Temperature: a macroscopic property (we can sense & measure), it is a consequence of microscopic behavior of matter.

Unit: °F (degree Fahrenheit) °C (degree Celsius) SI: °K (degree kelvin) (absolute zero is 0°K)

\[ T_F = \frac{9}{5} T_C + 32 \quad \text{→ (body temp. \( \sim \) 100°F)} \]

\[ T_C = \frac{5}{9} (T_F - 32) \quad \text{→ (boiling point of water \( \sim \) 100°C)} \]

\[ T_C = T_K - 273.16^\circ\text{K} \quad \text{Triple point of water (3 phases coexist @ this temperature)} \]

Bowl of soup 120°F

120°F in thermal contact (bowl & table can exchange heat)

60°F 60°F after 8 hours

they have arrived at thermodynamic equilibrium (T.D.) or same temperature by exchanging heat
Heat transfer rate: \[ H \rightarrow \text{how fast heat can be transferred or exchanged between two objects.} \]

1. Lower surface contact
2. Layer thickness \( dx \)
3. Lower thickness \( dx \)
4. Larger \( \Delta T \)

1 & 4: Larger \( \Delta T \) \( \rightarrow \) faster heat transfer rate \( \rightarrow \) \( H \)

5. Faster transfer depending on material:
   - Metal are faster heat conductors.
   - Metal are also good electrical conductors:
     - Free electrons
     - Energy carriers

"Temperature is a consequence of microscopic properties of matter."
Units for $H$: SI

- Length $\rightarrow L \rightarrow$ meter (m)
- Mass $\rightarrow M \rightarrow$ kilogram (kg)
- Temperature $\rightarrow T \rightarrow$ degree Kelvin (°K)
- Time $\rightarrow T \rightarrow$ second (s)
- Energy $\rightarrow M \frac{L^2}{T^2} \rightarrow$ joule (J)
  - Mechanical (kinetic, potential)
  - Thermal or heat
- Power $\rightarrow M \frac{L^2}{T^3} \rightarrow$ joule/s $= \text{watt (W)}$

Dimension of $H$

$[H] = \frac{[k][A][\frac{[OT]}{[DX]}]}{[Time]} \rightarrow \frac{[E]}{[Time][OT][A]}$

Unit for $k$: $\frac{J}{s \ m^0 K}$

**$K$**

- SI: $\frac{W}{m \ K}$
- British: $\frac{\text{Btu}}{\text{hr} \ \text{ft}^2 \ \text{°F}}$

**$R$-Factor**

$R = \frac{dx}{k}$ (larger $R$ better insulation)

$[H] = -\left(\frac{kA\Delta T}{DX}\right) = -\frac{A\Delta T}{R}$

Units:

- SI: $\frac{m^0 K}{W}$
- British: $\frac{ft^2 \ \text{°F}}{\text{Btu} \ \text{hr}}$

$R \leq 6 \rightarrow R = 6 \frac{ft^2 \ \text{°F}}{\text{Btu} \ \text{hr}}$
Mechanisms of heat transfer:

1. Initially: stove top is heated.
   Heat transfer: stove top → pan by [conduction] (via free electrons in metal pan)

2. Water at bottom gets heated
   Heat transfer: bottom water → top water by [convection] (circulation of water molecules)

3. Hand feels some heat
   Heat transfer: hot water → hand by [radiation] (does this require a medium? No! we feel heat from sun!)

   Convection:
   → Why?
   hot molecules & atoms tend to stay apart → lower density
   → tend to rise above liquid or gas, with higher density
   Reverse process for higher density → tend to sink

   Stefan–Boltzman law: heat loss rate (per unit time) by radiation of an object of area $A$ & temp. $T$:
   $P = e \sigma A T^4$
   $\sigma$ is Stefan–Boltzman constant = $5.67 \times 10^{-8}$ W m$^{-2}$K$^{-4}$

   Radiation:
Heat balance equation:

Assume: system water (in bucket) & copper is isolated:

\[ \Delta Q_{\text{Cu}} + \Delta Q_{\text{w}} = 0 \]

(heat lost by copper = heat gained by water)

\[ m_{\text{Cu}} \cdot c_{\text{Cu}} \cdot (T_f - T_{\text{in}}) + m_{\text{w}} \cdot c_{\text{w}} \cdot (T_f - T_{\text{w}}) = 0 \]

- Copper gets cooled down
- Water gets heated

Specific heat:

\[ c_{\text{w}} = 4184 \, \frac{J}{\text{kg} \cdot \text{K}} \]
\[ c_{\text{Cu}} = 386 \, \frac{J}{\text{kg} \cdot \text{K}} \]

Capacity to absorb heat per unit mass

Material-specific!
Ch 17  Thermal Behavior of Matter

Matter: 3 states

- Density
  - Low → Gas: almost free molecules
  - Medium → Liquid: some interaction between molecules
  - High → Solid: molecules are not free to move

Gas → Ideal Gases: completely free (non-interacting) molecules.

- Macroscopic quantities or properties:
  - \( T \) (temp., K)
  - \( P \) (pressure, Pa)
  - \( V \) (volume, m\(^3\))

- Microscopic quantity: \( N \) (num. of molecules)

\[ T, N, V, P \]

State 1

Assuming:
- Gas can't escape
- \( 1 \rightarrow 2 \) isothermal process (very slowly)

Equation of state for an ideal gas: \( 1 \rightarrow 2 \) \( N_1 T_1 = N_2 T_2 \)

\[ P_1 V_1 = k N_1 T_1 \]
\[ P_2 V_2 = k N_2 T_2 \]

\[ PV = kNT \]

\( k = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J/} \text{K} \)
Avogadro number: \( N_A = 6.022 \times 10^{23} \)

\( N = n \cdot N_A; \quad n: \text{number of mole} \)

\[ PV = k \cdot \frac{nN_A}{N} \cdot T = n \cdot kN_A \cdot T \]

\[ R = 8.314 \quad \text{J/k mol} \]

\( \Rightarrow PV = nRT \quad \text{Ideal Gas} \)

If we consider interactions b/w gas molecules \( \Rightarrow \text{Real Gas} \)

\( \Rightarrow \text{Eq. of state for Real Gas or Van der Waals equation} \)

\[ (P + \frac{n^2a}{V^2})(V - nb) = nRT \]

a, b are given constants, depending on the strength of interactions b/w molecules.

Closer look at the connection b/w microscopic behavior (that of single gas molecule \( \Rightarrow \) and its transfer of momentum to the container) and the macroscopic properties of the gas (such as its temperature, pressure).

Mechanics for a gas molecule \( \Rightarrow \) a particle of mass \( m \)

\( \begin{align*}
\text{System: one gas molecule + container } & \rightarrow \text{no net external force on this system, i.e., } \text{what about their } m^2 \text{ same in } 1, 2, 3 \rightarrow \text{will not change our result. } \\
\end{align*} \)

\( \rightarrow \text{motion along perpendicular directions are independent} \)
Newton's 2nd Law: \[
F_{net,x} = \frac{dp_x}{dt}
\]
\[
0 = \frac{dp_x}{dt}
\]
\[\rightarrow \text{\textit{p}}_x \text{ is conserved}
\]
\[mV_{x,T}: \text{linear momentum in the x-direction.}
\]

\[
(mV_x)^{\text{Total linear momentum}} = (mV_x)^{\text{Total linear momentum}}
\]

\[
mV_x + mV_x = -mV_x + 2mV_x
\]
\[\text{mass of gas molecule}
\]
\[
mV_x + mV_x = -mV_x + 2mV_x
\]
\[\text{mass of l.m. gas molecule}
\]
\[
\text{Conclusion}: \text{after hitting right wall, a gas molecule i has transferred } 2mV_x \text{ (momentum) to the container.}
\]

\[
P_i = \frac{F_i}{A} = \frac{\Delta p_i}{\Delta t} = \frac{2mV_x}{A} = \frac{mV_x^2}{A \cdot l} = \frac{mV_{x}^{2}}{\text{vol.}}
\]

\[
P = \sum_{i=1}^{N} P_i = m \frac{\Sigma}{\text{vol}} \sum_{i=1}^{N} \frac{V_x^2}{N}
\]
\[
P = mN \frac{\Sigma V_x^2}{\text{vol} \cdot N}
\]
\[
P = mN \frac{\Sigma V_x^2}{\text{vol}} \rightarrow \frac{P}{V} = \frac{mN \Sigma V_x^2}{\text{vol} \cdot V}
\]
\[
\text{\textit{p}}V = mN \overline{V_x^2}
\]
\[
\overline{V_x^2} \text{ is one of the 3 components of } \overline{V_x^2} = \frac{V_x^2}{3}
\]
$PV = mN \frac{v^2}{3}$

The av. of $v^2$ in one direction is the average of $v^2$ in 3D divided by 3.

Now relate this to the eq. of state for an ideal gas: $PV = kNT$

$kNT = mN \frac{v^2}{3} \rightarrow kT = \frac{1}{3}m \overline{v^2}$

$3kT = m \overline{v^2}$

$\frac{3}{2}kT = \frac{1}{2}m \overline{v^2}$

Proportional Average kinetic energy for one molecule

Recall: Temperature: a macroscopic property that is a consequence of microscopic behavior

We can use reverse connection as well:

Room: $T_1 = 298 \, ^\circ\text{K}$ (normal condition) \( \frac{1}{2}m \overline{v_1^2} < \frac{1}{2}m \overline{v_2^2} \)

$T_2 = 320 \, ^\circ\text{K}$
**Dimensions**:

\[ 3D \quad \rightarrow \quad \frac{3}{2} kT = \frac{1}{2} m \bar{v}^2 \]

\[ 1D \quad \rightarrow \text{average KE per molecule in 1D} = \frac{1}{2} kT \]

\[ 2D \quad \rightarrow \quad kT \]

\[ \text{av. KE of a gas molecule per degree of freedom is } \frac{1}{2} kT. \]

- Molecule formed of 2 atoms that are connected: \( O_2, N_2, \text{ etc.} \)

\[ \begin{array}{cc}
0 & 1 \\
1 & 2
\end{array} \quad \text{d.o.f.} = 5 \quad x, y, \dot{x}, \dot{y} \]

\[ \begin{array}{cc}
0 & \rightarrow \quad \text{d.o.f.} = 3 \\
1 & \rightarrow \quad \text{d.o.f.} = 3 \\
0 & 2 & \rightarrow \quad \text{d.o.f.} = 6
\end{array} \]

\[ \text{Av. KE per diatomic molecule is } \frac{5}{2} kT \]

\[ \text{Av. KE per monatomic molecule is } \frac{3}{2} kT \]

- \( x \): 0 moves left \( \rightarrow \) so will 2
- \( y \): 0 moves up, 2 not necessarily will move up
- \( z \): 0 moves out of pipe, 2 not necessarily will move out of pipe (can rotate with middle pivot)
milk \( m_m = 2 \text{ oz} \)

\( T_m = 3^\circ \text{C} \)


Cacao
\( T_c = 90^\circ \text{C} \)
\( m_c = 6 \text{ oz} \)

Cocoa
\( \Delta Q_c + \Delta Q_m = 0 \)

\( m_c c_c (T_f - T_c) + m_m c_m (T_f - T_m) = 0 \)

\[
\frac{m_c c_c}{c_m} \left( m_c c_c + m_m c_m \right) T_f = m_c c_c T_c + m_m c_m T_m
\]

\[
T_f = \frac{m_c T_c + m_m T_m}{m_c + m_m}
\]

**Heat Balance Equation:** (isolated system)

\[
\begin{align*}
T & \rightarrow \text{in K} \rightarrow T_c = 90^\circ + 273.16^\circ \\
& = 363.16^\circ \text{K}
\end{align*}
\]

\[
\begin{align*}
T_m & = 3^\circ + 273.16^\circ \\
& = 276.16^\circ \text{K}
\end{align*}
\]

\( m \rightarrow \text{in kg (not necessary in this particular case, as mass is above and below the fraction)} \)

\[
T_f = \frac{6 \times 363.16 + 2 \times 276.16}{8} = 341.46^\circ \text{K}
\]

Can go back to Celsius:

\[
T_f = 341.46 - 273.16 = 68.3^\circ \text{C}
\]

If I work in the Celsius:

\[
T_f = \frac{m_c (T_{c_k} - 273.16) + m_m (T_{m_k} - 273.16)}{m_c + m_m}
\]

\[
= \frac{m_c T_{c_k} + m_m T_{m_k}}{m_c + m_m} + \frac{(m_c + m_m)(-273.16)}{(m_c + m_m)}
\]

\[
T_f = \frac{T_{c_k}}{T_{m_k}} - 273.16
\]
Conversion factors (Appendix C)

\[
\begin{align*}
1 \text{ oz} &= 0.02835 \text{ kg} \\
1 \text{ lb} &= 0.454 \text{ kg} \\
1 \text{ in} &= 0.0254 \text{ m}
\end{align*}
\]

16.49

1) Write down the data
2) Write down the equation
3) Solve for what is needed

\[
\text{Fan \hspace{1cm} Heat capacity \hspace{1cm} } G = 1400 \frac{\text{J}}{\text{K}}
\]

\[
\text{Stove \hspace{1cm} Top \hspace{1cm} } P = 1000 \text{ W}
\]

\[
\text{Microwave \hspace{1cm} } P = 675 \text{ W}
\]

Heat capacity: \( G = c \cdot m \)  
\[ m \text{ specific mass} \text{ heat} \]

Relating to what we said: \( \Delta Q = m \cdot c \cdot \Delta T = G \cdot \Delta T \)

Question: How much water before it is quicker to heat using stove top?

Let's compare time it takes to heat an amount of water \( M \) so its temp increases \( \Delta T \), using stove top & microwave
Stove top

\[ \Delta t_s \]

\[ H_s = \frac{\Delta Q}{\Delta t_s} \]

Heat transfer per unit time

1000 W

\[ \Delta t_s = \frac{\Delta Q}{1000} = \frac{(c_{\text{pan}} + c_w) \Delta T}{1000} \]

Heat transferred from stove top (at 1000 W) is absorbed by pan & water

\[ c_{\text{pan}} = 1400 \ \frac{\text{J}}{\text{kg} \cdot \text{K}} \]

\[ c_w = 4184 \ \frac{\text{J}}{\text{kg} \cdot \text{K}} \]

\[ c_{\text{tw}} = m_w c_w \]

\[ \Delta t_s = \frac{(m_w c_w + c_{\text{pan}}) \Delta T}{1000} \]

When is quicker w/ stove top?

\[ \Delta t_s < \Delta t_m \]

@ what \( m_w \)?

\[ \frac{(m_w c_w + c_{\text{pan}}) \Delta T}{1000} < \frac{m_w c_w \Delta T}{625} \]

\[ \frac{c_{\text{pan}}}{1000} < m_w \left( \frac{c_w}{625} - \frac{c_w}{1000} \right) = m_w (0.375) \]

\[ \frac{c_{\text{pan}}}{1000 c_w 0.375} < m_w \]

\[ \frac{1400}{375 \times 4184} = 0.558 \text{ kg} < m_w \]

Microwave

\[ \Delta t_m \]

\[ H_m = 625 \ \frac{\text{W}}{\text{s}} = 625 \text{ W} \]

\[ \Delta t_m = \frac{\Delta Q}{625} = \frac{c_{\text{water}} \Delta T}{625} \]

Heat transferred in from microwave is absorbed by water (paper towel has \( c' \approx 0 \))
b) Rate of temperature increase, or \( \frac{\Delta T}{\Delta t} \)

\[ m_w = 0.558 \text{ kg} \rightarrow \frac{\Delta T}{\Delta t} = \frac{m_w c_w + C_{pan} \Delta T}{1000} \rightarrow \frac{\Delta T}{\Delta t} = \frac{1000}{m_w c_w + C_{pan}} \]

\[ \frac{\Delta T}{\Delta t} = \frac{1000}{0.558 \times 184 + 1400} = 0.268 \text{ \degree C/s} \]
Thermal Behavior of Matter (Cont.)

When heat is supplied to matter, what are the consequences?

1) $\Delta T$ ($\Delta Q$ produces a change of temp.):
   
   $\Delta Q = mc \Delta T$

2) Expansion:
   - Linear expansion coefficient $\alpha = \frac{\Delta L}{L \Delta T}$
   - Volume expansion coefficient $\beta = \frac{\Delta V}{V \Delta T}$

3) Change of phase:
   - $\Delta Q = m \Delta T$
     - "Latent heat"
     - $L_s$ (Latent heat of sublimation)
     - $L_f$ (Latent heat of fusion)
     - $L_v$ (Latent heat of vaporization)
Table 17.1:

Water:

\[ L_f = 334 \text{ kJ/kg} \]

\[ L_v = 9257 \text{ kJ/kg} \]

(it takes 334 kJ to melt one kg of ice)

Water temp. max is 100°C!

There is \( DT = 0 \) since water starts boiling until it evaporates.

(Note the different equation to use in a change of phase!)
Phase diagram: \( P \) vs. \( T \).

**Normal matter**
- At a fixed \( T_i \), if we increase \( P \): gas → liquid → solid.
- If we press on ice, it melts. (Ice skaters melt the ice!)

**Water**
- At fixed \( T_i \), if we increase \( P \): gas → liquid → solid.
- If we press on ice, it melts.

**Triple point (3 phase coexist at the same time)**

@ this point (fixed \( T_i \)) if we increase \( P \) further, solid goes back to liquid!

1st Law of Thermodynamics: \[ DU = Q - W \]

- Heat absorbed by a system: \( Q \)
- Work done by system: \( W \)
- Change of internal energy of system: \( DU \)

\[ DU = Q - W \]

**State variable** (does not depend on process)

- **State**: is a point on a \( P, V \) diagram. We have set values for \( P \) & \( V \) in a state.

**Process**: connects two different states.

\( DU \) or change of internal energy of a system \( y/w \) \( 1 \rightarrow 2 \) is the same whether the system went from \( 1 \) to \( 2 \) via isobaric + isovolumic or via an isothermal process.

\( Q \) & \( W \) \( y/w \) \( 1 \) & \( 2 \) individually depend on the particular process.
Work done by a system: \[ W = \int P \, dV \]

(Recall: \[ P = \frac{F}{A} \quad W = F \cdot \Delta x = P \cdot A \cdot \Delta x \frac{\Delta V}{\Delta V} \])

In a PV diagram:

\[ \int P \, dV \text{ is the area under the curve} \]
\[ \Rightarrow \text{Work done is this area.} \]

Processes:
1. Isothermal (\( T \) is const.)
2. Isochoric (\( V \) is const.)
3. Isobaric (\( P \) is const.)
4. Adiabatic (no heat is absorbed: \( Q = 0 \))

Since \( Q \) & \( W \) depend on a particular process:

Isothermal (\( T \) is const.)

\[ W = \int P \, dV = nRT \int \frac{dN}{V} = nRT [\ln(V_f) - \ln(V_i)] \]

Ideal gas:
\[ P = \frac{nRT}{V} \]

\[ W = nRT \ln \left( \frac{V_f}{V_i} \right) \]

\[ W = nRT (\ln V_f - \ln V_i) = nRT \ln \left( \frac{V_f}{V_i} \right) \]
To find $Q$ we will find $\Delta U$ in an isothermal process, then use the 1st Law of Thermodynamics: $\Delta U = Q - W = Q - N_l - l$

What is $\Delta U$ in an isothermal process on ideal gas?

**Ideal gas:**
1. no interaction between molecules
2. total internal energy comes from kinetic energy only ($E = \frac{1}{2}mv^2 + P.E.$)
3. $E = \frac{3}{2}kT$ (molecular)
   - $E = \frac{2}{2}kT$ (macro)

2. $\text{Av. KE per molecule in an ideal gas}$
   - is proportional to the temperature
   - $\Delta U_{12} = U_2 - U_1 = \begin{cases} \frac{3}{2}k(T_2 - T_1) = 0 & \text{if } T_2 = T_1 \\ \frac{2}{2}k(T_2 - T_1) = 0 & \text{if } T_2 = T_1 \end{cases}$

$\Delta U = 0$ in any isothermal process

$\Rightarrow [Q = W]$

In an isothermal process the heat absorbed is equal to the work done since $\Delta U = 0$

$\quad T_1 \quad \Rightarrow \quad Q$

$\quad T_2 \quad \Rightarrow \quad W = Q$ if $T_1 = T_2$
$W = \int_1^2 PdV = 0$

1st Law of TD: $\Delta U = Q - W \rightarrow [Q = \Delta U]$

What is $\Delta U$ in an isovolumic process in ideal gas?

$C_v$: specific heat at constant volume

$C_v = \frac{1}{n} \frac{Q}{\Delta T} = \frac{1}{n} \frac{\Delta U}{\Delta T}$

$Q = C_v n \Delta T = \Delta U$
Isobaric Process (const P.)

\[ P = P_1 = P_2 \]

\[ V \quad \Delta V \quad V \]

\[ W = \int_1^2 \frac{P_1}{P_2} dV = P \int_1^2 dV = P(V_2 - V_1) \]

What is \( Q \)?

\( \dot{Q} \): Specific heat at constant \( P \):

\[ \dot{Q} = \frac{1}{n} \frac{Q}{\delta T} \]

\[ Q = \dot{Q} \cdot n \cdot \delta T \]

Ideal gas:

\[ P \cdot V = n \cdot R \cdot T \]

\[ W \quad \text{(isobaric)} \]

\( \Delta U = Q - W \)

\[ n \cdot C_P \cdot T \]

\[ n \cdot R \cdot T \]

\[ C_V = C_P - R \]

\[ G = C_V + R \]

\( \text{This is a connection between an isobaric & isovolumic process} \)

\[ R = 8.3 kJ \text{ mol}^{-1} \text{ K}^{-1} \]

\[ C_P > C_V \]

More heat absorption capacity at constant \( P \)

Than at constant volume.
A system can absorb more heat @ constant P since it can do work (but not in a reversible process!)

\[ Q = 
\]

Adiabatic Process: \( Q = 0 \)

\[ W = -\Delta U \quad (1^{st} \text{ law of TD: } \Delta U = Q - W) \]

\[ W = \frac{P V_i - P V_f}{\gamma - 1} \]

\[ \gamma = \frac{C_p}{C_v} \]

Dimensionless coefficient

\[ \gamma \text{ depends on the type of gas: monatomic or diatomic:} \]

\[ C_v = \frac{1}{n} \frac{\Delta U}{\Delta T} = \begin{cases} \text{monatomic:} & \frac{1}{n} \frac{N \frac{3}{2} k_B T}{\Delta T} = \frac{3}{2} k \frac{N}{n} = \frac{3}{2} \frac{k N_A}{N} = \frac{3}{2} R \frac{N_A}{N} \\ \text{diatomic:} & \frac{5}{2} R \end{cases} \]

\[ U = \frac{N}{2} \frac{3}{2} k T \]

(Total internal energy for an ideal monatomic gas)

\[ N = \text{total # of molecules} \]

\[ n = \text{# of moles} = \frac{N}{N_A} \]

\[ N_A = 6.022 \times 10^{23} \text{ } \]

\[ k = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ } \frac{J}{\text{K}} \]

\[ R = k N_A = 8.314 \text{ } \frac{J}{\text{mol} \cdot \text{K}} \]

\[ \gamma = \begin{cases} \text{Monatomic:} & \frac{3}{2} \frac{R}{k} = \frac{3}{2} \frac{R}{3} = 1.67 \\ \text{Diatomic:} & \frac{5}{2} \frac{R}{k} = \frac{5}{2} \frac{R}{3} = 1.4 \end{cases} \]
**Adiabatic process:**

\[ P_1 V_1^\gamma = P_2 V_2^\gamma \]

Also, because \( PV = nRT \) and \( P = \frac{nRT}{V} \)

\[ nR \frac{T_1}{V_1} = nR \frac{T_2}{V_2} \implies \frac{T_1}{V_1}^{\gamma-1} = \frac{T_2}{V_2}^{\gamma-1} \]

Apply to an ideal gas in an adiabatic process.

---

**(17.39)**

@ \( T = 25^\circ C \) (room temp.) \( \rightarrow P = 180 \text{ atm.} \)

4) How many moles of air \( n \)

Assume ideal gas: \( n = \frac{PV}{RT} \)

\[ P = 180 \text{ atm} \times 1.013 \times 10^5 \text{ Pa} \]

\[ V = \pi (0.1)^2 \cdot 1 = \frac{n}{100} \text{ m}^3 \]

\[ R = 8.314 \frac{\text{J}}{\text{K} \cdot \text{mol}} \]

\[ T = 25 + 273.16 = 298.16 \text{ K} \]

\[ n = \frac{180 \times 1.013 \times 10^5 \times P}{8.314 \times 298.16} = 231 \text{ mol} \]

b) What would be the gas volume @ \( P = 1 \text{ atm} \) & \( T = 25^\circ C \)

\[ n = 231 \rightarrow \frac{PV}{RT} \]

\[ V = \frac{n}{100} \times 180 \text{ m}^3 = 5.65 \text{ m}^3 \]
Thermal resistance of fridge wall is \( R = 0.12 \frac{\text{K}}{\text{W}} \)

\[ R = \frac{R}{A} \]

\[ \Delta Q \]

\[ 15 \text{kg} @ 0^\circ \text{C} \]

\[ 20^\circ \text{C} \]

\[ 0^\circ \text{C} \]

\[ 20^\circ \text{C} \]

\[ \Delta Q \]

\[ \text{Freeze} \rightarrow \Delta Q \]

Heat will come in through walls only delayed by thermal resistance \( R \).

\[ R = \frac{R}{A} \rightarrow \text{R-factor} \]

\( R \rightarrow \text{determine } H \) (Heat transfer rate through fridge's walls)

\[ \text{hours} \rightarrow \text{hours} \]

\[ \Delta t = \frac{\Delta Q}{H} \]

(\( H = \frac{\Delta Q}{\Delta t} \))

\[ \Delta Q \text{ will go into melting the ice or into } \text{melt } L_f \]

\[ H = -kA \frac{\Delta T}{dx} = -\frac{\Delta Q}{R} = -\frac{\Delta Q}{RA} = -\frac{\Delta T}{R} \]

(Heat transferred from outside to inside \( \rightarrow H_{\text{absorbed}} = \frac{\Delta T}{R} \))

\[ \Delta t = \frac{\Delta Q}{H_{\text{absorbed}}} = \frac{m_{\text{ice}} L_f}{\Delta T} = \frac{15 \times 334 \times 10^3}{(20 - 0)} = 3 \times 10^4 \frac{\text{J}}{3600 \text{s}} = 8.35 \text{ h} \]

\( L_f \) (value) = 334 \( \frac{\text{kJ}}{\text{kg}} \)
Walk-in cooler

20°C  2.3m  26°C

20°C  3m  26°C

20°C  2m  26°C

\( R = \frac{A_0 T}{H} \)

\( \frac{R}{k} = \frac{0.08}{0.029} = 2.76 \)

\( Table 16.2 \)

\( A = 3\times2\times2 + 3\times2.3\times2 + 1.3\times2\times2 \)

Front & back  Left & right  Top & both

= 35 m²

\( H = -\frac{32 \times 16}{2.76} = -203 W \)
\[ \text{PV} = nRT \]
\[ \frac{P}{V} = \frac{nRT}{V} \]

Isothermal expansion \( A \rightarrow B \)

\( B \) will be on the right of \( A \)

Incorporate correct sign by comparing data with definitions of \( Q \) & \( W \)

\[ \Delta U = Q - W \]

- Work done by system \( \rightarrow + \)
- Work received \( \rightarrow - \)
- Heat absorbed \( \rightarrow + \)
- (Heat coming into system \( \rightarrow \) positive
  Heat coming out of system \( \rightarrow \) negative)

**Isobaric compression** \( B \rightarrow C \)

\( C \) will be to the left of \( B \)

on a horizontal line, with \( V_c = V_a \)

\( b) \) Work done or received by gas during complete cycle:

\[ A \rightarrow B \rightarrow C \rightarrow A \]

\[ W_{AB} \quad W_{BC} \quad W_{CA} = 0 \quad (V_c = V_a \rightarrow \Delta V = 0 \rightarrow W = 0) \]

Work \( W \) and heat \( Q \) are not state variables \( \rightarrow \) depend on the type of process.
A → B: Isothermal process: 
\[ W_{AB} = Q \quad (\text{DU} = 0 \quad \text{(ideal gas)}) \]
\[ = +35\, \text{J} \quad (\text{given}) \]

B → C: Isobaric compression: 
\[ W_{BC} = -22\, \text{J} \quad (\text{given}) \]

\[ W_{ABCA} = W_{AB} + W_{BC} = 35\, \text{J} - 22\, \text{J} = 13\, \text{J} \]

By the sign → work done by gas in one cycle is 13 J.

\( Q_{BCA} \)

\( Q_{ABCA} = Q_{ABCA} - Q_{AB} \) (linear superposition)

\[ Q_{AB} = +35\, \text{J} \]

\[ Q_{BC} = 13\, \text{J} - 35\, \text{J} = -22\, \text{J} \]

By the sign, heat in the amount of 22 J has been lost by the gas in B → C → A.

\( Q_{BCA} = Q_{BC} + Q_{CA} \)
Ch 19 2nd Law of Thermodynamics

In connection with heat engines working b/w 2 different heat reservoirs.

Heat reservoir: a source of heat, large, at constant temp.

Visual experiment: put a piston filled with an ideal gas in thermal contact with a heat reservoir (hot) at temperature $T_h$. Gas absorbs heat, expands, does work. No more expansion (no more work done) when gas reaches same temp. as the constant $T_h$ heat reservoir.

How to get more work done? Bring gas back to its original temp. by putting it in thermal contact with a cold reservoir @ temp. $T_c$ -> gas loses heat, compressed (receiving work).

Diagram:

Gas does work: $Q_h$ $A$ $Q_v$ $A$ $Q_c$ $T_h$ $T_c$ $Q_v$ (1st half of cycle)

Gas receives work: $Q_v$ $A$ $Q_c$ $T_c$ $Q_v$ (2nd half of cycle)
In one diagram:

Heat engine

\[ Q_h \]

Heat absorbed: \( Q_h \)

Heat lost: \( Q_c \)

More work done than received by gas \( \Rightarrow \) net work done is \( W \)

1st Law of T.D. for a heat engine:

\[ Q_{\text{in}} - W = Q_h - Q_c - W \]

\[ \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} \]

Efficiency of the heat engine:

\[ e = 1 - \frac{Q_c}{Q_h} \]

\( 0 < \frac{Q_c}{Q_h} < 1 \)

\[ e = 1 - \frac{10c}{10h} < 1 \]

There is no \( e > 1 \)

2nd Law of T.D.: it is impossible to build a heat engine operating in cycles that extracts heat from a heat reservoir (and returning some of it to a cold reservoir) that can deliver 100% efficiency \( (e < 1) \)
Gas mixture: \( \text{Ar} \) & \( \text{O}_2 \)

Fraction: \( f \) & \( 1-f \) \((0 < f < 1)\)

Possible paths:

\[ Q = 0 \]

\[ dU = -W \]

\[ P_1 V_1^\gamma = P_2 V_2^\gamma \]

\[ \gamma = \frac{C_p}{C_v} = \frac{C_p + R}{C_v} = 1.44 \]

\[ C_v = \frac{3}{2} R f + \frac{5}{2} R (1-f) \]

\[ \gamma = \frac{\frac{3}{2} R f + \frac{5}{2} R (1-f) + R}{\frac{3}{2} R f + \frac{5}{2} R (1-f)} = \frac{7}{2} - f \]

Now find \( \gamma \):

\[ \frac{P_1}{P_2} = \left( \frac{V_2}{V_1} \right)^\gamma \]

\[ \ln \left( \frac{P_1}{P_2} \right) = \gamma \ln \left( \frac{V_2}{V_1} \right) \]

\[ \ln \left( \frac{P_1}{P_2} \right) = \ln 3 \]

\[ \ln \left( \frac{V_2}{V_1} \right) = \ln 2 \]

\[ \gamma = \frac{\ln 3}{\ln 2} \approx 1.58 \]

(recall: monotonic: \( \gamma = 1.67 \); diatomic: \( \gamma = 1.4 \))

\[ \frac{5}{2} \gamma - \gamma f = \frac{7}{2} - f \]

\[ \frac{5}{2} \gamma - \gamma f = (\gamma-1)f \]

\[ f = \frac{\frac{5}{2} \times 1.58 - \frac{7}{2}}{1.58 - 1} \approx 0.78 \]

\[ \begin{align*}
A_n & \quad 78\% \\
O_2 & \quad 22\%
\end{align*} \]
Blood artery pressure
\[ P_h = 125 \text{ mm Hg} \]
\[ P_L = 80 \text{ mm Hg} \]
Gauge pressure.
Actual pressure → add atmospheric pressure:
\[ 760 \text{ mm Hg} \]

Air bubble: \[ d_L = 1.52 \text{ mm} \] @ \[ P_L \]

a) \[ d_h < d_L \] → gas volume will determine the diameter of the spherical bubble:

Ideal gas equation: \[ PV = nRT \]

\[ P_L V_L = P_h V_h \]
\[ V_h = \frac{P_L}{P_h} V_L \]

\[ \frac{4\pi}{3} \left( \frac{d_h}{2} \right)^3 = \frac{P_L}{P_h} \frac{4\pi}{3} \left( \frac{d_L}{2} \right)^3 \]

\[ d_h = \frac{P_L}{P_h} d_L \]

\[ d_h = \left( \frac{P_L}{P_h} \right)^{\frac{1}{3}} d_L \]

\[ \frac{(760 + 80)^{\frac{1}{3}}}{(760 + 125)^{\frac{1}{3}}} = 1.52 \text{ mm} \]

\[ d_h = \left( \frac{840}{885} \right)^{\frac{1}{3}} 1.52 \text{ mm} = 1.49 \text{ mm} \]

b) How much work does blood (in heart) do to compress the bubble?

(b) body temp. stays constant @ 37°C → isothermal process:

\[ W = nRT \ln \left( \frac{V_h}{V_L} \right) = \frac{nRT}{\frac{P_L}{V_L}} \ln \left( \frac{V_h}{V_L} \right) \]

\[ = \ln \left( \frac{(1.49)^3}{1.52} \right) \]

\[ = -12.312 \times 10^{-9} \text{ J} \]

Work done on gas in bubble
Since it is getting compressed.
Clock: brass pendulum: \( L = 0.2 \text{ m} @ 20^\circ \text{C} \) (good)

\( T_2 = 18^\circ \text{C} \) → how long until it makes a time error?

Since for a pendulum: # oscillations per second \( \omega = \sqrt{\frac{g}{L}} \)

Length will change with temp, according to the linear coefficient of expansion for brass: → Table 17.2: \( \alpha = 19 \times 10^{-6} \text{ deg}^{-1} \)

\[ \Delta L = \frac{AL}{L} \Rightarrow \Delta L = \alpha \Delta T \Rightarrow \Delta L = \alpha L \Delta T \]

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}} \]

\[ T_{20^\circ C} = \frac{2\pi}{\sqrt{\frac{L}{g}}} \]

\[ T_{18^\circ C} < T_{20^\circ C} \Rightarrow T_{20^\circ C} - T_{18^\circ C} = \frac{2\pi}{\sqrt{g}} \left[ \sqrt{L} - \sqrt{L(1 + \alpha \Delta T)} \right] \]

\[ = \frac{2\pi}{\sqrt{g}} \left[ 1 - \sqrt{1 - 2\alpha \Delta T} \right] \]

\[ \Delta T = -2^\circ \text{C} \]

\[ \sqrt{1 - 2\alpha} = \left( 1 - 2\alpha \right)^{1/2} \approx 1 - \frac{1}{2} \left( 2\alpha \right) = 1 - \alpha \]

\[ (1 + x)^{1/2} \approx 1 + \frac{1}{2} x + \ldots \]

\[ \text{negligible if } x \text{ very small compared to } 1 \]

\[ \alpha = 19 \times 10^{-6} \text{ deg}^{-1} \]

\[ T_{20^\circ C} - T_{18^\circ C} = \frac{2\pi}{\sqrt{g}} \left[ 1 - (1 - \alpha) \right] = \frac{2\pi}{\sqrt{g}} \alpha \text{ time error per period.} \]
How many periods of pendulum until a difference of 1 km?

\[
\frac{60s}{2\pi \sqrt{\frac{L}{g}} \alpha}
\]

How long until a difference of 60s? \[
\frac{60s}{m \sqrt{\frac{L}{g}} \alpha} = \frac{T_{202}}{2m \sqrt{\frac{L}{g}} \alpha}
\]

\[
e = \frac{60}{2m \sqrt{\frac{L}{g}} \alpha} = \frac{60}{19 \times 10^{-6}} \text{ s} \rightarrow 36.5 \text{ days}
\]