Physics II 2011

Ch16 Temperature & Heat

Temperature: a macroscopic quantity or property (we can sense and measure) that is a consequence of microscopic behavior of matter. (energy)

→ **Units:**

- °F (degree Fahrenheit) (Body temp. ≈ 100°F)
- °K (degree Kelvin) → Absolute zero temp. is 0°K
- °C (degree Celsius) — H_2O { Freezing : 0°C, Boiling : 100°C } → Normal conditions

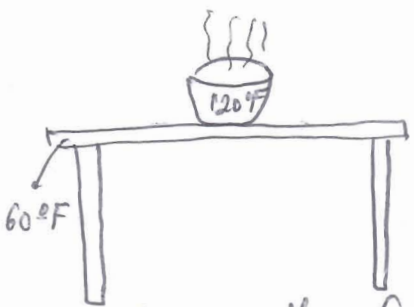
$$T_F = \frac{9}{5}T_C + 32$$

$$T_C = \frac{5}{9}(T_F - 32)$$

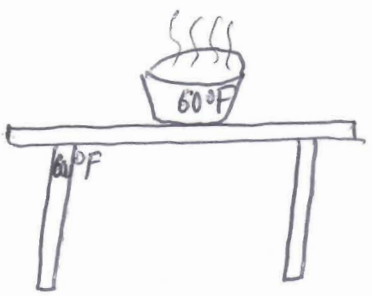
$$T_C = T_K - 273.16^\circ$$

Triple point of water (3 phases coexist @ this temperature)

Hot bowl of soup 120 °F



Now it is in thermal contact with the table: they can exchange heat

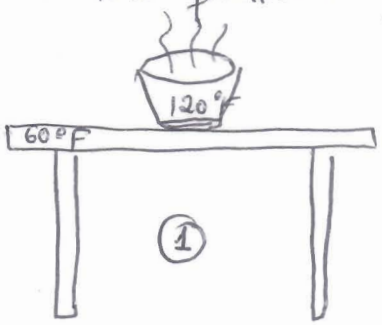


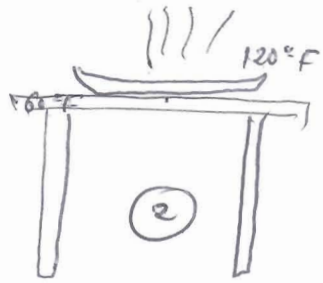
After 8 hours: Temperature has changed, they have arrived at a thermodynamic equilibrium (T.D. equilibrium):

1) $T_{final} = 60^\circ F$ (since table is in thermal contact with the room)
2) By exchanging heat

Heat exchange or transfer rate (heat transfer per unit time): H

H: how fast heat can be transferred b/w two objects; let's describe this quantitatively using our intuition:





→ Larger surface area (in thermal contact) → faster heat transfer
→ Also: faster heat transfer if plate or/and table were metal: → (free electrons → energy carriers)
thermal constant k

$$H = -k \frac{A}{\Delta x} \Delta T$$

↓

Thermal constant of material heat needs to travel through (hot matter)

(Temperature is a consequence of microscopic properties of matter)

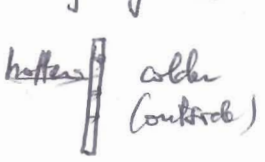
larger thickness Δx
↓
slower heat transfer rate!

larger ΔT (b/w soup & table)
→ faster heat transfer

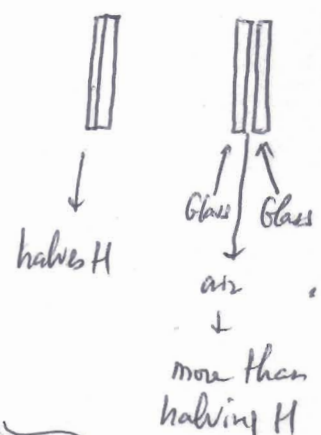
Thermal constant k:

Glass windows : k

Single pane



Double pane



Microscopic connection:
→ Air is poorer heat conductor b/c its lower density.

→ But some gas is needed b/w layers to hold against atmospheric pressure.

Units for H:

Quantity	Dimension	SI. unit
Length	L	meter (m)
Mass	M	kilogram (kg)
Temperature	T	degree Kelvin (°K)
Time	T	second (s)
Energy	$\frac{ML^2}{T^2}$	joule (J)
Power	$\frac{ML^2}{T^3}$	$\frac{\text{joule}}{s} = \text{watt}$ (W)

Energy → Mechanical: kinetic, potential
Thermal or heat

Dimension for H:

$$[H] = [k][A]\frac{[\Delta T]}{[\Delta x]}$$

$[H]$ → $\frac{J}{s} = W$

$[k]$ → $\frac{W}{m \cdot {}^\circ K}$ (SI)

$[A]$ → m^2

$[\Delta T]$ → ${}^\circ K$

$[\Delta x]$ → m

$$\text{Unit of } k: \begin{cases} \frac{W}{m \cdot {}^\circ K} & \text{(SI)} \\ \frac{\text{Btu} \cdot \text{in}}{h \cdot ft^2 \cdot {}^\circ F} & \text{(British)} \end{cases}$$

in → inch

h → hour

$\mathcal{R}$-factor or another way of writing H: $\mathcal{R} = \frac{\Delta x}{k}$

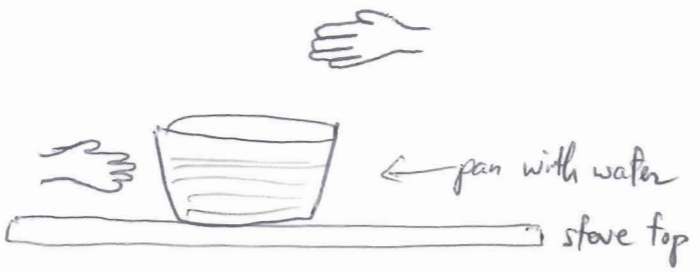
(Curly R)

$$\boxed{H = -k\frac{A\Delta T}{\Delta x} = -\frac{A\Delta T}{\mathcal{R}}}$$

→ Commonly used in insulator materials: ($\uparrow \mathcal{R}$ are more expensive and better insulator):

$\mathcal{R}$-21 means: $\mathcal{R} = 21\ \frac{ft^2 \cdot {}^\circ F}{\frac{\text{Btu}}{h}}$

Mechanisms of Heat Transfer :



Initially : turn on stove, stove top is heated, then heat gets transfered into water at bottom of pan : heat transfer by **conduction** (via free electrons in bottom of metal pan).

Water at bottom of pan gets heated, then water at top of pan gets heated : heat transfer by **convection** (in fluid : liquid or gas)

Air convection : { heaters : located low : hot air will rise & cold air will sink.
AC's : located high to allow the air convection.
so all parts of a room can be at a same temperature.

When placing your hand next to pan you feel the heat → by **radiation**
Does it require a medium with matter ?

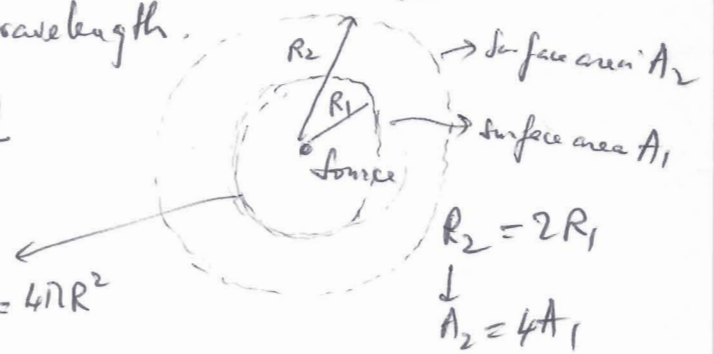
Sound waves need a medium : air

Radiation such as light (Sun) requires no medium to propagate : EM (electromagnetic) waves (infrared, microwaves, lights, depending on their wavelengths)

Radiowaves : $\lambda \sim 1m$

Can't get a good signal sufficiently far from station : because of reflections & interferences with structures of sizes comparable to its wavelength.

$$\text{Intensity} = \frac{\text{strength (wave amplitude)}}{\text{unit area}}$$



→ intensity goes down 4 fold as the radius is doubled
→ from spherical geometry (not wave reflection & interferences!)

Convection: in fluids there was a phenomenon we can take advantage for transportation: in air and in water (hot air balloons and ships:) buoyancy (lower density stays above higher density)

Radiation: Stefan-Boltzman Law: heat loss rate by radiation of an object of area A & temperature T

$$P = e \sigma A T^4$$

- P: energy loss per unit time or power
- e: emissivity coefficient
- σ: 'sigma' or Stefan-Boltzman constant $5.67 \times 10^{-8} \frac{W}{m^2 K}$

Heat Balance Equation:

Dropping (or bringing into thermal contact) a hot block of Cu and some water → by heat exchange arrives @ T.D. equilibrium → Both Cu & H_2O will have same final temp. of T_f

Heat exchange direction is $Cu \rightarrow H_2O$ $(T_w < T_f < T_{Cu})$

$$\underbrace{\Delta Q_{Cu}}_{\text{Heat loss by Cu}} + \underbrace{\Delta Q_w}_{\text{Heat gain by } H_2O} = 0$$

(Assuming system water & Cu is isolated)
↓
(Heat left Cu went into H_2O)

negative → $m_{Cu} C_{Cu} (T_f - T_{Cu})$; positive → $+ m_w C_w (T_f - T_w) = 0$

C_{Cu}: specific heat of copper

Specific heat: capacity to absorb heat per unit mass: specific to the material

$C_w = 4184 \frac{J}{kg\,°K}$

$C_{Cu} = 386 \frac{J}{kg\,°K}$

Ch 17 Thermal Behavior of Matter

Matter can exist in 3 states

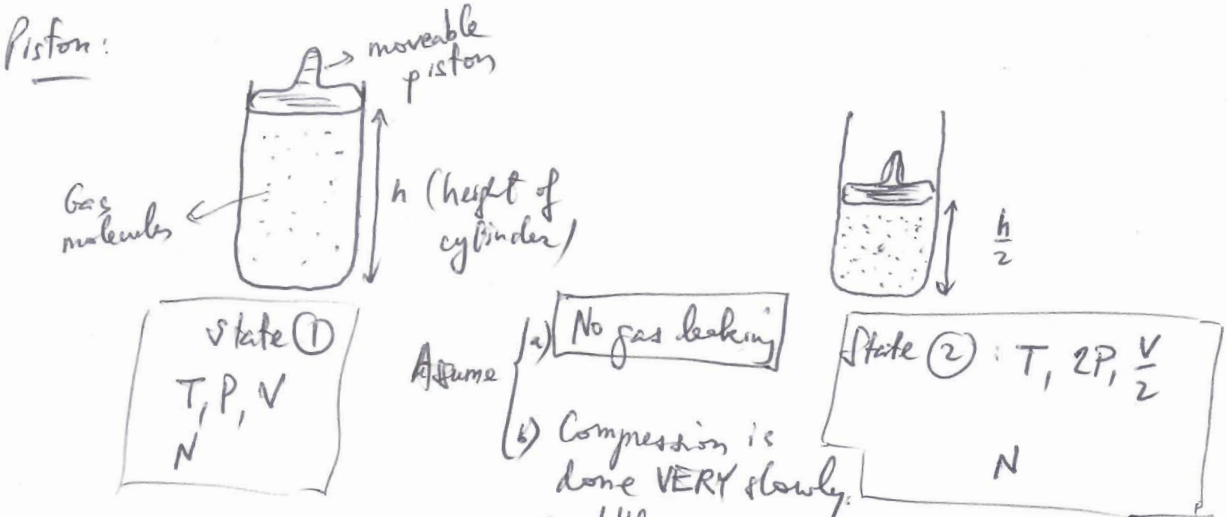
Density	State
low	Gas : almost free molecules (not interacting)
Medium	Liquid : some interaction b/w molecules
High	Solid : molecule / atom are not free move.

Microscopic properties will influence / determine macroscopic thermal behavior.

Thermal Behavior of Gases:
- Ideal gases → molecules are completely free!
- Real gases → some interaction b/w molecule

Ideal Gases
- Microscopic : N molecules or particles
- Macroscopic behavior
  - T (Temp, °K)
  - P (Pressure, Pa)
  - V (Volume, m^3)

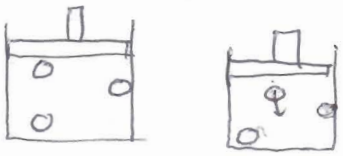
Piston:



→ Observations:
- P·V is the same in ① & ②
- N·T is also the same in ① & ②

$PV = (k)NT$

k = Boltzman constant $= 1.38 \times 10^{-23} \frac{J}{°K}$



if we goes slow enough we may hit any molecule → no additional KE for molecule
Although they will hit each other more frequently due to reduced volume they didn't receive any extra KE. (or may have enough time to transfer it back to the container)

Avogadro number: $N_A = 6.022 \times 10^{23}$

$N = n \cdot N_A$: n = number of moles

Ideal gas equation → $P.V = kNT = k n N_A T = n\, kN_A\, T = nRT$

$R = 8.314 \frac{J}{{}^\circ K\, mol}$

→ $\boxed{P.V = nRT}$ (Ideal gases where there is no interaction b/w molecules)

Real gases: with some interaction b/w molecules:

→ $\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$

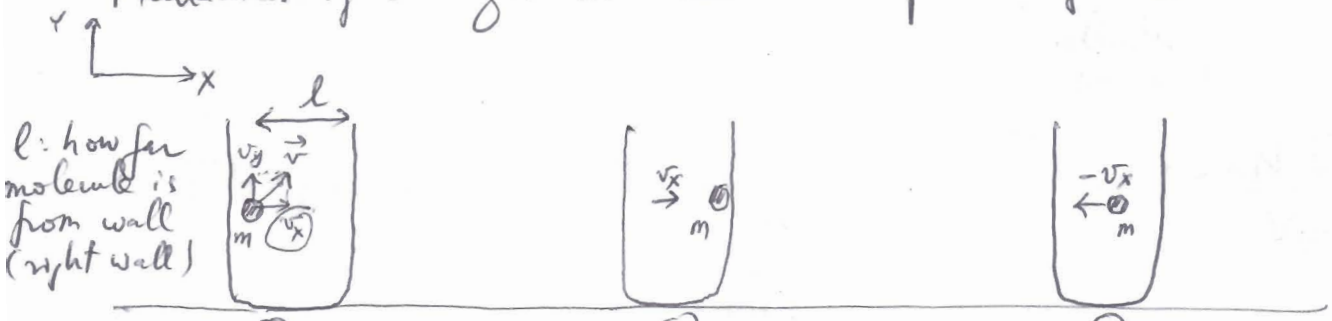
a, b are given constants, depending on the strength of interactions b/w molecules.

A closer look at microscopic behavior & macroscopic properties.

quantitative

- Transfer of momentum b/w gas molecule & container
- Temperature, Pressure

Mechanics of one gas molecule : a particle of mass m



① - Molecule travels in some direction inside a container.
- In term of momentum transfer to container → look at v_x: Molecule moving toward right wall @ v_x

② - Hits right wall
- Elastic collision → ③ - Rebounds with same speed in opposite direction

System = gas molecule + container

Net external force on system : in x-direction : $F_{net,x} = 0$
(mg is along y-direction)

2nd Newton's Law → $\frac{dp_x}{dt} = 0$

→ p_x of system is conserved

Before collision ①		After Collision ③
$mv_x + \underbrace{M \cdot 0}_{0}$	=	$-mv_x + \underbrace{Mv_{x_{fin}}}_{\parallel} \; 2mv_x$

After the elastic collision the molecule has transferred $2mv_x$ of momentum to the container :

Pressure: by one molecule → force applied per unit area

$$P_i = \frac{F_i}{A} = \frac{\frac{\Delta p_i}{\Delta t}}{A} = \frac{\frac{2m v_x}{\frac{2l}{v_x}}}{A} = \frac{m v_x^2}{A \cdot l} = \frac{m v_x^2}{\text{Vol.}}$$

($A \cdot l$ = Volume)

Pressure due to N molecules $P = \sum_{i=1}^{N} P_i = \frac{m}{vol} \sum_{i=1}^{N} v_{x_i}^2$

(all molecules have the same mass)

Total pressure $P = \frac{mN}{vol} \underbrace{\frac{\sum_{j=1}^{N} v_{x,j}^2}{N}}_{\text{average of } v_x^2 \equiv \overline{v_x^2}}$ $\rightarrow P = \frac{mN\,\overline{v_x^2}}{V}$ (V ← vol.)

$$\rightarrow \boxed{PV = mN\,\overline{v_x^2} = mN\frac{\overline{v^2}}{3}} \quad (2)$$

This came from $F = \frac{\Delta p}{\Delta t}$ 2nd Newton / Transfer of momentum.

In general v_x is one of the three components of the velocity $\vec{v}$ of magnitude v $\rightarrow \boxed{\overline{v_x^2} = \frac{1}{3}\overline{v^2}}$ Mathematically

Compare this with ideal gas equation: $\boxed{PV = kNT}$ (1)

→ RHS's are equal!

$$kNT = mN\frac{\overline{v^2}}{3}$$

$$3kT = m\overline{v^2}$$

$$\frac{3}{2}kT = \frac{1}{2}m\overline{v^2}$$

Gas Temperature → Macroscopic

Average KE per gas molecule → Microscopic

The microscopic average KE per molecule determines the macroscopic temperature of the gas

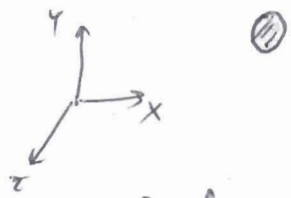
Dimensions & Degrees of Freedom (d.o.f)

3D: $\longrightarrow \frac{3}{2}kT = \frac{1}{2}m\overline{v^2}$ { Average KE per molecule in 3D is $\frac{3}{2}kT$

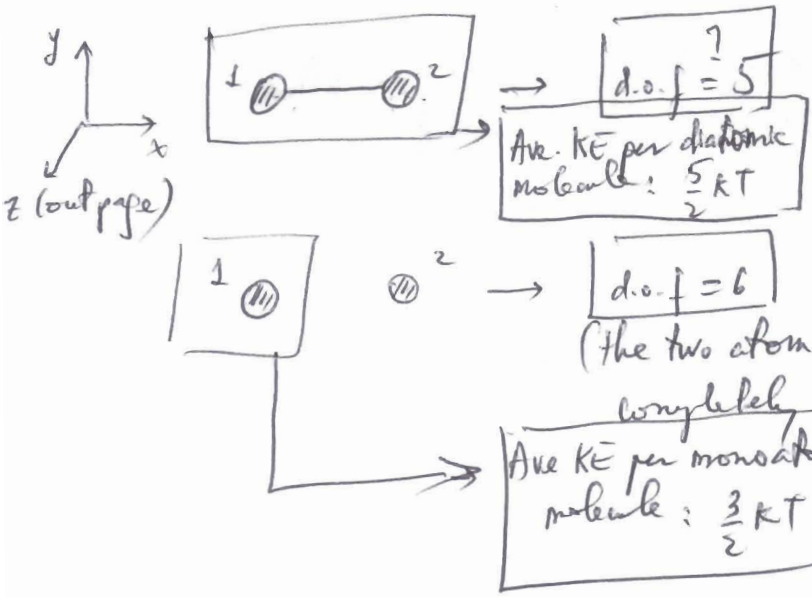
What is the average KE per molecule in 1D in term of the gas temperature? $\frac{1}{2}kT$ → Average KE per molecule per dimension or d.o.f is $\frac{1}{2}kT$

What is the average KE per molecule in 2D? kT

Single molecule: or monoatomic molecules (one atom) → d.o.f = 3



Diatomic molecule (e.g. O_2): two-atom molecule



1 → x-direction: (1) moves right to left (2)

2 → y-direction: (1) move up, (2) will not necessarily

2 → z-direction: (1) moves out, (2) will not necessarily

Thermal Behavior of Matter (Cont.)

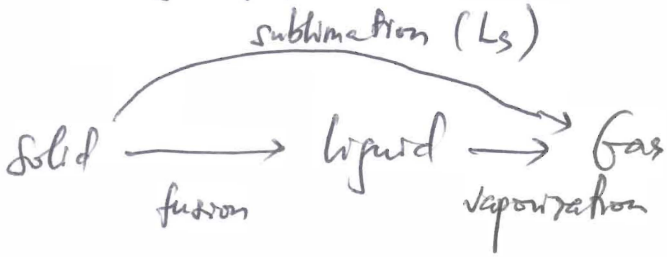
When heat is applied to matter what are the consequences?

heat → ΔQ

1) Temperature increase: $\Delta T \rightarrow \Delta Q = mc\Delta T$ (Ch16)

2) Expansion
- Linear expansion coefficient: $\alpha = \frac{\frac{\Delta L}{L}}{\Delta T}$ (Length = L; temp = T)
- volumic expansion coefficient: $\beta = \frac{\frac{\Delta V}{V}}{\Delta T}$ (Volume = V, temp = T)

3) Change of phase:



$\Delta Q = mL$
- m: mass
- L: Latent heat L_s, L_f, L_v

Different depending on the particular change of phase

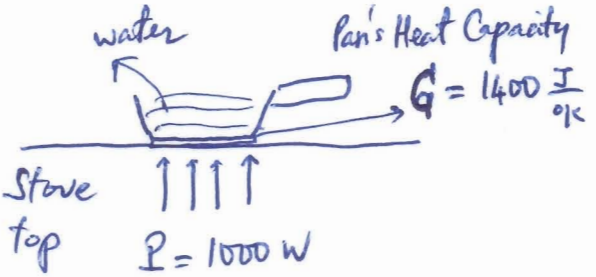
Table 17.1

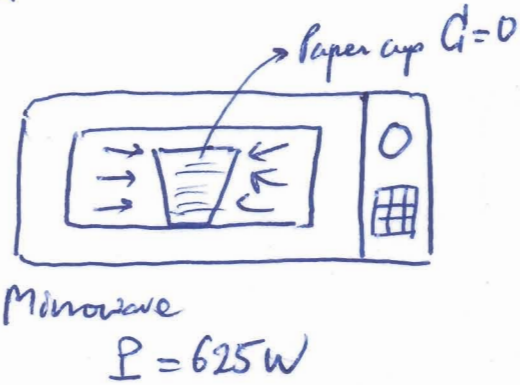
water
- $L_f = 334 \frac{kJ}{kg}$
- $L_v = 2257 \frac{kJ}{kg}$

16.49/

1) Write down data & making a sketch
2) Write down the appropriate equation
3) Solve for needed information

1)





Heat Capacity: $G = c \cdot m$

(c: specific heat; m: mass)

Relating to the expression for heat exchange:

$$\Delta Q = mc\Delta T = G\Delta T$$

a) Question: How much water before it is **faster** to heat using the stove top?

→ Start with an amount of water of mass m

→ Find how long it would take to increase the temperature by ΔT using stove Δt_s and using the microwave: Δt_m and then set $\Delta t_s < \Delta t_m$ → find that critical mass

Stove

2)(3) Δt_s ← time to heat m_w so its temperature rises ΔT

$$H_s = 1000\,W = \frac{\Delta Q}{\Delta t_s}$$

$$\Delta t_s = \frac{\Delta Q}{1000} = \frac{(C_w + C_{pan})\Delta T}{1000}$$

Heat supplied by stove top is absorbed by both water & pan

We need mass of water: m_w

$$\rightarrow C_w = m_w c_w$$

$$\Delta t_s = \frac{(m_w 4184 + 1400)\cdot \Delta T}{1000}$$

Microwave

Δt_m →

$$H_m = 625\,W = \frac{\Delta Q}{\Delta t_m}$$

$$\Delta t_m = \frac{\Delta Q}{625} = \frac{(C_w + \overset{\text{paper cup}}{\uparrow 0})\Delta T}{625}$$

Heat supplied by the microwave is absorbed only by water (paper cup's heat capacity $C \approx 0$)

$$c_w = 4184\ \frac{J}{kg\,K}\ ;\ C_{pan} = 1400\ \frac{J}{^\circ K}$$

$$\Delta t_m = \frac{(m_w 4184)\Delta T}{625}$$

3) When is it faster to heat m_w with stove?

$$\Delta t_s < \Delta t_m$$

$$\frac{(m_w 4184 + 1400)\cancel{\Delta T}}{1000} < \frac{m_w 4184\ \cancel{\Delta T}}{625}$$

$$1.4 < m_w \underbrace{\left(\frac{4184}{625} - \frac{4184}{1000}\right)}_{4184 \times 0.375}$$

$$\frac{1.4}{4184 \cdot 0.375} < m_w \quad \text{or} \quad \boxed{0.558\,kg < m_w}$$

5) What is the rate at which the temperature of this much water rises? $\frac{\Delta T}{\Delta t}$

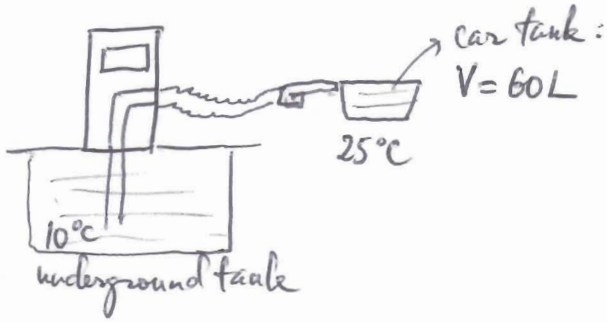
this much → $m_w = 0.558\,kg$.

$$\Delta t_s = \frac{(C_w + C_{pan})\Delta T}{1000} \rightarrow \frac{1000}{C_w + C_{pan}} = \frac{\Delta T}{\Delta t_s}$$

$$\rightarrow \frac{\Delta T}{\Delta t_s} = \frac{1000}{0.558 \cdot 4184 + 1400} = 0.268\,\frac{^\circ K}{s}$$

17.63

1)



2) Coefficient of volume expansion $\beta \equiv \frac{\frac{\Delta V}{V}}{\Delta T}$

$$\beta \Delta T = \frac{\Delta V}{V} \rightarrow \Delta V = \beta \Delta T V$$

ΔV: additional volume by thermal expansion

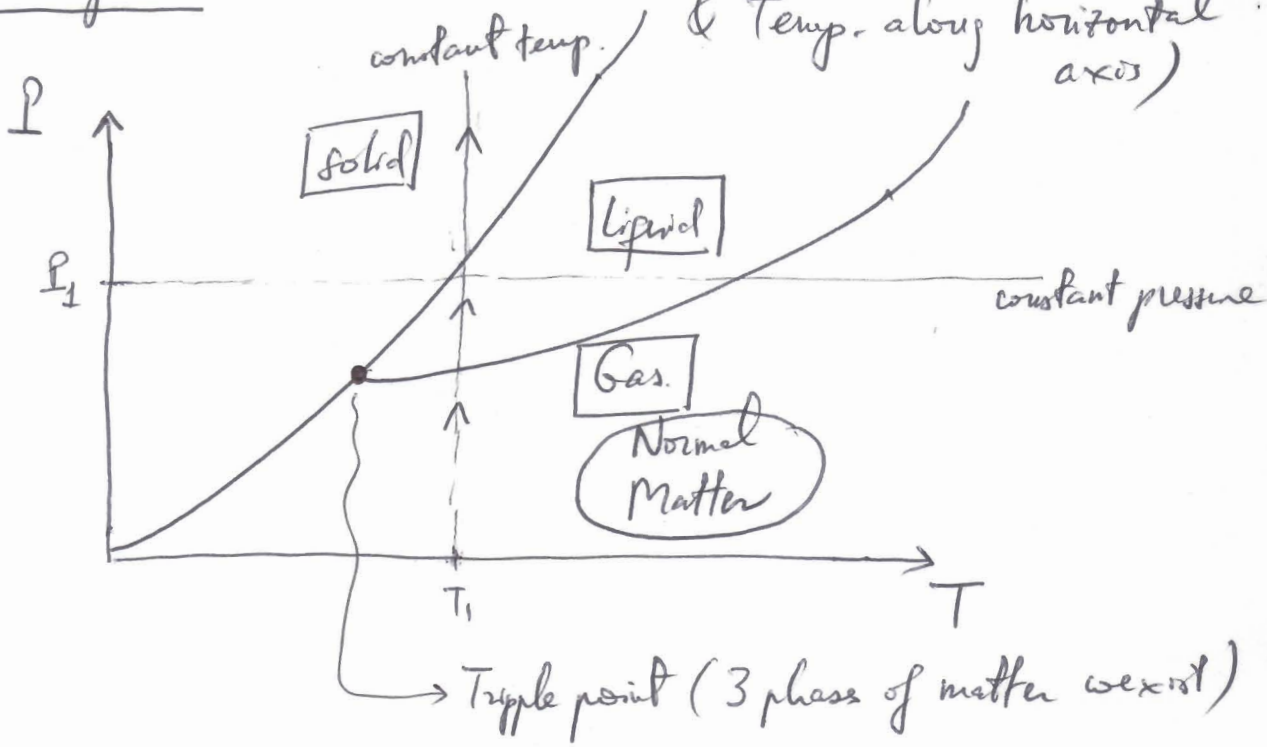
V: original volume of gasoline or the volume we can pump in

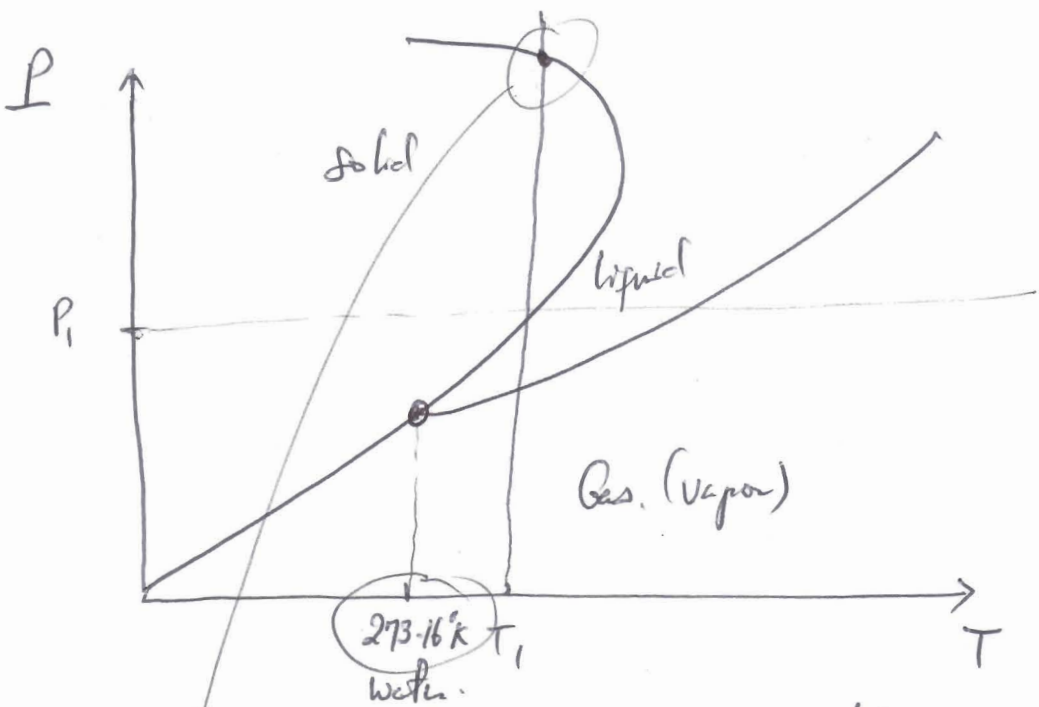
$$60 = V + \Delta V = V + \beta \Delta T V = V(1 + \beta \Delta T)$$

$$\boxed{V = \frac{60L}{1 + \beta \Delta T} = \frac{60L}{1 + 95 \times 10^{-5} \times 15} = 59.2L}$$

$\beta_{Gasoline}$ (data) $= 95 \times 10^{-5}\,^\circ K^{-1}$; $\Delta T = T_H - T_C = 15^\circ C$ or $^\circ K$

Phase Diagram : P vs. T (Pressure along vertical axis & Temp. along horizontal axis)





→ Water anomaly { If we increase pressure here, solid goes into liquid!

Ch 18 Heat, Work, 1st Law of Thermodynamics

1st Law of Thermodynamics:

$$\Delta U = Q - W$$

(Conservation of energy)

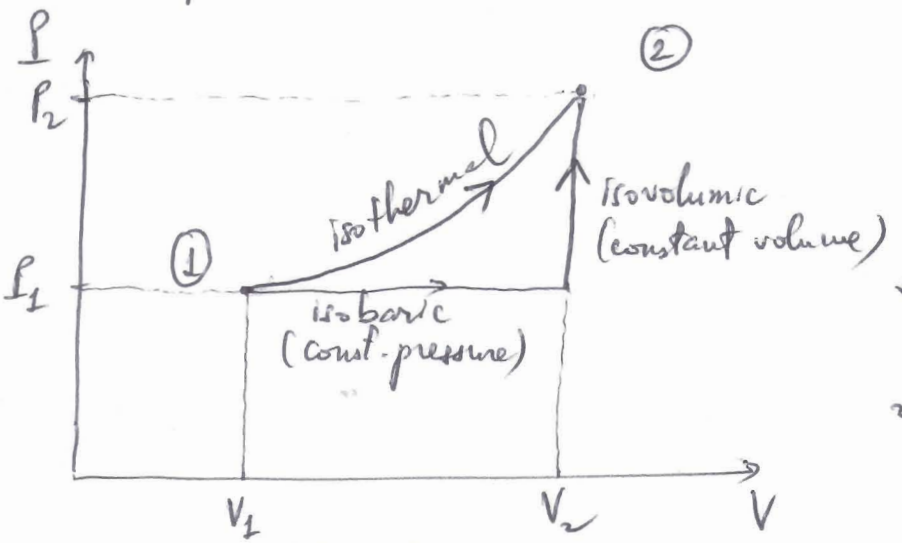
- ΔU → State variable
- $Q - W$ → Not state variables

ΔU: change of internal energy of a system

Q: heat absorbed by system

W: work done by system.

A state is a point on a PV diagram (P along vertical axis, V along horizontal axis). ΔU is a state variable since given a state ΔU is determined uniquely. Now Q & W are not state variables, they depend on a particular process b/w states!



Two alternative way to ① → ②

1) Via an isothermal process (fixed T)

2) Isobaric + Isovolumic processes

Process: connects 2 different states.

ΔU_{12}: change of internal energy of a system b/w ① & ②. It is unique regardless of which process it followed b/w ① & ②

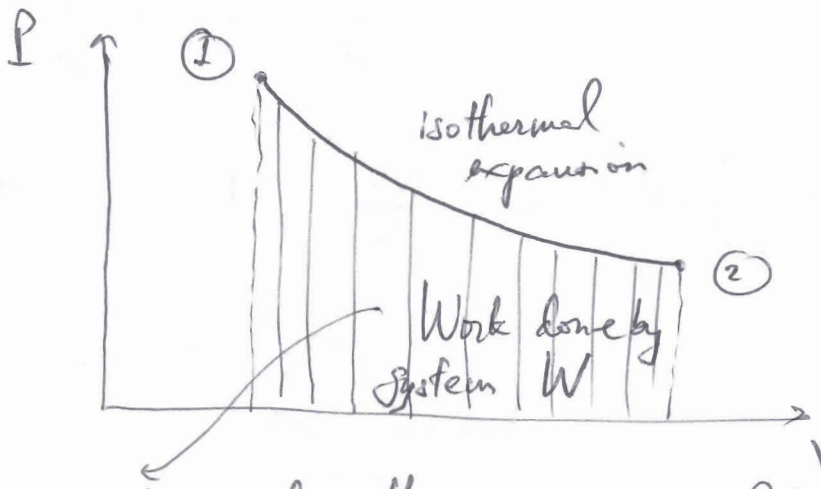
Q & W b/w ① & ② depend on whether it ~~chose~~ followed isothermal or isobaric + isovolumic process.

→ Work done by a system: $W = \int P\,dV$

Quick check: $W = F \cdot \Delta x = P \cdot \underbrace{A \cdot \Delta x}_{\Delta V}$

F: force applied; Δx: displacement

Pressure $P = \frac{F}{A}$



Area under the curve in a PV diagram is $W = \int P\,dV$

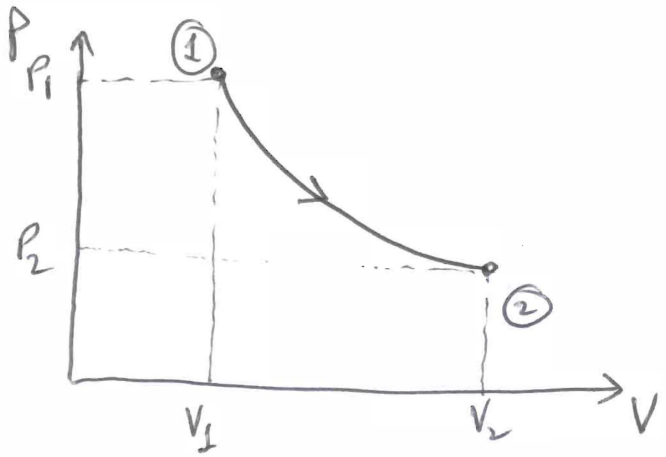
Processes:

1) Isothermal (T is constant)
2) Isovolumic (V is constant)
3) Isobaric (P is constant)
4) Adiabatic (No heat absorbed or Q = 0)

↳ Q & W depends on a particular process!

(ideal gas)

Isothermal (T is fixed)



$V_2 > V_1$, $P_2 < P_1$: Isothermal expansion

$$\rightarrow W = \int_1^2 P\,dV = nRT \int_1^2 \frac{dV}{V} = nRT\left[\ln V\right]_1^2 = nRT\underbrace{\left(\ln V_2 - \ln V_1\right)}_{\ln\left(\frac{V_2}{V_1}\right)}$$

ideal gas: $PV = nRT$

This is constant! (isothermal)

$$\rightarrow P = \frac{nRT}{V}$$

$$\boxed{W = nRT\ln\left(\frac{V_2}{V_1}\right)}$$ (work done by an ideal gas in an isothermal process)

$\rightarrow Q = \Delta U + W \rightarrow$ What is ΔU in an isothermal process for ideal gas?

1st Law $\Delta U = Q - W$

Ideal gas:

1) No interactions b/w molecules $\leftrightarrow$ no potential energy $\leftrightarrow$ total internal energy of gas comes from kinetic energy only

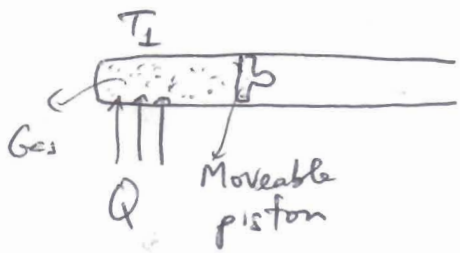
2) Average KE per molecule in an ideal gas is $\begin{cases} \frac{3}{2}kT & \text{monoatomic molecule} \\ \frac{5}{2}kT & \text{diatomic molecule} \end{cases}$

$\Delta U = 0$ for isothermal process

3) $\Delta U_{12} = U_2 - U_1 = \begin{cases} \frac{3}{2}k(T_2 - T_1)N = 0 \\ \frac{5}{2}k(T_2 - T_1)N = 0 \end{cases}$ Isothermal process b/w ① & ② $T_2 = T_1$

$\rightarrow \boxed{Q = W}$ in isothermal process (ideal gas)

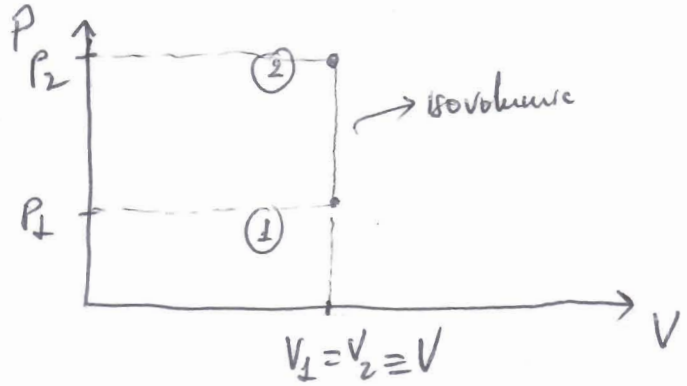
Gas piston expansion:



Gas isothermal expansion

Work done by piston $W = Q$
(since $\Delta U = 0$ for isothermal process for ideal gas)

Isovolumic (Constant V)



$$\boxed{W = \int_1^2 P\,dV = 0}$$

$$\boxed{Q = \Delta U}$$ → What is ΔU in isovolumic process in ideal gas?

1st Law: $\Delta U = Q - W$

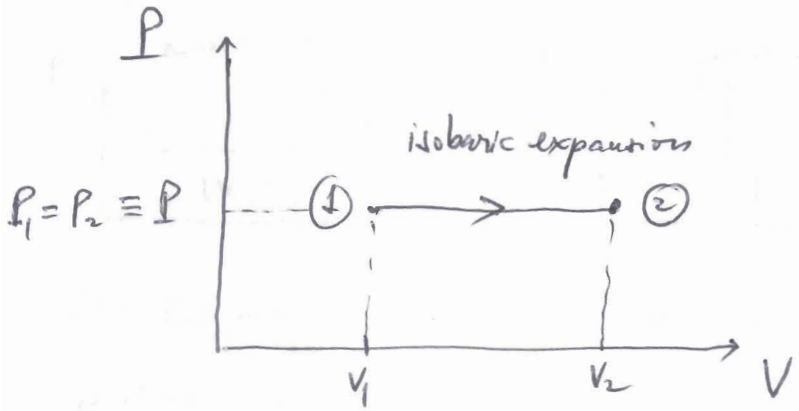
↳ c_v = specific heat at constant volume

$$C_v \equiv \frac{1}{n}\frac{Q}{\Delta T} = \frac{1}{n}\frac{\Delta U}{\Delta T}$$

(# mole)

$$\boxed{Q = c_v n \Delta T = \Delta U}$$

Isobaric Process (Const. Pressure):



$$\boxed{W = \int_1^2 P\,dV = P\int_1^2 dV = P(\underbrace{V_2 - V_1}_{\Delta V}) = P.\Delta V}$$

↓ Valid for any gas (we did not use ideal gas equation!)

$Q = ?$

c_p = specific heat @ const pressure : $c_p = \frac{1}{n}\frac{Q}{\Delta T}$

$$\boxed{Q = nc_p\Delta T}$$

Ideal gas: $P\Delta V = nR\Delta T$

1st Law of T.D: $\Delta U = Q - W$

$nc_v\Delta T$ $\quad$ $nc_p\Delta T$ $\quad$ $P.\Delta V = nR\Delta T \rightarrow \boxed{C_v = C_p - R}$

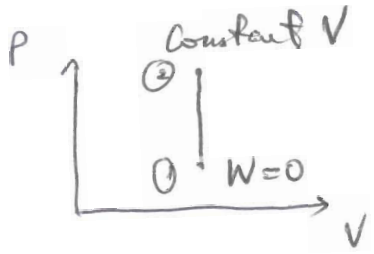
or $\boxed{C_p = C_v + R}$

$R = 8.314 \frac{J}{mol\,°K}$

↓ There is more heat absorption capacity @ constant pressure than @ constant volume

$C_p = C_v + R$

constant P



$Q = \Delta U$

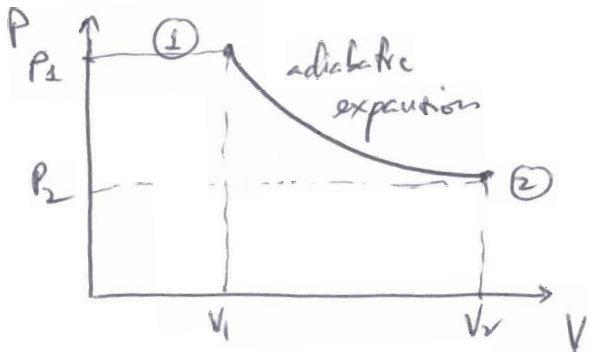
$Q = \Delta U + W$

If it can do work it can keep absorbing heat!

Adiabatic Process ($Q = 0$)

1st Law of T.D. : $Q = \Delta U + W$

$$\boxed{W = -\Delta U}$$



$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

γ → dimensionless coefficient $\gamma \equiv \frac{C_p}{C_v}$

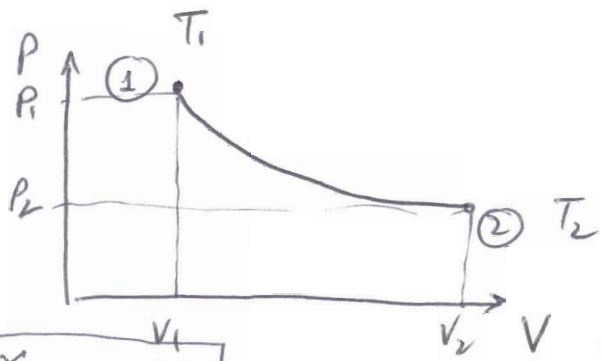
Since C_v take on different values depending on the type of gas:

$$C_v = \frac{1}{n}\frac{\Delta U}{\Delta T} = \begin{cases} \text{monoatomic } C_v = \frac{1}{n}\frac{N\frac{3}{2}k\Delta T}{\Delta T} = \frac{3}{2}k\frac{N}{\frac{n}{N_A}} = \frac{3}{2}kN_A = \frac{3}{2}R \\ \text{diatomic} \quad C_p = \frac{5}{2}R \end{cases}$$

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$$

- Monoatomic: $\gamma = \dfrac{\frac{3}{2}R + R}{\frac{3}{2}R} = \dfrac{5}{3} = 1.67$
- Diatomic: $\gamma = \dfrac{\frac{5}{2}R + R}{\frac{5}{2}R} = \dfrac{7}{5} = 1.4$

Adiabatic Process:



$$\boxed{P_1 V_1^{\gamma} = P_2 V_2^{\gamma}}$$

Ideal gas: $PV = nRT$ or $P = \dfrac{nRT}{V}$

$$\boxed{T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}}$$

(16.75)

$\begin{cases} m_w = 5.4\times10^6\ kg \\ \Delta T = 350°C - 10°C = 340°K \end{cases}$ (water remains liquid → no change of phase: no vaporization)

Thermal power output $P = 1.42\times10^9\ W$

How long would it take to heat m_w from 10°C to 350°?

Δt? → 1) Total energy needed: $\Delta Q = m_w c_w \Delta T = 5.4\times10^6 \times 4184 \times 340\ J$

2) Use heat transfer rate to find $\Delta t = \frac{\Delta Q}{P} = \frac{5.4\times10^6\times4184\times340}{1.42\times10^9}$

$= 5400\ s = 1.5\ hrs.$

(18.33) Air bubble in blood: (a) $d_L = 1.52\ mm$ @ $P_L = 80\ mm\ Hg$; (b) d_H? @ $P_H = 125\ mm\ Hg$

Notes:
- These values for P_L & P_H are gauge pressures (excesses above the atmospheric pressure: 760 mm Hg or 1 atm)
- We expect an air bubble to be smaller ($d_H < d_L$) when blood pressure is @ P_H

a) Body temp. is constant @ 37°C → so will be the air temp. inside bubble: → assume it's an ideal gas: $\begin{cases} P_L V_L = nRT \\ P_H V_H = nRT \end{cases}$ $\boxed{P_L V_L = P_H V_H}$

→ $V_H = \frac{P_L V_L}{P_H}$; volume of sphere in term of its diameter: $\boxed{V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi}{6}d^3}$

$\frac{\pi}{6} d_H^3 = \frac{(80+760)\frac{\pi}{6}d_L^3}{(125+760)}$; Note: mm of Hg to Pa (Pascal, SI unit for Pressure) is only scaling factor that will cancel up & down in the fraction $\frac{P_L}{P_H}$

To solve for d_H : elevate both sides to the power of $\frac{1}{3}$

$$d_H = \left(\frac{840}{885}\right)^{\frac{1}{3}} d_L = \left(\frac{840}{885}\right)^{1/3} 1.52\,mm = 1.49^{38}\,mm$$

(Note: $d_H < d_L$ as expected)

b) Work
- As the bubble compresses (by 0.03 mm) the blood & heart have done some work! this work is received by gas inside bubble.
- Gas undergoes an isothermal process ① → ② or (L) → (H) (P_L, V_L → P_H, V_H)

$$W = nRT\ln\left(\frac{V_2}{V_1}\right) = P_L V_L \ln\frac{V_H}{V_L}$$

↓ work done by gas

$$= \underbrace{840 \cdot \frac{1.013\times10^5}{760}}_{P_L} \cdot \underbrace{\frac{4\pi}{3}\left(\frac{1.52\times10^{-3}}{2}\right)^3}_{V_L} \cdot \underbrace{\ln\left[\frac{(1.49)^3}{(1.52)^3}\right]}_{\ln\left(\frac{V_H}{V_L}\right)}$$

↓ Work done by heart is $-W$

$$= -12.3\times10^{-6}\,J = -12.3\,\mu J \;?\; (-10.7\,\mu J)$$

if using $d_H = 1.4938\,mm$

$\left\{\begin{array}{l} 1\,Atm = 1.013\times10^5\,Pa \\ \text{same as } 760\,mm\,Hg. \end{array}\right\}$ $P_L = 840\,\cancel{mm\,Hg} \cdot \frac{1.013\times10^5\,Pa}{760\,\cancel{mm\,Hg}}$

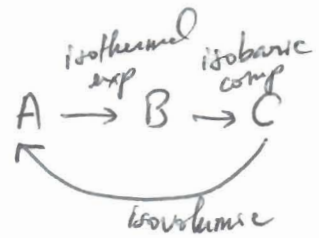
10^{-6} = micro = μ

Note: Work done by gas in going (L) → (H) is negative since it was compressed in this process ($V_H < V_L$) so it received work.

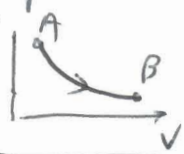
Work done by heart is $-W = +12.3\,\mu J$

18.42

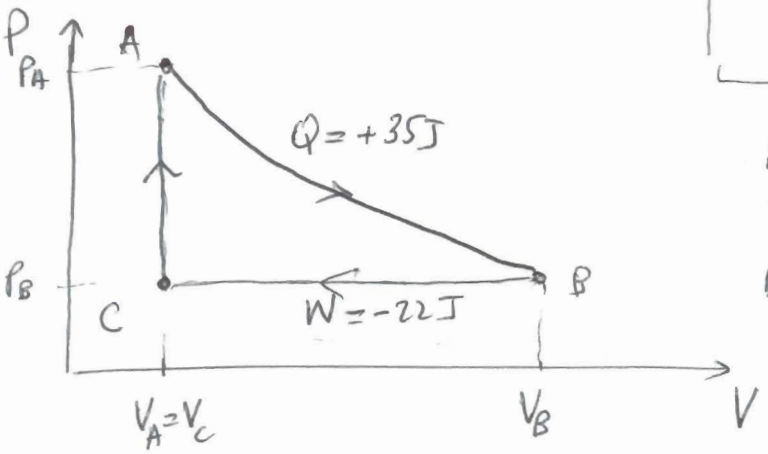
Gas: undergoes a cycle



isothermal expansion always comes with a decrease in pressure: $PV = nRT = \text{const.}$



a)



A → B: gas absorbs heat, Q is positive!

B → C: isobaric @ P_B

$V_C = V_A$

Work done on gas is 22 J

W = is work done by gas $W = -22$ J

C → A: isovolumic

b) How much [work] is done on or by gas during complete cycle?

(work: depends on the process)

A → B → C → A

W_{AB} ; $W_{BC} = -22$J ; $W_{CA} = 0$ (isovolumic)

$W_{ABCA} = W_{AB} + W_{BC}$

W_{AB}: ? ; W_{BC}: -22 J

W_{AB}: 1st Law: $\Delta U_{AB} = Q_{AB} - W_{AB}$

Ideal gas → $U = \frac{1}{2}mv^2 = \frac{3}{2}kT$

$T_A = T_B$

$0 = 35\text{J} - W_{AB}$

$\rightarrow \boxed{W_{AB} = 35\text{J}}$

$W_{ABCA} = 35\text{J} - 22\text{J} = (+)13\text{J}$

work done by gas.

c) How much heat is transferred to or from gas B → C → A (B → C: isobaric; C → A: isovolumic)

$Q_{BCA} = Q_{ABCA} - Q_{AB}$

Q_{ABCA} ? $\quad 1^{st}$ Law of TD : $\Delta U_{ABCA} = Q_{ABCA} - W_{ABCA}$

$\Delta U_{ABCA} = 0$; $W_{ABCA} = 13\,J$

$0 = Q_{ABCA} - 13\,J$

$Q_{ABCA} = 13\,J$

$Q_{BCA} = 13\,J - Q_{AB} = -22\,J$;

$Q_{AB} = 35\,J$

↳ Heat absorbed is negative → gas lost 22 J of heat during $B \to C \to A$.

Ch 19. 2nd Law of Thermodynamics

I

Heat engines working b/w 2 different reservoirs

Heat reservoir: source of heat, large, at constant temperature

Visual experiment: place a piston filled with ideal gas in thermal contact with a heat reservoir (hot) at temperature T_h. Gas absorbs heat, expands, does work. When it absorbs heat, its temperature will increase. When its temperature reaches T_h: stops heat absorption, stops expansion, stops doing work. How do you make this into a heat engine or for the gas to continue doing work?

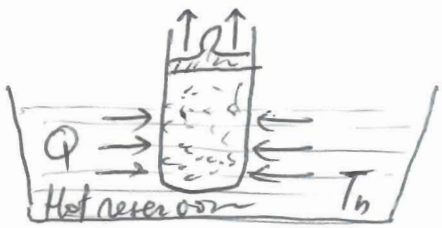
1) Keep raising the temperature T_h: not practical

2) Cooling it down to a lower temperature T_c (placing it in thermal contact with a cold reservoir) before placing it in contact again with the hot reservoir. Then repeat the cycle. Gas needs to receive work as it compresses while cooling, our goal is to get a net work done per cycle.

Diagrams:

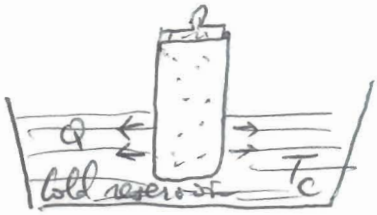
Cycle

1st half



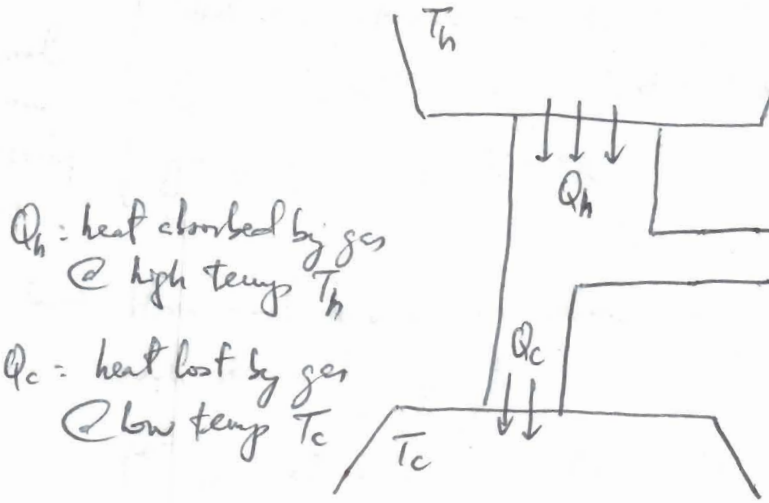
Gas absorbs heat, does work

2nd half



Gas ejecting heat, receives work

Heat engine in one diagram:



Q_h : heat absorbed by gas @ high temp T_h

Q_c : heat lost by gas @ low temp T_c

Net work done by gas (work done minus work reserved)

1st Law of T.D. on a heat engine:

$$\underbrace{\Delta U}_{0} = Q_{net} - W = \underbrace{Q_h - Q_c}_{Q_{net}} - W = 0$$

After one cycle: (back to its initial state)

$$\boxed{Q_h - Q_c = W}$$ → Net heat absorbed is net work done

Efficiency of a heat engine: $e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$

$$\rightarrow e = 1 - \frac{|Q_c|}{|Q_h|} < 1$$

↳ Absolute value are to ensure $e<1$ b/c:

2nd Law of T.D.: it is impossible to build a heat engine operating in cycles that extracts heat from a hot reservoir and returning some of it to a cold reservoir, that can deliver 100 % efficiency ($e=1$ is not possible! → $e<1$)

(18.52)

$n = 21$

External forces were compressing a gas.

- ideal monoatomic $\rightarrow C_V = \frac{3}{2}R$
- $Q = -15\,kJ$ (gas lost 15kJ of heat to its surroundings)
- $\Delta T = T_f - T_i = 160\,°K$

↳ How much work was done on gas? $-W$ (where W = work done by gas)

1st Law of T.D: $\Delta U = Q - W$

- ΔU: change of internal energy of gas
- Q: heat absorbed by gas
- W: Work done by gas.

$$\rightarrow -W = \Delta U - Q$$

$$= nC_V\Delta T - Q$$

$$= 21 \times \frac{3}{2} \times 8.314 \times 160 - (-15000)$$

$$= 56900\,J$$

→ Work done on gas is $+56.9\,kJ$

(18.57)

Gas mixture: Ar & O_2 → adiabatic expansion ① → ②

$V_1 \rightarrow V_2 = 2V_1$

$P_1 \rightarrow P_2 = \frac{P_1}{3}$

fraction: f $1-f$

$0 < f < 1$ to describe composition of mixture

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$\gamma = \frac{C_p}{C_V}$

γ, C_p, C_V are for the mixture

Strategy: 1) f will appear in expression of γ for the mixture

2) Find value of γ from adiab. expansion → find f

Mixture: $C_v = \underbrace{\frac{3}{2}Rf}_{\text{monoatomic } Ar} + \underbrace{\frac{5}{2}R(1-f)}_{\text{diatomic } O_2}$

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{\frac{3}{2}Rf + \frac{5}{2}R(1-f) + R}{\frac{3}{2}Rf + \frac{5}{2}R(1-f)}$$

$$\gamma = \frac{\frac{7}{2} - f}{\frac{5}{2} - f}$$

Adiabatic expansion: $P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \rightarrow \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$

$$\ln\left(\frac{P_1}{P_2}\right) = \gamma \ln \frac{V_2}{V_1}$$

$$\gamma = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln 3}{\ln 2}$$

$$\gamma = 1.58$$

$$\gamma = 1.58 = \frac{\frac{7}{2} - f}{\frac{5}{2} - f} \rightarrow \frac{5}{2}\gamma - \gamma f = \frac{7}{2} - f$$

$$\frac{5}{2}\gamma - \frac{7}{2} = (\gamma - 1) f$$

$$\rightarrow f = \frac{\frac{5}{2}\gamma - \frac{7}{2}}{\gamma - 1}$$

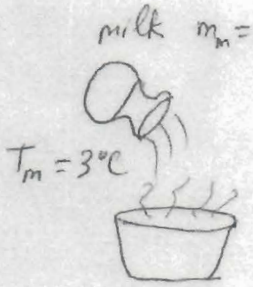
$$= \frac{\frac{5}{2} \cdot 1.58 - \frac{7}{2}}{0.58}$$

$f = 0.78$ (Ar)

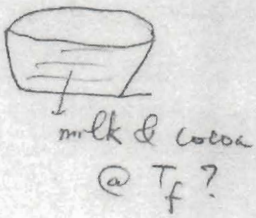
$Ar = 78\%$

$O_2 = 22\%$

16.54



$C_m = C_c = C_{water} = 4184 \frac{J}{kg\,°K}$



Heat balance equation: (isolated system)

$$\Delta Q_c + \Delta Q_m = 0$$

$$m_c c_c (T_f - T_c) + m_m c_m (T_f - T_m) = 0$$

($c_c \approx c_w$, $c_m \approx c_w$)

$$(m_c c_w + m_m c_w) T_f = m_c c_w T_c + m_m c_w T_m$$

$$T_f = \frac{m_c T_c + m_m T_m}{m_c + m_m}$$

SI:
- $T \rightarrow$ in °K $\rightarrow$ $T_c = 90° + 273.16° = 363.16\ °K$
- $T_m = 3° + 273.16° = 276.16\ °K$
- $m \rightarrow$ in kg (not necessary in this particular ex. $\rightarrow$ masses showing above & below the fraction!)

$$T_f = \frac{6 \times 363.16 + 2 \times 276.16}{8} = 341.46\ °K$$

Can go back to celsius: $T_f = 341.46 - 273.16 = 68.3°C$

If I work in ~~cels~~ celsius:

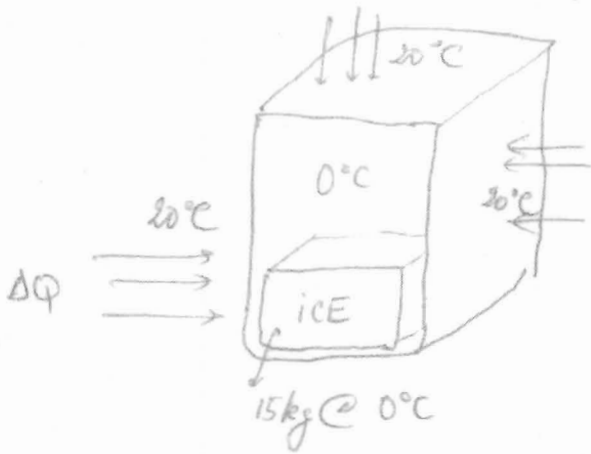
$$T_f = \frac{m_c (T_{c_K} - 273.16) + m_m (T_{m_K} - 273.16)}{m_c + m_m}$$

$$= \frac{m_c T_{c_K} + m_m T_{m_K}}{m_c + m_m} + \frac{(m_c + m_m)(-273.16)}{(m_c + m_m)}$$

$$T_f = T_{f_K} - 273.16$$

(17.65)

Thermal resistance of fridge wall is $R = 0.12 \frac{°K}{W}$



$$R \equiv \frac{\mathcal{R}}{A}$$ → $\mathcal{R}$ factor; → area

R → determine H (Heat transfer rate through fridge's walls)

→ how long will it last $\Delta t = \frac{\Delta Q}{H}$ $\left(H = \frac{\Delta Q}{\Delta t}\right)$

ΔQ will go into melting the ice or $m_{ice} L_f$

$$H = -kA\frac{\Delta T}{\Delta x} = -\frac{A\Delta T}{\mathcal{R}} = -\frac{A\Delta T}{RA} = -\frac{\Delta T}{R}$$

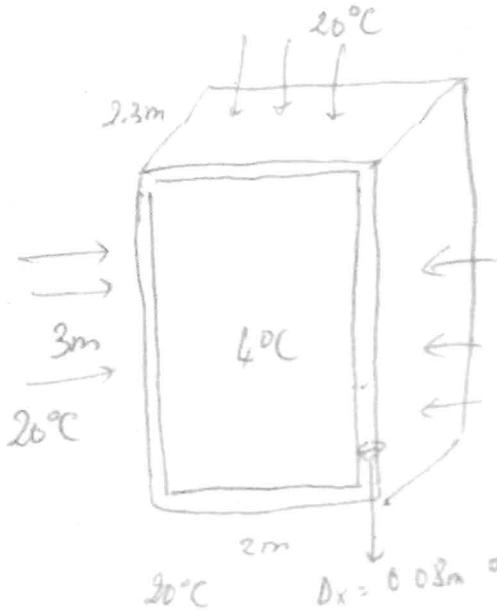
(Heat transfered from outside to inside → $H_{absorbed} = \frac{\Delta T}{R}$)

$$\Delta t = \frac{\Delta Q}{H_{abs.}} = \frac{m_{ice} L_f}{\frac{\Delta T}{R}} = \frac{15 \times 334 \times 10^3}{\frac{(20-0)}{0.12}} = 3 \times 10^4 s \frac{1h}{3600s}$$

$$= 8.35 h$$

L_f (water) = $334 \frac{kJ}{kg}$

16.56



Walk-in cooler

$$H = -\frac{A\Delta T}{\mathcal{R}}$$

$$\mathcal{R} = \frac{\Delta x}{k} = \frac{0.08}{0.029} = 2.76$$

(Table 16.2)

$A = \underbrace{3\times2\times2}_{\text{Front \& Back}} + \underbrace{3\times1.3\times2}_{\text{Left \& right}} + \underbrace{1.3\times2\times2}_{\text{Top \& bottom}}$

$= 35\,m^2$

$$H = -\frac{32\times16}{2.76} = -203\,W$$

(17.71) Clock: brass pendulum: $L = 0.2\,m$ @ $20^\circ C$ (good)
(calibrated to work as a clock @ $20^\circ C$.

↓

$T_2 = 18^\circ C$ ⟶ how long until it makes a 1min error?

Since for a pendulum: # oscillations per second $\omega = \sqrt{\frac{g}{L}}$
Length will change with temp, according to the linear coefficient of expansion for brass: → Table 17.2: $\alpha = 19 \times 10^{-6}\,{}^\circ K^{-1}$

$$\alpha = \frac{\frac{\Delta L}{L}}{\Delta T} \rightarrow \frac{\Delta L}{L} = \alpha \Delta T \rightarrow \Delta L = \alpha L \Delta T$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \begin{cases} T_{20^\circ C} = 2\pi\sqrt{\frac{L}{g}} \\ T_{18^\circ C} = 2\pi\sqrt{\frac{L+\Delta L}{g}} = 2\pi\sqrt{\frac{L+\alpha L \Delta T}{g}} \end{cases}$$

($-2^\circ K$)

$$= 2\pi\sqrt{\frac{L(1+\alpha\Delta T)}{g}}$$

$$T_{18^\circ C} < T_{20^\circ C} \rightarrow T_{20^\circ C} - T_{18^\circ C} = \frac{2\pi}{\sqrt{g}}\left[\sqrt{L} - \sqrt{L(1+\alpha\Delta T)}\right]$$

$$= 2\pi\sqrt{\frac{L}{g}}\left[1 - \sqrt{1-2\alpha}\right]$$

($\Delta T = -2^\circ K$)

$$\sqrt{1-2\alpha} = (1-2\alpha)^{1/2} \cong 1 - \tfrac{1}{2}(2\alpha) = 1-\alpha$$

$$(1+x)^{1/2} \simeq 1 + \tfrac{1}{2}x + \ldots$$

(negligible if x very small compared to 1)

$\alpha = 19 \times 10^{-6}\,{}^\circ K^{-1}$

$$\boxed{T_{20^\circ} - T_{18^\circ} = 2\pi\sqrt{\frac{L}{g}}\left[1 - (1-\alpha)\right] = 2\pi\sqrt{\frac{L}{g}}\,\alpha}$$ time error per period.

How many periods of pendulum until a difference of 1 min.

$$\frac{60s}{2\pi\sqrt{\frac{L}{g}}\,\alpha}$$

How long until a difference of 60s? $\frac{60s}{2\pi\sqrt{\frac{L}{g}}\,\alpha}\,T_{20°}$

$$= \frac{60}{2\pi\sqrt{\frac{L}{g}}\,\alpha}\,2\pi\sqrt{\frac{L}{g}}$$

$$= \frac{60}{\alpha} = \frac{60}{19\times10^{-6}}\,s \rightarrow \underline{\underline{36.5\,days}}$$

17.39

@ T = 25°C (room temp.) → P = 180 atm.

a) How many moles of air n

Assume ideal gas. $n = \frac{PV}{RT}$

$P = 180\,\text{atm}\cdot\frac{1.013\times10^5\,Pa}{1\,\text{atm}}$

$V = \pi(0.1)^2\cdot 1 = \frac{\pi}{100}\,m^3$

$R = 8.314\,\frac{J}{°K\,mol}$

$T = 25 + 273.16 = 298.16°K$

$$n = \frac{180\times1.013\times10^5\times\frac{\pi}{100}}{8.314\times298.16} = 231\,mol$$

b) What would be the gas volume @ $n = 231 = \frac{PV}{RT}$

P = 1 atm & T = 25°C

if P decreases 180 fold

→ V increases 180 times

$$V = \frac{\pi}{100}\times180\,m^3 = 5.65\,m^3$$