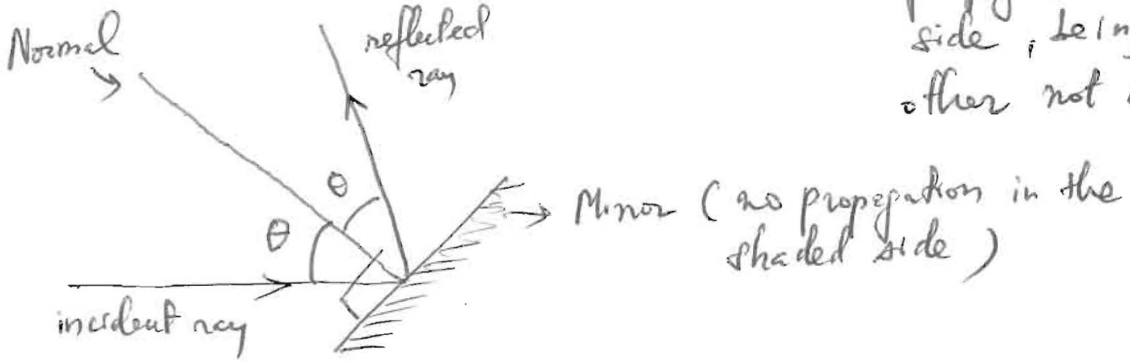
Ch 30: Reflection & Refraction

Optics
- → **Geometrical Optics** (Ch 30, 31): propagation of light rays in straight lines using geometry
- → **Physical Optics** (Ch 32): use wave properties of light in addition to the geometry of the problem.

Reflection: light ray incident upon a mirror
↳ light can only propagate in one side, being the other not accessible

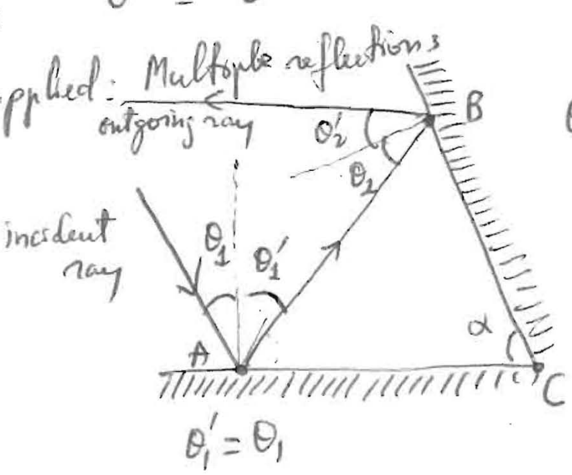


Incident angle θ: angle b/w incident ray & the normal to the mirror surface

Reflected angle θ': b/w the reflected ray & the normal

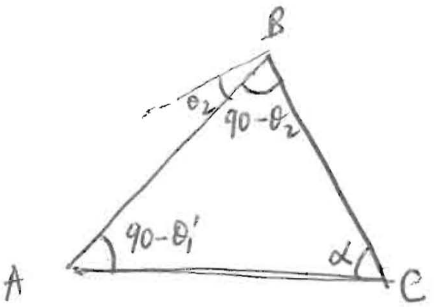
Law of reflection: $\theta' = \theta$

When geometry is applied: Multiple reflections



$\theta_2' = \theta_2$

$\boxed{\text{Need to relate } \theta_2' \text{ to } \theta_1}$
↓
Use geometry (Triangle ABC)



<u>In any triangle</u>:
$$90^\circ - \theta_1' + \alpha + 90^\circ - \theta_2 = 180^\circ$$
$$\alpha - \theta_1' - \theta_2 = 0$$
$$\theta_2 = \alpha - \theta_1'$$

$$\underbrace{\theta_2' = \theta_2}_{\text{Law of Reflection on mirror BC}} \underbrace{=}_{\text{Triangular geometry}} \alpha - \theta_1' \underbrace{=}_{\text{Law of Reflection on mirror AC}} \alpha - \theta_1$$

$\boxed{\text{Refraction:}}$ when light rays travel from one medium to another.

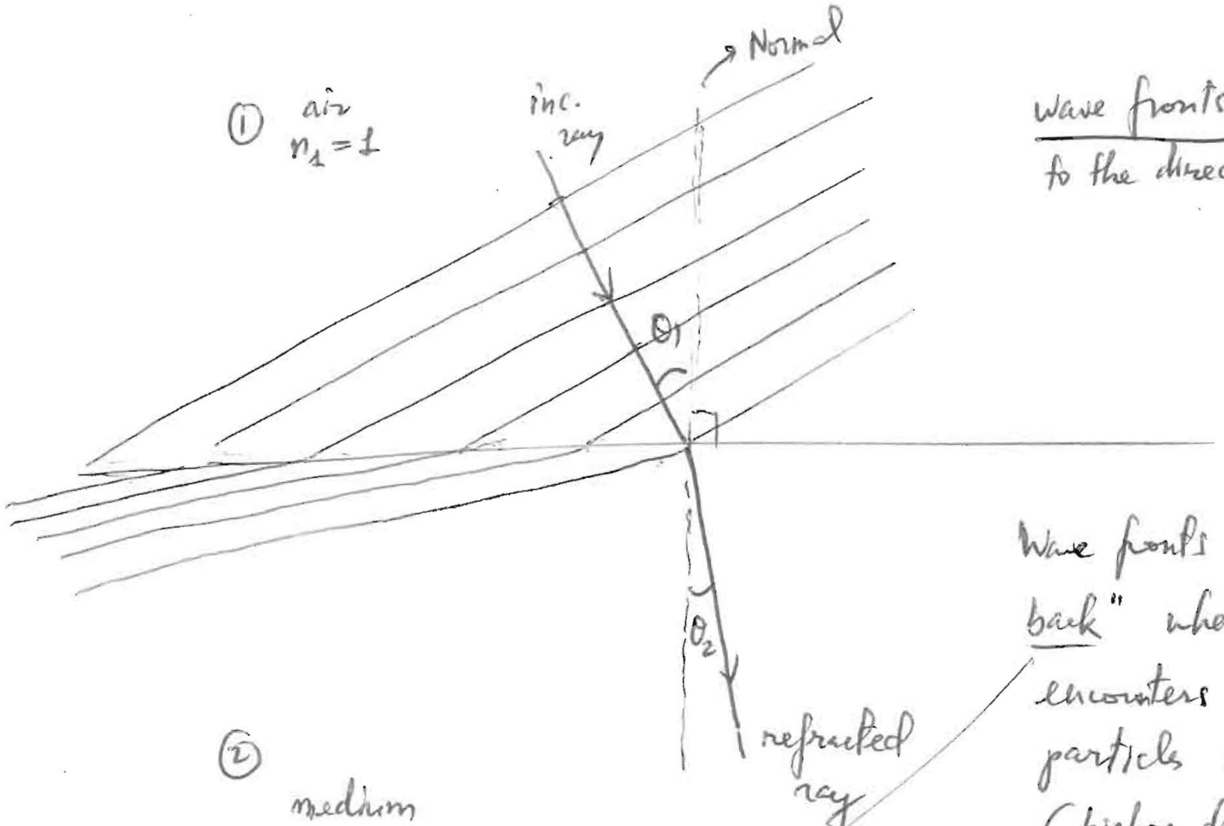
→ light speed in vacuum : $c = 3\times10^8$ m/s

→ light speed in water : $c_w = \frac{c}{n_w}$ (n_w = index of refraction for water : 1.333)

<u>Daily phenomenon</u>: → rainbow (different refraction indices for different colors or wave lengths)

→ broken straw:

<u>wave fronts</u> are perpendicular to the direction of propagation.



②

medium

$n_2 = n > 1$

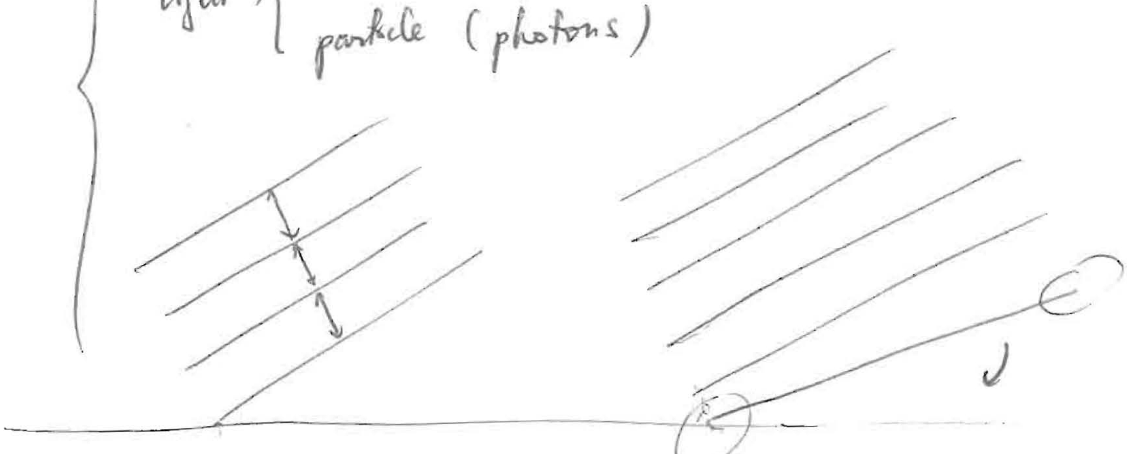
(Larger index normally due to higher density)

Wave fronts are "pushed back" when light encounters more particles in the way (higher density or higher index of refraction)

↓

$\theta_2 < \theta_1$ when going from lower to higher index

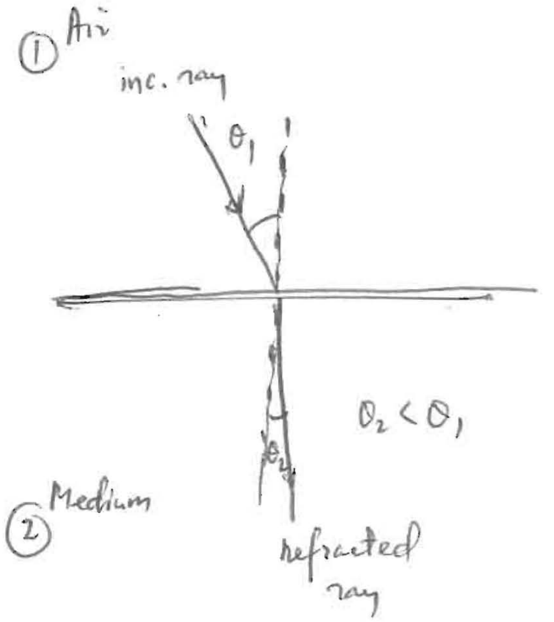
light → { wave (EM waves)
particle (photons) }

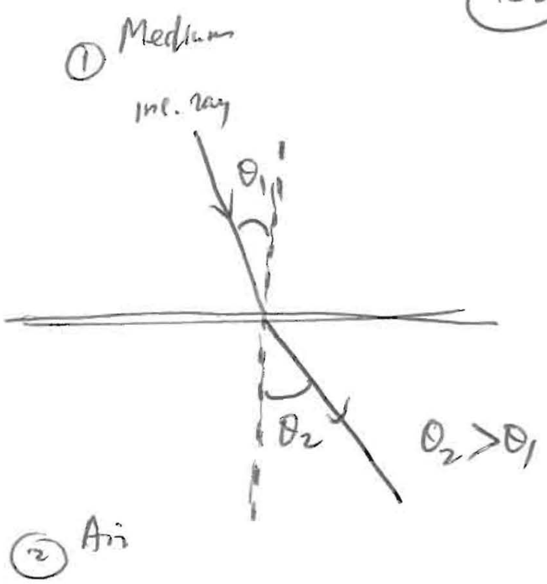


This side is still in medium 1

↓

This side of wave front is changing medium (higher index) → gets pushed back. → Rotation of wave front.



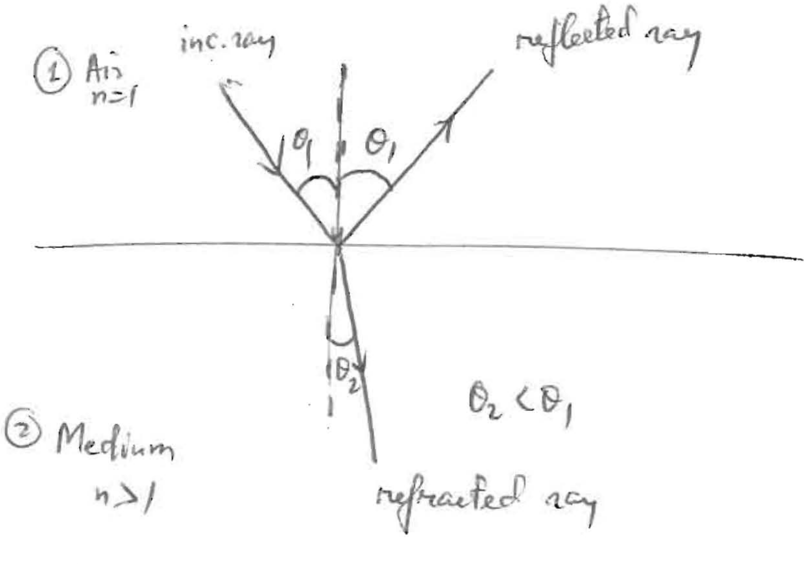


Law of refraction : Snell's Law :

$$\boxed{n_1 \sin(\theta_1) = n_2 \sin(\theta_2)}$$

n_1: index of medium 1 — θ_1: incident angle — n_2: index of medium 2 — θ_2: refracted angle.

In practice:



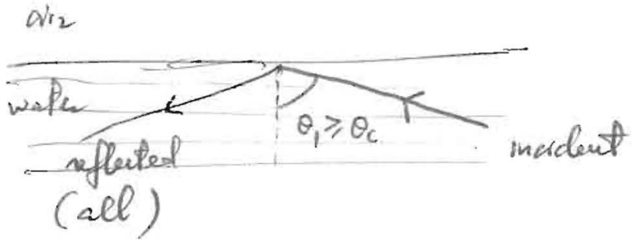
→ Normally: there are more refraction than reflection.

→ Beyond a critical angle: $\theta_1 > \theta_c$: all incident light is reflected (none is refracted)

Normally there are more refraction, but it decrease when θ_1 is getting larger towards the θ_c. Refraction disappears @ and beyond θ_c.

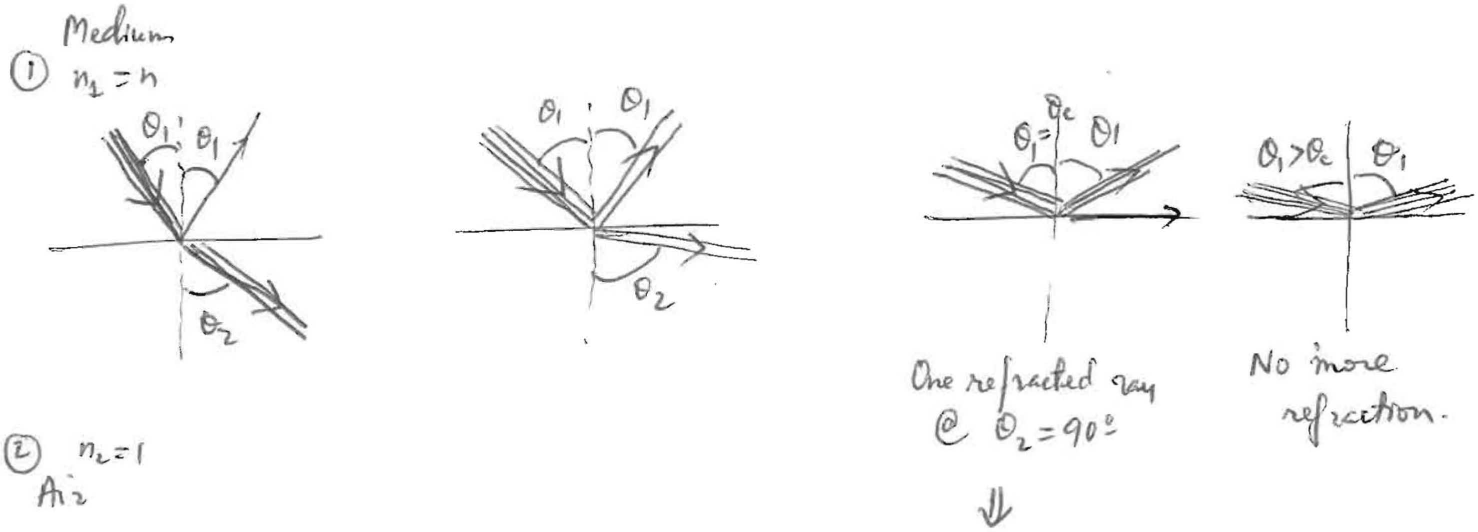
Total Internal Reflection

↳ Happens when light travels from higher index to lower index



→ light gets confined
→ Used to carry information (internet, phone, etc).
↓
Not @ c but faster & more reliable than electrical current in cables.

Critical angle:



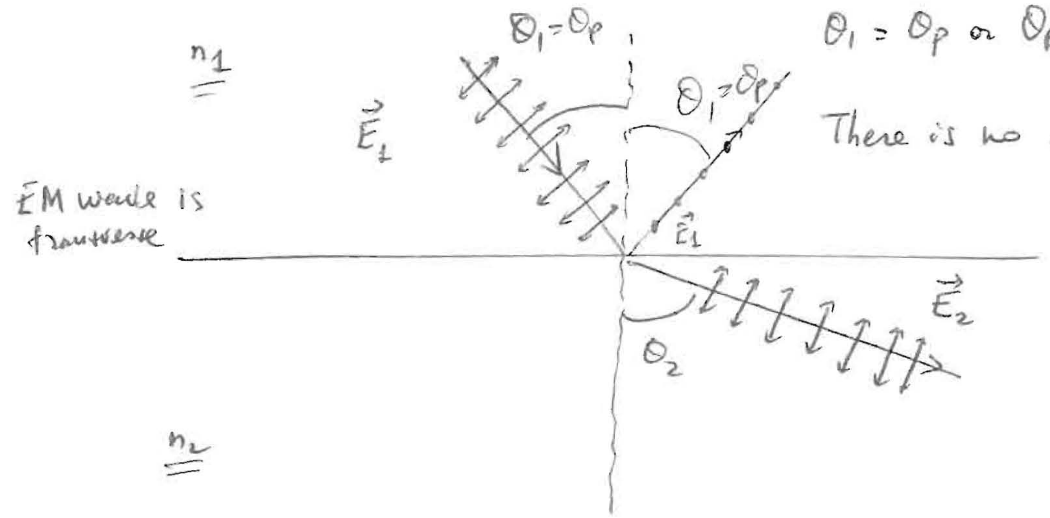
⇓

$$\theta_c = \theta_1 \text{ when } \theta_2 = 90°$$

$$n_1 \sin\theta_c = n_2 \sin 90° = n_2$$

$$\boxed{\theta_c = \sin^{-1}\frac{n_2}{n_1}}$$

Is there any incident angle @ which there is no reflection and all incident light is refracted? → The <u>polarizing angle</u> or <u>Brewster angle</u>.



$\theta_1 = \theta_p$ or $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

There is no reflection when $\theta_1 = \theta_p$

<u>Observation</u>:

1) If $\vec{E}$ has a component that is perpendicular the page, that component is not affected even if $\theta_1 = \theta_p$ (or θ_B): there is reflection for that component.

2) When $\vec{E}$ has $E_{\parallel}$ (parallel to page) & $E_{\perp}$ (perpendicular to page) light refracted into medium 2 will have $\vec{E}$ polarized in direction parallel to page (if $\theta_1 = \theta_B$ (or θ_p))

3) This is useful when $n_2 > n_1$: taking picture of something behind a glass window.

Mona Lisa

Ch 31 Images & Optical Instruments

↳ Mirrors, Lenses } Geometrical Optics

↓

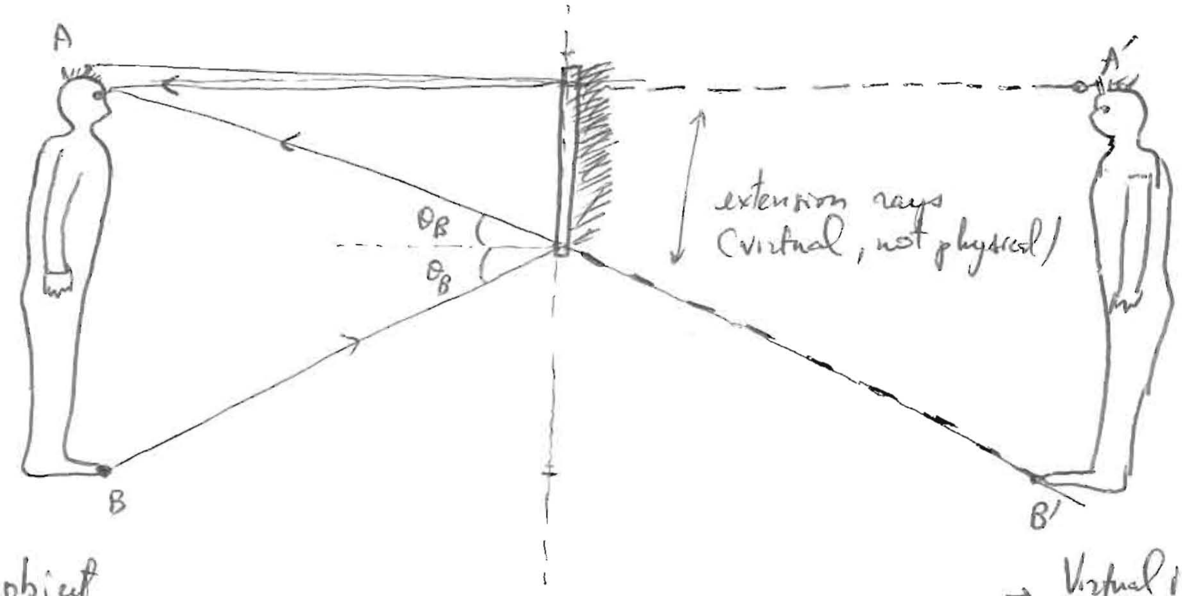
How to form the image of an object off a mirror or through a lense?

Image formation by a mirror:

↓

How tall a mirror (height h_m) we need so to see our whole body? (height is h_b)

1) $h_m = h_b$
2) $h_m = \frac{1}{2} h_b$ ✓
3) $h_m = \frac{2}{3} h_b$ ✓



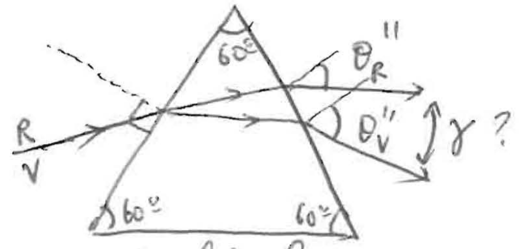
object

→ Virtual image
light cannot travel behind the mirror.

→ Formed by extension rays.

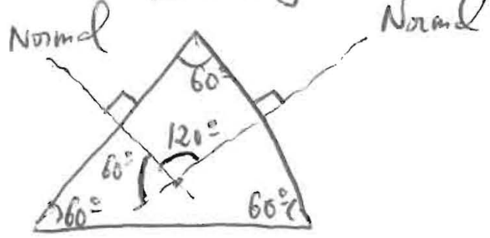
30.28
30.57

30.28



Equilateral.

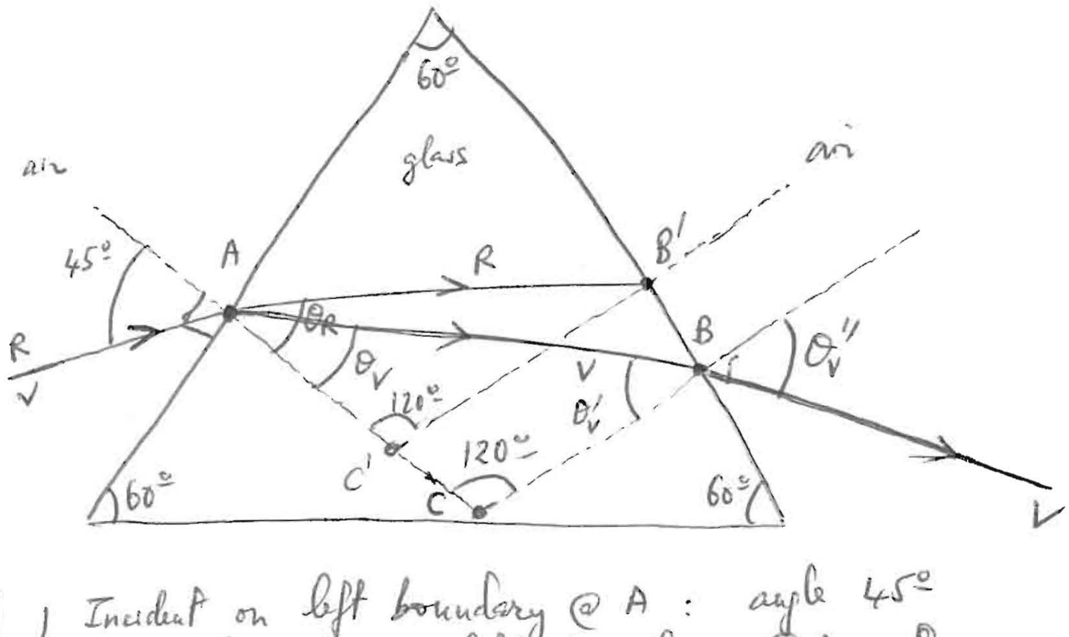
Geometry:



Prism $\begin{cases} n_R = 1.582 \\ n_V = 1.633 \end{cases}$ → Dispersion: different colors come out @ different directions the other side of prism.

(Different indices of refraction for different wavelengths)

$\gamma \equiv \theta_V'' - \theta_R''$?



$\boxed{\text{Violet ray:}}$

- Incident on left boundary @ A : angle 45°
- Refracted angle on left boundary @ A : θ_V
- Incident on right boundary @ B : θ_V'
- Refracted angle @ B : θ_V''

- Snell's Law @ A: $\underbrace{1 \sin 45^\circ}_{\frac{1}{\sqrt{2}}} = 1.633 \sin \theta_V \rightarrow \theta_V = \sin^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{1.633}\right) = 25.5^\circ$

- Snell's Law @ B: $1.633 \sin \theta_V' = 1 \sin \theta_V''$

Need one more eq. → from geometry: triangle ABC → $\theta_V + \theta_V' + 120^\circ = 180^\circ$

$\rightarrow \theta_V + \theta_V' = 60°$ $\rightarrow \theta_V' = 60° - \theta_V = 60° - 25.5° = 34.5°$

$$\boxed{\theta_V'' = \sin^{-1}(1.633 \sin 34.5°) = 67.7°}$$

$\boxed{\text{Red ray:}}$ same calculations except $n_R = 1.582$

Snell's Law @ A: $1 \sin 45° = 1.582 \sin \theta_R \rightarrow \theta_R = \sin^{-1} \frac{\frac{1}{\sqrt{2}}}{1.582} = 26.5°$

Snell's Law @ B': $1.582 \sin \theta_R' = 1 \sin \theta_R''$

From triangle AB'C': $\theta_R + \theta_R' + 120° = 180°$

$\theta_R' = 60 - \theta_R = 60 - 26.5°$

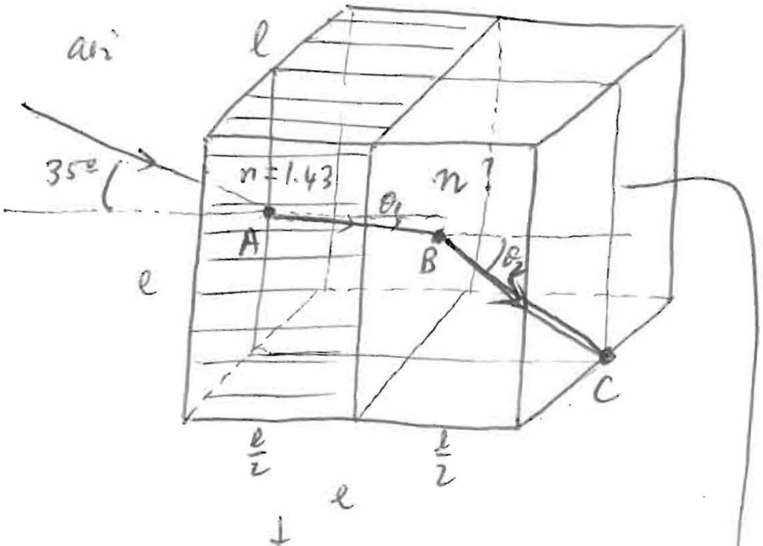
$\theta_R' = 33.5°$

$$\boxed{\theta_R'' = \sin^{-1}(1.582 \sin 33.5°) = 60.8°}$$

$\rightarrow$ Angular dispersion $\delta = \theta_V'' - \theta_R'' = 67.7° - 60.8° = 6.85°$.

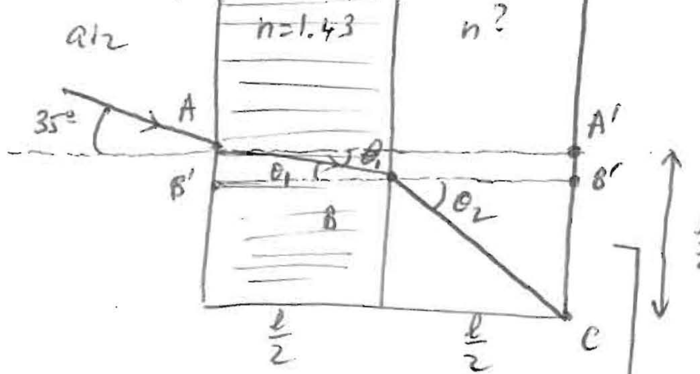
For the beam: Red & Violet are the outer wavelengths of the visible spectrum: other colors ~~behave~~ are between these: $1.582 < n < 1.633$.

30.57



1st refraction @ A:

Front view:



Snell's Law:

@ A: $1 \sin 35^\circ = 1.43 \sin\theta_1$

$$\theta_1 = \sin^{-1}\left(\frac{\sin 35^\circ}{1.43}\right)$$

$$\theta_1 = 23.6^\circ$$

@ B: $1.43 \sin 23.6^\circ = n \sin\theta_2$ → 2 unknowns: need one more equation from the geometry of the problem:

$$\tan\theta_2 = \frac{B'C}{\frac{l}{2}}$$

$$A'C = \frac{l}{2} \rightarrow B'C = A'C - A'B'$$

Now find $A'B'$: $\frac{A'B'}{\frac{l}{2}} = \tan\theta_1$

$$A'B' = \frac{l}{2}\tan\theta_1$$

$$\boxed{\tan\theta_2 = \frac{\frac{l}{2} - \frac{l}{2}\tan\theta_1}{\frac{l}{2}} = 1 - \tan\theta_1}$$

$$\theta_2 = \tan^{-1}(1 - \tan 23.6^\circ)$$

$$\boxed{\theta_2 = 29.3^\circ}$$

$$\boxed{n = \frac{1.43\sin(23.6^\circ)}{\sin(29.3^\circ)} = 1.17}\ *$$

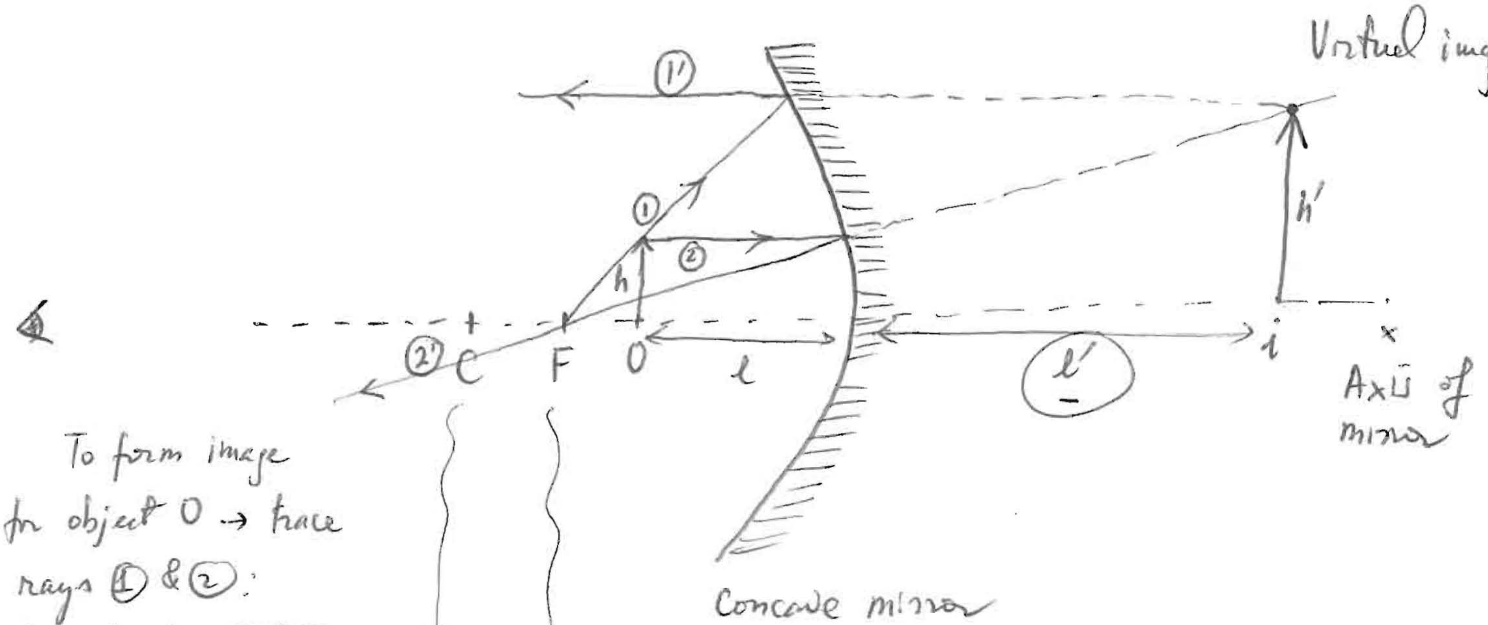
* From Figure given as data $\theta_2 > \theta_1$ → we expect $n < 1.43$ → checked

Ch 31 Image Formation in Optical Instruments

Mirrors & lenses.

Virtual image by a flat mirror ✓

Virtual image by a curved mirror:



To form image for object O → trace rays 1 & 2: By extending 1' & 2' behind the mirror we found location of Virtual image @ i

In this example: we assume F is b/w C and the midpoint of mirror (where axis goes through mirror)

F: Focal point
1) Incident rays ∥ axis will reflect through F
2) Incident rays through F will reflect parallel to axis

Mirror equation:

$$\boxed{\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}}$$ ← (From geometry of rays)

Magnification factor: $M = \frac{h'}{h} = -\frac{\ell'}{\ell}$

Sign convention for mirrors

f: + concave mirror; − convex mirror

ℓ': + image located same side of mirror as object (real image); − image located in the other side of mirror (Virtual image)

Can we get a real image using a concave mirror?

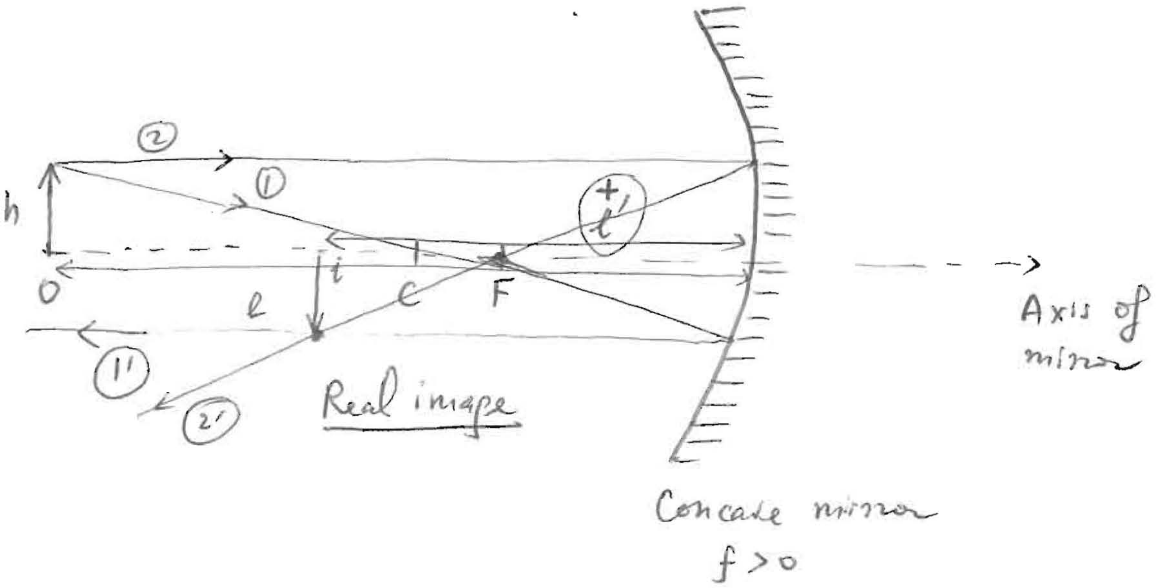
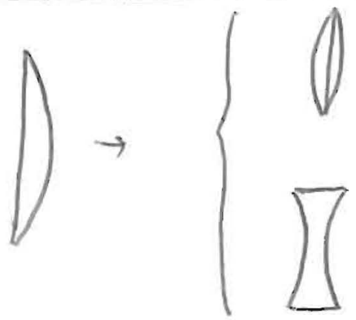


Image formation in lenses:



Converging lense or convex lense

diverging lense or concave lense

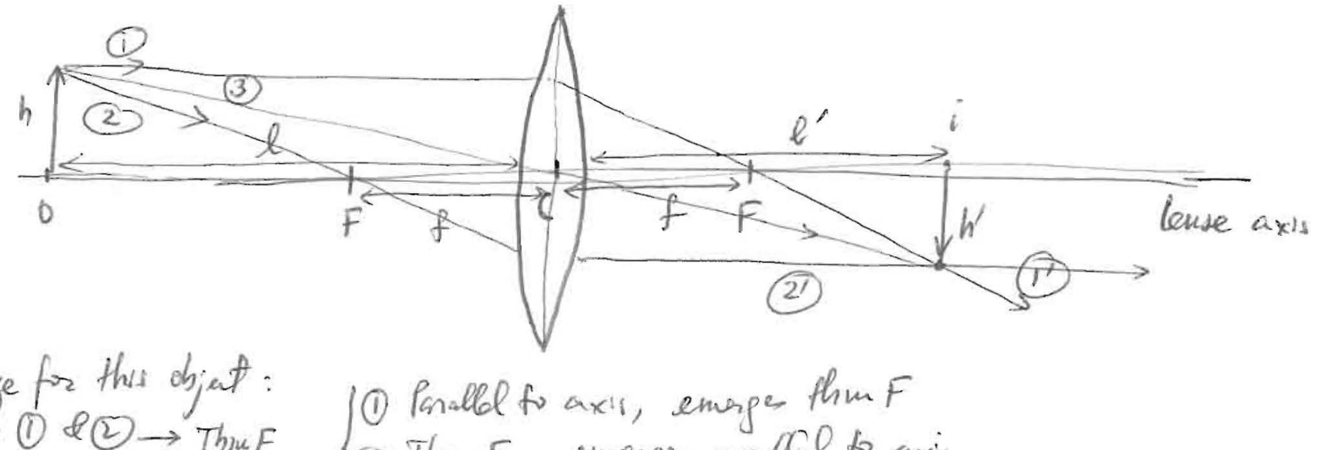


Image for this object:
rays ① & ② → Thru F
~~That~~ Parallel to axis

① Parallel to axis, emerges thru F
② Thru F, emerges parallel to axis
③ Thru C → straight through. (not essential after ① & ②)

Lense equation: $\boxed{\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}}$

Magnification factor: $M = \frac{h'}{h} = -\frac{\ell'}{\ell}$

Sign convention for lense:

- f:
  - $-$ concave lense (diverging)
  - $+$ convex lense (converging)
- ℓ':
  - $+$ image located the other side of lense
  - $-$ image located same side of lense as object

Types of lense:

1) Air - Glass - Air
(Medium)

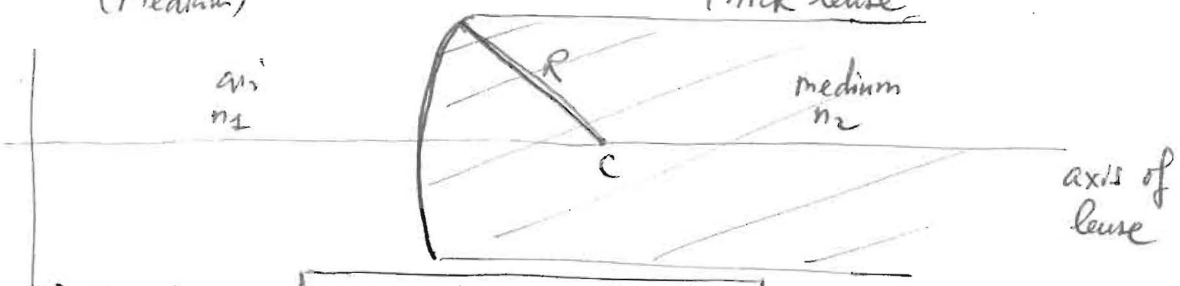
or

(Thin lense)

$\boxed{\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}}$

2) Air - Glass:
(Medium)

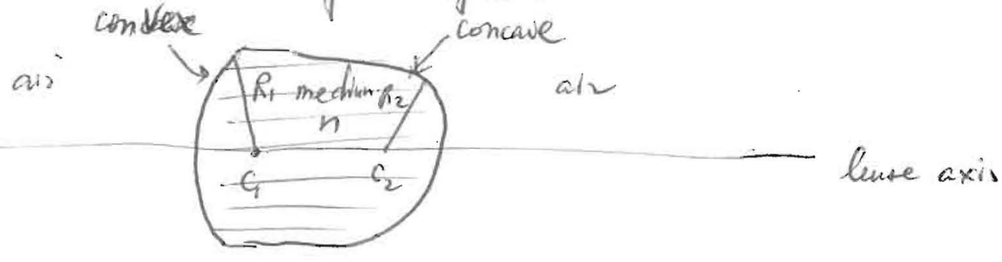
Thick lense



Equation: $\boxed{\frac{n_1}{\ell} + \frac{n_2}{\ell'} = \frac{n_2 - n_1}{R}}$

Sign convention for R:
- $+$ convex
- $-$ concave

3) Air - Medium - Air: with different radii of curvature left & right.



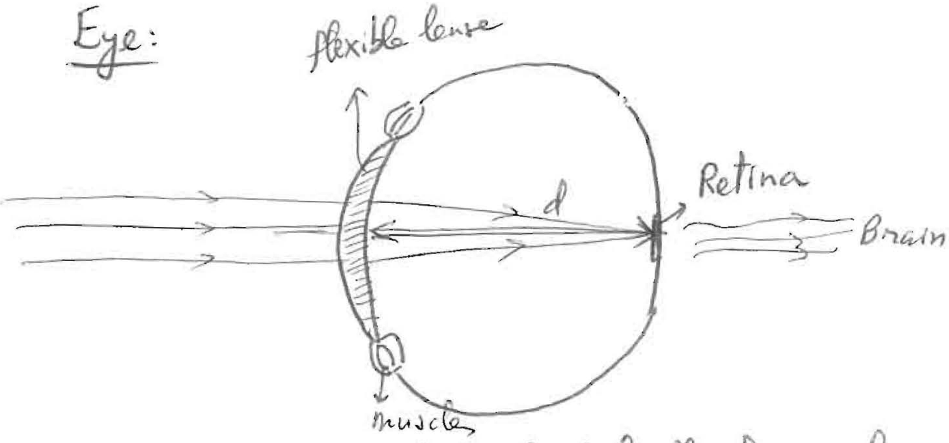
Equation: "Lense maker's equation"

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

$\frac{1}{f}$ → inverse of focal length.

R_1 & R_2 sign convention $\begin{cases} + & \text{convex} \\ - & \text{concave} \end{cases}$

Eye:



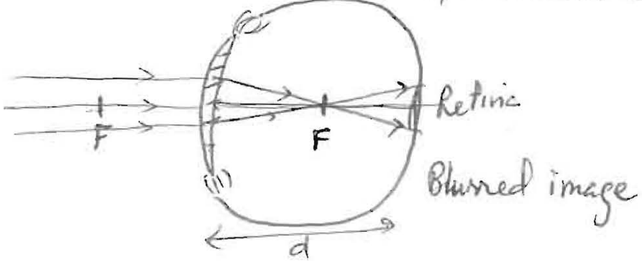
controlling focal length of our lense: trying to set $f = d$

(we will see a clear & focused far away objects)

→ Good eye.

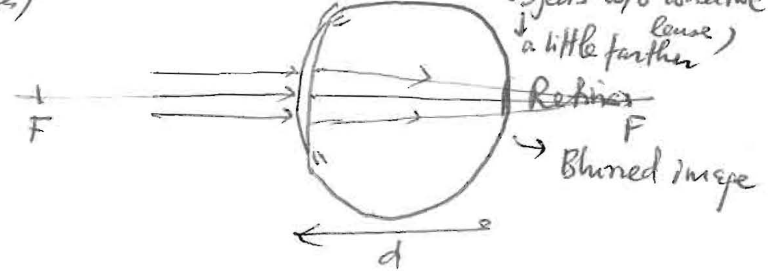
Near sighted (myopic)

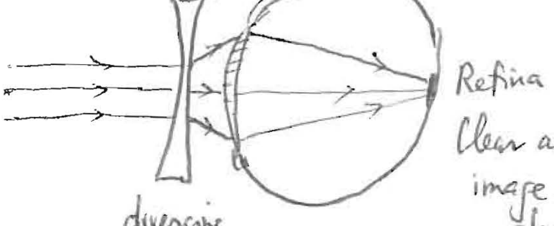
$\boxed{f < d}$ (can see closer objects w/o corrective lense)



Far sighted (hyperopic)

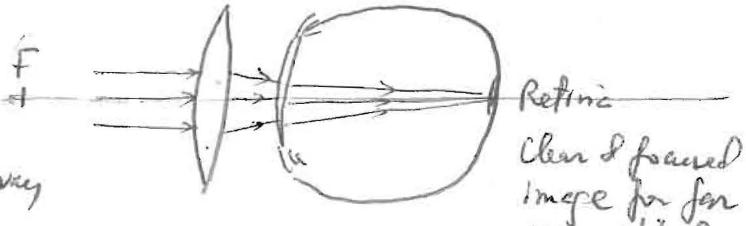
$\boxed{f > d}$ (can see ~~far away~~ objects w/o corrective lense) a little farther





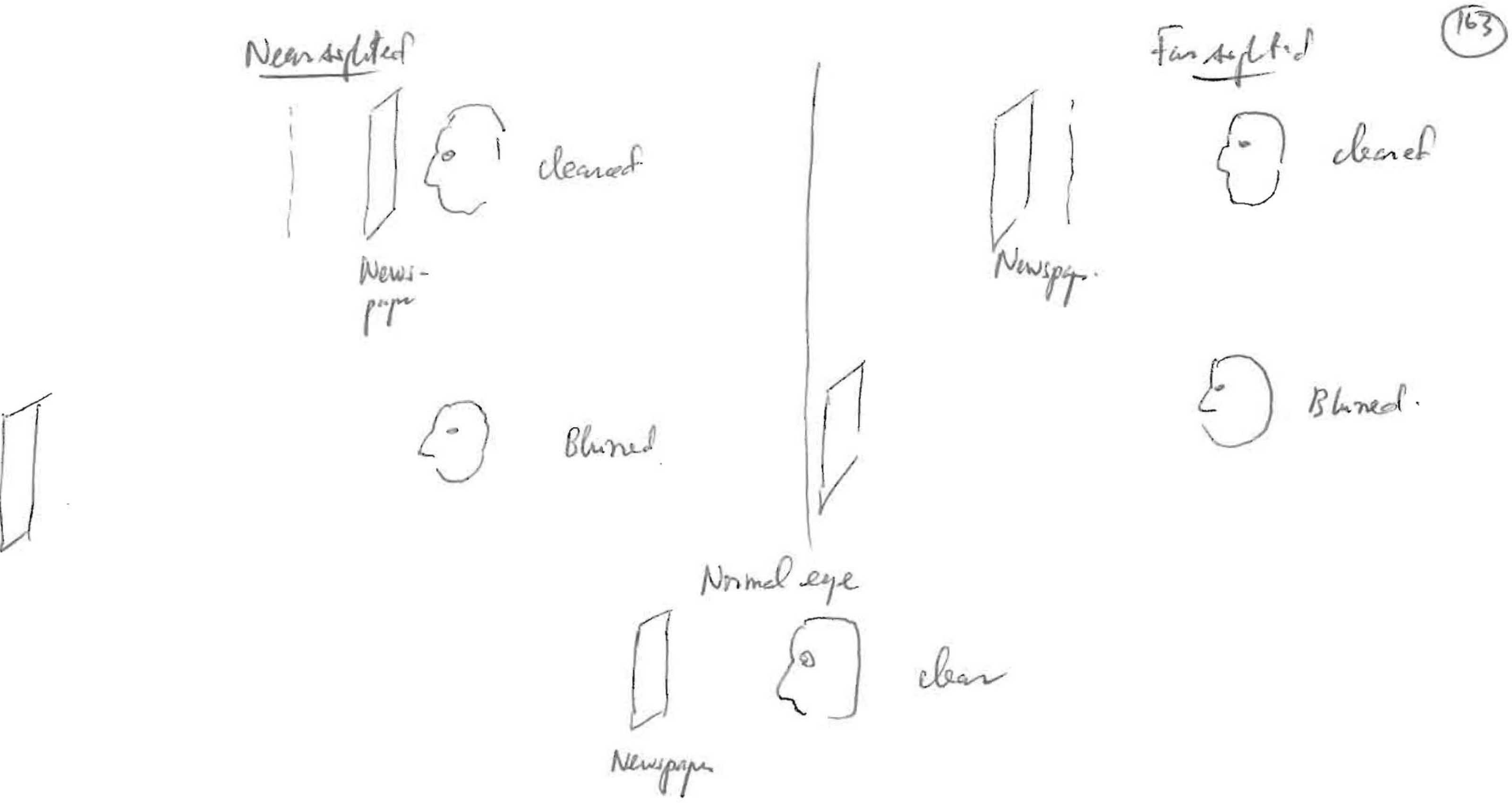
Clear and focused image for far away object.

diverging corrective lense → $\boxed{f < 0}$ $\boxed{\text{diopter} = \frac{1}{f}}$ → meters.



Clear & focused image for far away objects

converging lense $\boxed{f > 0}$ → $\boxed{\text{diopter} = \frac{1}{f}}$ meter.



30,44

$n = 1.52$

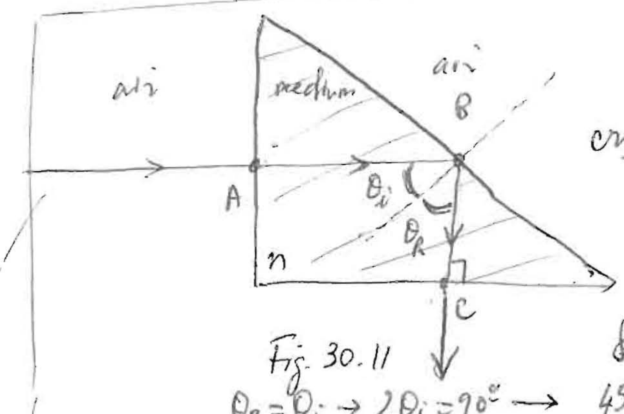


Fig. 30.11

critical angle: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

@ B

$\left\{\begin{array}{l} n_2 = 1 \\ n_1 = n = 1.52 \end{array}\right.$ $\theta_c = \sin^{-1}\left(\frac{1}{1.52}\right) = 41°$

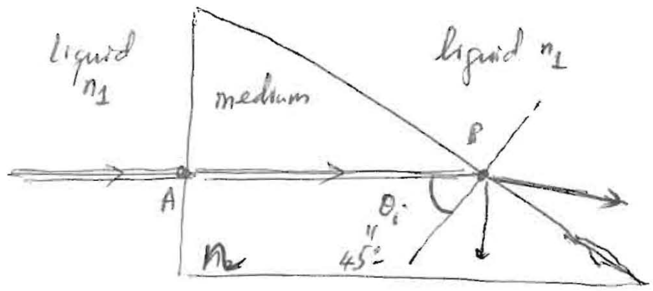
there is Total internal reflection (TIR)

$\theta_R = \theta_i \rightarrow 2\theta_i = 90° \longrightarrow 45° = \boxed{\theta_i \geq 41°}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_1 = 0 \Rightarrow \theta_2 = 0$

Now inmersed in liquid: no more Total reflection @ B



$n_2 = 1.52$

What minimum n_1 so there is some refracted light out to liquid?

Since w/ or w/o liquid $\theta_i = 45^\circ$ → no longer T.I.R. if

θ_i now is $\leqslant \theta_c^{liq}$ → $\sin\theta_i = \sin 45^\circ = \frac{1}{\sqrt{2}} \leq \sin\theta_c^{liq}$.

θ_c^{liq} : new critical angle w/ liquid outside (@ B)

$$\theta_c^{liq} = \sin^{-1}\left(\frac{n_1}{n}\right) = \sin^{-1}\left(\frac{n_1}{1.52}\right)$$

$$\sin\theta_c^{liq} = \frac{n_1}{1.52}$$

$$\frac{1}{\sqrt{2}} \leq \frac{n_1}{1.52} \rightarrow \boxed{n_1 \geqslant \frac{1.52}{\sqrt{2}} = 1.07}$$

Ch 32 Interference & Diffraction :

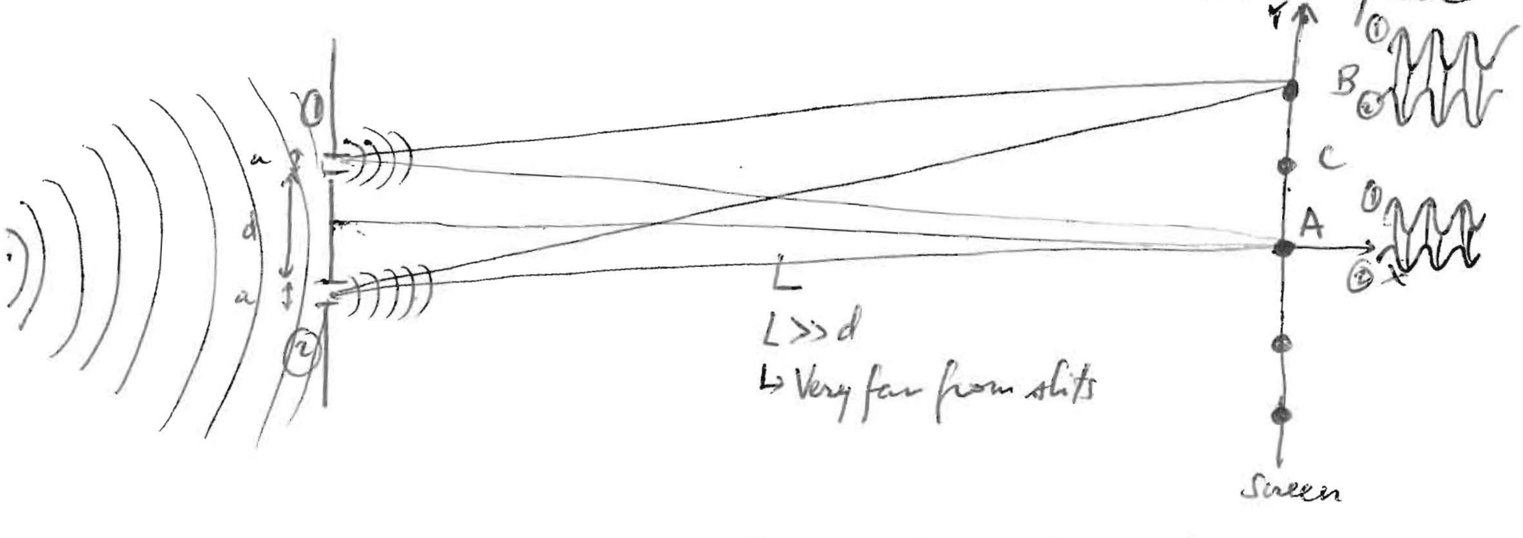
↓

Physical optics: using wave properties of light in addition to the geometry of problem.

↳ superposition of waves { constructive (in phase) / destructive (out of phase or 180°) }

Double-slit Interference :

→ One source of wave → 2 slits → 2 identical waves coherent or in phase

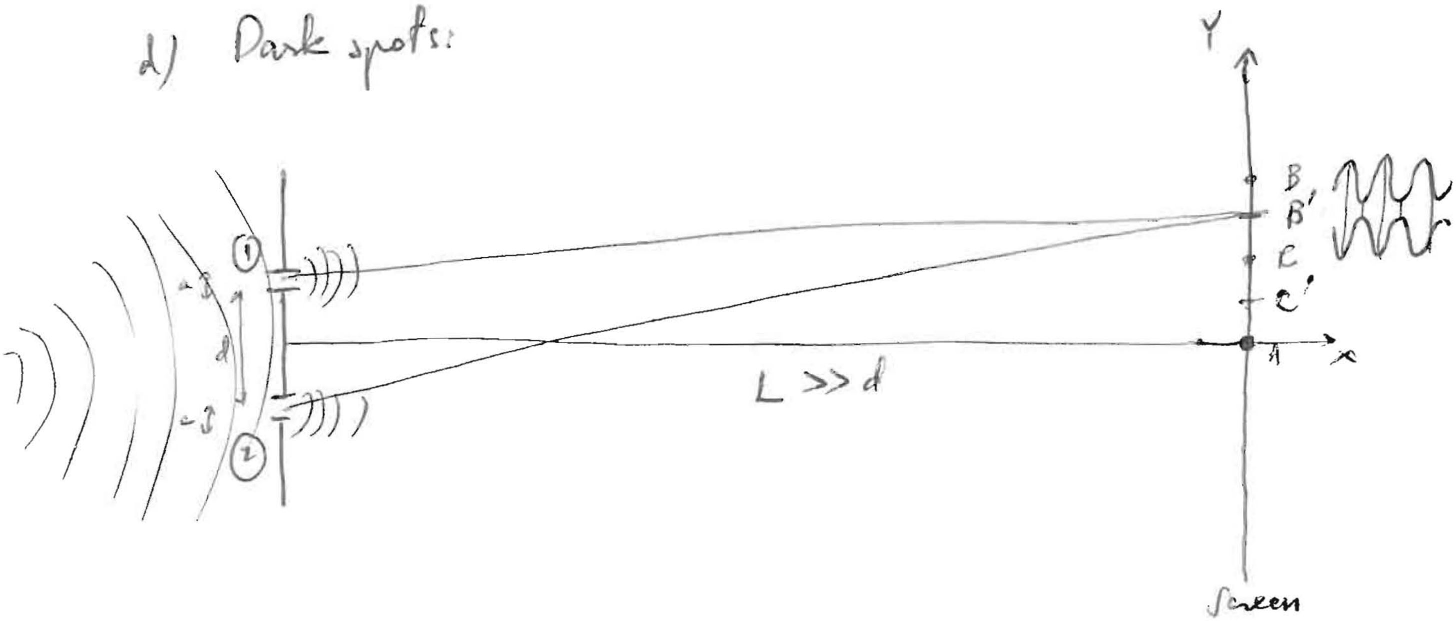


a) 1A = 2A → waves 1 & 2 in phase initially → also in phase at A
→ waves 1 & 2 combine constructively @ A → bright spot @ A.

b) 1B < 2B → if $2B - 1B = n\lambda$ → waves 1 & 2 are still in phase @ B → combine constructively → bright spot.

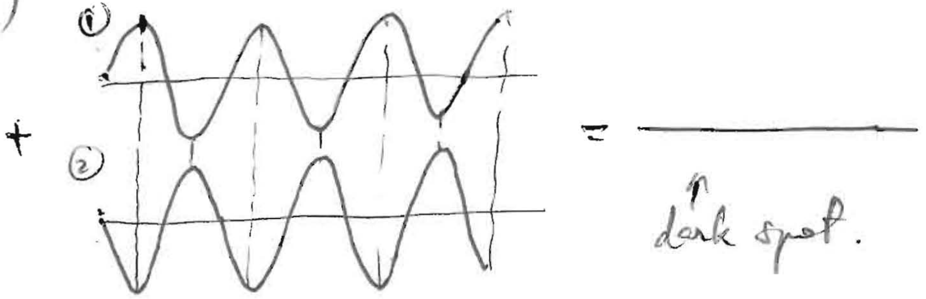
c) 1st bright spot beyond A : $2C - 1C = \lambda \quad (n=1)$
2nd " " " A , $2B - 1B = 2\lambda \quad (n=1)$

d) Dark spots:



$2B' > 1B' \rightarrow \quad 2B' - 1B' = \underbrace{(2m+1)\frac{\lambda}{2}}_{\text{odd multiple of half wavelengths}} \quad (m = 0, 1, 2, 3, \ldots)$

waves ① & ② arrive @ B' out of phase (max of ① coincides with min. of ②, ...)

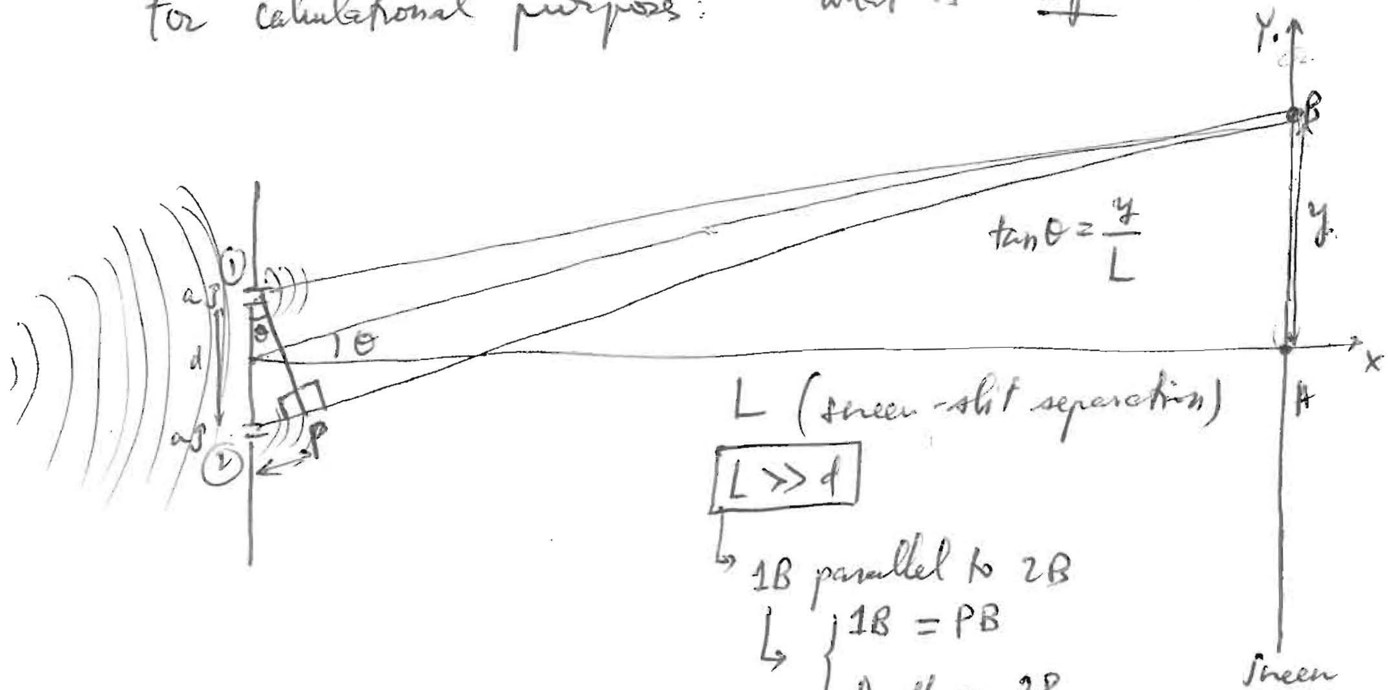


1st dark spot: $\quad \underbrace{2C' - 1C'} = \frac{\lambda}{2} \quad (n=0)$

2nd dark spot: $\quad \underbrace{2B' - 1B'} = 3\frac{\lambda}{2} \quad (n=1)$

5th dark spot: $\quad \underbrace{\Delta path}_{\text{difference in paths travelled by waves ① & ②}} = 9\frac{\lambda}{2} \quad (n=4)$

For calculational purposes: what is $\underline{\Delta path}$?

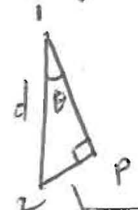


L (screen-slit separation)

$\boxed{L >> d}$

↳ 1B parallel to 2B

↳ $\begin{cases} 1B = PB \\ \Delta path = 2P \end{cases}$



$\boxed{\Delta path = d\sin\theta}$

Constructive interference: $\Delta path = n\lambda$ $(n = 0, 1, 2, 3 \ldots)$

$n = 0$ ↓ central bright spot; $n = 1, 2, 3$ ↑ 1st, 2nd, 3rd

$d\sin\theta_n = n\lambda \rightarrow d\sin\left(\tan^{-1}\frac{y_n}{L}\right) = n\lambda$

$\theta_n = \tan^{-1}\frac{y_n}{L}$

or loc. of bright spot of order n:

$$\boxed{y_n = L\tan\theta_n = L\tan\left[\sin^{-1}\left(\frac{n\lambda}{d}\right)\right]}$$

$$\boxed{\text{if } \lambda << d \rightarrow y_n \simeq \frac{n\lambda L}{d}} \quad (\sin\theta_n \simeq \tan\theta_n)$$

Destructive interference:

$\Delta path = (2n+1)\frac{\lambda}{2}$ $(n = 0, 1, 2, 3, \ldots)$

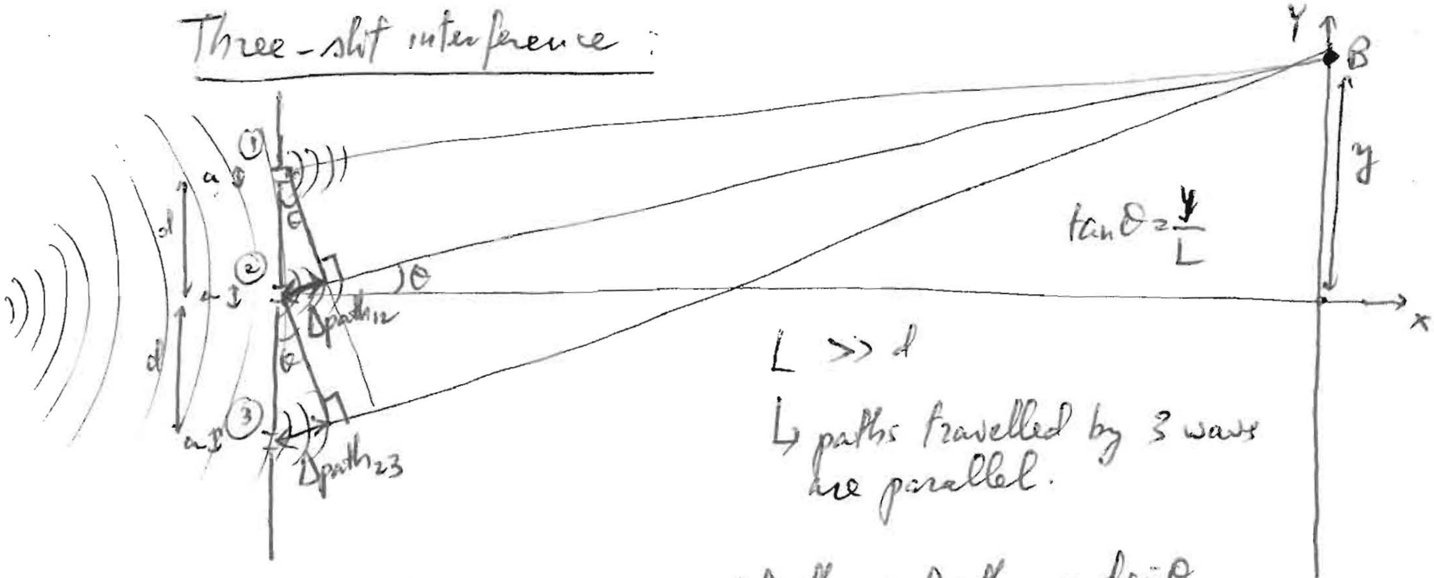
$d\sin\theta_n = (2n+1)\frac{\lambda}{2}$ → loc of dark spot of order $(n+1)$

is

$$\boxed{y_n = L\tan\theta_n = L\tan\left[\sin^{-1}\left(\frac{(2n+1)\lambda}{2d}\right)\right]}$$

$$\boxed{\text{if } \lambda << d \rightarrow y_n = \frac{(2n+1)\lambda L}{2d}} \quad (\sin\theta_n \simeq \tan\theta_n)$$

Three-slit interference:



$\tan\theta = \frac{y}{L}$

$L >> d$

↳ paths travelled by 3 waves are parallel.

$$\begin{cases} \Delta path_{12} = \Delta path_{23} = d\sin\theta \\ \Delta path_{13} = 2d\sin\theta \end{cases}$$

one source → 3 identical waves.

Constructive interference @ B: ⟶ For our 3 waves:

$$\left.\begin{array}{lll} ① \& ② : & d\sin\theta_m & = m\lambda \\ ② \& ③ : & d\sin\theta_m & = m\lambda \\ ① \& ③ : & \cancel{2}d\sin\theta_m & = \cancel{2}m\lambda \end{array}\right\}$$

$$\boxed{d\sin\theta_m = m\lambda \quad (m = 0, 1, 2, 3, \ldots)}$$

Destructive interference @ B

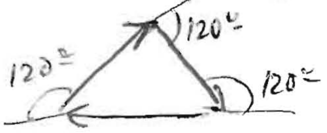
↳ 2 slits: $\Delta path = (2n+1)\frac{\lambda}{2} = (n+\frac{1}{2})\lambda \quad (n = 0, 1, 2, 3, \ldots)$

~~3 slits~~ Two waves are $180°$ out of phase $\uparrow\downarrow = 0$

↳ $\frac{360°}{2}$

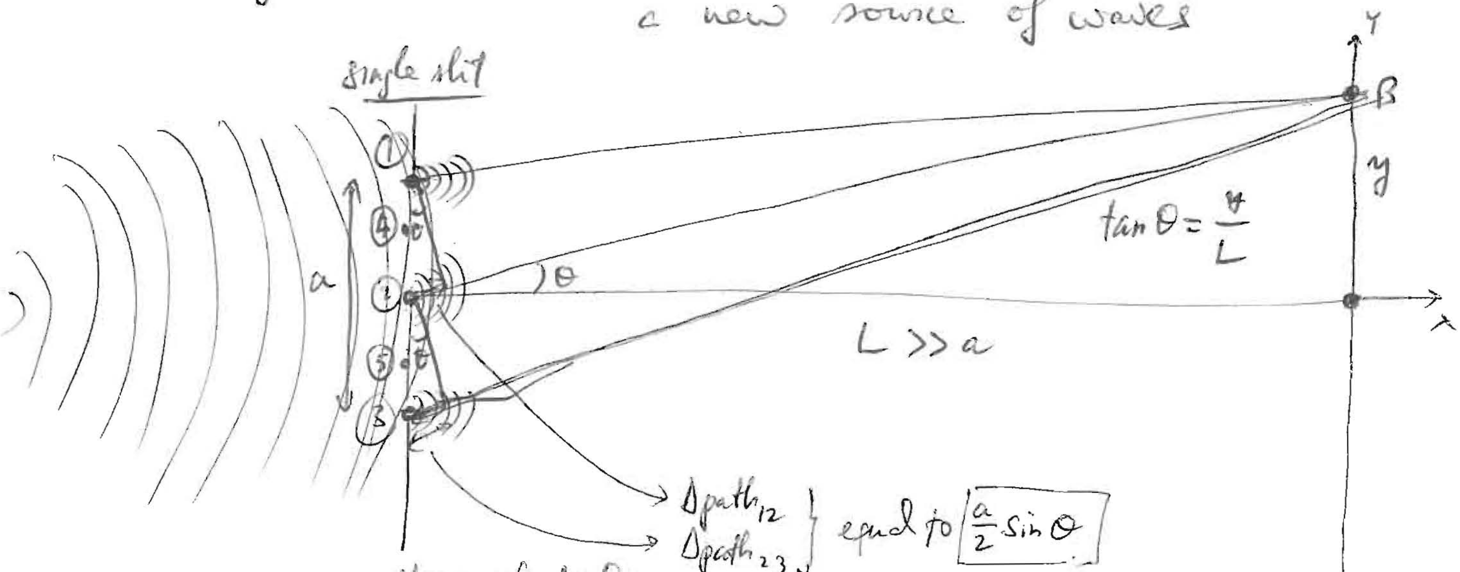
3 slits: $\Delta path = (n+\frac{1}{3})\lambda \quad (n = 0, 1, 2, 3, \ldots)$ $\frac{360°}{3}$

Three waves ~~should~~ be $120°$ out of phase:



Diffraction in a single slit: superposition of waves from different points along the slit:

Huyghens' principle: each point on a wavefront can become a new source of waves



e.g. three identical waves come out of points along slit

Destructive interference @ B:

waves ① & ②, ② & ③:

$$\Delta path = (2n+1)\frac{\lambda}{2} \quad (n=0,1,2,\ldots)$$

$$\frac{a}{\cancel{2}}\sin\theta_n = (2n+1)\frac{\lambda}{\cancel{2}}$$

$$\boxed{a\sin\theta_n = (2n+1)\lambda \quad (n=0,1,2,\ldots)}$$

waves ① & ③: $\Delta path$ ①&③ $= a\sin\theta$

$$a\sin\theta_n = \cancel{2}\,(2n+1)\frac{\lambda}{\cancel{2}} \quad (n=0,1,2,\ldots)$$

$$\boxed{a\sin\theta_n = (2n+1)\lambda \quad (n=0,1,2,\ldots)}$$

For all 3 waves:

$$\boxed{a\sin\theta_n = (2n+1)\lambda \quad (n=0,1,2,3,\ldots)}$$

$\lambda; 3\lambda; 5\lambda; 7\lambda \ldots$

Dark spots for ∞ waves	$a\sin\theta_n = n\lambda$ $(n=1,2,3,\ldots)$
	$n=0$ is a bright spot

Now waves ① & ④: $\frac{a}{4}\sin\theta_n = (2n+1)\frac{\lambda}{2} \rightarrow a\sin\theta_2 = 2(2n+1)\lambda = 2\lambda; 6\lambda; 10\lambda; 14\lambda \ldots$

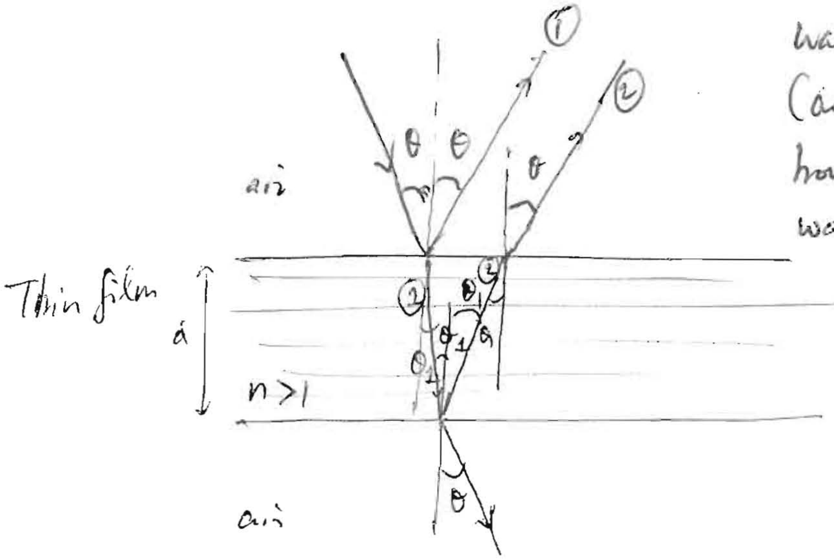
waves ① & ⑤: $\frac{3}{4}a\sin\theta_n = 2(2n+1)\frac{\lambda}{2} \rightarrow a\sin\theta_n = \frac{4}{3}(2n+1)\lambda = 4\lambda\ (n=1); 12\lambda\ (n=3);$ etc.

→ Diffraction limit in optical instruments:

$$\underbrace{\theta_{min}}_{\text{minimum angle b/w objects we can distinguish through a lense}} = \frac{1.22\lambda}{D}$$

D is diameter of slit or lense.

Thin-film interference : (rainbow on thin layer of oil on water)



waves ① & ② come out parallel (as in the double slit experiment), however wave ① stayed in air while wave ② has travelled approximately 2d in the medium

Wave ① : because of a reflection off a higher index medium (like a wave reaching the fixed end of a string → gets inverted) : gets inverted → gets a phase shift of 180° or $\frac{\lambda}{2}$

wave ② : has travelled an additional 2d.

→ b/w ① & ②

constructive interference :

$$2d = m\lambda + \frac{\lambda}{2}$$

$$= (m+\tfrac{1}{2})\lambda \quad (m=0,1,2 ...)$$

→ wavelength in film

destructive interference :

$$2d = (2m+1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$= 2m\frac{\lambda}{2} + \lambda$$

$$= (m+1)\lambda \quad (m=0,1,2,...)$$

→ wavelength in film

32.42 $\quad \underbrace{400\,nm}_{\lambda_v \text{ (violet)}} < \underset{\text{visible light}}{\lambda} < \underbrace{700\,nm}_{\lambda_r \text{ (red)}}$

Visible spectra dispersed by a grating ↔ slit diffraction

↓

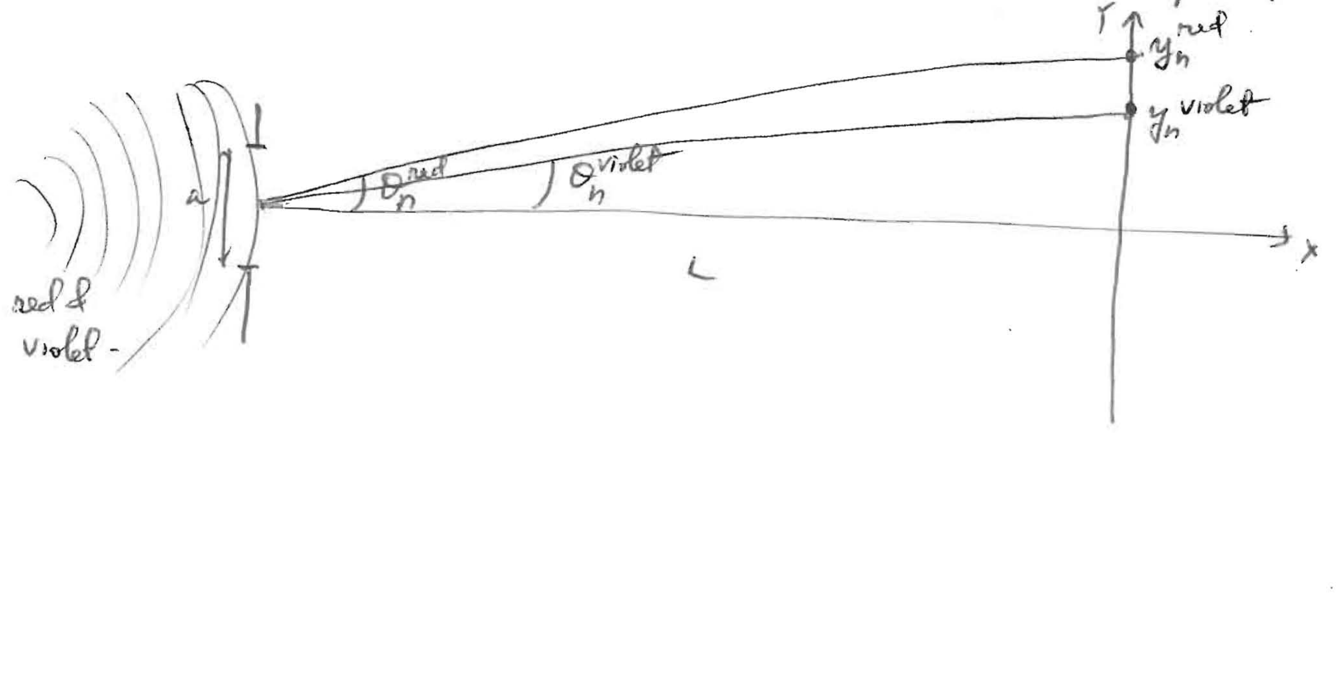
$\boxed{\text{Dark spot}}$ on a screen for a single slit diffraction:

$$a \sin \theta_n = n\lambda$$

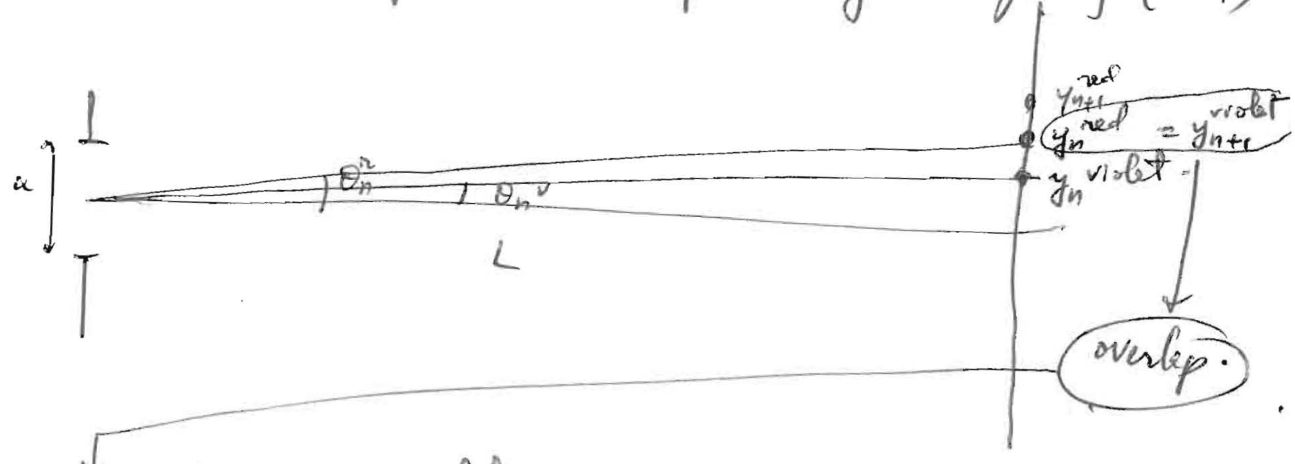
$y_n = L \tan \theta_n$ location on screen of a dark spot of order n

$$\begin{cases} \theta_n^{red} = \sin^{-1}\left(\dfrac{n\lambda_{red}}{a}\right) \\ \theta_n^{violet} = \sin^{-1}\left(\dfrac{n\lambda_v}{a}\right) \end{cases}$$

For a same order n a dark spot for red light is further up from the x-axis (midline) than the dark spot for violet.



Question: lowest pair of consecutive orders for some overlap of visible spectra as dispersed by the grating (slit)



Since θ_n & y_n are related.

$$\sin\theta_n^{red} = \sin\theta_{n+1}^{violet}$$

$$\frac{n\lambda_{red}}{\cancel{a}} = \frac{(n+1)\lambda_{violet}}{\cancel{a}} \rightarrow n\lambda_{red} - n\lambda_{violet} = \lambda_{violet}$$

$$\rightarrow n = \frac{\lambda_v}{\lambda_r - \lambda_v} = \frac{400\,nm}{700\,nm - 400\,nm} = \frac{4}{3} = 1.33$$

can only be an integer $\rightarrow$ $\begin{cases} n = 2 \\ n+1 = 3 \end{cases}$

$$y_2^{red} = y_3^{violet}$$

(loc. of 2nd dark spot for red coincides with loc. of 3rd dark spot for violet).

32-38 : $\lambda = 633\,nm$; $d = 6.5\,\mu m$; $L = 1.7\,m$

a) $y_2 - y_1$ b) $y_4 - y_3$

a) $$y_2 - y_1 = L\left\{\tan\left[\sin^{-1}\frac{2\lambda}{d}\right] - \tan\left[\sin^{-1}\frac{\lambda}{d}\right]\right\}$$

$$= 1.7\left\{\tan\left[\sin^{-1}\frac{2\times 633\times10^{-9}}{6.5\times10^{-6}}\right] - \tan\left[\sin^{-1}\frac{633\times10^{-9}}{6.5\times10^{-6}}\right]\right\}$$

$$= 17.17\ cm$$

b) $$y_4 - y_3 = L\left\{\tan\left[\sin^{-1}\frac{4\lambda}{d}\right] - \tan\left[\sin^{-1}\frac{3\lambda}{d}\right]\right\}$$

$$= 20\ cm$$

If we use $\lambda \ll d$ ($\lambda \to 0.633\,\mu m$, $d \to 6.5\,\mu m$) $\xrightarrow{\text{Not so good}}$

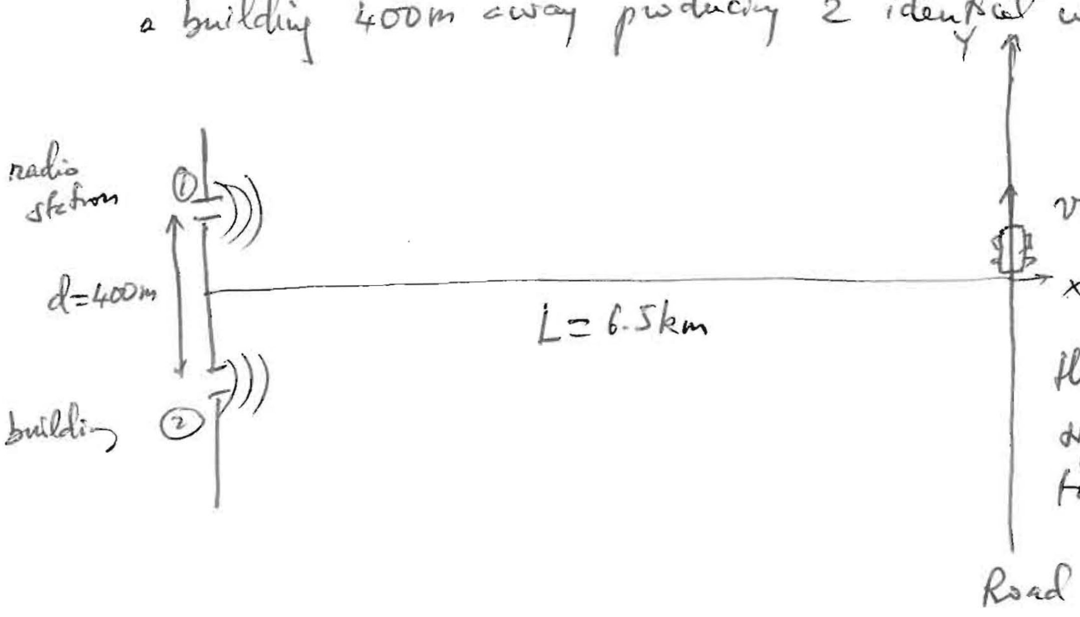
$$y_2 - y_1 = (2-1)\frac{\lambda L}{d} = \frac{\lambda L}{d} = 17\ cm$$

$$y_4 - y_3 = (4-3)\frac{\lambda L}{d}$$

$$= \frac{\lambda L}{d} = \frac{633\times10^{-9}\times1.7}{6.5\times10^{-6}}$$

$$= 17\ cm$$

32.70 Signal from a 103.9 MHz FM radio station reflects off a building 400m away producing 2 identical waves



How often signal appears to fade → crossing a "dark" spot for radio waves interference.

$\lambda \qquad d = 400m$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{103.9\times10^6} = \frac{300}{103.9} = 2.89m$$

$$\boxed{\lambda \ll d} \rightarrow \quad y_{n+1} - y_n =$$

$$\frac{(2(n+1)+1)\lambda L}{2d} - \frac{(2n+1)\lambda L}{2d}$$

$$\underbrace{\left[2n+3-(2n+1)\right]}_{2} \frac{\lambda L}{2d}$$

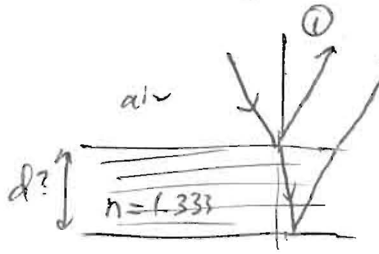
$$\underbrace{y_{n+1} - y_n}_{\text{separation b/w two consecutive fading spots}} = \frac{\cancel{2}\lambda L}{\cancel{2}d}$$

Time b/w two consecutive fadings: $\dfrac{y_{n+1}-y_n}{v} = \dfrac{\frac{\lambda L}{d}}{v}$

$$= \frac{2.89 \times 6500}{400\times\frac{60}{3.6}} = 2.82s$$

How often $= \dfrac{1}{2.82s}$

32-21 Find d_{min} ($n = 1.333$ for film) for $\boxed{\lambda_0 = 550\,nm}$ wavelength in vacuum.
to undergo constructive interference.



→ Constructive interference:

$$2d = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2 \ldots)$$

λ → wavelength in the film.

$\boxed{\lambda = \frac{\lambda_0}{n}}$ → n: index refraction of film.

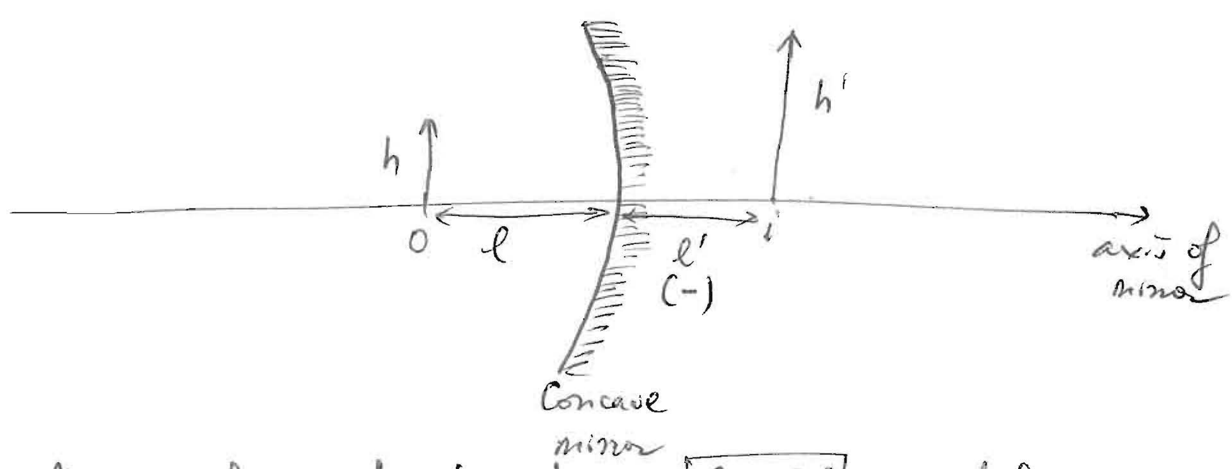
$$2d = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n}$$ → Min thickness: smallest m: → $m = 0$

$$\boxed{d_{min} = \frac{\lambda_0}{2n\,2} = \frac{\lambda_0}{4n} = \frac{550\,nm}{4 \times 1.333} = 103\,nm.}$$

31.32 31.42 31.50

31.42 concave mirror R?

$h' = 9.5\,cm$ (virtual image)
$h = 5.7\,cm$ (object)
$\ell = 22\,cm$ (from mirror).



Minor equation: $\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}$ $\boxed{R = 2f}$ → need f:

Magnification factor $M = \frac{h'}{h} = -\frac{\ell'}{\ell}$ → $\boxed{\ell' = -\ell\frac{h'}{h}}$

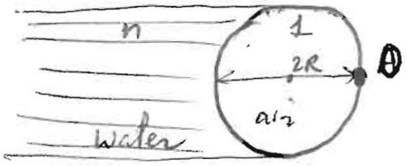
$$\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}$$

$$\frac{1}{\ell} - \frac{1}{\ell \frac{h'}{h}} = \frac{1}{\ell}\left[1 - \frac{1}{\frac{h'}{h}}\right] = \frac{1}{\ell}\left[1 - \frac{h}{h'}\right] = \frac{1}{f}$$

$$R = 2f = \frac{2\ell}{1 - \frac{h}{h'}} = \frac{2 \times 22\,\text{cm}}{1 - \frac{5.7}{9.5}} = 110\,\text{cm}.$$

31.32

<u>Bubble underwater:</u>



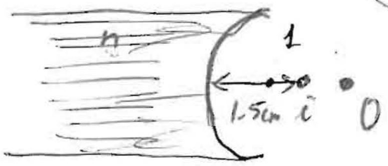
$$\frac{n_1}{\ell} + \frac{n_2}{\ell'} = \frac{n_2 - n_1}{R}$$ ← acts like a thick lense

concave → (R<0)

$n_2 = n = 1.333$ (medium)

$n_1 = 1$

$\ell = 2R$ (always +)



$$\frac{1}{2R} + \frac{1.333}{\ell'} = \frac{0.333}{-R}$$

Bubble appears to be 1.5cm in diameter → $\ell' = -1.5$ cm

sign convention for lens when image is in same side as object.

$$\frac{1}{2R} + \frac{0.333}{R} = -\frac{1.333}{\ell'} = +\frac{1.333}{+1.5\,\text{cm}}$$

$$\frac{1}{R}(0.5 + 0.333) = \frac{1.333}{1.5} \rightarrow R = \frac{0.833 \times 1.5}{1.333} = 0.938\,\text{cm}$$

→ Actual diameter is $\boxed{2R = 1.87\,\text{cm}}$

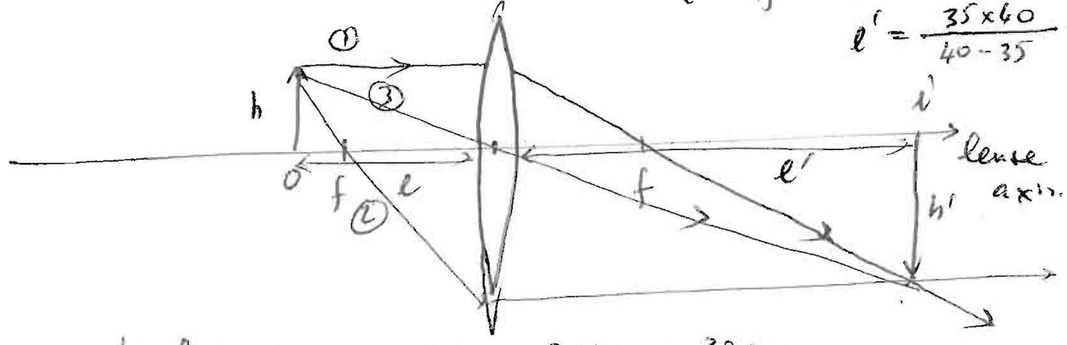
31.50

Converging lense: → thin lense equation : $\frac{1}{l} + \frac{1}{l'} = \frac{1}{f}$

$f = +35\,cm$

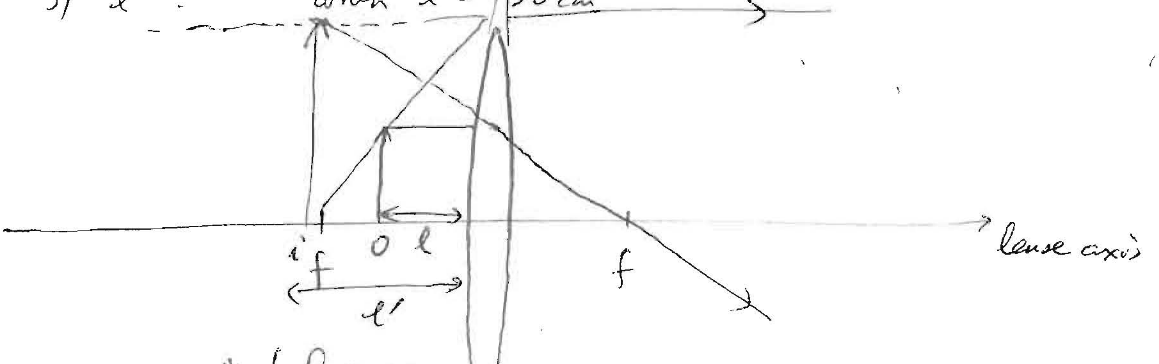
a) l'? when $l = 40\,cm$ → $\frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = \frac{1}{35cm} - \frac{1}{40cm}$

$l' = \frac{35 \times 40}{40 - 35} = 280\,cm$



object to image = 40cm + 280cm = 320cm.

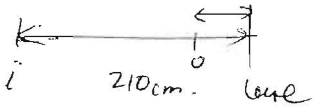
b) l'? when $l = 30\,cm$



Virtual image
$l'(-)$

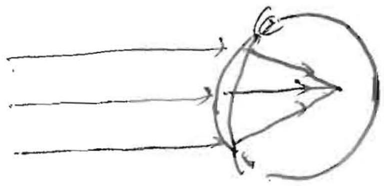
$\frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = \frac{1}{35} - \frac{1}{30} \rightarrow l' = \frac{35 \times 30}{-5} = -210\,cm$

object to image : 180cm.



31.36 Eye : $\begin{cases} f = 2.0\,cm \\ f_{required} = 2.2\,cm \end{cases}$ (sharply focused image on retina)

a) Image for faraway objects is formed before retina



Near sighted.

b) Power of corrective lense needed :

↳ Diopter = $\begin{cases} \text{Eye} : \dfrac{1}{(f)_{\to \text{ in } m}} = \dfrac{1}{0.02\,m} = 50 \text{ diopters.} \\ \text{Eye (good)} \ \dfrac{1}{0.022\,m} = 45.5 \text{ diopters.} \\ \text{Corrective lense will have } -4.5 \text{ diopters.} \end{cases}$