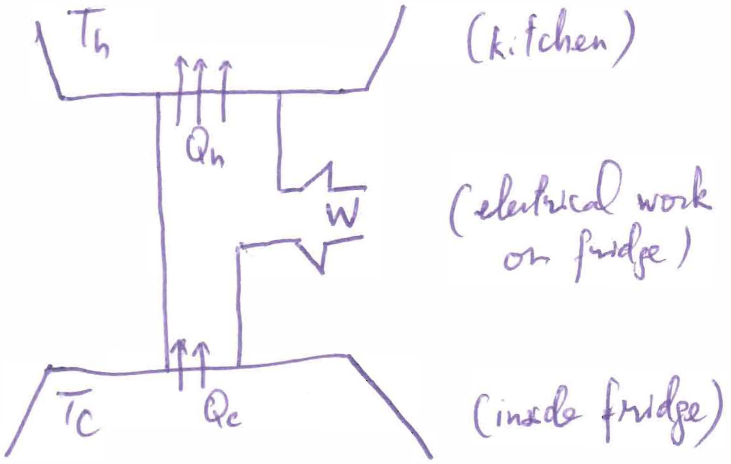
Ch 19 (Cont.)

Refrigerators: reversed heat engine



C.O.P. (Coefficient of performance): $\text{C.O.P} \equiv \dfrac{Q_c}{W}$

2nd Law of T.D.: it is impossible to transfer heat from a cold reservoir to a hot reservoir without requiring any work

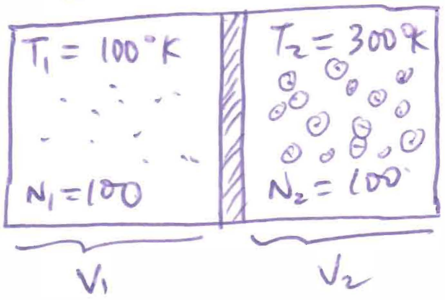
---

3rd Law of T.D.

Entropy: $$\Delta S \equiv \int_1^2 \frac{dQ}{T}$$ Change of entropy b/w states 1 & 2

→ Entropy of a closed system can never decrease (or $\Delta S \geq 0$)

A

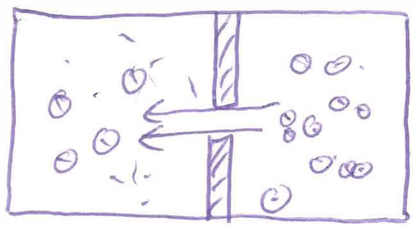


$V_1 = V_2$

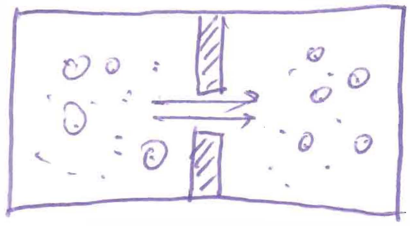
P_1 $P_2 > P_1$

($PV = nRT$ ideal gas)

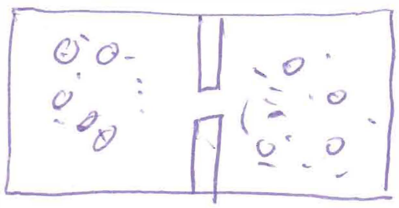
B



C



D



classified: · in left side
⊙ in right side.

both types are mixed
(entropy got increased,
order got decreased.

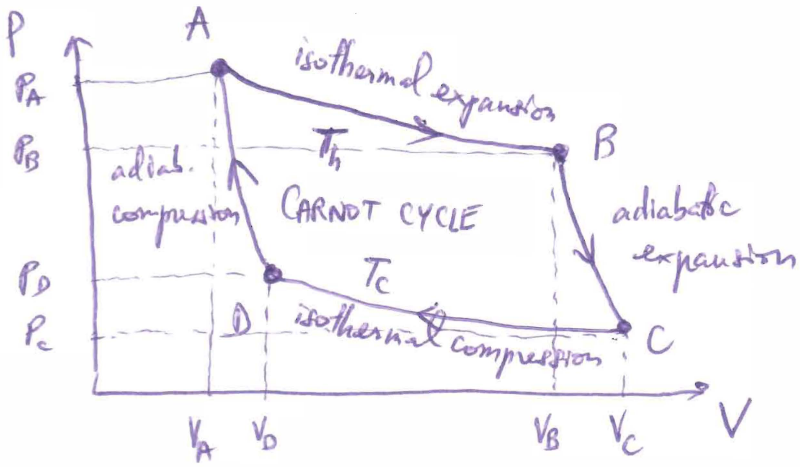
→ $\boxed{\text{entropy} \sim \text{degree of disorder}}$

3rd law of T.D.: disorder in the universe gets increased.

Heat engines: operate in cycles (2nd half of cycle is to bring engine or system back to original state) → representation in PV diagram is a closed loop!

→ Types
- → Carnot engines: 4 reversible processes (2 isothermal, 2 adiabatic)
- → Otto cycle: 4 reversible processes (2 adiabatic, 2 isovolumic)

Carnot Engine: efficiency by a Carnot engine is the maximum achievable so far. $e_{Carnot} = e_{max}$



$$e_{Carnot} = e_{max} = 1 - \underbrace{\frac{|Q_c|}{|Q_h|}}_{\text{on Carnot cycle}}$$ (def. of efficiency: $e = \frac{W}{Q_h}$ & 1st Law of T.D.)

$$= 1 - \frac{|Q_c|_{CD}}{|Q_h|_{AB}}$$

Isothermal process: $\Delta U_{CD} = 0 \rightarrow Q_{CD} = W_{CD} = nRT_c \underbrace{\ln\left(\frac{V_D}{V_C}\right)}_{-}$

$$|Q_c| = |Q_{CD}| = nRT_c \ln\left(\frac{V_C}{V_D}\right)$$

ΔU_{AB} = change of total energy b/w A & B, ideal gas $\rightarrow U \propto T$

$T_B = T_A$ (isothermal) $\rightarrow \Delta U_{AB} = 0 \rightarrow Q_{AB} = W_{AB}$ ($\downarrow$ 1st Law of T.D.)

$$Q_{AB} = nRT_h \underbrace{\ln\left(\frac{V_B}{V_A}\right)}_{+}$$

$$|Q_h| = |Q_{AB}| = nRT_h \ln\left(\frac{V_B}{V_A}\right)$$

Before plugging these $|Q_h|$, $|Q_c|$ into the e_{Carnot}, we will derive the relationships b/w V_A, V_B, V_C, V_D: they are related because $\left\{\begin{matrix} B \to C \\ D \to A \end{matrix}\right\}$ are adiabatic:

$B \to C$: adiabatic expansion:

$$T V^{\gamma-1} = \text{constant}$$

$$T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$$

$$\left(\frac{V_B}{V_C}\right)^{\gamma-1} = \frac{T_C}{T_B} = \frac{T_c}{T_h}$$

$D \to A$: adiab. compression:

$$T_D V_D^{\gamma-1} = T_A V_A^{\gamma-1}$$

$$\left(\frac{V_D}{V_A}\right)^{\gamma-1} = \frac{T_A}{T_D} = \frac{T_h}{T_c}$$

$$\left(\frac{V_B}{V_C}\right)^{\gamma-1} = \left(\frac{V_A}{V_D}\right)^{\gamma-1}$$

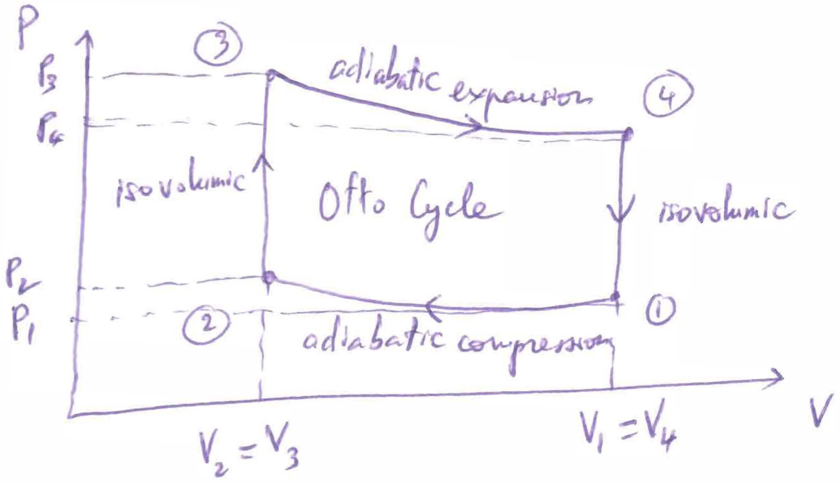
$$\boxed{\frac{V_B}{V_C} = \frac{V_A}{V_D}}$$

$$\boxed{\frac{V_B}{V_A} = \frac{V_C}{V_D}}$$

$$e_{max} = e_{Carnot} = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{\cancel{nR} T_c \ln\left(\cancel{\frac{V_C}{V_D}}\right)}{\cancel{nR} T_h \ln\left(\cancel{\frac{V_B}{V_A}}\right)} = \boxed{1 - \frac{T_c}{T_h}}$$

Max efficiency for any heat engine

Otto Cycle Engines :



$$e_{Otto} < e_{Carnot} = e_{max}$$

Entropy: $\Delta S_{12} = \int_1^2 \frac{dQ}{T}$

1) Isothermal:

$$\Delta S_{12} = \frac{1}{T} \underbrace{\int_1^2 dQ}_{Q_2 - Q_1} = \frac{\Delta Q}{T}$$

2) Isovolumic: $c_V \equiv \frac{1}{n} \frac{dQ}{dT}$

$$\rightarrow dQ = n c_v \, dT$$

$$\Delta S_{12} = \int_1^2 \frac{dQ}{T} = n c_v \int_1^2 \frac{dT}{T}$$

$$= n c_v \ln\left(\frac{T_2}{T_1}\right)$$

End of block of T.D.

Ch ~~16~~ 18 : Temp & Heat

Ch ~~17~~ 19 : 1st Law of T.D : $\Delta U = Q - W$

Ch 18 : ~~2nd~~ Thermal Behavior of Matter : $Q \rightarrow$ 1) $\Delta T \uparrow$; 2) Change of phase; 3) Expansion α, β

Ch 19 : 2nd (heat engines, refrigerators)

3rd

} Laws of T.D.

Ch 20 Electric Charge, Force, Field

- Charge : is a multiple of e^- or e^+ (Coulomb → C)
- Charge distribution : a discrete or continuous group of charges
- Fields (electric) : charges interact through their electric fields. ($\frac{N}{C}$)
- Force (electric) : $\vec{F} = q_{test}\vec{E}$ (N) : force felt by a test charge of value q_{test} in the presence of $\vec{E}$

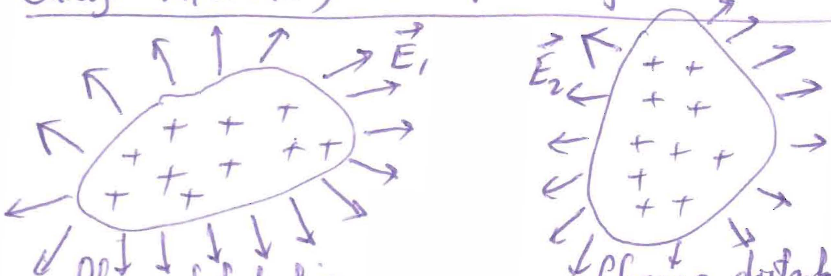
Electron is the elementary charge : $e^- = -1.6 \times 10^{-19}$ C

↑ SI unit for a charge (Coulomb)

↳ We have electrons, we are neutral or not electric, → there are also positive charges.
The proton has a positive charge $e^+ = +1.6 \times 10^{-19}$ C

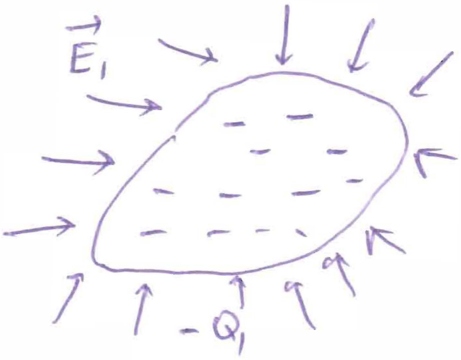
↳ 2 type of charges (electrical) : + and −

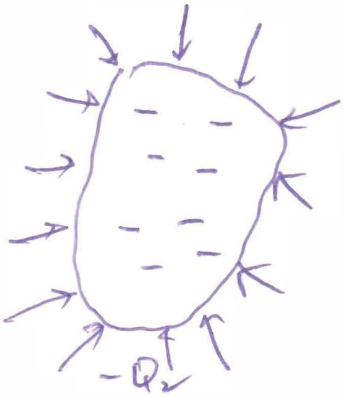
Charge distributions interact through their electric fields:



Charge distribution $+Q_1$	Charge distribution $+Q_2$
↓	↓
Electric field $\vec{E}_1$ pointing away from distribution	Electric field $\vec{E}_2$ also away from the distribution (+ charge)

These two charge distributions through their electric fields $\vec{E}_1$ & $\vec{E}_2$ will repel each other





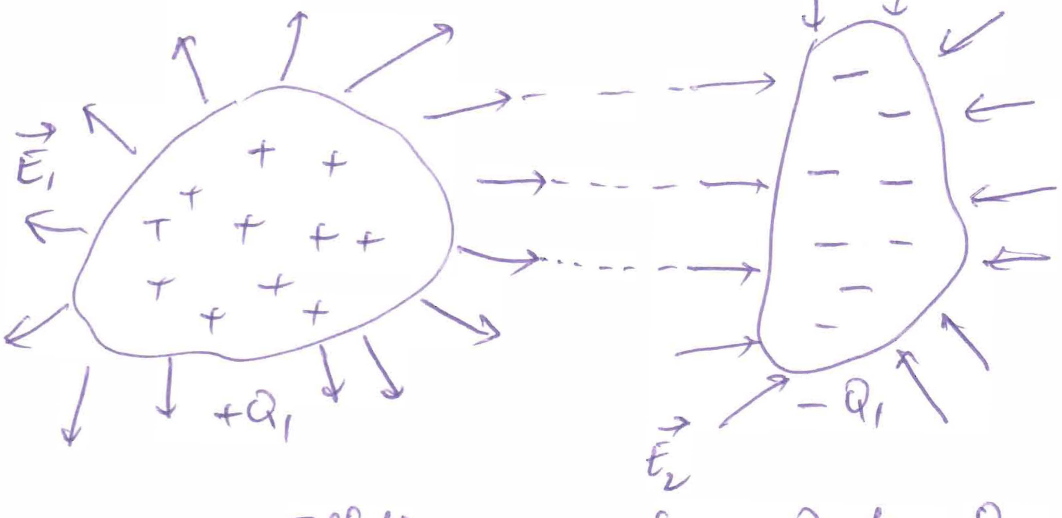
$\vec{E}_1$ pointing towards the charge distribution (negative)

$\vec{E}_2$ pointing towards charge distribution

These two distributions through their electric fields $\vec{E}_1$ & $\vec{E}_2$ will repel each other.

→ Same type of charges (+ & + or − & −) repel each other.

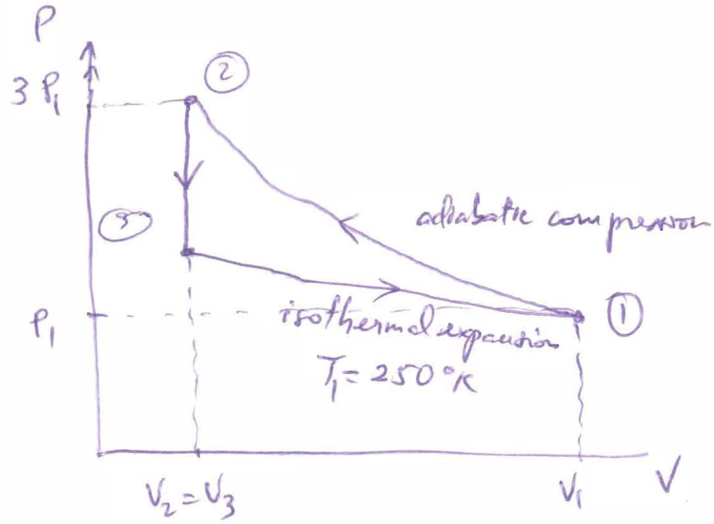
→ Opposite type of charges (+ & − or − & +) attract each other.



Field lines can go from $+Q_1$ to $-Q_2$ w/o any problem → can stay as close as possible

→ + & − charges attract each other through their electric fields.

18.49



ideal gas $\gamma = 1.67$ (monoatomic)

$T_1 = 250° K$

$P_1 = 50$ kPa

$V_1 = 25$ L

a) How much work on gas in this cycle $1 \to 2 \to 3 \to 1$?

$$W_{1231} = \underbrace{W_{12}}_{\text{adiabatic}} + \underbrace{W_{23}}_{0} + \underbrace{W_{31}}_{\text{isothermal}}$$

$$W_{31} = nRT_1 \ln\left(\frac{V_1}{V_3}\right) = nRT \ln\left(\frac{V_1}{V_2}\right) = P_1V_1 \ln\left(\frac{V_1}{V_2}\right)$$

$$1 \to 2: \text{ adiabatic: } \quad P_1V_1^{\gamma} = P_2V_2^{\gamma} \to \left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{P_2}{P_1} \to \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = 3^{\frac{1}{1.67}}$$

$$\to W_{31} = 50 \times 10^3 \times \frac{25}{1000} \ln\left(3^{\frac{1}{1.67}}\right) = \frac{50 \times 25}{1.67} \ln 3 = 822 \text{ J}$$

1000L in 1 m^3

$$W_{12} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{1 - 3 \times \left(\frac{1}{3^{1/1.67}}\right)}{0.67} P_1V_1 = \frac{1 - \left(\frac{3}{3^{1/1.67}}\right)}{0.67} 50 \times 10^{\cancel{3}} \frac{25}{10^{\cancel{3}}}$$

$$= \left[\frac{1 - 3^{\left(1 - \frac{1}{\gamma}\right)}}{\gamma - 1}\right] 1250 = -1033 \text{ J}$$

$$\to W_{1231} = 822 \text{ J} - 1033 \text{ J} = -211 \text{ J}$$

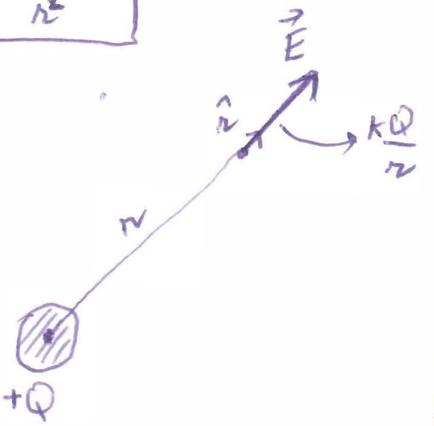
work received by gas is 211 J

18.52 Work done on gas is 56900 J

Quantitative description of the electric field $\vec{E}$ → direction is important

Electric field due to a charge Q @ a point r from Q has intensity $k\frac{Q}{r^2}$, direction along the radial direction b/w Q and point r : $\hat{r}$ (unit vector)
↓
length is normalized to 1

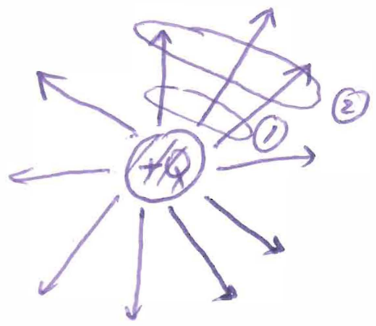
$$\boxed{\vec{E} = k\frac{Q}{r^2}\,\hat{r}}$$

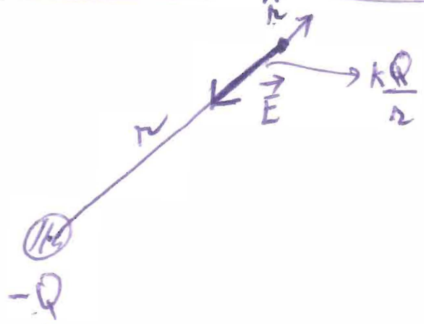


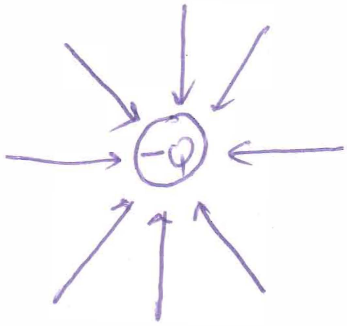
K = electric constant = $9\times10^{9}\ \frac{Nm^2}{C^2}$ (SI)
Q = net charge creating the field
r : separation from charge to point r where field is probed
$\hat{r}$: radial unit vector (always points away from the charge)

Electric field around a +Q :

Higher line density @ ① (compared to ②) indicate stronger electric field @ smaller separation r to the charge







Note: radial unit vector $\hat{r}$ always points away from the charge or center where the charge is located. Direction of $\vec{E}$ is parallel to $\hat{r}$

$$\begin{cases} \text{same direction if } Q \text{ is positive} \\ \text{opposite direction if } Q \text{ is negative} \end{cases}$$

Electric field	Gravitational field
$\vec{E} = k \frac{Q}{r^2} \hat{r}$	$\vec{g} = G \frac{M}{r^2} \hat{r}$

Similarities:

- Inverse square law	- Inverse square law
- Radial direction ($\hat{r}$)	- Radial direction ($\hat{r}$)
- Electric constant	- Gravitational constant
- Proportional to charge creating field	- Proportional to the mass creating the field.

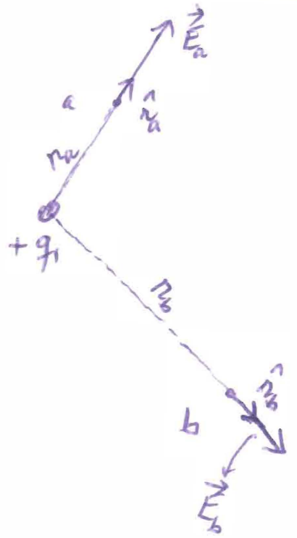
Differences

→ Charge can be + or − ↳ Field can be attractive ($Q<0$) or repulsive ($Q>0$)	→ Mass has no sign. ↳ Grav. field is always attractive.
→ $k = 9 \times 10^{9} \frac{Nm^2}{C^2}$	→ $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

Calculation of the Electric field (Direct method):

- Due to one charge
- Due to two charges (dipole)
- Continuous ring of charge
- Infinite line of charge

Electric field by one charge: q_1



$$\vec{E}_a = k\frac{q_1}{r_a^2}\hat{r}_a$$

$$\vec{E}_b = k\frac{q_1}{r_b^2}\hat{r}_b$$

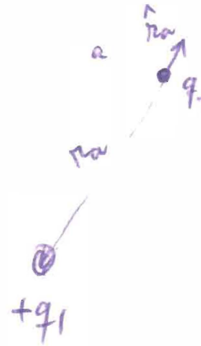
Can calculate $\vec{E}$ @ any point around q_1

Force: when a second charge (test charge) q_{test} comes into the picture (field created q_1) it will feel a force:

$$\vec{F} = q_{test}\vec{E} \quad \begin{cases} \text{- repulsive if } q_{test} > 0 \\ \text{- attractive if } q_{test} < 0 \end{cases} \text{ in this field } \vec{E} \text{ created by } +q_1$$

In a field created by $-q_1$; the test charge q_{test} would feel a force:

$$\vec{F} = q_{test}\vec{E} \quad \begin{cases} \text{- attractive if } q_{test} > 0 \\ \text{- repulsive if } q_{test} < 0 \end{cases} \text{ in a field } \vec{E} \text{ created by } -q_1$$

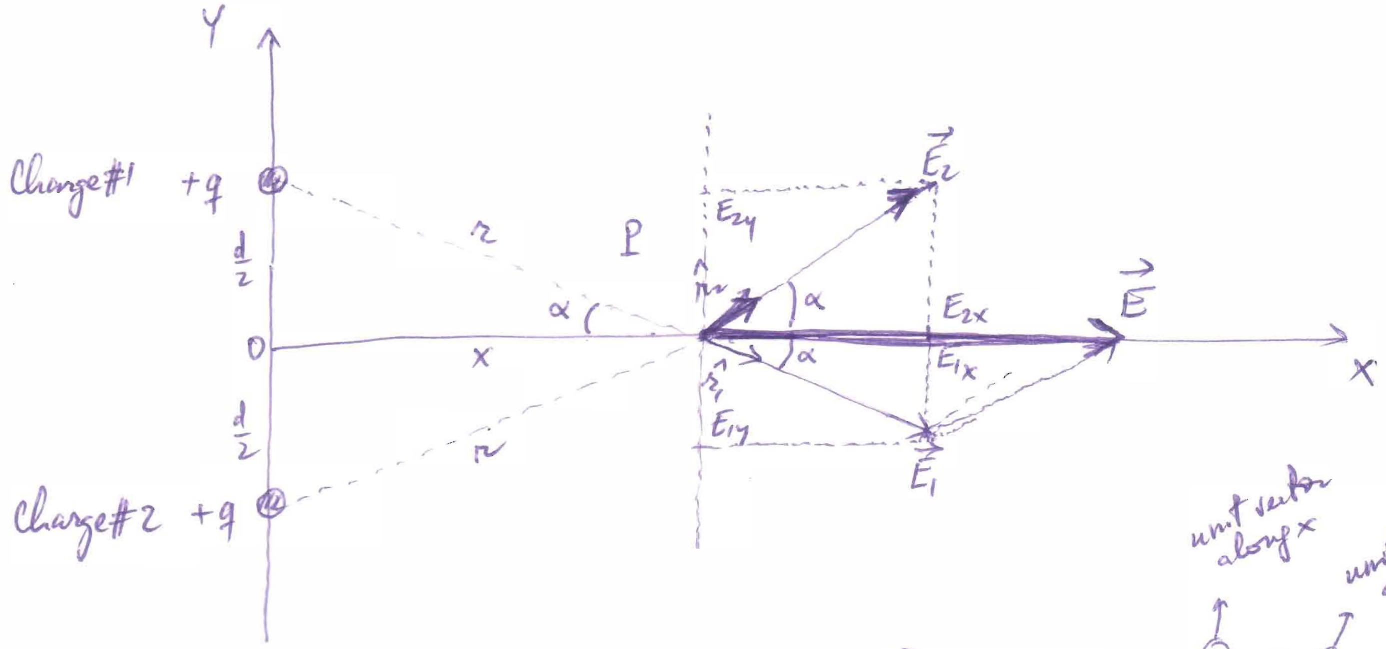


$q_{test} \rightarrow$ Feels force $\vec{F} = q_{test}\,\vec{E}_a = k\,\dfrac{q_1 q_{test}}{r_a^2}\,\hat{r}_a$

- This is the force applied by q_1 on q_{test}
- By 3rd Newton's Law (action & reaction): q_{test} applies a same force on q_1, in the opposite direction:

$$\vec{F} = -k\,\frac{q_1 q_{test}}{r_a^2}\,\hat{r}_a$$

Electric field by two positive charges: along the mid line b/w (x-axis) the two charges.



unit vector along x — unit vector along y

Electric field @ P due to charge #1 $\vec{E}_1 = k\,\dfrac{q}{r^2}\,\hat{r}_1 = E_{1x}\,\hat{i} + E_{1y}\,\hat{j} = E_1\cos\alpha\,\hat{i} - E_1\sin\alpha\,\hat{j}$

" " " " " #2 $\vec{E}_2 = k\,\dfrac{q}{r^2}\,\hat{r}_2 = E_{2x}\,\hat{i} + E_{2y}\,\hat{j} = E_1\cos\alpha\,\hat{i} + E_1\sin\alpha\,\hat{j}$

$\underbrace{\qquad\qquad}_{\text{same magnitude!} \rightarrow E_1 = E_2}$

Total electric field @ P $\vec{E} = \vec{E}_1 + \vec{E}_2 = 2E_1\cos\alpha\,\hat{i}$ (only x-component!)

Back to polar form: (using separation r instead of cartesian coordinates x & y)

$$\vec{E} = 2E_1 \cos\alpha \ \hat{i} = 2\frac{kq}{r^2}\frac{x}{r}\hat{i} = \frac{2kqx}{r^3}\hat{i}$$

$$\cos\alpha = \frac{\text{adjacent}}{\text{hypothenuse}} = \frac{x}{r}$$

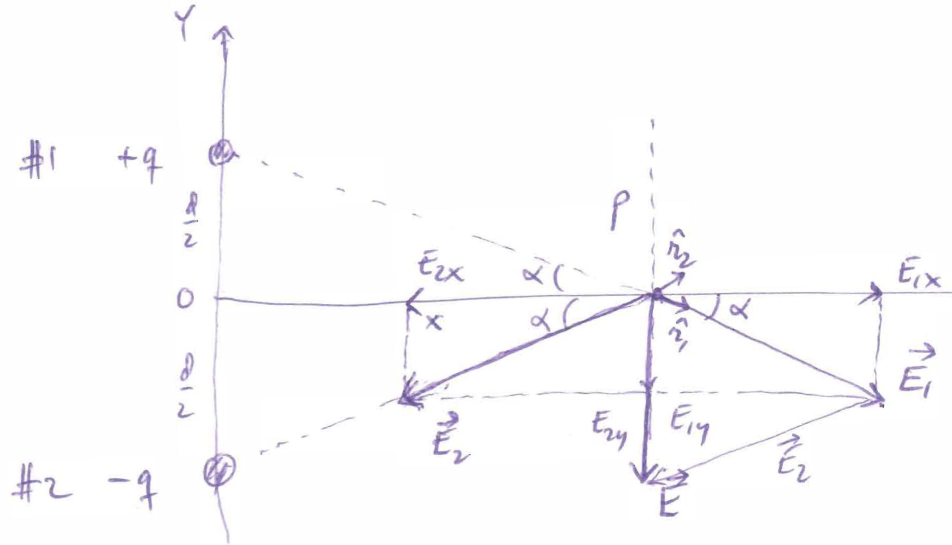
$$\vec{E} = \longrightarrow \boxed{\vec{E} = \frac{2kqx}{\left(x^2+\left(\frac{d}{2}\right)^2\right)^{3/2}}\hat{i}}$$

$$r = \sqrt{x^2+\left(\frac{d}{2}\right)^2}$$

Electric field by 2 charges of value $+q$ @ $\pm\frac{d}{2}$ along the y-axis

Unit: $\frac{N}{C}$ (S.I.)

Electric field by a dipole: along mid-line b/w the 2 charges.



$E_2 = E_1$ (magnitudes)

$$\vec{E}_1 = k\frac{q}{r^2}\hat{r}_1 = E_1\cos\alpha\,\hat{i} - E_1\sin\alpha\,\hat{j}$$

$$\vec{E}_2 = -k\frac{q}{r^2}\hat{r}_2 = -E_1\cos\alpha\,\hat{i} - E_1\sin\alpha\,\hat{j}$$

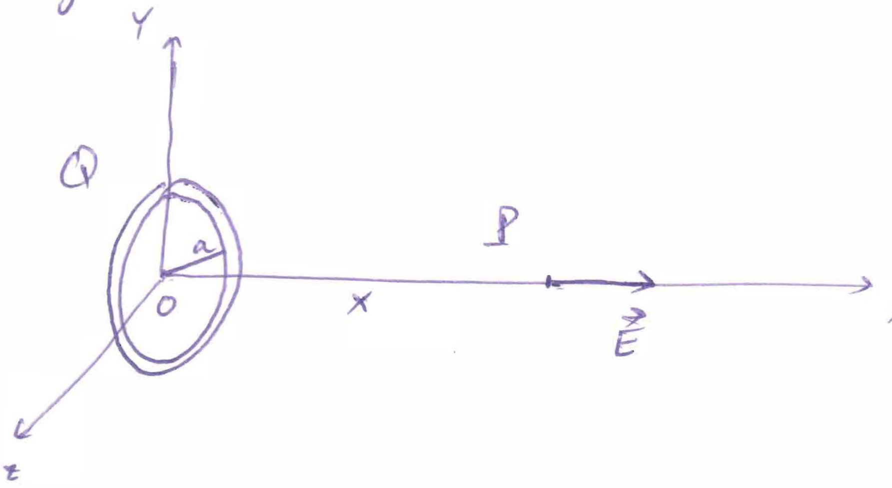
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = -2E_1\sin\alpha\,\hat{j}$$

$$\vec{E} = -\cancel{2}\frac{kq}{r^2}\frac{d}{\cancel{2}r}\hat{j} = -\frac{kqd}{r^3}\hat{j} = \boxed{\frac{-kqd}{\left[x^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}\hat{j}}$$

(if I switch $+q$ & $-q$ → $\vec{E}$ points along $+\hat{j}$)

$\sin\alpha = \frac{d}{2}/r$

Electric field due to a continuous ring of charge: @ a point along its axis

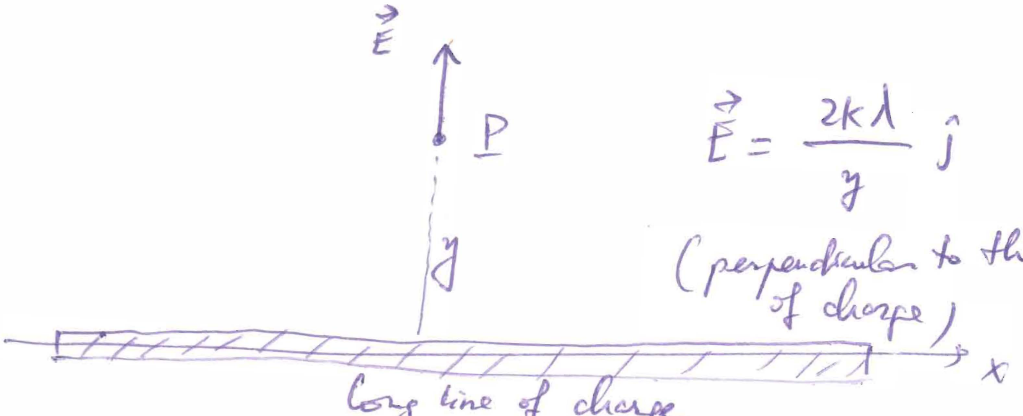


(ring axis)

(ring is perpendicular to the x-axis)

$$\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{i} \qquad (N/C)$$

Electric field by an infinite (very long) line of charge;



$$\vec{E} = \frac{2k\lambda}{y}\,\hat{j}$$

(perpendicular to the line of charge)

Long line of charge

linear density of charge is $+\lambda$
(charge per unit length)

19.46

Ideal, diatomic $C_V = \frac{5}{2}R$ $\begin{cases} n = 5 \\ P = 1\,atm \\ T_1 = 300\,°K \end{cases}$

ΔS_{12} $\begin{cases} \text{a) constant vol.} \\ \text{b) constant } P \\ \text{c) adiabatically} \rightarrow \Delta S_{12} = 0 \end{cases}$

$T_2 = 500\,°K$

$$\Delta S_{12} = \int_1^2 \frac{dQ}{T} \begin{cases} \text{const-volume: } c_V = \frac{1}{n}\frac{dQ}{dT} \rightarrow dQ = c_V n\, dT \\ \text{constant pressure: } c_p = \frac{1}{n}\frac{dQ}{dT} \rightarrow dQ = c_p n\, dT \\ \text{adiabatic } dQ = 0 \rightarrow \Delta S_{12} = 0 \end{cases}$$

a) Isovolumic: $\Delta S_{12} = \int_1^2 \frac{c_V n\, dT}{T} = n c_V \int_1^2 \frac{dT}{T} = n c_V \ln\left(\frac{T_2}{T_1}\right)$

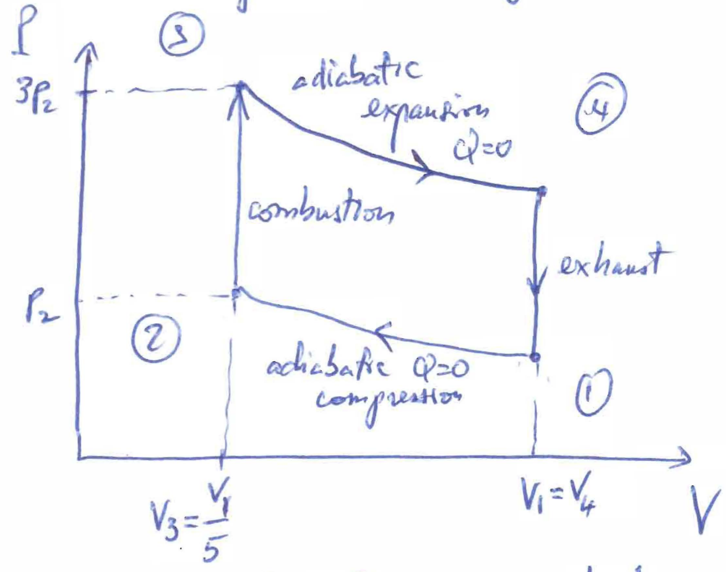
$$= 5 \times \frac{5}{2} \times 8.314 \ln \frac{500}{300} = 53.1 \frac{J}{°K}$$

b) Isobaric $\Delta S_{12} = n c_p \ln\left(\frac{T_2}{T_1}\right) = 53.1 \times \frac{7}{5} \frac{J}{°K} = 74.3 \frac{J}{°K}$

$c_p = c_V + R = \frac{7}{2}R$

19.54

Gasoline engine in Otto cycle:



Given in the problem

a) $e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{|Q_c|}{|Q_h|} < 1$

Heat engine

cycle

$\Delta U = 0 \rightarrow W = Q$

$\left(e = 1 - \frac{T_c}{T_h} \right.$ only for a Carnot engine! max.$\left. \right)$

$$\left.\begin{aligned} Q_c &= Q_{41} = nc_v \Delta T = nc_v (T_1 - T_4) \\ Q_h &= Q_{23} = nc_v \Delta T = nc_v (T_3 - T_2) \end{aligned}\right\} \rightarrow e = 1 - \frac{|T_1 - T_4|}{|T_3 - T_2|}$$

→ Need relationships b/w T's: → using the adiabatic processes equations

$$TV^{\gamma-1} = \text{const.} \left\{ \begin{aligned} 1 \rightarrow 2 &\left\{ T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \right. \\ 3 - 4 &\left\{ T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \right. \end{aligned} \right\} \begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ \boxed{T_4 V_4^{\gamma-1} = T_3 V_3^{\gamma-1}} \end{aligned}$$

$$\left.\begin{aligned} V_1 &= V_4 \\ V_2 &= V_3 \end{aligned}\right\} \rightarrow \boxed{\frac{T_1}{T_4} = \frac{T_2}{T_3}}$$

$$e = 1 - \frac{\left|T_4 \cancel{\left(\frac{T_1}{T_4} - 1\right)}\right|}{\left|T_3 \cancel{\left(1 - \frac{T_2}{T_3}\right)}\right|} \Rightarrow \boxed{e = 1 - \frac{|T_4|}{|T_3|}}$$

(5)

$$T_4 V_4^{\gamma-1} = T_3 V_3^{\gamma-1} \rightarrow \left[\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \overset{\text{data}}{=} \left(\frac{\frac{\cancel{V_1}}{5}}{\cancel{V_1}}\right)^{\gamma-1} = \frac{1}{5^{\gamma-1}} = 5^{1-\gamma}\right]$$

$$\boxed{e = 1 - \left|\frac{T_4}{T_3}\right| = 1 - 5^{1-\gamma}} \rightarrow \underline{\text{Otto Cycle}}$$

b) Find $\underset{\substack{\| \\ T_3}}{T_{max}}$ in term of $\underset{\substack{\| \\ T_1}}{T_{min}}$

$\rightarrow$ ideal gas $PV = nRT$ $\left\{ \begin{array}{l} ② \; P_2V_2 = nRT_2 \\ ③ \; \underset{\substack{\| \\ 3P_2V_2}}{P_3V_3} = nRT_3 \end{array} \right\}$ $\boxed{T_3 = 3T_2}$

$$\rightarrow \frac{T_4}{T_3} = 5^{1-\gamma} \rightarrow T_3 = T_4\, 5^{\gamma-1} = 3T_1 \frac{5^{\gamma-1}}{\cancel{3}}$$

can we relate T_4 & T_1 ?

Previous page. $\left\{ \begin{array}{l} ④ \\ ① \end{array} \right.$ adiab $\begin{array}{l} 1\to2 \\ 3\to4 \end{array} \left\{ \frac{T_1}{T_4} = \frac{T_2}{T_3} = \frac{1}{3} \rightarrow \boxed{T_4 = \frac{3T_1}{\cancel{3}}} \right.$

$$\boxed{T_3 = 3 \times 5^{\gamma-1} T_1}$$

max. min.

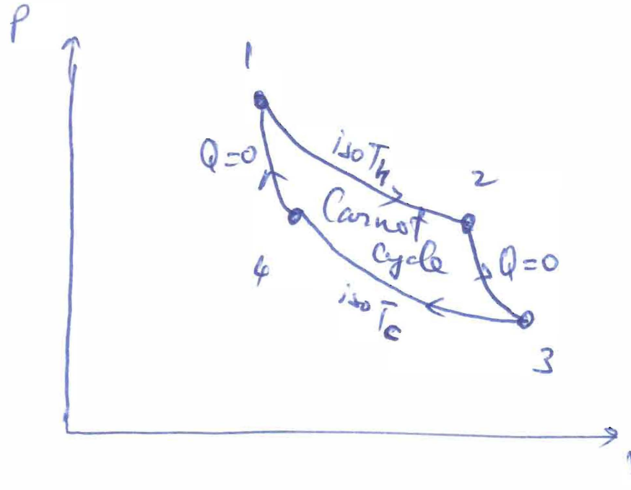
c) For a Carnot engine $\left\{ \begin{array}{l} T_h = T_3 \\ T_c = T_1 \end{array} \right\} \rightarrow e_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_1}{T_3}$

$$\boxed{e_{max} = 1 - \frac{1}{3 \times 5^{\gamma-1}} = 1 - \frac{5^{1-\gamma}}{3}}$$

$$e_{Otto} = 1 - 5^{1-\gamma} < e_{max}$$

(19.42)

$n = 0.2$
ideal gas



$P_1 = 8\,atm$
$V_1 = 1L$
$P_2 = 4\,atm$
$V_2 = 2L$
$P_3 = 2.050\,atm$
$V_3 = 3.224L$
$P_4 = 4.1\,atm$
$V_4 = 1.612L$

a) $Q_h = Q_{12} = nRT_h \ln\left(\frac{V_2}{V_1}\right) = \cancel{0.2 \times 8.314 \times}$

isothermal
$\Delta U = 0 \rightarrow Q = W$

$$= P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = 8 \times 1.013 \times 10^{5} \times \frac{1}{10^{3}} \ln 2 = 561.7\,J$$

b) $Q_c = Q_{34} = P_3 V_3 \ln\left(\frac{V_4}{V_3}\right) = 2.050 \times 1.013 \times 10^5 \frac{3.224}{10^3} \underbrace{\ln\left(\frac{1.612}{3.224}\right)}_{-}$

$= (-)464.1\,J$
heat ejected.

c) Work done: W
Cycle: $\Delta U = 0 \rightarrow W = Q_{net} = |Q_h| - |Q_c| = 561.7 - 464.1 = 97.66\,J$
or $= Q_h + Q_c$

d) $e = \frac{W}{Q_h} = \frac{97.66}{561.7} = 0.1739$ or 17.39%

e) Compare with: $e = 1 - \frac{T_c}{T_h} = 1 - \frac{\frac{P_3 V_3}{nR}}{\frac{P_2 V_2}{nR}} = 1 - \frac{P_3 V_3}{P_2 V_2}$

$T_h = \frac{P_2 V_2}{nR} = \frac{2.05 \times 1.013 \times 10^5 \times \frac{3.224}{10^3}}{0.2 \times 8.314} = 487.4°K$ $\quad = 1 - \frac{2.05 \times 3.224}{4 \times 2}$

$T_c = \frac{P_3 V_3}{nR} = 402.6°K.$ $\quad$ = same answer.

$\vec{F} = q_{test}\vec{E}$ → can find interactions (electric) b/w objects by knowing their electric fields.

⇒ How to calculate the electric field?

1) <u>Direct method</u>: vector superposition (e.g. $\vec{E}$ by 2 charges: $\vec{E} = \vec{E}_1 + \vec{E}_2$). (Ch 20)

2) <u>Using Gauss Law</u> (symmetry) (Ch 21)

3) <u>Using Electric Potential</u> (using derivatives, similar to mechanics: $F = -\frac{dU}{dx}$) (Ch 22)

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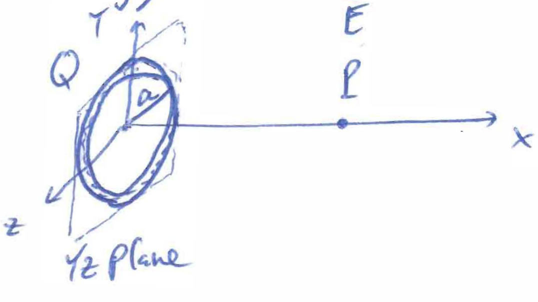
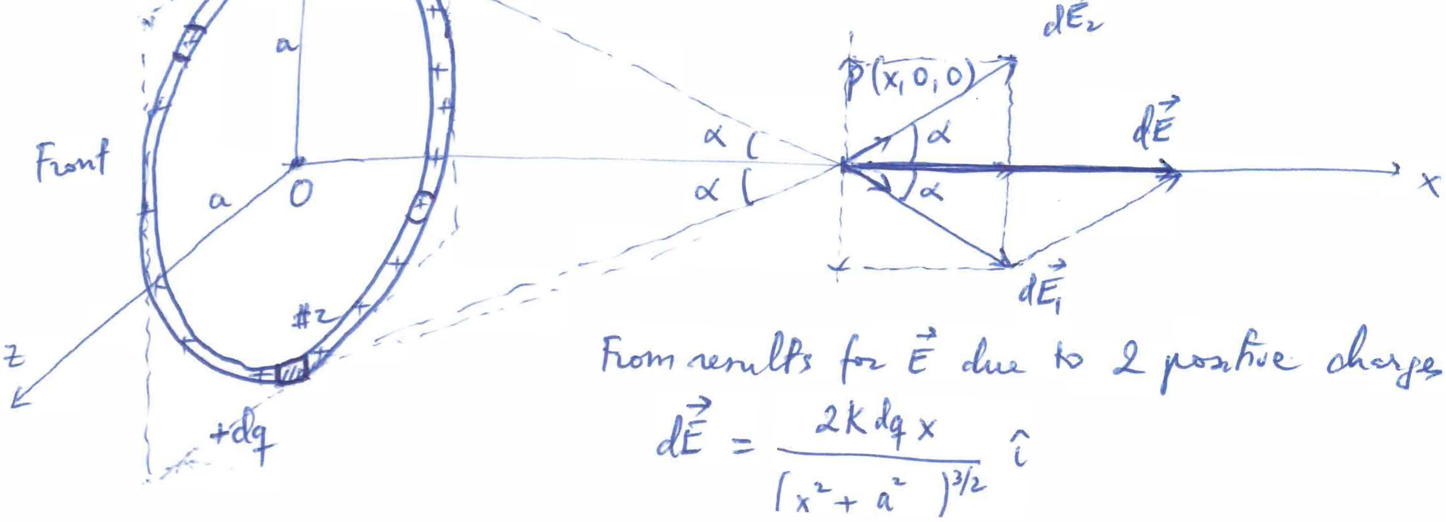
1) <u>Direct Method</u>: (vector superposition)

→ Electric field due to a continuous ring of charge, at a point along its axis (perpendicular to the ring)



Ring center is @ origin of coordinates.



From results for $\vec{E}$ due to 2 positive charges:

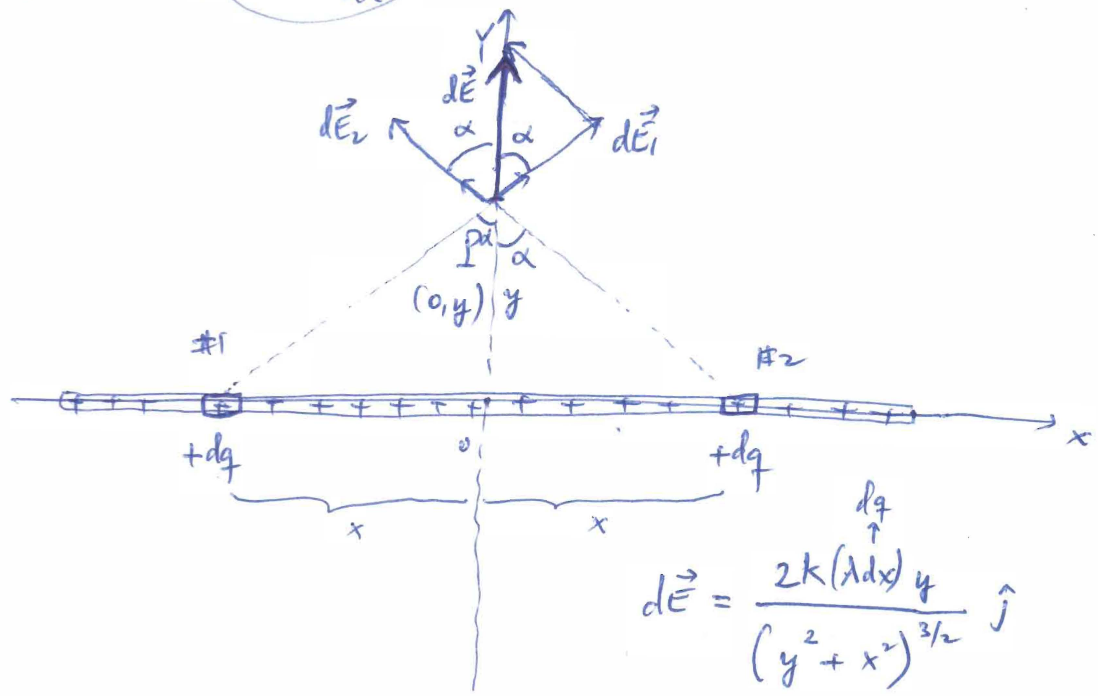
$$d\vec{E} = \frac{2k\,dq\,x}{(x^2 + a^2)^{3/2}}\,\hat{i}$$

To get $\vec{E}$ by the whole ring: $\vec{E} = \int_{\text{Half Ring}} d\vec{E} = \frac{2Kx}{(x^2+a^2)^{3/2}}\hat{i}\underbrace{\int_{\text{half ring}} dq}_{\frac{Q}{2}}$

(Uniformly distributed charge on the ring)

$$\boxed{\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\hat{i}} \quad \left(\frac{N}{C}\right)$$

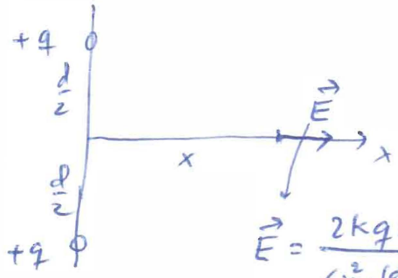
→ Electric field due to a very long line of charge (with linear charge density $\lambda = \frac{dq}{dx}$) → $dq = \lambda dx$



$$d\vec{E} = \frac{2k(\overset{dq}{\overbrace{\lambda dx}})\,y}{(y^2+x^2)^{3/2}}\hat{j}$$

↓

$$\vec{E} = \int_{\text{Half Line}} d\vec{E} = 2K\lambda y\hat{j}\int_{\text{Half Line}} \frac{dx}{(x^2+y^2)^{3/2}}$$



$$\vec{E} = \frac{2kqx}{\left(x^2+\left(\frac{d}{2}\right)^2\right)^{3/2}}\hat{i}$$

Table for integrals:

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

$$\boxed{\vec{E}_{\infty\,\text{line}}} = 2k\lambda y\,\hat{j}\left[\underbrace{\frac{x}{y^2\left(x^2+y^2\right)^{1/2}}}_{\frac{1}{y^2}-0}\right]_{x=0}^{x=\infty} = \boxed{\frac{2k\lambda}{y}\,\hat{j}}$$

(Unlike a finite charge distribution, the field decreases as $\frac{1}{y}$ not $\frac{1}{y^2}$!)

Ch21: Method #2: Using Gauss Law:

Electric flux: $\phi \equiv \oint_{\substack{\text{closed}\\\text{surface}\\\text{integral}}} \vec{E}\cdot d\vec{A}$

"Phi"

$\vec{E}$: Electric field

$d\vec{A}$: element of surface area

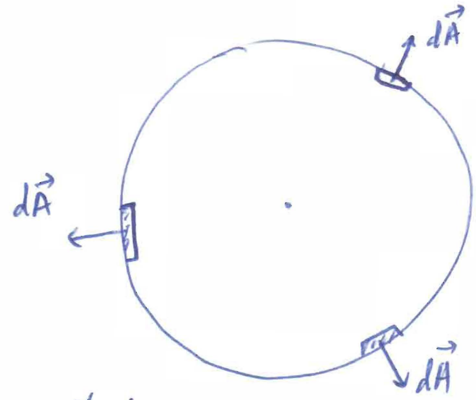
$\cdot$: scalar product b/w two vectors $\vec{A}$ & $\vec{B}$

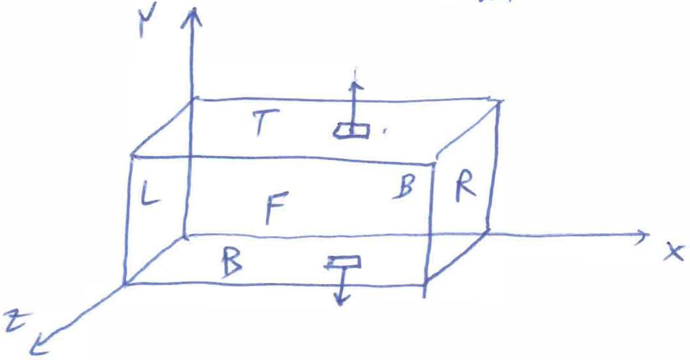
$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$A = |\vec{A}|$, $B = |\vec{B}|$, θ = angle b/w $\vec{A}$ & $\vec{B}$

(Examples: $W = \vec{F}\cdot\Delta\vec{r}$)

Element of surface area: is perpendicular to the element of area. For a spherical surface $d\vec{A}$ points along the radial direction: $d\vec{A} = dA\,\hat{r}$





Top: $d\vec{A} = dA\,\hat{j}$

Bottom: $d\vec{A} = -dA\,\hat{j}$

Left: $d\vec{A} = -dA\,\hat{i}$

Right: $d\vec{A} = dA\,\hat{i}$

Front: $d\vec{A} = dA\,\hat{k}$

Back: $d\vec{A} = -dA\,\hat{k}$

Electric flux: $\phi \equiv \oint_{\substack{\text{closed} \\ \text{surface} \\ \text{integral}}} \vec{E} \cdot d\vec{A} = \oint_{\substack{\text{closed} \\ \text{surface}}} E_\perp \, dA = E_\perp \oint_{\text{surface}} dA$

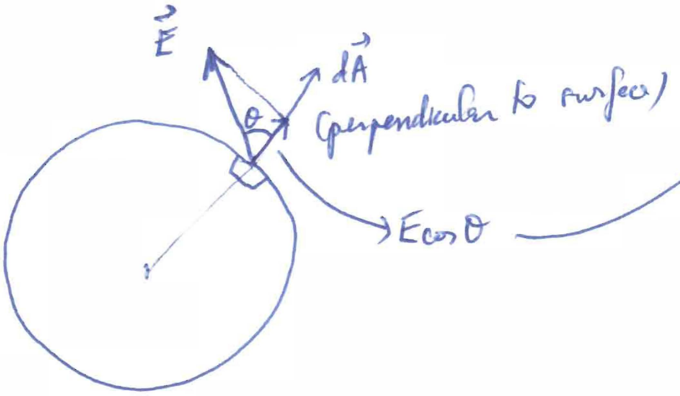
If there is symmetry so that $E_\perp$ is constant over the surface

$= E_\perp A$

$\vec{E} \cdot d\vec{A} = E . dA . \cos\theta = \overbrace{\underbrace{E\cos\theta}_{}}^{E_\perp} \, dA$

$\vec{E} \cdot d\vec{A}$: $d\vec{A}$ perpendicular to surface

$E\cos\theta$: component of $\vec{E}$ that is perpendicular to the surface.



We will use Gauss Law to calculate electric fields in these simple symmetry situations.

Gauss Law:

$$\boxed{\phi_{\substack{\text{closed} \\ \text{surface}}} = \frac{q_{\text{enclosed by surface}}}{\varepsilon_0}}$$

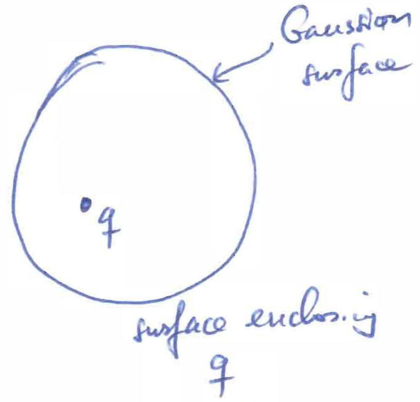
ε_0 = dielectric constant in vacuum

$\varepsilon_0 = \dfrac{1}{4\pi k} = 8.85 \times 10^{-12} \, \dfrac{C^2}{N m^2}$ or $\boxed{k = \dfrac{1}{4\pi\varepsilon_0}}$

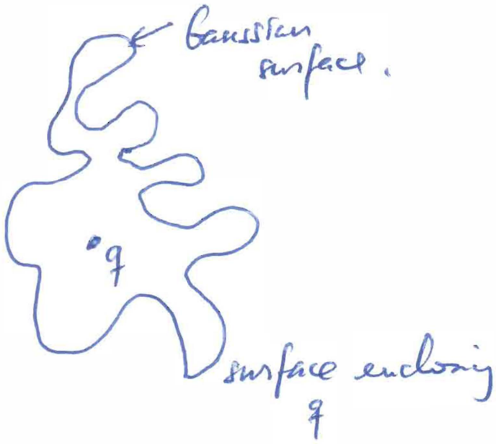
k: electric constant

$k = 9 \times 10^9 \, \dfrac{Nm^2}{C^2}$

Meaning of Gauss Law:



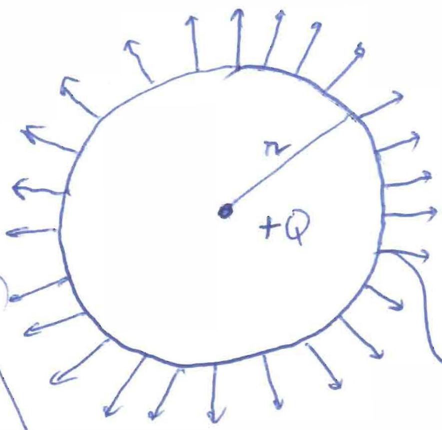
$$\phi_{\substack{\text{closed}\\ \text{surface}}} = \frac{q}{\varepsilon_0}$$



$$\phi_{\substack{\text{closed}\\ \text{surface}}} = \frac{q}{\varepsilon_0}$$

However, to calculate $\vec{E}$ using Gauss law, our Gaussian surface exhibits high symmetry.

1) Using Gauss Law to calculate $\vec{E}$ due to a point charge



First of all: determine the Gaussian surface (with high symmetry so

$$\phi = E_{\perp} A$$

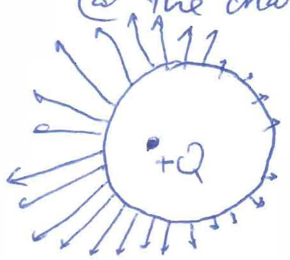
otherwise it will take additional efforts to calculate E)

Gaussian surface: sphere centered @ the charge.

$A = 4\pi r^2$

$$E_{\perp} A = \frac{Q}{\varepsilon_0}$$

$$E_{\perp} = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$



off centered spheres will not allow

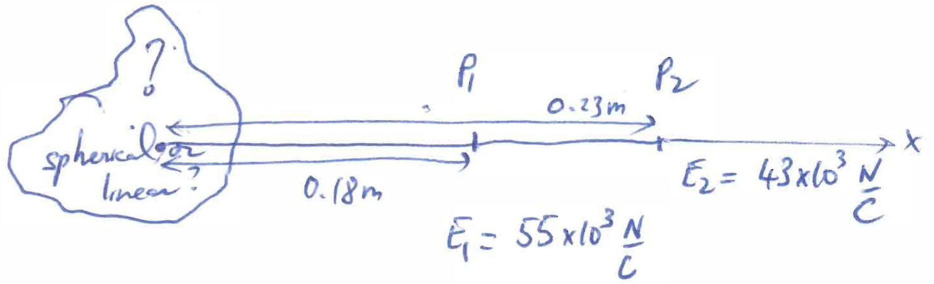
$$\phi = E_{\perp} \oint dA$$

Also for this Gaussian surface

$E_{\perp} = E$ ($\vec{E}$ is radial so it is perpendicular to the surface)

Using Gauss Law and a highly symmetrical Gaussian surface (sphere centered @ charge) we have derived an expression for the electric field due to a point charge $E = \frac{kQ}{r^2}$ that agrees with what we know from Chapter 20.

21-33



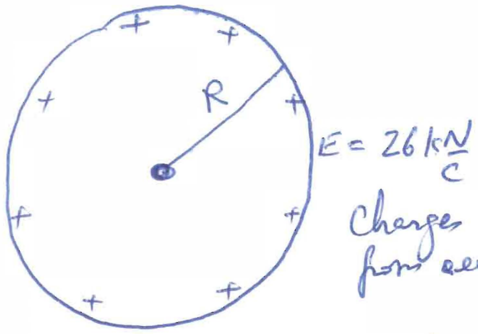
spherical: $E = \frac{kQ}{x^2} \longrightarrow \frac{E_2}{E_1} = \frac{x_1^2}{x_2^2}$ $\left\{ \frac{0.18^2}{0.23^2} = ? \right.$

$\boxed{\text{linear}:}$ $E = \frac{2k\lambda}{x} \longrightarrow \frac{E_2}{E_1} = \frac{x_1}{x_2}$ $\left\{ \frac{0.18}{0.23} = ? \right.$ ✓

$\frac{E_2}{E_1} = \frac{43}{55}$

21.47

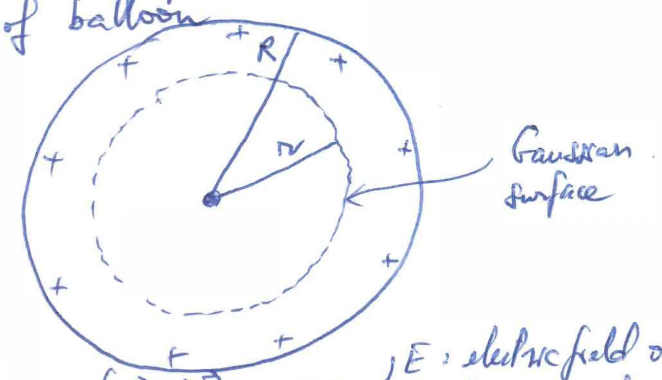
$R = 0.7m$



Charges stay on surface of balloon @ $R=0.7m$ from center.

a) E ($r=0.5m$ or inside balloon)

Using Gauss law → 1) Det. Gaussian surface → sphere centered @ center of balloon



2) $\phi_{\text{Gaussian surface}} = \oint_{G.S.} \vec{E} \cdot d\vec{A} = EA$ $\left\{ \begin{array}{l} E: \text{electric field on Gaussian surface} \\ A: \text{area of Gaussian surface} = 4\pi r^2 \end{array} \right.$

$= E 4\pi r^2$

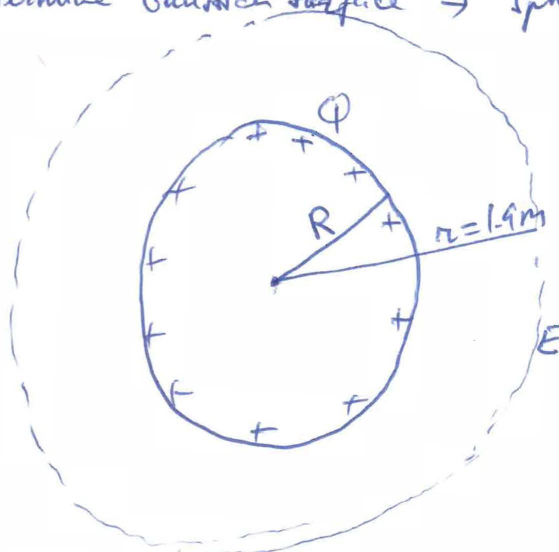
3) Gauss Law : $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$

$$\downarrow$$

$$E 4\pi r^2 = \frac{0}{\varepsilon_0} \rightarrow \boxed{E(r=0.5m) = 0}$$

b) $E(r = 1.9m$, or outside balloon$)$

1) Determine Gaussian surface → sphere centered @ center of balloon



2) $\phi = E\, 4\pi r^2$

3) $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$ (Gauss Law)

$$E 4\pi r^2 = \frac{Q}{\varepsilon_0} \rightarrow E(r>R) = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$

(like that of a point charge!)

Alternatives : find Q, then $E(r=1.9m)$

observation: $E(r=R) = \dfrac{kQ}{0.7^2} = 26\ \dfrac{kN}{C}$

$$\boxed{E(r=1.9m) = \frac{kQ}{1.9^2}}$$

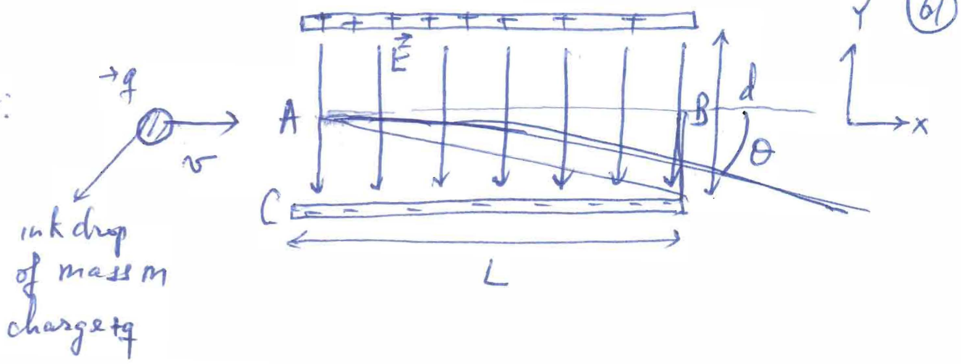
$$\frac{E(r=1.9m)}{E(r=0.7m)} = \frac{0.7^2}{1.9^2} \rightarrow E(r=1.9m) = \frac{0.7^2}{1.9^2}\, 26\ \frac{kN}{C}$$

$$= \boxed{3.53\ \frac{kN}{C}}$$

c) Net charge on balloon $Q = \dfrac{1.9^2 \times 3.53 \times 10^3}{9 \times 10^9} = 1.42\ \mu C$

$$\mu \downarrow 10^{-6}$$

20.78 Ink jet printer:



Ink drop while crossing field region, feels a downward force → a downward acceleration → $a_y = \frac{F}{m} = \frac{qE}{m}$ → constant downward acceleration!

<u>Min v for ink drop to make it through field region:</u>

During time it takes to go A→B (x ~~component~~ direction) it should be going not more than AC $\left(\frac{d}{2}\right)$ (y direction)

x direction : $t_{AB} = \frac{L}{v}$

Motion in x direction is NOT affected by $\vec{E}$ → uniform motion.

y-direction: constant acceleration motion: $y = \frac{1}{2} a_y t^2$

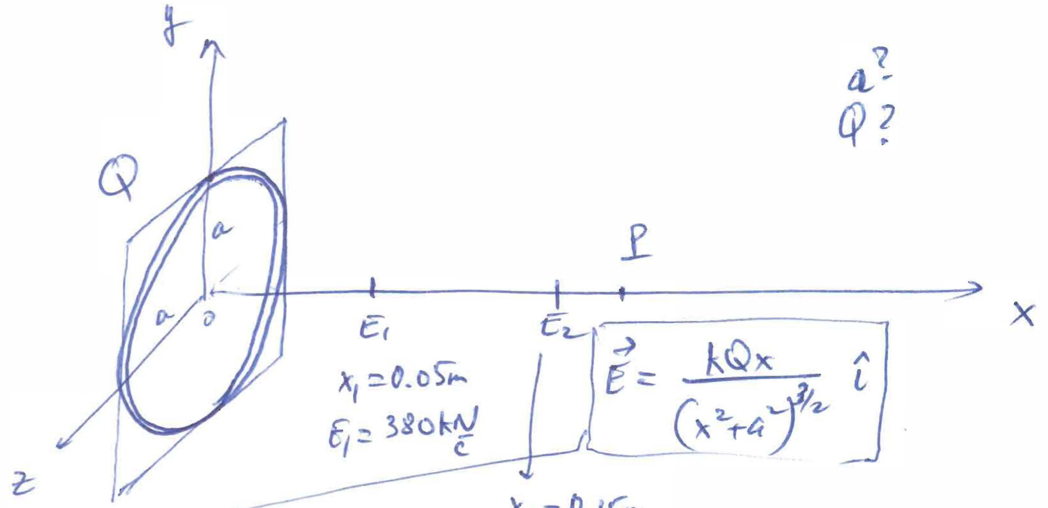
$$y = \frac{1}{2} a_y t_{AB}^2 < \frac{d}{2}$$

$$\frac{1}{\cancel{2}} \frac{qE}{m} \frac{L^2}{v^2} < \frac{d}{\cancel{2}} \rightarrow \frac{qEL^2}{md} < v^2$$

$$\boxed{L\sqrt{\frac{qE}{md}} < v}$$

$$\boxed{v_{min} = L\sqrt{\frac{qE}{md}}}$$

20.65



a? Q?

$x_1 = 0.05\,m$

$E_1 = 380\,\frac{kN}{C}$

$$\boxed{\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{i}}$$

$x_2 = 0.15\,m$

$E_2 = 160\,\frac{kN}{C}$

a) $$\left[\frac{E_1}{E_2} = \frac{380}{160} = \frac{x_1}{x_2}\cdot\frac{(x_2^2+a^2)^{3/2}}{(x_1^2+a^2)^{3/2}}\right]^{2/3}$$

$$\left[\frac{380}{160}\right]^{2/3} = \left(\frac{1}{3}\right)^{2/3}\frac{0.15^2+a^2}{0.05^2+a^2} \rightarrow \boxed{a = 0.07\,m}$$

b) $$E_1 = \frac{kQx_1}{(x_1^2+a^2)^{3/2}} \rightarrow Q = \frac{E_1(x_1^2+a^2)^{3/2}}{kx_1}$$

$$= \frac{380\times10^3\,(0.05^2+0.07^2)^{3/2}}{9\times10^9\times0.05}$$

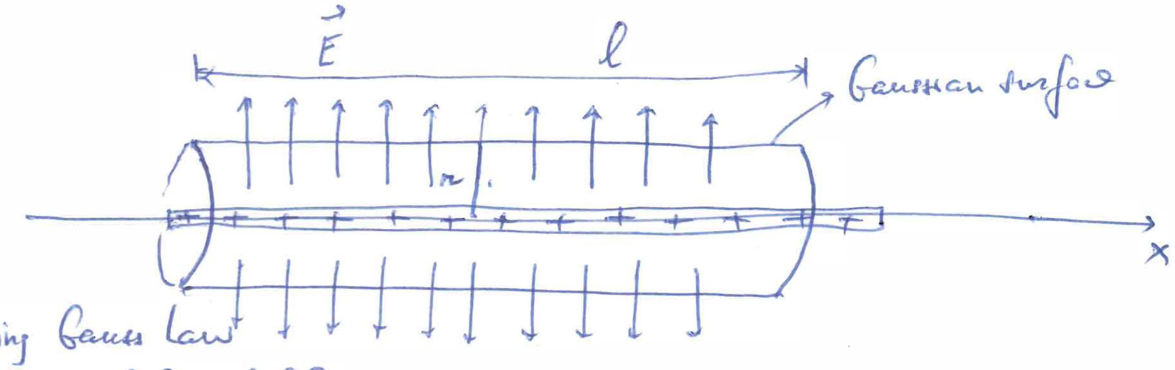
$$\boxed{Q = 538\,nC}$$

$\downarrow$ 10^{-9}

Method #2 : Calculation of $\vec{E}$ using Gauss Law :

Example #2 : Very long line of charge (linear charge density λ)

$$\lambda = \frac{dq}{dx}$$



Using Gauss law to find electric field:

1) Gaussian surface: such that E is constant on the surface.

$$\phi = \oint \vec{E} \cdot d\vec{A} = E_\perp A$$

→ A cylinder of radius r with its axis along the line of charge.

2) Gaussian surface $\begin{cases} \text{Body} : E_\perp = E \\ \text{Left side} : E_\perp = 0 \\ \text{Right side} : \end{cases}$ (E perpendicular to the left side has to point along $-x$, since all electric fields are perpendicular to x)

$\vec{E}$

Similarly : $E_\perp = 0$

$$\phi = E_\perp A = E_\perp A_{Body} + \underbrace{E_{\perp L}}_{0} A_{Left} + \underbrace{E_{\perp R}}_{0} A_{Right} = E_\perp A_{Body} = E A_{Body}$$

$$\phi = E \cdot 2\pi r l$$

3) Gauss law : $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$

$$E\, 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0} \rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{2k\lambda}{r}}$$

agrees with vector superposition result.

Method #3 Electric Potential (Ch 22)

Electric Potential

Potential energy difference b/w points A & B in mechanics:

$$\Delta U_{AB} = - \int_A^B \vec{F} \odot d\vec{l}$$

force applied

scalar product

infinitesimal displacement

Electric interaction : $\vec{F} = q' \vec{E}$

test charge

Electric potential energy difference b/w points A & B:

$$\Delta U_{AB} = - q' \int_A^B \vec{E} \cdot d\vec{l} \qquad (\text{unit SI: J})$$

Electric potential difference b/w points A & B:

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q'} = - \int_A^B \vec{E} \cdot d\vec{l} \qquad \left(\text{unit SI: } \frac{J}{C}\right)$$

V (Volt)

$$\vec{E} = -\vec{\nabla}(\Delta V_{AB})$$

$\vec{\nabla}$: gradient vector $= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$

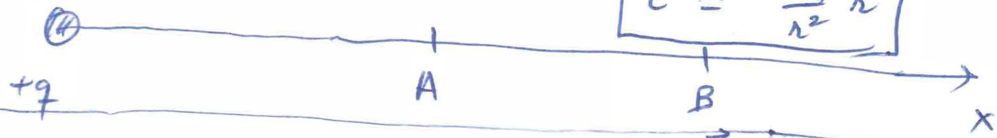
Electric field can be calculated by differentiating the electric potential with a minus sign.

Example #1: Calculation of $\vec{E}$ for a point charge using the Electric Potential:

For a point charge q: $V(r) = \frac{kq}{r} \rightarrow \vec{E} = -\vec{\nabla} V = -\left(\frac{\partial}{\partial r} V\right)\hat{r}$

$$= -\frac{\partial}{\partial r}\left(\frac{kq}{r}\right)\hat{r} = -kq\overbrace{\left(\frac{\partial}{\partial r}\frac{1}{r}\right)}^{-\frac{1}{r^2}}\hat{r}$$

$$\boxed{\vec{E} = \frac{kq}{r^2}\hat{r}}$$



First time contact with electric potential

$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{l} = -\int_A^B \overbrace{\frac{kq}{x^2}\hat{i}}^{\vec{E}} \underbrace{\cdot}_{1.1\cos 0 = 1} \overbrace{\hat{i}\,dx}^{d\vec{l}} = -kq\underbrace{\int_A^B \frac{dx}{x^2}}_{\left[-\frac{1}{x}\right]_A^B}$$

$$\Rightarrow \Delta V_{AB} = kq\left(\frac{1}{x_B} - \frac{1}{x_A}\right)$$

Use a reference point (zero potential : $x_A \Rightarrow \infty$)

$$\Delta V_{\infty B} = kq\frac{1}{x_B}$$

$\vec{x}$ could be any direction.

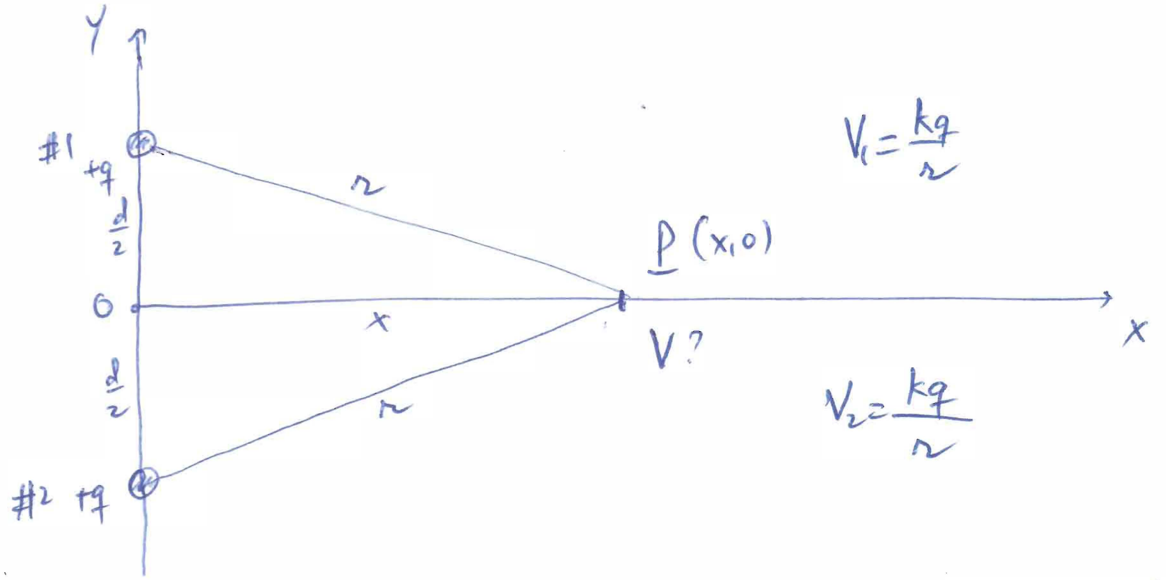
$$\Delta V_{\infty B} = \frac{kq}{r_B}$$

Always same reference point @ ∞ $\rightarrow$ Convention is

$$\boxed{V(r) = \frac{kq}{r}}$$

Electric potential due to a point charge, is a scaler (unit $\frac{J}{C}$ or V)

Example #2: Calculation of $\vec{E}$ due to 2 point charges @ a point P along the midline b/w the 2 charges.



What is V @ P, due to 2 point charges? → $V = V_1 + V_2$

V_1: electric pot. due to charge #1; V_2: elect. pot. due to charge #2

Strength for Method #3: adding numbers instead vectors

$$V(@P) = \frac{2kq}{r} = \frac{2kq}{\left(x^2 + \left(\frac{d}{2}\right)^2\right)^{1/2}}$$

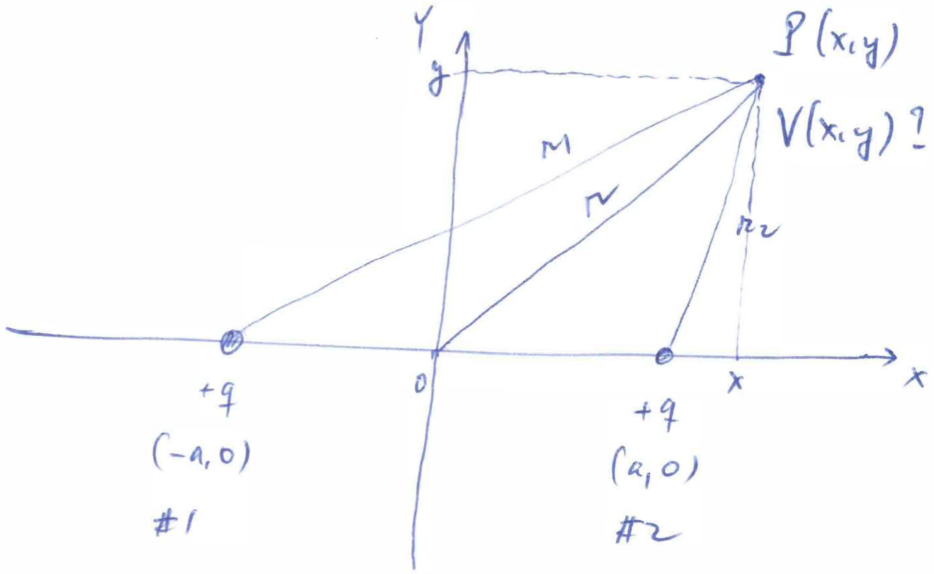
$$\vec{E}(@P) = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\,\hat{\imath}$$

$$= -2kq \frac{\partial}{\partial x} \frac{1}{\left[x^2 + \frac{d^2}{4}\right]^{1/2}}\,\hat{\imath} = -2kq \frac{\partial}{\partial x}\left[x^2 + \frac{d^2}{4}\right]^{-\frac{1}{2}}\hat{\imath}$$

$$= kq\left[x^2 + \frac{d^2}{4}\right]^{\left(-\frac{1}{2}-1\right)} 2x\,\hat{\imath} = 2kqx\left[x^2 + \frac{d^2}{4}\right]^{-\frac{3}{2}}\hat{\imath}$$

$$\boxed{\vec{E} = \frac{2kqx}{\left[x^2 + \frac{d^2}{4}\right]^{3/2}}\,\hat{\imath}}$$

22.53



a) $V(x,y) = \underset{\text{due charge \#1}}{V_1(x,y)} + \underset{\text{due to charge \#2}}{V_2(x,y)}$

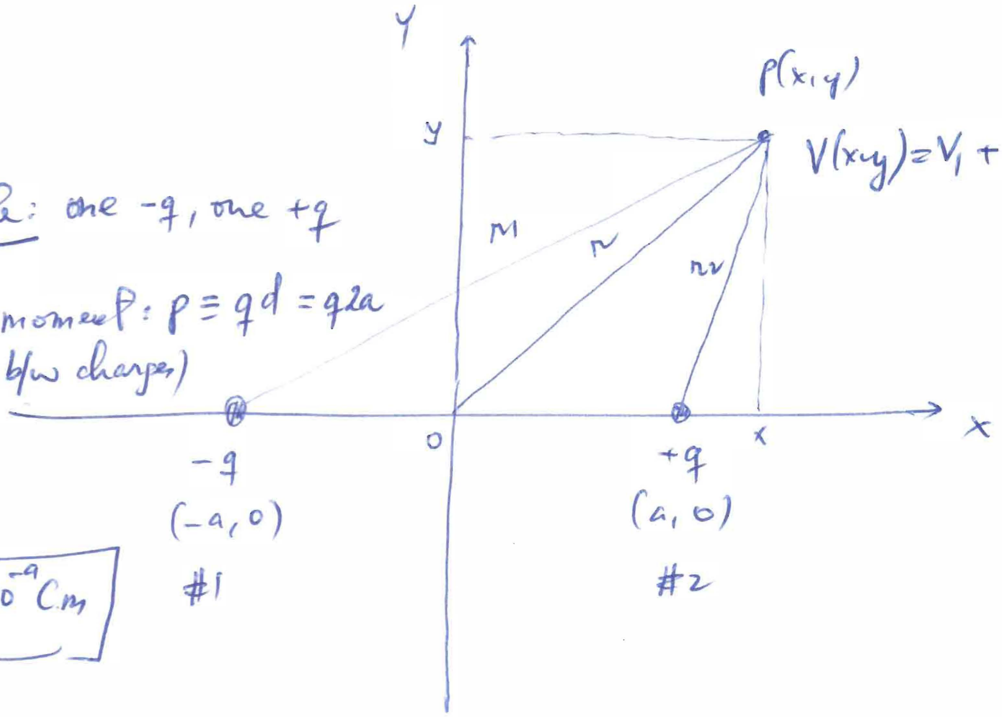
$$= \frac{kq}{r_1} + \frac{kq}{r_2} = \frac{kq}{\left[(x+a)^2+y^2\right]^{1/2}} + \frac{kq}{\left[(x-a)^2+y^2\right]^{1/2}}$$

b) What is $V(x,y)$ approximately if P is very far away from the two charges : $x \gg a$ & $y \gg a$

$$V(x,y) \simeq \frac{kq}{(x^2+y^2)^{1/2}} + \frac{kq}{(x^2+y^2)^{1/2}} = \frac{2kq}{(x^2+y^2)^{1/2}}$$

$$= \frac{2kq}{r}$$ → Far away the electric potential is that of one point charge of value $2q$

22.54



$V(x,y) = V_1 + V_2$

Dipole: one $-q$, one $+q$

↓

Dipole moment: $p \equiv qd = q2a$

(d: sep. b/w charges)

$\boxed{p = 2.9 \times 10^{-9}\ \mathrm{Cm}}$

$$V(x,y) = kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = kq\left[\frac{1}{[(x-a)^2+y^2]^{1/2}} - \frac{1}{[(x+a)^2+y^2]^{1/2}}\right]$$

a) P Along dipole axis or x-axis → $y=0$

$$V(x,0) = kq\left[\frac{1}{x-a} - \frac{1}{x+a}\right] = kq\left[\frac{x+a-x+a}{(x-a)(x+a)}\right]$$

$$= \frac{kq\,2a}{x^2-a^2} = \frac{kp}{x^2-a^2} \simeq \frac{kp}{x^2} = \frac{9\times10^{9} \times 2.9\times10^{-9}}{0.1^2}$$

{0° to axis
r = 0.1 m.
r >> a

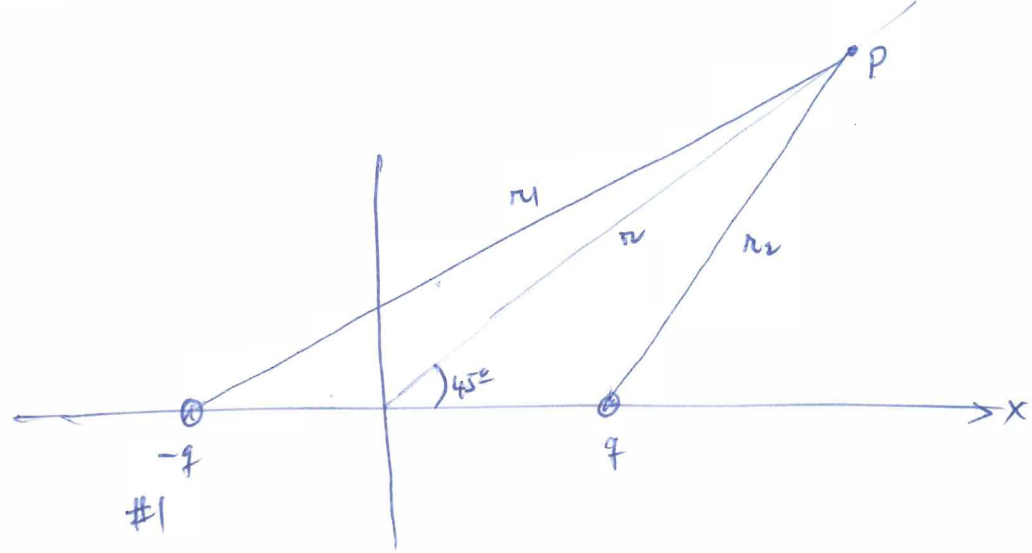
dipole sep. (2a) << x = 10 cm (data)

$= 2.61 \times 10^{3}$ V

b) P @ {45° to axis, r = 0.1 m., r >> a} $x = y = r\cos 45°$

$$V(x,y) = kq\left[\frac{1}{((x-a)^2+y^2)^{1/2}} - \frac{1}{((x+a)^2+y^2)^{1/2}}\right]$$

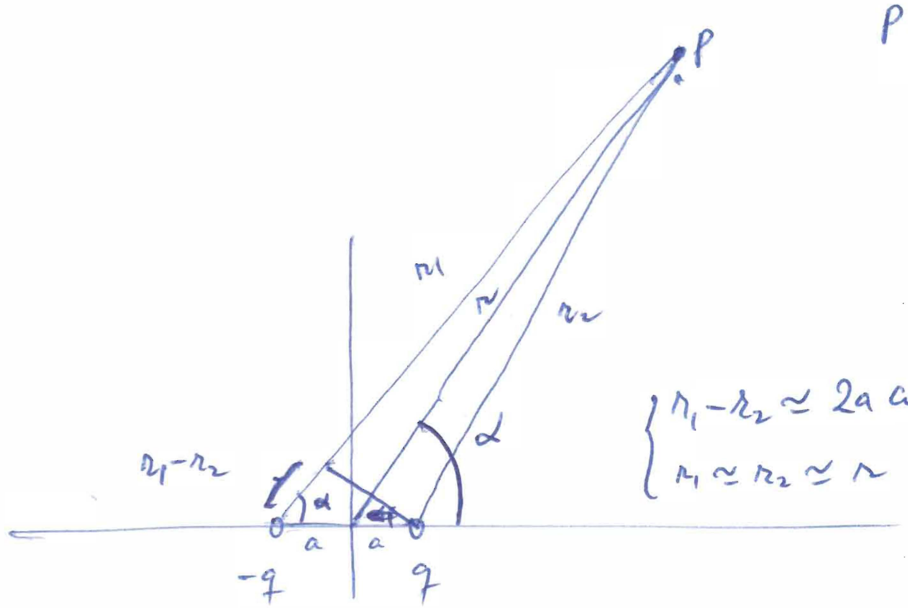
31, 70, 67



Here, polar coordinates are more useful.

$$V(x,y) = kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = kq\,\frac{r_1 - r_2}{r_1 r_2} \simeq kq\,\frac{2a\cos\alpha}{r^2}$$

↓ P very far away



$$\begin{cases} r_1 - r_2 \simeq 2a\cos\alpha \\ r_1 \simeq r_2 \simeq r \end{cases}$$

$\alpha = 45°$

$r = 0.1\,m$

$$V(x,y) = \frac{k\overbrace{(q\,2a)}^{p}\cos\alpha}{r^2} = \frac{9\times10^{9}\times 2.9\times10^{-9}\cos 45°}{0.1^2} = 1.85\,kV$$

c) P along bisector: $\alpha = 90°$:

$$V(x,y) = \frac{9\times10^{9}\times 2.9\times10^{-9}\,\overbrace{\cos 90°}^{0}}{0.1^2} = 0$$

22-31 $V(x,y,z) = 2xy - 3zx + 5y^2$

$P(x = 1m, y = 1m, z = 1m)$

a) $V(1,1,1) = 2 - 3 + 5 = 4V$

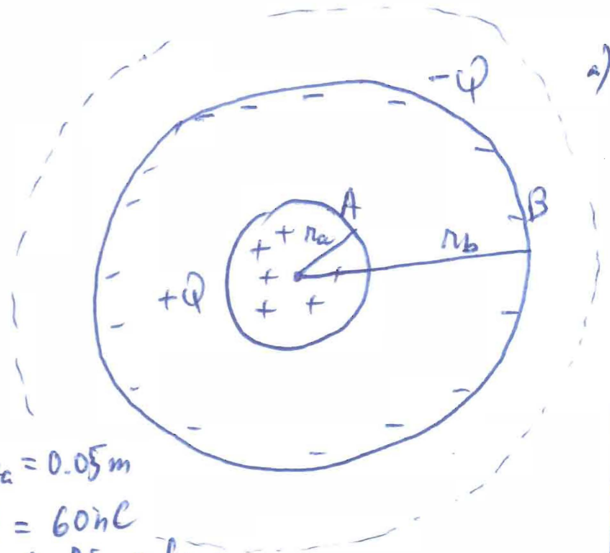
b) $\vec{E} = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$= -\hat{i}(2y - 3z) - \hat{j}(2x + 10y) - \hat{k}(-3x)$

$\vec{E}(1,1,1) = -\hat{i}(-1) - \hat{j}(12) - \hat{k}(-3)$

$= \hat{i} - 12\hat{j} + 3\hat{k} \quad \left(\frac{N}{C}\right)$

22-70



$\begin{cases} r_a = 0.05m \\ Q = 60nC \\ \text{conducting sphere} \end{cases}$

$\begin{cases} r_b = 0.15m \\ Q = -60nC \\ \text{conducting shell} \end{cases}$

a) Find V @ $r = r_a$

$V(r = r_a) = \Delta V_{\infty A} = -\int_{\infty}^{A} \frac{kQ}{r^2} dr$

$= +kQ\left[\frac{1}{r}\right]_{\infty}^{A} = \frac{kQ}{r_A}$

(True if there was no outer shell)

$V(r=r_a) = \Delta V_{\infty A} = \underbrace{\Delta V_{\infty B}}_{} + \Delta V_{BA}$

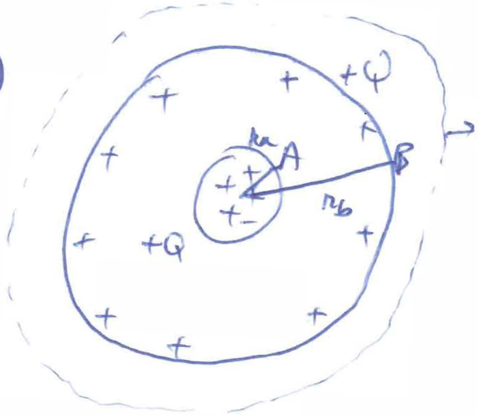
E outside outer shell is 0! { Gaussian surface enclosing shell + sphere contains +Q−Q = 0

$= -\int_{B}^{A} \frac{kQ}{r^2} dr = kQ\left[\frac{1}{r}\right]_{B}^{A}$

$= kQ\left[\frac{1}{r_A} - \frac{1}{r_B}\right] = 9\times10^9 \times 60\times10^{-9} \left[\frac{1}{0.05} - \frac{1}{0.15}\right]$

$\boxed{V(r=r_a) = 7200\ V}$

b)



E here outside shell + sphere is not zero.

$$V(r=r_a) = \Delta V_{\infty A} = \underbrace{\Delta V_{\infty B}}_{} + \underbrace{\Delta V_{BA}}_{}$$

ΔV_{BA}: 7200 V (only changing charge on shell!)

$\Delta V_{\infty B}$:

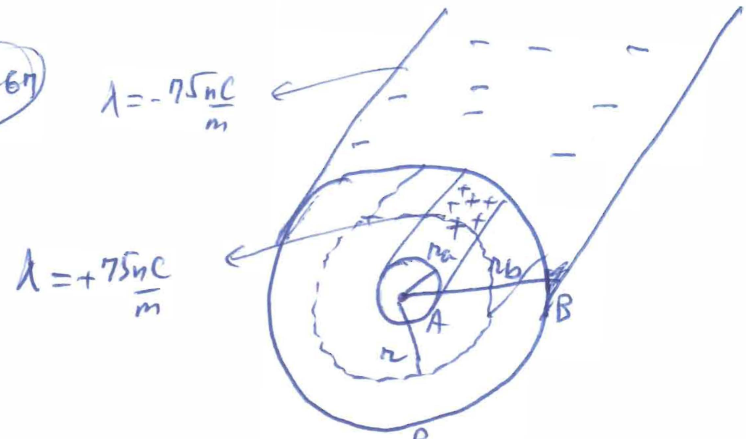
$$-\int_{\infty}^{B} \frac{k2Q}{r^2}\,dr$$

$$= k2Q\left[\frac{1}{r}\right]_{\infty}^{B} = \frac{k2Q}{r_B} = \frac{9\times10^9\times2\times60\times10^{-9}}{0.15}$$

$$= 7200\ V$$

$$V(r=r_a) = 7200 + 7200 = 14400\ V.$$

22-67



Coaxial cable { inner cylinder + outer shell with the same axis

c) $$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{r}$$

Field b/w inner & outer conductor { Gaussian Law with G. surface of radius r $r_a < r < r_b$

↓

Electric due to inner conductor (very long wire)

$$\rightarrow E = \frac{2k\lambda}{r}\hat{r}$$

72

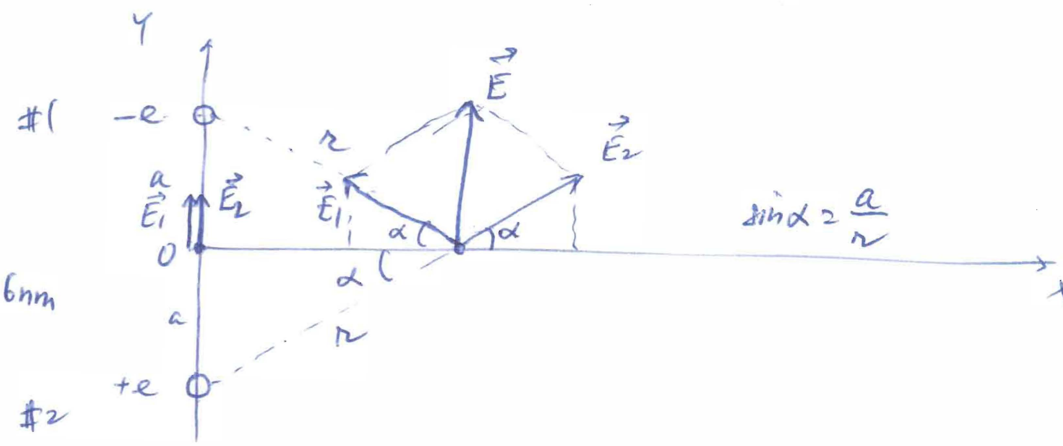
$$\Delta V_{AB} = -\int_A^B \frac{2k\lambda}{r}\,dr = -2k\lambda \int_A^B \frac{dr}{r} = -2k\lambda \ln\left(\frac{r_B}{r_A}\right)$$

$$= -2\times 9\times10^9 \times 75\times10^{-9} \ln\left(\frac{10\,mm}{2\,mm}\right) = -2170\,V$$

b) if λ for outer conductor changes to $+150\frac{nC}{m}$

ΔV_{AB} = same = $-2170V$ (since this would not change the electric b/w inner & outer conductor. It only change field outside outer conductor)

2.50



$a = 0.6nm$

a) $\vec{E}(x=0, y=0) = \vec{E}_1 + \vec{E}_2 = \frac{ke}{a^2}\times 2\,\hat{j} = \frac{9\times10^9\times1.6\times10^{-19}\times2}{0.6\times10^{-9}}\,\hat{j}\left(\frac{N}{C}\right)$

$= 8\times10^9\,\hat{j}\left(\frac{N}{C}\right)$

b) $\vec{E}(x=2nm, y=0) = 2E_y\,\hat{j} = 2\left(\frac{ke}{r^2}\right)\sin\alpha\,\hat{j} = 2\frac{kea}{r^3}\,\hat{j}$

$= 2\frac{kea}{(x^2+a^2)^{3/2}}\,\hat{j} = \hat{j}\,2\frac{9\times10^9\times1.6\times10^{-19}\times0.6\times10^{-9}}{[(2\times10^{-9})^2+(0.6\times10^{-9})^2]^{3/2}}$

$r = (x^2+a^2)^{1/2}$

$= \hat{j}\,190\times10^6\,\frac{N}{C}$

c) $\vec{E}(x=-20nm, y=0) = \hat{j}\,2\frac{9\times10^9\times1.6\times10^{-19}\times0.6\times10^{-9}}{[(20\times10^{-9})^2+(0.6\times10^{-9})^2]^{3/2}} = 216\times10^3\,\frac{N}{C}$