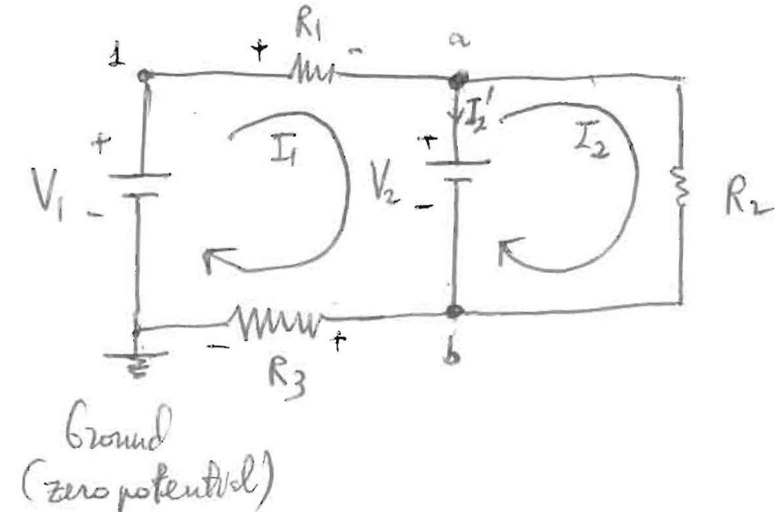


Loop analysis (2 loops)

→ I_1 & I_2 in clockwise (CW)

Loop 1) $+V_1 - I_1R_1 - V_2 - I_1R_3 = 0$

Loop 2) $+V_2 - I_2R_2 = 0$

Solve for I_1 & I_2:

→ $\boxed{I_2 = \frac{V_2}{R_2}}$

→ $V_1 - V_2 = I_1(R_1+R_3)$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1+R_3}}$$

Node analysis (2 nodes)

Node a) $I_1 - I_2' - I_2 = 0$

Node b) $-I_1 + I_2' + I_2 = 0$

only one effective node!

→ $I_1 = \frac{V_1 - V_a}{R_1}$ (Ohm's law)

$I_2 = \frac{V_a - V_b}{R_2}$

→ $$\boxed{\frac{V_1 - V_a}{R_1} - I_2' - \frac{V_a - V_b}{R_2} = 0}$$

$V_a = V_2 + I_1R_3$ $(a \to b \to 0)$

$V_a = -I_1R_1 + V_1$ $(a \to 1 \to 0)$

$V_2 + I_1R_3 = -I_1R_1 + V_1$

$I_1(R_1+R_3) = V_1 - V_2 \to \boxed{I_1 = \frac{V_1 - V_2}{R_1+R_3}}$

→ $\frac{V_1 - V_a}{R_1} - I_2' - \frac{V_2}{R_2} = 0$

$$I_1 - I_2 = I_2' = \frac{V_1 - V_a}{R_1} - \frac{V_2}{R_2} = \frac{V_1 - (V_2 + I_1R_3)}{R_1} - \frac{V_2}{R_2}$$

$$= \frac{V_1 - V_2}{R_1} - I_1\frac{R_3}{R_1} - \frac{V_2}{R_2}$$

$$I_1\left(1 + \frac{R_3}{R_1}\right) - \underbrace{I_2}_{\frac{V_2}{R_2}} = \frac{V_1}{R_1} - V_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

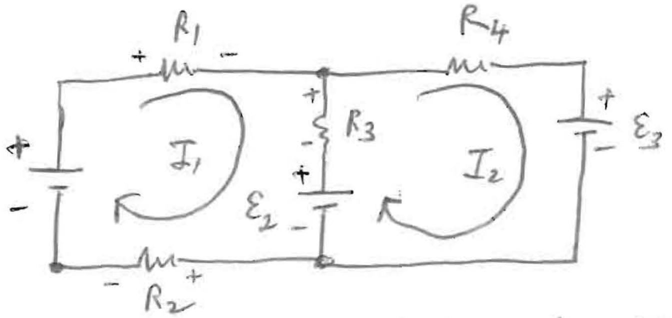
$$I_1\left(1+\frac{R_3}{R_1}\right) = \frac{V_1}{R_1} + \cancel{\frac{V_1}{R_2}} - \frac{V_2}{R_1} - \cancel{\frac{V_2}{R_2}}$$

$$I_1 = \frac{V_1 - V_2}{R_1}\,\frac{1}{1+\frac{R_3}{R_1}}$$

$$= \frac{V_1 - V_2}{\cancel{R_1}}\,\frac{1}{\frac{R_1+R_3}{\cancel{R_1}}}$$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

25.53



$\mathcal{E}_1 = 6V$; $\mathcal{E}_2 = 1.5V$; $\mathcal{E}_3 = 4.5V$
$R_1 = 270\Omega$; $R_2 = 150\Omega$;
$R_3 = 560\Omega$; $R_4 = 820\Omega$

Find current in R_3 including direction (up or down)
Will solve using 2 alternatives : Loop & Node analyses

$\boxed{\text{Loop Analysis:}}$ 2 loops → I_1 & I_2 in CW ⇒ Current through R_3 is $(I_1 - I_2)$

Loop 1) $\quad + \mathcal{E}_1 - I_1R_1 - (I_1 - I_2)R_3 - I_1R_2 - \mathcal{E}_2 = 0$
+
Loop 2) $\quad + \mathcal{E}_2 - (I_2 - I_1)R_3 - I_2R_4 - \mathcal{E}_3 = 0$

$$\mathcal{E}_1 - \mathcal{E}_3 - I_1(R_1+R_2) - I_2R_4 = 0$$

$$I_1 = \frac{\mathcal{E}_1 - \mathcal{E}_3 - I_2R_4}{R_1 + R_2}$$

$$\boxed{I_1 = \frac{1.5 - 820\,I_2}{420}}$$

Plug this into eq 2)

$$\mathcal{E}_2 - \mathcal{E}_3 - I_2(R_3+R_4) + I_1R_3 = 0$$

$$-3 - 1380\,I_2 + 560\,I_1 = 0$$

$$\to -3 - 1380\,I_2 + \frac{560}{420}(1.5 - 820\,I_2) = 0$$

$$-1 - 2473.2\, I_2 = 0$$

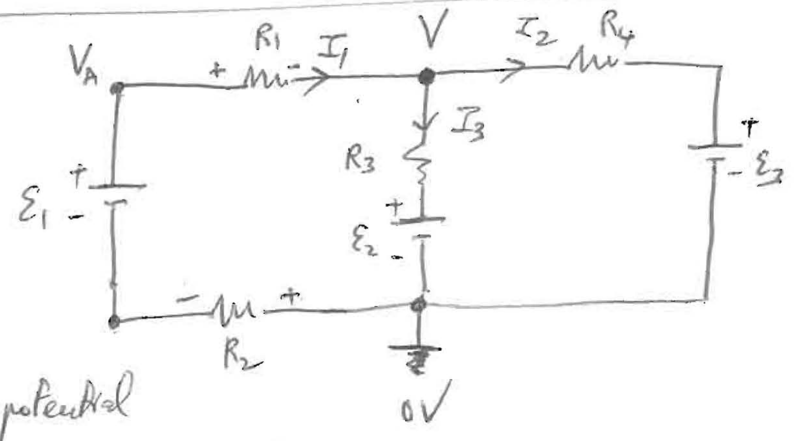
$$\boxed{I_2 = -\underbrace{0.4\,m}_{10^{-3}}A}$$

$$\boxed{I_1 = \frac{1.5 - 820\,(-0.4\times10^{-3})}{420} = +4.36\,mA}$$

Current thru R_3: $I_1 - I_2$ (net downward current, from our assumption of directions for I_1 & I_2: both CW)

$$I_1 - I_2 = 4.36\,mA - (-0.4\,mA) = \underbrace{+}_{\text{downward current through } R_3\,!}4.76\,mA$$

$\boxed{\text{Node Analysis}}$



1) Define zero potential
2) Identify node: only one left
2) Assume directions for I_1, I_2, I_3 by drawing on circuit
4) Node equation: $I_1 - I_2 - I_3 = 0$
5) Write node equations using voltage:

$$I_1 = \underbrace{\frac{V_A - V}{R_1}}_{\text{Ohm's law}} = \frac{\mathcal{E}_1 - I_1R_2 - V}{R_1} \rightarrow I_1R_1 = \mathcal{E}_1 - I_1R_2 - V$$

$$V_A = \mathcal{E}_1 - I_1R_2$$

$$\boxed{I_1 = \frac{\mathcal{E}_1 - V}{R_1 + R_2}}$$

$$I_2 = \frac{V - \mathcal{E}_3}{R_4}$$

$$I_3 = \frac{V - \mathcal{E}_2}{R_3}$$

Back into node equation:

$$\frac{\mathcal{E}_1 - V}{R_1 + R_2} - \frac{V - \mathcal{E}_3}{R_4} - \frac{V - \mathcal{E}_2}{R_3} = 0$$

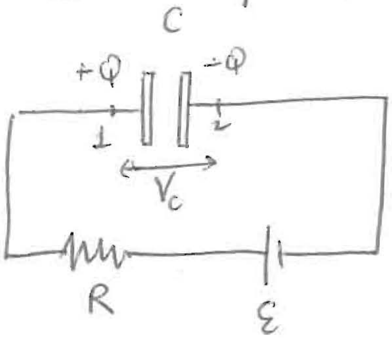
$$\frac{6 - V}{420} - \frac{V - 4.5}{820} - \frac{V - 1.5}{560} = 0$$

$$\boxed{V \doteq 4.17V}$$

Current through R_3: $\boxed{I_3 = \frac{V - \mathcal{E}_2}{R_3} = \frac{4.17 - 1.5}{560} = +4.76 mA}$

↓ downward. @ R_3

Circuits involving resistors & capacitors:



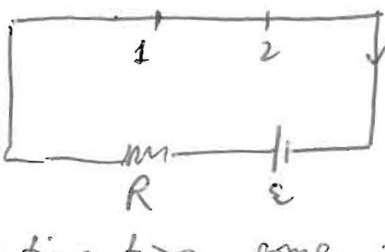
a) @ time $t=0$ we connect the uncharged capacitor C to a circuit with $\mathcal{E}$ & R

$\left\{\begin{array}{l} \text{Initial charge } Q = 0 \\ \text{Initial potential } V_c = 0 \end{array}\right\}$ The capacitor behaves effectively like a wire

1 — 2

$V_1 = V_2$

@ $t=0$ c behaves like:

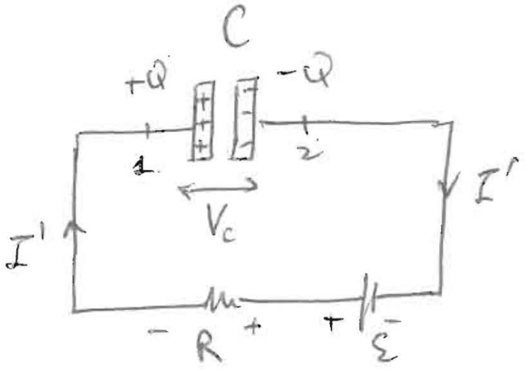


$I = \frac{\mathcal{E}}{R}$ (Ohm's Law)

b) @ time $t>0$ some + charges are transferred from right plate ~~thro~~ to left plate via the circuit → capacitor is being charged.

@ t>0

Actually negative charges are transferred in the opposite directions !!

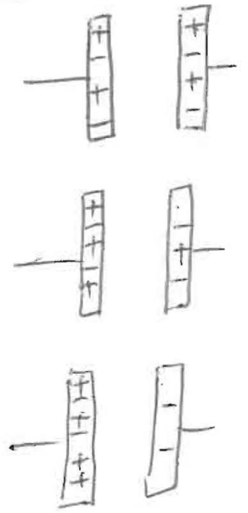


$$\mathcal{E} - I'R - V_c = 0$$

(loop equation for RC circuit)

$$I' = \frac{\mathcal{E} - V_c}{R}$$

* I' over time : getting smaller & smaller :



$I \rightarrow$ max

I' (decreasing)

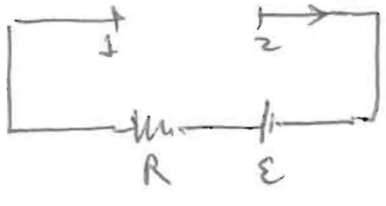
$I' = 0$ (capacitor is fully charged!)

(also it was harder to transfer later charges since we need to go against stronger fields !)

* V_c over time : getting larger & larger.

c) @ time $t = \infty$ (very long time after circuit is connected)

$\begin{cases} I' = 0 \\ V_c = \text{max.} \end{cases}$ $\longrightarrow$ Capacitor effectively behaves like an open circuit !



Quantitative description: RC circuit $\begin{cases} @\, t=0 & I' = \frac{\mathcal{E}}{R} \\ @\, t=\infty & I' = 0 \end{cases}$

$\downarrow$

$I'(t)$

$$\frac{d}{dt}\left[\mathcal{E} - I'R - V_c = 0 \right] \longrightarrow -R\frac{dI'}{dt} - \frac{1}{C}\underbrace{\frac{dQ}{dt}}_{I'} = 0$$

$\frac{d\mathcal{E}}{dt} = 0$ ($\mathcal{E}$ is constant)

$$V_c = \frac{Q(t)}{C}$$

C = capacitance, constant.

$$-R\frac{dI'}{dt} - \frac{1}{C}I' = 0 \rightarrow \frac{dI'}{dt} = -\frac{1}{RC}I' \rightarrow \frac{dI'}{I'} = -\frac{1}{RC}dt$$

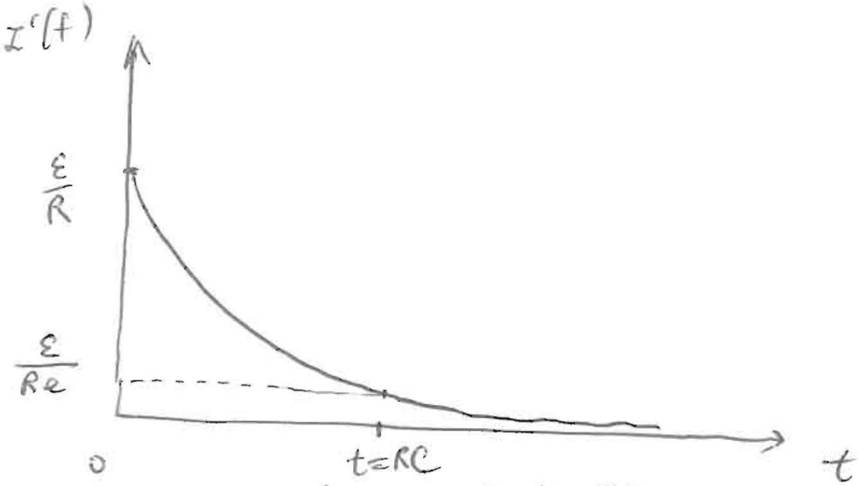
$$\rightarrow \ln I' = -\frac{1}{RC}t + \text{const.}$$

from indefinite integral

$$\rightarrow \underbrace{e^{\ln I'}} = \underbrace{e^{\text{const}}} \cdot e^{-\frac{1}{RC}t}$$

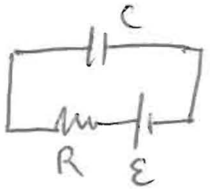
$$\boxed{I' = \underbrace{I'(t=0)}_{} \, e^{-\frac{t}{RC}}} \rightarrow \boxed{I'(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}}$$

qualitative $\rightarrow \frac{\mathcal{E}}{R}$

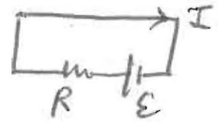


(time constant: RC → SI unit: s)

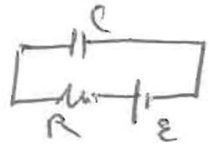
RC circuit:



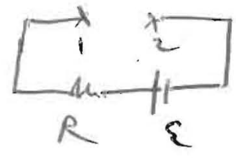
@ $t=0$ $\quad I = \frac{\varepsilon}{R}$; $V_c = 0$ → Capacitor is in short circuit



@ $0<t<\infty$ $\quad I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$



@ $t=\infty$ $\quad I = 0$ → Capacitor in open circuit.



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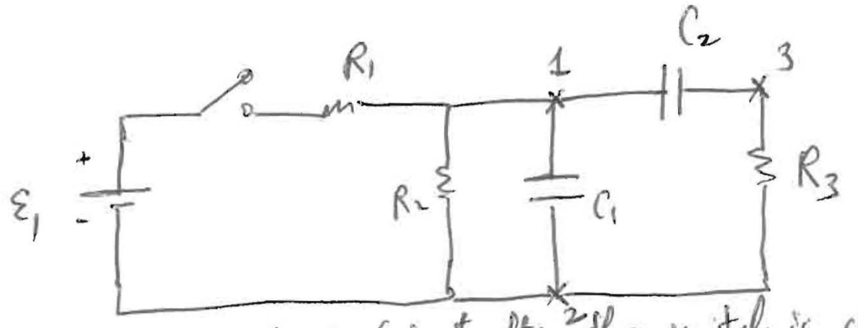
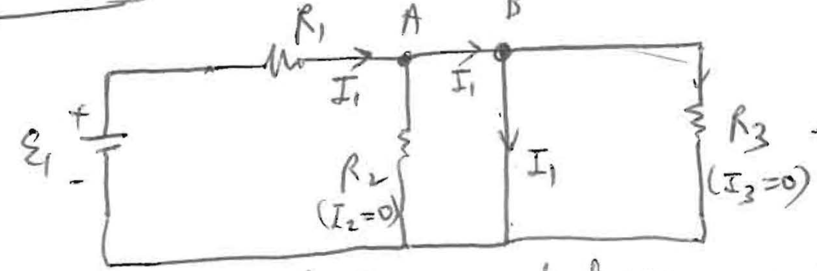


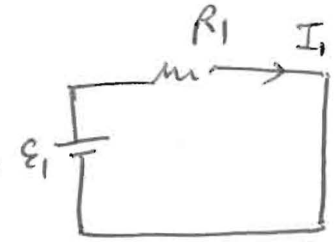
Data:
- switch initially open.
- C_1 & C_2 uncharged
- $R_1 = R_2 = R_3 = R$

Find: current in R_2
- a) @ $t=0$ (just after the switch is closed)
- b) @ $t=\infty$ (long after " " " ")
- c) current in R_3 qualitatively

a) $\boxed{@\ t=0}$ → Capacitors behave as short circuit:



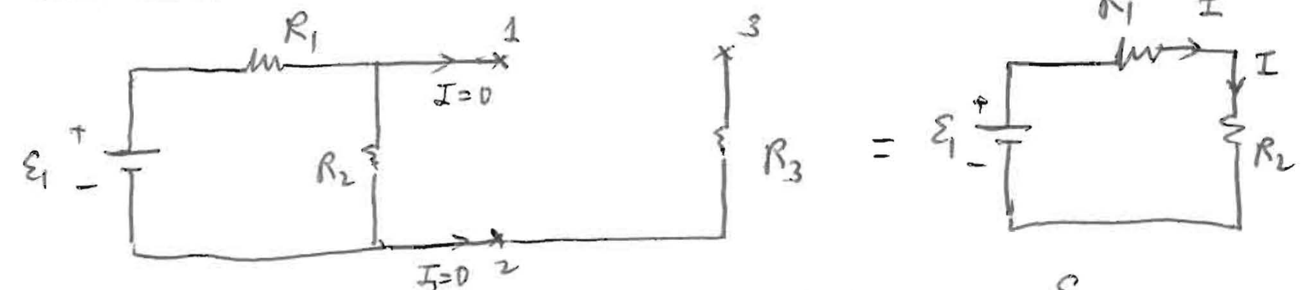
→



@ Node B: current division: current in one branch is proportional to the ~~current~~ resistance in the other branch. or current always prefer the least resistance path

→ Current through R_2 @ $t=0$ is 0.

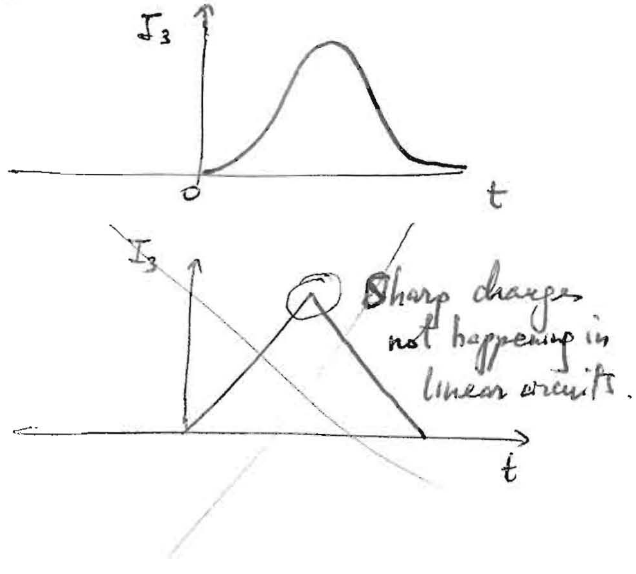
b) $\boxed{@\ t=\infty}$ → Capacitors behave as an open circuit



Current through R_2 is $I = \dfrac{\varepsilon_1}{R_1 + R_2}$ ↓ Ohm's Law.

c) Current in R_3 qualitatively:

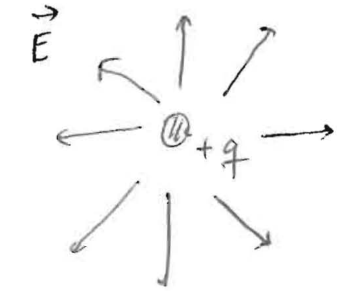
$@\ t=0 \rightarrow I_3 = 0$
$@\ t=\infty \rightarrow I_3 = 0$ } but it is not 0 all the time:

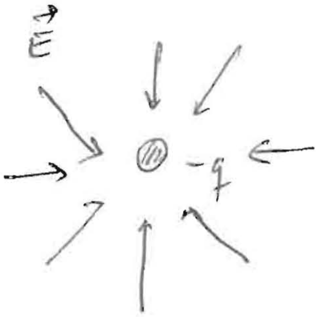


Ch 26 Magnetic Field:

Electric Magnetic Electromagnetic

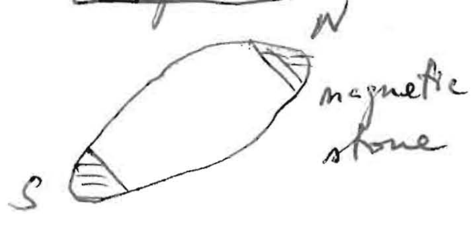
$\boxed{\text{Electric}}$

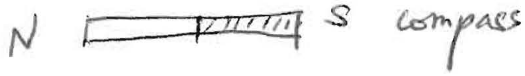


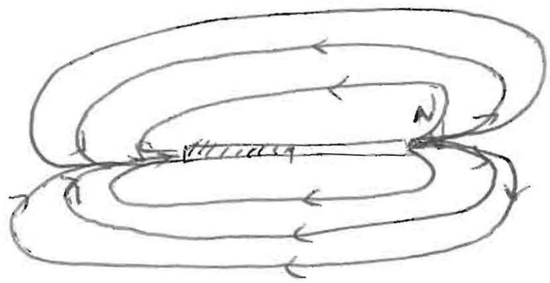


- Two types of charges: +, -
- Equal charges repel
  Opposite charges attract
- Electric field lines are open in general

$\boxed{\text{Magnetic}}$



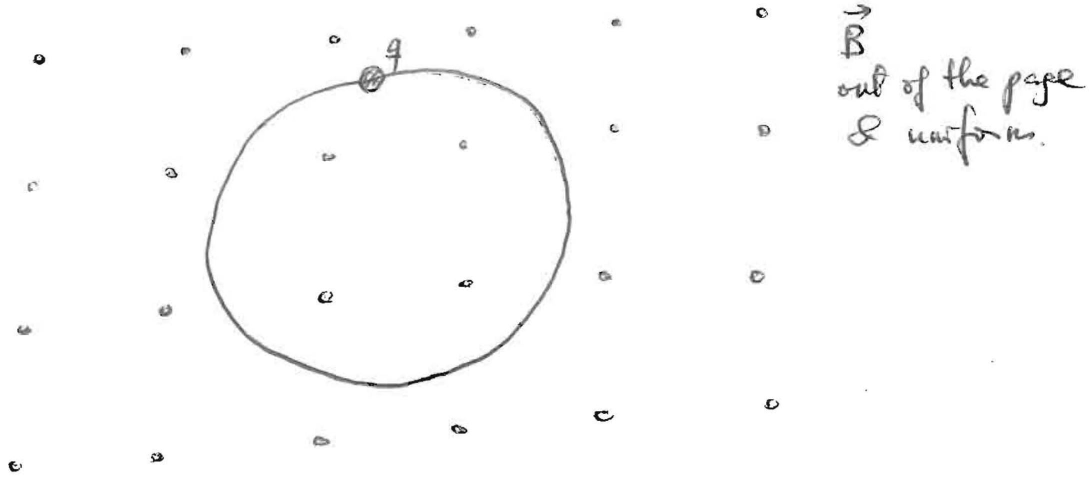




- Two types of magnetic poles: N, S
- They are always attached
  (magnetic monopole have not been found yet)
- Equal poles repel
  Opposite poles attract
- Magnetic field lines are always closed (N & S are attached together)

Effects of the magnetic field:

On a moving charge (of value q) @ velocity $\vec{v}$ on the ~~paper~~ plane of this page, a uniform magnetic field $\vec{B}$ pointing out the page (perpendicular to the plane of charge motion) would bend the trajectory of that charge into circles:



Applications: 1) particles confinement or trap
2) cyclotron → synchrotron (medical applications, particle physics research)

Closer look:
1) If charge q instead moves in/out of page
→ This $\vec{B}$ has no effect on that ~~part~~ charge!
2) When charge q is moving in the page
→ This $\vec{B}$ has max. effect on the charge.
3) Intermediate effect if $\vec{v}$ forms another angle θ with $\vec{B}$ $0 < \theta < 90°$

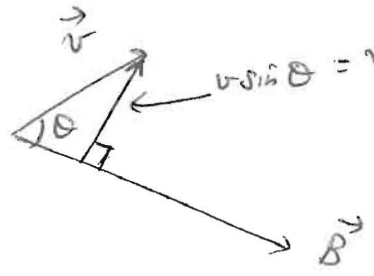
→ $$\vec{F}_{magnetic} = q\, \vec{v} \otimes \vec{B}$$

$\otimes$ ↓ vector cross-product b/w $\vec{v}$ & $\vec{B}$

* Vector cross product $\vec{v} \times \vec{B}$ is another vector that is perpendicular to both $\vec{v}$ & $\vec{B}$, its direction is given by the Right Hand Rule (RHR): when you turn your right hand fingers from the 1st vector ($\vec{v}$) to the 2nd vector ($\vec{B}$), your thumb indicates direction of $\vec{v} \times \vec{B}$. Magnitude is $vB \sin\theta$ (θ is the angle b/w $\vec{v}$ & $\vec{B}$)

* Interpretation: magnitude of cross product is

$$vB\sin\theta = \underbrace{v\sin\theta}_{v_\perp} \; B$$



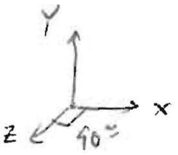
$v\sin\theta = v_\perp \rightarrow$ the perpendicular component of $\vec{v}$ to the magnetic field $\vec{B}$

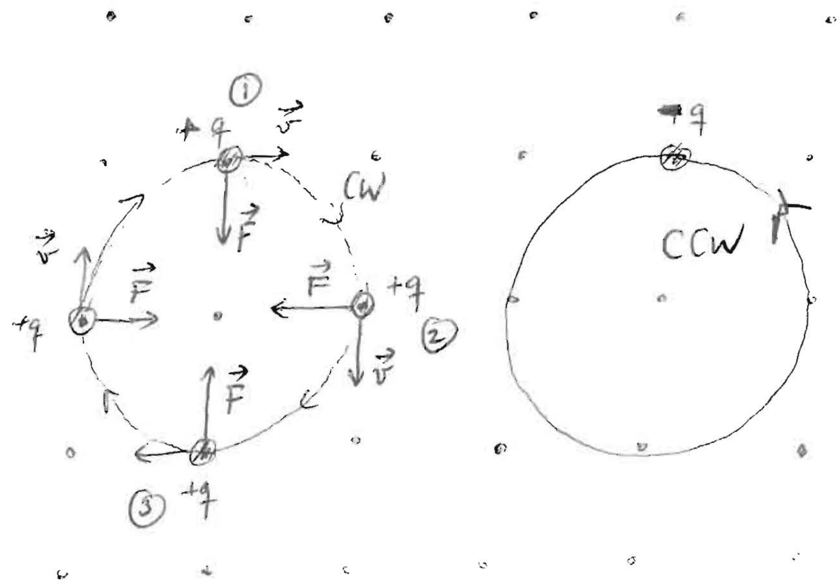
$F_{magnetic} = q\, vB\sin\theta = q\, v_\perp B \rightarrow$ only the perpendicular component of the velocity with respect to the magnetic field contributes to the magnetic force! This agrees with:

1) Particle moving parallel to field $\vec{B} \rightarrow v_\perp = 0$
$\rightarrow F_{magnetic} = 0 \rightarrow$ Field has no effect.

2) Particle moving perpendicular to field $\vec{B} \Rightarrow v_\perp = v$
$\rightarrow F_{magnetic} = \text{max} \rightarrow$ Field has max effect on charge.

* Direction





$\vec{B}$: uniform & out of page
$\vec{B} = B\hat{k}$

① A charge $+q$ moving in $+x$ direction
$\vec{v} = v\hat{i}$ → $\vec{F} = q\vec{v}\times\vec{B}$ points
downward: $\vec{F} = qvB\,(-\hat{j})$ $(\sin 90^\circ = 1)$

② $\vec{F} = q\vec{v}\times\vec{B} = qvB\,(-\hat{i})$

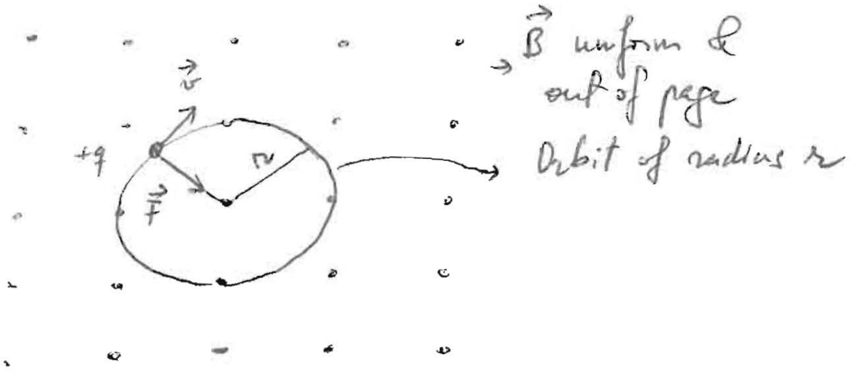
③ $\vec{F} = q\vec{v}\times\vec{B} = qvB\,(\hat{j})$

④ $\vec{F} = q\vec{v}\times\vec{B} = qvB\,(\hat{i})$

Consequences:

$+q$ & $\vec{B}$ out of page	→	CW
$-q$ & " " "	→	CCW
$+q$ & $\vec{B}$ into page	→	CCW
$-q$ & $\vec{B}$ " "	→	CW

* Charge in a magnetic field (uniform) → follows uniform circular motion (UCM): speed along circular trajectory is constant, but velocity is changing direction due to a radial acceleration given by the magnetic force ($F = ma$ or 2nd Newton's Law)



Net force on +q is $F = vqB$ ($\sin\theta = \sin 90° = 1$) along the radial direction → $F = ma = m\dfrac{v^2}{r}$ ↓ UCM

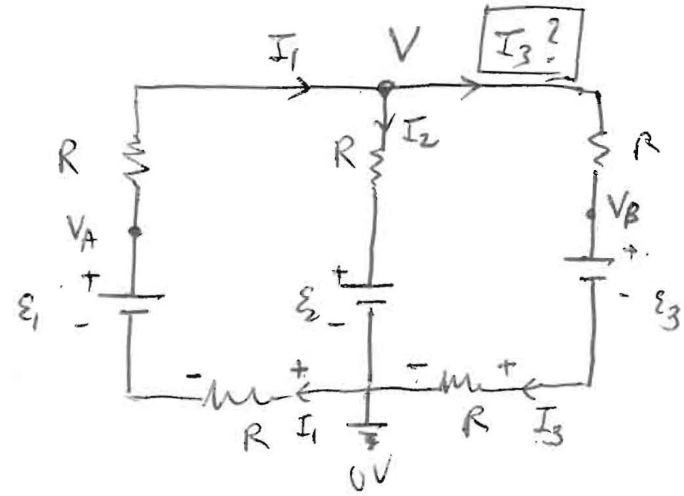
$$\cancel{v}qB = m\frac{v^{\cancel{2}}}{r} \rightarrow \boxed{r = \frac{mv}{qB}}$$

Observation:

- Particle confinement (smaller orbits or small r) → Large B (current limitation in fusion energy) $B_{max} = 10\,T$ → need much larger B)
- Orbital period: time to complete one turn.

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi \cancel{v}}{\frac{qB\cancel{v}}{m}} = \frac{2\pi m}{qB}}$$

25.75



Data = $\begin{cases} R = 1.5\times10^{6}\,\Omega \\ \mathcal{E}_1 = 75\times10^{-3}\,V \\ \mathcal{E}_2 = 45\times10^{-3}\,V \\ \mathcal{E}_3 = 20\times10^{-3}\,V \end{cases}$

Node analysis:

1) Set zero potential
2) Select the node : voltage V
3) Assume directions for currents @ node : I_1 (in), I_2 (out), I_3 (out)
4) Write node equation: $I_1 - I_2 - I_3 = 0$
5) Write currents in terms of the voltages: $\boxed{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3 \,\&\, V}$

$$I_1 = \frac{V_A - V}{R} = \frac{(\mathcal{E}_1 - I_1R) - V}{R} \rightarrow I_1R = \mathcal{E}_1 - I_1R - V$$

$\downarrow$ ohm's law

$$2I_1R = \mathcal{E}_1 - V$$

$$\boxed{I_1 = \frac{\mathcal{E}_1 - V}{2R}}$$

$$\boxed{I_2 = \frac{V - \mathcal{E}_2}{R}}$$

$$I_3 = \frac{V - V_B}{R} = \frac{V - (\mathcal{E}_3 + I_3R)}{R} \rightarrow I_3R = V - \mathcal{E}_3 - I_3R$$

$$2I_3R = V - \mathcal{E}_3$$

$$\boxed{I_3 = \frac{V - \mathcal{E}_3}{2R}}$$

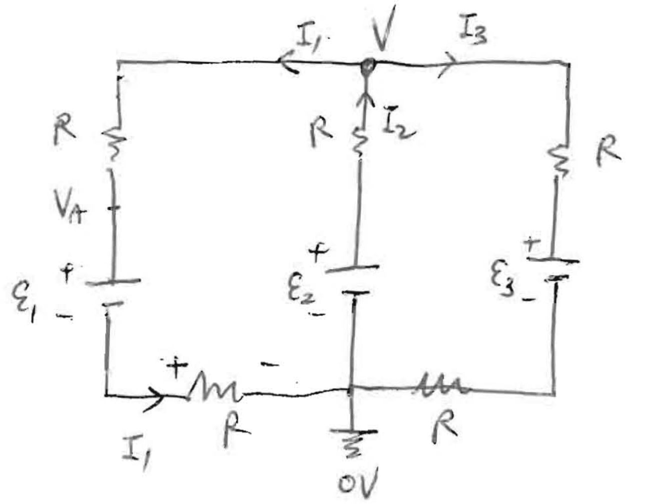
Back into the node equation: $\frac{\mathcal{E}_1 - V}{2\cancel{R}} - \frac{V - \mathcal{E}_2}{\cancel{R}} - \frac{V - \mathcal{E}_3}{2\cancel{R}} = 0$

$\rightarrow \mathcal{E}_1 - V - 2V + 2\mathcal{E}_2 - V + \mathcal{E}_3 = 0$

$\mathcal{E}_1 + 2\mathcal{E}_2 + \mathcal{E}_3 - 4V = 0 \rightarrow V = \frac{\mathcal{E}_1 + 2\mathcal{E}_2 + \mathcal{E}_3}{4} =$

$V = \frac{75 + 2\times45 + 20}{4}\,mV = \frac{185}{4}\,mV = 46.25\,mV \rightarrow I_3 = \frac{(46.25 - 20)10^{-3}}{2\times1.5\times10^{6}} = 8.75\,nA$

$\downarrow$ 10^{-9}



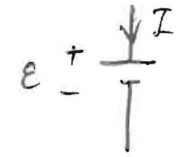
another perfectly valid assumptions for currents @ node V.
(Actual directions will be given by the signs of these currents after we solve the equations)

$$5)\quad I_1 = \frac{V - V_A}{R} = \frac{V - \mathcal{E}_1 - I_1 R}{R}$$

$$V_A = \mathcal{E}_1 + I_1 R$$

Across a battery:

→ Gain energy



→ loose energy.

Across resistance:

→ loose energy.

→ loose energy.

Think of friction.

↓

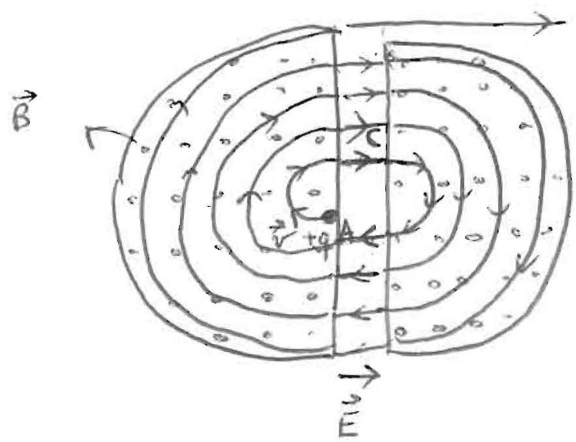
allways a potential drop across a resistance + to −

Applications of effect of magnetic field on a moving charge:

1) **Cyclotron:** (modern version is synchrotron)

Goal is to accelerate charged particle to very high speed using the magnetic & the electric fields

Applications: . study of subatomic structure (CERN)
. lower energy range: medical applications.



→ 2 D-shaped chambers filled with a uniform magnetic field out of page

→ Alternating electric field $\vec{E}$ in the gap b/w chambers: flipping directions left and right

→ Charge +q is pushed in @ A, ~~get heat~~ follows a circular trajectory

→ @ C charge has higher speed due to the $\vec{E}$ in the gap. $r = \frac{mv}{qB}$

→ larger circular orbit.

Higher v, larger orbit until it exits cyclotron.

[When v gets close to $c = 3\times10^8$ m/s → relativistic corrections (synchrotron)]

Max K.E? What feature of the cyclotron most affects Max KE?

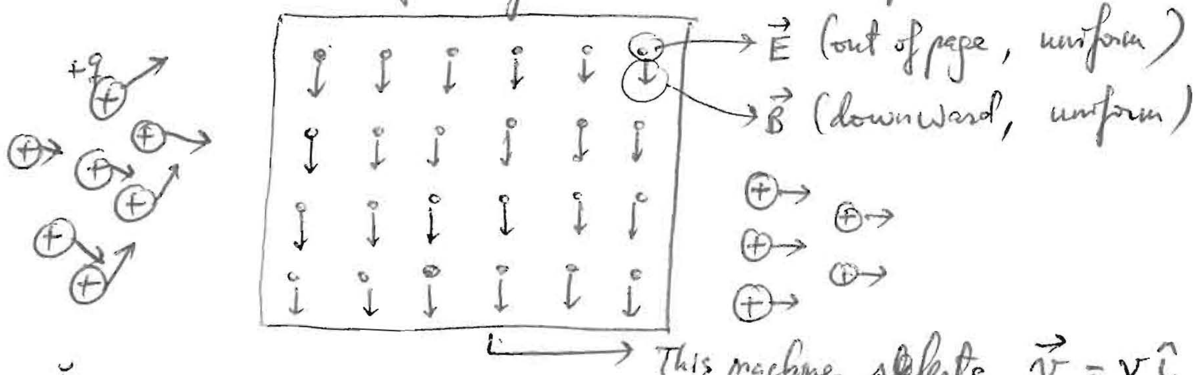
$\vec{E}$ but most importantly the radius of the cyclotron:

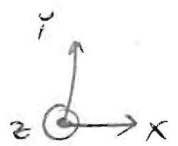
$$\frac{1}{2}mv_{max}^2 = \frac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m}$$

Max radius of an orbit is also the radius of the cyclotron!

2) Velocity selector:

↳ Goal: to pick out among a bunch of ions of different velocities those with a desired velocity by using a combination of magnetic & electric fields.





$\vec{E} = E\,\hat{k}$

$\vec{B} = B(-\hat{\jmath})$

Why does this arrangement of $\vec{E}$ & $\vec{B}$ select $\vec{v} = v\hat{\imath}$?

An ion going through will feel two forces:

$$\begin{cases} \vec{F}_E = +q\vec{E} = qE\,\hat{k} & \text{(along } +z \text{ direction)} \\ \vec{F}_B = +q\,\vec{v}\times\vec{B} = qvB\ \hat{\imath}\times(-\hat{\jmath}) = -qvB\,\underbrace{(\hat{\imath}\times\hat{\jmath})}_{\text{by RHR } = \hat{k}} \end{cases}$$

Net force on ion is $\vec{F}_E + \vec{F}_B = (\underbrace{qE - qvB}) \hat{k}$

if $v = \dfrac{E}{B}$ ⟶ 0 or net force is zero!

Those ions with $\vec{v} = \dfrac{E}{B}\hat{\imath}$ the machine has no effect! → they would pass through! Ions with $\vec{v} = v\hat{\imath}$ where $v \neq \dfrac{E}{B}$ get pushed off along $+z$. Ions with y or z components of velocity will also get pushed off.

Calculate the magnetic field from currents:
(Sources of the magnetic fields are currents!)

Electric field: (sources are charges):

$$d\vec{E} = k\frac{dq}{r^2}\hat{r}$$ inverse-square law or Coulomb's Law

$k = 9\times10^9$ (SI)

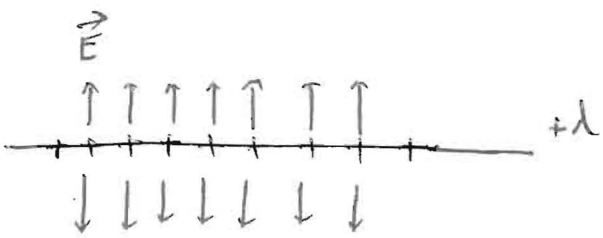
Magnetic field: (sources are current)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$ Inverse-square law or Biot-Savart's Law

μ_0 = permeability in vacuum
$4\pi\times10^{-7}\ \frac{N}{A^2}$

→ Magnetic field due to a line of current (along a straight wire) is wrapping around the current:

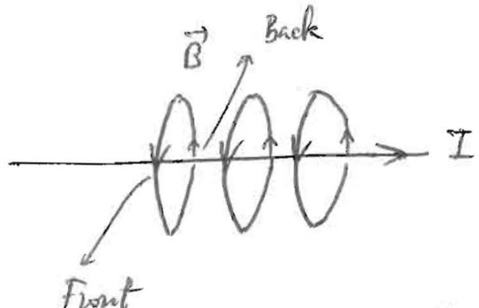
line of charge



$E = \frac{2k\lambda}{r}$

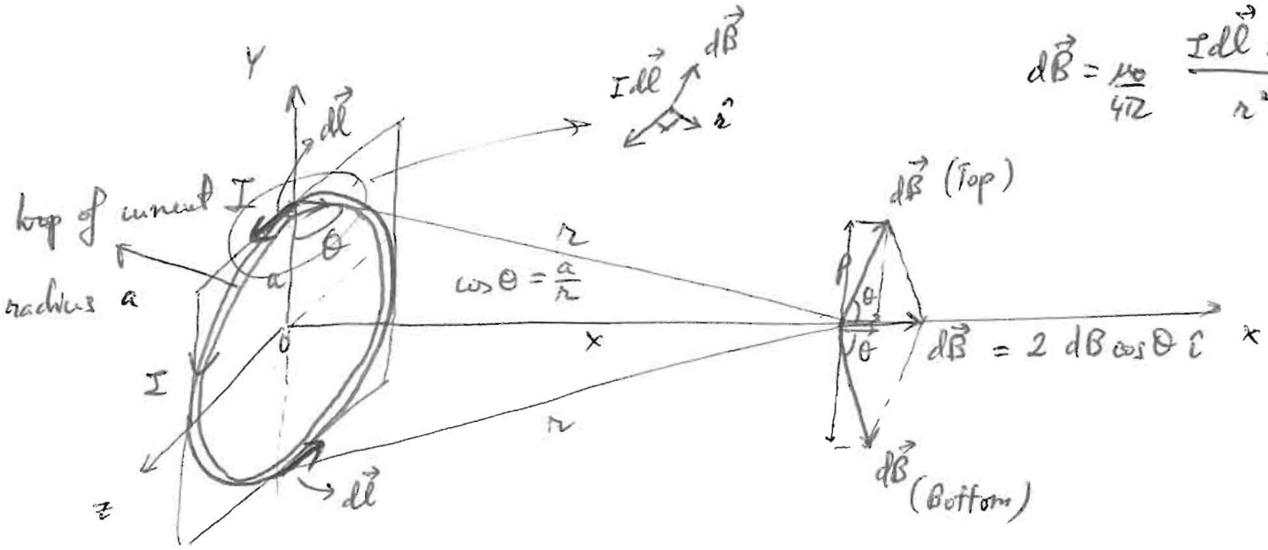
(r = sep. from the line)

line of current



Magnetic field lines are closed! Direction given by RHR: Thumb in direction of I, right hand fingers will turn in direction of the magnetic field.

Calculation of magnetic field due to a loop of current (in the YZ plane) @ a point along the axis of the loop (x-axis) : center of loop @ origin of coordinates



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$\underbrace{d\vec{B}_{Total}}_{Top + Bottom} = 2 dB \cos\theta\, \hat{i} = 2 \frac{\mu_0}{4\pi} \frac{I\, dl}{r^2} \cos\theta\, \hat{i} = \frac{2\mu_0}{4\pi} \frac{I\, dl\, a}{r^3}\, \hat{i}$$

$$\vec{B} = \int_{\substack{Half\\Loop}} d\vec{B}_{Total} = \frac{2\mu_0 I a}{4\pi r^3}\, \hat{i} \overbrace{\boxed{\int_{\substack{Half\\Loop}} dl}}^{\text{Half circumference} = \pi a} = \frac{2\mu_0 I a^2 \pi}{4\pi r^3}\, \hat{i}$$

sep. of all points on loop to P is r

$$\boxed{\vec{B} = \frac{\mu_0 I a^2}{2 r^3}\, \hat{i} = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}\, \hat{i}}$$

Magnetic field created by a circular loop of current I (radius a) @ point P along the x-axis (loop axis)

Observation.

→ Very far away: $x >> a$ $\quad (x^2+a^2)^{3/2} \simeq (x^2)^{3/2} = x^3$

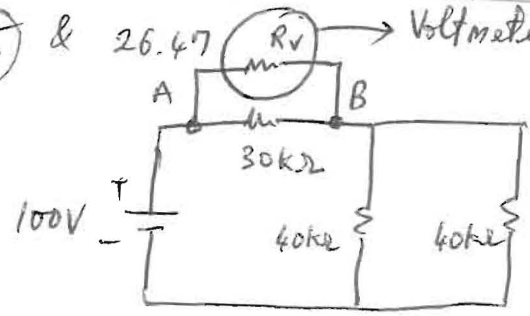
$$B \simeq \frac{\mu_0 I a^2}{2x^3}$$ (inverse-cube!) far from loop.

→ E due to a dipole $x >>$ dipole size: was also an inverse-cube law!

→ Loop of current is magnetic counterpart of the electric dipole!

25.55 & 26.47 R_V → Voltmeter w/ internal resistance R_V

What is the reading of voltage across $30k\Omega$



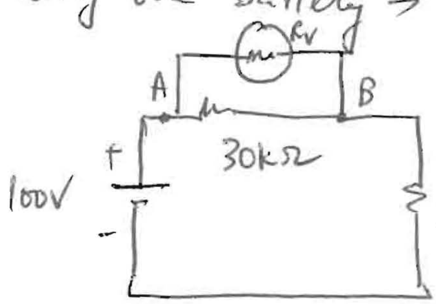
a) If $R_V = 50 k\Omega$

b) $R_V = 250 k\Omega$

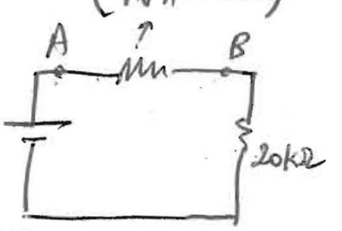
c) $R_V = 10 M\Omega$

→ Find equation giving V_{AB} in term of R_V

→ Only one battery → can use series & parallel combination.



$20k\Omega = (40k\Omega \parallel 40k\Omega)$ =



$$V_{AB} = 100V\left[\frac{R_V \parallel 30k\Omega}{(R_V \parallel 30k\Omega) + 20k\Omega}\right] = 100\,\frac{\frac{R_V 30}{R_V+30}}{\frac{R_V 30}{R_V+30}+20}$$

↓ Voltage division

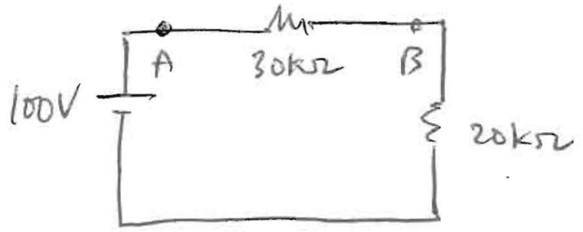
$$V_{AB} = 100 \frac{30R_V}{30R_V + 20R_V + 600} = 100 \frac{30R_V}{50R_V + 600}$$

a) $R_V = 50\ k\Omega \rightarrow V_{AB} = 100 \frac{30 \times 50}{50 \times 50 + 600} = 48.39V$

b) $R_V = 250\ k\Omega \rightarrow V_{AB} = 57.25\ V$

c) $R_V = 10\ M\Omega = 10^4\ k\Omega \rightarrow V_{AB} = \frac{10^4 \times 30}{10^4 \times 50 + 600} = 59.93\ V$

Observation: what is the theoretical value for V_{AB}?
(before we use voltmeter across A & B)



$\rightarrow V_{AB} = 100V \frac{30k\Omega}{30k\Omega + 20k\Omega}$

$= 100 \frac{3}{5} = 60V$

→ Conclusion: larger internal resistance for voltmeter gives better measurement → since it will draw less current from the circuit.

26.47

Data: $v = 185\,m/s$; $q = 1.4\,\mu C$

$$\vec{F}_B = (2.5\hat{i} + 7\hat{j})\ \mu N$$

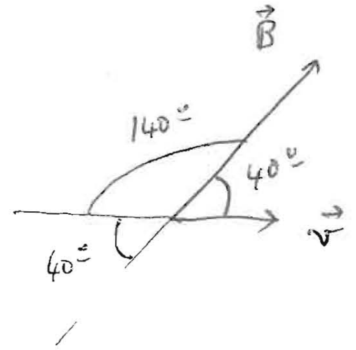
$$\vec{B} = (42\hat{i} - 15\hat{j})\ mT$$

Find θ b/w $\vec{v}$ & $\vec{B}$

$$\vec{F} = q\vec{v} \times \vec{B} \longrightarrow \text{Magnitude}: \left\{ \begin{array}{l} F = qvB\sin\theta \\ \sin\theta = \dfrac{F}{qvB} \\ F = |\vec{F}| = \sqrt{F_x^2 + F_y^2} \\ \quad = \sqrt{2.5^2 + 7^2}\ 10^{-6}\,N \\ B = |\vec{B}| = \sqrt{42^2 + 15^2}\ 10^{-3}\,T \end{array} \right.$$

$$\sin\theta = \frac{\sqrt{2.5^2 + 7^2}\ 10^{-6}}{1.4 \times 10^{-6} \times 185 \times \sqrt{42^2 + 15^2}\ 10^{-3}} = 0.644 \rightarrow \theta = \sin^{-1} 0.644$$

$$\boxed{\theta = 40.1^\circ}$$



Calculation of fields (simple)

Electric	Magnetic
1) → Vector superposition ($\vec{E}$ due to 2 charges, ring of charges, etc.)	1) Vector superposition using Biot-Savart Law (loop of current I, radius a)
2) → Gauss Law $$\underbrace{\oint_{\text{Gaussian surface}} \vec{E}\cdot d\vec{A}}_{\phi \text{ (electric flux)}} = \frac{q_{enclosed}}{\varepsilon_0}$$	2) Ampere Law: $$\oint_{\text{Amperian Loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$
3) → Using the Electric Potential V $$\vec{E} = -\vec{\nabla} V$$ $\vec{\nabla}$ ↓ derivative operator	3) Using the vector potential $\vec{A}$ $$\vec{B} = \underbrace{\vec{\nabla}\times}_{\text{rotational or curl of } \vec{A}} \vec{A}$$ (derivative operator & cross-product)

Calculation of the magnetic field using Ampere's Law:

1) Determine the Amperian Loop (closed loop) taking advantage of the symmetry so:

$$\oint \vec{B} \cdot d\vec{l} = \vec{B} \cdot \oint d\vec{l} = BL$$

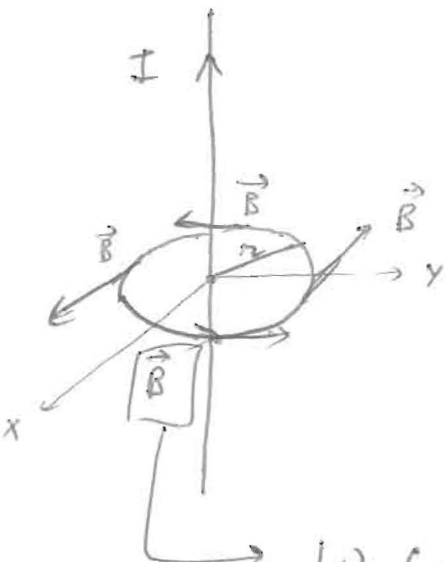
→ $\vec{B}$ constant along the Amperian Loop, and tangential to all points along the Loop. "L" length of loop.

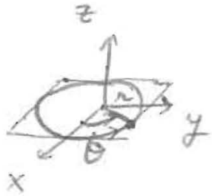
2) Determine the current enclosed by that loop

3) Apply Ampere's Law: $BL = \mu_0 I_{enclosed}$

$$B = \frac{\mu_0 I_{enclosed}}{L}$$

Magnetic field due to a long line of current:





I along z-axis, magnetic field wraps around this current (on xy plane), direction CCW seen from above (given by RHR)

↳ Thumb in direction of current, field wraps around as the right hand fingers)

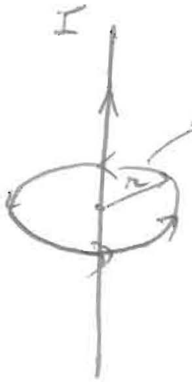
$\vec{B} = B\,\hat{\theta}$

↓

unit vector for angle θ

1) Constant @ all points along a circular loop centered at the current (fixed r)
2) Tangential to all points of this loop.

1) Determine Amperian loop: circles centered @ current.



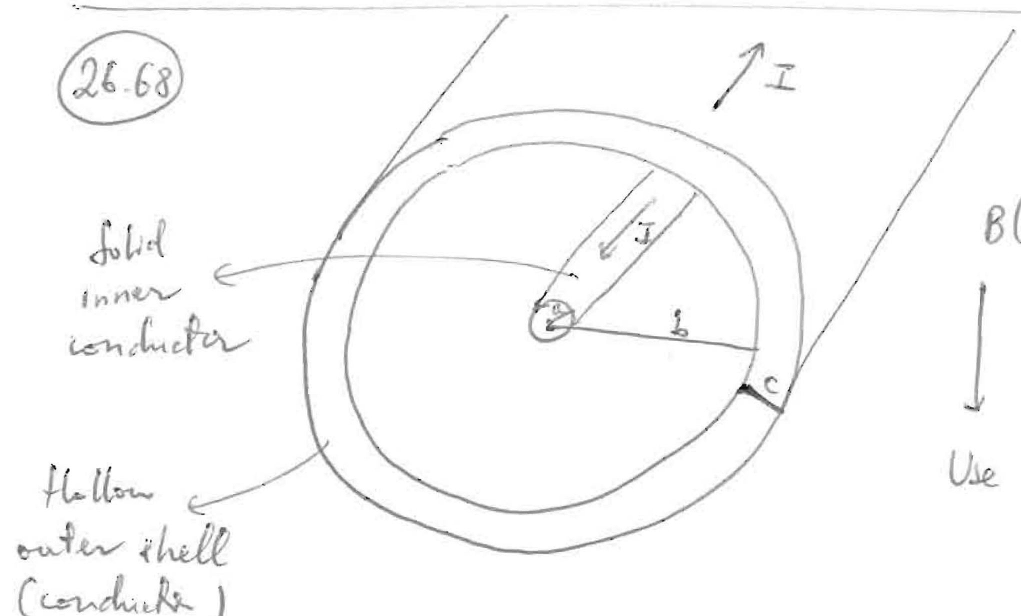
2) $I_{enclosed} = I$

3) Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = BL = \mu_0 I_{enclosed} = \mu_0 I$

$$\rightarrow \boxed{B = \frac{\mu_0 I}{L} = \frac{\mu_0 I}{2\pi r}}$$

$$\Rightarrow \vec{B}(r) = B(r)\,\hat{\theta} = \frac{\mu_0 I}{2\pi r}\,\hat{\theta}$$

26.68



$B(r)$? a) $r < a$; b) $a < r < b$; c) $r > b + c$

$\downarrow$

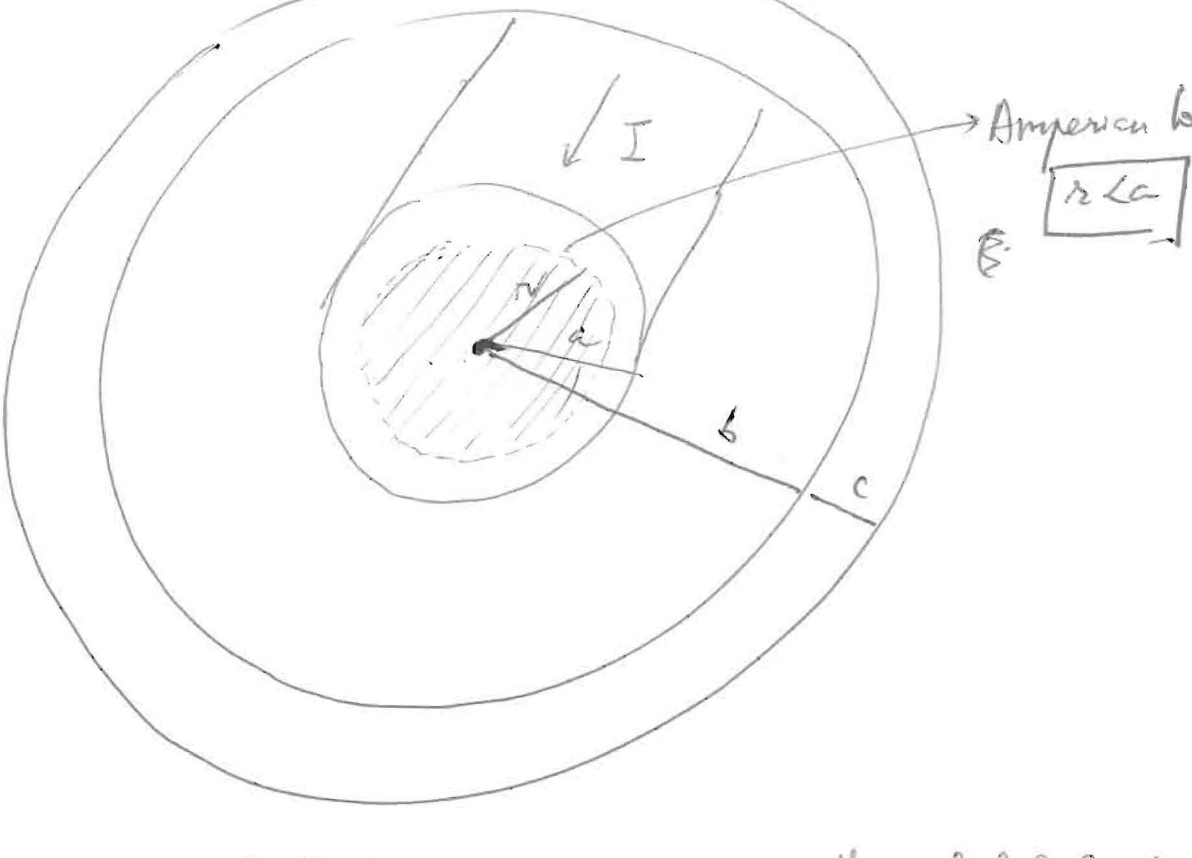
Use Ampere's Law.

Application of Ampere's Law to calculate $B(r)$:

a) 1) Det. the Amperian Loop:

Criteria $\begin{cases} 1) \ \vec{B} \text{ constant along loop} \\ 2) \ \vec{B} \text{ tangential to loop.} \end{cases}$ ∴ since inner conductor resembles a long line of current:

→ Circle of radius r centered @ ~~center~~ axis of conductor.



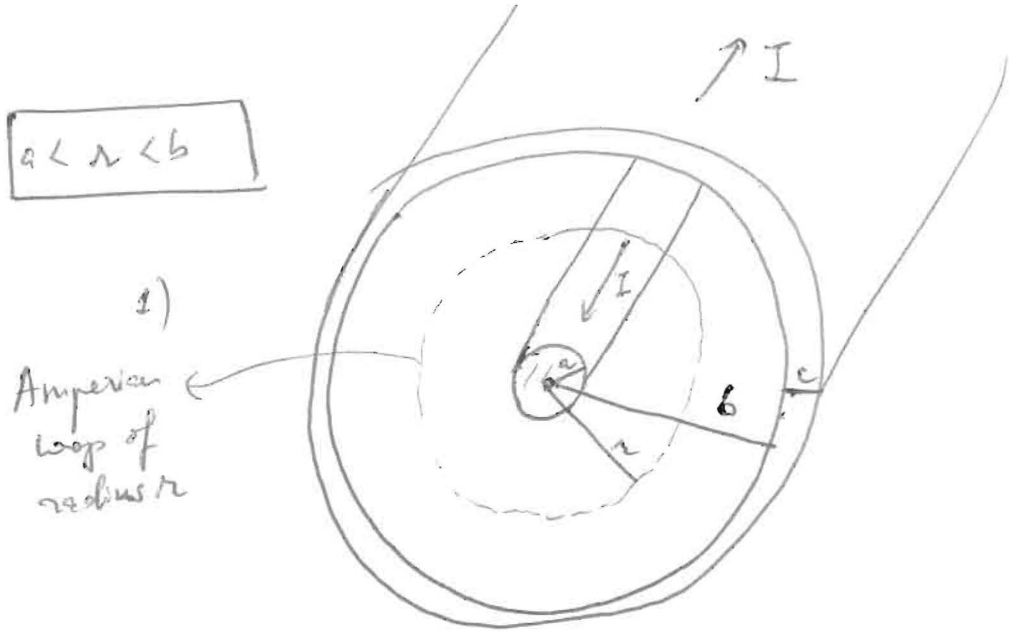
2) Current enclosed by amperian loop: the shaded fraction!

$$I_{enclosed} = I\,\frac{\pi r^2}{\pi a^2}$$

3) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} \longrightarrow$ $B\cdot \overset{2\pi r}{\uparrow}{L} = \mu_0 I \frac{r^2}{a^2}$

$$\boxed{B = \frac{\mu_0 I r^{\cancel{2}}}{2\pi \cancel{r} a^2} = \frac{\mu_0 I}{2\pi a^2}\, r \qquad r < a \text{ : within inner conductor !}}$$

b) $\boxed{a < r < b}$



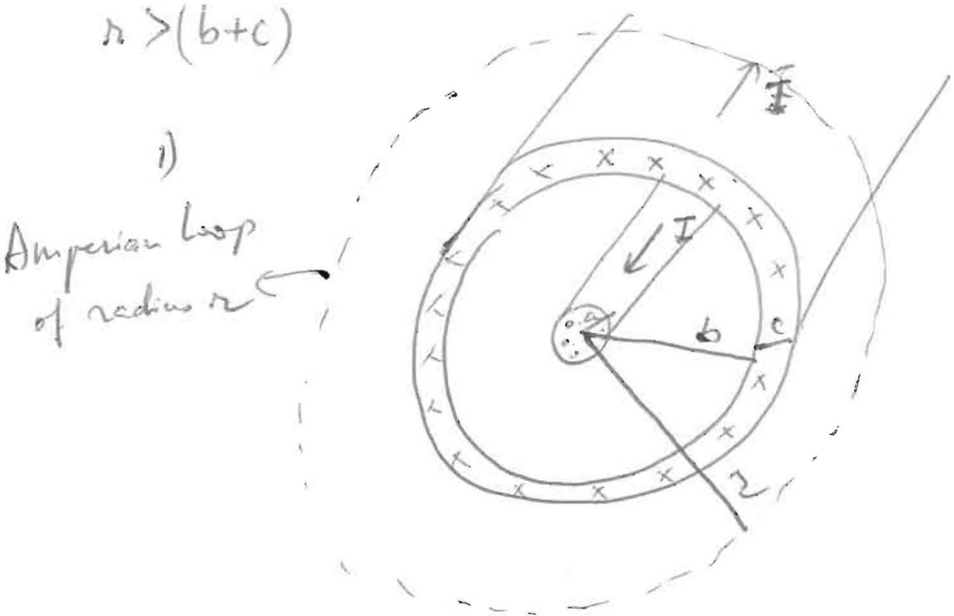
2). $I_{enclosed} = I$

3) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$

$$B\,L = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r} \quad ; \quad a < r < b}$$

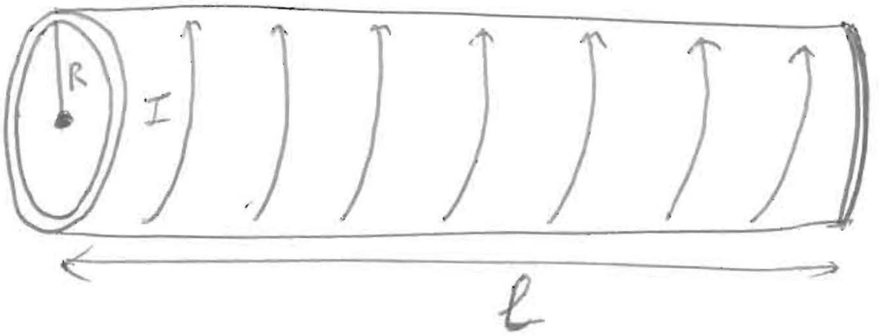
c) $r > (b+c)$



2) $I_{enclosed} = 0$

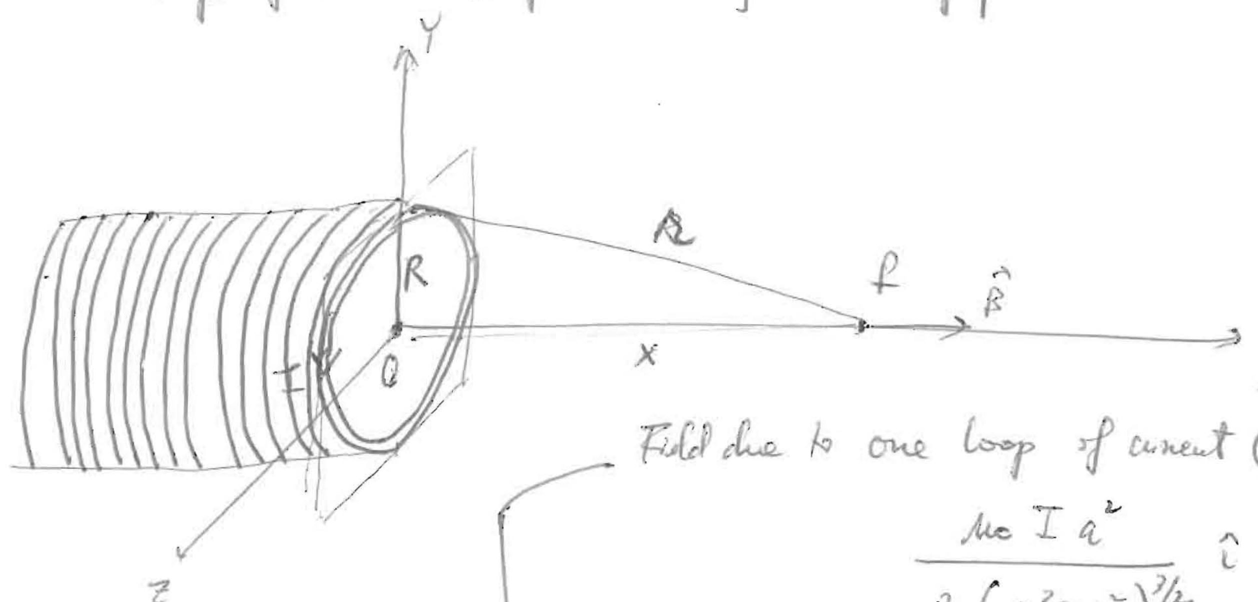
3) $B = 0 \quad r > b+c \quad \rightarrow$ Coaxial cable: confines the magnetic within the cable.

26.76 $\boxed{\text{Long}}$ hollow pipe



$B(r)$?
$r < R$
& $r > R$

Is this related to anything we have done? → Stacking many loops of current to form a long hollow pipe



Field due to one loop of current @ P:

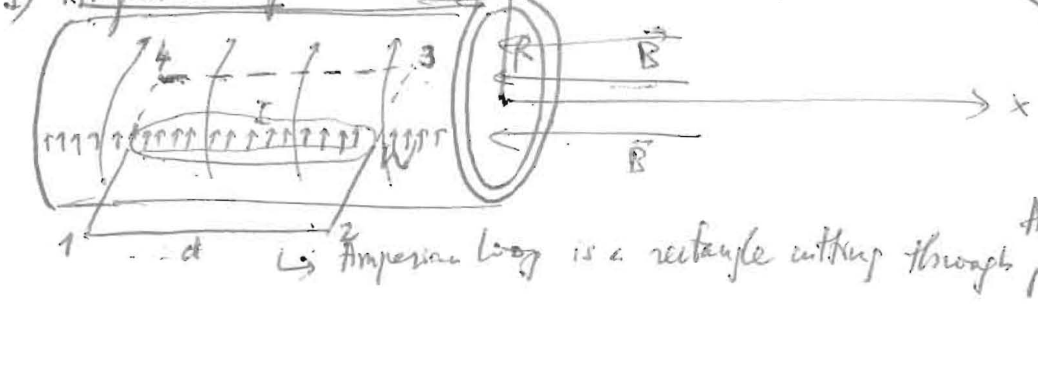
$$\frac{\mu_0 I a^2}{2\left(x^2+a^2\right)^{3/2}}\,\hat{\imath}$$

P @ origin = $x = 0$; $r = R$; $a = R$

$$\frac{\mu_0 I R^2}{2\left(R^2\right)^{3/2}}\,\hat{\imath} = \frac{\mu_0 I}{2R}\,\hat{\imath}$$

Find B using Ampere's law for the long hollow pipe:

1) Amperian loop:



$\vec{B}$ parallel to x (−x direction)

↓

$\vec{B}$ tangential to the Amperian loop.

↳ Amperian loop is a rectangle cutting through pipe.

$$\oint \vec{B}\cdot d\vec{l} = \underbrace{B\,d}_{\text{side } 34} + \underbrace{0\cdot d}_{(\text{side } 12)}$$

scalar product: $\begin{cases} \text{if } \vec{B} \perp d\vec{l} \rightarrow \text{zero contribution (sides 14 \& 23)} \\ \text{if } \vec{B} \parallel d\vec{l} \rightarrow \text{max. contribution } (\cos 0 = 1) \text{ (sides 12 \& 34)} \end{cases}$

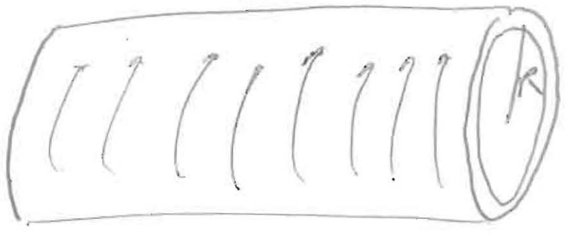
2) <u>Current enclosed by this loop</u>: $I_{enclosed} = I \dfrac{d}{\ell}$

3) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

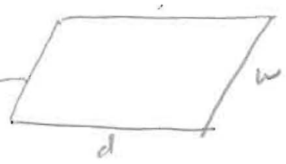
$$B \cancel{d} = \mu_0 I \frac{\cancel{d}}{\ell} \rightarrow \boxed{B = \mu_0 \frac{I}{\ell}}$$

inside the hollow pipe.

$B_{outside}$:

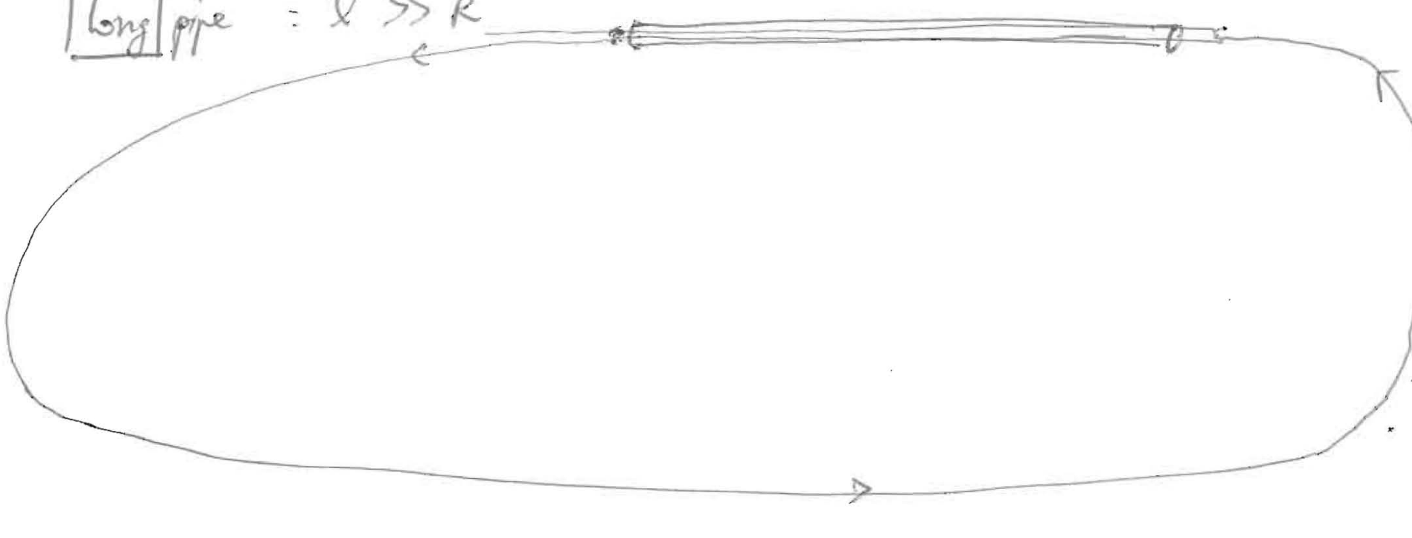


Amperian loop ←



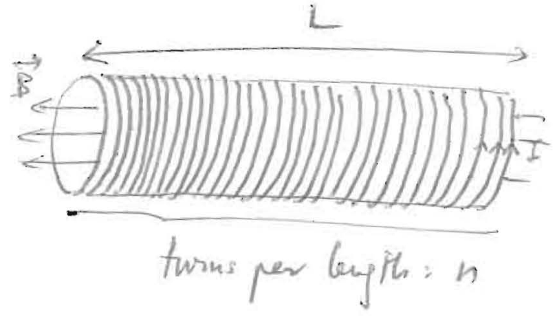
doesn't enclose any current → $B = 0$

$\boxed{\text{long}}$ pipe : $\ell \gg R$



26.44 Superconducting solenoid (highest B achieved so far)

$n = \frac{N}{L}$: # of turns per unit length ; $\begin{cases} n = 3300 \ \frac{\text{turns}}{m} \\ I = 4100\,A \end{cases}$

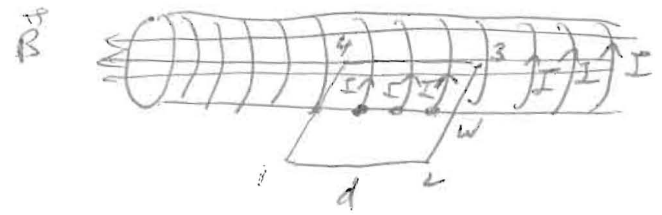


a long superconducting wire is wrapped around a cylinder many turns ; a current I goes through each turn.

Relating to $\vec{B}$ by one loop of current → $\vec{B}$ parallel to axis of solenoid

(→ long line of current $\vec{B}$ wraps around current
→ wrapping current $\vec{B}$ straight line, along axis)

1) Amperian loop → rectangle cutting through body of solenoid:



$$\oint \vec{B}\cdot d\vec{l} = \underbrace{Bd}_{\text{side } 34} + \underbrace{0\cdot d}_{\text{side } 12} + \underbrace{\underbrace{0}_{\text{side } 14} + \underbrace{0}_{\text{side } 23}}_{\vec{B} \perp d\vec{l}}$$

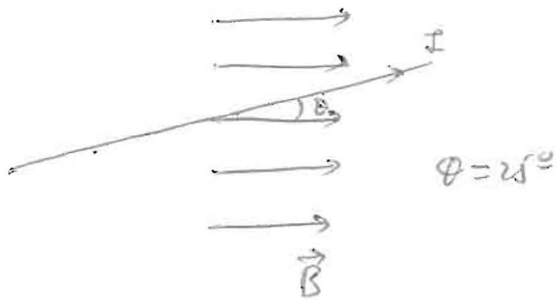
2) Current enclosed: $I_{enclosed} = \underbrace{nd}_{\text{total \# turns enclosed by Amperian loop}} I$

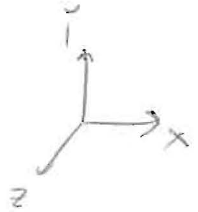
3) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

$$B\cancel{d} = \mu_0 n \cancel{d} I \rightarrow \boxed{B = \mu_0 n I}$$

$$B = 4\pi\times10^{-7} \times 3300 \times 4100 = 17\,T$$

28.29 Wire w/ $I = 15\,A$ @ $25°$ with a uniform magnetic field. $\frac{F_m}{L} = 0.31 \frac{N}{m}$ B?





a) → Current consists of moving charges: → B will apply a force on each charge → force on current or wire.

$$\boxed{\vec{F} = q\vec{v}\times\vec{B} = q\underbrace{\frac{\vec{l}}{t}}_{\vec{v}}\times\vec{B} = \underbrace{\frac{q}{t}}_{I}\,\vec{l}\times\vec{B} = I\vec{l}\times\vec{B}}$$

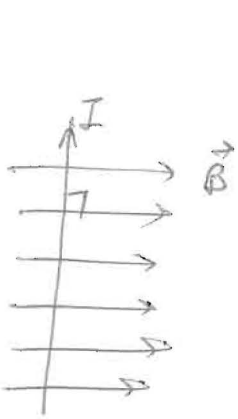
$F = I l B \sin\theta$ (θ angle b/w wire & field)

$\frac{F}{l} = I B \sin\theta \rightarrow B = \frac{F/l}{I \sin\theta} = \frac{0.31}{15 \times \sin 25°}$

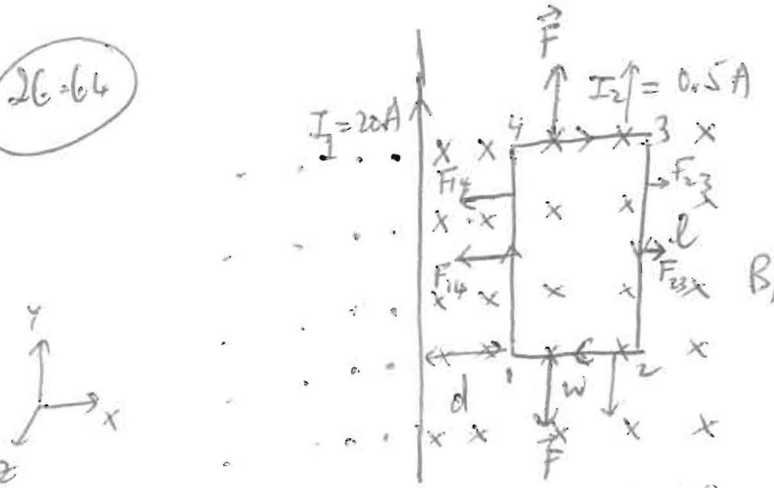
$B = 48.9\ mT$

b) Max $\frac{F}{l}$ when $\sin\theta = 1$ or $\theta = 90°$

$\frac{F}{l} = IB = 15 \times 48.9\times10^{-3} = 0.734 \frac{N}{m}$



26-64



d = 2 cm
w = 5 cm
l = 10 cm.

Net F_m on loop?

Current I_1 creates field B_1 (wrapping around it) → magnetic force by B_1 on the current I_2 or loop: 4 sides:

a) 1 2 & 3 4 : opposite forces at same separation to wire → No net force on 12 & 34.

$\vec{F} = I_2 \vec{l} \times \vec{B}_1$

b) $\begin{cases} \vec{F}_{14} = I_2 l B_1(d)(-\hat{\imath}) \\ \vec{F}_{23} = I_2 l B_1(d+w)(\hat{\imath}) \end{cases}$ Net force $= \vec{F}_{14} + \vec{F}_{23}$

$\vec{F}_{net} = I_2 l \hat{\imath} \underbrace{\left(B_1(d+w) - B_1(d) \right)}_{<0}$

attractive force.

$B_1 = \frac{\mu_0 I_1}{2\pi r}$ (r: sep to I_1)

↓
magn. field by a long line of current

$\vec{F}_{net} = I_2 l \hat{\imath} \frac{\mu_0 I_1}{2\pi} \left(\frac{1}{d+w} - \frac{1}{d} \right) = -7.14 \times 10^{-6} N \hat{\imath}$

Ch 27 Electromagnetic Induction

Faraday's Law: $\boxed{\mathcal{E} = -\dfrac{d\Phi_B}{dt}}$

$B(t)$ { B const but direction changes

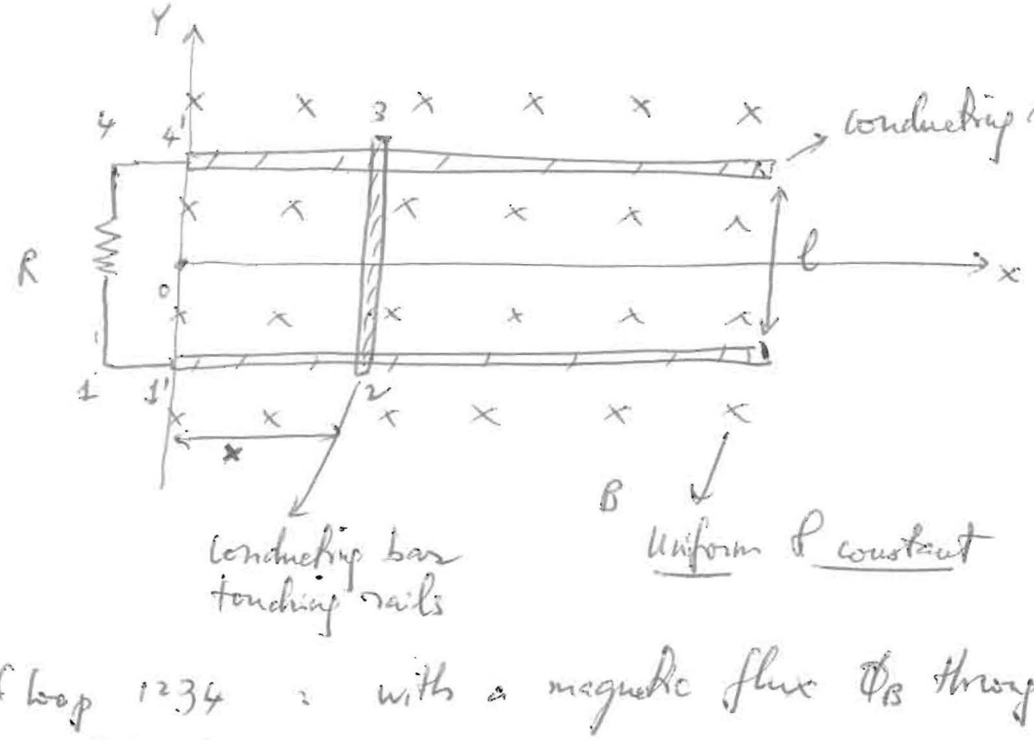
$\vec{B}$ changes with time

A changes with time

$\Phi_B = \int \vec{B} \cdot d\vec{A}$ = magnetic flux → change

$\mathcal{E}$ = induced e.m.f or induced voltage.

27.47



Closed loop 1234 : with a magnetic flux Φ_B through

$\Phi_B = \oint \vec{B} \cdot d\vec{A} = BA = B(x\ell)$

↳ area of loop with magnetic field → 1'234'

B & ℓ are constant, but if we move the conducting bar 23 or changing x → then $\dfrac{d\Phi_B}{dt} \neq 0$ → there is an induced $\mathcal{E}$ in the loop (acts like a battery) → a current $I = \dfrac{\mathcal{E}}{R}$ will show up in the loop.

a) Direction of current in resistor?

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

the induced $\mathcal{E}$ will oppose the change in Φ_B

1) $\Phi_B \uparrow \rightarrow \mathcal{E}$ will be such that it reduces Φ_B

2) $\Phi_B \downarrow \rightarrow \mathcal{E}$ will be such that it increase Φ_B

If conducting bar 23 goes right $\rightarrow \Phi_B \uparrow \rightarrow \mathcal{E}$ will tend to reduce Φ_B by creating a current in the loop that produces a induced magnetic field out of page to reduce the original field and so to reduce the Φ_B despite an increase in A due to the conducting bar moving to the right.

$\rightarrow$ I induced will go $4 \rightarrow 1$ across the resistor (downward)

b) What power (work per unit time) is need to pull the bar 23?

$$\underbrace{P = \underbrace{I}_{\text{induced current}} . V = I^2 R}_{\text{Ohm's Law}} = \left(\frac{\mathcal{E}}{R}\right)^2 R = \frac{\mathcal{E}^2}{R} = \frac{\left(\frac{d\Phi_B}{dt}\right)^2}{R}$$

$$\Phi_B = B x l \rightarrow \frac{d\Phi_B}{dt} = Bl\frac{dx}{dt} = Bl\underbrace{v}_{\text{speed of bar 23}}$$

$$\boxed{P = \frac{(Blv)^2}{R}}$$