Ch23 Electrostatic Energy & Capacitors.

Store electrostatic energy by separating charges of opposite types or bringing charges of the same types together.
(Similarly: water tends to fall to lower height due to gravitational attraction of the Earth, bringing water to certain height = storing hydrostatic energy)

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q_{test}}$$

U: electric potential energy (J)

V: electric potential $\left(\frac{J}{C}\right)$ or V for Volt

$$\Delta U_{AB} = -W_{AB} = -\int_A^B \vec{F} \cdot \vec{dl}$$

ΔU_{AB}: change in potential energy is minus the work done

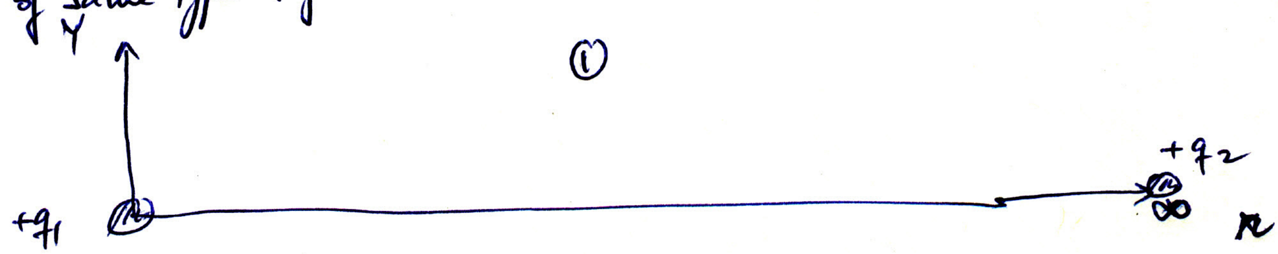
(Relating this to the 1st Law of T.D.: $Q=0$)

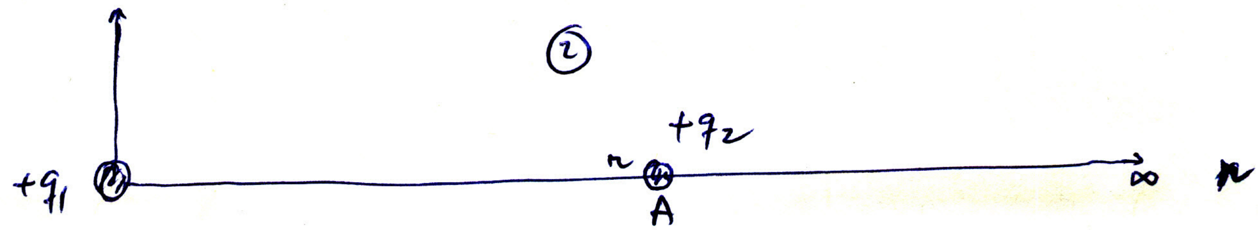
$\vec{F}$: Force applied

$\cdot$: scalar product

$\vec{dl}$: infinitesimal displacement

To separate charges of ~~same~~ opposite types we need to apply a force $\vec{F}$ $\left(k\frac{q_1 q_2}{r^2}\right)$ b/w A & B. ~~so~~ By doing this we are storing certain amount of energy. Similar thing happens when we bring charges of same type together:





- Charge $+q_1$ creates a field pointing in all directions around it (away from it since q_1 is positive)
- While bringing $+q_2$ from ∞ to A, we are going against the field created by $+q_1$, need work : $\Delta U_{\infty A}$

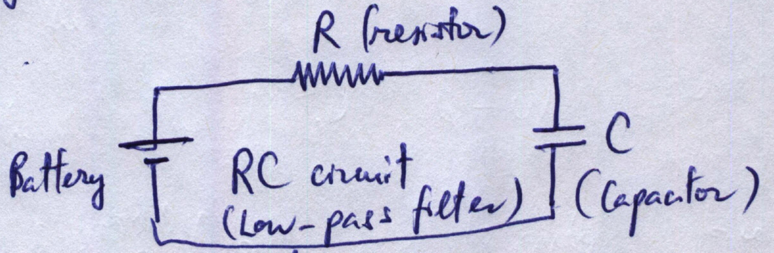
$$\Delta U_{\infty A} = \underset{\substack{\downarrow \\ q_{test}}}{q_2} \Delta V_{\infty A} = q_2 \, k q_1 \left(\frac{1}{r} - \underset{\substack{\downarrow \\ 0}}{\frac{1}{\infty}} \right) = k \frac{q_1 q_2}{r}$$

<u>Observation</u>:
- a) Smaller final separation r → more energy stored
- b) Larger q_2 → more energy stored
- c) larger q_1 → more energy stored.

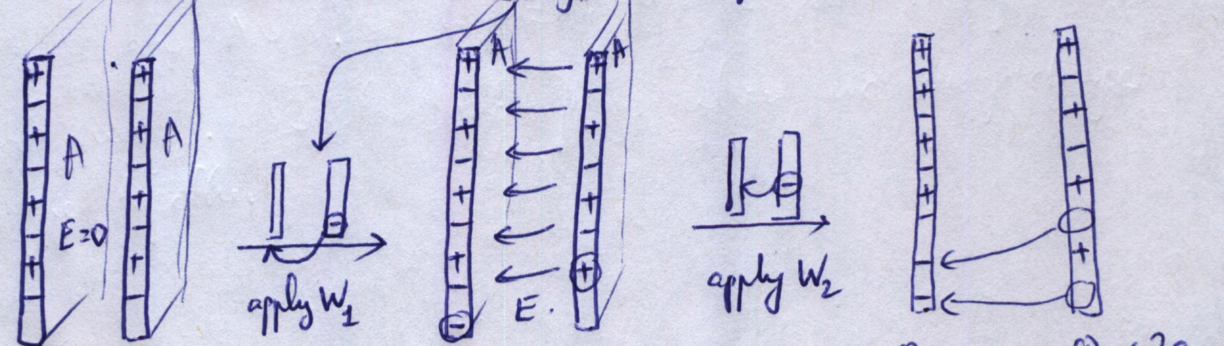
<u>Electrostatic Energy Storage Devices: Capacitors</u> – Parallel Plates

Symbol: ||

In a circuit:



↳ RC circuit: used to <u>charge</u> the capacitor:



Q=0 Q=0 Q=−e Q=+e Q=−2e Q=+2e

d d

Equal amount of each type of charges in either plate

Assumption: $d \ll \sqrt{A}$
A is the cross sectional area of each plate
What we see here is just a front view

} Use E as that → created by an ∞ plate

$W_2 > W_1$ (to move 2^{nd} e^- we work against the field created by the 1^{st} e^- that was moved to the left plate)

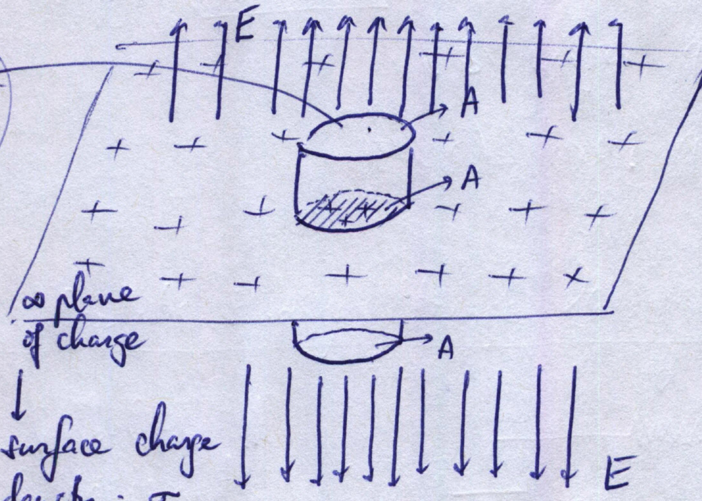
$\downarrow$

Energy stored get larger & larger in the charging process.

<u>What is the electric field b/w plates?</u> (assumption $d \ll \sqrt{A}$)

→ plates look ∞ @ this small d approximation:

Gaussian Surface



→ Horizontal plane.

↓ surface charge density : σ (sigma)

$\sigma \equiv \frac{Q}{\text{Area}}$

Gauss law: 1) Gauss surface: cylinder as shown

2) $\phi = \left\{ \begin{array}{l} \text{Top} : EA \\ \text{Bottom} : EA \\ \text{Body} \Rightarrow \phi = 0 \end{array} \right\} = 2EA$

3) $\phi = \frac{q_{enclosed}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$

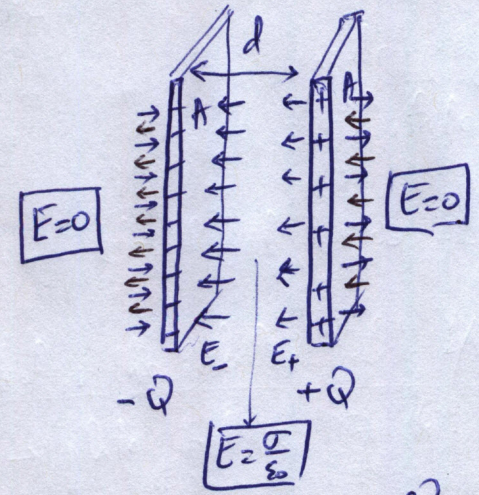
charge on ∞ plane sitting within the intersection of Gaussian cylinder with the plane → area A

$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$ Electric field due to an ∞ plane of charge (of charge density σ)

↓

Applies to the parallel plates capacitor (for <u>one plate</u>!)

Electric field b/w plates of a capacitor :



$d \leq \sqrt{A}$

$\sigma = \frac{Q}{A}$

$$\left.\begin{aligned} E_+ &= \frac{\sigma}{2\varepsilon_0} \\ E_- &= \frac{\sigma}{2\varepsilon_0} \end{aligned}\right\} \text{ same direction} \rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Capacitance : $C \equiv \frac{Q}{V}$ (charge on either plate over the electric potential V b/w plates)

↳ Parallel plate: $\left[C = \frac{Q}{Ed} = \frac{Q}{\frac{\sigma}{\varepsilon_0} d} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0} d} = \frac{A\varepsilon_0}{d} \right]$

How do we write V in term of E ?

Unit (SI) : F (Farad) $\begin{cases} \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \\ A \ (m^2) \\ d \ (m) \end{cases}$

$\left(k = \frac{1}{4\pi\varepsilon_0} \right)$

Total energy stored in a fully charged capacitor ? $\sigma = \frac{q}{A}$

$dU = -dW = -\underbrace{dq}_{\text{infinitesimal test charge}} V = dq\, Ed = dq \frac{\sigma}{\varepsilon_0} d = dq \frac{q}{A\varepsilon_0} d$

$V = -\int \vec{E} \cdot d\vec{l} = -Ed$

$dU = \frac{d}{A\varepsilon_0} q\, dq \rightarrow U = \int dU = \frac{d}{A\varepsilon_0} \underbrace{\int_0^Q q\, dq}_{\left[\frac{q^2}{2}\right]_0^Q} = \frac{1}{2} \frac{d}{A\varepsilon_0} Q^2$

Total energy stored in a parallel plate capacitor:

$$U = \frac{1}{2}\frac{d}{A\varepsilon_0}Q^2 = \frac{1}{2}\frac{A\varepsilon_0 d}{(A\varepsilon_0)^2}Q^2 = \frac{1}{2}A\varepsilon_0 d \frac{Q^2}{A^2\varepsilon_0^2}$$

$\frac{Q^2}{A^2} \rightarrow \sigma^2$

$$= \frac{1}{2}A\varepsilon_0 d \left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{1}{2}\varepsilon_0 E^2 \underbrace{(Ad)}_{\text{volume b/w plates}}$$

$\frac{\sigma}{\varepsilon_0} \rightarrow E$

$$\boxed{\frac{U}{vol} = \frac{1}{2}\varepsilon_0 E^2}$$

→ Total energy stored per unit volume (b/w plates) is $\frac{1}{2}\varepsilon_0 E^2 \rightarrow \left(\frac{J}{m^3}\right)$

(recall: $\frac{1}{2}mv^2$; $\frac{1}{2}kx^2$)

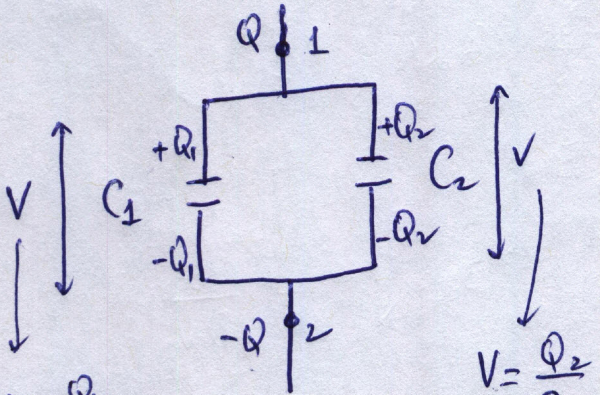
Alternative expression:

$$U = \frac{1}{2}\varepsilon_0 E^2\, vol = \frac{1}{2}\varepsilon_0 E^2 Ad \frac{d}{d} = \frac{1}{2}\underbrace{\frac{A\varepsilon_0}{d}}_{C}\underbrace{E^2 d^2}_{V^2}$$

$$\boxed{U = \frac{1}{2}CV^2}$$

How to connect capacitors:

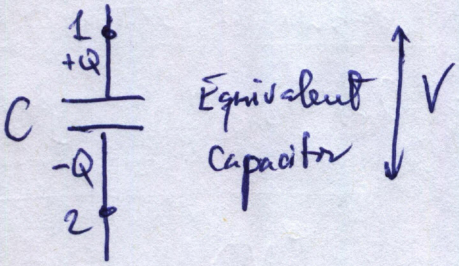
Parallel



$V = \frac{Q_1}{C_1}$

$V = \frac{Q_2}{C_2}$

$$\begin{cases} Q = Q_1 + Q_2 \\ \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \end{cases}$$



$$\boxed{C \equiv \frac{Q}{V}}$$

$$V = \frac{Q}{C}$$

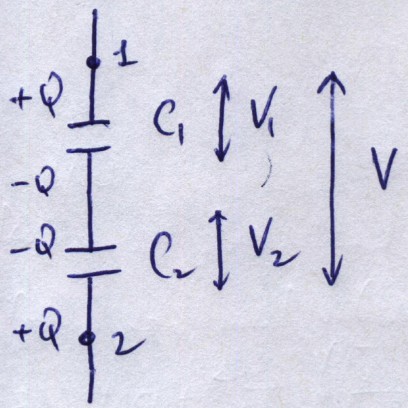
$$C = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \underbrace{\frac{Q_1}{V}}_{C_1} + \underbrace{\frac{Q_2}{V}}_{C_2}$$

$$\boxed{C = C_1 + C_2}$$

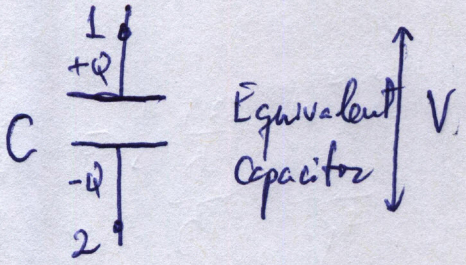
Series

$$V_1 = \frac{Q}{C_1} \begin{cases} +Q \\ -Q \end{cases}$$

$$V_2 = \frac{Q}{C_2} \begin{cases} -Q \\ +Q \end{cases}$$



$$V = V_1 + V_2$$



$$C = \frac{Q}{V} = \frac{Q}{V_1 + V_2} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2}}$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}} \text{ or } \Rightarrow \boxed{C = \frac{C_1 C_2}{C_1 + C_2}}$$

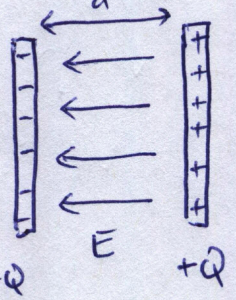
(79)

How to increase the capacitance ?

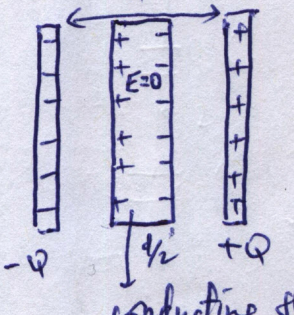
1) Connecting 2 or more capacitors in parallel

2) $C_1 = \frac{A\varepsilon_0}{d}$ (parallel plate capacitors)

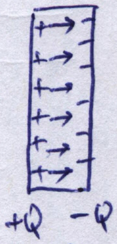
2a) Decrease d : inserting a conducting slab b/w plates:



$V = E \cdot d$



conducting slab
free e^- → rearrange
so



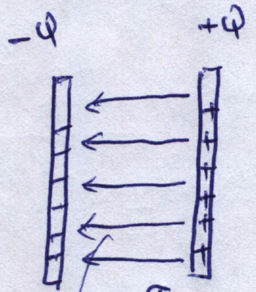
$\underbrace{V' = E \cdot \frac{d}{2}}$

The field now inside the conducting slab (width $\frac{d}{2}$) is 0
→ we have effectively reduced the separation in half
By inserting a conducting slab of width $\frac{d}{2}$ → the capacitance is doubled $C' = \frac{A\varepsilon_0}{\frac{d}{2}} = 2C$

2b) Notice $\underset{\uparrow \text{vacuum}}{\varepsilon_0}$: dielectric constant in vacuum (air)

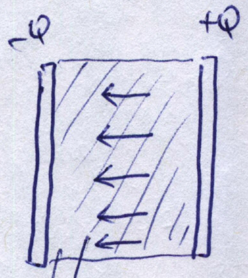
In a medium $\varepsilon = \underset{\downarrow \text{Kappa}}{K}\varepsilon_0$; $K > 1$ → K can help increase the capacitance: if we insert a dielectric of $\varepsilon = K\varepsilon_0$ b/w the plates

Dielectric: not many free e^- as in a conductor $\rightarrow$
we don't get a perfect rearrangement of charges as in a conductor
so the field within the dielectric insert is only reduced, not 0
(by a factor of K)



$E = \frac{\sigma}{\varepsilon_0}$

air $\rightarrow \varepsilon = \varepsilon_0$



Filled w/ dielectric insert $\varepsilon = K\varepsilon_0$

$E' = \frac{\sigma}{\varepsilon} = \frac{\sigma}{K\varepsilon_0} = \frac{E}{K}$

$C = \frac{A\varepsilon_0}{d}$ $\qquad$ $C' = \frac{AK\varepsilon_0}{d} = KC$

$V = Ed$ $\qquad$ $V' = \frac{E}{K}d = \frac{V}{K}$

$C = \frac{Q}{V}$ $\qquad$ $C' = \frac{Q}{V'} = \frac{Q}{\frac{V}{K}} = K\frac{Q}{V} = KC$

23-68 From example 23.4: energy in storm is 140 GJ ($G = 10^9$)

Data: Lightning flash $\begin{cases} \text{Happens every } 5\,s \\ Q = 30\,C\ ;\quad V = 30\,MV \quad (M = 10^6) \end{cases}$

Question: How long will lightning last?

Answer: 1) How much energy per lightning flash:

$U = q_{test} V$ since lightning moves $q_{test} = 30C$ across a potential of $30MV$ → lightning has energy $= 30 \times 30 \times 10^6 = 9 \times 10^8 J$

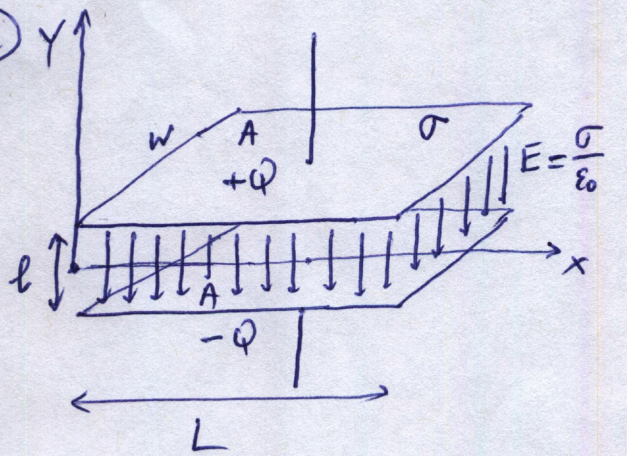
2) # of flashes: N

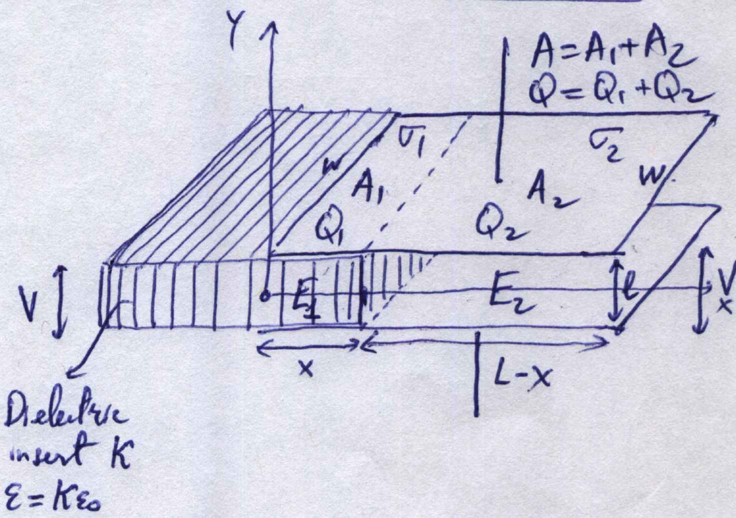
$$N = \frac{\text{Total energy in storm}}{\text{Energy in each lightning flash}} = \frac{140 \times 10^9 J}{9 \times 10^8 J} = 156 \text{ flashes}$$

3) Total time = # flashes × 5s

$= 156 \times 5\,s \times \frac{1 min}{60 s} = 13$ min.

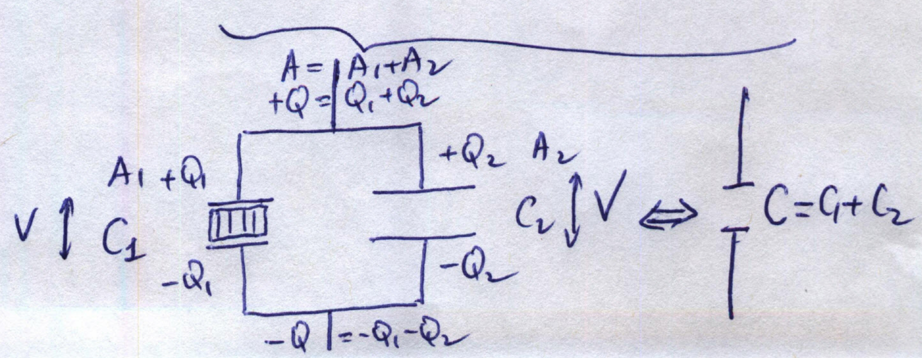
23.72



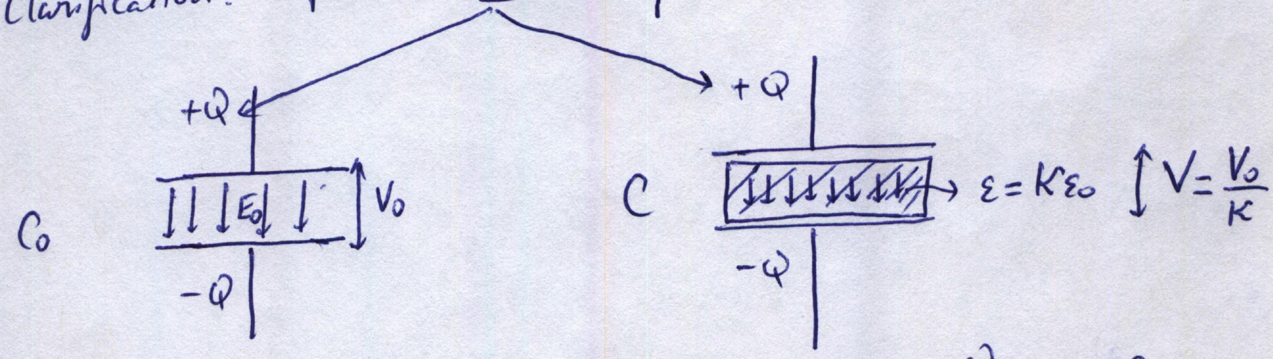


C_0, V_0,

U_0



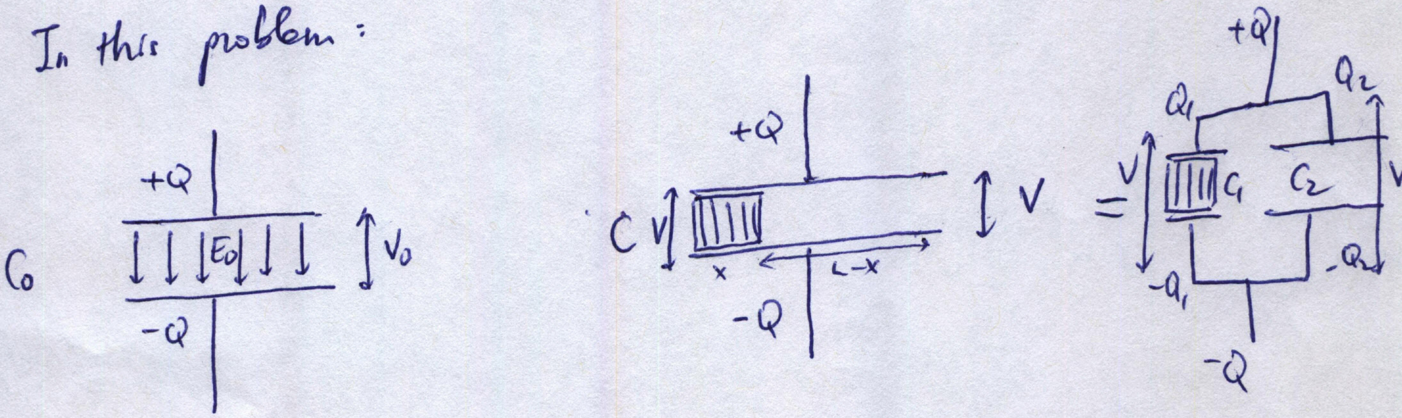
Clarification: for an isolated capacitor



$$C_0 = \frac{Q}{V_0}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{V_0}{K}} = KC_0$$

In this problem:



$$C = C_1 + C_2$$

~~Total capacitance when dielectric is inserted a distance x:~~

$$C = C_1 + C_2 = \frac{Q_1}{V} + \frac{Q_2}{V} = \frac{Q_1 + Q_2}{V}$$

Find Q_1 in term of Q_2: by looking @ E_1 & E_2:

$$E_1 = \frac{\sigma_1}{K\varepsilon_0} = \frac{\frac{Q_1}{A_1}}{K\varepsilon_0} = \frac{Q_1}{xwK\varepsilon_0}$$

$$E_2 = \frac{\sigma_2}{\varepsilon_0} = \frac{\frac{Q_2}{A_2}}{\varepsilon_0} = \frac{Q_2}{(L-x)w\varepsilon_0}$$

There is no discontinuity @ x → field should be continuous → $E_1 = E_2$

$$\frac{Q_1}{xwK\varepsilon_0} = \frac{Q_2}{(L-x)w\varepsilon_0} \rightarrow Q_1 = Q_2 \frac{x\cancel{w}K\cancel{\varepsilon_0}}{(L-x)\cancel{w}\cancel{\varepsilon_0}} = \boxed{Q_2 \frac{Kx}{L-x}}$$

$$C = \frac{Q_2 \frac{Kx}{L-x} + Q_2}{V} = \frac{Q_2}{V}\left(\frac{Kx}{L-x} + 1\right)$$

What is $\frac{Q_2}{V}$ in terms of L, x, w, l ...

$$\frac{Q_2}{V} = \frac{Q_2}{E_2 l} = \frac{Q_2}{\frac{\sigma_2}{\varepsilon_0} l} = \frac{\cancel{Q_2}}{\frac{\cancel{Q_2}}{A_2 \varepsilon_0} l} = \frac{A_2 \varepsilon_0}{l} = \frac{(L-x) w \varepsilon_0}{l}$$

$$C(x) = \frac{(L-x) w \varepsilon_0}{l} \left(\frac{Kx}{L-x} + 1 \right) = \frac{w \varepsilon_0}{l} \left(Kx + L - x \right)$$

$$\boxed{C(x) = \frac{w\varepsilon_0}{l} [\underbrace{x(K-1)}_{\text{positive!}} + L]}$$

a) $$C\left(x = \frac{L}{2}\right) = \frac{w\varepsilon_0}{l} \left[\frac{L}{2}(K-1) + L \right] = \frac{w\varepsilon_0 L}{l} \underbrace{\left[\frac{K-1}{2} + 1 \right]}_{\frac{K-1+2}{2}}$$

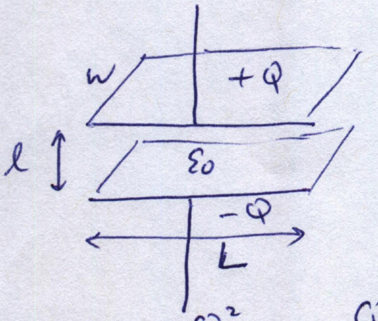
$$= \frac{w \varepsilon_0 L}{2l} (K+1)$$

b) $$U\left(x = \frac{L}{2}\right) = ?$$

$$U(x) = \frac{1}{2} C V^2 = \frac{1}{2} \cancel{C} \frac{Q^2}{C^{\cancel{2}}} = \frac{Q^2}{2C}$$

$$C = \frac{Q}{V} \rightarrow V = \frac{Q}{C}$$

$$U(x) = \frac{Q^2}{2 \frac{w\varepsilon_0}{l} [x(K-1) + L]} = \underbrace{\frac{Q^2}{\frac{2wL\varepsilon_0}{l}}}_{U_0} \frac{L}{[x(K-1) + L]}$$



$$\frac{1}{2} C_0 V_0^2 = U_0 = \frac{Q^2}{2C_0} = \frac{Q^2}{2 \frac{wL\varepsilon_0}{l}}$$

$C_0 = \frac{A\varepsilon_0}{d}$ (parallel plate, d: spacing)

$$U(x) = U_0 \left[\frac{L}{x(K-1) + L} \right]$$

$$U\left(x = \frac{L}{2}\right) = U_0 \left[\frac{\cancel{L}}{\frac{\cancel{L}}{2}(K-1) + \cancel{L}} \right]$$

$$\boxed{U\left(x = \frac{L}{2}\right) = U_0 \frac{2}{K+1}} = \frac{C_0 V_0^2}{K+1}$$

c) Force on dielectric slab?
Why there is a force? Capacitor will not suck in the dielectric slab, a force needs to apply:

$$F = -\frac{dU}{dx} \qquad \left(\text{since} \quad E = -\frac{dV}{dx} \rightarrow \underbrace{q_{test} E}_{F} = -\frac{d(q_{test} V)}{dx} \right.$$
$$\left. F = -\frac{dU}{dx} \right)$$

$\downarrow$

This is why we needed to work out $C(x) \rightarrow U(x)$ + etc..

$$\vec{F} = -\frac{d}{dx}\left(\frac{U_0 L}{x(K-1)+L}\right)\hat{i} = U_0 L \frac{K-1}{(x(K-1)+L)^2}\,\hat{i}$$

$\downarrow$

{ → gradient operator : $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$
{ → b/c pushing in along +x.

$$\vec{F}\left(x=\frac{L}{2}\right) = U_0 L \frac{K-1}{\left(\frac{L}{2}(K-1)+L\right)^2}\,\hat{i} = \frac{U_0\,(K-1)}{L\left[\frac{K-1}{2}+1\right]^2}\,\hat{i}$$

$$= \frac{4U_0(K-1)}{L\,(K+1)^2}\,\hat{i} = \frac{2C_0V_0^2\,(K-1)}{L\,(K+1)^2}\,\hat{i}$$

$\downarrow$

$U_0 = \frac{1}{2}C_0V_0^2$

(Total force required to hold dielectric slab inserted halfway in the spacing)

Ch. 24 Electric Current.

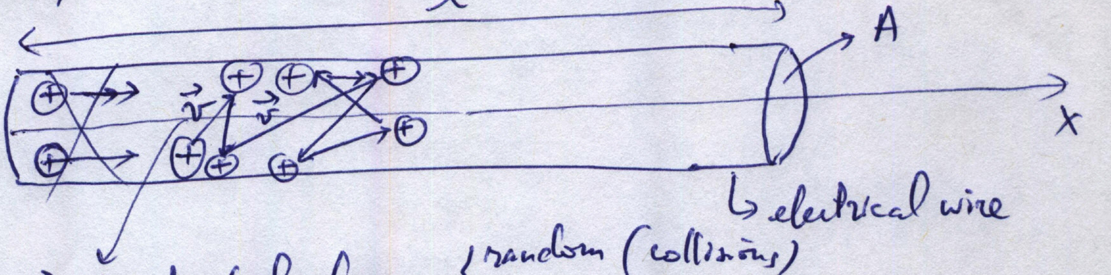
Motion of charges $\rightarrow \frac{\Delta q}{\Delta t} = I_{average}$.

Electric current $I \equiv \frac{dq}{dt}$ (SI: $\frac{C}{s} = A$ for Amp)

↳ Macroscopic (measurable with simple instruments : ampmeter)

Microscopic motion of charges:

Drift velocity: v_d average velocity along a wire (in the x-direction)



$\vec{v}$ of individual charges $\begin{cases} \text{random (collisions)} \\ \text{high} \end{cases}$

$\vec{v}_d \begin{cases} \text{forward (along } x) \\ \text{low} \end{cases}$ net motion of charges along x-axis (in the forward direction)

Number of charges per unit volume : n

Individual charge : q

$$\underbrace{I}_{\text{macroscopic}} = \frac{\Delta q}{\Delta t} = \frac{n A \cancel{l} q}{\frac{\cancel{l}}{v_d}} = \underbrace{n q A v_d}_{\text{microscopic}}$$

n → Very large

→ What would be v_d in a Copper wire $A = 1\,mm^2$; $I = 5A$;
each atom of Copper contributes $1.3e$ of charge ?

data $\begin{cases} \rho_{copper} = 8920\, kg/m^3 \quad (\rho_w = 1000\, kg/m^3) \\ \text{Periodic table: mass for one atom of copper: } 63.55 \text{ a.u. (atomic unit)} \end{cases}$

Conversion factor: $1\,au = 1.66 \times 10^{-27}\,kg$

$$v_d = \frac{I}{n_e q A} \quad \left\{ \begin{array}{l} I = 5A \;;\; A = 1\,mm^2 \\ q = \text{charge of one electron} = 1.6\times10^{-19}\,C \\ n_e = \text{number of electrons per unit volume ?} \end{array} \right.$$

Need n_{Cu} or # of atoms of copper per unit volume:
$(n_e = n_{Cu}\, 1.3)$

$$n_{Cu} = \frac{\rho_{Cu}}{m_{Cu}} = \frac{8920 \frac{kg}{m^3}}{63.55 \times 1.66 \times 10^{-27}\,kg} = 8.5 \times 10^{28} \frac{\text{atoms Cu}}{m^3} \text{ (large!!)}$$

$$n_e = 1.3\, n_{Cu} = 11.05 \times 10^{28} \frac{\text{electrons}}{m^3}$$

$$v_d = \frac{5A}{11.05\times10^{28}\frac{1}{m^3} \times 1.6\times10^{-19}\,C \times 10^{-6}} = \frac{5}{11.05\times10^{3} \times 1.6} = 0.283\,\frac{mm}{s}$$

(walking speed $0.3\frac{m}{s} = 300\frac{mm}{s}$)

Can you estimate actual speed of electrons (random)?

$\downarrow$ rough estimate: using equipartition theorem:

$\hookrightarrow$ average KE per molecule is $\frac{3}{2}kT$ (monatomic)

$$\downarrow$$
$$\frac{1}{2}mv^2$$

$$v = \sqrt{\frac{3kT}{m_e}} = \sqrt{\frac{3\times1.38\times10^{-23}\times298.16}{\cancel{63.55\times1.66\times10^{-27}}\; 9.11\times10^{-31}}}$$

$T = 298.16\,°K$

9.11×10^{-31} $\downarrow$ mass of one electron

$$\cong \sqrt{\frac{1500}{10}}\; 10^4 = 12\times10^4\,\frac{m}{s} = 120000\,\frac{m}{s}$$

Ohm's Law & Electric Power:

$$\boxed{I = \frac{V}{R}}$$

$V \rightarrow$ electrical potential

$R \rightarrow$ Resistance (e^- will find difficulty pushing through atoms in the electrical wire)

SI: Ohms Ω

For a material: resistivity ρ (rho)

wire

ℓ , A

material w/ resistivity ρ

$$\boxed{R = \rho \frac{\ell}{A}}$$

wire for dryer is normally short

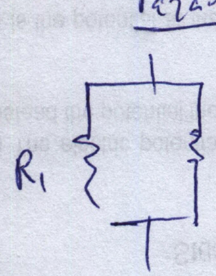
$$\boxed{\text{Power} = I \cdot V} = \left\{ \begin{array}{l} \frac{V}{R} V = \frac{V^2}{R} \\ I \, IR = I^2 R \end{array} \right\} (\text{SI unit} = W)$$

$$\frac{U}{\Delta t} = \frac{q V}{\Delta t}$$

$\frac{q}{\Delta t} = I$

Resistors

Parallel

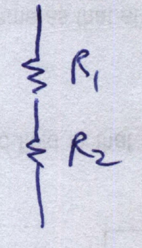


R_1 R_2

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$\text{or } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Series



R_1

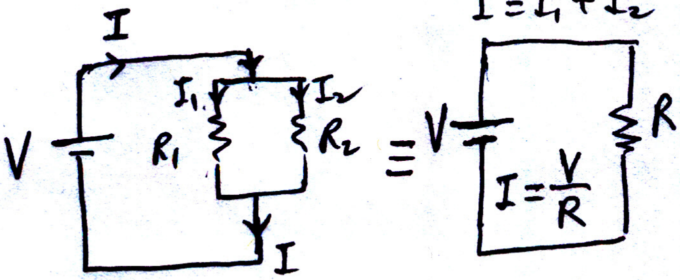
R_2

$$R = R_1 + R_2$$

Resistors

Parallel ↔ Current division

$I = I_1 + I_2$



R: is the equivalent resistor for R_1 & R_2:

$$I = I_1 + I_2$$

Ohm's Law:

$$= \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

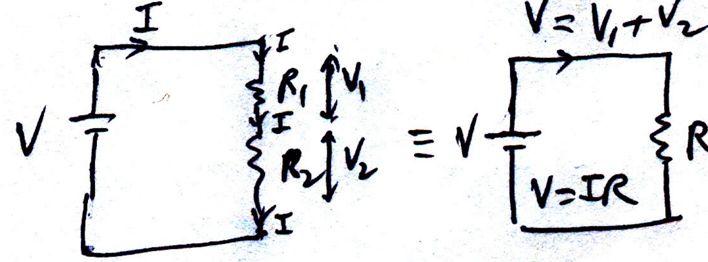
$$I = \frac{V}{R}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

or $\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$

Series ↔ Voltage division

$V = V_1 + V_2$



R: is the equivalent resistor for R_1 & R_2:

$$\begin{cases} V = V_1 + V_2 = IR_1 + IR_2 = I(R_1+R_2) \\ V = IR \end{cases}$$

$$\boxed{R = R_1 + R_2}$$

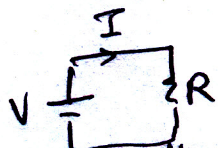
Voltage division:

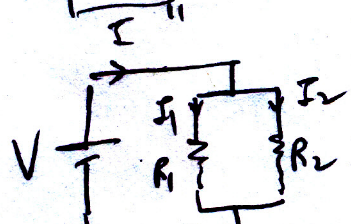
$$V_1 = IR_1 = \underbrace{\frac{V}{R_1+R_2}}_{I} R_1 = \underbrace{\frac{R_1}{R_1+R_2}}_{<1} V$$

$$V_2 = IR_2 = \frac{R_2}{R_1+R_2} V$$

$$V_1 + V_2 = \left(\frac{R_1}{R_1+R_2} + \frac{R_2}{R_1+R_2}\right) V = V$$

Power Consumption





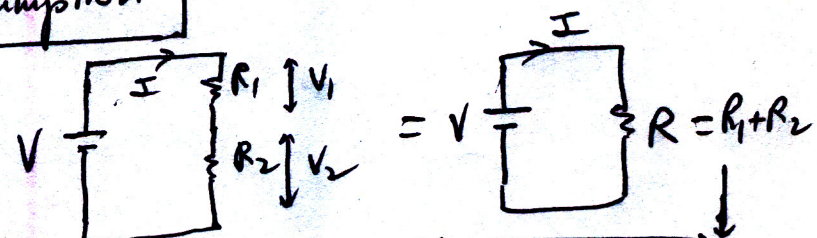
$\boxed{R_1 = R_2}$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1^2}{2R_1}$$

P_1 : power consumed @ R_1 :

$$P_1 = I_1 V = \frac{I}{2} V = \frac{V^2}{2R}$$

$$R = \frac{R_1}{2} \rightarrow \boxed{P_1 = \frac{V^2}{R_1}}$$



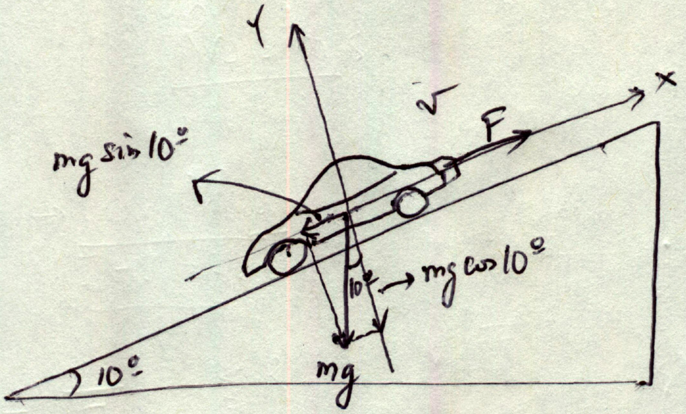
$\boxed{R_1 = R_2} \rightarrow V_1 = \frac{V}{2} = V_2$ $\qquad R = 2R_1$

P_1 : power consumed @ R_1 :

$$P_1 = \cancel{I} V_1 = \frac{V}{R} V_1 = \frac{V}{R}\frac{V}{2} = \frac{V^2}{2R}$$

$$R = 2R_1 \rightarrow \boxed{P_1 = \frac{V^2}{4R_1}}$$

24.69



Data:

$m = 1500\,kg$ | $v = 45\,km/h = \frac{45}{3.6}\,m/s$

$g = 9.81\,m/s^2$

26 12V batteries → Total 312V

each battery delivers $Q = 100\,A\cdot h = 3.6\times10^5\,C$

There is a downhill force of $mg\sin 10°$

→ For the car to go uphill @ const. speed v $(a=0)$ engine needs to apply at least $F = mg\sin 10°$

→ Power consumed by engine $F.v$

(Why? $F.v = \frac{F.\Delta x}{\Delta t} = \frac{\text{energy}}{\text{time}}$ = power)

or Fv is the rate of energy consumption
↓
per unit time.

→ How long it will last @ speed v ? $= t$

$$t = \frac{\text{total } \cancel{\text{electrical}} \text{ mechanical energy available } \cancel{\text{(from battery)}}}{\text{rate of energy consumed by engine}}$$

$$= \frac{0.85 \times \text{total electrical energy available (from batteries)}}{\text{rate of energy consumed by engine}}$$

$$= \frac{0.85 \times V \times Q}{F.v} = \frac{0.85 \times 312 \times 3.6\times10^5}{1500 \times 9.81 \times \sin 10° \times \frac{45}{3.6}}$$

$$= 2989\,s = 49.8\,min$$

Total electrical energy E

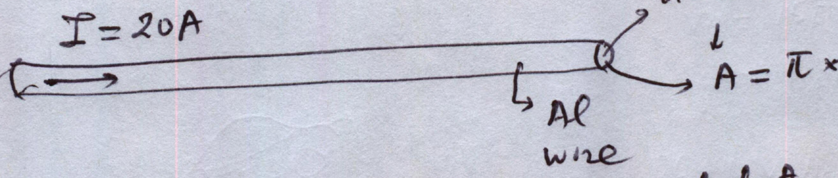
$E = P\Delta t$

$= IV\Delta t$

$= I\Delta t\, V$

$= QV$

24-61



$A = \pi \times (1.05\times10^{-3})^2$

$v_d = ?$ (if each Al atom contributes 3.5 electrons)

$$v_d = \frac{I}{n\,q\,A}$$

I → current in the wire ✓

A → cross-sectional area of the wire ✓

q → charge of one electron ✓

n → # electrons per unit volume

→ $\underbrace{\text{\# of AL atoms per unit volume}} \times 3.5$

$$\frac{\rho_{AL}}{m_{AL}} = \frac{\text{mass density of AL}}{\text{mass of one atom of AL}} = \frac{2702\,kg/m^3}{26.98\,amu \; \frac{1.66\times10^{-27}\,kg}{amu}}$$

(Tables or on-line)

$$\rightarrow n = \frac{2702}{26.98\times1.66\times10^{-27}} \times 3.5$$

$$v_d = \frac{20\,A}{\frac{2702\times3.5}{26.98\times1.66\times10^{-27}}\,1.6\times10^{-19}\times\pi\times(1.05\times10^{-3})^2} = 0.171\,\frac{mm}{s}$$

(Low!)

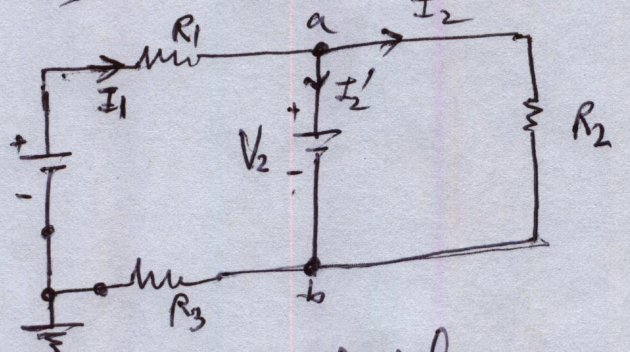
Next find I_2 & I_3 as fractions of I ($I = I_2 + I_3$)

↳ use { current div. / voltage div. / Ohm's law

$$\left\{\begin{array}{l} I_2 = \dfrac{V_{23}}{R_2} = \dfrac{I R_{23}}{R_2} = \dfrac{R_3}{R_2+R_3} I \\ \text{Ohm's law} \\ I_3 = \dfrac{V_{23}}{R_3} = \dfrac{R_2}{R_2+R_3} I \end{array}\right.$$

Note: { In a current division I_2 is proportional to the resistance R_3 in the other branch!

1b)



Ground (zero potential point)

a) R_1 & R_2 are not in series neither in parallel

{ I_1 thru R_1 / I_2 thru R_2 }

{ $I_1 R_1$ across R_1 / V_2 across R_2

Loop Analysis

Total voltage difference across elements in a closed loop is zero (conservation of energy)

There 2 independent loops in this circuit.

- ~~Introduce~~ Assume directions for your variables (currents & voltages) → ~~Actual~~ directions will be given by the equation via signs for numeric answers.

Node & loop analysis → assume directions for currents.

Node analysis.

Total current @ any node is zero (conservation of charge)

Node: where 3 branches converge (There are 2 nodes: a & b)

Ch 25 Electrical Circuits:

Linear circuits (b/c those involving elements with linear relationships b/w voltage V & current I)

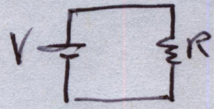
R, L, C

R → resistors; L → inductors; C → capacitors

2 types of linear circuits

1) Resistors only

1a) Those circuits that are reducible to

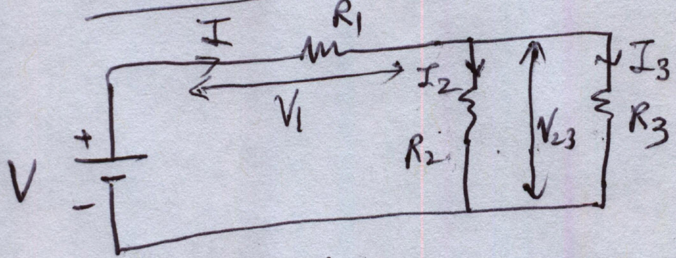


using series and/or parallel connections

1b) Series/parallel are not useful but Loop or Node analysis.

2) Resistors & capacitors

1a) Can reduce to one voltage & one resistor using series/parallel connections

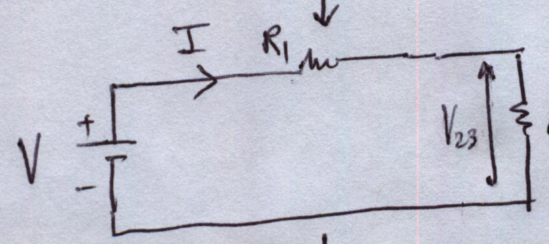


(R_1 is in series with the parallel equivalent of R_2 & R_3)

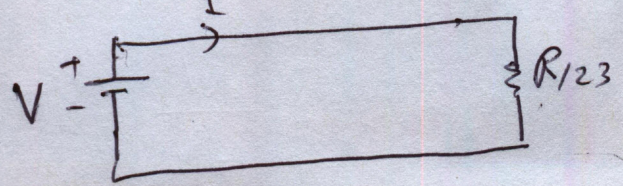
~~Circu~~

Circuit analysis:

solve for unknowns: I, I_2, I_3



$$R_{23} \equiv \frac{R_2 R_3}{R_2 + R_3}$$



$$R_{123} = R_1 + \frac{R_2 R_3}{R_2 + R_3} \rightarrow I = \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

Loop Analysis

Once directions for currents are assumed:

Sign convention for voltages:

1) V @ a battery will have + sign if a current goes from − to + thru the battery

2) V will have − sign if current that you assumed goes ~~from~~ + to −

3) V across a resistor is negative

$$\boxed{\sum_i V_i = 0}$$

Node Analysis

Sign convention for currents.

1) Current into node → +

2) Current leaving node → −

$$\boxed{\sum_i I_i = 0}$$