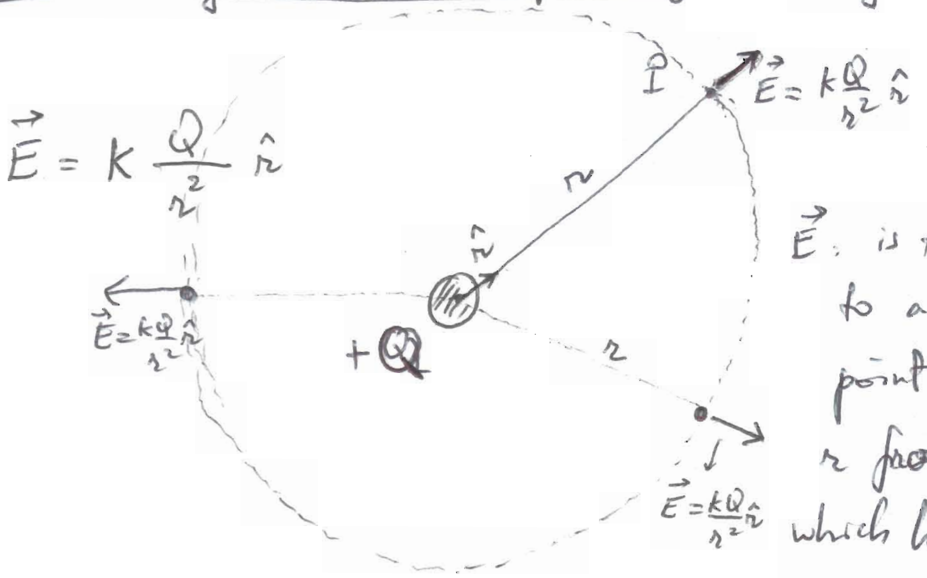
2) These fieldlines can't connect! $-Q_2$ will feel a repulsive force from $-Q_1$ via its electric field $\vec{E}_1$ & vice versa.

Opposite type of charges attract Similar type of charges repel

over the distance via the electric field.

From previous discussions: once we know the electric field by a charge distribution (e.g. $+Q_1$), it is straightforward to calculate the electric force on any test charge.

Calculations of the electric field by a charge distribution: $\vec{E}$ (vector)



$\vec{E}$: is the electric field due to a charge $+Q$ @ a point P that is a distance r from the charge Q, which has magnitude $k\frac{Q}{r^2}$, direction is radial from $+Q$ to P

Notes:

k: electric constant $= 9 \times 10^9 \ \frac{Nm^2}{C^2}$ (S.I.)

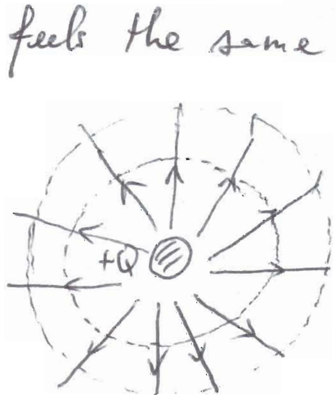
Q: net charge that creates the field

r: separation from Q to point P where the field is probed.

$\hat{r}$: radial unit vector (its length is 1)

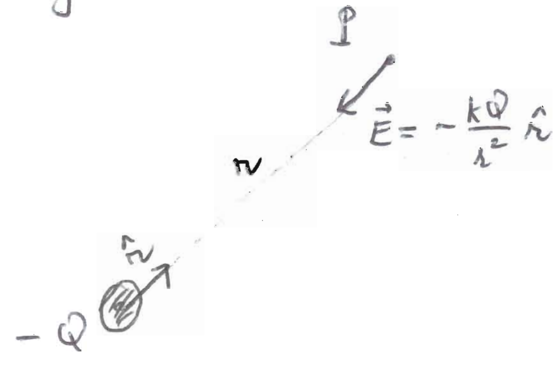
→ Dotted circle of radius r around the charge: any point on this circle feels the same electric field strength $\frac{kQ}{r^2}$.

→ Electric field lines:



on each of these circles: same density of field lines or same field strength!

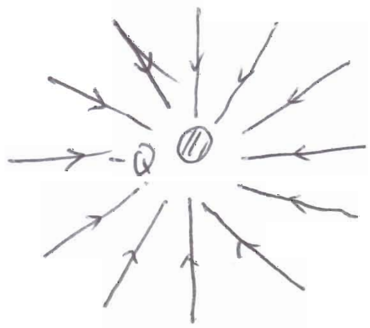
For a negative charge $-Q$



Notes:

$\hat{r}$: radial unit vector always point away from the charge (radial direction)

$\vec{E}$: now points in the opposite direction as $\hat{r}$ due to the negative sign of the charge. $\vec{E}$ points toward a negative charge.



Conclusion: $\vec{E}$ is parallel to the radial unit vector $\hat{r}$
- Same direction if charge is + (away from charge)
- Opposite direction if charge is − (toward the charge)

Observations:

Electric field

$$\vec{E} = k\frac{Q}{r^2}\,\hat{r}$$

Gravitational field

$$\vec{g} = -G\frac{M}{r^2}\,\hat{r}$$

Similarities

- Inverse square law $(\sim \frac{1}{r^2})$
- Pointing in radial direction $(\hat{r})$
- Proportional constant: k, G
- Proportional to a Mass or Charge that creates the field

Difference

- Charge Q can be + or −
  ↳ E-Field can be attractive or repulsive
- $k = 9\times10^{9}\ \frac{Nm^2}{C^2}$

- Mass M is always positive +
  ↳ Grav. field is always attractive!
- $G = 6.67\times10^{-11}\ \frac{Nm^2}{kg^2}$

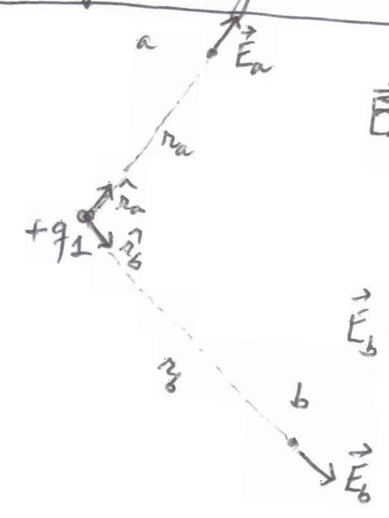
(gravitation is weaker, a lot weaker than electric interaction)
↓
Thanks to a balance of + & − charges we are here.

Calculation of the electric field due to a charge distribution:

Methods.
1) Direct calculation: vector superposition
2) Gauss' Law
3) Electric potential

$\boxed{\text{Direct Calculation}}$ one charge; two charges (dipole); continuous ring of charge; ∞ line of charge.

Electric field by one charge q_1:



$$\vec{E}_a = k\frac{q_1}{r_a^2}\hat{r}_a$$

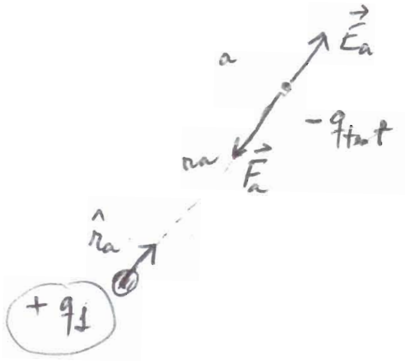
$$\vec{E}_b = k\frac{q_1}{r_b^2}\hat{r}_b$$

Can calculate $\vec{E}$ @ any point around q_1.

Electric force: $\vec{F} = q_{test}\vec{E}$ a test q_{test} will feel this force when brought into the vicinity of q_1. In this case:

$\vec{F}$ $\begin{cases} \text{repulsive if } q_{test} > 0 \\ \text{attractive if } q_{test} < 0 \end{cases}$

If q_1 was negative $\rightarrow$ $\vec{E}$ is toward $-q_1$ $\rightarrow$ $\vec{F}$ $\begin{cases} \text{repulsive if } q_{test} < 0 \\ \text{attractive if } q_{test} > 0 \end{cases}$



→ Test charge here feels:

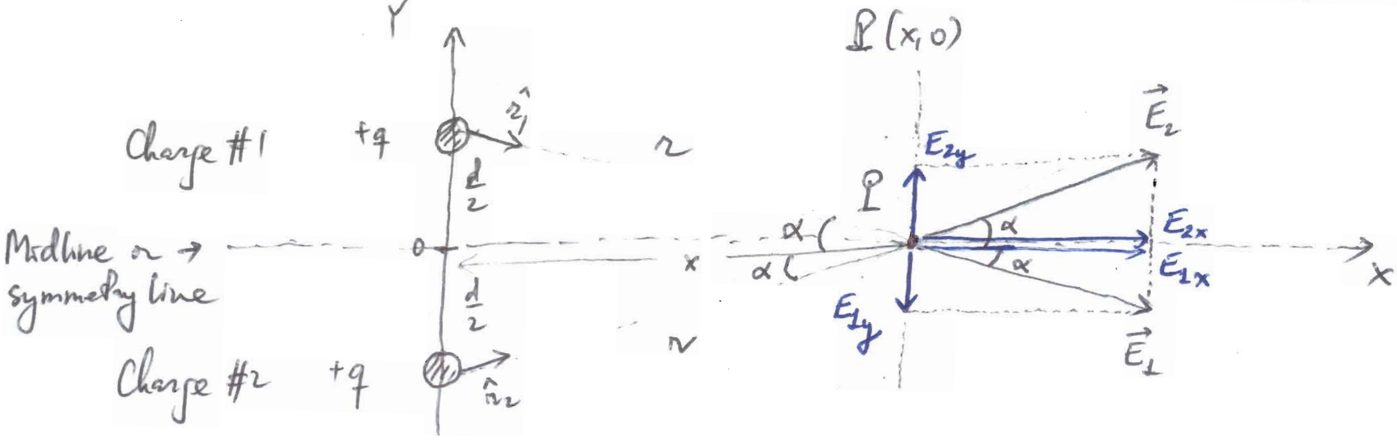
$$\vec{F}_a = -q_{test}\,\vec{E}_a = -k\,\frac{q_1 q_{test}}{r_a^2}\,\hat{r}_a$$

This is the force applied by q_1 on $-q_{test}$

→ 3rd Newton's law also applies to electric interactions: action & reaction: $-q_{test}$ applies a same force on q_1 but in the opposite direction (through the electric field that $-q_{test}$ creates which points toward $-q_{test}$!). In the electric field created by $-q_{test}$, the test charge is $+q_1$ → electric force of reaction is toward $-q_{test}$: (same as $\hat{r}_a$)

$$\vec{F}_a^{\,Reaction} = k\,\frac{q_1 q_{test}}{r_a^2}\,\hat{r}_a$$

Electric field due to two positive charges @ P along the midline b/w them
a symmetry line



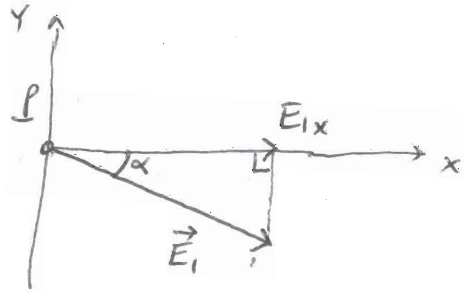
Total electric field $\vec{E}$ @ P: superposition of the fields by each charge @ P

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \underbrace{\left(k\,\frac{q}{r^2}\right)}_{E_1}\hat{r}_1 + \underbrace{\left(k\,\frac{q}{r^2}\right)}_{E_2 = E_1}\hat{r}_2$$

Vectors can be added easily using Cartesian components: vectors point in different directions but their components point in the same directions!

$$\vec{E} = \left(E_{1x}\,\hat{\imath} + E_{1y}\,\hat{\jmath}\right) + \left(E_{2x}\,\hat{\imath} + E_{2y}\,\hat{\jmath}\right) = \underbrace{\left(E_{1x} + E_{2x}\right)}_{2E_{1x}}\hat{\imath} + \underbrace{\left(E_{1y} + E_{2y}\right)}_{0}\hat{\jmath}$$

$$\rightarrow \begin{cases} E_{1x} = E_{2x} \\ E_{1y} = -E_{2y} \end{cases}$$

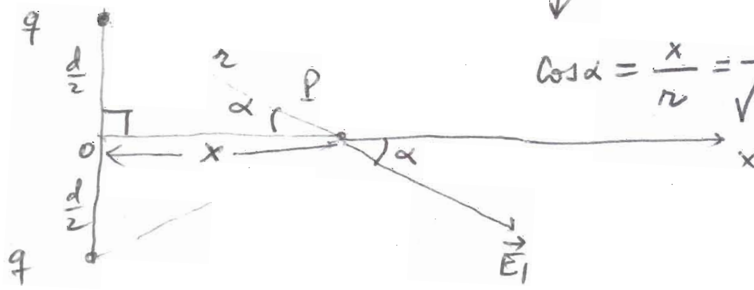


$$\vec{E} = 2E_{1x}\,\hat{\imath}$$

$$= 2E_1 \cos\alpha \; \hat{\imath}$$

$\hat{\imath}$: unit vector in x-direction

$$E_{1x} = E_1 \boxed{\cos\alpha}$$



$$\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2 + \left(\frac{d}{2}\right)^2}}$$

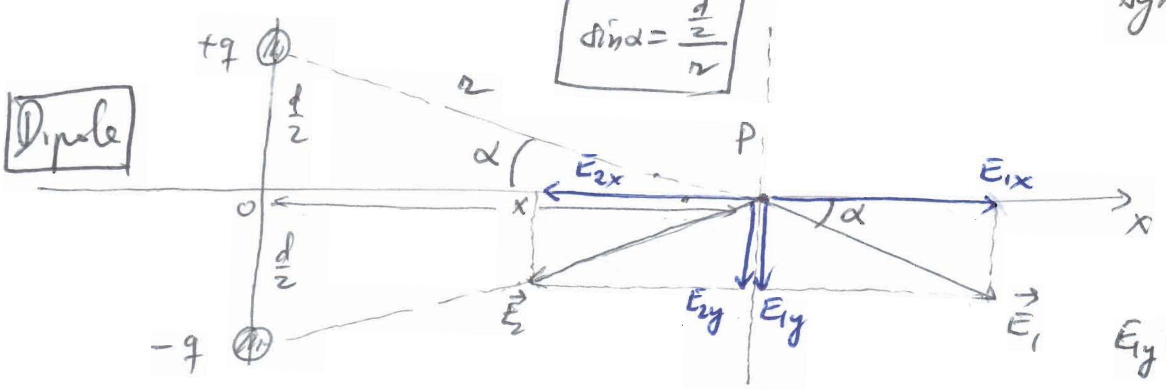
$$\vec{E} = 2 \cdot \frac{kq}{r^2}\,\frac{x}{r}\,\hat{\imath} = \frac{2kqx}{r^3}\,\hat{\imath}$$

Electric field by two charges of value q each at a point P along the midline.

$$= \frac{2kqx}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}\,\hat{\imath} \quad \left(\frac{N}{C}\right)$$

$\boxed{\text{Dipole}}$

Electric field due to a positive and a negative charge @ P along the symmetry line



$E_y = E_1 \sin\alpha$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = -2E_{1y}\,\hat{j} = -2E_1\sin\alpha\,\hat{j}$$

unit vector in y-direction

Total field is parallel to axis of the 2 charges (from + to −)

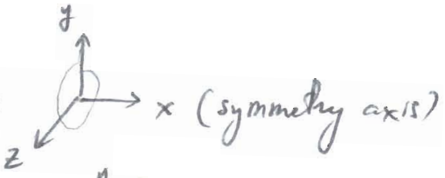
$$\vec{E} = -2\,\frac{kq}{r^2}\,\frac{d}{2r}\,\hat{j} = -\,\frac{kqd}{r^3}\,\hat{j}$$

$$\vec{E} = -\,\frac{kqd}{\left[x^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}\,\hat{j} \qquad \left(\frac{N}{C}\right)$$

Electric field due to a dipole along the symmetry line.

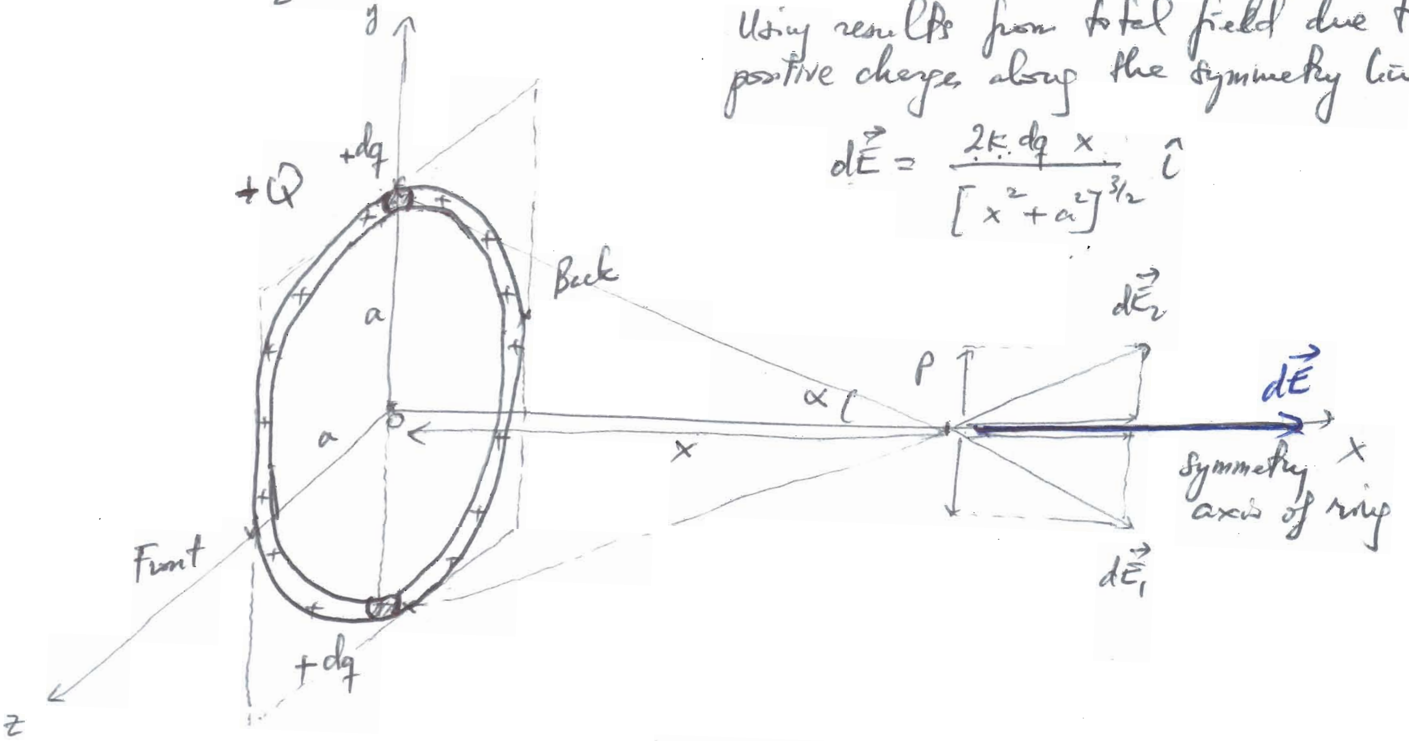
Electric field due to a continuous ring of charge: @ point P along its symmetry axis.

3D axis :



Using results from total field due to 2 positive charges along the symmetry line:

$$d\vec{E} = \frac{2k\, dq\, x}{[x^2+a^2]^{3/2}}\,\hat{\imath}$$

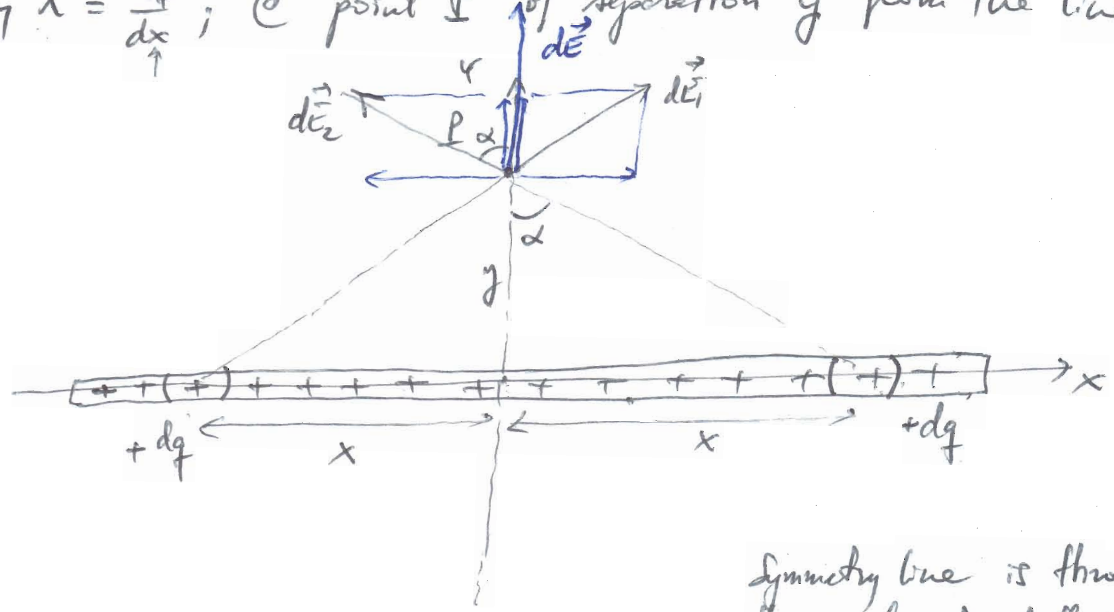


To find total $\vec{E}$ due to whole ring:

$$\vec{E} = \int_{\text{Half ring}} d\vec{E} = \frac{2kx\,\hat{\imath}}{[x^2+a^2]^{3/2}} \underbrace{\int_{\text{Half Ring}} dq}_{\frac{Q}{2}}$$

$$\boxed{\vec{E} = \frac{kQx}{[x^2+a^2]^{3/2}}\,\hat{\imath} \quad \left(\frac{N}{C}\right)}$$

Electric field due to a very long (∞) line of charge (with linear charge density $\lambda = \frac{dq}{dx}$; @ point P of separation y from the line)



Symmetry line is through the midpoint of the line of charge!

Using results from total field by 2 positive charges along symmetry line:

$$d\vec{E} = \frac{2k\,dq\,y}{[y^2 + x^2]^{3/2}}\,\hat{j} \underset{dq = \lambda dx}{=} \frac{2k\lambda y\,dx}{[y^2+x^2]^{3/2}}\,\hat{j}$$

$$\vec{E} = \int_{\substack{\text{Half}\\\text{length}}} d\vec{E} = 2k\lambda y\,\hat{j}\int_{\substack{\text{Half}\\\text{length}\\0\to\infty}} \frac{dx}{[y^2+x^2]^{3/2}} = 2k\lambda y\,\hat{j}\underbrace{\left[\frac{x}{y^2(x^2+y^2)^{1/2}}\right]_{0=x}^{x=\infty}}_{\frac{1}{y^2} - 0}$$

Table of integrals: $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$

$$\boxed{\vec{E} = \frac{2k\lambda}{y}\,\hat{j} \quad \left(\frac{N}{C}\right)}$$

Ch 21: Method of Calculation of $\vec{E}$ #2: Gauss Law

Electric flux: $\phi \equiv \int \vec{E} \cdot d\vec{A}$

"Phi"

dot product or scalar product b/w two vectors:

$\vec{A} \cdot \vec{B} = AB \cos\alpha$

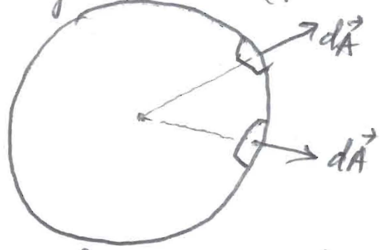
length of $\vec{A}$; length of $\vec{B}$; angle b/w $\vec{A}$ & $\vec{B}$

$\vec{E}$ = electric field

$d\vec{A}$: element of surface area.

Element of surface area:

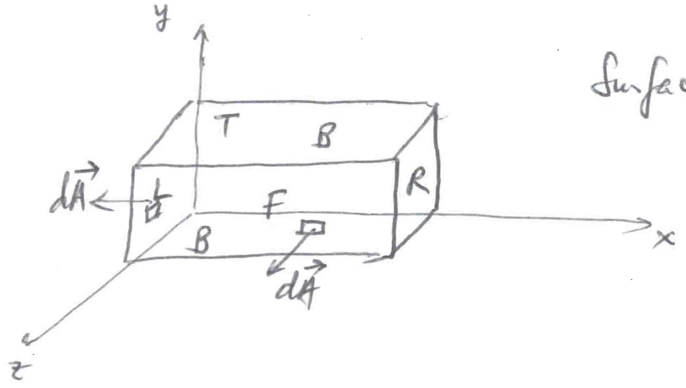
- Spherical surface area. (to enclose a spherical charge)



$d\vec{A}$ points in the radial direction

$d\vec{A} = dA\,\hat{r}$

- Rectangular surface area: 6 ~~side~~ face: F, B, T, B, L, R



Surface areas point away!

Top: $d\vec{A} = dA\,\hat{j}$

Bottom $d\vec{A} = -dA\,\hat{j}$

Front $d\vec{A} = dA\,\hat{k}$

Left $d\vec{A} = dA\,(-\hat{i})$

...

Gauss Law:

$$\boxed{\phi_{\substack{\text{closed}\\\text{surface}}} = \frac{q_{\text{enclosed by same surface}}}{\varepsilon_0}}$$

ε_0 : dielectric constant in vacuum

$$\varepsilon_0 = \frac{1}{4\pi K} = \frac{1}{4\pi \times 9 \times 10^9} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

since $\phi = \oint \underbrace{\vec{E} \cdot d\vec{A}}_{\substack{\text{only } E_\perp\\\text{will contribute}}} = \oint E_\perp \, dA = E_\perp \underbrace{\oint dA}_{A} = E_\perp . A$

find a closed surface on which E has the same value;

$$E_\perp A = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$\boxed{E_\perp = \frac{q_{\text{enclosed}}}{\varepsilon_0 A}}$$

Very important.

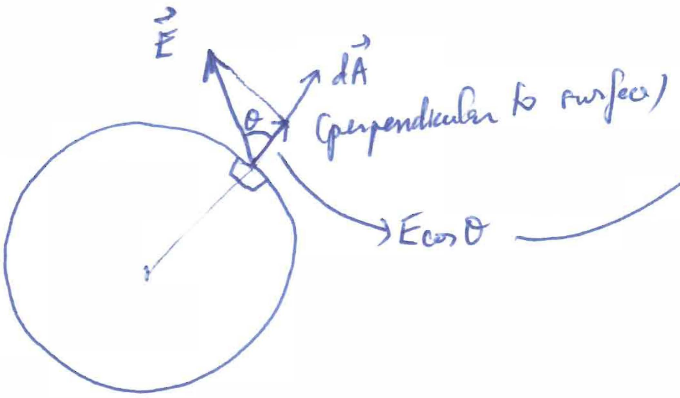
Electric flux: $\phi \equiv \oint_{\substack{\text{closed}\\\text{surface}\\\text{integral}}} \vec{E} \cdot d\vec{A} = \oint_{\substack{\text{closed}\\\text{surface}}} E_\perp \, dA = E_\perp \oint_{\text{surface}} dA$

If there is symmetry so that $E_\perp$ is constant over the surface

$= E_\perp A$

$\vec{E} \cdot d\vec{A} = E \, dA \cos\theta = \overbrace{\underbrace{E\cos\theta}_{\text{component of } \vec{E} \text{ that is perpendicular to the surface.}}}^{E_\perp} dA$

$d\vec{A}$: perpendicular to surface



We will use Gauss Law to calculate electric fields in these simple symmetry situations.

Gauss Law:

$$\boxed{\phi_{\substack{\text{closed}\\\text{surface}}} = \frac{q_{\text{enclosed by surface}}}{\varepsilon_0}}$$

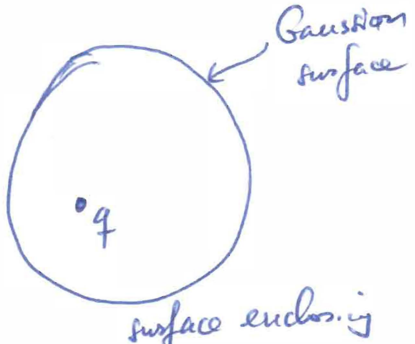
ε_0 = dielectric constant in vacuum

$\varepsilon_0 = \dfrac{1}{4\pi k} = 8.85 \times 10^{-12} \dfrac{C^2}{Nm^2}$ or $\boxed{k = \dfrac{1}{4\pi\varepsilon_0}}$

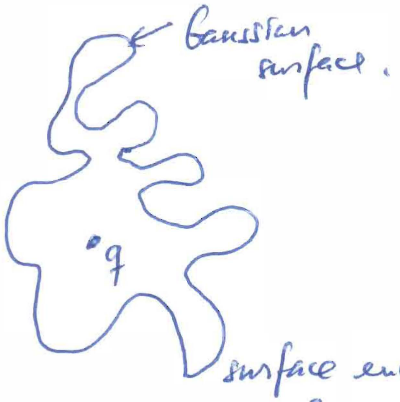
k: electric constant

$k = 9 \times 10^9 \dfrac{Nm^2}{C^2}$

Meaning of Gauss Law:



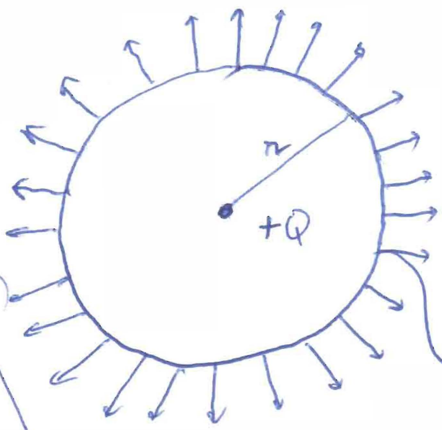
$$\phi_{\substack{\text{closed}\\ \text{surface}}} = \frac{q}{\varepsilon_0}$$



$$\phi_{\substack{\text{closed}\\ \text{surface}}} = \frac{q}{\varepsilon_0}$$

However, to calculate $\vec{E}$ using Gauss law, our Gaussian surface exhibits high symmetry.

1) Using Gauss Law to calculate $\vec{E}$ due to a point charge



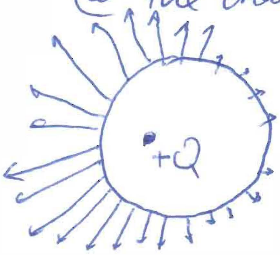
$A = 4\pi r^2$

First of all: determine the Gaussian surface (with high symmetry so

$$\phi = E_\perp A$$

otherwise it will take additional efforts to calculate E)

Gaussian surface: sphere centered @ the charge.



off centered sphere will not allow

$$\phi = E_\perp \oint dA$$

$$E_\perp A = \frac{Q}{\varepsilon_0}$$

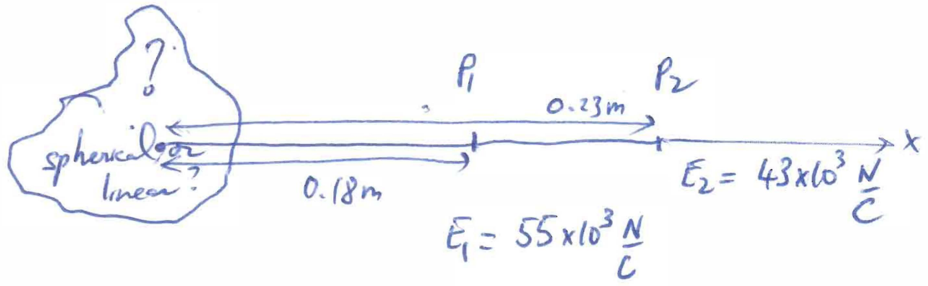
$$E_\perp = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$

Also for this Gaussian surface

$E_\perp = E$ ($\vec{E}$ is radial so it is perpendicular to the surface)

Using Gauss Law and a highly symmetrical Gaussian surface (sphere centered @ charge) we have derived an expression for the electric field due to a point charge $E = \frac{kQ}{r^2}$ that agrees with what we know from Chapter 20.

21-33

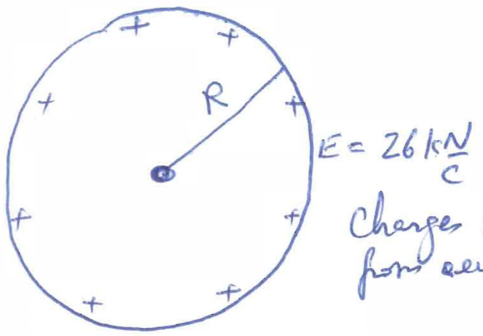


spherical : $E = \frac{kQ}{x^2} \longrightarrow \frac{E_2}{E_1} = \frac{x_1^2}{x_2^2}$ $\left\{ \frac{0.18^2}{0.23^2} = ? \right.$

$\boxed{\text{linear :}}$ $E = \frac{2k\lambda}{x} \longrightarrow \frac{E_2}{E_1} = \frac{x_1}{x_2}$ $\left\{ \frac{0.18}{0.23} = ? \right.$ ✓

$\frac{E_2}{E_1} = \frac{43}{55}$

21.47

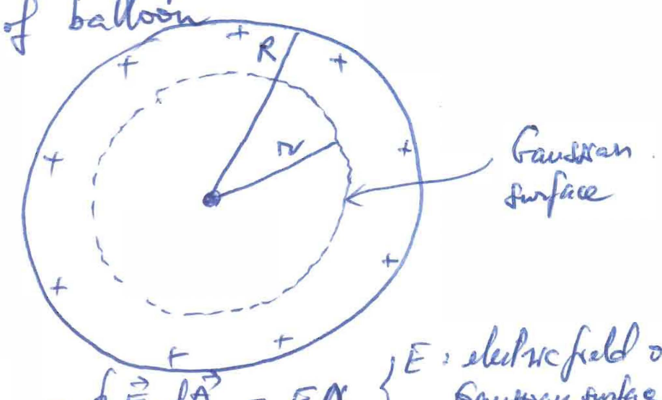


$R = 0.7m$

Charges stay on surface of balloon @ $R = 0.7m$ from center.

a) E ($r = 0.5m$ or inside balloon)

Using Gauss law → 1) Det. Gaussian surface → sphere centered @ center of balloon



2) $\phi_{\text{Gaussian surface}} = \oint_{G.S.} \vec{E} \cdot d\vec{A} = EA$ $\left\{ \begin{array}{l} E : \text{electric field on Gaussian surface} \\ A : \text{area of Gaussian surface} = 4\pi r^2 \end{array} \right.$

$= E 4\pi r^2$

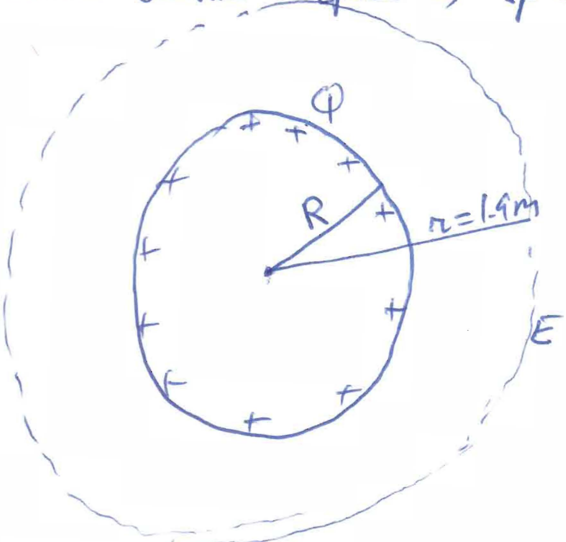
3) Gauss Law: $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$

$\downarrow$

$$E\,4\pi r^2 = \frac{0}{\varepsilon_0} \rightarrow \boxed{E(r=0.5m) = 0}$$

b) $E(r=1.9m$, or outside balloon$)$

1) Determine Gaussian surface → sphere centered @ center of balloon



2) $\phi = E\,4\pi r^2$

3) $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$ (Gauss Law)

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_0} \rightarrow E(r>R) = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$

(like that of a point charge!)

Alternatives: find Q, then $E(r=1.9m)$

observation:

$$E(r=R) = \frac{kQ}{0.7^2} = 26\,\frac{kN}{C}$$

$$\boxed{E(r=1.9m) = \frac{kQ}{1.9^2}}$$

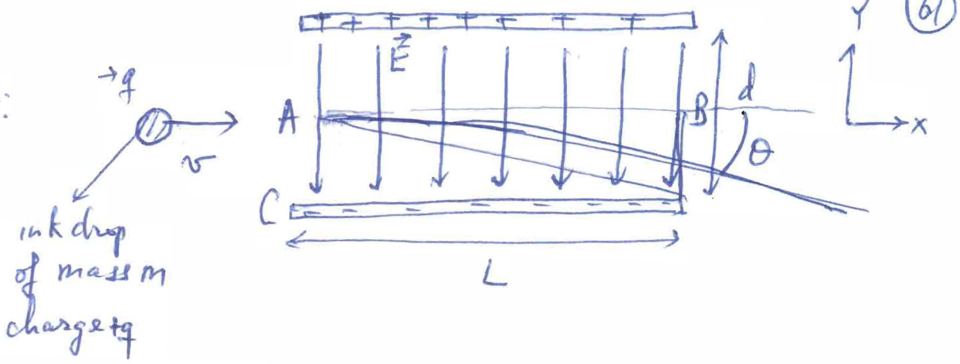
$$\frac{E(r=1.9m)}{E(r=0.7m)} = \frac{0.7^2}{1.9^2} \rightarrow E(r=1.9m) = \frac{0.7^2}{1.9^2}\,26\,\frac{kN}{C} = \boxed{3.53\,\frac{kN}{C}}$$

c) Net charge on balloon $Q = \dfrac{1.9^2 \times 3.53\times10^3}{9\times10^9} = 1.42\,\mu C$

$\downarrow$

10^{-6}

20.78 Ink jet printer:



Ink drop while crossing field region, feels a downward force → a downward acceleration → $a_y = \frac{F}{m} = \frac{qE}{m}$ → constant downward acceleration!

Min v for ink drop to make it through field region:

During time it takes to go $A \to B$ (x ~~component~~ direction) it should be going not more than AC ($\frac{d}{2}$) (y direction)

x direction : $t_{AB} = \frac{L}{v}$

↓

Motion in x direction is NOT affected by $\vec{E}$ → uniform motion.

y-direction: constant acceleration motion : $y = \frac{1}{2} a_y t^2$

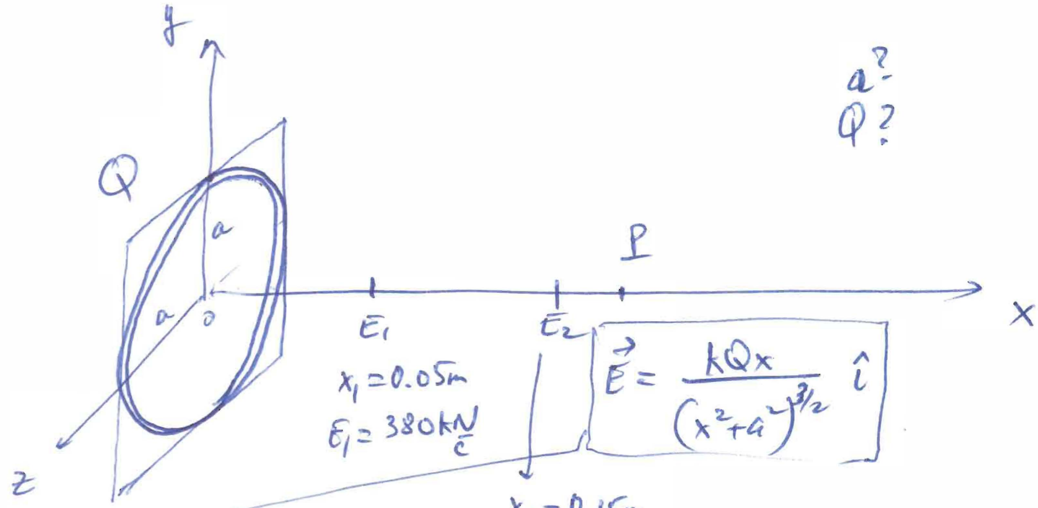
$$y = \frac{1}{2} a_y t_{AB}^2 < \frac{d}{2}$$

$$\frac{1}{\cancel{2}} \frac{qE}{m} \frac{L^2}{v^2} < \frac{d}{\cancel{2}} \quad \rightarrow \quad \frac{qEL^2}{md} < v^2$$

$$\boxed{L\sqrt{\frac{qE}{md}} < v}$$

$$\boxed{v_{min} = L\sqrt{\frac{qE}{md}}}$$

20.65



a?
Q?

$x_1 = 0.05m$
$E_1 = 380 \frac{kN}{C}$

$$\boxed{\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}} \hat{i}}$$

$x_2 = 0.15m$
$E_2 = 160 \frac{kN}{C}$

a)

$$\left[\frac{E_1}{E_2} = \frac{380}{160} = \frac{x_1}{x_2} \cdot \frac{(x_2^2+a^2)^{3/2}}{(x_1^2+a^2)^{3/2}}\right]^{2/3}$$

$$\left[\frac{380}{160}\right]^{2/3} = \left(\frac{1}{3}\right)^{2/3} \frac{0.15^2+a^2}{0.05^2+a^2} \rightarrow \boxed{a = 0.07m}$$

b)

$$E_1 = \frac{kQx_1}{(x_1^2+a^2)^{3/2}} \rightarrow Q = \frac{E_1(x_1^2+a^2)^{3/2}}{kx_1}$$

$$= \frac{380\times10^3(0.05^2+0.07^2)^{3/2}}{9\times10^9\times0.05}$$

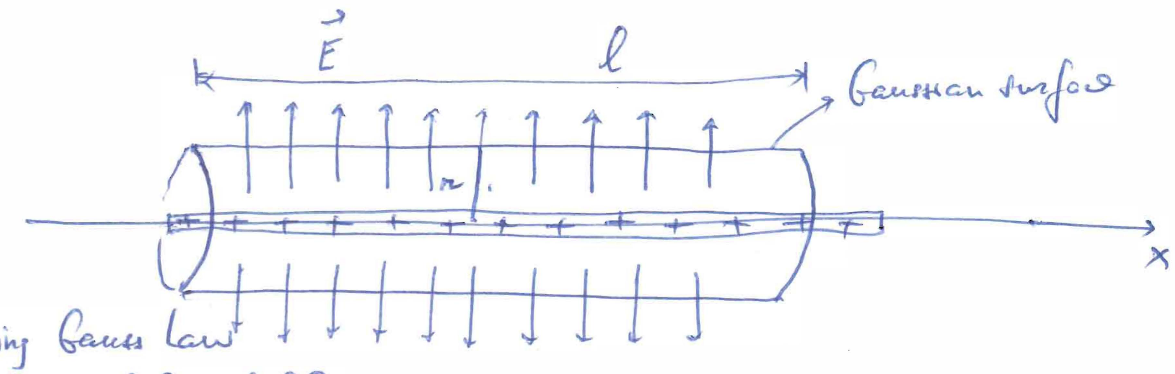
$$\boxed{Q = 538\,nC}$$

$\downarrow$ 10^{-9}

Method #2 : Calculation of $\vec{E}$ using Gauss Law :

Example #2 : Very long line of charge (linear charge density λ)

$$\lambda = \frac{dq}{dx}$$



Using Gauss law to find electric field:

1) Gaussian surface: such that E is constant on the surface.

$$\phi = \oint \vec{E} \cdot d\vec{A} = E_\perp A$$

→ A cylinder of radius r with its axis along the line of charge.

2) Gaussian surface $\begin{cases} \text{Body} : E_\perp = E \\ \text{Left side} : E_\perp = 0 \\ \text{Right side} : \end{cases}$ (E perpendicular to the left side has to point along $-x$, since all electric fields are perpendicular to x)

$\vec{E}$

Similarly : $E_\perp = 0$

$$\phi = E_\perp A = E_\perp A_{Body} + \underbrace{E_{\perp L}}_{0} A_{Left} + \underbrace{E_{\perp R}}_{0} A_{Right} = E_\perp A_{Body} = E A_{Body}$$

$$\phi = E . 2\pi r l$$

3) Gauss law : $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

$$E\, 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0} \rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{2k\lambda}{r}}$$

agrees with vector superposition result.

Method #3 Electric Potential (Ch 22)

Electric Potential

Potential energy difference b/w points A & B in mechanics:

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}$$

force applied

scalar product

infinitesimal displacement

Electric interaction : $\vec{F} = q' \vec{E}$

test charge

Electric potential energy difference b/w points A & B:

$$\Delta U_{AB} = -q' \int_A^B \vec{E} \cdot d\vec{l} \qquad (\text{unit SI}: J)$$

Electric potential difference b/w points A & B:

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q'} = -\int_A^B \vec{E} \cdot d\vec{l} \qquad \left(\text{unit SI}: \frac{J}{C}\right)$$

$\to V$ (Volt)

$$\vec{E} = -\vec{\nabla}(\Delta V_{AB})$$

$\vec{\nabla}$: gradient vector $= \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)$

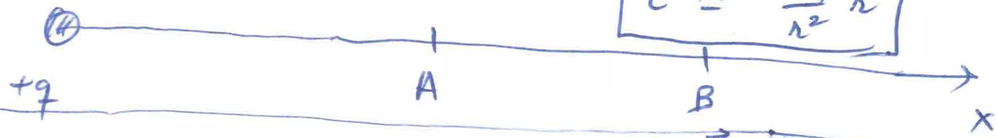
Electric field can be calculated by differentiating the electric potential with a minus sign.

<u>Example #1</u>: Calculation of $\vec{E}$ for a point charge using the Electric Potential.

For a point charge q: $V(r) = \frac{kq}{r} \rightarrow \vec{E} = -\vec{\nabla} V = -\left(\frac{\partial}{\partial r} V\right)\hat{r}$

$$= -\frac{\partial}{\partial r}\left(\frac{kq}{r}\right)\hat{r} = -kq\underbrace{\left(\frac{\partial}{\partial r}\frac{1}{r}\right)}_{-\frac{1}{r^2}}\hat{r}$$

$$\boxed{\vec{E} = \frac{kq}{r^2}\hat{r}}$$



First time contact with electric potential

$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{l} = -\int_A^B \underbrace{\frac{kq}{x^2}\hat{i}}_{\vec{E}} \cdot \underbrace{\hat{i}\,dx}_{d\vec{l}} = -kq\underbrace{\int_A^B \frac{dx}{x^2}}_{\left[-\frac{1}{x}\right]_A^B}$$

$\hat{i}\cdot\hat{i}$: $1.1\cos 0 = 1$

$$\Rightarrow \Delta V_{AB} = kq\left(\frac{1}{x_B} - \frac{1}{x_A}\right)$$

Use a reference point (zero potential : $x_A \Rightarrow \infty$)

$$\Delta V_{\infty B} = kq\frac{1}{x_B} \qquad \Delta V_{\infty B} = \frac{kq}{r_B}$$

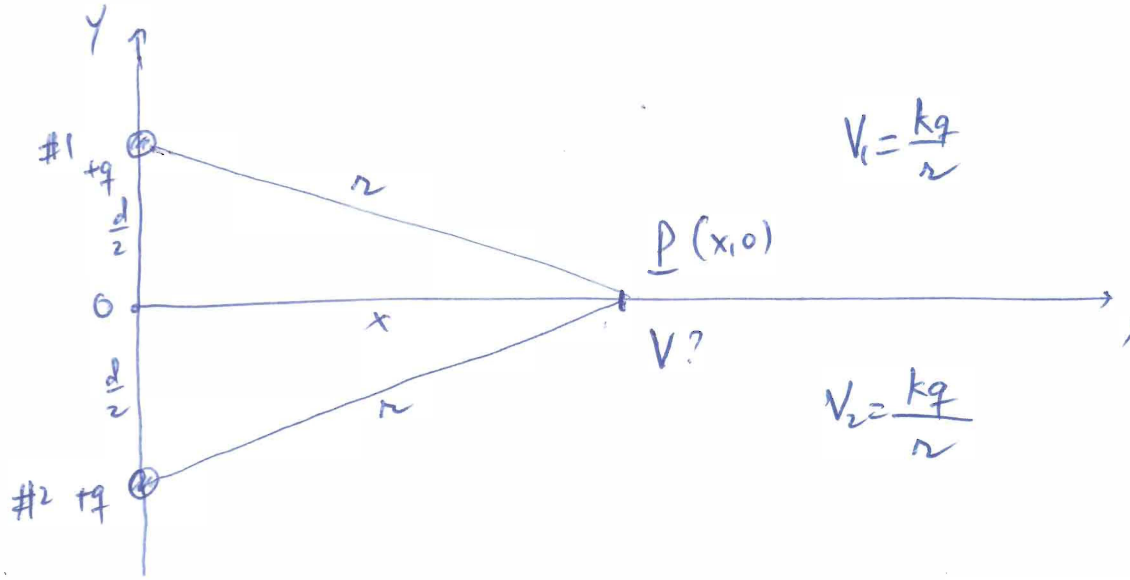
$\vec{x}$ could be any direction.

Always same reference point @ ∞ → Convention is

$$\boxed{V(r) = \frac{kq}{r}}$$

Electric potential due to a point charge, <u>is a scaler</u> (unit $\frac{J}{C}$ or V)

Example #2: Calculation of $\vec{E}$ due to 2 point charges @ a point P along the midline b/w the 2 charges.



What is V @ P, due to 2 point charges? $\rightarrow V = V_1 + V_2$

V_1: electric pot. due to charge #1; V_2: elect. pot. due to charge #2

Strength for Method #3 adding numbers instead vectors

$$V(@P) = \frac{2kq}{r} = \frac{2kq}{\left(x^2 + \left(\frac{d}{2}\right)^2\right)^{1/2}}$$

$$\vec{E}(@P) = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\,\hat{\imath}$$

$$= -2kq\,\frac{\partial}{\partial x}\,\frac{1}{\left[x^2 + \frac{d^2}{4}\right]^{1/2}}\,\hat{\imath} = -2kq\,\frac{\partial}{\partial x}\left[x^2 + \frac{d^2}{4}\right]^{-\frac{1}{2}}\hat{\imath}$$

$$= kq\left[x^2 + \frac{d^2}{4}\right]^{\left(-\frac{1}{2}-1\right)} 2x\,\hat{\imath} = 2kqx\left[x^2 + \frac{d^2}{4}\right]^{-\frac{3}{2}}\hat{\imath}$$

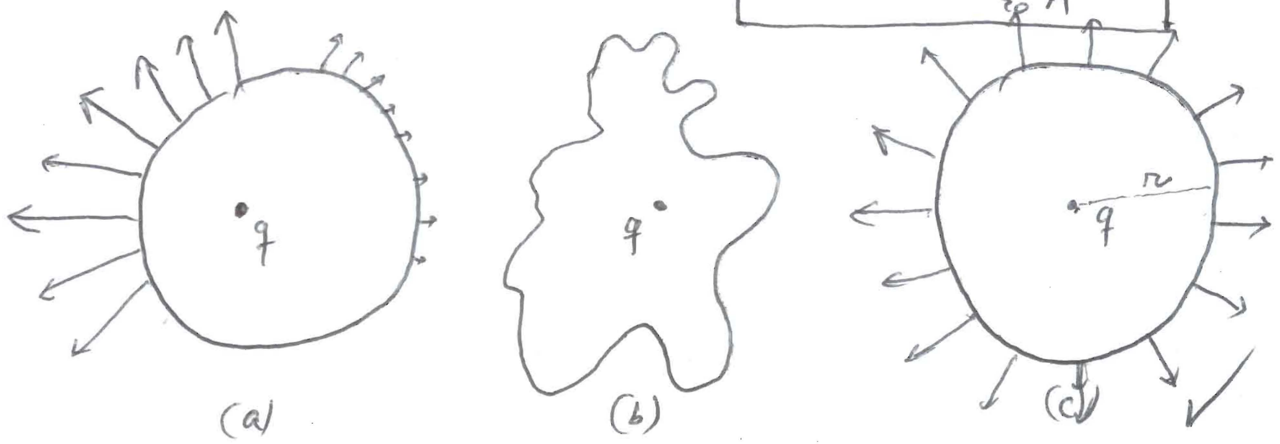
$$\boxed{\vec{E} = \frac{2kqx}{\left[x^2 + \frac{d^2}{4}\right]^{3/2}}\,\hat{\imath}}$$

Gauss Law : Calculation of the electric field due to a point charge

1) Define the Gaussian surface : highly symmetric surface to enclose the charge distribution so the field by charge distribution will be ⊥ surface and has the same strength on that surface.

$$\rightarrow \quad E_\perp \cdot A = \phi_{\substack{\text{Gaussian} \\ \text{surface}}} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\rightarrow \quad \boxed{E_\perp = \frac{q_{enclosed}}{\varepsilon_0 A}}$$



(a) (b) (c)

(i) Total charge enclosed by (a), (b) or (c) is q

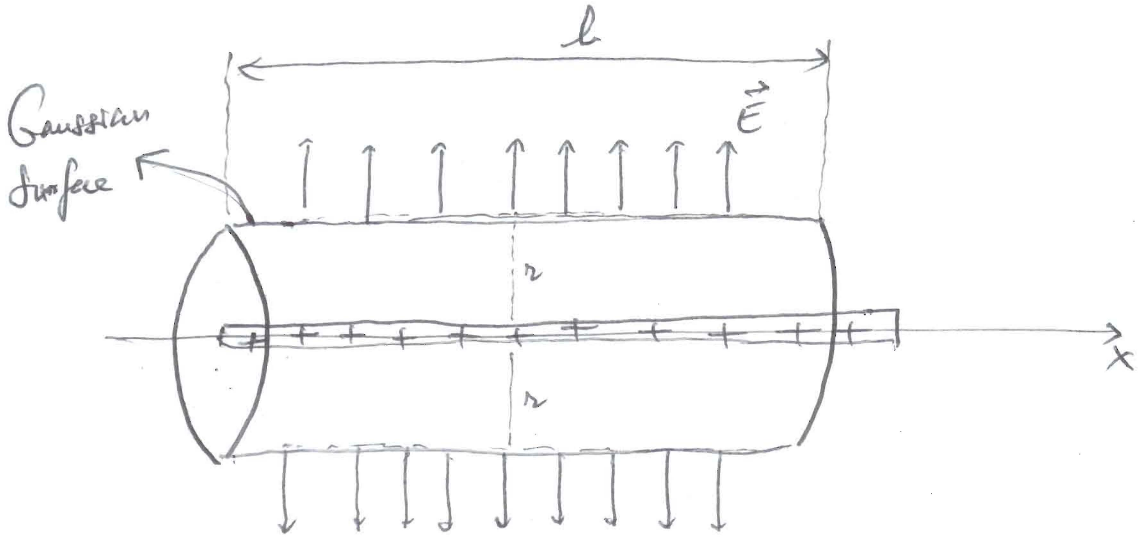
(ii) (c) $\rightarrow$ Gauss law $E_\perp = \frac{q}{\varepsilon_0 4\pi r^2} = \frac{kq}{r^2}$ (same as before !)

$k = \frac{1}{4\pi\varepsilon_0}$

(iii) Direction is ⊥ spherical surface centered @ charge $\rightarrow$ in the radial direction $\rightarrow$ $\vec{E} = \frac{kq}{r^2}\hat{r}$

Gauss Law: Calculation of the electric field for a very long line of charge

$\lambda = \frac{dq}{dx}$: linear charge density



1) Define the Gaussian surface: highly symmetric so field is $\perp$ that surface AND has the same strength on surface.
(From the superposition or direct calculation Ch20: $\vec{E}$ points away and $\perp$ the line of charge) It would have same strength a distance r from the line of charge, in any direction: above & below (as shown), in front, etc..

→ Gaussian surface: cylinder centered at line of charge with a radius r:
- Body: E is $\perp$ to body: $\boxed{E_\perp = E}$
- Left & Right sides: E is $\parallel$ these sides $\boxed{E_\perp = 0}$ → No contribution to $\Phi_{enclosed}$ on Left & Right sides!

$$\Phi_{enclosed} = E_\perp A_{\substack{Body\\ of\ cylinder}} = E \cdot 2\pi r l$$

2) Gauss law:

$$\Phi_{enclosed} = \frac{q_{enclosed}}{\varepsilon_0}$$

$q_{enclosed}$ → length l of line of charge: λl

$$E \cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{2\lambda k}{r}$$

→ Direction: $\perp$ and away from line of charge.

Calculation of Electric Field : Methods
1) Vector superposition (Direct)
2) Gauss Law
3) Electric potential

Electric Potential :

Potential energy b/w A & B: $\Delta U_{AB} = - \int_A^B \vec{F} \cdot d\vec{l}$

Electric interaction : $\vec{F} = q'\vec{E}$
↳ test charge

→ Electric potential energy b/w A & B: $\Delta U_{AB} = -q' \int_A^B \vec{E} \cdot d\vec{l}$ (J)

→ Electric potential b/w A & B = $\boxed{\Delta V_{AB} = \frac{\Delta U_{AB}}{q'} = - \int_A^B \vec{E} \cdot d\vec{l}}$ $\left(\frac{J}{C}\right)$

V for Volt

Can calculate $\vec{E}$: → differentiate electric potential:

$$\vec{E} = - \vec{\nabla}(\Delta V)$$

vector derivative
or gradient

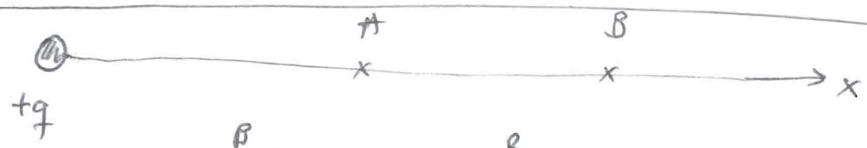
$$\vec{E} = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] V$$

(Taking derivatives of electric potential with a minus sign)

Example #1: Calculation of Electric field due to a point charge using electric potential:

$$V(r) = \frac{kq}{r} \rightarrow \vec{E} = -\hat{r}\frac{\partial}{\partial r}(V) = -\hat{r}kq\frac{\partial}{\partial r}\left(\frac{1}{r}\right) = \hat{r}\,\frac{kq}{r^2} \quad \left(\frac{J}{C\cdot m} = \frac{N\cdot m}{C\cdot m}\right)$$

$\left(\frac{J}{C}\right)$



$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{l} = -\int_A^B \frac{kq}{x^2}\underbrace{\hat{i}\cdot\hat{i}}_{1}dx = -kq\int_A^B \frac{dx}{x^2} = -kq\left[-\frac{1}{x}\right]_A^B$$

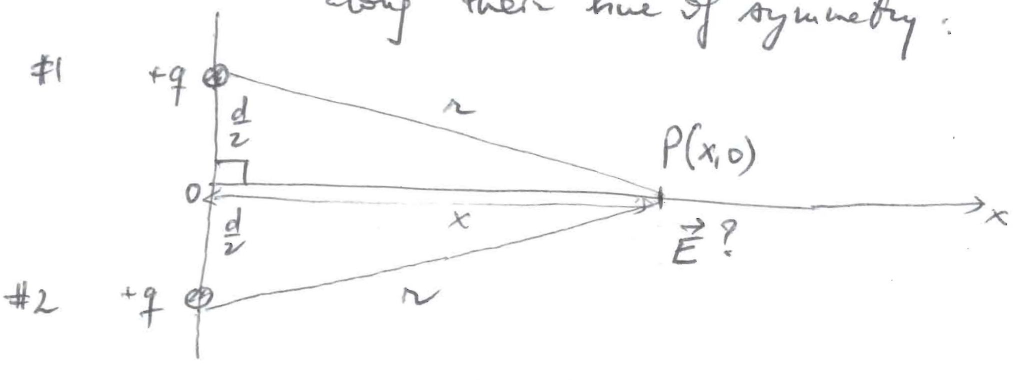
$$\Delta V_{AB} = kq\left(\frac{1}{x_B} - \frac{1}{x_A}\right)$$

Reference Point (0 potential point)

$\hookrightarrow x_A = \infty$

$\rightarrow \boxed{V(r) = \frac{kq}{r}}$ Electric Potential due to a point charge q is $\frac{kq}{r}$

Example #2: Calculation of $\vec{E}$ due to 2 point charges @ a point P along their line of symmetry:



1) Calculate the total electric potential @ P: $V = \underbrace{V_1}_{} + \underbrace{V_2}_{}$

potential due to charge #1   potential due to charge #2

$$V_1 = \frac{kq}{r} = V_2 \rightarrow V = 2V_1 = \frac{2kq}{r} = \frac{2kq}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}$$

(so simple! since the electric potential is a scaler!)

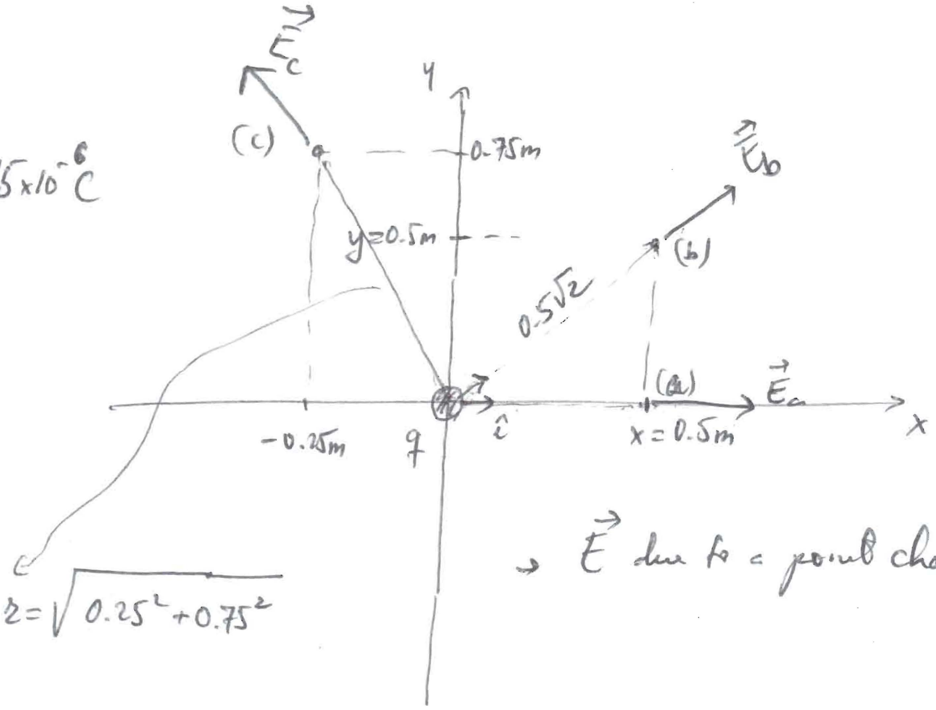
$$2)\ \vec{E} = -\hat{i}\frac{\partial}{\partial x}V = -2kq\,\hat{i}\frac{\partial}{\partial x}\frac{1}{\left[x^2+\left(\frac{d}{2}\right)^2\right]^{1/2}} = -2kq\hat{i}\frac{\partial}{\partial x}\left[x^2+\left(\frac{d}{2}\right)^2\right]^{-\frac{1}{2}}$$

$$= kq\hat{i}\left[x^2+\left(\frac{d}{2}\right)^2\right]^{-\frac{3}{2}}2x = \boxed{\frac{2kqx}{\left[x^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}\hat{i}}$$

Same as with direct method.

(20.46)

$q = 65 \times 10^{-6}$ C



→ $\vec{E}$ due to a point charge is $\frac{kq}{r^2}\hat{r}$ ↓ radial unit vector from charge to probe point.

a) $\vec{E}_a = \frac{kq}{0.5^2}\hat{i} = \frac{9\times10^9 \times 65\times10^{-6}}{0.5^2}\hat{i} = 2.34\times10^6\,\hat{i}\,\frac{N}{C}$

b) $\vec{E}_b = \frac{kq}{(0.5\sqrt{2})^2}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{9\times10^9 \times 65\times10^{-6}}{2\cdot\sqrt{2}\cdot 0.5^2}(\hat{i}+\hat{j})\,\frac{N}{C} = 827\times10^3(\hat{i}+\hat{j})\,\frac{N}{C}$

$\hat{r} = \frac{\hat{i}+\hat{j}}{\sqrt{2}} = \frac{\cancel{0.5}\hat{i}+\cancel{0.5}\hat{j}}{\sqrt{2}\cdot\cancel{0.5}}$

$\hat{i}$: unit vector in x-direction: length is 1
$\hat{j}$: unit vector in y-direction: length is 1

$\hat{i}+\hat{j}$, $\hat{i}$, $\hat{j}$, length is $\sqrt{2}$

c) $\vec{E}_c = \frac{kq}{(0.25^2+0.75^2)}\hat{r} = \underbrace{\frac{\overbrace{9\times10^9\times65\times10^{-6}}^{5.85\times10^5\,?}}{(0.25^2+0.75^2)^{3/2}}}_{1.184\times10^6}\left(-0.25\hat{i}+0.75\hat{j}\right)\frac{N}{C}$

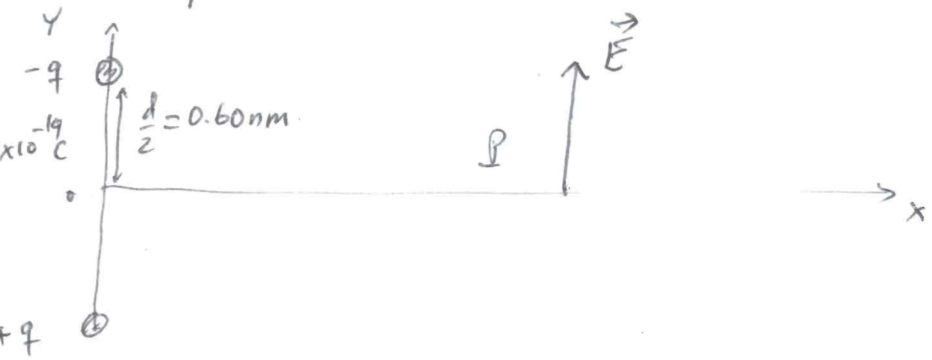
$\hat{r} = \frac{-0.25\hat{i} + 0.75\hat{j}}{\sqrt{0.25^2+0.75^2}}$ (we can get a unit vector by dividing the vector by its length.)

12

20.50 Electric dipole

Electron:
$-q = -e = -1.6\times10^{-19}\,C$

Proton:
$q = e$



$$\vec{E} = +\frac{kqd}{\left[x^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}\,\hat{j}\quad\left(\frac{N}{C}\right)$$

a) Midpoint b/w 2 charges: $x=0 \rightarrow \vec{E} = \dfrac{9\times10^{9}\times1.6\times10^{-19}\times1.2\times10^{-9}}{(0.6\times10^{-9})^3}\,\hat{j}$

$$= 8\times10^{9}\,\hat{j}\,\left(\frac{N}{C}\right)$$

b) @ $x = 2nm,\ y=0 \rightarrow \vec{E} = \dfrac{9\times10^{9}\times1.6\times10^{-19}\times1.2\times10^{-9}}{\left[(2\times10^{-9})^2+(0.6\times10^{-9})^2\right]^{3/2}}\,\hat{j}$?

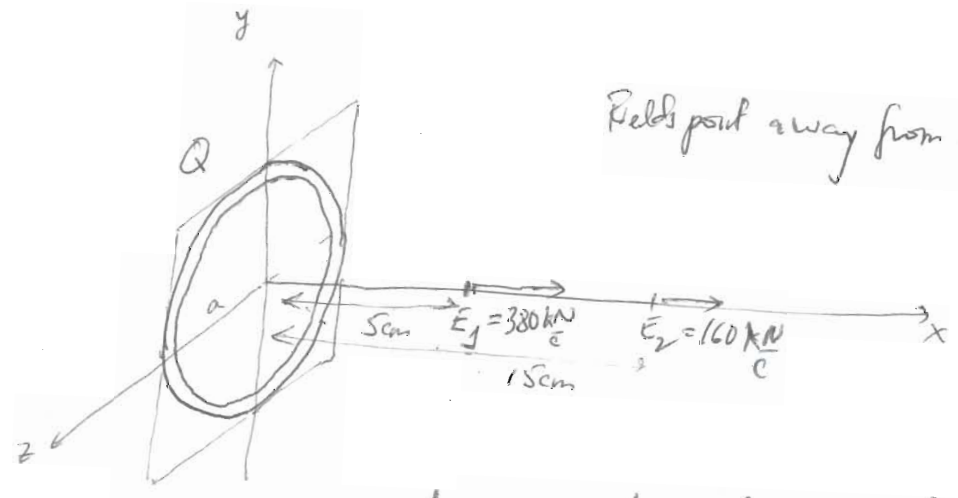
$$= 190\times10^{6}\,\hat{j}\,\left(\frac{N}{C}\right) \quad = 1.89\times10^{8}$$

c) @ $x = -20nm\ \ y=0 \rightarrow \vec{E} = 216\times10^{3}\,\hat{j}\,\left(\frac{N}{C}\right)$

Gaussian surface of cross sectional area A $\quad \varepsilon_0 \quad 2\varepsilon_0 \quad = 31.0\ \frac{}{m^2}$

20.65

Field point away from ring → $Q > 0$



$$\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{\imath}$$

Loc. of a probe point on axis of symmetry of ring

ch 20

a) Radius a? Ratio of field strengths to eliminate Q:

$$\left[\frac{E_1}{E_2} = \frac{380}{160} = \frac{x_1}{(x_1^2+a^2)^{3/2}} \cdot \frac{(x_2^2+a^2)^{3/2}}{x_2}\right]^{2/3}$$

To solve for a

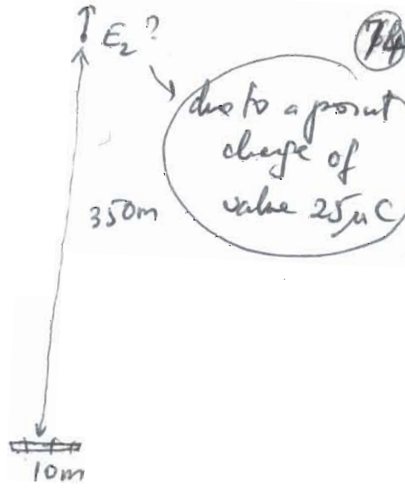
$$\left(\frac{380}{160}\right)^{2/3} = \underbrace{\left(\frac{0.05}{0.15}\right)^{2/3}}_{\left(\frac{1}{3}\right)^{2/3}} \cdot \frac{(0.15^2+a^2)}{(0.05^2+a^2)} \rightarrow a = 0.07\,m$$

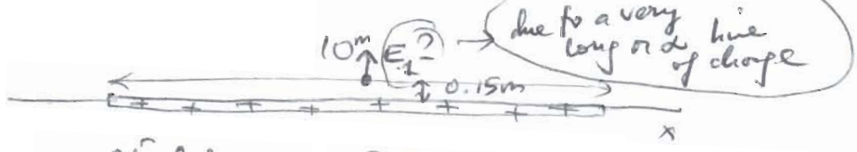
b) $Q = ?$ → Use E_1 or E_2:

$$E_1 \rightarrow Q = \frac{E_1 (x_1^2+a^2)^{3/2}}{k x_1} = \frac{380\times10^3 (0.05^2+0.07^2)^{3/2}}{9\times10^9 \times 0.05} = 538\times10^{-9}\,C$$

$$= 538\,nC$$

(20.67)



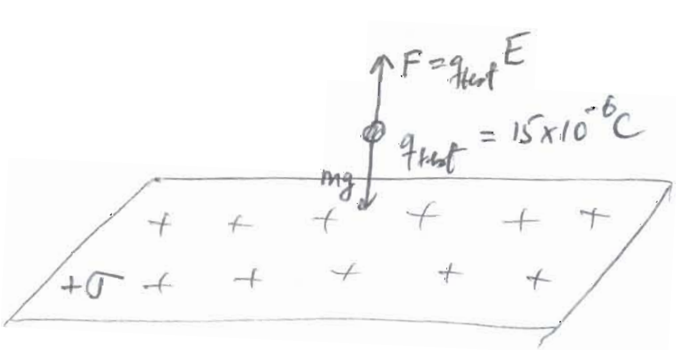


a) $\left.\begin{matrix}25\mu C\\ 10m\end{matrix}\right\}$ $\lambda = \dfrac{25\mu C}{10m} = 2.5\,\dfrac{\mu C}{m}$

b) $E_1 = \dfrac{2k\lambda}{r} = \dfrac{2\times 9\times 10^9 \times 2.5\times 10^{-6}}{0.15} = 300\ k\dfrac{N}{C}$

c) $E_2 = \dfrac{kq}{r^2} = \dfrac{9\times 10^9 \times 25\times 10^{-6}}{350^2} = 1.84\,\dfrac{N}{C}$

(21.56)



Particle with $q_{test} = 15\times 10^{-6}C$ and mass $m = 5\times 10^{-3}$ kg will suspend:

$$q_{test}E = mg$$
$$E = \frac{mg}{q_{test}}$$

Surface charge density:
$$\sigma = \frac{Q}{A}$$

E as a function of σ can be calculated using Gauss law:
$$E = \frac{\sigma}{2\varepsilon_0}$$

$$\sigma = 2\varepsilon_0 \frac{mg}{q_{test}} =$$
$$= \frac{2\times 8.85\times 10^{-12}\times 5\times 10^{-3}\times 9.81}{15\times 10^{-6}}$$
$$= 57.8\ \frac{nC}{m^2}$$

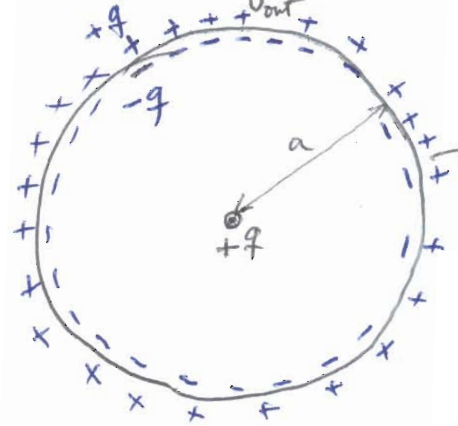
Gaussian surface of cross sectional area A

E ⊥ plane → Φ only gets contribution from top & bottom sides of Gaussian surface.

$$\boxed{\Phi = EA\times 2} = \frac{\sigma A}{\varepsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Conductors - (i) no field inside, (ii) free electrons → attracted by + charges.

(21.60)



→ spherical conducting shell of radius a

$q = 250\,nC$

$a = 0.2\,m$

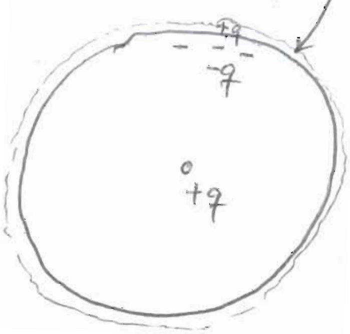
a) Surface charge density on outer surface of shell $\sigma_{out} = \frac{+q}{A} = \frac{250\times10^{-9}}{4\pi\, 0.2^2}$

$A = 4\pi a^2$

$= 497\ \frac{nC}{m^2}$

b) Electric field strength at the shell outer surface:

Place a spherical Gaussian surface on the outer shell:



$$\phi = \frac{q_{enclosed}}{\varepsilon_0} = \frac{(+q-q+q)}{\varepsilon_0}$$

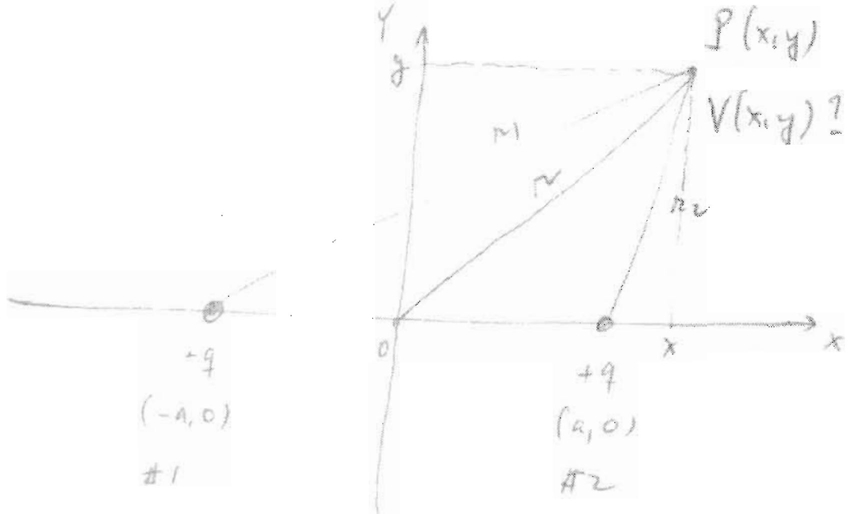
$$\varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{Nm^2}$$

$$E \cdot A_{Gaussian} = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{A_{Gauss}\cdot\varepsilon_0} = \frac{\sigma_{out}}{\varepsilon_0}$$

$$= \frac{497\times10^{-9}}{8.85\times10^{-12}} = 56.2\ \frac{kN}{C}$$

22.53



a) $V(x,y) = \underset{\text{due charge \#1}}{V_1(x,y)} + \underset{\text{due to charge \#2}}{V_2(x,y)}$

$$= \frac{kq}{r_1} + \frac{kq}{r_2} = \frac{kq}{\left[(x+a)^2+y^2\right]^{1/2}} + \frac{kq}{\left[(x-a)^2+y^2\right]^{1/2}}$$

b) What is $V(x,y)$ approximately if P is very far away from the two charges: $x \gg a$ & $y \gg a$

$$V(x,y) \simeq \frac{kq}{(x^2+y^2)^{1/2}} + \frac{kq}{(x^2+y^2)^{1/2}} = \frac{2kq}{(x^2+y^2)^{1/2}}$$

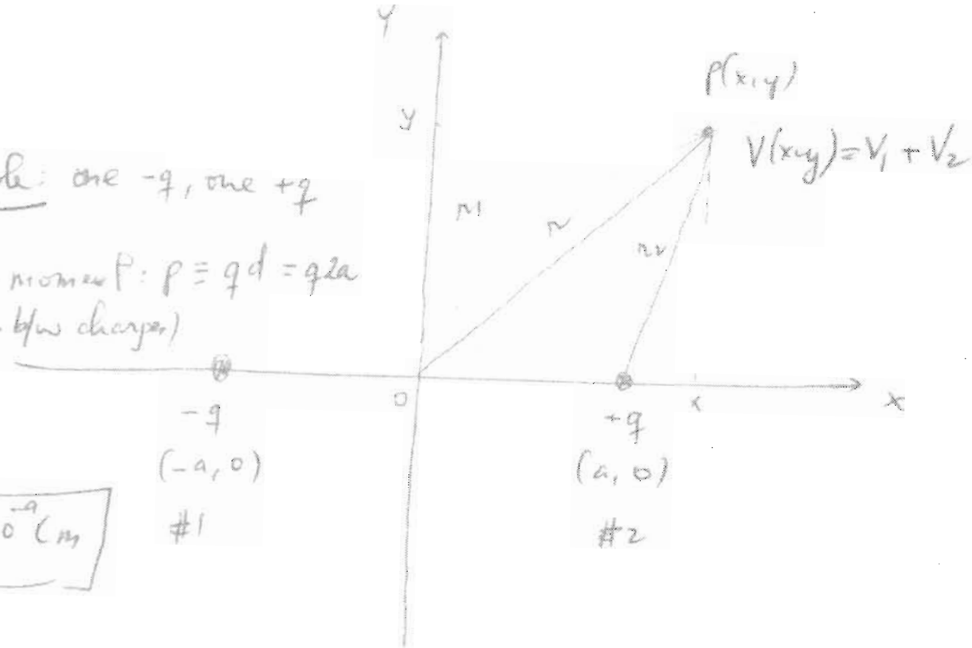
$$= \frac{2kq}{r}$$ $\rightarrow$ Far away the electric potential is that of one point charge of value $2q$

22.54

Dipole: one $-q$, one $+q$

Dipole moment P: $p \equiv qd = q2a$
(d: sep. b/w charges)

$\boxed{p = 2.9\times10^{-9}\ \text{Cm}}$



$$V(x,y) = kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = kq\left[\frac{1}{[(x-a)^2+y^2]^{1/2}} - \frac{1}{[(x+a)^2+y^2]^{1/2}}\right]$$

a) P Along dipole axis or x-axis $\rightarrow y = 0$

$$V(x,0) = kq\left[\frac{1}{x-a} - \frac{1}{x+a}\right] = kq\left[\frac{x+a-x+a}{(x-a)(x+a)}\right]$$

$$= \frac{kq\,2a}{x^2-a^2} = \frac{kp}{x^2-a^2} \simeq \frac{kp}{x^2} = \frac{9\times10^{9}\times2.9\times10^{-9}}{0.1^2}$$

{0° to axis
{r = 0.1 m
{r >> a

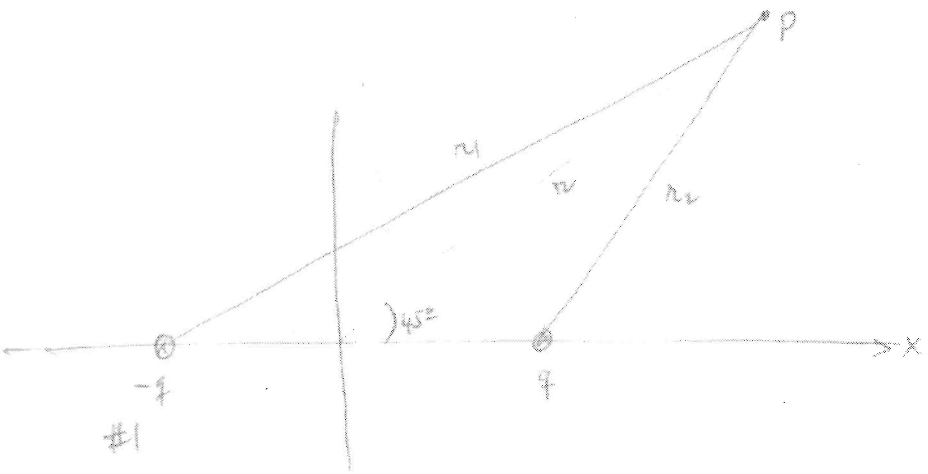
dipole sep. (2a) << x = 10 cm (data)

$= 2.61\times10^{3}$ V

b) P @ {45° to axis, r = 0.1 m, r >> a} $x = y = r\cos45°$

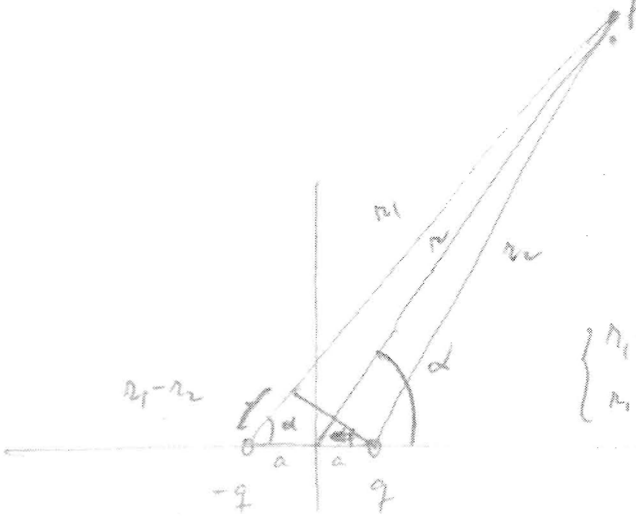
$$V(x,y) = kq\left[\frac{1}{((x-a)^2+y^2)^{1/2}} - \frac{1}{((x+a)^2+y^2)^{1/2}}\right]$$

31, 70, 67



Here polar coordinates are more useful.

$$V(x,y) = kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = kq\,\frac{r_1 - r_2}{r_1 r_2} \underset{\text{P very far away}}{\simeq} kq\,\frac{2a\cos\alpha}{r^2}$$



$$\begin{cases} r_1 - r_2 \simeq 2a\cos\alpha \\ r_1 \simeq r_2 \simeq r \end{cases}$$

$\alpha = 45°$
$r = 0.1\,m$

$$V(x,y) = \frac{k\overbrace{q(2a)}^{p}\cos\alpha}{r^2} = \frac{9\times10^{9} \times 2.9\times10^{-9} \cos 45°}{0.1^2} = 1.85\,kV$$

c) P along bisector: $\alpha = 90°$:

$$V(x,y) = \frac{9\times10^{9} \times 2.9\times10^{-9}\, \underbrace{\cos 90°}_{\to 0}}{0.1^2} = 0$$

22.31

$$V(x,y,z) = 2xy - 3zx + 5y^2$$

$P(x = 1m, y = 1m, z = 1m)$

a) $V(1,1,1) = 2 - 3 + 5 = 4V$

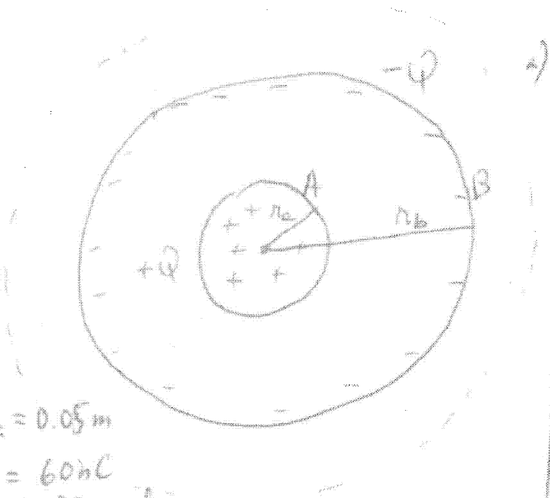
b) $\vec{E} = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$$= -\hat{i}(2y - 3z) - \hat{j}(2x + 10y) - \hat{k}(-3x)$$

$$\vec{E}(1,1,1) = -\hat{i}(-1) - \hat{j}(12) - \hat{k}(-3)$$

$$= \hat{i} - 12\hat{j} + 3\hat{k} \quad \left(\frac{N}{C}\right)$$

22.70



$r_a = 0.05 m$
$Q = 60 nC$
conducting sphere

$r_b = 0.15 m$
$Q = -60 nC$
conducting shell

a) Find V @ $r = r_a$

$$V(r = r_a) = \Delta V_{\infty A} = -\int_{\infty}^{A} \frac{kQ}{r^2}\,dr$$

$$= +kQ\left[\frac{1}{r}\right]_{\infty}^{A} = \frac{kQ}{r_A}$$

(True if there was no outer shell)

$$V(r = r_a) = \Delta V_{\infty A} = \underbrace{\Delta V_{\infty B}}_{} + \Delta V_{BA}$$

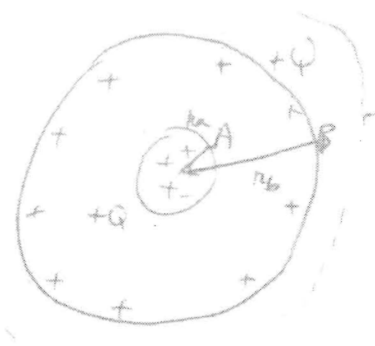
E outside outer shell is 0 ! } Gaussian surface enclosing shell + sphere contains $+Q - Q = 0$

$$= -\int_{B}^{A} \frac{kQ}{r^2}\,dr = kQ\left[\frac{1}{r}\right]_{B}^{A}$$

$$= kQ\left[\frac{1}{r_A} - \frac{1}{r_B}\right] = 9\times10^9 \times 60\times10^{-9}\left[\frac{1}{0.05} - \frac{1}{0.15}\right]$$

$$\boxed{V(r = r_a) = 7200\ V}$$

b)



E here outside shell + sphere is not zero.

$$V(r=r_a) = \Delta V_{\infty A} = \underbrace{\Delta V_{\infty B}}_{\downarrow} + \underbrace{\Delta V_{BA}}_{7200\,V}$$

7200 V (only changing charge on shell!)

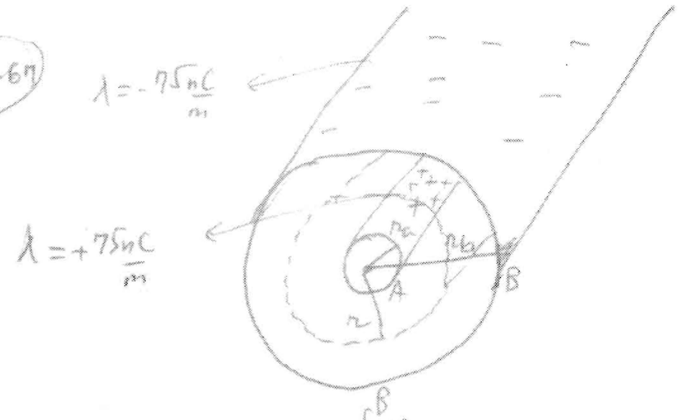
$$-\int_{\infty}^{B} \frac{k2Q}{r^2}\,dr$$

$$= k2Q\left[\frac{1}{r}\right]_{\infty}^{B} = \frac{k2Q}{r_B} = \frac{9\times10^9 \times 2 \times 60\times10^{-9}}{0.15}$$

$$= 7200\,V$$

$$V(r=r_a) = 7200 + 7200 = 14400\,V.$$

22-67



Coaxial cable { inner cylinder + outer shell with the same axis

c) $$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{r}$$

Field b/w inner & outer conductor { Gaussian Law with G. Surface of radius r $r_a < r < r_b$

↓

Electric due to inner conductor (very long wire)

$$\rightarrow E = \frac{2k\lambda}{r}\hat{r}$$

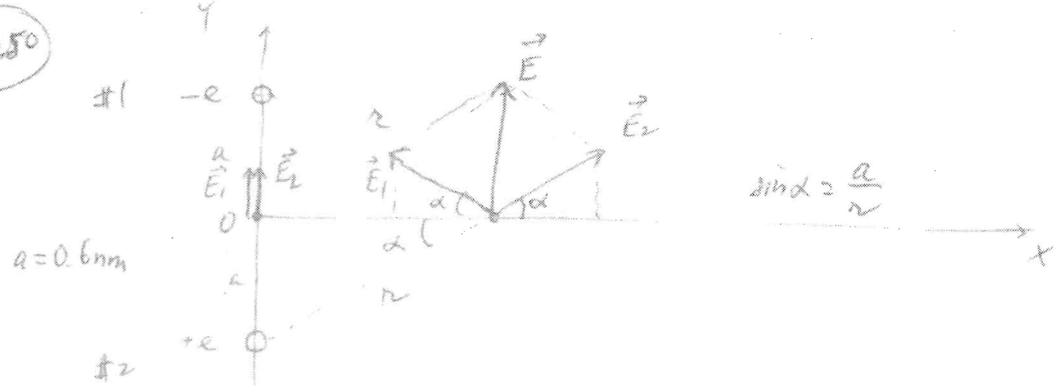
$$\Delta V_{AB} = -\int_A^B \frac{2k\lambda}{r}\,dr = -2k\lambda \int_A^B \frac{dr}{r} = -2k\lambda \ln\left(\frac{r_B}{r_A}\right)$$

$$= -2\times 9\times10^9 \times 75\times10^{-9} \ln\left(\frac{10\,mm}{2\,mm}\right) = -2170\,V$$

b) if λ for outer conductor changes to $+150\frac{nC}{m}$

ΔV_{AB} = same = $-2170V$ (since this would not change the electric b/w inner & outer conductor. It only change field outside outer conductor)

2.150



a) $\vec{E}(x=0, y=0) = \vec{E}_1 + \vec{E}_2 = \frac{ke}{a^2}\times 2\,\hat{j} = \frac{9\times10^9\times1.6\times10^{-19}\times2}{0.6\times10^{-9}}\,\hat{j}\,\left(\frac{N}{C}\right)$

$= 8\times10^9\,\hat{j}\,\left(\frac{N}{C}\right)$

b) $\vec{E}(x=2nm, y=0) = 2E_y\,\hat{j} = 2\left(\frac{ke}{r^2}\right)\sin\alpha\,\hat{j} = 2\frac{kea}{r^3}\,\hat{j}$

$= 2\frac{kea}{(x^2+a^2)^{3/2}}\,\hat{j} = \hat{j}\,2\frac{9\times10^9\times1.6\times10^{-19}\times0.6\times10^{-9}}{\left[(2\times10^{-9})^2+(0.6\times10^{-9})^2\right]^{3/2}}$

$r=(x^2+a^2)^{1/2}$

$= \hat{j}\,190\times10^6\,\frac{N}{C}$

c) $\vec{E}(x=-20nm, y=0) = \hat{j}\,2\frac{9\times10^9\times1.6\times10^{-19}\times0.6\times10^{-9}}{\left[(20\times10^{-9})^2+(0.6\times10^{-9})^2\right]^{3/2}} = 216\times10^3\,\frac{N}{C}$