Ground (zero potential)

**Loop analysis (2 loops)**

$\rightarrow I_1 \& I_2$ in clockwise (CW)

**Loop 1**

$V_1 - I_1R_1 - V_2 - I_2R_3 = 0$

**Loop 2**

$V_2 - I_3R_2 = 0$

Solve for $I_1$ and $I_2$:

$I_2 = \frac{V_2}{R_2}$

$V_1 - V_2 = I_1(R_1 + R_3)$

$I_1 = \frac{V_1 - V_2}{R_1 + R_3}$

**Node analysis (2 nodes)**

Node a)

$I_1 - I_2' - I_2 = 0 \quad \text{only one electric node}$

Node b)

$I_1 + I_2' + I_2 = 0$

$I_i = \frac{V_i - V_a}{R_i}$ (Ohm's law)

$I_1 = \frac{V_a - V_b}{R_1}$

$V_i - V_a - I_2' - \frac{V_a - V_b}{R_2} = 0$

$V_a = V_a + I_1R_3 \quad (a \rightarrow b \rightarrow 0)$

$V_a = -I_1R_1 + V_1 \quad (a \rightarrow 1 \rightarrow 0)$

$V_2 + I_1R_3 = -I_1R_1 + V_1$

$I_1(R_1 + R_3) = V_1 - V_2 \Rightarrow I_1 = \frac{V_1 - V_2}{R_1 + R_3}$

$\frac{V_1 - V_a}{R_1} - I_2' - \frac{V_2}{R_2} = 0$

$I_1 - I_2' = I_2 = \frac{V_1 - V_a}{R_1} - \frac{V_a - V_b}{R_2} = \frac{V_2}{R_1} - I_1 \frac{R_3}{R_1} - \frac{V_a}{R_2}$

$I_1(1 + \frac{R_3}{R_1}) - I_2 = \frac{V_1}{R_1} - V_2(\frac{1}{R_1} + \frac{1}{R_2})$


\[ I_1(1 + \frac{R_3}{R_1}) = \frac{V_1}{R_1} + \frac{V_2}{R_2} - \frac{V_2}{R_1} - \frac{V_3}{R_2} \]

\[ I_1 = \frac{V_1 - V_2}{R_1} \frac{1}{1 + \frac{R_3}{R_1}} \]

\[ = \frac{V_1 - V_2}{R_1} \frac{1}{\frac{R_1 + R_3}{R_1}} \]

\[ I_1 = \frac{V_1 - V_2}{R_1 + R_3} \]

\[ \varepsilon_1 = 6V; \quad \varepsilon_2 = 1.5V; \quad \varepsilon_3 = 4.5V \]

\[ R_1 = 270.2\Omega; \quad R_2 = 150.2\Omega; \quad R_3 = 560.2\Omega; \quad R_4 = 820.2\Omega \]

Find current in \( R_3 \) including direction (up or down)

Will solve using 2 alternatives: Loop & Node analysis

**Loop Analysis:**

2 loops \( I_1 \) & \( I_2 \) in CW \( \Rightarrow \) current through \( R_3 \) is \((I_1 - I_2)\)

\[ \text{loop 1)} \quad + \varepsilon_1 - I_1R_1 - (I_1 - I_2)R_3 - I_1R_2 - \varepsilon_2 = 0 \]

\[ \text{loop 2)} \quad + \varepsilon_3 - (I_2 - I_1)R_3 - I_2R_4 - \varepsilon_3 \quad = 0 \]

\[ \varepsilon_1 - \varepsilon_3 - I_1(R_1 + R_2) - I_2R_4 \quad = 0 \]

\[ I_1 = \frac{\varepsilon_1 - \varepsilon_3 - I_2R_4}{R_1 + R_2} \]

\[ I_1 = \frac{1.5 - 820I_2}{420} \]

\[ \text{plug this into eq 2)} \]

\[ \varepsilon_2 - \varepsilon_3 - I_2(R_3 + R_4) + I_1R_3 \quad = 0 \]

\[ -3 - 1380I_2 + 560I_1 \quad = 0 \]

\[ \Rightarrow -3 - 1380I_2 + \frac{560}{420}(1.5 - 820I_2) \quad = 0 \]
\[ -1 - 24.73 \times 2I_2 = 0 \]

\[ I_2 = -0.4 \times 10^{-3} \text{ mA} \]

\[ I_1 = \frac{1.5 - 820 \times (-0.4 \times 10^{-3})}{420} = +4.36 \text{ mA} \]

Current thru R3: \[ I_1 - I_2 \] (not downward current, from our assumption of directions for \( I_1 \) & \( I_2 \) = both (a))

\[ I_1 - I_2 = 4.36 \text{ mA} - (-0.4 \text{ mA}) = +4.76 \text{ mA} \] (downward current through R3)

Node Analysis

1. Define zero potential
2. Identify node: only one left
3. Assume directions for \( I_1, I_2, I_3 \) by drawing on circuit
4. Node equation: \[ I_1 - I_2 - I_3 = 0 \]
5. Write node equations using voltage:

\[ I_1 = \frac{V_A - V}{R_1} \]
\[ \text{Ohm's law} \quad V_A = E_1 - I_1R_2 \]
\[ I_2 = \frac{V - E_2}{R_4} \]
\[ I_3 = \frac{V - E_2}{R_3} \]

\[ I_1 = \frac{E_1 - V}{R_1 + R_2} \]
Back into node equation:

\[
\frac{\varepsilon_1 - V}{R_1 + R_2} - \frac{V - \varepsilon_3}{R_u} - \frac{V - \varepsilon_2}{R_3} = 0
\]

\[
\frac{6 - V}{420} - \frac{V - 4.5}{820} - \frac{V - 1.5}{560} = 0
\]

\[V \approx 4.17\text{V}\]

Current through \(R_3\):

\[I_3 = \frac{V - \varepsilon_2}{R_3} = \frac{4.17 - 1.5}{560} = +4.76\text{mA}\]

\[\downarrow\]

\[\text{downwards, \(\varepsilon\)}\]

\[\text{at } R_3\]

Circuits involving resistors & capacitors:

(a) At time \(t=0\) we connect the uncharged capacitor \(C\) to a circuit with \(\varepsilon\) & \(R\)

\[\text{Initial charge } Q = 0\]
\[\text{Initial potential } V_c = 0\]

The capacitor behaves effectively like a wire.

(b) At \(t=0\) we "stop" charge:

\[V_1 = V_2\]

\[I = \frac{V_c}{R} \quad \text{(Ohm's Law)}\]

(b) At time \(t>0\) some + charges are transferred from right plate to left plate via the circuit \(\Rightarrow\) capacitor is being charged.
@ t > 0

Actually negative charges are transferred in the opposite directions!!

\[
\begin{align*}
\mathcal{E} - IR - V_c &= 0 \\
(\text{loop equation for RC circuit}) \quad & \quad V_c' = \frac{\mathcal{E} - V_c}{R}
\end{align*}
\]

* I' over time: getting smaller
  - Smaller: \( I \to \max \)
  - I' (decaying)
  - \( I' = 0 \) (capacitor is fully charged!)
  (also it was harder to transfer later charges since we need to go against stronger fields!)

* V_c over time: getting larger & larger.

\[ \begin{align*}
@ \text{time } t = \infty & \quad \text{(very long time after circuit is connected)} \\
\begin{cases}
I' = 0 \quad \to \quad \text{Capacitor effectively behaves like an open circuit!} \\
V_c = \max .
\end{cases}
\end{align*} \]
Quantitative description: RC circuit

\[ I'(t) \]
\[ \frac{d}{dt} \left[ \mathcal{E} - I' R - V_c = 0 \right] \rightarrow -R \frac{dI'}{dt} - \frac{1}{C} \frac{dV_c}{dt} = 0 \]
\[ \frac{dV_c}{dt} = 0 \]
\[ V_c = \frac{\mathcal{O}(t)}{C} \text{ (C is constant)} \]

\[ C = \text{capacitance, constant.} \]

\[ -R \frac{dI'}{dt} - \frac{1}{C} I' = 0. \rightarrow \frac{dI'}{dt} = -\frac{1}{RC} I' \rightarrow \frac{dI'}{I'} = -\frac{1}{RC} \frac{dt}{I'} \]
\[ \ln I' = -\frac{R}{RC} t + \text{const.} \]
\[ e^{-\frac{1}{RC} t} = e^{\text{const.}} e^{-\frac{1}{RC} t} \]
\[ I' = I'(t=0) e^{-\frac{t}{RC}} \rightarrow I'(t) = \frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}} \]

Qualitative \[ \frac{\mathcal{E}}{R} \]

Diagram: \[ t = RC \text{ (time constant: } RC \text{ is time unit: s)} \]
RC circuit:

@ t=0  \quad I = \frac{\varepsilon}{R}; \quad V_c = 0 \rightarrow \text{Capacitor is in short circuit.}

@ t> \infty  \quad I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}

@ t=\infty  \quad I = 0 \rightarrow \text{Capacitor is open circuit.}

25.64

\begin{center}
\begin{tikzpicture}
\end{tikzpicture}
\end{center}

\text{Data:}
- \text{Switch initially open.}
- \text{C}_1 \text{ and } \text{C}_2 \text{ unchanged.}
- \text{R}_1 = \text{R}_2 = \text{R}_3 = \text{R}

\text{Find:}
\begin{enumerate}
\item @ t=0 (just after the switch is closed)
\item @ t=\infty (long after "...")
\item Current in \text{R}_3 \text{ quantitatively}
\end{enumerate}

\text{a)} \quad @ t=0 \rightarrow \text{Capacitors behave in short circuit.}

\text{Node B: current division: current in one branch is proportional to the resistances in the other branch.}

\text{Current always prefers the least resistance path.}

\text{Current through } \text{R}_2 \text{ at } t=0 \text{ is 0.}
b) \( t = \infty \) \rightarrow \text{Capacitors behave as an open circuit}

\[ R_1 \quad I = 0 \]

Current through \( R_2 \) is \( I = \frac{\varepsilon_1}{R_1 + R_2} \)

Ohm's Law.

c) \text{Current in } R_3 \text{ qualitatively:}
\begin{align*}
@ t = 0 & \rightarrow I_3 = 0 \quad \text{but it is not } 0 \text{ all the time:} \\
@ t = \infty & \rightarrow I_3 = 0
\end{align*}

\text{Sharp charge, not happening in linear circuits.}
Ch 26 Magnetic Field

Electric  Magnetic  Electromagnetic

- Two types of charges: +, -
- Equal charges repel
  Opposite charges attract
- Electric field lines are open in general

- Two types of magnetic poles: N, S
- They are always attached
  (Magnetic monopole have not been found yet)
- Equal poles repel
  Opposite poles attract
- Magnetic field lines are always closed (N & S are attached together)
Effects of the magnetic field:

On a moving charge (of value $q$) @ velocity $\mathbf{v}$ on the page plane of this page, a uniform magnetic field $\mathbf{B}$ pointing out the page (perpendicular to the plane of charge motion) would bend the trajectory of that charge into a circle.

![Diagram of a charged particle moving in a magnetic field](image.png)

Applications:
1. Particle confinement or trap
2. Cyclotron - synchronization (medical applications, particle physics research)

Closer look:

1. If charge $q$ instead moves in/out of page
   $\Rightarrow$ This $\mathbf{B}$ has no effect on that particle!  

2. When charge $q$ is moving on the page
   $\Rightarrow$ This $\mathbf{B}$ has max effect on the charge.

3. Intermediate effect of $\mathbf{v}$ forms another angle $\theta$ with $\mathbf{B}$, $0 < \theta < 90^\circ$

$$\mathbf{F}_{\text{magnetic}} = q \mathbf{v} \times \mathbf{B}$$

Note: Vector cross-product.

$\frac{1}{m} \mathbf{v} = \mathbf{F} \times \mathbf{B}$
* Vector cross product $\vec{v} \times \vec{B}$ is another vector that is perpendicular to both $\vec{v}$ & $\vec{B}$, its direction is given by the Right Hand Rule (RHR): when you turn your right hand fingers from the 1st vector ($\vec{v}$) to the 2nd vector ($\vec{B}$), your thumb indicates direction of $\vec{v} \times \vec{B}$. Magnitude is $vB \sin \theta$ ($\theta$ is the angle b/w $\vec{v}$ & $\vec{B}$)

* Interpretation: magnitude of cross product is $vB \sin \theta = \frac{v_B \sin \theta}{v}$

$$\begin{align*}
\vec{v} \times \vec{B} &= \vec{v}_\perp \quad \text{the perpendicular component of } \vec{v} \text{ to the magnetic field } \vec{B} \\
F_{\text{magnetic}} &= q (v_B \sin \theta) = q v_\perp B \quad \text{only the perpendicular component of the velocity with respect to the magnetic field contributes to the magnetic force! This agrees with:}
\end{align*}$$

1) Particle moving parallel to field $\vec{B} \rightarrow v_\parallel = 0$  
   $\rightarrow F_{\text{magnetic}} = 0 \rightarrow$ Field has no effect.

2) Particle moving perpendicular to field $\vec{B} \rightarrow v_\perp = v$  
   $\rightarrow F_{\text{magnetic}} = \text{max} \rightarrow$ Field has max effect on charge.
1. A charge \( q \) moving in \( +x \) direction:
\[ \vec{v} = v \hat{i} \rightarrow \vec{F} = q \vec{v} \times \vec{B} \text{ points downward: } \vec{F} = qvB \hat{j} \quad (\sin 90^\circ = 1) \]

2. \( \vec{F} = q \vec{v} \times \vec{B} = qvB \hat{i} \)

3. \( \vec{F} = q \vec{v} \times \vec{B} = qvB \hat{j} \)

4. \( \vec{F} = q \vec{v} \times \vec{B} = qvB \hat{i} \)

Consequence:
- \( +q \) & \( B \text{ out of page} \rightarrow \text{ CW} \)
- \( -q \) & \( \text{ } \rightarrow \text{ CCW} \)
- \( +q \) & \( \vec{B} \text{ in/into page} \rightarrow \text{ CCW} \)
- \( -q \) & \( \vec{B} \text{ } \rightarrow \text{ CW} \)
* Charge in a magnetic field (uniform) → follows uniform circular motion (UCM): speed along circular trajectory is constant, but velocity is changing direction due to a radial acceleration given by the magnetic force (\( F = ma \) or 2nd Newton's Law)

Net force on \( q \) is \( F = qv_B \) (\( \sin \theta = \frac{v}{v_B} \frac{r}{r} \approx 1 \)) along the radial direction → \( F = ma = m \frac{v^2}{r} \) → UCM

\[ v_B = \frac{m v^2}{qB} \rightarrow r = \frac{mv^2}{qB} \]

Observation:

- Particle confinement (smaller orbits or small \( r \))
  - Large \( B \) (current limitation in fusion energy)
  - \( B_{\text{max}} = 10T \) → need much larger \( B \)

- Orbital period: time to complete one turn
  \[
  T = \frac{2\pi m}{v} = \frac{2\pi m}{qB} = \frac{2\pi m}{qB}
  \]
Node analysis:

1) Set zero potential
2) Select the node: voltage V
3) Assume directions for currents Q node: \( I_1, I_2, I_3 \) (in) (out) (out)
4) Write node equation: \( I_1 - I_2 - I_3 = 0 \)
5) Write currents in terms of the voltage: \( \varepsilon_1, \varepsilon_2, \varepsilon_3 + V \)

\[
I_1 = \frac{V_1 - V}{R} = \frac{(\varepsilon_1 - I_1R) - V}{R} \quad \rightarrow \quad I_1R = \varepsilon_1 - I_1R - V
\]

\[
I_2 = \frac{V - \varepsilon_2}{R} \quad \rightarrow \quad I_2R = \varepsilon_2 - I_2R
\]

\[
I_3 = \frac{V - \varepsilon_3}{R} \quad \rightarrow \quad I_3R = \varepsilon_3 - I_3R
\]

Back into the node equation:

\[
\varepsilon_1 - V - 2V + 2\varepsilon_2 - V + \varepsilon_3 = 0
\]

\[
\varepsilon_1 + 2\varepsilon_2 + \varepsilon_3 - 4V = 0 \quad \rightarrow \quad V = \frac{\varepsilon_1 + 2\varepsilon_2 + \varepsilon_3}{4}
\]

\[
V = \frac{75 + 2 	imes 45 + 20}{4} \text{mV} = \frac{185}{4} \text{mV} = 46.25 \text{mV}
\]

\[
I_3 = \frac{46.25 - 20}{2 \times 1.5 \times 10^6} \text{mA} = 8.75 \text{mA}
\]
5) \[ I_1 = \frac{V - V_A}{R} = \frac{V - \varepsilon_1 - I_1R}{R} \]

\[ V_A = \varepsilon_1 + I_1R \]

Across a battery:
\[ \varepsilon + \frac{1}{I} \rightarrow \text{Gain energy} \]
\[ \varepsilon - \frac{1}{I} \rightarrow \text{Lose energy} \]

Across resistance:
\[ \frac{1}{R} \frac{1}{I} \rightarrow \text{Lose energy} \]
\[ R \frac{1}{I} \rightarrow \text{Lose energy} \]

Always, a potential drop across a resistance.

Another perfectly valid assumptions for currents at node V.

(Actual directions will be given by the signs of these currents after we solve the equations.)
Applications of effect of magnetic field on a moving charge:

1) Cyclotron: (modern version is synchrotron)

Goal is to accelerate charged particle to very high speed using the magnetic & the electric fields

Applications:
- study of subatomic structure (CERN)
- high energy range: medical applications

2) D-shaped chambers filled with a uniform magnetic field out of page

- Alternating electric field \( \vec{E} \) in the gap between chambers: flipping directions left and right

- Charge +q is pushed in @ A, follows a circular trajectory

- C charge has higher speed due to the \( \vec{E} \) in the gap: \( r = \frac{mv}{qB} \)

- Larger circular orbit, higher v, larger orbit until it exits cycle.

Max K.E.? What feature of \( \vec{E} \) the cyclotron most affects Max K.E.?

\[ v^2 = \frac{1}{2} (\frac{qBR}{m})^2 = \frac{q^2B^2R^2}{2m} \]

Max radius of an orbit is also the radius of the cyclotron!
Velocity selector:

Goal: to pick out among a bunch of ions of different velocities, those with a desired velocity by using a combination of magnetic & electric fields.

\[ \vec{E} = E \hat{k}, \quad \vec{B} = B \hat{j} \]

This machine selects \( v = v \hat{E} \)

Why does this arrangement of \( \vec{E} \) & \( \vec{B} \) select \( v = v \hat{E} \)?

An ion going through will feel two forces:

\[ \vec{F}_E = +q \vec{E} = qE \hat{k} \quad \text{(along } +z \text{ direction)} \]

\[ \vec{F}_B = +q \vec{v} \times \vec{B} = qvB \hat{i} \times (-\hat{j}) = -qvB \hat{e}_x \]

Net force on ion is \( \vec{F}_E + \vec{F}_B = (qE - qvB) \hat{k} \),

if \( v = \frac{E}{B} \) \quad \rightarrow \quad 0 \quad \text{or net force is zero!} \]

Those ions with \( v = \frac{E}{B} \) the machine has no effect! \( \rightarrow \) they would pass through! Ions with \( v = v\hat{E} \) where \( v \neq \frac{E}{B} \) get pushed off along \( +z \). Ions with \( +y \) or \( +z \) components of velocity will also get pushed off.
Calculate the magnetic field from currents:
(source of the magnetic fields are currents!)

Electric field: (source are charges):
\[ dE = k \frac{dq}{r^2} \]
inverse-square law
\[ k = 9 \times 10^9 \text{ (SI)} \]
Coulomb's law

Magnetic field: (source are current)
\[ dB = \frac{\mu_0}{4\pi} \frac{Il \times \hat{n}}{r^2} \]
inverse-square law
Biot-Savart's law

\[ \mu_0 = \text{permability in vacuum} \]
\[ 4\pi \times 10^{-7} \frac{N}{A^2} \]

Magnetic field due to a line of current (along a straight wire) is
wrapping around the current:

\[ E = \frac{2k\lambda}{r} \]
(\( r \): sep. from the line)

\[ \text{line of charge} \]
\[ \text{line of current} \]

Magnetic field lines are closed!
Direction given by \( \text{RHRL} \):
Thumb in direction of \( I \),
right hand fingers will turn in direction of the magnetic field.
Calculation of magnetic field due to a loop of current $I$ in the $yz$ plane.

At a point along the axis of the loop ($x$-axis) at the origin of coordinates,

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{\vec{I} \times \hat{z}}{r^2}$$

where $r = \frac{\alpha}{\pi}$.

The magnetic field $\vec{B}$ at a point on the loop is given by

$$\vec{B} = \frac{\mu_0 I a^2}{2 \pi r^3} \hat{z} = \frac{\mu_0 I a^2}{2 \left(x^2 + a^2\right)^{3/2}} \hat{z}$$

for a circular loop of current $I$ (radius $a$) at a point along the $x$-axis (loop axis).
Observations:

Very far away: $x \gg a$

$$\left(x^2 + a^2\right)^{\frac{3}{2}} \approx (x^2)^{\frac{3}{2}} = x^3$$

$$\beta \approx \frac{\mu_0 I a^2}{2 x^2}$$

(inverse-cube!)

far from loop.

$E$ due to a dipole $x \gg$ dipole size: was also an inverse-cube law!

Loop of current is magnetic counterpart of the electric dipole!

25.55

& 26.47

\begin{align*}
\text{Voltmeter with internal resistance } R_v \\
\text{What is the reading of voltage across } 20k\Omega \\
\end{align*}

a) If $R_v = 50k\Omega$

b) $R_v = 250k\Omega$

c) $R_v = 10M\Omega$

Find equation giving $V_{AB}$ in term of $R_v$

Only one battery → can use series & parallel combination:

$$V_{AB} = 100V \left[ \frac{R_v \parallel 30k\Omega}{(R_v \parallel 30k\Omega) + 20k\Omega} \right] = 100 \left( \frac{R_v \parallel 30k\Omega}{R_v + 30} \right)$$
\[ V_{AB} = \frac{100 \cdot 30R}{30R + 20R + 600} = 100 \frac{30 \cdot 50}{50 \times 50 + 600} \]

\[ V_{AB} = 57.25 \text{ V} \]

\[ V_{AB} = \frac{10^4 \times 30}{10^4 \times 50 + 600} \]

**Observation:** what is the theoretical value for \( V_{AB} \)?

(before we use voltmeter across \( AB \))

\[ V_{AB} = \frac{100 \cdot 30k}{30k + 20k} \]

\[ V_{AB} = 100 \frac{3}{5} = 60 \text{ V} \]

**Conclusion:**higher internal resistance for voltmeter gives better measurement since it will draw less current from the circuit:
Data: \( v = 185 \text{ m/s} \); \( q = 1.4 \mu \text{C} \)
\[
\vec{F}_B = (2.5 \hat{i} + 7 \hat{j}) \text{ \mu N}
\]
\[
\vec{B} = (42 \hat{i} - 15 \hat{j}) \text{ mT}
\]

Find \( \Theta \) \& \( u \) \& \( \vec{B} \)

\[
\vec{F} = q \vec{v} \times \vec{B} \quad \rightarrow \quad \text{Magnitude:} \quad F = qvB \sin \Theta
\]

\[
\sin \Theta = \frac{F}{qvB}
\]

\[
F = \sqrt{F_x^2 + F_y^2} = \sqrt{2.5 \times 10^{-6} + 7 \times 10^{-6}} \quad \text{N}
\]

\[
B = 10^{-3} \text{ T}
\]

\[
\sin \Theta = \frac{\sqrt{2.5^2 + 7^2} \times 10^{-6}}{1.4 \times 10^{-6} \times 85 \times \sqrt{42^2 + 15^2} \times 10^{-3}} = 0.644 \quad \rightarrow \quad \Theta = \sin^{-1} 0.644
\]

\[\Theta = 40.1^\circ\]
Electric

1) Vector superposition
   \( \vec{E} \) due to \( 2 \) charges,
   ring of charges, \( \vec{d}q \)

2) Gauss Law
   \[ \int \vec{E} \cdot dA = \frac{\text{Gaussian surf}e}{\Phi} \]
   \( \Phi \) (electric flux)

3) Using the Electric Potential \( V \)
   \[ \vec{E} = -\vec{\nabla}V \]
   derivative operator

Magnetic

4) Vector superposition
   using Biot-Savart Law
   (loop of current \( I \), radius \( r \))

2) Ampere Law:
   \[ \int \vec{B} \cdot d\ell = \mu_0 I \text{enclosed} \]
   Amperean loop

3) Using the vector potential \( \vec{A} \)
   \[ \vec{B} = \vec{\nabla} \times \vec{A} \]
   rotational or curl of \( \vec{A} \)
   (derivative operator & cross-product)
Calculation of the magnetic field using Ampere's Law:

1) Determine the Amperian loop (closed loop) taking advantage of the symmetry so:
\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mathbf{B} \cdot \int d\mathbf{l} = BL \]
\[ B \text{ constant along the Amperian loop, and tangential to all points along the loop.} \]
\[ L \text{' length of loop.} \]

2) Determine the current enclosed by the loop.

3) Apply Ampere's Law:
\[ BL = \mu_0 I_{\text{enclosed}} \]
\[ B = \frac{\mu_0 I_{\text{enclosed}}}{L} \]

Magnetic field due to a long line of current:

I along z-axis, magnetic field wraps around this current (on xy plane), direction CCW seen from above (given by RHR)

1) Constant @ all points along a circular loop centered at the current (fixed) 
2) Tangential to all points off this loop.
1) **Determine Ampere's loop:** circle centered on current.

\[ \text{Ampere loop:} \]

2) **\( I_{\text{enclosed}} = I \)**

3) **Ampere's Law:**

\[ \oint B \cdot dl = B_L = \mu_0 I_{\text{enclosed}} = \mu_0 I \]

\[ B = \frac{\mu_0 I}{2\pi r} \]

\[ B(r) = \frac{\mu_0 I}{2\pi r} \]

**Use Ampere's law.**
Application of Ampere's Law to calculate $B(r)$:

1) Def: the Amperean Loop:

Criteria
1) $B$ constant along loop & have inner conductor
2) $B$ tangent to loop. Resemble a long line of current

Circle of radius $r$ centered @ center axis of conductor.

2) Current enclosed by Amperean Loop: $I_{enclosed}$ = $I - \frac{N I}{Na}$

3) $\oint B \cdot dl = \mu_0 I_{enclosed}$

$B \cdot L = \mu_0 \frac{I \frac{r^2}{a^2}}{2\pi r} = \frac{\mu_0 I}{2\pi a^2} r$

$r < a$, within inner conductor
b) \( a < r < b \)

1. Amperian loop of radius \( r \)

2. \( I_{\text{enclosed}} = I \)

3. \( \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}} \)

\[ B L = \mu_0 I \]

\[ B = \frac{\mu_0 I}{2\pi r} ; \quad a < r < b \]

c) \( r > (b+c) \)

1. Amperian loop of radius \( r \)

2. \( I_{\text{enclosed}} = 0 \)

3. \( B = 0 \quad r > b+c \quad \rightarrow \text{Coaxial cable: confines the magnetic within the cable.} \)
Long hollow pipe

Is this related to anything we have done? → Stacking many loops of current to form a long hollow pipe

Field due to one loop of current @ $P$:

$$B = \frac{\mu_0 I R}{2 \sqrt{R^2 + x^2 + z^2}} \hat{z}$$

$P$ @ origin: $x = 0$, $z = R$, $\varphi = R$

$$B = \frac{\mu_0 I R^2}{2 (R^2)\sqrt{R^2}} \hat{z} = \frac{\mu_0 I I}{2R} \hat{z}$$

Find $B$ using Ampere's law for the long hollow pipe:

1) Ampere's loop:

2) $B$ parallel to $x$ (negative direction)

3) $B$ tangential to the Ampere loop
\[
\int \vec{B} \cdot d\vec{l} = B d + \nabla \cdot \vec{A} \quad (\text{side 34})
\]

Scalar product: \[
\begin{cases}
\int \vec{B} \parallel d\vec{l} \rightarrow \text{zero contribution} \quad (\text{sides 14 \\& 23}) \\
\int \vec{B} \perp d\vec{l} \rightarrow \text{max. contribution} \quad (\mu_0 = 1)
\end{cases}
\quad (\text{side 12 \\& 24})
\]

2) Current enclosed by this loop: \[
I_{\text{enclosed}} = I \frac{d}{l}
\]

3) \[
\int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}
\]

\[
B \times = \mu_0 I \frac{d}{l} \rightarrow \vec{B} = \mu_0 I \frac{d}{l}
\]
inside the hollow pipe.

**Boundary:**

[Diagram of a hollow cylindrical pipe with a current circulating inside.]
Supercconducting solenoid (highest \( B \) achieved so far)

\[
n = \frac{N}{L} = \text{# of turns per unit length}; \quad \{ n = 3300 \text{ turns/m} \quad I = 4\,100 \text{ A} \}
\]

A long superconducting wire is wrapped around a cylinder many times; a current \( I \) goes through each turn.

Relate \( \vec{B} \) by one loop of current \( \rightarrow \vec{B} \) parallel to axis of solenoid \( \rightarrow \) long line of current \( \vec{B} \) wraps around

1) Amperean loop \( \rightarrow \) rectangle cutting through body of solenoid.

\[
\oint \vec{B} \cdot d\vec{l} = B_1 \, l + 0 \, l + 0 + 0 = B_1 \, l
\]

2) Current enclosed:

\[
I_{\text{enclosed}} = \frac{n \, d \, l}{\text{total # turns enclosed}}
\]

by Amperean loop.

3) \( \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} \)

\[
B_1 \, l = \mu_0 \, n \, d \, I \rightarrow [B = \mu_0 n \, I]
\]

\[
B = 4\pi \times 10^{-7} \times 3300 \times 4100 = 17 \, \text{T}
\]
Wire \( I = 15 \, \text{A} \) at \( 25^\circ \) with a uniform magnetic field. \( \frac{F_{\text{m}}}{L} = 0.31 \, \text{N/m} \). B?

\[ F = q \frac{v}{t} \times B = q \frac{\ell}{t} \times \vec{B} = \frac{q \ell}{t} \times B = I \ell \times B \]

\[ F = I \ell B \sin \theta \] (\( \theta \) angle between wire \( \ell \), field)

\[ \frac{F}{\ell} = I B \sin \theta \Rightarrow B = \frac{F}{I \ell \sin \theta} = \frac{0.31}{15 \times \sin 25^\circ} \]

\[ B = 48.9 \, \text{mT} \]

b) \( \max \frac{F}{\ell} \) when \( \sin \theta = 1 \) or \( \theta = 90^\circ \)

\[ \frac{F}{\ell} = I B = 15 \times 48.9 \times 10^{-3} = 0.734 \, \text{N/m} \]
Net $F_m$ on loop?

Current $I_1$, vector field $B_1$ (wrapping around it) → magnetic force by $B_1$ on the current $I_2$ in a loop: 4 sides:

2) $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$; opposite force at some separation to wire $\Rightarrow$ no net force on sides 12 & 34.

\[
\begin{align*}
\vec{F}_{14} &= I_2 l B_1 \theta (-\hat{i}) \\
\vec{F}_{23} &= I_2 l B_1 \theta (\hat{i}) \\
\text{Net force} &= \vec{F}_{14} + \vec{F}_{23} \\
\text{Net force} &= I_2 l \hat{i} \left( B_1 (d+w) - B_1 (d) \right) \\
&< 0 \quad \text{Attractive force.}
\end{align*}
\]

\[
B_1 = \frac{\mu_0 I_1}{2\pi r} \quad (r: \text{sep to } I_1)
\]

\[
\vec{F}_{\text{Lorentz}} = I_2 l \hat{i} \frac{\mu_0 I_1}{2\pi} \left( \frac{1}{d+w} - \frac{1}{d} \right) = -7.14 \times 10^{-6} \text{ N} \cdot \text{C}
\]
Ch. 27 Electromagnetic Induction

Ferdinand's Law: \[ E = -\frac{d\Phi_B}{dt} \]

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} = \text{magnetic flux} \]

\[ E = \text{induced e.m.f. or induced voltage} \]

Closed loop 1234 with a magnetic flux \( \Phi_B \) through

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} = BA \]

Area of loop with magnetic field \( \rightarrow 1234 \)

\( B \& l \) are constant, but if we move the conducting bar 23 or change \( x \) then \( \frac{d\Phi_B}{dt} \neq 0 \) there is an induced \( E \) in the loop (acts like a battery)

\[ \text{current } I = \frac{E}{R} \text{ will show up in the loop.} \]
a) Direction of current in rector?

\[ E = \frac{d\Phi_B}{dt} \]

The induced \( E \) will oppose the change in \( \Phi_B \)

1. If \( \Phi_B \) increases \( \rightarrow \Phi_B \) will be such that it reduces.
2. If \( \Phi_B \) decreases \( \rightarrow \Phi_B \) will be such that it increases.

b) Conduction of a bar \( 23 \)

If conducting bar moves right \( \rightarrow \Phi_B \) increases \( \rightarrow \Phi_B \) will tend to reduce \( \Phi_B \) by creating a current in the loop that produces a induced magnetic field out of phase to reduce the original field and so to reduce the \( \Phi_B \) despite an increase in \( A \) due to the conducting bar moving to the right.

3. I induced will go \( 4 \rightarrow 1 \) across the rector (downward).

b) What power (work per unit time) is need to pull the bar \( 23 \)?

\[ P = I \cdot V = I^2R = \left( \frac{E}{R} \right)^2R = \frac{E^2}{R} = \frac{\left( \frac{d\Phi_B}{dt} \right)^2}{R} \]

\( \Phi_B = B \times L \rightarrow \frac{d\Phi_B}{dt} = BL \frac{dx}{dt} = BLv \) (speed of bar \( 23 \))

\[ P = \frac{(BLv)^2}{R} \]