Ch 27: Electromagnetic Induction (CONT.)

Faraday's Law: \[ E = -\frac{d\Phi_B}{dt} \]

Lenz's Law

\( \Phi_B \): magnetic flux: flux of magnetic field \( \vec{B} \)
through a surface enclosed by a loop

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cdot dA \]

scalar product: \( B dA \cos \theta = B \cdot dA \)
perpendicular to surface area

\( \frac{d\Phi_B}{dt} \): change of magnetic flux per unit time

change can come from \( \frac{d\Phi_B}{dt} \) \{- change in magnitude \}
\{- change in direction \}

\( E \): voltage induced along the loop

\( E \) will be such that it opposes the change in magnetic flux: by creating an induced magnetic field \( E \) pointing in opposite direction as the original field

- if \( \Phi_B \) was increasing:
- pointing in some direction as the original field
- if \( \Phi_B \) was decreasing.
Visual experiment: a magnet bar on a fireproofed track

1) Slide magnet towards loop: as magnet approaches, loop, there is an increased magnet flux through loop, → induced E in loop will create a magnetic field (induced) pointing along (→)
induced current goes up @ front side of loop (according to RHR)

2) Magnet continues sliding beyond the loop: as magnet leaves, there is a decreased magnetic flux through loop, → induced E in loop will now create an induced magnetic field pointing along +E
Assume magnet was given a push and let go: will slide on its own along track. Will it have same speed @ A, B, C

\[ \begin{align*}
\vec{v}_B &< \vec{v}_A \\
\vec{v}_B &< \vec{v}_A
\end{align*} \]

(Conservation of energy: initially there was no current in loop, induced current draws energy from moving magnet)

\[ \begin{align*}
\vec{v}_D &< \vec{v}_C \\
\vec{v}_D &< \vec{v}_C
\end{align*} \]

(Conservation of energy, energy transferred into loop is being returned (part of it) to the magnet)

27.60 Uniform magnetic field into page:

Data:

\[ B(t=0) = 2T \]
\[ B(t=2t) = 0 \]

→ There is a magnetic flux through surface area \( A \) enclosed by the square loop.

\( \phi_B \) can change over time

\[ \phi_B = \int B \cdot dA \]

\( B \), changes over time

\( \phi_B \), changes over time

A changes over time
a) Since magnitude of $\mathbf{B}$ changes over time \[ E = -\frac{d}{dt} \left[ \frac{B_y - B_z}{\Delta t} \right] \]
\[ E = -A \left( \frac{0 - 2}{\Delta t} \right) = \frac{2A}{\Delta t^2} \]

Light bulb is 6V bulb: needs 6V for it to shine in full brightness.

\[ G = \frac{2A}{\Delta t} \rightarrow \Delta t = \frac{2A}{G} = \frac{2(3 \times 3)}{6} = 3s \rightarrow B \text{ needs to change from } 2T \text{ to } 0T \text{ in } \Delta t \leq 3s \text{ for light bulb to shine in full brightness.} \]

b) What is direction of the induced current in the loop?

CW to create an "induced" magnetic field into pipe (same as original field) to compensate for the decreased magnetic flux (from the decreased magnetic field).
Inductance & Magnetic Energy

Electric

Capacitors: storage device for electrostatic energy

\[ C = \frac{Q}{V} \]

Magentic

Inductors: device for magnetic energy

\[ L = \frac{\Phi}{I} \]

Mutual inductance

\[ M = \frac{\Phi_2}{I_1} \]

(interaction of 2 inductors)

Two inductors interaction:

By Ampere's Law: \( B_1 = n_1 I_1 \)
\( n_1 \): turns per unit length in solenoid or inductor #1

Solenoid #2 is within the field \( B_1 \) created by solenoid #1 → receives a magnetic flux \( \Phi_2 \).

\( \Phi_2 \) is sufficiently small so \( B_1 \) is uniform over this small area \( A_2 \)

\[ \Phi_2 = B_1 A_2 \]

\( \Phi_2 \) change over time \( \frac{d}{dt} \)

\( B_1 \), magnitude changes with time

\( B_1 \) change dependence changes over time

\( A_2 \) change over time

\( \Phi_2 \) change with time
\[ \varepsilon_2 = -\frac{d\Phi_2}{dt} = -\frac{d}{dt} (B_2 A_2) = -A_2 \frac{d\Phi_1}{dt} = -A_2 \frac{d\Phi_2}{dt} = -A_2 \frac{dI_2}{dt} \]

**M:** Mutual inductance - relates the induced voltage in inductor \#2 due to a changing current in inductor \#1. And vice versa!

**Note:**
\[ \frac{dM}{dt} = -\frac{d(MI_1)}{dt} \Rightarrow \phi_2 = MI_1 \text{ or } M = \frac{\phi_2}{I_1} \]

**SI Unit:**
\[ \varepsilon_2 = -M \frac{dI_2}{dt} \Rightarrow \text{unit } M = \frac{V}{A} = \frac{V}{s} = \text{H (Henry)} \]

**Question #1:** What about effect of \( \vec{B}_2 \) by solenoid \#2 on solenoid \#1?

\[ \varepsilon_1 = -A_1 \frac{d\Phi_2}{dt} \]

**M:** same mutual inductance!

**Question #2:** What about effect of \( \vec{B}_1 \) (by \#1) on itself?

Since \( \vec{B}_1 \) goes through its own cross-sectional area \( A_1 \), there is a magnetic flux \( \Phi_1 = B_1 A_1 \), \( \Phi_1 = \Phi_2 \) (ideally small so \( \vec{B}_1 \) is uniform over \( A_1 \)). If \( I_1 \) changes over time \( \rightarrow \vec{B}_1 \) changes over time \( \rightarrow \) induced voltage \( \varepsilon = -\frac{d\Phi_1}{dt} = -\frac{d(B_1 A_1)}{dt} = -A_1 \frac{dI_1}{dt} = -A_1 \frac{dI_2}{dt} \) (self-inductance)
$L$: self-inductance: relates self-induced voltage with a changing current in the same solenoid.

$I$ unit: same as for $M \rightarrow H$ (Henry)

$$\phi \text{self} = LI \quad \Rightarrow \quad L = \frac{\phi \text{self}}{I}$$

**RC Circuits**

$E$ \hspace{2cm} $C$ \hspace{2cm} $V_c$

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>switch just closed $\Rightarrow$ short-circuit across $C$</th>
<th>$V_C = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = \infty$</td>
<td>long after switch the circuit</td>
<td>$V_C = 0$</td>
</tr>
</tbody>
</table>

$I_C = \frac{E}{R} - \frac{t}{RC}$

$\tau = \frac{1}{RC}$: time constant

**RL Circuits**

$E$ \hspace{2cm} $L$ \hspace{2cm} $I_L$ (inductor)

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>switch just closed $\Rightarrow$ open-circuit across $L$</th>
<th>$V_L = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = \infty$</td>
<td>long after switch is opened</td>
<td>$I_L = 0$</td>
</tr>
</tbody>
</table>

$0 < t < \infty$  

$V_L = E - \frac{t}{L}$

$\tau = \frac{L}{R}$: time constant
Energy Storage

**Electrostatic Energy & Capacitors**

\[ q = \frac{C}{V} \]

Energy: \( U_C = \frac{1}{2} CV^2 \) (J)

SI: \( F \cdot V \) (Farad) (Volt)

Energy density: \( u_C = \frac{U_C}{Ad} = \frac{1}{2} \varepsilon_0 E^2 \) (J/m³)

Volume per plate: \( A d \)

Cross-sectional area of each plate

**Magnetic Energy & Inductors**

\[ I \frac{d}{dt} A \]

\[ L \]

Faraday's Law:

\[ V_L = -L \frac{dI}{dt} \]

Energy:

\[ U_L = \int_0^t \frac{1}{2} L I^2 \, dt \]

\[ L = \frac{1}{2} \mu_0 n^2 A l \]

Energy density:

\[ u_L = \frac{U_L}{Al} = \frac{1}{2} \mu_0 n^2 I^2 \]

Volume: cylinder inside coil: \( A l \)

Length of coil: \( n l \)

Magnetic field created by coil: \( B = \frac{\mu_0 I}{2\pi r} \)

\# turns per unit length, current through each turn

\( n \)
Self-inductance:

\[ L = N_2 A_1 n_1 \mu_0 = \mu_0 A_1 \frac{N_2^2}{L} \]

Length of coil or solenoid:

\[ n_1 = \frac{N_1}{L} \]

Electric:

\[ \frac{1}{2} C V_e^2 : C \text{ is an inertia for } V_e \]

Magnetic:

\[ \frac{1}{2} L I_i^2 : L \text{ is an inertia for } I_i \]

Kinetic:

\[ \frac{1}{2} m v^2 : m \text{ is an inertia for speed} \]

27.62

\[ R_1, R_2, R_3, L \]

\[ E_0 = 12 \text{V} \]

\[ R_1 = 4 \Omega \]

\[ R_2 = 8 \Omega \]

\[ R_3 = 2 \Omega \]

\[ L = 2 \text{H} \]

1) \[ I_3 \] right after switch is closed

Inductor: inducts to current before switch was closed \( I_3 = 0 \), right after switch is closed \( I_3 = 0 \rightarrow \) open circuit across \( L \)
2) \( I_2? \) Long after switch is closed: \( L \) behaves as a short-circuit \( \Rightarrow t = \infty \) \( (I_2 \text{ is max}) \)

- \( \text{Find } I_1 \)
- \( \text{Find } I_2 = I_1 \frac{R_3}{R_2 + R_3} \)

Current division

\[ R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} \]

\[ E_0 = \frac{E_0}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{12}{4 + \frac{8.2}{8+2}} = 2.14 \text{ A} \]

\[ I_2 = I_1 \frac{R_3}{R_2 + R_3} = 2.14 \frac{2}{8+2} = \frac{2.14}{5} = 0.429 \text{ A} \]

3) \( I_2? \) Long after switch is closed, it is reopened.

Initially, inrush to current \( \Rightarrow t = \infty \): \( I_L \text{ max} \) \( (L \text{ behaves as a short-circuit}) \)

- Switch is reopened \( \Rightarrow I_L \text{ max} \rightarrow L \text{ still a short-circuit} \)
- \( t = \infty \)
- \( L \text{ is still in there} \)

\[ I_3 R_2 \rightarrow I_2 = 0.429 \text{ A} \]

\[ \text{Find current across } L \text{ resists to change} \]

\( I_2 = -0.429 \text{ A} \)

\( \text{Energy from magnetic energy stored b/w } t = 0 \text{ & } \infty \)
Current will decrease to zero when all energy stored in the inductor is dissipated at \((R_2 + R_3)\).

**We used parallel & series connection.**

---

**Loop analysis:** 2 loops: \(I_a\) & \(I_b\)

1) \[
E_0 - R_1 I_a - (I_a - I_b) R_2 = 0
\]

2) \[
(I_b - I_a) R_2 - R_3 I_b = 0
\]

\[
E_0 - R_1 I_a - R_3 I_b = 0 \rightarrow \quad I_a = \frac{E_0 - R_3 I_b}{R_1}
\]

2) \[
-I_b (R_2 + R_3) + I_a R_2 = 0
\]

\[
I_b (R_2 + R_3) = \frac{R_2}{R_1} (E_0 - R_3 I_b) = 0
\]

\[
I_b (R_2 + R_3 + \frac{R_1 R_2}{R_1}) = \frac{R_2}{R_1} E_0
\]

\[
I_b = \frac{E_0}{R_2 + R_3 + \frac{R_1 R_2}{R_1}}
\]

\[
I_b = \frac{12}{7} \text{ A}
\]

Current across \(R_2 = (I_a - I_b)\)

\[
I_a = \frac{12 - 2 \times \frac{12}{7}}{4} = 3 \left(1 - \frac{2}{7}\right) = \frac{15}{7} \text{ A}
\]

\[
\frac{15}{7} - \frac{12}{7} = \frac{3}{7} = 0.4285 \text{ A}
\]

**Conclusion:** If there is only one battery, shortest solution is to do parallel & series reduction.
Some electric & magnetic fields have same energy density \[ u_c = u_L \]. Find \( \frac{E}{B} \)?

\[ \frac{\gamma_0 E^2}{\gamma_0 B^2} \to \frac{E^2}{B^2} = \frac{1}{\varepsilon_0 \mu_0} \]

\[ \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 2.99 \times 10^8 \text{ m/s} \]

\[ \frac{E}{B} = c \]
Ampere's law as modified by Maxwell:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \int \frac{d\mathbf{E}}{dt} \cdot dA$$

Observations:

1) Now we can say \( \frac{d}{dt} \) creates \( B \)

\( \frac{\partial B}{\partial t} \) creates \( E \)

This explains the fact that EM waves (e.g. sun light) can propagate in vacuum: \( E \rightarrow B \rightarrow E \). ... Hence the importance of Maxwell.

Other example of EM waves: cell phone signals, signals from space probes (takes time to travel)

2) Maxwell's term also explains a technicality about a measured magnetic field around a capacitor in an RC circuit:

![Diagram of an RC circuit with a capacitor and a loop](image)

No physical current b/w the plates.

![Diagram of an Ampere's law example](image)

Ampere loop parallel to left plate: current enclosed by this loop is 0 (since I doesn't go through this loop!)

Old Ampere's law: \( \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}} \)

\( B \cdot 2\pi r = \mu_0 O \rightarrow B \approx 0 \)

This does not agree with experiment measurements that...
Chapter 29  Maxwell's Equations & EM waves.

So far we have seen some connection f... the electric & magnetic fields via Ampere’s & Faraday’s Laws:

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int \mathbf{J} \cdot dA \quad (\mathbf{E} \rightarrow \mathbf{J} \rightarrow \mathbf{B}) \]

Current enclosed by the Amperean loop.

\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot dA \quad (\frac{\partial \mathbf{B}}{\partial t} \rightarrow \mathbf{E}) \]

Induced voltage

Ultimate connection was discovered by Maxwell:

1) Gauss' Law: \( \oint \mathbf{E} \cdot dA = \frac{\text{enclosed}}{\varepsilon_0} \)

Electric flux through a Gaussian surface = no magnetic monopole discovered yet.

Maxwell’s equations

2) “Gauss’ law for B”: \( \oint \mathbf{B} \cdot d\mathbf{A} = 0 \leftarrow \text{Maxwell's term} \)

3) Ampere’s law: \( \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \text{I}_{\text{enclosed}} + \mu_0 \text{I}_{\text{displacement}} \)

4) Faraday’s law: \( \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A} \)

Induced voltage

Displacement current: \( \text{I}_{\text{displacement}} = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \int \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{A} \)

This does not agree with experiment measurements that
confirm \( B \neq 0 \) around the plate.

With Maxwell's equations:

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \mathcal{I}_{\text{enclosed}} + \mu_0 \varepsilon_0 \oint \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{A}
\]

\[
\mathbf{B}(\mathbf{r}) = 0 + \mu_0 \varepsilon_0 \oint \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{A}
\]

\[\Rightarrow B \neq 0.\]

(not by the current enclosed but by a changing electric field)

RC circuit connected to an AC voltage source (Alternating Current) → current is switching direction 60 times per second

\( f = 60\text{Hz} \)

---

Maxwell's equations:

1) Starting from Maxwell's equations → EM wave equations:

\[
\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}
\]

\[
\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}
\]

\[
\frac{\partial^2 y}{\partial x^2} = \omega^2 \frac{\partial^2 y}{\partial t^2}
\]

Transverse wave along a string

2) \( \mathbf{E} \& \mathbf{B} \) are vectors → directions make a difference

→ Polarization of EM wave,

is Sun glasses: reduces intensity of sun light by allowing only \( \mathbf{E} \) along certain orientations to pass through.
If $I = 95 \text{ A}$, what is $t$ since switch has been closed?

Inductor presents inerter to current $\rightarrow I(t=0) = 0$

$\text{switch is closed.}$

$0 < t < \infty : \quad V_L = E_0 e^{-\frac{t}{L}}$

Time constant $\tau = \frac{L}{R}$

By def: when $t = \tau \rightarrow V_L = E_0 e^{-1} = \frac{E_0}{e}$ ($e = 2.711...$)

Also @ an inductor: $\quad V_L = L \frac{dI}{dt}$ (self-induced voltage)

$\frac{dI}{dt} = \frac{E_0}{L} e^{-\frac{t}{R}} - \frac{t}{R}$

$\int dt \rightarrow I = \frac{E_0}{L} \int e^{-\frac{t}{R}} dt + C$

$I(t=0) = 0 \rightarrow C = + \frac{E_0}{R} \rightarrow \frac{E_0}{R} e^{-0} + C = 0, \quad I(t) = \frac{E_0}{R} (e^{-\frac{t}{R}} + 1)$

$t = \frac{45}{3.3} \left[ e^{-\frac{2.1}{3.3}} + 1 \right]

\ln \left( 1 - \frac{95 \times 3.3}{45} \right) = - \frac{t}{2.1 \times 3.3} \rightarrow t = \frac{2.1}{3.3} \ln \left( \frac{9.5 \times 3.3}{45} \right) \text{s}

$t = \approx 23.3 \text{s}$
Eq. 17.8: \[ I = I_0 \, e^{-\frac{t}{L}} \]

\[ \begin{tikzpicture}
    \node (v) at (0,0) {$E_0$};
    \node (r) at (2,0) {$V_R$};
    \node (l) at (2,-1) {$L$};
    \draw (v) -- (r) -- (l) -- cycle;
    \node (i) at (1.5,-.5) {$I$};
    \node (i1) at (1.5,0) {$I$};
    \draw[->] (v) -- (r) node[midway,above] {$I$};
    \draw[->] (r) -- (l) node[midway,above] {$I$};
\end{tikzpicture} \]

a) Use Eq. 17.8 to find \( P_R(t) \)

\[ P_R(t) = I \cdot V_R = I^2 R = I_0^2 R e^{-\frac{2t}{L}} \]  

Ohm's Law

b) Total Energy dissipated @ \( t = \infty \):

\[ \int_0^\infty P_R(t) \, dt = I_0^2 R \int_0^\infty e^{-\frac{2t}{L}} \, dt \]

\[ = I_0^2 R \left[ \frac{e^{-\frac{2t}{L}}}{\left( -\frac{2}{L} \right)} \right]_t=0 = I_0^2 R \left[ 0 - \frac{1}{2R} \right] \]

\[ = I_0^2 R \left( \frac{1}{2RL} \right) = \frac{1}{2} \frac{L}{R} I_0^2 \]

Total energy initially stored in the inductor.

---

Flame → Al foil of mass \( m = 30 \mu g \) and cross-sectional area \( 5 \text{A} \)

Later beam (light → EM wave)

\[ \text{Power needed to do this (held the \text{C} piece of Al foil in the air)} \]

\[ \text{Flame} - mg = 0 \]

\[ \begin{array}{lr}
\text{Radiation pressure:} & P = \text{Flame} = F \cdot A \\
\text{Radiation intensity:} & S = \frac{\text{Power}}{A} \leftrightarrow \text{Rad. intensity Average } = \overline{S} = \frac{\text{Power}}{A} \\
\end{array} \]
\[ P = \frac{F}{A} = \frac{\frac{\Delta p}{\Delta t}}{A} = \frac{\frac{i}{c} \frac{\Delta u}{\Delta t}}{A} = \frac{1}{c} \frac{\text{Power}}{A} = \frac{1}{c} \frac{S}{c} \]

Linear momentum
Radiation momentum: \[ p = \frac{u}{c} \]

Flux: \[ F_{\text{flux}} = mg \]

Pressure: \[ P_{\text{A}} = mg \]

\[ \frac{1}{S} \frac{2}{c} \cdot A = mg \quad \text{Since } S = \frac{\text{Power}}{\text{Area}} \quad \frac{\text{Power}}{cA} \cdot 2A = mg \quad \text{Power} = \frac{mgc}{2} \]

\[ \rightarrow \text{Power} = \frac{30 \times 10^{-9} \times 9.81 \times 3 \times 10^{8}}{2} = 44.1 \text{ W} \]

Why? Remember: microscopically molecules & macroscopically temp.

\[ \frac{mv}{\theta} \rightarrow \text{Mom.} = 0 \]

All foils reflect all light = "elastic collisions" for radiation.

For momentum transfer from radiation (gas molecule counterpart) to Al foil (container wall counterpart)
Electromagnetic wave: (Cont.)

1) Vector nature of \( \vec{E} \) & \( \vec{B} \)

- Field points along +y
- Propagation is along x-axis
- Minus sign b/w \( kx \) & \( wt \) \rightarrow \text{propagation is in } +x \text{ direction}
- Wavelength \( \lambda = \frac{2\pi}{k} \)
  \[ \text{sep. b/w convective peaks or troughs in space} \]
- Period \( T = \frac{2\pi}{\omega} \)
  \[ \text{sep. b/w convective peaks or troughs in time} \]

- \( \vec{B} \) has to be perpendicular to \( \vec{E} \)
- EM wave = transverse wave in a string
  \[ \text{perturbation perpendicular to direction of propagation} \]
- \( \vec{B} = B \sin (kx - wt) \hat{k} \)
- Also: direction of propagation is given by \( \vec{E} \times \vec{B} \) using RHR vector product

Two consecutive same zeros (both increasing or decreasing) are also separated by \( \lambda \)
Unpolarized light with intensity $S_0$ incident on 3 polarizers (a polarizer only allows light with $\vec{E}$ pointing along the polarizer axis to pass through).

Unpolarized light $S_0$

\[ S_1 = \frac{S_0}{2} \]

\[ S_2 = \frac{S_0}{4} \]

\[ S_3 = \frac{S_0}{8} \]

Axis of $#1$ is @ Vertical axis

Axis of $#2$ is @ Horizontal axis @ 45°

Axis of $#3$ is @ 45° to Vertical $\rightarrow E_{\parallel}$, will pass through $#3$.

Intensity $S = \frac{E_x E_y}{c}$

Axis of $#2$ is @ 45° to the $\vec{E}$ that passed through $#3$

\[ S_2 = S_1 \cos^2 45° = \frac{S_0}{4} \times \frac{1}{2} = \frac{S_0}{8} \]
Radio waves are EM waves.

Max. Magnitude of $E$ (at 1.5 km) = $350 \text{ mV/m}$

a) Power of the transmitter?

\[
\text{Radiation intensity: } I = \frac{\text{Power}}{\text{Area}}
\]
\[
I = \frac{\text{Energy}}{\text{Time} \times \text{Area} \times \text{Length}} = \frac{\text{Energy}}{\text{Vol.} \times \text{Vol.}} = \frac{\text{Energy}}{\text{Vol.}}
\]
\[
I = \left( \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} B^2 \right) c = \epsilon_0 E^2 c
\]
\[
\epsilon_0 = \frac{1}{c^2 \mu_0}
\]
\[
\frac{E}{B} = c
\]

\[\downarrow\]

\[
\text{Power} = IS, \quad \text{Area} = \epsilon_0 E^2 c \cdot \text{Area}
\]

\[\downarrow\]

\[
\text{Power} = \epsilon_0 E^2 c \cdot \text{Area} = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 \cdot 4\pi R^2
\]

\[
\frac{1}{2} E_{\text{max}}^2
\]

\[
\text{Sphere of radius } R \text{ centered at the transmitter} \quad \text{(transmits in all directions)}
\]

\[
\text{Power} = 4.59 \text{ kW}
\]
Motor rocket emits beam of light instead of hot gas. Need thrust (force upward) of $35 \times 10^6 N \rightarrow$

Power needed for light?

Radiation pressure $= \frac{\text{Av. Rel. intensity}}{c}$ or $P = \frac{\gamma}{c}

\begin{align*}
F_{\text{up}} & = P \cdot A = \frac{\gamma}{c} A = \frac{\text{Power}}{c} A = \frac{\text{Power}}{c} \\
\text{Power} & = F_{\text{up}} \cdot c = 35 \times 10^6 \times 3 \times 10^8 = 10^{16} W
\end{align*}

All of our power generating capability is only $10^{12} W$