

Ch16 Temperature & Heat

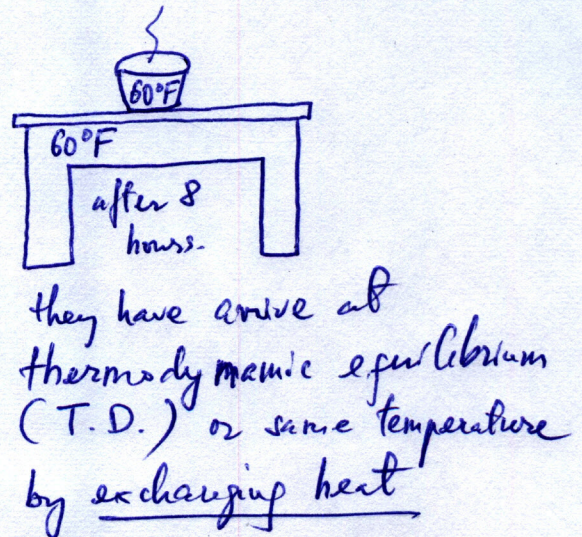
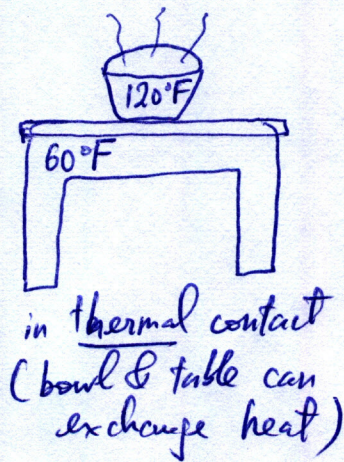
Temperature: a macroscopic property (we can sense & measure), it is a consequence of microscopic behavior of matter.

Unit: OF (degree Fahrenheit) SI: °K (degree kelvin) (absolute zero is 0°K)
 °C (degree Celsius)

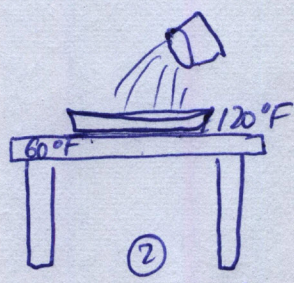
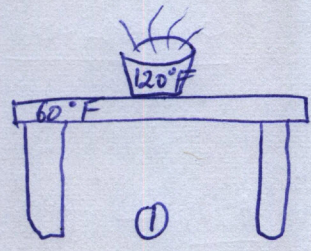
$$T_F = \frac{9}{5} T_C + 32 \rightarrow (\text{body temp} \sim 100^\circ\text{F})$$

$$T_C = \frac{5}{9} (T_F - 32) \rightarrow (\text{boiling point of water} \sim 100^\circ\text{C})$$

$$T_C = T_K - \underbrace{273.16^\circ\text{K}}_{\text{Triple point of water (3 phases coexist @ this temperature)}}$$



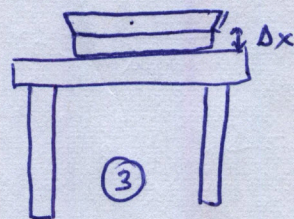
Heat transfer rate : $H \rightarrow$ how fast heat can be transferred or exchanged b/w 2 objects.
↳ per unit time



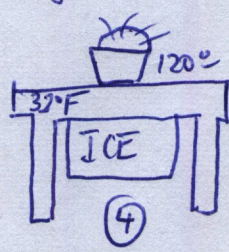
larger contact surface A

$$H = -k \frac{A}{\Delta x} \Delta T$$

Thermal constant (depending on materials)

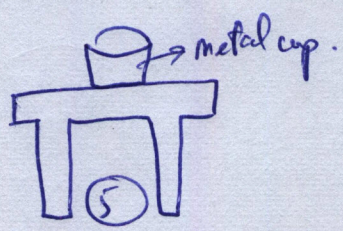


larger thickness Δx



larger ΔT

① & ④ : Larger $\Delta T \rightarrow$ faster heat transfer rate $\rightarrow \uparrow H$



① & ⑤

faster transfer depending on material
metal are faster heat conductors.

metal are also good electrical conductors

free electrons
↓
energy carriers
↓

"Temperature is a consequence of microscopic properties of matter"

Units for H: SI

- Length $\rightarrow L \rightarrow$ meter (m)
- Mass $\rightarrow M \rightarrow$ kilogram (kg)
- Temperature $\rightarrow T \rightarrow$ degree kelvin ($^{\circ}K$)
- Time $\rightarrow T \rightarrow$ second (s)
- Energy $\rightarrow M \frac{L^2}{T^2} \rightarrow$ joule (J)
 - \rightarrow Mechanical (kinetic, potential)
 - \rightarrow Thermal or heat
- Power $\rightarrow M \frac{L^2}{T^3} \rightarrow$ joule/s = watt (W)

Dimension of H

$$[H] = [k] [A] \frac{[\Delta T]}{[\Delta x]} \rightarrow [k] = \frac{[E]}{[Time]} \frac{[\Delta x]}{[\Delta T]} \frac{1}{[A]}$$

$$\downarrow$$

$$\frac{[E]}{[Time]}$$

$$\downarrow$$

$$\text{Unit for } k = \frac{J}{s} \frac{m}{^{\circ}K} \frac{1}{m^2} = \frac{W}{m^{\circ}K}$$

k \rightarrow (Thermal constant)

- SI: $\frac{W}{m^{\circ}K}$
- British: $\frac{Btu \cdot in}{h \cdot ft^2 \cdot ^{\circ}F}$ (Btu: ~~both~~ British thermal unit)

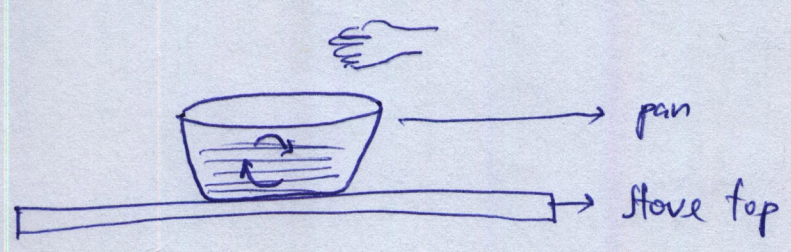
R-factor: $R = \frac{\Delta x}{k}$ (larger R better insulation)

$$H = - \frac{k A \Delta T}{\Delta x} = - \frac{A \Delta T}{R}$$

Units: SI: $\frac{m^2 \cdot ^{\circ}K}{W}$
 British: $\frac{ft^2 \cdot ^{\circ}F}{\frac{Btu}{h}}$

$\leftarrow R-21 \rightarrow R=21 \frac{ft^2 \cdot ^{\circ}F}{\frac{Btu}{h}}$

Mechanisms of heat transfer:



- Initially: stove top is heated.

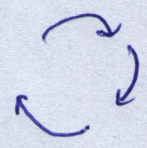
Heat transfer: stove top \rightarrow pan by conduction (via free electrons in metal pan)

- Water at bottom gets heated

Heat transfer: bottom water \rightarrow top water by convection (circulation of water molecules)

\rightarrow Hand feels some heat

Heat transfer: hot water \rightarrow hand by radiation (does this require a medium \rightarrow No: we feel heat from sun!)
 \downarrow
propagation of radiation via electromagnetic waves

Convection:  Why?

\downarrow
buoyancy

hot molecules & atoms tend to stay apart \rightarrow lower density.
 \rightarrow tend to rise above liquid or gases with higher density.
Reverse process for higher density \rightarrow tend to sink.

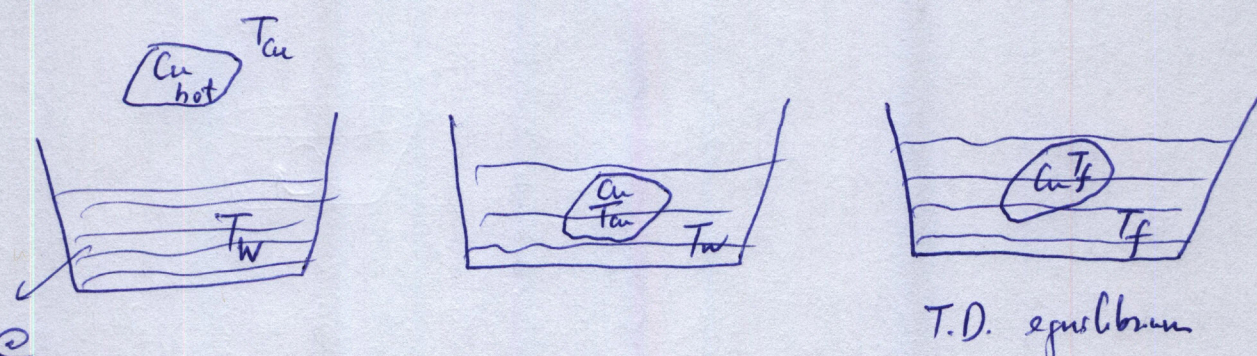
Radiation:

Stefan-Boltzman Law: heat loss rate (per unit time) by radiation of an object of area A & temp. T:

$$P = e \sigma A T^4$$

\downarrow emissivity (no unit) \downarrow Stefan-Boltzman constant = $5.67 \times 10^{-8} \frac{W}{m^2 \cdot K}$

Heat balance equation:



water @ room temp.

Assume: system water (& bucket) & copper is isolated:

$$\underbrace{\Delta Q_{Cu}}_{\text{Heat loss by Copper}} + \underbrace{\Delta Q_w}_{\text{Heat gain by water}} = 0 \quad (\text{heat left copper } \cancel{\text{with}} \text{ went into water})$$

$$= m_{Cu} \underbrace{C_{Cu}}_{\substack{\text{specific heat} \\ \text{of Copper}}} (T_f - T_{Cu}) + m_w C_w (T_f - T_w) = 0$$

Copper gets cooled down

+ water gets heated

Specific heat

$$\left\{ \begin{array}{l} C_w = 4184 \frac{J}{kg \cdot K} \\ C_{Cu} = 386 \frac{J}{kg \cdot K} \end{array} \right.$$

Capacity to absorb heat per unit mass
Material-specific!

Ch 17 Thermal Behavior of Matter

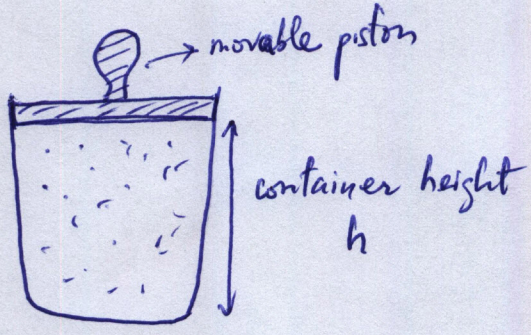
Matter : 3 states

Density	State
low →	<u>Gas</u> : almost free molecules. ↓ not interacting
Medium →	<u>Liquid</u> : some interaction b/w molecules
High →	<u>Solid</u> : molecules are not free to move

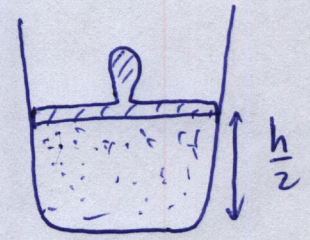
Gas → Ideal Gases : completely free (non-interacting) molecules.

Macroscopic quantities or properties: $\begin{cases} T \text{ (temp., } ^\circ\text{K)} \\ P \text{ (pressure, Pa)} \\ V \text{ (volume, m}^3\text{)} \end{cases}$

Microscopic quantity : N (num. of molecules)



T, N, V, P
state ①



$T, N, \frac{V}{2}, 2P$
state ②

Assuming :
 - Gas can't escape
 - ① → ② isothermal process (very slowly)

Equation of state for an ideal gas: ① → ② $\begin{cases} N_1 T_1 = N_2 T_2 \\ P_1 V_1 = P_2 V_2 \end{cases}$

$\Rightarrow P_1 V_1 = k N_1 T_1 \quad \& \quad P_2 V_2 = k N_2 T_2$

$PV = kNT$ $k = \text{Boltzmann constant} \rightarrow 1.38 \times 10^{-23} \frac{\text{J}}{^\circ\text{K}}$

Avogadro number : $N_A = 6.022 \times 10^{23}$

$N = n N_A$; n : number of mole

$$PV = k \frac{n N_A}{N} T = n \underbrace{k N_A}_{R} T \Rightarrow \boxed{PV = nRT}$$

$R = 8.314 \frac{J}{K \text{ mol}}$

Ideal Gas

If we consider interactions b/w gas molecules, \rightarrow Real Gas

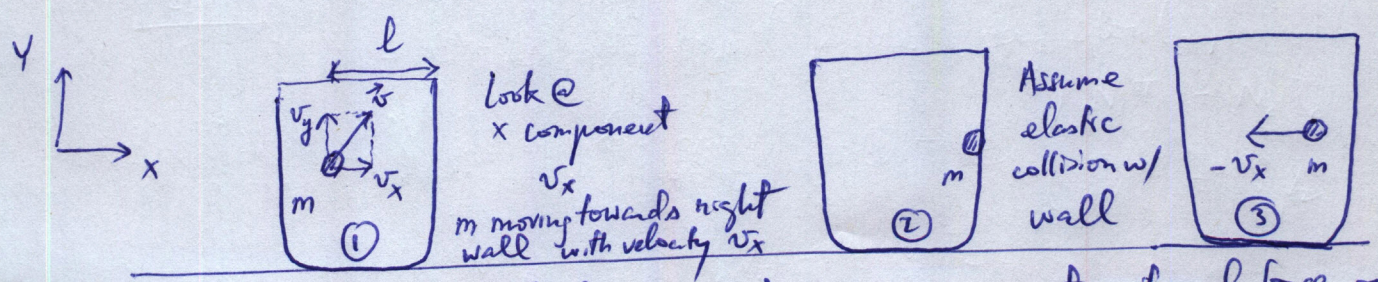
\rightarrow Eq. of state for Real Gas or Van der Waals equation

$$\left(P + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

a, b are given constants, depending on the strength of interactions b/w molecules.

Closer look at the connection b/w microscopic behavior (that of single gas molecule \rightarrow and its transfer of momentum to the container) and the macroscopic property of the gas (such as its temperature, pressure)

Mechanics for a gas molecule \rightarrow a particle of mass m



System: one gas molecule + container \rightarrow no net external force on this system in x (\rightarrow what about mg ? same in ①, ②, ③ \rightarrow will not change our result) \rightarrow motion along perpendicular directions are independent

Newton's 2nd Law: $F_{net,x} = \frac{dp_x}{dt}$

$0 = \frac{dp_x}{dt} \rightarrow \boxed{p_x \text{ is conserved}}$
 \downarrow
 total
 $m v_{xT}$: linear momentum
 in the x-direction.

$(m v_{xT})_{(1)} = (m v_{xT})_{(3)}$

Total linear momentum @ (1) = Total linear momentum @ (3)

$m v_x + M \cdot 0 = -m v_x + \frac{2 m v_x}{\text{linear momentum of container}} = M v_{xc} \rightarrow v_{xc} = \frac{2m}{M} v_x$
 \uparrow 10⁻²⁷ kg
 \downarrow mass of container = 1 kg
 ≈ 0

Conclusion: after hitting right wall, a gas molecule i has transferred $2m v_x$ (momentum) to the container.

\downarrow
 $P_i = \frac{F_i}{A} = \frac{\frac{\Delta p_i}{\Delta t}}{A} = \frac{\frac{2m v_x}{\frac{2l}{v_x}}}{A} = \frac{m v_x^2}{A \cdot l} = \frac{m v_x^2}{\text{vol.}}$

\downarrow
 Pressure due to N molecules: $P = \sum_{i=1}^N P_i = \frac{m}{\text{vol}} \sum_{i=1}^N v_{xi}^2$

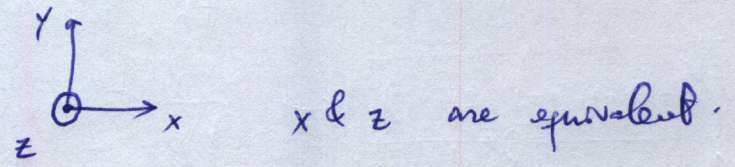
$\left. \begin{aligned} P &= \frac{mN}{\text{vol}} \frac{\sum_{i=1}^N v_{xi}^2}{N} \\ &\text{average of } v_x^2 \rightarrow \overline{v_x^2} \end{aligned} \right\} \rightarrow P = \frac{mN}{\text{vol}} \overline{v_x^2}$
 $\text{vol} \rightarrow V$

$PV = mN \overline{v_x^2}$
 $\xrightarrow{\text{v}_x \text{ is one of the 3 components of } \vec{v}} \overline{v_x^2} = \frac{\overline{v^2}}{3}$

$$PV = mN \frac{\overline{v^2}}{3}$$

$$\overline{v_x^2} = \frac{\overline{v^2}}{3}$$

The av. of v^2 in one direction is the average of v^2 in 3D divided by 3



$v_y \rightarrow$ there is molecule weight of mg since $m \sim 10^{-27}$ kg, for this derivation, we assume it is negligible $\rightarrow x, y, z$ are equivalent.

Now relate this to the eq. of state for an ideal gas: $PV = KNT$

$$KNT = mN \frac{\overline{v^2}}{3} \rightarrow KT = \frac{1}{3} m \overline{v^2}$$

$$3KT = m \overline{v^2}$$

$\frac{3}{2}KT$	$=$	$\frac{1}{2}m\overline{v^2}$
proportional to temp		Average kinetic energy for one molecule.
↓		↓
Macroscopic		Microscopic

Recall: Temperature: a macroscopic property that is a consequence of microscopic behavior

\rightarrow We can use reverse connection as well:

Room	$\left\{ \begin{array}{l} T_1 = 298 \text{ }^\circ\text{K} \text{ (normal condition)} \\ T_2 = 320 \text{ }^\circ\text{K} \end{array} \right\}$	$\frac{1}{2}m\overline{v_1^2} < \frac{1}{2}m\overline{v_2^2}$

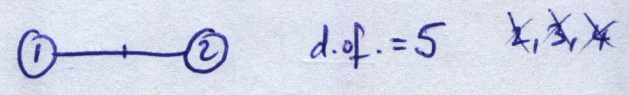
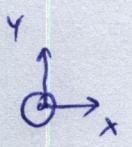
Dimensions :

$$3D \longrightarrow \frac{3}{2} kT = \frac{1}{2} m \overline{v^2}$$

$\left\{ \begin{array}{l} 1D \rightarrow \text{What is the } \text{avg} \text{ average KE per molecule in } 1D \rightarrow \frac{1}{2} kT \\ 2D \rightarrow kT \end{array} \right.$

↳ av. KE of a gas molecule per degree of freedom is $\frac{1}{2} kT$.

↳ Molecule formed of 2 atoms that are connected:
O₂, N₂, etc...



- $\left\{ \begin{array}{l} ① \rightarrow \text{d.o.f.} : 3 \\ ② \rightarrow \text{d.o.f.} : 3 \end{array} \right.$
- $\left\{ \begin{array}{l} ① \quad ② \rightarrow \text{d.o.f.} = 6 \end{array} \right.$

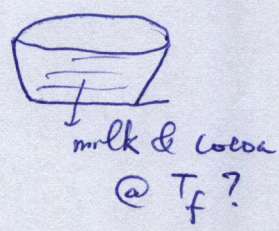
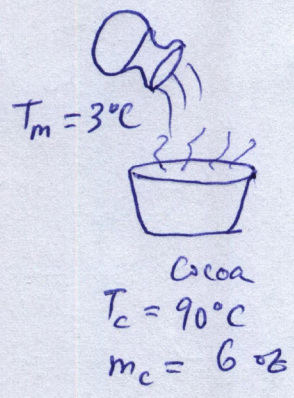
- $x :$ ① moves left \rightarrow so will ②
- $y :$ ① moves up, ② not necessarily will move up
- $z :$ ① moves out of page
② not necessarily will move out of page
(can rotate w.r.t middle pivot)

$\left\{ \begin{array}{l} \text{Av. KE per diatomic molecule is } \frac{5}{2} kT \\ \text{Av. KE per monoatomic molecule is } \frac{3}{2} kT \end{array} \right.$

16-54

milk $m_m = 2 \text{ oz}$

$$C_m = C_c = C_{\text{water}} = 4184 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$



Heat balance equation:
(isolated system)

$$\Delta Q_c + \Delta Q_m = 0$$

$$m_c c_c (T_f - T_c) + m_m c_m (T_f - T_m) = 0$$

$$(m_c c_w + m_m c_w) T_f = m_c c_w T_c + m_m c_w T_m$$

$$T_f = \frac{m_c T_c + m_m T_m}{m_c + m_m}$$

SI

$$\left\{ \begin{array}{l} T \rightarrow \text{in } ^\circ\text{K} \rightarrow \begin{cases} T_c = 90^\circ + 273.16^\circ \\ = 363.16^\circ\text{K} \\ T_m = 3^\circ + 273.16^\circ \\ = 276.16^\circ\text{K} \end{cases} \\ m \rightarrow \text{in kg} \end{array} \right.$$

(not necessary in this particular case) masses showing above & below the fraction!

$$T_f = \frac{6 \times 363.16 + 2 \times 276.16}{8} = 341.46^\circ\text{K}$$

Can go back to Celsius: $T_f = 341.46 - 273.16 = 68.3^\circ\text{C}$

If I work in Celsius:

$$T_f = \frac{m_c (T_{cK} - 273.16) + m_m (T_{mK} - 273.16)}{m_c + m_m}$$

$$= \frac{m_c T_{cK} + m_m T_{mK}}{m_c + m_m} + \frac{(m_c + m_m)(-273.16)}{(m_c + m_m)}$$

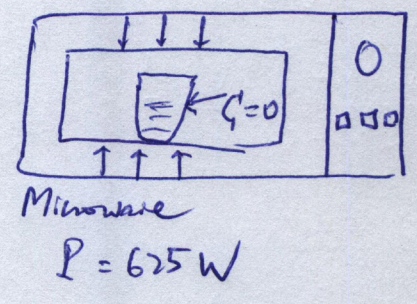
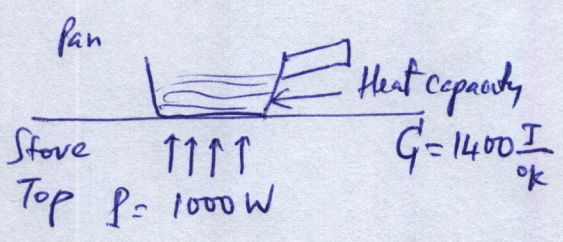
$$T_f = T_{fK} - 273.16$$

Conversion factors (Appendix C)

- 1 oz = 0.02835 kg
- 1 lb = 0.454 kg
- 1 in = 0.0254 m

116.49

- 1) Write down the data
- 2) Write down the equation
- 3) Solve for what is needed



Heat capacity: $G = c \cdot m$

\downarrow \downarrow
 specific mass
 heat heat

Relating to what we said: $\Delta Q = mc \Delta T = G \Delta T$

Question: How much water before it is quicker to heat using stove top? shorter time

Let's compare time it takes to heat an amount of water M so its temp increases ΔT , using stove top & microwave

Stove top

$$\Delta t_s = ?$$

$$H_s = \frac{\Delta Q}{\Delta t_s}$$

↓
heat transfer per unit time

"
1000 W

$$\rightarrow \Delta t_s = \frac{\Delta Q}{1000} = \frac{(C_w + C_{pan}) \Delta T}{1000}$$

↓
Heat transferred from stove top (@ 1000 J/s) is absorbed by pan & water

$$C_{pan} = 1400 \frac{J}{^{\circ}K}$$

$$C_w = 4184 \frac{J}{kg^{\circ}K}$$

$$C_w = m_w C_w$$

$$\Delta t_s = \frac{(m_w C_w + C_{pan}) \Delta T}{1000}$$

Microwave

$$\Delta t_m = ?$$

$$H_m = 625 \frac{J}{s} = 625 W$$

$$\Delta t_m = \frac{\Delta Q}{625} = \frac{C_w \Delta T}{625}$$

↓
Heat transferred in from microwave is absorbed by water (paper cup has $C \approx 0$)

$$\Delta t_m = \frac{m_w C_w \Delta T}{625}$$

When is it quicker w/ stove top?
 $\Delta t_s < \Delta t_m$ @ what m_w ?

$$\frac{(m_w C_w + C_{pan}) \Delta T}{1000} < \frac{m_w C_w \Delta T}{625}$$

$$\frac{C_{pan}}{1000} < m_w \left(\frac{C_w}{625} - \frac{C_w}{1000} \right) = m_w C_w (0.375)$$

$$\frac{C_{pan}}{1000 C_w 0.375} < m_w$$

$$\frac{1400}{375 \times 4184} = 0.558 kg < m_w$$

a)

b) Rate of temperature increase? or $\frac{\Delta T}{\Delta t}$

$$m_w = 0.558 \text{ kg} \rightarrow \Delta t_s = \frac{m_w c_w + C_{pan} \Delta T}{1000} \rightarrow \frac{\Delta T}{\Delta t_s} = \frac{1000}{m_w c_w + C_{pan}}$$

$$\frac{\Delta T}{\Delta t_s} = \frac{1000}{0.558 \times 184 + 1400} = 0.268 \frac{\text{°K}}{\text{s}}$$

Thermal Behavior of Matter (Cont.)

15

When heat is supplied to matter, what are the

(ΔQ)

↓
delta \leftrightarrow increment or
change of

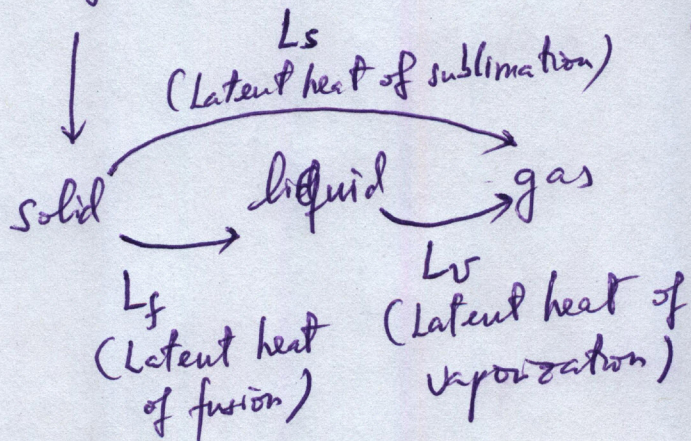
consequences?

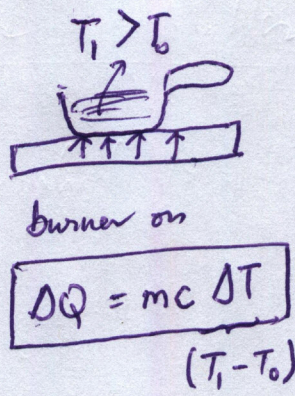
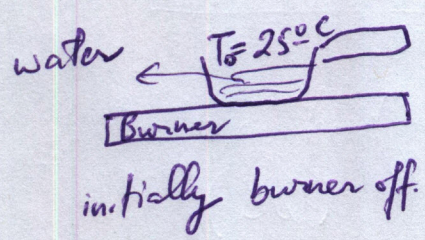
1) ΔT (ΔQ produces a change of temp.):

$$\Delta Q = mc \Delta T$$

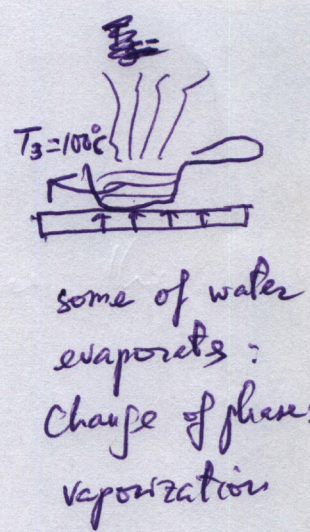
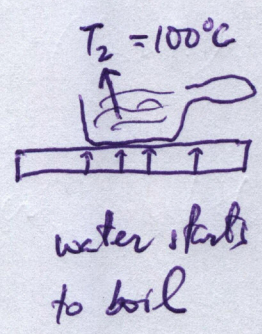
2) Expansion: $\left\{ \begin{array}{l} \text{Linear expansion coefficient } \alpha = \frac{\frac{\Delta L}{L}}{\Delta T} \\ \text{(L: length, T: temp.)} \\ \text{Volumic expansion coefficient } \beta = \frac{\frac{\Delta V}{V}}{\Delta T} \\ \text{(V: vol, T: temp.)} \end{array} \right.$

3) Change of phase: $\Delta Q = m L$
"Latent heat"





Heat supplied
↓
increase in
temperature



$\Delta Q = mL_v$

Water temp. max is 100°C !
there is $\Delta T = 0$ ~~for~~
since water starts boiling
until it evaporates.
(Note the different equation
to use in a change of
phase!)

Table 17.1 :

Water:

$L_f = 334 \frac{\text{kJ}}{\text{kg}}$

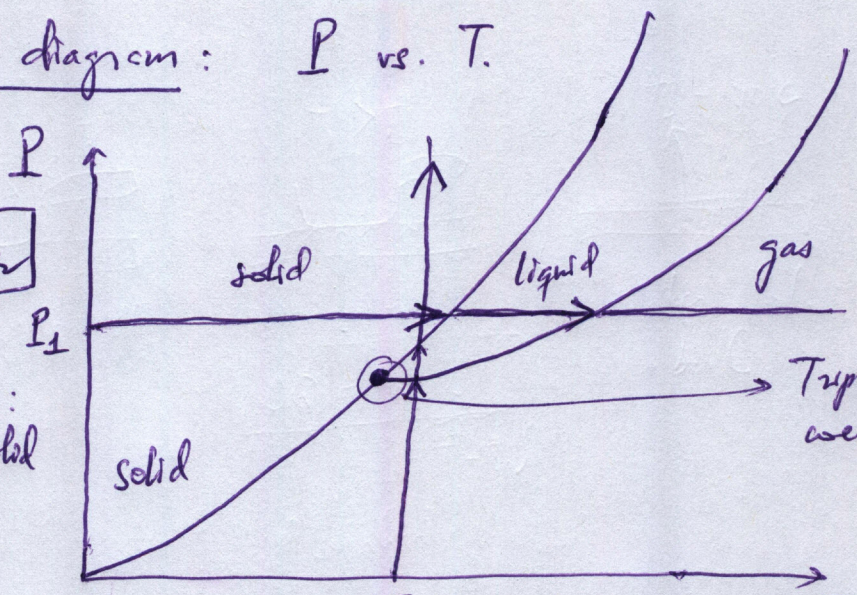
(it takes 334kJ to melt
one kg of ice)

$L_v = 2257 \frac{\text{kJ}}{\text{kg}}$

Phase diagram: P vs. T.

Normal matter

↳ at a fix T_1 ,
if we increase P :
gas → liquid → solid

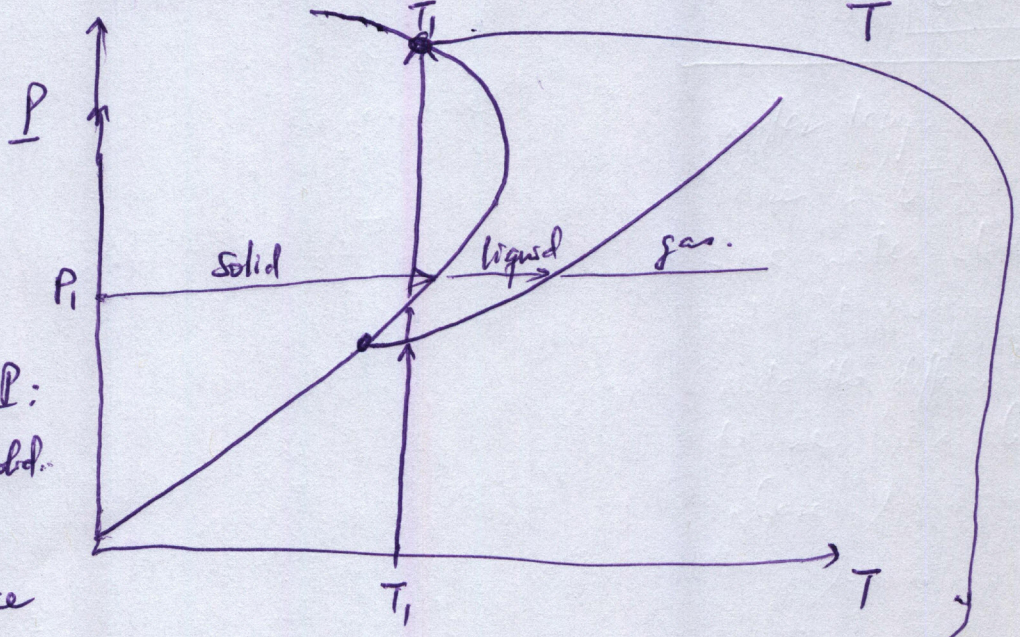


Triple point (3 phases coexist at the same time)

Water

↳ at fixed T_1 ,
if we increase P :
gas → liquid → solid.

↓
if we press on ice
it melts
(ice skates melt
the ice!)



@ this point (fixed T_1) if we increase P further, solid goes back to liquid!

Ch 18: Heat, Work, 1st Law of Thermodynamics. (18)

1st Law of Thermodynamics:

$$\Delta U = Q - W$$

Conservation of energy

Heat absorbed by a system: Q

Work done by system: W

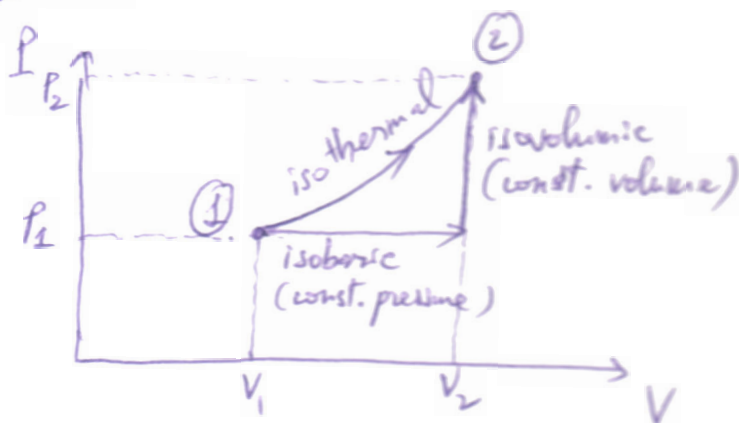
Change of internal energy of system: ΔU

$$\Delta U = Q - W$$

State Variable
(does not depend on process)

not state variables \rightarrow they depend on a particular process.

State: is a point on a P, V diagram. We have set values for P & V in a state.



Alternatives ① \rightarrow ②

- 1) Isobaric + Isochoric
- 2) Isothermal: constant temp.

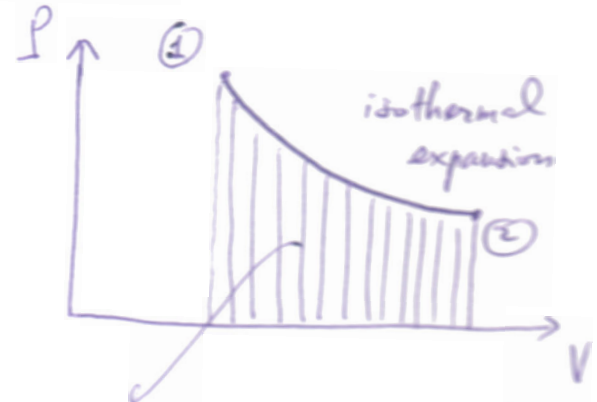
Process: connects two different states.

ΔU_{12} or change of internal energy of a system b/w ① & ② is the same whether the system went from ① to ② via isobaric + isochoric or via an isothermal process.

Q & W b/w ① & ② individually depend on the particular process!

Work done by a system: $W = \int P dV$
 (Recall: $P = \frac{F}{A}$; $W = F \cdot \Delta x = P \cdot \frac{A \cdot \Delta x}{\Delta V}$)

in a PV diagram:

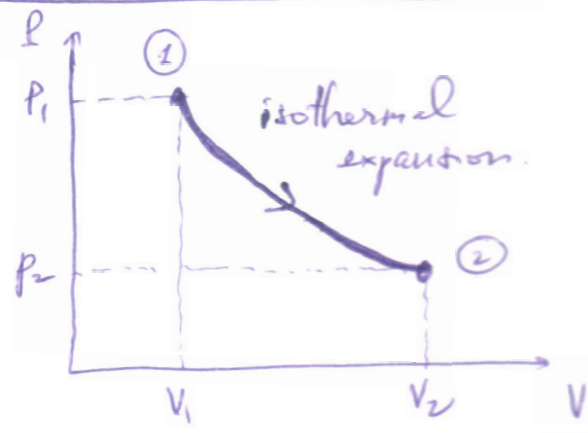


$\int P dV$ is the area under the curve
 → Work done is this area.

- Processes:
- 1) Isothermal (T is const)
 - 2) Isochoric (V is const)
 - 3) Isobaric (P is const.)
 - 4) Adiabatic (no heat is absorbed: $Q = 0$)

Since Q & W depend on a particular process:

Isothermal (T is const.)



$$W = \int_1^2 P dV = nRT \int_1^2 \frac{dV}{V} = nRT [\ln V]_1^2$$

↓
 Ideal gas
 $PV = nRT$
 constant in isothermal process!

$$W = nRT (\ln V_2 - \ln V_1) = nRT \ln \left(\frac{V_2}{V_1} \right)$$

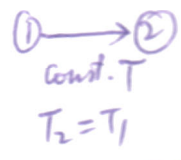
To find Q we will find ΔU in an isothermal process, then use the 1st Law of Thermodynamics. : $\Delta U = Q - W \rightarrow Q = \Delta U + W$

What is ΔU in an isothermal process on ideal gas?

Ideal gas: 1) ↔ no interaction b/w molecules ↔ no potential energy ↔ total internal energy comes from kinetic energy only ($E = \frac{1}{2}mv^2 + P.E.$)

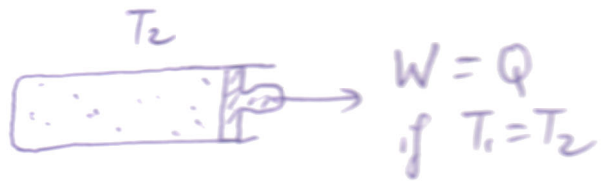
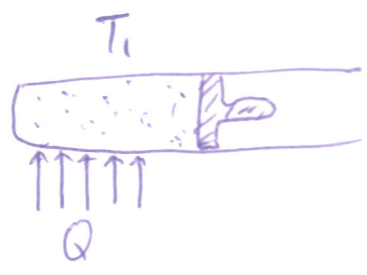
2) Av. KE per molecule in an ideal gas is proportional to the temperature $\begin{cases} \frac{3}{2}kT \text{ (mono-atomic)} \\ \frac{5}{2}kT \text{ (diatomic)} \end{cases}$

3) $\Delta U_{12} = U_2 - U_1 = \begin{cases} \frac{3}{2}k(T_2 - T_1) = 0 \\ \frac{5}{2}k(T_2 - T_1) = 0 \end{cases} = 0$

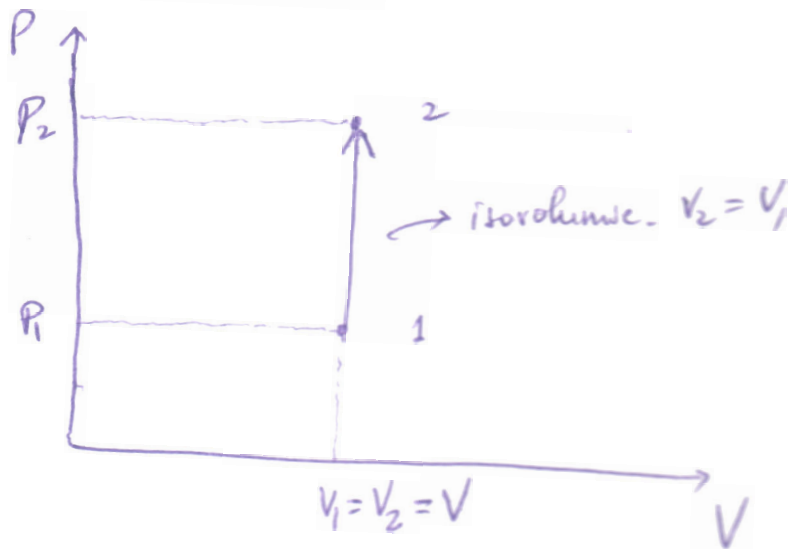


$\Delta U = 0$ in any isothermal process

⇒ $Q = W$ In an isothermal process the heat absorbed is equal to work done since $\Delta U = 0$



Isochoric (const. V)



$$W = \int_1^2 P dV = 0$$

1st Law of T.D : $\Delta U = Q - W \rightarrow \boxed{Q = \Delta U}$

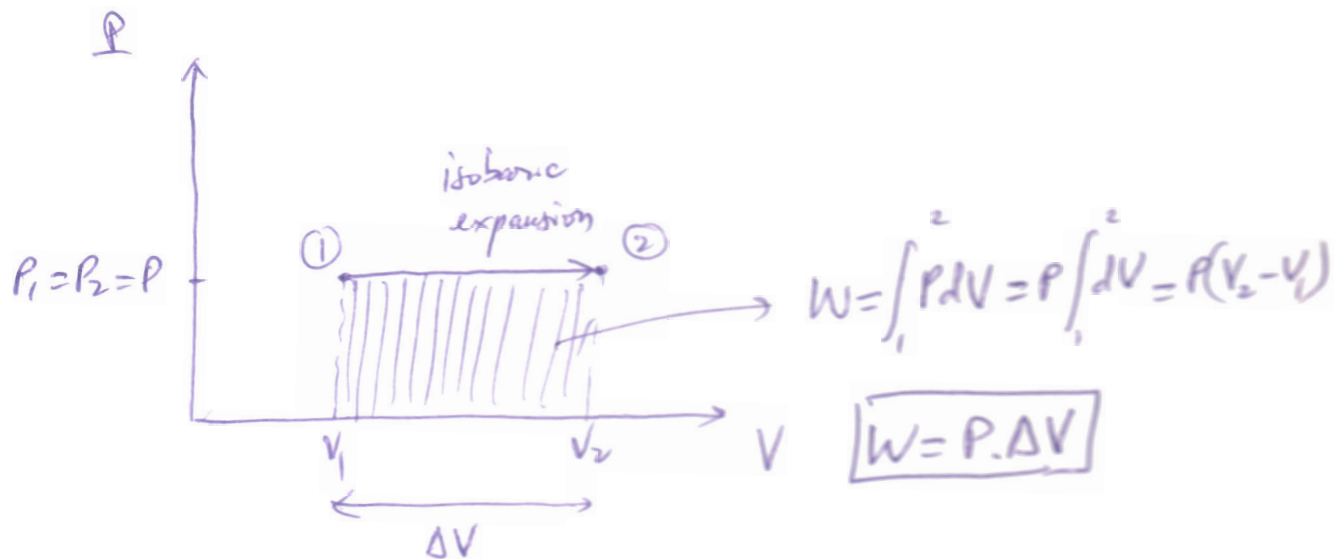
What is ΔU in an isochoric process in ideal gas?

\downarrow
 c_v : specific heat at constant volume

$$c_v \equiv \frac{1}{n} \frac{Q}{\Delta T} = \frac{1}{n} \frac{\Delta U}{\Delta T}$$

$$\boxed{Q = c_v n \Delta T = \Delta U}$$

Isobaric Process (const P.)



What is Q?

c_p = specific heat @ const. P : $c_p \equiv \frac{1}{n} \frac{Q}{\Delta T}$

$Q = c_p n \Delta T$

Ideal gas: $P \Delta V = n R \Delta T$

W
(isobaric)

1st Law of TD: $\Delta U = Q - W$

$n c_v \Delta T$ $n c_p \Delta T$ $n R \Delta T$

$c_v = c_p - R$

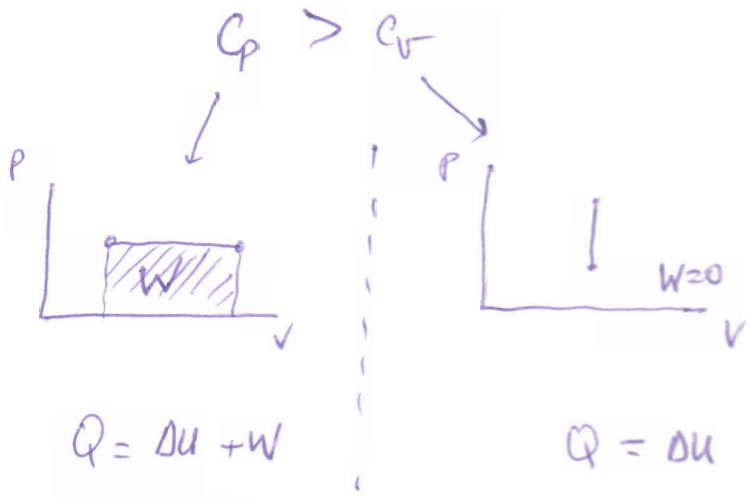
$c_p = c_v + R$

This is a connection b/w a isobaric & isovolumic process

$R = 8.314 \frac{J}{mol \cdot K}$

$c_p > c_v$

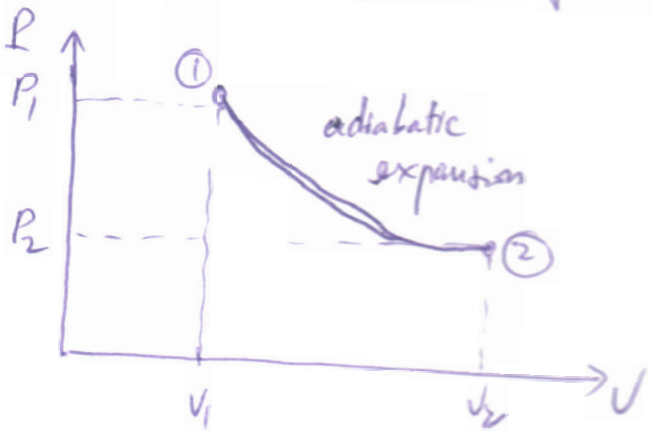
More heat absorption capacity @ constant P than @ constant volume



A system can absorb more heat @ constant P since it can do work (but not in a isovolume process!)

Adiabatic Process: (Q=0)

$W = -\Delta U$ (1st law of TD: $\Delta U = Q - W$)



$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$
 dimensionless coefficient
 $\gamma = \frac{C_p}{C_v}$

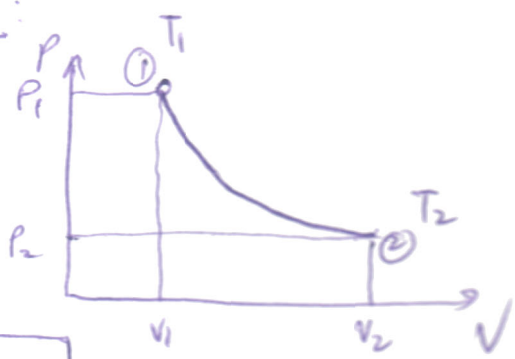
γ depends on the type of gas: monatomic or diatomic:
 $C_v = \frac{1}{n} \frac{\Delta U}{\Delta T} = \begin{cases} \text{monatomic: } \frac{1}{n} \frac{N \frac{3}{2} k \Delta T}{\Delta T} = \frac{3}{2} k \frac{N}{n} = \frac{3}{2} k N_A = \frac{3}{2} R \\ \text{diatomic: } \frac{5}{2} R \end{cases}$

$U = N \frac{3}{2} kT$
 (Total internal energy for an ideal monatomic gas)
 $N = \text{total \# of molecules}$
 $n = \text{\# of moles} = \frac{N}{N_A}$

$N_A = 6.022 \times 10^{23}$
 $k = \text{Boltzmann const.} = 1.38 \times 10^{-23} \frac{J}{^{\circ}K}$
 $R = k N_A = 8.314 \frac{J}{mol \cdot ^{\circ}K}$

$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \begin{cases} \text{Monatomic} = \frac{\frac{3}{2}R + R}{\frac{3}{2}R} = \frac{5}{3} = 1.67 \\ \text{Diatomic} = \frac{\frac{5}{2}R + R}{\frac{5}{2}R} = \frac{7}{5} = 1.4 \end{cases}$

Adiabatic process:



$P_1 V_1 \neq P_2 V_2$ since $T_1 \neq T_2$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

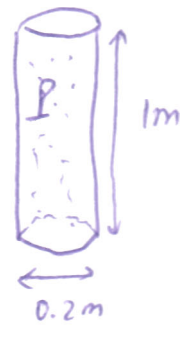
also because $PV = nRT \Rightarrow P = \frac{nRT}{V}$

$$nR \frac{T_1}{V_1} V_1^\gamma = nR \frac{T_2}{V_2} V_2^\gamma$$

$$\Leftrightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

Apply to an ideal gas in an adiabatic process.

17.39



@ $T = 25^\circ\text{C}$ (room temp.) $\rightarrow P = 180 \text{ atm}$.

a) How many moles of air n

Assume ideal gas: $n = \frac{PV}{RT}$

$$P = 180 \frac{\text{atm}}{1 \text{ atm}} \cdot \frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}}$$

$$V = \pi (0.1)^2 \cdot 1 = \frac{\pi}{100} \text{ m}^3$$

$$R = 8.314 \frac{\text{J}}{\text{K mol}}$$

$$T = 25 + 273.16 = 298.16^\circ\text{K}$$

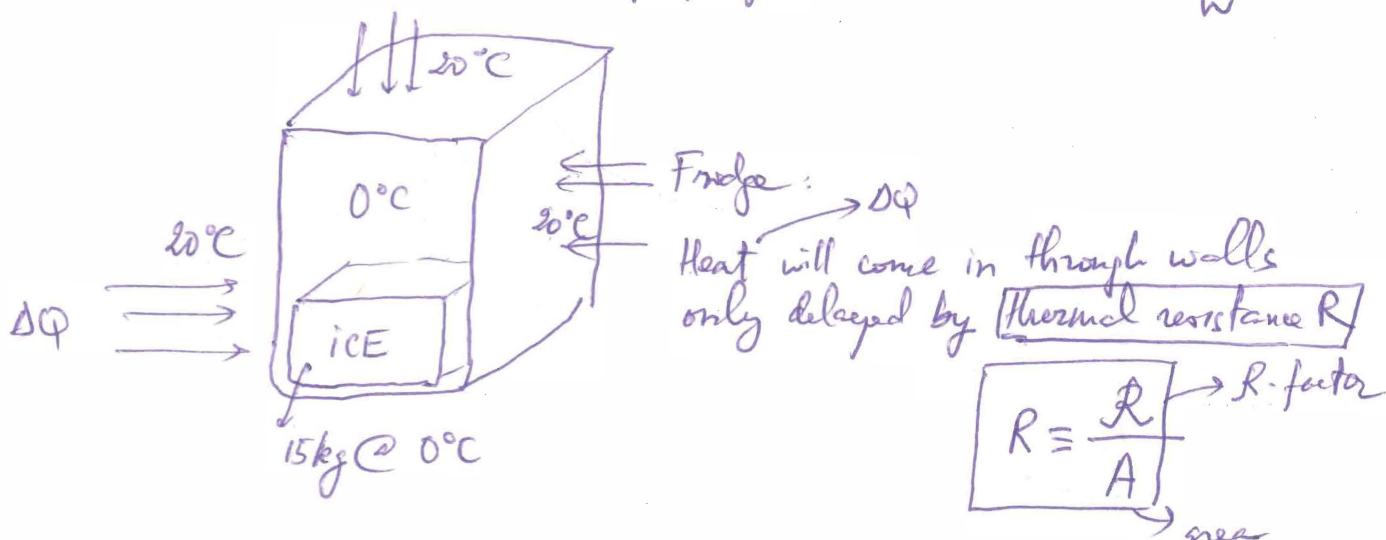
$$n = \frac{180 \times 1.013 \times 10^5 \times \frac{\pi}{100}}{8.314 \times 298.16} = 231 \text{ mol}$$

b) What would be the gas volume @ $P = 1 \text{ atm}$ & $T = 25^\circ\text{C}$
 $\rightarrow n = 231 \Rightarrow \frac{PV}{RT}$ } if P decreases 180 fold $\rightarrow V$ increases 180 times

$$V = \frac{\pi}{100} \times 180 \text{ m}^3 = 5.65 \text{ m}^3$$

17.65

Thermal resistance of fridge wall is $R = 0.12 \frac{^{\circ}\text{K}}{\text{W}}$



R → determine H (Heat transfer rate through fridge's walls)

→ how long will last $\Delta t = \frac{\Delta Q}{H}$ ($H = \frac{\Delta Q}{\Delta t}$)

ΔQ will go into melting the ice or $m_{ice} L_f$

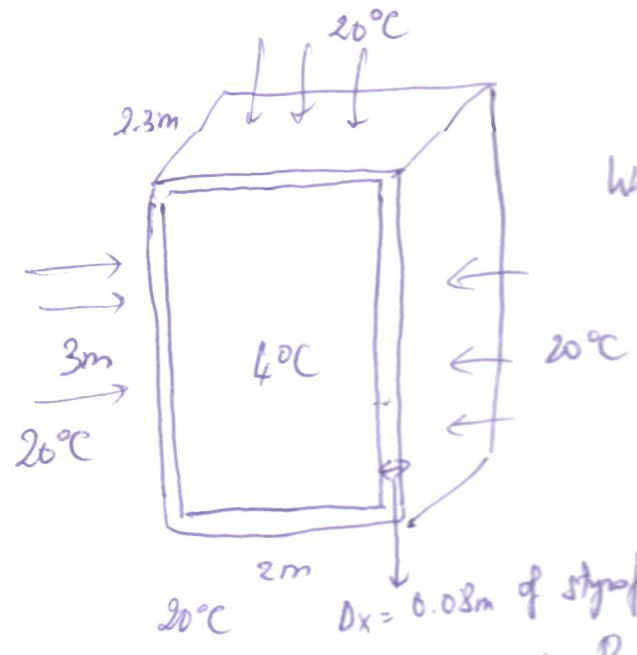
$$H = -kA \frac{dT}{dx} = -\frac{A \Delta T}{\mathcal{R}} = -\frac{A \Delta T}{R A} = -\frac{\Delta T}{R}$$

(Heat transferred from outside to inside → $H_{absorbed} = \frac{\Delta T}{R}$)

$$\Delta t = \frac{\Delta Q}{H_{abs.}} = \frac{m_{ice} L_f}{\frac{\Delta T}{R}} = \frac{15 \times 334 \times 10^3}{\frac{(20-0)}{0.12}} = 3 \times 10^4 \text{ s} \frac{1 \text{ h}}{3600 \text{ s}} = 8.35 \text{ h.}$$

$L_f (\text{water}) = 334 \frac{\text{kJ}}{\text{kg}}$

16.56



Walk-in cooler

$$H = - \frac{A \Delta T}{R}$$

$$R = \frac{\Delta x}{k} = \frac{0.08}{0.029} = 2.76$$

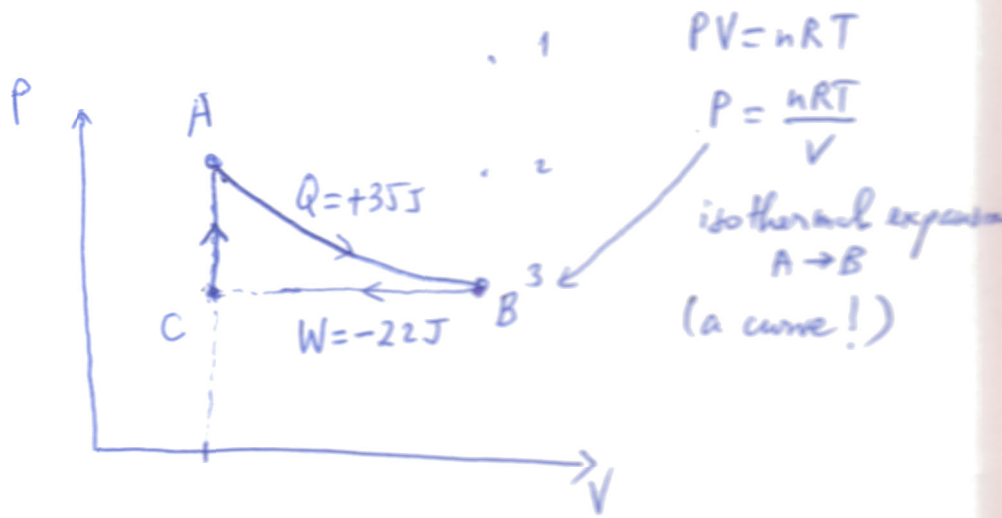
Table 16.2

$$A = \underbrace{3 \times 2 \times 2}_{\text{Front \& back}} + \underbrace{3 \times 2.3 \times 2}_{\text{Left \& right}} + \underbrace{2.3 \times 2 \times 2}_{\text{Top \& bottom}} = 35 \text{ m}^2$$

$$H = - \frac{32 \times 16}{2.76} = - 203 \text{ W}$$

18.42

a)



* expansion $A \rightarrow B$
 (B will be on the right of A)

Incorporate correct signs by comparing data with definitions of Q & W

$\Delta U = Q - W$	Work done by system	} Work done $\rightarrow +$ Work received $\rightarrow -$
\downarrow <div style="border: 1px solid black; padding: 2px; display: inline-block;">Heat absorbed</div> \downarrow $+$		
(Heat coming into system \rightarrow positive Heat coming out of system \rightarrow negative)		

* isobaric compression $B \rightarrow C$
 (C will be to the left of B)
 on a horizontal line, with $V_c = V_A$

b) Work done or received by gas during complete cycle:

$$\begin{array}{c}
 A \rightarrow B \rightarrow C \rightarrow A \\
 \downarrow \quad \downarrow \quad \downarrow \\
 W_{AB} \quad W_{BC} \quad W_{CA} = 0 \quad (V_c = V_A \rightarrow \Delta V = 0 \rightarrow W = 0)
 \end{array}$$

Work W and heat Q are not state variables! \rightarrow depend on the type of process.

A → B: isothermal process: $W_{AB} = Q$ ($\Delta U = 0$ ideal gas) (given).
 $= +35 \text{ J}$

B → C: isobaric compression: $W_{BC} = -22 \text{ J}$ (given)

$$W_{ABCA} = W_{AB} + W_{BC} = 35 \text{ J} - 22 \text{ J} = 13 \text{ J}$$

By the sign → work done by gas in one cycle is 13 J.

c) (1st) $Q_{BCA} = Q_{ABCA} - Q_{AB}$ (linear superposition)

$$Q_{ABCA} = ? \left\{ \begin{array}{l} \Delta U \text{ is a state variable} \rightarrow \Delta U_{ABCA} = 0 \\ \text{1st Law: } \Delta U_{ABCA} = Q_{ABCA} - W_{ABCA} \\ 0 = Q_{ABCA} - 13 \text{ J} \end{array} \right.$$

$$\boxed{Q_{ABCA} = 13 \text{ J}}$$

$$Q_{AB} = +35 \text{ J}$$

$$\rightarrow Q_{BCA} = 13 \text{ J} - 35 \text{ J} = -22 \text{ J}.$$

By the sign heat in the amount of 22 J has been lost by the gas in B → C → A.

(2nd) $Q_{BCA} = Q_{BC} + Q_{CA}$

Ch 19 2nd Law of Thermodynamics

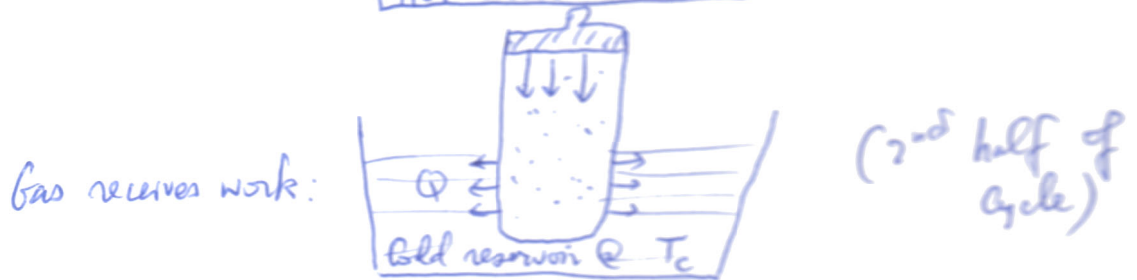
In connection with heat engines working b/w 2 different heat reservoirs.

Heat reservoir: a source of heat, large, at constant temp.

Visual experiment: put a piston filled with an ideal gas in thermal contact with a heat reservoir (hot) at temperature T_h . Gas absorbs heat, expands, does work. No more expansion (no more work done) when gas reaches same temp. as the constant T_h heat reservoir.

How to get more work done? \rightarrow Bring gas back to its original temp. by putting it in thermal contact with a cold reservoir @ temp. T_c \rightarrow gas loses heat, compressed (receiving work).

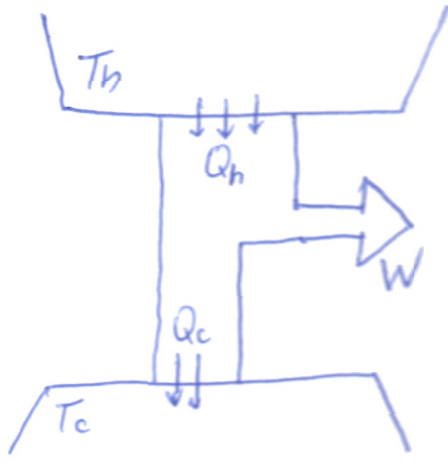
Diagrams:



In one diagram: Heat engine

Q_{ac} {

Heat absorbed = Q_h
Heat lost = Q_c



More work done than received by gas \rightarrow net work done is W

1st Law of T.D. for a heat engine:

$$\Delta U = Q_{net} - W = Q_h - Q_c - W$$

In one complete cycle
(the 2nd half brings gas back to original condition or state)

$$Q_h - Q_c = W$$

Efficiency of the heat engine: $e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$

$$0 < \frac{Q_c}{Q_h} < 1$$

$$\rightarrow e = 1 - \frac{|Q_c|}{|Q_h|} < 1$$

There is not $e > 1$

2nd Law of T.D.: it is impossible to build a heat engine operating in cycles that extracts heat from a hot reservoir (and returning some of it to a cold reservoir) that can deliver 100% efficiency ($e < 1$)

18.59, 33, 17.7/

18.59

Gas mixture: Ar & O₂

Fraction: f 1-f
(0 < f < 1)

$$\begin{cases} V_1 \rightarrow V_2 = 2V_1 \\ P_1 \rightarrow P_2 = \frac{P_1}{3} \end{cases}$$

Adiabatic process (expansion)

Possible paths:

$$\begin{aligned} Q &= 0 \quad \times \\ \Delta U &= -W \quad \times \\ P_1 V_1^\gamma &= P_2 V_2^\gamma \quad \checkmark \quad \gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} \Rightarrow \Delta U \end{aligned}$$

$$C_v = \frac{3}{2}Rf + \frac{5}{2}R(1-f)$$

monoatomic diatomic.

$$\gamma = \frac{\frac{3}{2}Rf + \frac{5}{2}R(1-f) + R}{\frac{3}{2}Rf + \frac{5}{2}R(1-f)} = \frac{\frac{7}{2} - f}{\frac{5}{2} - f}$$

Now find γ :

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma \rightarrow \ln\left(\frac{P_1}{P_2}\right) = \gamma \ln\left(\frac{V_2}{V_1}\right)$$

$$\rightarrow \gamma = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln 3}{\ln 2} = 1.58$$

(recall: monoatomic: $\gamma = 1.67$; diatomic $\gamma = 1.4$)

$$\frac{5}{2}\gamma - \gamma f = \frac{7}{2} - f \rightarrow \frac{5}{2}\gamma - \frac{7}{2} = (\gamma - 1)f \rightarrow f = \frac{\frac{5}{2}\gamma - \frac{7}{2}}{\gamma - 1}$$

$$f = \frac{\frac{5}{2} \cdot 1.58 - \frac{7}{2}}{1.58 - 1} = 0.78 \rightarrow \begin{cases} \text{Ar } 78\% \\ \text{O}_2 \text{ } 22\% \end{cases}$$

18.33

Blood artery pressure $\left\{ \begin{array}{l} P_H = 125 \text{ mm Hg} \\ P_L = 80 \text{ mm Hg} \end{array} \right\}$ gauge pressures.
 actual pressures \rightarrow add atmospheric pressure = 1 atm
 \downarrow
 760 mm Hg

Air bubble : $d_L = 1.52 \text{ mm} @ P_L$

a) d_H ? ($d_H < d_L$) \rightarrow gas volume will determine the diameter of the spherical bubble :

Ideal gas equation: $PV = nRT$ $\left\{ \begin{array}{l} P_L V_L = nRT \\ P_H V_H = nRT \end{array} \right\}$ same body temp.

$\hookrightarrow P_L V_L = P_H V_H \rightarrow V_H = \frac{P_L}{P_H} V_L$

$\frac{4}{3}\pi\left(\frac{d_H}{2}\right)^3 = \frac{P_L}{P_H} \frac{4}{3}\pi\left(\frac{d_L}{2}\right)^3$

$d_H^3 = \frac{P_L}{P_H} d_L^3$

$d_H = \left(\frac{P_L}{P_H}\right)^{1/3} d_L$

$= \frac{(760 + 80)^{1/3}}{(760 + 125)^{1/3}} 1.52 \text{ mm}$

$760 \text{ mm Hg} \rightarrow 1 \text{ atm} \rightarrow 1.013 \times 10^5 \text{ Pa}$

$d_H = \left(\frac{840}{885}\right)^{1/3} 1.52 \text{ mm} = 1.49 \text{ mm}$

b) How much work does blood (so heart) do to compress the bubble?
 (body temp stays const. @ 37°C \rightarrow isothermal process :

$W = nRT \ln\left(\frac{V_2}{V_1}\right) = \frac{nRT}{P_L V_L} \ln\left(\frac{V_H}{V_L}\right) = \left\{ \begin{array}{l} 840 \cdot \frac{1.013 \times 10^5}{760} \times \frac{4}{3}\pi\left(\frac{1.52 \times 10^{-3}}{2}\right)^3 \times \\ \ln\left[\left(\frac{1.49}{1.52}\right)^3\right] \\ = -12312 \times 10^{-9} \text{ J} \end{array} \right.$

\downarrow $= -12 \mu\text{J}$
 Work done on gas or bubble since it is getting compressed.
 \downarrow micro or 10^{-6}

17.71

clock: brass pendulum: $L = 0.2 \text{ m}$ @ 20°C (good)
calibrated to work as a clock @ 20°C .

↓
 $T_2 = 18^\circ\text{C}$ → how long until it makes a 1min error?

Since for a pendulum: # oscillations per second $\omega = \sqrt{\frac{g}{L}}$
Length will change with temp, according to the linear coefficient
of expansion for brass: → Table 17.2: $\alpha = 19 \times 10^{-6} \text{ }^\circ\text{K}^{-1}$

$$\alpha = \frac{\frac{\Delta L}{L}}{\Delta T} \rightarrow \frac{\Delta L}{L} = \alpha \Delta T \rightarrow \Delta L = \alpha L \Delta T$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}} \quad \left\{ \begin{array}{l} T_{20^\circ\text{C}} = 2\pi \sqrt{\frac{L}{g}} \\ T_{18^\circ\text{C}} = 2\pi \sqrt{\frac{L + \Delta L}{g}} = 2\pi \sqrt{\frac{L + \alpha L \Delta T}{g}} \end{array} \right.$$

$$T_{18^\circ\text{C}} = 2\pi \sqrt{\frac{L + \Delta L}{g}} = 2\pi \sqrt{\frac{L + \alpha L \Delta T}{g}} \quad \begin{array}{l} \uparrow \\ -2^\circ\text{K} \end{array}$$
$$= 2\pi \sqrt{\frac{L(1 + \alpha \Delta T)}{g}}$$

$$T_{18^\circ\text{C}} < T_{20^\circ\text{C}} \rightarrow T_{20^\circ\text{C}} - T_{18^\circ\text{C}} = \frac{2\pi}{\sqrt{g}} \left[\sqrt{L} - \sqrt{L(1 + \alpha \Delta T)} \right]$$
$$= \frac{2\pi}{\sqrt{g}} \sqrt{L} \left[1 - \sqrt{1 - 2\alpha} \right]$$

\downarrow
 $\Delta T = -2^\circ\text{K}$

$$\sqrt{1 - 2\alpha} = (1 - 2\alpha)^{1/2} \cong 1 - \frac{1}{2}(2\alpha) = 1 - \alpha$$
$$(1 + x)^{1/2} \cong 1 + \frac{1}{2}x + \dots$$

negligible if x very
small compared to 1

$$\boxed{T_{20^\circ\text{C}} - T_{18^\circ\text{C}} = 2\pi \sqrt{\frac{L}{g}} [1 - (1 - \alpha)] = 2\pi \sqrt{\frac{L}{g}} \alpha} \quad \begin{array}{l} \text{time} \\ \text{error per period.} \end{array}$$

How many periods of pendulum until a difference of 1 min:

$$\frac{60s}{2\pi\sqrt{\frac{L}{g}} \propto}$$

How long until a difference of 60s? $\frac{60s}{2\pi\sqrt{\frac{L}{g}}} T_{20\%}$

$$= \frac{60}{2\pi\sqrt{\frac{L}{g}}} \cancel{2\pi\sqrt{\frac{L}{g}}}$$

$$= \frac{60}{\alpha} = \frac{60}{19 \times 10^{-6}} s \rightarrow \underline{\underline{36.5 \text{ days}}}$$