Ch 30: Reflection & Refraction

Optics
\[ \text{Geometrical Optics: (Ch 30, 31)} \]
\[ \text{Physical Optics: (Ch 32)} \]

Reflection:
light ray incident upon a mirror

Normal → reflected ray

incident ray → Minor (no propagation in the shaded side)

Incident angle \( \theta \): angle \( \theta \) between incident ray & the normal to the mirror surface

Reflected angle \( \theta' \): angle \( \theta' \) between the reflected ray & the normal

Law of reflection: \( \theta' = \theta \)

When geometry is applied: Multiple reflections

\[ \theta_2 = \theta_2' \]

Need to relate \( \theta_2' \) to \( \theta_1 \)

Use geometry (Triangle ABC)
In any triangle: \( 90^\circ - \theta_1' + \alpha + 90^\circ \Rightarrow \theta_2 = 180^\circ \)
\[ \theta_1 - \theta_1' - \theta_2 = 0 \]
\[ \theta_2 = \alpha - \theta_1' \]

\[ \theta_1' = \theta_2 = \alpha - \theta_1' = \alpha - \theta_1 \]
\[ \text{Law of Reflection} \quad \text{Law of Reflection} \]
\[ \text{on minor BC} \quad \text{on minor AC} \]

**Refraction:** when light rays travel from one medium to another:
- Light speed in vacuum: \( C = 3 \times 10^8 \text{ m/s} \)
- Light speed in water: \( C_w = \frac{C}{n_w} \) (\( n_w \) = index of refraction for water: 1.333)

Daily phenomena: rainbow (different refraction indices for different colors or wave lengths)
- Broken straw:

\[ \text{\textbullet \text{empty} \quad \text{\textbullet \text{water}}.} \]

1. Air $n_1 = 1$
   - Wave fronts are perpendicular to the direction of propagation.

2. Medium $n_2 = n > 1$
   - Larger index (normally due to higher density)

   - Light $\rightarrow$ wave (EM waves)
   - Light $\rightarrow$ particle (photons)

   - Wave fronts are "pushed back" when light encounters medium 2 in the way of a higher density or higher index of refraction.

   - $n_2 < n_1$ when going from lower to higher index.

   - This side is still in medium 1.

   - This side of wave front is changing medium (higher index) $\rightarrow$ gets pushed back.
Law of refraction: Snell's Law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

In practice:

1. Air
2. Medium

\( n > 1 \)

\( \theta_2 < \theta_1 \)

Normally, there are more refraction than reflection.

Beyond a critical angle: \( \theta_1 > \theta_c \), all incident light is reflected (none is refracted). Normally, there are more refraction, but it decreases when \( \theta_1 \) is getting larger towards the \( \theta_c \). Refraction disappears @ and beyond \( \theta_c \).
**Total Internal Reflection**

Happens when light travels from higher index to lower index

\[ \theta_1 > \theta_c \]

- Air
- Water
- Reflected
- Incident

- Fiber Optics
  - Light gets confined
  - Used to carry information (internet, phone, etc.)
  - Not as fast but faster & more reliable than electrical current in cables.

**Critical Angle**

1. \( n_2 = n \)

2. \( n_c = 1 \)

- Medium

\[ \theta_1, \theta_2 \]

\[ \frac{n_2 \sin \theta_c}{n_1} = n_2 \sin 90^\circ = n_2 \]

\[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]
any incident

Is there an angle at which there is no reflection and all incident light is refracted? \( \rightarrow \) The polarizing angle or Brewster angle.

\[
\theta_1 = \theta_p \text{ or } \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)
\]
There is no reflection when \( \theta_1 = \theta_p \)

\[
E_1 = E_p \quad \text{(EM wave is transverse)}
\]

Observation:
1) If \( E \) has a component that is perpendicular to the page, that component is not affected even if \( \theta_1 = \theta_p \text{ or } \theta_B \): there is reflection for that component.
2) When \( E \) has \( E_{||} \) (parallel to page) & \( E_\perp \) (perpendicular to page) light refracted into medium 2 will have \( E \) polarized in direction parallel to page (if \( \theta_1 = \theta_B \text{ or } \theta_p \))
3) This is useful when \( n_2 > n_1 \): taking picture of something behind a glass window

Mona Lisa
Ch 31  Image & Optical Instruments

{ Mirrors  }  { Lenses  }  { Geometrical Optics }

How to form the image of an object off a mirror or through a lens?

Image formation by a mirror:

How tall a mirror we need so to see our whole body?

(height is \( h_b \))

1) \[ h_m = h_b \]

2) \[ h_m = \frac{1}{2} h_b \]  \( \checkmark \)

3) \[ h_m = \frac{2}{3} h_b \]  \( \checkmark \)

---

object

---

Virtual Image

Light cannot travel behind the mirror.

Formed by extension rays.
Prism \( n_R = 1.582 \)
\( n_V = 1.633 \)

Dispersion: different colors come out at different directions the other side of prism.
\( \gamma = \theta'' - \theta''' \)

| Violet ray | Incident on left boundary at A: angle 45° | Refracted angle on left boundary at A: \( \theta_V \)
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<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>Incident on right boundary at B: ( \theta'' )</td>
<td>Refracted angle at B: ( \theta''' )</td>
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Shell's Law at A:
\( \frac{1}{n_R} \cdot \sin 45° = 1.633 \sin \theta_V \Rightarrow \theta_V = \sin^{-1}\left(\frac{1}{\sqrt{2}} \cdot \frac{1}{1.633}\right) = 25.5° \)

Shell's Law at B:
\( 1.633 \sin \theta'' = 1 \sin \theta''' \)

Need one more eq. from geometry: triangle ABC \( \Rightarrow \theta_V + \theta'' + 120° = 180° \)
\[ \theta_v + \theta_v' = 60^\circ \quad \Rightarrow \quad \theta_v' = 60^\circ - \theta_v = 60^\circ - 25.5^\circ = 34.5^\circ \\
\text{[}\theta_v'' = \sin^{-1}(1.633 \sin 34.5^\circ) = 67.7^\circ\text{]} \]

Red ray:
- Same calculation except \( n_0 = 1.582 \)

Shell's law at A:
\[ 1 \sin 45^\circ = 1.582 \sin \theta_R \Rightarrow \theta_R = \frac{\sin^{-1} \frac{1}{1.582}}{1.582} = 26.5^\circ \]

Shell's law at B:
\[ 1.582 \sin \theta_R = 1 \sin \theta_K \]

From triangle ABC:
\[ \theta_R + \theta_K + 126^\circ = 180^\circ \]
\[ \theta_K = 60 - \theta_R = 60 - 26.5^\circ \]
\[ \theta_K = 33.5^\circ \]
\[ \theta_K'' = \sin^{-1}(1.582 \sin 33.5^\circ) = 60.8^\circ \text{[]} \]

Angular dispersion:
\[ \delta = \theta_v'' - \theta_K'' = 67.7^\circ - 60.8^\circ = 6.85^\circ \text{[]} \]

For the beam:
Red & Violet are the outer wavelengths
of the visible spectrum; other colors
behave as between these: \( 1.582 < n < 1.633 \text{[]} \)
1st refraction \( OA \):  

Shell's Law:

\[ \angle A = \sin 35^\circ = 1.43 \sin \theta_1 \]

\[ \theta_1 = \sin^{-1} \left( \frac{\sin 35^\circ}{1.43} \right) \]

\[ \theta_1 = 23.6^\circ \]

\[ \angle B = 1.43 \sin 23.6^\circ = n \sin \theta_2 \rightarrow 2 \text{ unknown: need one more equation from the geometry of the problem:} \]

\[ \tan \theta_2 = \frac{BC}{\frac{L}{2}} \]

\[ A'C = \frac{L}{2} \rightarrow BC = A'C - A'B' \]

Now find \( A'B' \):

\[ \frac{A'B'}{\frac{L}{2}} = \tan \theta_1 \]

\[ \frac{A'B'}{\frac{L}{2}} = \tan \theta_1 \]

\[ \tan \theta_2 = \frac{\frac{L}{2} - \frac{L}{2} \tan \theta_1}{\frac{L}{2}} = 1 - \tan \theta_1 \]

*From figure given as data \( \theta_2 > \theta_1 \rightarrow \) we expect \( n < 1.43 \rightarrow \) checked*
Virtual image by a flat mirror
Virtual image by a curved mirror.

To form image
for object O → trace rays 1 & 2:
By extending 1' & 2' behind the mirror
we found location of virtual image

Mirror equation:
\[
\frac{1}{e} + \frac{1}{e'} = \frac{1}{f}
\] (From geometry of rays)

Magnification factor:
\[
M = \frac{h'}{h} = -\frac{e'}{e}
\]

In this example: we assume F is by C and the midpoint of minor (where axis goes through minor)
F: Focal point
1) Incident rays II axis will reflect through F
2) Incident rays through F will reflect parallel to axis

Sign convention for mirrors:
\[
|f| \begin{cases}
+ \text{concave mirror} \\
- \text{convex mirror} \\
\end{cases}
\]
\[
l' \begin{cases}
+ \text{image located same side of mirror as object (real image)} \\
- \text{image located in the other side of mirror (virtual image)} \\
\end{cases}
\]
Can we get a real image using a concave mirror?

Image formation in lenses:

- Converging lens or convex lens
- Diverging lens or concave lens

Image for this object:

1. Rays 1 and 2 after parallel to axis

- Parallel to axis, emerge from F
- Through F, emerge parallel to axis
- Through C, straight through (not essential after 0.85)
Lens equation: \[ \frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \]

Magnification factor: \[ M = \frac{h'}{h} = -\frac{l^0}{l} \]

Sign convention for lens:
- \[ f \] \quad convex lens (diverging)
- \[ l' \] \quad convex lens (converging)
- Image located on the other side of lens of \( l \)
- Image located same side as object

Types of lenses:

1) Air - Glass - Air (Medium)

2) Air - Glass (Medium)

3) Air - Medium - Air: with different radii of curvature left & right.

Thick lens:

Equation:
\[ \frac{n_1}{l} + \frac{n_2}{l'} = \frac{n_2 - n_1}{R} \]

Sign convention for \( R \):
- \[ + \] concave
- \[ - \] convex

Thin lens:
\[ \frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \]
Equation: "Lense maker's equation"

\[
\frac{1}{f} = (n-1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]

- \( \frac{1}{f} \) = inverse of focal length.

\[ R_1 \quad \text{and} \quad R_2 \quad \text{sign convention} \]
- \( + \) convex
- \( - \) concave

**Eye:**

- flexible lens
  - muscles: controlling focal length of our lens: trying to set \( f = d \)
  - we will see a clear & focused far away objects
  - good eye

**Near sighted (myopic):**

\[ f < d \] (can see closer objects w/o corrective lens)

- Retina
  - blurred image

**Far sighted (hyperopic):**

\[ f > d \] (can see objects w/o corrective lens a little farther)

- Retina
  - blurred image

**Diverging (negative)**

\[ f < 0 \] \( \text{diameter} = \frac{1}{f} \) meters

**Converging (positive)**

\[ f > 0 \] \( \text{diameter} = \frac{1}{f} \) meters
Near sighted

Far sighted

Normal eye

\[ n \sin \theta_i = n \sin \theta_r \]
\[ \theta_i = 0 \Rightarrow \theta_r = 0 \]

Critical angle: \[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]
\[ \theta_c = \sin^{-1} \left( \frac{1}{1.52} \right) = 41^\circ \]
\[ n_1 = n = 1.52 \]

There is total internal reflection (TIR)

Now immersed in liquid; no more total reflection at \( B \)

What minimum \( n_2 \) so there is some reflected light out to liquid?

\( \text{Incident w/ or w/o liquid } \theta_i = 45^\circ \rightarrow \text{no longer TIR if} \)

\( \theta_i \) now is \( \leq \theta_c^{\text{i/f}} \rightarrow \sin \theta_i = \sin 45^\circ = \frac{1}{\sqrt{2}} \leq \sin \theta_c^{\text{i/f}} \).

\( \theta_c^{\text{i/f}} \): new critical angle w/ liquid outside \( \text{at } B \)

\[ \theta_c^{\text{i/f}} = \sin^{-1} \left( \frac{n_l}{n} \right) = \sin^{-1} \left( \frac{n_l}{1.52} \right) \]

\[ \sin \theta_c^{\text{i/f}} = \frac{n_l}{1.52} \]

\[ \frac{1}{\sqrt{2}} \leq \frac{n_l}{1.52} \rightarrow \left[ n_l > \frac{1.52}{\sqrt{2}} = 1.07 \right] \]
Ch 32 Interference & Diffraction:

Physical optics: using wave properties of light in addition to the geometry of the problem.

- Superposition of waves
  - Constructive
    - In phase
  - Destructive
    - Out of phase or 180°

Double-slit Interference:

- One source of wave → 2 slits → 2 identical waves coherent or in phase

Diagram:

- L >> d
- L very far from slits

1) \( L = 2A \) → waves 1 & 2 in phase initially → also in phase at A → waves 1 & 2 combine constructively at A → bright spot at A.

2) \( L < 2B \) → if \( 2B - 1B = n\lambda \) → waves 1 & 2 are still in phase at B → combine constructively → bright spot.

3) 1st bright spot beyond A: \( 2C - 1C = \lambda \) (n=1)

2nd bright spot beyond A: \( 2B - 1B = 2\lambda \) (n=1)
1) Dark spots:

\[ 2B' - 1B' = \frac{(2m+1)\lambda}{2} \quad (n = 0, 1, 2, 3, \ldots) \]

where \( \lambda \) is an odd multiple of half wavelength.

Waves 1 & 2 are in and \( B' \) out of phase (max of 1 coincides with min of 2, \ldots)

\[ \begin{array}{c}
1^{st} \text{ dark spot:} \\
2^{nd} \text{ dark spot:} \\
5^{th} \text{ dark spot:} \\
\end{array} \]

\[ \begin{align*}
2B' - 1B' & = \frac{\lambda}{2} \\
2B' - 1B' & = 3 \frac{\lambda}{2} \\
\Delta \text{path} & = 9 \frac{\lambda}{2} \\
\end{align*} \]

\( \Delta \) path difference in path travelled by waves 1 & 2.
For calculational purpose: what is $A_{\text{path}}$?

\[ \tan \theta = \frac{y}{L} \]

$L \gg \lambda$

1B parallel to 2B.

$1B = PB$

$A_{\text{path}} = 2P$

Constructive interference:

\[ d \sin \theta_n = n \lambda \quad \Rightarrow \quad d \sin (\tan^{-1} \frac{y_n}{L}) = n \lambda \]

loc. of bright spot of order $n$:

\[ y_n = L \tan \theta_n = L \tan \left( \sin^{-1} \left( \frac{n \lambda}{L} \right) \right) \]

if $\lambda \ll d 
\Rightarrow y_n \approx \frac{n \lambda L}{(\sin \theta_n \tan \theta_n)}

Destructive interference:

$A_{\text{path}} = (2n+1) \frac{\Delta}{2}$

\[ d \sin \theta_n = (2n+1) \frac{\Delta}{2} \quad \Rightarrow \quad \text{loc. of dark spot of order } (n+1) \]

is

\[ y_n = L \tan \theta_n = L \tan \left( \sin^{-1} \left( \frac{(2n+1) \lambda}{2d} \right) \right) \]

if $\lambda \ll d 
\Rightarrow y_n \approx \frac{(2n+1) \lambda L}{2d}$
Three-slit interference:

One source → 3 identical waves.

Constructive interference

\[ \tan \theta = \frac{y}{L} \]

\[ L \gg d \]

\[ L \text{ paths travelled by 3 waves are parallel.} \]

\[ \begin{align*}
\text{Path}_{12} &= \text{Path}_{23} = d \sin \theta \\
\text{Path}_{3} &= 2d \sin \theta
\end{align*} \]

For our 3 waves:

\[ \tan \theta_m = m \lambda \]  \[ m = 0, 1, 2, 3, \ldots \]

Destructive interference

2 slits:

\[ \text{Path} = (2n+1) \frac{\lambda}{2} = (n + \frac{1}{2}) \lambda \]  \[ n = 0, 1, 2, 3, \ldots \]

2 slits:

\[ \text{Two waves are } 180^\circ \text{ out of phase} \quad \downarrow = 0 \]

3 slits:

\[ \text{Path} = (n + \frac{1}{3}) \lambda \]  \[ n = 0, 1, 2, 3, \ldots \]

\[ \text{Three waves should be } 120^\circ \text{ out of phase:} \]

\[ 120^\circ \]
Diffraction in a single slit: superposition of waves from different points along the slit.

Huygens' principle: each point on a wavefront can become a new source of waves.

\[ \tan \theta = \frac{y}{L} \]

E.g. three identical waves come out of points along slit.

Constructive interference \( C \sim \beta \):

For all 3 waves:

\[ \alpha \Phi + \Phi_n = (2n+1) \frac{\lambda}{2} \quad (n=0,1,2,\ldots) \]

2nd spot for zero waves:

\[ \alpha \Phi + \Phi_n = m \lambda (n=1,2,3,\ldots) \]

Now waves \( \Phi \):

\[ \frac{1}{2} \pi \sin \theta_n = (2n+1) \frac{\lambda}{2} \rightarrow \alpha \Phi + \Phi_n = 2(2n+1) \lambda = 2\lambda; 6\lambda; 10\lambda; 14\lambda; \ldots \]

Waves \( \Phi \):

\[ \frac{1}{2} \pi \sin \theta_n = 2(2n+1) \frac{\lambda}{2} \rightarrow \alpha \Phi + \Phi_n = \frac{3}{2}(2n+1) \lambda = 3\lambda (n=1); 5\lambda (n=2); 7\lambda (n=3) \ldots \]

Diffraction limit in optical instruments:

\[ \theta_{\text{min}} = \frac{1.22 \lambda}{D} \]

\( \theta_{\text{min}} \) is the minimum angle between objects we can distinguish through a lens.
Thin film interference: (rainbows on thin layer of oil on water)

Wave 1 & 2 come out parallel (as in the double slit experiment), however wave 1 stayed in air while wave 2 has travelled approximately 2d in the medium.

Wave 1: because of a reflection off a higher index medium (like a wave reaching the fixed end of a string & getting inverted), gets inverted & gets a phase shift of 180° or \( \frac{\lambda}{2} \)

Wave 2: has travelled an additional 2d.

Constructive interference:

\[
2d = m\lambda + \frac{\lambda}{2}
\]

Destructive interference:

\[
2d = (m+1)\frac{\lambda}{2} + \frac{\lambda}{2}
\]
Visible spectrum dispersed by a grating $\rightarrow$ slit diffraction.

Dark spot on a screen for a single slit diffraction:

$$
\sin \theta_n = \frac{n \lambda}{d}
$$

$$
\gamma_n = \left( \tan \theta_n \right), \text{ location on screen of a dark spot of order } n
$$

$$
\theta_{n \text{ red}} = \sin^{-1} \left( \frac{n \lambda_{\text{red}}}{a} \right), \text{ For a same order } n
$$

$$
\theta_{n \text{ violet}} = \sin^{-1} \left( \frac{n \lambda_{\text{violet}}}{a} \right), \text{ a dark spot for red light is further up from the } x\text{-axis (midline) than the dark spot for violet.}
$$
Question: lowest pair of consecutive orders for some overlap of visible spectrum as dispersed by the grating (slit).

\[ \sin \theta_n \text{ red} = \sin \theta_n \text{ violet} \]

\[ \frac{n \lambda_n \text{ red}}{\theta} = \frac{(n+1) \lambda_n \text{ violet}}{\theta} \rightarrow n \lambda_n \text{ red} - (n+1) \lambda_n \text{ violet} = \lambda_n \text{ violet} \]

\[ n = \frac{\lambda_n \text{ violet}}{\lambda_n \text{ red} - \lambda_n \text{ violet}} = \frac{400 \text{ nm}}{700 \text{ nm} - 400 \text{ nm}} = \frac{4}{3} \approx 1.33 \]

Since \( n \) and \( y_n \) are related,

\[ \sin \theta_n \text{ red} = \frac{y_n \text{ red}}{2} \rightarrow \sin \theta_n \text{ violet} = \frac{y_n \text{ violet}}{2} \]

so \( n + 1 \text{ red} = n + 1 \text{ violet} \)

Thus, any integer \( n \) can only be an integer

\[ n = 2 \quad \text{or} \quad n + 1 = 3 \]

\[ y_2 = y_3 \]

(loc. of 2nd dark spot for red coincides with loc. of 3rd dark spot for violet).
\( \lambda = 633\, \text{nm} \); \( d = 6.5\, \text{mm} \); \( L = 1.7\, \text{m} \)

a) \( y_2 - y_1 \)
b) \( y_4 - y_3 \)

\[
y_2 - y_1 = L \left( \tan \left[ \tan^{-1} \left( \frac{2\lambda}{d} \right) \right] - \tan \left[ \tan^{-1} \left( \frac{\lambda}{d} \right) \right] \right)
= 1.7 \left( \tan \left[ \tan^{-1} \left( \frac{2 \times 6.33 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] - \tan \left[ \tan^{-1} \left( \frac{6.33 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] \right)
= 17.17\, \text{cm}
\]

b) \( y_4 - y_3 \)

\[
y_4 - y_3 = L \left( \tan \left[ \tan^{-1} \left( \frac{4\lambda}{d} \right) \right] - \tan \left[ \tan^{-1} \left( \frac{3\lambda}{d} \right) \right] \right)
= 20\, \text{cm}
\]

If we use \( \lambda \ll d \),

\[
0.633\, \text{mm} \ll 6.5\, \text{mm}
\]

\[
y_2 - y_1 = (2-1) \frac{\lambda L}{d} = \frac{\lambda L}{d} = 17\, \text{cm}
\]

\[
y_4 - y_3 = (4-3) \frac{\lambda L}{d} = \frac{\lambda L}{d} = \frac{633 \times 10^{-9} \times 1.7}{6.5 \times 10^{-6}} = 17\, \text{cm}
\]
Signal from a 103.9 MHz FM radio station reflects off a building 400m away producing 2 identical waves.

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{103.9 \times 10^6} = 2.89 \text{ m} \]

\[ n + 1 \text{ } \text{ } n \nonumber \]

\[ \frac{2n+3 - (2n+1)}{2} \frac{\lambda L}{2d} \]

\[ \frac{(2n+1)\lambda L}{2d} - \frac{(2n+3)\lambda L}{2d} \nonumber \]

\[ \frac{2.89 \times 6500}{200 \times 60} \nonumber = 2.82 \text{ s} \]

\[ \text{How often signal appears to fade:} \]

\[ \frac{\lambda L}{2d} \]

\[ \frac{2 \lambda L}{2d} \nonumber \]

\[ \text{How often:} \frac{1}{2.82 \text{ s}} \]
32.21

Find the thickness (n = 1.333 for film) for \( \lambda = 550 \text{ nm} \) to undergo constructive interference.

\[
2d = \left( m + \frac{1}{2} \right) \frac{\lambda_0}{n} \quad (m = 0, 1, 2, \ldots)
\]

\[
d_{\text{min}} = \frac{\lambda_0}{2n} = \frac{\lambda_0}{4n} = \frac{550 \text{ nm}}{4 \times 1.333} = 103 \text{ nm}.
\]

31.32 31.42 31.50

Concave mirror \( R? \)

\( h' = 9.5 \text{ cm} \) (virtual image)
\( h = 5.7 \text{ cm} \) (object)
\( l = 22 \text{ cm} \) (from mirror)

Mirror equation:
\[
\frac{1}{l} + \frac{1}{l'} = \frac{1}{f}
\]

\[
R = 2f ightarrow \text{need } f:\]

Magnification factor:
\[
M = \frac{h'}{h} = -\frac{l'}{l} \rightarrow l' = -\frac{h'f}{h}
\]
\[
\frac{1}{l} + \frac{1}{l'} = \frac{1}{f}
\]
\[
\frac{1}{l} - \frac{1}{\frac{l h'}{h}} = \frac{1}{l} \left[ 1 - \frac{1}{\frac{h'}{h}} \right] = \frac{1}{l} \left[ 1 - \frac{h}{h'} \right] = \frac{1}{f}
\]

\[R = 2f = \frac{2l}{1 - \frac{h}{h'}} = \frac{2 \times 22 \text{cm}}{1 - \frac{5.7}{9.5}} = 110 \text{ cm}.
\]

Bubble underwater:

\[
\begin{align*}
\frac{n_2}{l} + \frac{n_1}{l'} &= \frac{n_2 - n_1}{R} \\
n_2 &= n = 1.333 \text{ (medium)} \\
n_1 &= 1 \\
l &= 2R \text{ (always +)}
\end{align*}
\]

\[\frac{1}{2R} + \frac{1.333}{l'} = 0.333 \quad \text{(R)}
\]

Bubble appears to be 1.5 cm in diameter \( \rightarrow \) \( l' = 1.5 \) cm

Form convention for lens when image is in same size as object.

\[
\frac{1}{R} \left( 0.5 + 0.333 \right) = \frac{1.333}{1.5} \quad \rightarrow \quad R = \frac{0.833 \times 1.5}{1.333} = 0.938 \text{ cm}.
\]

\[\text{Actual diameter is} \quad 2R = 1.87 \text{ cm}\]
Converging lens: \[
\frac{1}{f'} + \frac{1}{l'} = \frac{1}{f}
\]

\[f = +35 \text{ cm}\]

a) \(l'\) when \(l = 40 \text{ cm}\):

\[
\frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = \frac{1}{35} - \frac{1}{40} = \frac{1}{40 - 35} = \frac{1}{5}
\]

\[
l' = \frac{35 \times 60}{40} = 280 \text{ cm}
\]

Object to image: \(40 \text{ cm} + 280 \text{ cm} = 320 \text{ cm}\).

b) \(l'\) when \(l = 30 \text{ cm}\):

Virtual image \(l' (-)\):

\[
\frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = \frac{1}{35} - \frac{1}{30} = \frac{35 \times 30}{35 - 30} = \frac{1050}{5} = -210 \text{ cm}
\]

Object to image: \(180 \text{ cm}\).
31.36 \( f = 2.0 \text{ cm} \)

\( f_{\text{required}} = 2.2 \text{ cm} \) (sharply focused image on retina)

\( \text{a) Image for faraway objects is formed before retina} \)

\[ \text{Reflex} \quad \text{Near sighted.} \]

\( \text{b) Power of corrective lens needed:} \)

\[ \text{Diopter} = \begin{cases} \frac{1}{f_{\text{required}}} = \frac{1}{0.02 \text{ m}} = 50 \text{ diopters}, \\ \text{Eye (good)} \frac{1}{0.022 \text{ m}} = 45.5 \text{ diopters}. \\ \text{Corrective lens will have} \ -4.5 \text{ dipters.} \end{cases} \]