Ch 27. Electromagnetic Induction (cont.)

Faraday's Law:

$$\mathcal{E} = \boxed{-}\,\frac{d\Phi_B}{dt}$$

Lenz's Law

Φ_B = magnetic flux: flux of magnetic field $\vec{B}$ through a surface enclosed by a loop

* $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B_\perp \, dA$

scalar product: $B\,dA \cos\theta \equiv \underbrace{B_\perp}_{\text{perpendicular to surface area}} dA$

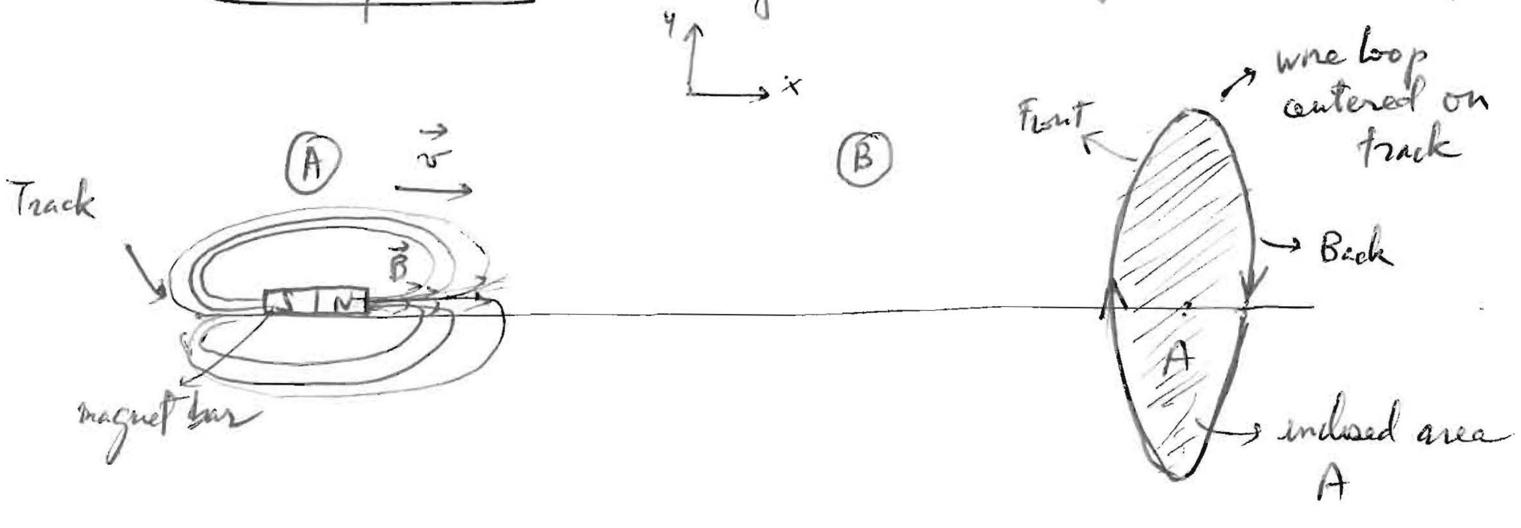
* $\frac{d\Phi_B}{dt}$, change of magnetic flux per unit time

can come from
- change of $\vec{B}$
  - change in magnitude
  - change in direction.
- change of A

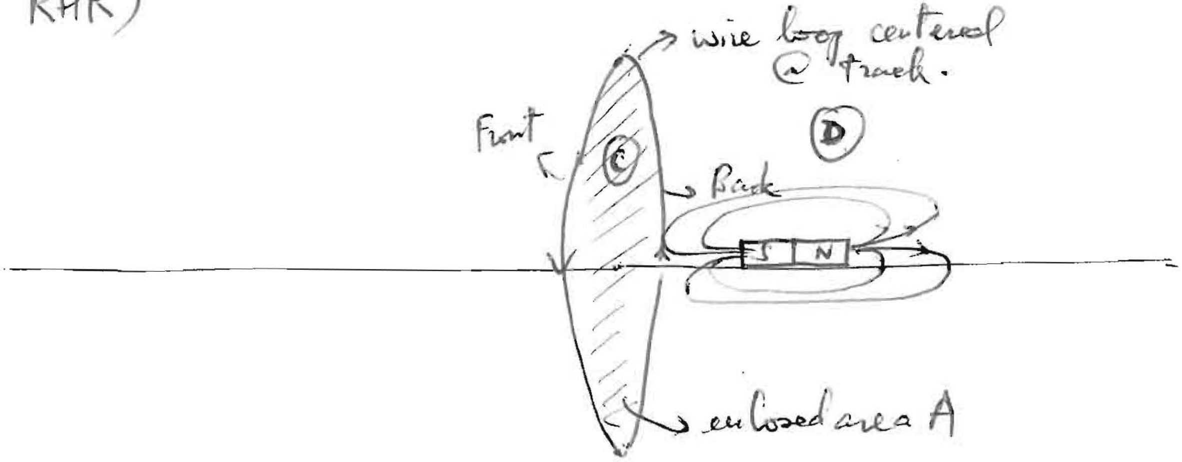
$\mathcal{E}$: voltage induced along the loop

$\boxed{-}$: $\mathcal{E}$ will be such that it opposes the change in magnetic flux: by creating an induced magnetic field
- → pointing in opposite direction as the original field if Φ_B was increasing
- → pointing in same direction as the original field if Φ_B was decreasing.

Visual experiment: a magnet bar on a $\boxed{\text{frictionless track}}$



1) Slide magnet towards loop: as magnet approaches loop, there is an increased magnet flux through loop → induced $\mathcal{E}$ in loop will create a magnetic field (induced) pointing along $(-\hat{\imath})$ → induced current goes up @ front side of loop (according to RHR)



2) Magnet continues sliding beyond the loop: as magnet leaves, there is a decreased magnetic flux through loop → induced $\mathcal{E}$ in loop will now create an induced magnetic field pointing along $+\hat{\imath}$

created by a ~~current~~ induced current (by the induced voltage) in the loop.

Assume magnet was given a push and let go: will slide on its own along track. Will it have same speed @ A, B, C

$$v_B \begin{cases} > v_A \\ = v_A \\ < v_A \checkmark \end{cases}$$

(Conservation of energy: initially there was no current in loop, induced current draws energy from moving magnet)

$$v_D \begin{cases} > v_C \checkmark \\ = v_C \\ < v_C \end{cases}$$

(Conservation of energy, energy transferred into loop is being returned (part of it) to the magnet)

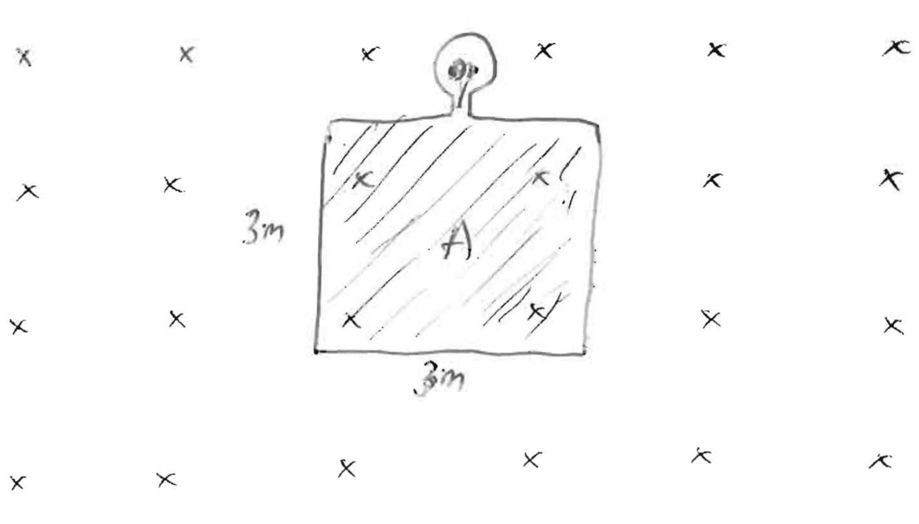
27.40 Uniform magnetic field into page:

Data:

$B(t=0) = 2T$

$\downarrow$

$B(t=\Delta t) = 0$



→ There is a magnetic flux through surface area A enclosed by the square loop.

Φ_B can change over time

$\Phi_B = \int B_\perp \, dA$

- $\vec{B}$ changes over time
  - Magnitude changes over t
  - Direction changes over t
- A changes over time

a) Since magnitude of $\vec{B}$ changes over time $\rightarrow$ $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$

$$= -\frac{d}{dt}\left[\int B_{\perp}\, dA\right]$$

$$= -\frac{d}{dt}\left[B \underbrace{\int dA}_{A}\right]$$

$$= -\frac{d}{dt}(BA)$$

$$= -A\frac{dB}{dt}$$

$$\mathcal{E} = -A\frac{(B_f - B_i)}{\Delta t}$$

$$\mathcal{E} = -A\frac{(0-2)}{\Delta t} = \frac{2A}{\Delta t}$$

Lightbulb is 6V bulb : needs 6V for it ~~to~~ shine in full brightness.

$6 = \dfrac{2A}{\Delta t} \rightarrow \Delta t = \dfrac{2A}{6} = \dfrac{2(3\times 3)}{6} = 3s \rightarrow$ B needs to change from 2T to 0T in $\Delta t \leq 3s$ for lightbulb to shine in full brightness.

b) What is direction of the induced current in the loop ?

CW to create an "induced" magnetic field into page (same as original field) to compensate for the decreased magnetic flux (from the decreased magnetic field)

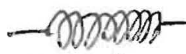
Inductance & Magnetic Energy

Electric

Capacitors : storage device for electrostatic energy

Capacitance $C = \dfrac{Q}{V}$

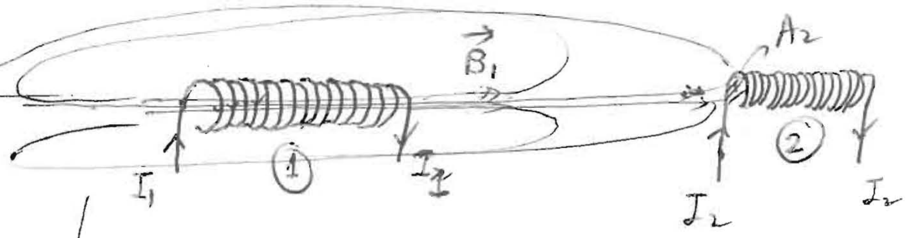
Magnetic



Inductors = storage device for magnetic energy

Inductance:
- self inductance $L = \dfrac{\phi}{I}$
- $\boxed{\text{Mutual inductance}}$ $M = \dfrac{\phi_2}{I_1}$ (interaction b/w 2 inductors)

Two inductors interaction:



cross sectional surface area A_2

By Ampere's Law: $B_1 = \mu_0 n_1 I_1$

n_1: # turns per unit length in solenoid or inductor #1

Solenoid #2 is within the field $\vec{B}_1$ created by solenoid #1 $\Rightarrow$ receives a magnetic flux ϕ_2.

A_2 is sufficiently small so $\vec{B}_1$ is uniform over this small area A_2

$\phi_2 = B_1 A_2$

ϕ_2 changes over time if
- $\boxed{B_1 \text{ magnitude changes with time}}$
- B_1 direction changes w/ time
- A_2 changes with time

B_1 changes over time if I_1 change over time

$B_1 = \mu_0 n_1 I_1$

→ If I_1 changes over time → induced voltage in ~~solenoid~~ solenoid #2 :

$$\mathcal{E}_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(B_1 A_2 N_2) = -A_2 N_2 \frac{dB_1}{dt} = -\underbrace{A_2 N_2 \mu_0 n_1}_{\substack{M \\ \text{Mutual inductance}}} \frac{dI_1}{dt}$$

M: mutual inductance : relates the induced voltage in inductor #2 due to a changing current in inductor #1 - And vice versa!

Note: $-\frac{d\phi_2}{dt} = -\frac{d(MI_1)}{dt} \rightarrow \phi_2 = M I_1$ or $\boxed{M = \frac{\phi_2}{I_1}}$

SI Unit: $\mathcal{E}_2 = -M\frac{dI_1}{dt} \rightarrow$ unit $M = \frac{V}{\frac{A}{s}} = \frac{Vs}{A} \equiv H$ (Henry)

Question #1: what about effect of $\vec{B}_2$ by solenoid #2 on solenoid #1 ?

$$\mathcal{E}_1 = -\underbrace{A_1 N_1 \mu_0 n_2}_{M}\frac{dI_2}{dt}$$

M → same mutual inductance !

Question #2: what about effect of $\vec{B}_1$ (by #1) on itself ?

Since $\vec{B}_1$ goes through its own cross-sectional area A_1 → there is a magnetic flux $\phi_1 = B_1 A_1$ ($B_{\perp 1} = B_1$ & A_1 sufficiently small so B_1 is uniform over A_1). If I_1 changes over time → B_1 changes over time ($B_1 = \mu_0 n_1 I_1$) → ϕ_1 changes over time → self-induced voltage

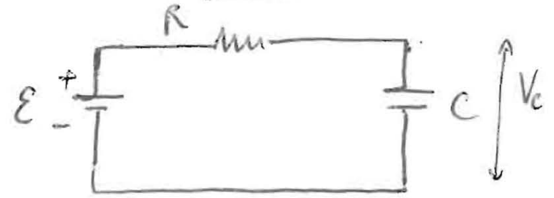
$$\mathcal{E} = -\frac{d\phi_{self}}{dt} = -\frac{d(B_1 A_1 N_1)}{dt} = -A_1 N_1 \frac{dB_1}{dt} = -\underbrace{A_1 \mu_0 n_1 N_1}_{\substack{L \\ \text{self inductance}}}\frac{dI_1}{dt}$$

L: self inductance: relates self-induced voltage with a changing current in the same solenoid.

SI unit: same as for M $\rightarrow$ H (Henry)

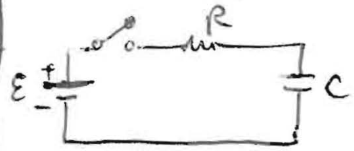
$$\phi_{self} = L I \quad \text{or} \quad \boxed{L = \frac{\phi_{self}}{I}}$$

RC Circuits



V_c doesn't change instantaneously as switch is closed:

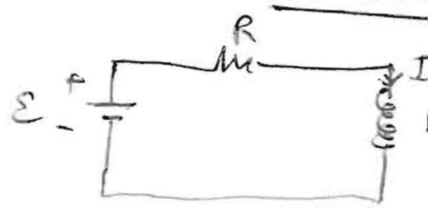
- $t=0$ switch just closed $\rightarrow$ Short-circuit across $C \rightarrow V_c = 0$



- $t=\infty$ long after switch was closed $\rightarrow$ Open circuit $I_c = 0$
- $0<t<\infty$ $\quad I_c = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$

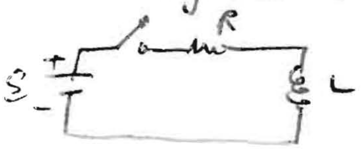
$\tau \equiv \frac{1}{RC}$ = time constant

RL Circuits



I_L does not change instantaneously as switch is closed:

- $t=0$ switch just closed. $\rightarrow$ Open-circuit across L $\downarrow$ $I_L = 0$

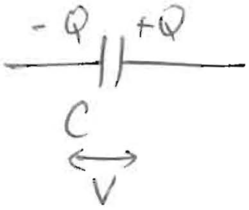


- $t=\infty$ long after $\rightarrow$ Short-circuit across L $V_L = 0$
- $0<t<\infty$ $\quad V_L = \mathcal{E} e^{-\frac{t}{(L/R)}}$

$\tau = \frac{R}{L}$: time constant.

Energy Storage

Electrostatic energy & Capacitors



↳ Energy: $U_C = \frac{1}{2} C V^2$ (J)

SI: F (Farad), V (Volt)

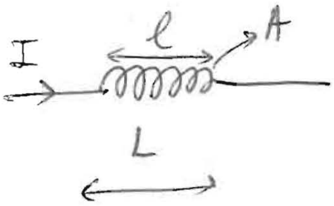
Energy density: $u_C = \frac{U_C}{Ad} = \frac{1}{2}\varepsilon_0 E^2$ $\left(\frac{J}{m^3}\right)$

(energy per unit volume)

volume b/w plate $= Ad$

- A: cross-sectional area of each plate
- d: spacing b/w plates.

Magnetic energy & inductors



V_L → Faraday's Law: $V_L = -L\frac{dI}{dt}$

Energy: $\boxed{U_L = \int_0^t P_L \, dt} = \int_0^t I |V_L| \, dt$

P_L: power or energy per unit time

$= L \underbrace{\int_0^t I \frac{dI}{\cancel{dt}} \cancel{dt}}_{\left[\frac{I^2}{2}\right]_{t=0}^{t=t} = \frac{I^2}{2}} = \boxed{\frac{1}{2} L I^2}$ (J)

$\boxed{L = A n \mu_0 N}$

Energy density: $\boxed{u_L = \frac{U_L}{A\ell}} = \frac{\frac{1}{2} N \mu_0 n \cancel{A} I^2}{\cancel{A}\ell}$

$= \frac{1}{2}\mu_0 n^2 I^2$

$= \frac{1}{2\mu_0}\mu_0^2 n^2 I^2$

$= \boxed{\frac{1}{2\mu_0} B^2}$

volume: cylinder inside coil: $A\ell$

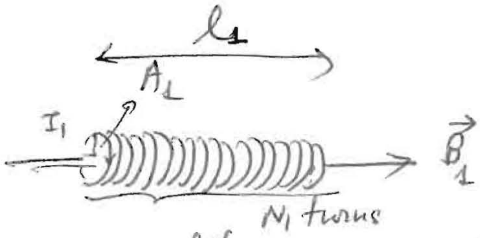
- A: cross-sectional area of coil
- ℓ: coil length

Magnetic field created by coil or solenoid

$\boxed{B = \mu_0 n I} \rightarrow I = \frac{B}{\mu_0 n}$

- n: # turns per unit length.
- I: current through each turn

<u>Self inductance</u>:



$(B_1 = \mu_0 n_1 I_1)$

Self-induced voltage: $\mathcal{E} = -\dfrac{d\Phi_{self}}{dt} = -\dfrac{d}{dt}(N_1 B_1 A_1) = -N_1 A_1 \dfrac{dB_1}{dt}$

$$= -\underbrace{N_1 A_1 \mu_0 n_1}_{\text{Self-inductance } L} \frac{dI_1}{dt}$$

$$\boxed{L = N_1 A_1 n_1 \mu_0 = \mu_0 A_1 \frac{N_1^2}{l_1}}$$

$n_1 = \dfrac{N_1}{l_1}$

$l_1 \to$ length of coil or solenoid

<u>Electric</u>	:	$\frac{1}{2} C V_C^2$	: C is an inertia for V_C
<u>Magnetic</u>	:	$\frac{1}{2} L I_L^2$	: L is an inertia for I_L
<u>Kinetic</u>	:	$\frac{1}{2} m v^2$	: m is an inertia for speed

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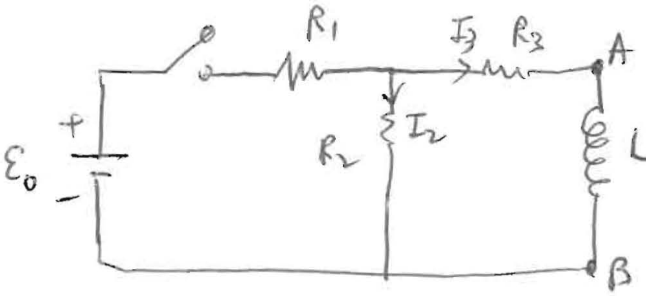
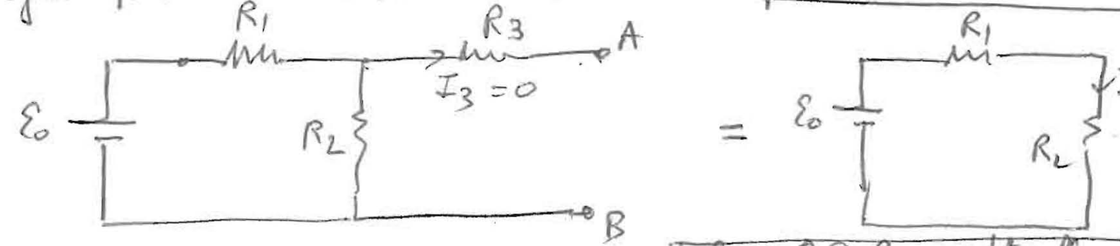
27.62



Data
$\mathcal{E}_0 = 12V$
$R_1 = 4\Omega$
$R_2 = 8\Omega$
$R_3 = 2\Omega$
$L = 2H$

1) $\boxed{I_2 ?}$ <u>right after switch is closed</u>: looks @ response @ L @ $t=0$

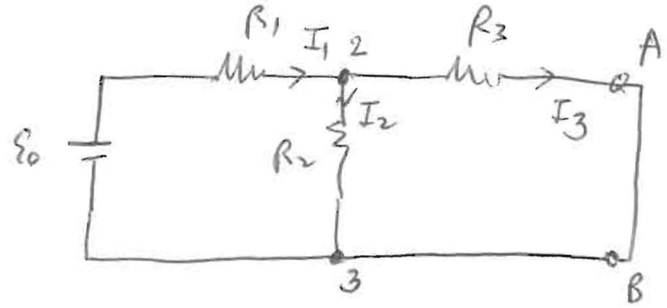
(Inductor : inertia to current, before switch was closed $I_3 = 0$, right after switch is closed $I_3 = 0 \to$ <u>open circuit across L</u>)



$I_2 = \dfrac{\mathcal{E}_0}{R_1 + R_2} = \dfrac{12}{4+8} = 1A$

$\boxed{\text{Current @ } R_2 \text{ right after switch is closed is } 1A}$

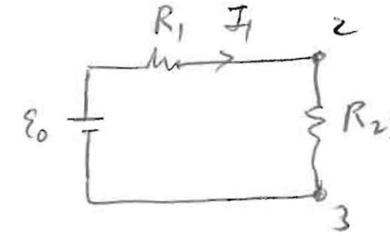
2) $\boxed{I_2 ?}$ Long after switch is closed: L behaves as a short-circuit @ $t=\infty$ (I_L is max.)



- Find I_1 :
- Find $I_2 = I_1 \dfrac{R_3}{R_2+R_3}$

current division

Find I_1:



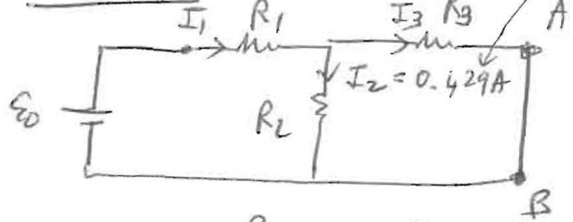
$R_{23} = R_2 \parallel R_3 = \dfrac{R_2 R_3}{R_2+R_3}$

Ohm's Law $I_1 = \dfrac{\mathcal{E}_0}{R_1 + R_{23}} = \dfrac{\mathcal{E}_0}{R_1 + \dfrac{R_2R_3}{R_2+R_3}} = \dfrac{12}{4 + \dfrac{8.2}{8+2}} = 2.14\,A$

$$\boxed{I_2 = I_1 \frac{R_3}{R_2+R_3} = 2.14 \frac{2}{8+2} = \frac{2.14}{5} = 0.429\,A}$$

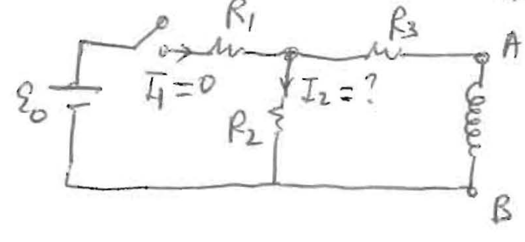
3) $\boxed{I_2 ?}$ long after switch is closed, it is reopened.
Inductor: inertia to current → $t=\infty$: I_L max (L behaves as a short-circuit) Right after switch is reopened → I_L max → L still a short-circuit

$\boxed{t=\infty}$ L behaves as a short-circuit



$t=\infty$
L is still in there

Right after switch is reopened



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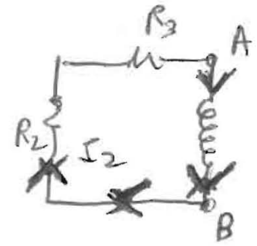
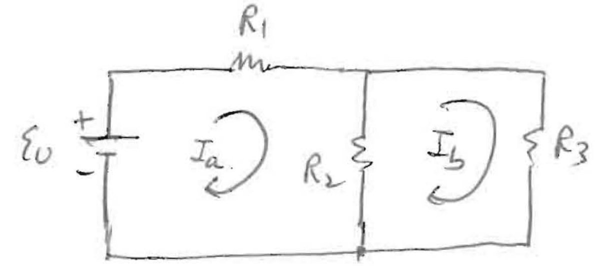


$I_2 = -0.429A$ since current across L resists to change.
(energy from magnetic energy stored b/w $t=0$ & $t=\infty$)

Current will decrease to zero when all energy stored in the inductor is dissipated @ $(R_2 + R_3)$.

Curiosity:



→ We used parallel & series connection.

Loop analysis: 2 loops: I_a & I_b

$$1)\quad +\mathcal{E}_0 - R_1 I_a - (I_a - I_b)R_2 = 0$$

$$+\quad 2)\quad -(I_b - I_a)R_2 - R_3 I_b = 0$$

$$\mathcal{E}_0 - R_1 I_a - R_3 I_b = 0 \rightarrow \boxed{I_a = \frac{\mathcal{E}_0 - R_3 I_b}{R_1}}$$

$$2)\quad - I_b(R_2+R_3) + I_a R_2 = 0$$

$$I_b(R_2+R_3) - \frac{R_2}{R_1}(\mathcal{E}_0 - R_3 I_b) = 0$$

$$I_b\left(R_2 + R_3 + \frac{R_2R_3}{R_1}\right) = \frac{R_2}{R_1}\mathcal{E}_0$$

$$I_b = \frac{\frac{R_2}{R_1}\mathcal{E}_0}{R_2 + R_3 + \frac{R_2R_3}{R_1}}$$

$$= \frac{\frac{8}{4}12}{8 + 2 + \frac{8\times 2}{4}}$$

$$\boxed{I_b = \frac{12}{7}\text{ A}}$$

Current across $R_2 = (I_a - I_b)$

$$I_a = \frac{12 - 2\times\frac{12}{7}}{4} = 3\left(1 - \frac{2}{7}\right) = \frac{15}{7}\text{ A}$$

$$\rightarrow \frac{15}{7} - \frac{12}{7} = \frac{3}{7} = \boxed{0.429\text{A}}$$

Conclusion: If there is only one battery, shortest solution is to do parallel & series reduction

27-90 Same electric & magnetic fields have same energy density $\boxed{u_C = u_L}$. Find $\frac{E}{B}$?

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2\mu_0} B^2 \rightarrow \frac{E^2}{B^2} = \frac{1}{\varepsilon_0 \mu_0}$$

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = \underbrace{2.99 \times 10^{8}}_{\text{speed of light: } c} \text{ m/s}$$

$$\boxed{\frac{E}{B} = c}$$

Ampere's Law as modified by Maxwell:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}$$

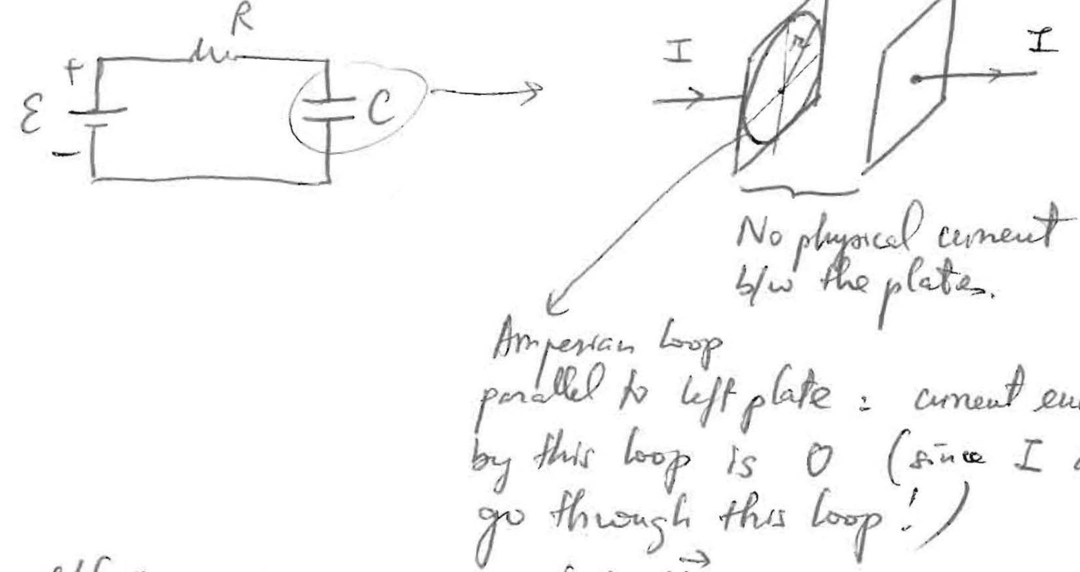
Observations:

1) Now we can say $\left\{ \begin{array}{l} \frac{\partial \vec{E}}{\partial t} \text{ creates } \vec{B} \\ \frac{\partial \vec{B}}{\partial t} \text{ creates } \vec{E} \end{array} \right.$

This explains the fact that EM waves (e.g. sun light) can propagate in vacuum : $\vec{E} \to \vec{B} \to \vec{E} \ldots$ Hence the importance of Maxwell.

Other example of EM waves: cell phone signals, signals from space probes (takes time to travel)

2) Maxwell's term also explains a technicality about a measured magnetic field around a capacitor in an RC circuit:



Old Ampere's Law : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

$$B \cdot 2\pi r = \mu_0 0 \to B = 0$$

This does not agree with experiment measurements that

Ch 29 Maxwell's Equations & EM wave.

So far we have seen ~~some~~ connection b/w the electric & magnetic fields via Ampere's & Faraday's Laws

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \underbrace{\int \vec{J}\cdot d\vec{A}}_{\text{current enclosed by the Amperian loop.}} \qquad (\vec{E}\to\vec{J}\to\vec{B})$$

$$\underbrace{\oint \vec{E}\cdot d\vec{l}}_{\text{induced voltage}} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A} \qquad \left(\frac{\partial \vec{B}}{\partial t}\to\vec{E}\right)$$

Ultimate connection was discovered by Maxwell:

Maxwell's equations

1) Gauss' Law: $\underbrace{\oint \vec{E}\cdot d\vec{A}}_{\text{Electric flux through a Gaussian surface}} = \dfrac{q_{enclosed}}{\varepsilon_0}$

2) "Gauss' Law for B": $\oint \vec{B}\cdot d\vec{A} = 0 \leftarrow$ no magnetic monopole discovered yet.

Maxwell's term.

3) Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} \boxed{+\ \mu_0 I_{displacement}}$

4) Faraday's Law: $\underbrace{\oint \vec{E}\cdot d\vec{l}}_{\text{induced voltage}} = -\dfrac{d\Phi_B}{dt} = -\int \dfrac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$

Displacement current: $\boxed{I_{displacement} = \varepsilon_0 \dfrac{d\Phi_E}{dt} = \varepsilon_0 \int \dfrac{\partial \vec{E}}{\partial t}\cdot d\vec{A}}$

electric flux.

This does not agree with experiment measurements that confirm $B \neq 0$ around the plate.

With Maxwell's ~~[illegible]~~ correction:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot dA$$

$$B\,2\pi r = 0 + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

$\Rightarrow B \neq 0.$ (not by the current enclosed but by a changing electric field)

$\downarrow$

RC circuit connect to an AC voltage source (Alternating Current $\to$ current is switching direction 60 times per second $f = 60Hz$)

<u>Maxwell's equations:</u>

1) Starting from Maxwell's equations $\longleftrightarrow$ EM wave equations:

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \\ \frac{\partial^2 B}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \end{aligned}\right\} \qquad \underbrace{\frac{\partial^2 y}{\partial x^2} = \omega^2 \frac{\partial^2 y}{\partial t^2}}_{\text{Transverse wave along a string}}$$

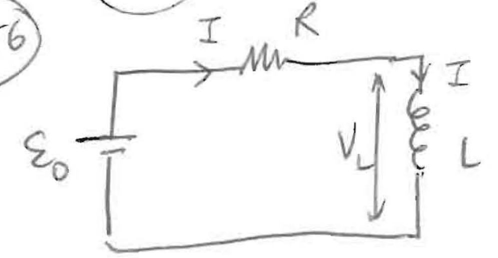
2) $\vec{E}$ & $\vec{B}$ are vectors $\to$ directions make a difference

$\to$ Polarization of EM waves.

$\hookrightarrow$ Sun glasses: reduces intensity of sun light by allowing only $\vec{E}$ along certain orientations to pass through

27.69 (& 53)

27.56



Data. $\begin{cases} \mathcal{E}_0 = 45V \\ R = 3.3\Omega \\ L = 2.1H \end{cases}$

If $\boxed{I = 9.5A}$ what is t since switch has been closed.?

→ Inductor presents inertia for current → $\boxed{I(\underbrace{t=0}_{\text{switch is closed.}}) = 0}$

→ $0 < t < \infty$: $\boxed{V_L = \mathcal{E}_0 \, e^{-\frac{t}{(\frac{L}{R})}}}$

Time constant $\tau = \frac{L}{R}$

↳ By def: when $t = \tau$ → $V_L = \mathcal{E}_0 e^{-1} = \frac{\mathcal{E}_0}{e}$ $(e = 2.71\ldots)$

Also @ an inductor: $\boxed{V_L = L\frac{dI}{dt}}$ (self induced voltage)

$$\frac{dI}{dt} = \frac{\mathcal{E}_0}{L} e^{-\frac{t}{\frac{L}{R}}} \xrightarrow{\int dt} I = \frac{\mathcal{E}_0}{L}\int e^{-\frac{t}{\frac{L}{R}}} dt + C$$

$$= \frac{\mathcal{E}_0}{\cancel{L}}\left[\frac{e^{-\frac{t}{\frac{L}{R}}}}{-\frac{1}{\frac{L}{R}}}\right] + C$$

$$\boxed{I(t) = \frac{-\mathcal{E}_0}{R} e^{-\frac{t}{\frac{L}{R}}} + C}$$

$$\boxed{\begin{aligned} &\underbrace{I(t=0)}=0 \rightarrow C = +\frac{\mathcal{E}_0}{R} \rightarrow \\ &-\frac{\mathcal{E}_0}{R}e^{-0} + C = 0 \quad I(t) = \frac{\mathcal{E}_0}{R}\left(-e^{-\frac{t}{\frac{L}{R}}} + 1\right) \end{aligned}}$$

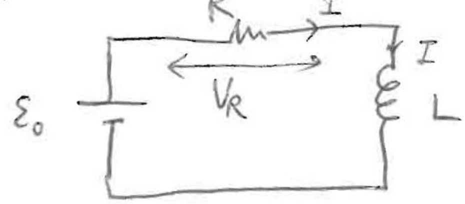
$$9.5 = \frac{45}{3.3}\left[-e^{-\frac{t}{\frac{2.1}{3.3}}} + 1\right]$$

$$\ln\left(1 - \frac{9.5 \times 3.3}{45}\right) = -\frac{t}{\frac{2.1}{3.3}} \rightarrow t = -\frac{2.1}{3.3}\ln\left(1 - \frac{9.5 \times 3.3}{45}\right) \text{ s}$$

$\boxed{t = \cancel{0.23} \text{ s}}$

27.69

Eq. 27-8: $I = I_0 e^{-\frac{t}{L/R}}$



a) Use Eq 27.8 to find $P_R(t)$

$$P_R(t) = I V_R = I\underbrace{(IR)}_{\text{Ohm's Law}} = I^2 R = I_0^2 R\, e^{-\frac{2t}{L/R}}$$

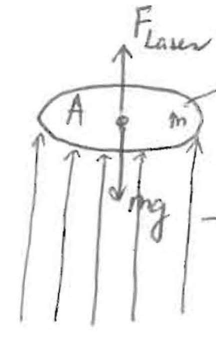
b) Total Energy dissipated @ resistor $= \int_0^\infty P_R(t)\,dt = I_0^2 R \int_0^\infty e^{-\frac{2Rt}{L}}\,dt$

$$= I_0^2 R \left[\frac{e^{-\frac{2Rt}{L}}}{\left(-\frac{2R}{L}\right)}\right]_{t=0}^{t=\infty} = I_0^2 R\left[0 - \frac{1}{-\frac{2R}{L}}\right]$$

$$= I_0^2 \cancel{R}\,\frac{L}{2\cancel{R}} = \boxed{\frac{1}{2} L I_0^2}$$

Total energy initially stored in the inductor.

29.56



Al foil of mass $m = 30\,\mu g$ and cross sectional area; A

Laser beam (light → EM wave)
Power needed to do this (hold the piece of Al foil in the air) ?

$F_{Laser} - mg = 0$

$\boxed{\text{Radiation pressure}}$: $P \rightarrow F_{Laser} = P \cdot A$

$\boxed{\text{Radiation intensity}}$ $S \equiv \frac{\text{Power}}{A} \leftrightarrow$ Rad. intensity Average = $\bar{S} = \frac{\text{Power}}{A}$

$\boxed{P = \frac{\bar{S}}{c}}$

$$\boxed{P = \frac{\overline{S}}{c}}$$

P: Rad. Pressure

$\overline{S}$: Average Rad. Intensity.

$$P = \frac{\overline{F}}{A} = \frac{\overline{\frac{dp}{dt}}}{A} = \frac{\frac{1}{c}\overline{\frac{dU}{dt}}}{A} = \frac{\frac{1}{c}\overline{Power}}{A} = \frac{1}{c}\overline{S}$$

Linear momentum

Radiation momentum: $\boxed{p = \frac{U}{c}}$

$$F_{Laser} = mg$$

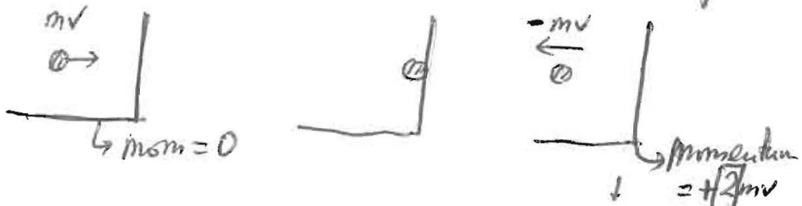
$$P_r .\ A = mg$$

$$\frac{1}{c}\overline{S}\cdot\boxed{2}\cdot A = mg$$

] Since $\overline{S} = \frac{\overline{Power}}{Area}$: $\frac{\overline{Power}}{c\cancel{A}}\,2\cancel{A} = mg \rightarrow \overline{Power} = \frac{mgc}{2}$

$\rightarrow \overline{Power} = \frac{30\times10^{-9}\times 9.81\times 3\times10^{8}}{2} = 44.1\ W$

Why? → Remember: microscopic molecules & macroscopic temp.



Al foils reflect all light = "elastic collision" for radiation.

Momentum transferred to wall

For momentum transfer from radiation (gas molecule counterpart) to Al foil (container wall counterpart)

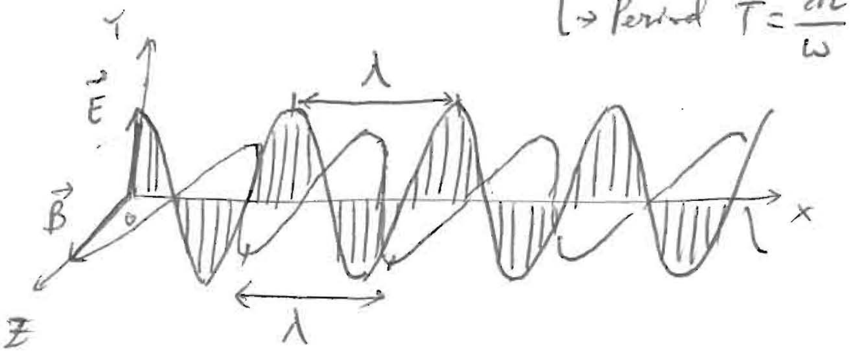
Electromagnetic waves: (Cont.)

1) $\boxed{\text{Vector nature of } \vec{E} \,\&\, \vec{B}}$ & Wave properties

$$\vec{E} = E \sin(kx - \omega t)\,\hat{j}$$

- → Field points along $+y$
- → Propagation is along x-axis
- → Minus sign b/w kx & ωt → propagation is in $+x$ direction
- → Wavelength $\lambda = \frac{2\pi}{k}$: sep. b/w consecutive peaks or troughs in space
- → Period $T = \frac{2\pi}{\omega}$: sep. b/w consecutive peaks or troughs in time.

E ↓ Max. Magnitude

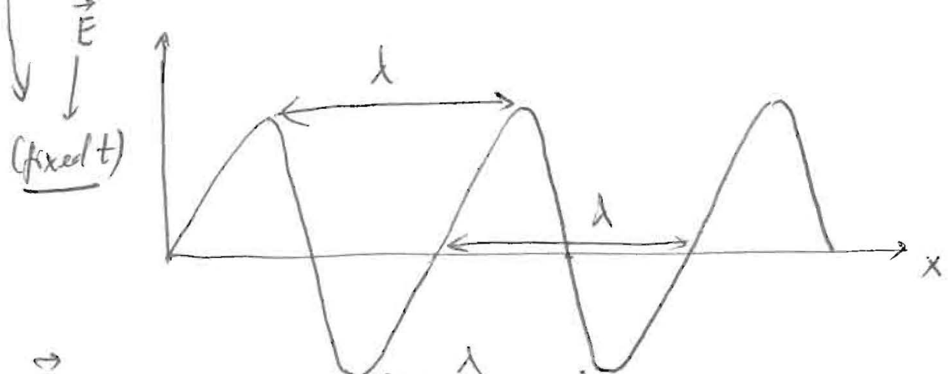


$\vec{B}$ has to be perpendicular to $\vec{E}$

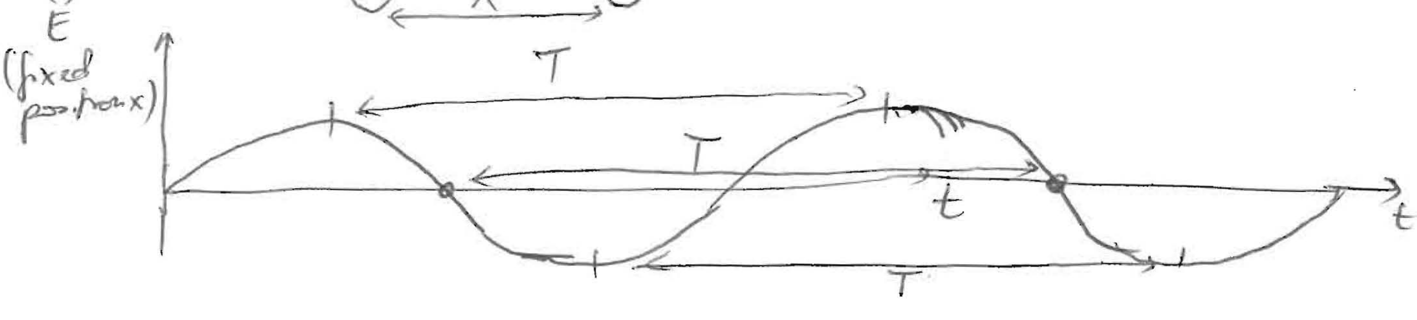
EM wave ~ transverse wave in a string
↳ perturbation perpendicular to direction of propagation

↳ $\vec{B} = B \sin(kx - \omega t)\,\hat{k}$

→ Also: direction of propagation is given by direction of $\vec{V}\,\vec{E} \otimes \vec{B}$ using RHR
↓ cross product



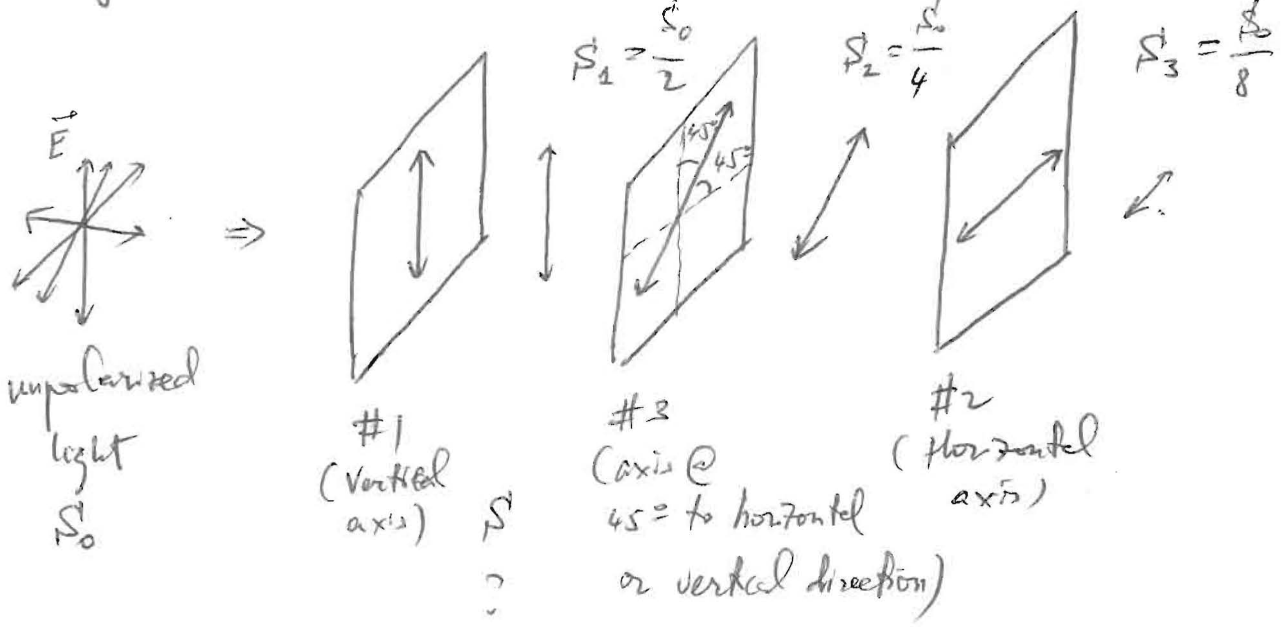
Two consecutive same zeroes (both increasing or decreasing) are also separated by λ



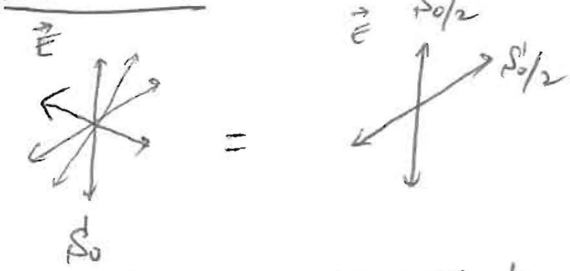
2) Vector nature of $\vec{E}$ & $\vec{B}$ & polarization

(29.44) **Unpolarized** ($\vec{E}$ pointing along all possible directions)

Unpolarized light with intensity S_0 incident on 3 **polarizers** (a polarizer only allows light with $\vec{E}$ pointing along the <u>polarizer axis</u> to pass through)

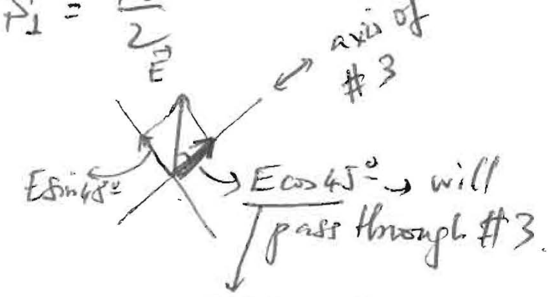


observation:



$\Rightarrow S_1 = \frac{S_0}{2}$

Axis of #3 is @ 45° to vertical →



Field amplitude

↓

Intensity ($\vec{S} = \frac{\vec{E} \times \vec{B}}{c}$)

↓

(Field)^2

$$S_2 = S_1 \cos^2 45^\circ$$

$$= \frac{S_0}{2} \underbrace{\cos^2 45^\circ}_{\frac{1}{2}} = \frac{S_0}{4}$$

Axis of #2 is @ 45° to the $\vec{E}$ that passed through #3

$$\rightarrow S_3 = S_2 \cos^2 45^\circ = \frac{S_0}{4} \cdot \frac{1}{2} = \frac{S_0}{8}$$

29.50 Radio waves are EM waves!

Max. Magnitude of $\vec{E}$ (@ 1.5 km) = $350 \frac{mV}{m}$

a) Power of the transmitter?

Rad. intensity: $S = \frac{\text{Power}}{\text{Area}}$

$S = \frac{\text{Energy} \cdot \overset{c}{\text{length}}}{\text{time} \cdot \underbrace{\text{Area} \times \text{length}}_{\text{Vol.}}} = \frac{\text{Energy}}{\text{vol}} c$ = energy density × c

$= \underbrace{\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right)}_{\text{Total EM energy density}} c = \varepsilon_0 E^2 c$

$c^2 = \frac{1}{\mu_0 \varepsilon_0} \leftrightarrow \varepsilon_0 = \frac{1}{c^2 \mu_0}$

$\frac{E}{B} = c$

$\rightarrow$ Power = S . Area = $\varepsilon_0 E^2 c$. Area

$\rightarrow \overline{\text{Power}} = \varepsilon_0 \overline{E^2}\, c \, . \text{Area} = \frac{1}{2} c \varepsilon_0 E_{max}^2 \cdot 4\pi r^2$

$\overline{E^2} \rightarrow \frac{1}{2} E_{max}^2$

Area: sphere of radius r ~~around~~ centered at the transmitter (transmits in all directions)

$\boxed{\overline{\text{Power}} = 4.59\ kW}$

29.59 Photon rocket emits beam of light instead of hot gas. Need thrust (force upward) of $35 \times 10^6 N \rightarrow$ Power needed for light?

Radiation pressure $= \dfrac{\text{Av. Rad. intensity}}{c}$ or $P = \dfrac{\bar{S}}{c}$

$$F_{lift} = P \cdot A = \frac{\bar{S}}{c} A = \frac{\overline{\text{Power}}}{\cancel{A} c} \cancel{A} = \frac{\overline{\text{Power}}}{c}$$

$$\overline{\text{Power}} = F_{lift} \cdot c = 35 \times 10^{6} \times 3 \times 10^{8} = 10^{16} W$$

All of our power generating capability is only $10^{12} W$