Ch 27: Inductance & Magnetic Energy

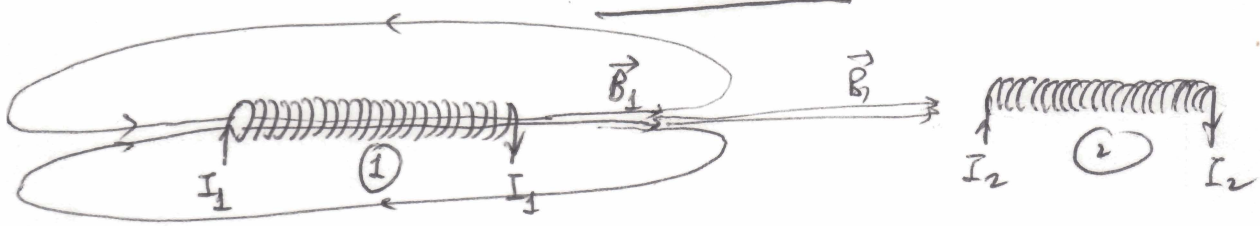
Capacitors: storage devices for electric energy

Capacitance $C = \frac{Q}{V}$

Inductors: storage devices for magnetic energy

$\left\{ \begin{array}{l} \underline{\text{Mutual inductance}} \quad M = \dfrac{\phi_2}{I_1} \\ \underline{\text{self inductance}}: \quad L = \dfrac{\phi}{I} \end{array} \right.$

Two solenoids



By Ampere's Law: $B_1 = \mu_0 \underbrace{n_1}_{\substack{\text{\# turns per} \\ \text{unit length in} \\ \text{solenoid \#1}}} I_1$

Solenoid #2 is exposed to B_1 (field created by solenoid #1) $\rightarrow$ ϕ_2 : magnetic flux going through solenoid #2 by the field B_1 created by #1

If I_1 changes with time $\rightarrow$ ϕ_2 changes with time $\rightarrow$ EM induction (Faraday's law): induced voltage $\quad -\varepsilon_2 = \frac{d\phi_2}{dt} = \underbrace{A_2}_{\substack{\text{Total crossectional} \\ \text{area of solenoid \#2}}} \frac{dB_1}{dt}$

$= \underbrace{A_2 \mu_0 n_1}_{M} \frac{dI_1}{dt} = \frac{d(MI_1)}{dt}$

M : Mutual Inductance (relates the induced electric voltage in #2 with a changing current in #1)

$\phi_2 = M I_1$ (M also relates ϕ_2 with I_1)

$\Rightarrow \boxed{M = \frac{\phi_2}{I_1}}$

Question: What about the effect of the magnetic field B_2 created by solenoid #2 on solenoid #1? Similar:

$$-\mathcal{E}_1 = \underbrace{A_1 \mu_0 n_2}_{M} \frac{dI_2}{dt}$$

$M \rightarrow$ Same mutual inductance!

Unit: $$M = \frac{[\mathcal{E}]}{\frac{[I]}{[time]}} = \frac{Volt}{\frac{A}{s}} = H \text{ (Henry)} \quad SI.$$

Question: What about the effect of the magnetic field B_1 created by solenoid #1 on itself?

$B_1 = \mu_0 n_1 I_1$ clearly goes through solenoid #1 $\rightarrow$ creating a self magnetic flux ϕ. If I_1 change with time $\rightarrow \phi$ change with time $\rightarrow$ creating a self-induced voltage:

$$-\mathcal{E} = \frac{d\phi}{dt} = A \frac{dB}{dt} = \underbrace{A \mu_0 n_1}_{L} \frac{dI_1}{dt}$$

L : self-inductance

$$\boxed{-\mathcal{E} = L \frac{dI}{dt}} = \boxed{\frac{d}{dt}(LI)}$$

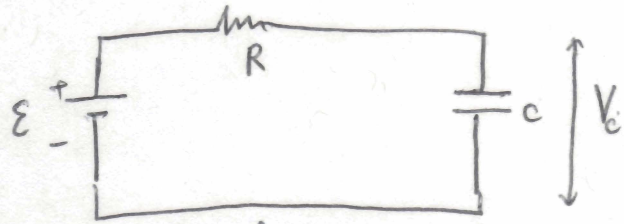
Unit: also H (Henry) in SI.

$$\phi = LI \quad \text{or} \quad \boxed{L = \frac{\phi}{I}}$$

self magnetic flux

resistor capacitor

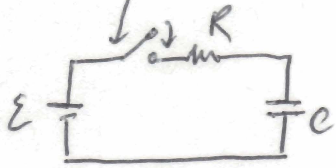
RC Circuit



Used to charge a capacitor

(*) $t=0$ (circuit is just closed) : $V_c = 0$

Short circuit across C



as circuit is closed V_c doesnot change instantaneously → electric inertia

$t=\infty$ (long after) : $I_c = 0$

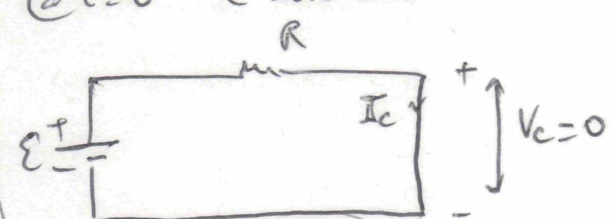
open circuit across C

$$I_c(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$$

is $I_c(t=0)$ max-current

$\tau \equiv \frac{1}{RC}$: "time constant" in s

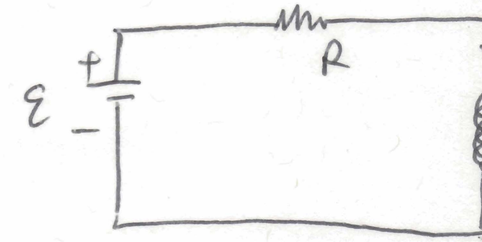
@ $t=0$ C acts like a short circuit:



$I_c = \frac{\mathcal{E}}{R}$

resistor inductor

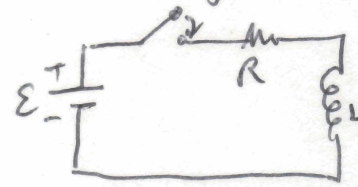
RL Circuit



helix for inductors

(*) $t=0$ (circuit is just closed) : $I_L = 0$

Open circuit across the inductor L



as circuit is closed. I_L doesnot change instantaneously → magnetic inertia

$t=\infty$ (long after) : $V_L = 0$

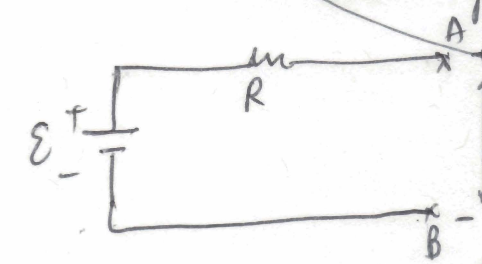
Short-circuit across L

$$V_L(t) = \mathcal{E} e^{-\frac{t}{(\frac{L}{R})}}$$

is $V_L(t=0)$ is max voltage

$\tau \equiv \frac{1}{\frac{L}{R}} = \frac{R}{L}$ (= "time constant) in s.

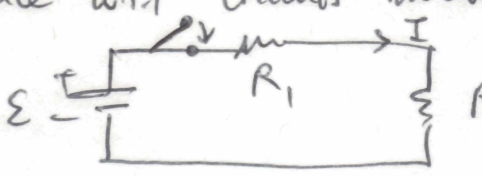
@ $t=0$: L acts like an open circuit.



$V_L = \mathcal{E}$

Note: Ohm's law applies only to closed circuits

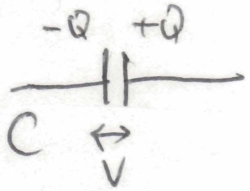
Main difference w.r.t circuits involving only resistors:



As circuit is closed : V_{R_1} & V_{R_2} as well as I come up to their final values instantaneously.

Magnetic Energy

Electric energy & Capacitors



Energy: $U_c = \boxed{\frac{1}{2}CV^2}$ (J)

C in F, V in Volt

Energy density: $u_c = \frac{U_c}{Ad} = \boxed{\frac{1}{2}\varepsilon_0 E^2}$ $\left(\frac{J}{m^3}\right)$

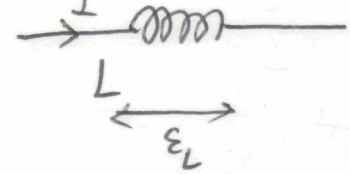
Ad: Vol b/w plates

A: cross sectional area

d: separation b/w plates.

Parallel plates: $C = \frac{A\varepsilon_0}{d}$

Magnetic energy & Inductors



$\mathcal{E}_L = -L\frac{dI}{dt}$ (Faraday's law)

Energy: $U_L = \int_0^t P_L\, dt = \int_0^t I|\mathcal{E}_L|\, dt = L\int_0^t I\frac{dI}{dt}\, dt$

P_L: energy per unit time or power

$= \frac{1}{2}L\left[I^2\right]_{t=0}^{t=t} = \boxed{\frac{1}{2}LI^2}$ (J)

L in H, I in A

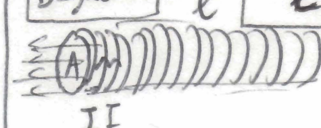
Energy density: $u_L = \frac{U_L}{A\ell} = \frac{\frac{1}{2}LI^2}{A\ell} = \frac{1}{2}\frac{\mu_0 N^2 \cancel{A}}{\cancel{A}\ell^2} = \boxed{\frac{1}{2}\frac{1}{\mu_0}B^2}$

$A\ell$: vol. in solenoid

A: cross-sectional area

ℓ: length

Ampere's law: $\boxed{B = \mu_0 n I}$



ℓ

$\boxed{n = \frac{N}{\ell}}$ N → # turns, ℓ → length

What is the self-inductance L?

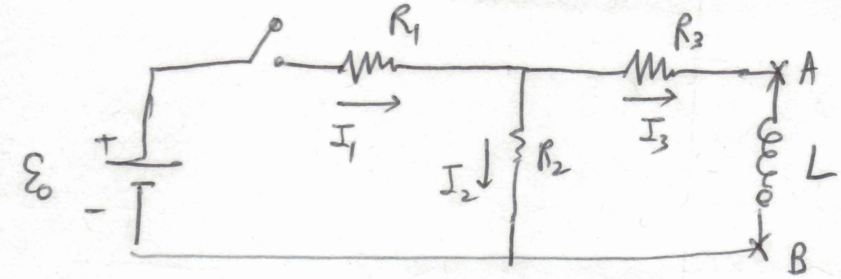
$$\boxed{L = \frac{\Phi}{I} = \frac{NBA}{I} = \frac{N\left(\mu_0\frac{N}{\ell}\cancel{I}\right)A}{\cancel{I}} = \mu_0\frac{N^2A}{\ell}}$$

Φ: Magnetic flux: $\oint \vec{B}\cdot d\vec{A} = BA$ in this case

- B uniform
- B ⊥ to the cross-sectional area ($\vec{B} \parallel \vec{A}$)

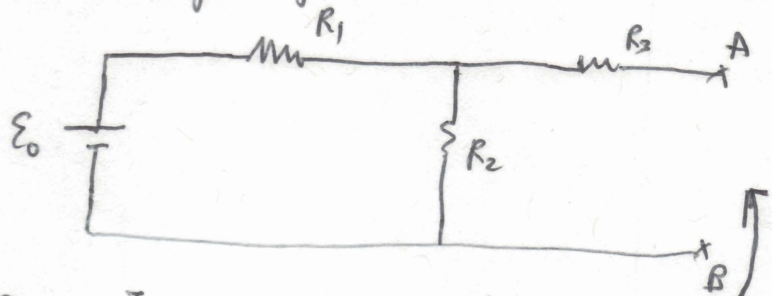
Flux through N turns is NBA

27.62



$$\begin{cases} \mathcal{E}_0 = 12V \\ R_1 = 4\Omega \\ R_2 = 8\Omega \\ R_3 = 2\Omega \\ L = 2H \end{cases}$$

a) Find I_2 right after $(t=0)$ switch is closed



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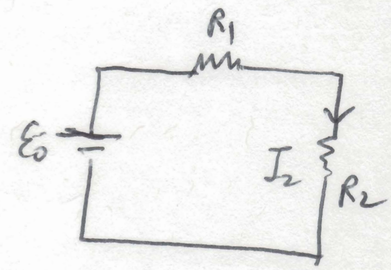
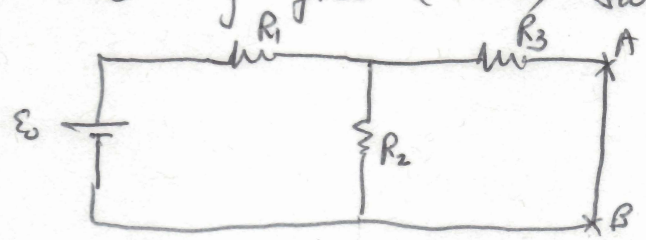


@ $t=0$: $I_L = 0 \rightarrow$ open circuit across L

$$I_2 = \frac{\mathcal{E}_0}{R_1+R_2} = \frac{12}{4+8}$$

$$\boxed{I_2 = 1A}$$

b) Find I_2 long after $(t=\infty)$ switch is closed:



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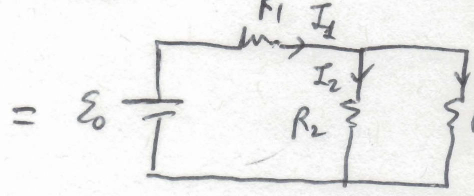
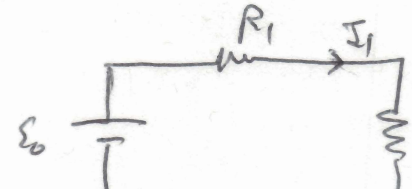


@ $t=\infty \rightarrow V_L = 0 \rightarrow$ short-circuit across L

Current division:

$$I_2 = I_1 \frac{R_3}{R_2+R_3}$$

To find I_1:



$\frac{R_2 R_3}{R_2+R_3}$ (Parallel combination b/w R_2 & R_3)

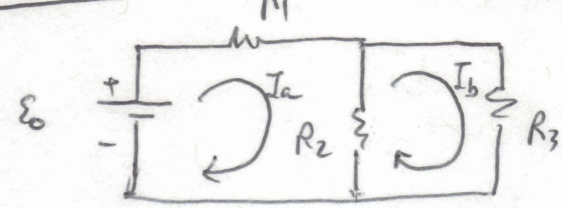
$$I_1 = \frac{\mathcal{E}_0}{R_1 + \frac{R_2R_3}{R_2+R_3}} = \frac{12}{4+\frac{8\times 2}{8+2}} = \frac{12}{4+1.6} = 2.14A$$

$$\boxed{I_2 = 2.14 \frac{2}{8+2} = \frac{2.14}{5} = 0.429A}$$

As a check: $I_3 = I_1 \frac{R_2}{R_2+R_3} = 2.14 \frac{8}{8+2} = 2.14 \frac{8}{10} = 1.71\,A$

$I_2 + I_3 = 0.429 + 1.71 = 2.14\,A = I_1$

Parenthesis: (Not needed to solve this problem)



Loop Analysis:

1) $\mathcal{E}_0 - I_a R_1 - (I_a - I_b) R_2 = 0$

\+ 2) $-(I_b - I_a) R_2 - I_b R_3 = 0$

$\mathcal{E}_0 - I_a R_1 - I_b R_3 = 0$

$$\boxed{I_a = \frac{\mathcal{E}_0 - I_b R_3}{R_1}}$$

2): $I_b (R_2 + R_3) - I_a R_2 = 0$

$I_b (R_2 + R_3) - \frac{R_2}{R_1}(\mathcal{E}_0 - I_b R_3) = 0$

$$I_b \left(R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) = \frac{R_2}{R_1} \mathcal{E}_0$$

$$I_b = \frac{\frac{R_2}{R_1}\mathcal{E}_0}{R_2 + R_3 + \frac{R_2 R_3}{R_1}}$$

$$= \frac{\frac{8}{4} 12}{8 + 2 + \frac{8 \times 2}{4}} = \frac{24}{14}$$

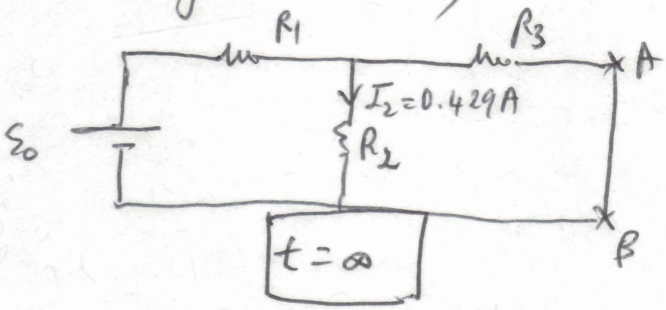
$$\boxed{I_b = \frac{12}{7}\,A}$$

$\rightarrow I_a = \frac{12 - \frac{12}{7} 2}{4} = 3\left(1 - \frac{2}{7}\right) = 3 \times \frac{5}{7}\,A$

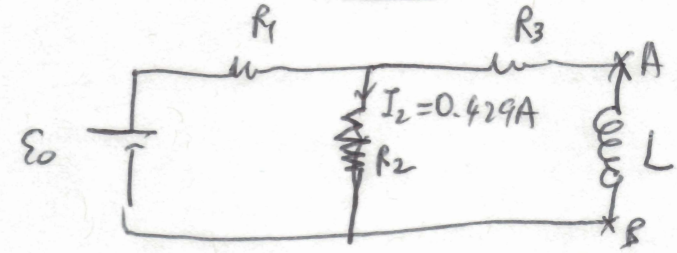
$\rightarrow I$ thru R_2: $I_a - I_b = \frac{15}{7} - \frac{12}{7} = \boxed{\frac{3}{7}\,A = 0.429\,A}$

Note: when there is only 1 battery in a circuit → less algebra if we use series & parallel ...

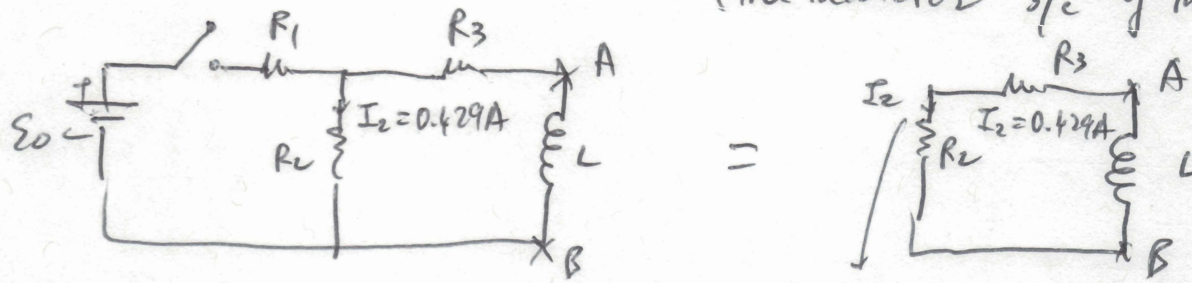
c) If long after, now circuit is reopened → what is I_2?
(due to magnetic inertia @ the inductor: if V_L was 0 at $t=\infty$
→ when the circuit is reopened: → V_L will stay 0 by
magnetic inertia)!



Now: L acts like a short-circuit
but it is still there physically!



When switch is reopened now: nothing is changed yet across AB
(the inductor b/c of m-inertia)



Current is still 0.429A.

How can I_2 stay @ 0.429A for a moment when the battery $\mathcal{E}_0$ has been disconnected? → Energy stored in L is being used! Current 0.429A will decrease to 0A when all stored energy is dissipated in R_2 & R_3

27.70 Some E & B have same <u>energy density</u> : obtain an expression for $\frac{E}{B}$ and evaluate this.

$$u_C = \frac{1}{2}\varepsilon_0 E^2 \qquad u_L = \frac{1}{2}\frac{1}{\mu_0}B^2$$

$$\rightarrow \quad u_C = u_L \rightarrow \cancel{\frac{1}{2}}\varepsilon_0 E^2 = \cancel{\frac{1}{2}}\frac{1}{\mu_0}B^2 \rightarrow \frac{E}{B} = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$$

$$\boxed{\frac{E}{B} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}} = \frac{1}{\sqrt{4\pi\times10^{-7}\,\cancel{\frac{N}{A^2}} \times 8.85\times10^{-12}\frac{C^2}{\cancel{N}m^2}}} = \frac{2.99\times10^8}{\sqrt{\frac{C^2}{A^2m^2}}}$$

$$= 2.99\times10^8 \; \frac{Am}{C} = 2.99\times10^8\,\frac{m}{s} = c$$

$$\frac{Am}{C} \rightarrow \frac{\frac{\cancel{C}}{s}m}{\cancel{C}}$$

$c \rightarrow$ $\boxed{\text{speed of light}}$

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	mass velocity	
K.E.	$\frac{1}{2}mv^2$	: mass is an inertia for change in speed

capacitance electric potential

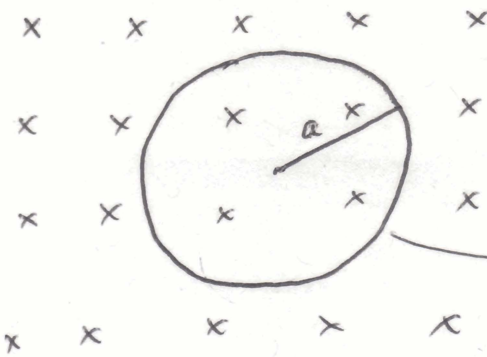
E.E $\quad \frac{1}{2}CV^2$: capacitor is an inertia for change in electric potential

inductance current

M.E $\quad \frac{1}{2}LI^2$: inductor is an inertia for changes in electric current

(delayed switch, ...)

27-53



B uniform, into page
changes from $B_1 \rightarrow B_2$

wire loop of resistance R

Show: total charge moves around ring is $Q = \frac{\pi a^2}{R}(B_2 - B_1)$
(Tip: integrate loop current over time)

$\hookrightarrow I = \frac{dq}{dt} \rightarrow \int I\, dt = \int \frac{dq}{\cancel{dt}} \cancel{dt} = Q$

$$Q = \int I_{induced}\, dt = \int \frac{\mathcal{E}}{R}\, dt = \int \frac{\left|\frac{d\phi}{dt}\right|}{R}\, dt =$$

when B changes $B_1 \rightarrow B_2 \rightarrow -\mathcal{E} = \frac{d\phi}{dt} \rightarrow I_{induced} = \frac{\mathcal{E}}{R}$

induced voltage

Ohm's law

$$= \frac{1}{R}\left|\int d\phi\right| = \frac{1}{R}\left|\Phi\right|_1^2 = \frac{\pi a^2}{R}|B_2 - B_1|$$

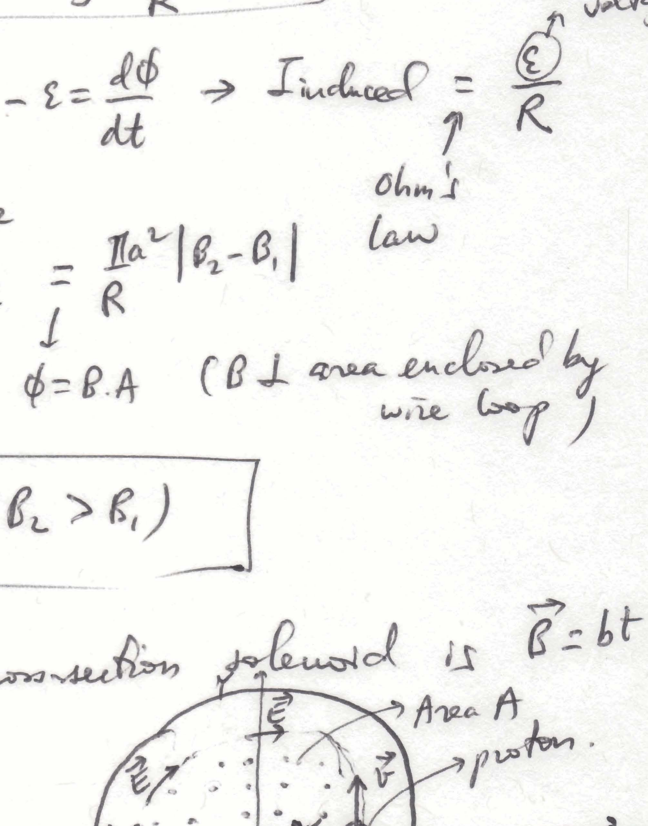
$\phi = B.A$ ($B \perp$ area enclosed by wire loop)

$$\rightarrow \boxed{Q = \frac{\pi a^2}{R}(B_2 - B_1) \qquad (B_2 > B_1)}$$

27-51

B inside a circular cross-section solenoid is $\vec{B} = bt\,\hat{k}$
$\left(b = 2.1\, T/ms\right) = 2100\, \frac{T}{s}$

$ms \rightarrow 10^{-3}s$



@ $t = 0.4\, \mu s$
proton @ $(x = 0.05m, y = 0, z = 0)$
$\vec{v} = 4.8 \times 10^6\, \hat{j}$ m/s
Find net electromagnetic force on proton

Electromagnetic force?

$$\boxed{\vec{F}_{EM} = q\vec{E} + q\vec{v}\times\vec{B}}$$

Magnetic force: $\vec{F}_B = q\vec{v}\times\vec{B}$ ✓

Electric force: from induced electric field: $\vec{F}_E = q\vec{E}$ since $\vec{B}$ is changing with time $\rightarrow \phi$ changing w/ time $\rightarrow$ induced electric field $\vec{E}$ $\rightarrow$ if there is a wire loop $\rightarrow$ induced current

Faraday's law: $\mathcal{E} = -\dfrac{d\phi}{dt} = -\dfrac{d(BA)}{dt} = -A\dfrac{dB}{dt}$

$$= -\pi r^2 \frac{dB}{dt}$$

$\mathcal{E} = -\nabla E$

$$\underbrace{-\int_{\text{along loop}} \vec{E}\cdot d\vec{l} = \bar{E}\cdot 2\pi r}$$

$$\left[E = \frac{r}{2}\frac{dB}{dt} = \frac{rb}{2}\right] \rightarrow \text{induced electric field.}$$

$B = bt$

$$\vec{F}_{EM} = q\left[\frac{rb}{2}(-\hat{j}) + v\underbrace{bt}_{B}(\hat{i})\right]$$

$$= \left[\frac{1.6\times10^{-19}\times 0.05\times 2100}{2}(-\hat{j}) + (\hat{i})\,1.6\times10^{-19}\times 4.8\times10^{6}\times 2100\times 0.4\times10^{-6}\right]$$

$$= \left[-8.4\times10^{-18}\,\hat{j} + 6.451\times10^{-16}\,\hat{i}\right] N$$

Ch 29: Maxwell's Equations & EM waves.

EM waves: unique: can propagate in empty space! Thanks to the ultimate connection b/w the electric & the magnetic field.

So far we have seen some connection b/w E & B through the EM induction (Faraday's law). Ultimate connection was done by Maxwell:

Maxwell's Equations

1) Gauss' Law : $\oint_{\text{closed surface or Gaussian surface}} \vec{E} \cdot d\vec{A} = \dfrac{q_{enclosed}}{\varepsilon_0}$

2) Gauss' Law for Magnetic field : $\oint_{\text{closed surface}} \vec{B} \cdot d\vec{A} = 0$ ← no magnetic monopole

3) Ampere's law : $\oint_{\text{closed loop or Amperian loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} \underbrace{\boxed{+ \mu_0 I_{displacement}}}_{\text{Maxwell's term}}$

4) Faraday's law : $\underbrace{\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{induced voltage } \mathcal{E}} = -\dfrac{d\Phi_B}{dt} = -\int \dfrac{\partial \vec{B}}{\partial t} \cdot d\vec{A}}_{\text{a time varying magnetic field can create an electric field (by induction)}}$

→ Maxwell added a displacement current term in the RHS of Ampere's law: $I_{displacement} = \varepsilon_0 \dfrac{d\Phi_E}{dt} = \varepsilon_0 \int \dfrac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$

↓ Φ_E electric flux

assuming constant area

Now the modified Ampere's Law reads:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

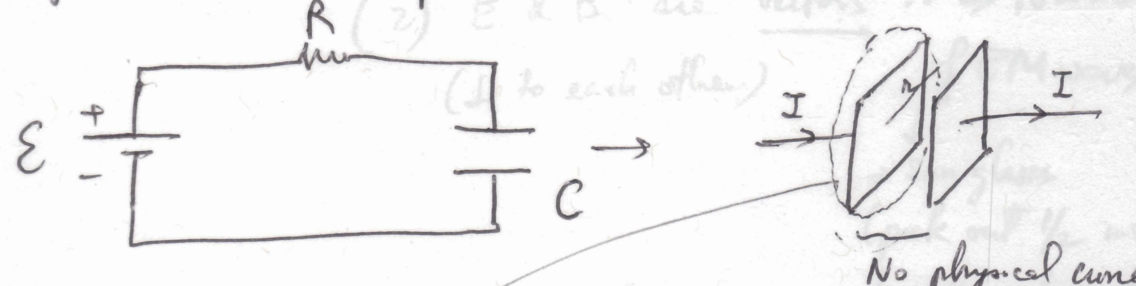
1) Now we can also say: a time-varying electric field can also create a magnetic field!

Important consequence: $E \to B \to E \to B \to \dots$

so EM waves can propagate in vacuum!

- Sun light
- Signals from space probes
- Cell phone signals.
- etc.

2) Technicality: provides an explanation for the measured magnetic field around a capacitor in an RC circuit:



No physical current b/w the plates

If the original Ampere's Law is used on an Amperian loop that is in plane with the left plate:

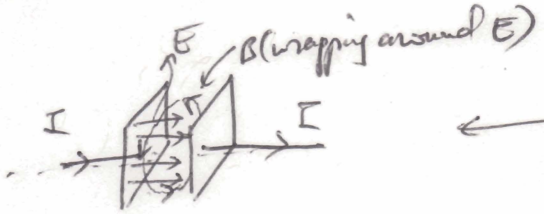
$$\oint_{\text{Amperian loop}} \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 \underbrace{I_{enclosed}}_{=0} \to B = 0$$

since I stops @ left plate w/o crossing the Amperian loop.

This $B=0$ contradicts w/ measurement around the plate when an AC voltage is applied → can measure B around the plates of a capacitor! → only explained by Maxwell's

Alternating current: switching at certain frequency
(standard power outlet $f = 60\,Hz$)

additional term: $\mu_0 I_{displacement} = \mu_0 \varepsilon_0 \int \underbrace{\frac{\partial \vec{E}}{\partial t}} \cdot d\vec{A}$



this time-varying E creates the B measured around the plates.

B { natural magnet (S N) $\vec{B}$; electromagnet: solenoid → B; time-varying E; current I }

Maxwell's equations → 1) Propagation of EM waves in vacuum
2) $\vec{E}$ & $\vec{B}$ are vectors !! → Polarization of EM waves.
(⊥ to each other)

→ Sun glasses
(pick out 1/2 intensity due to polarization).

Electromagnetic wave in Vacuum:

↳ no materials: no charges, no wires → so no currents.

But yes $\vec{E}$ & $\vec{B}$.

Maxwell's equations →

1) Gauss law: $\oint_{\text{Gaussian surface}} \vec{E} \cdot d\vec{A} = 0$

2) $\oint \vec{B} \cdot d\vec{A} = 0$

3) Modified Ampere's law: $\oint_{\text{Amperian loop}} \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$

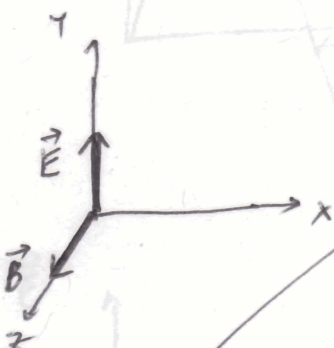
4) Faraday's law: $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

↳ Very similar!!

$\vec{E} \to \vec{B} \to \vec{E} \to \vec{B} \to \cdots$ EM wave.

time varying

perpendicular to each other:



$\vec{E} = E_p \sin(kx - \omega t)\ \hat{j}$

$\vec{B} = B_p \underbrace{\sin(kx - \omega t)}_{\text{wave}}\ \hat{k}$

wave: propagation in the (+) x-direction
↳ direction given by $\vec{E} \times \vec{B}$ (RHR)

(right hand fingers are aligned with $\vec{E}$, as these fingers turn toward $\vec{B}$, thumb points in the direction of propagation)

Amplitude/Magnitude

Wave number: $\frac{2\pi}{\lambda}$

λ ↓ wavelength

Angular frequency

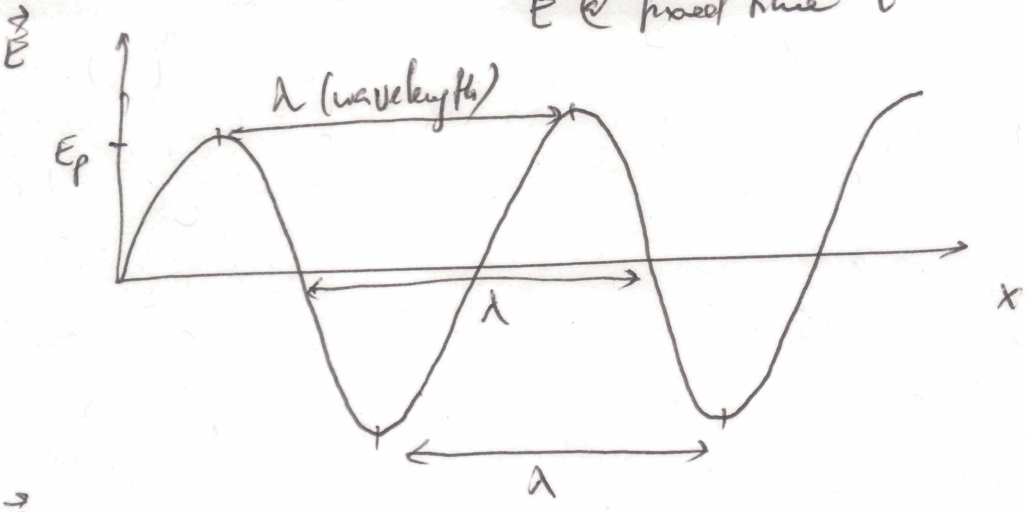
$\omega = \frac{2\pi}{T} = 2\pi f$

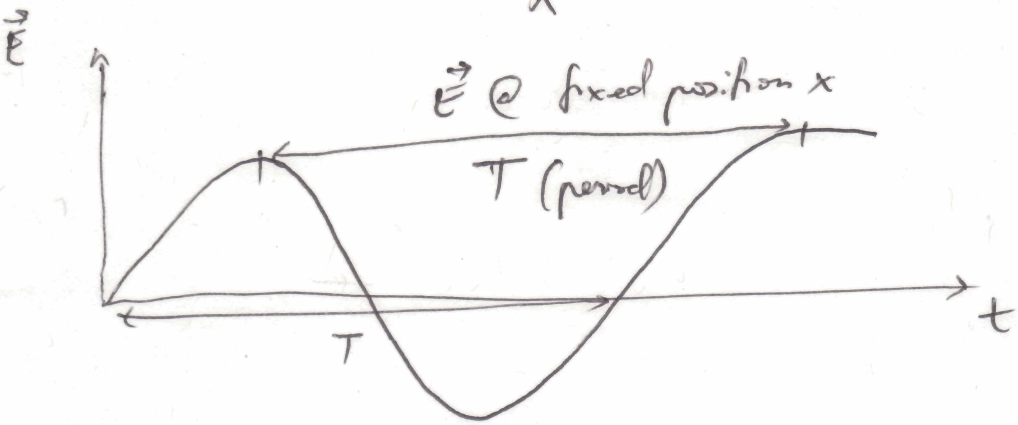
ω: rad/s

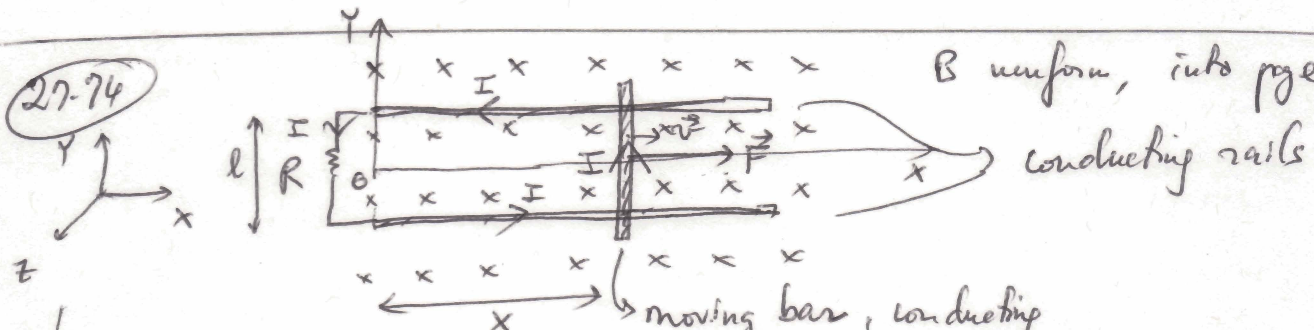
T: period

f: linear frequency Hz (Hertz)

$\vec{E}$ @ fixed time t







$\vec{F}$ constant, applied to bar. Formulate Newton's Law

for bar, find $v(t)$

$$* \begin{cases} \vec{F} = F\hat{i} \\ \vec{v} = v(t)\,\hat{i} \\ \vec{B} = -B\hat{k} \end{cases}$$

$$\vec{F}_{net_{on\,bar}} = m\vec{a}$$

$$\hookrightarrow \begin{cases} \vec{F} = F(\hat{i}) \\ \vec{F}_B \to \text{EM induction} \to I \to I\vec{l}\times\vec{B} \\ \qquad = IlB\,(-\hat{i}) \\ \vec{F}_E = q\vec{E}_{induced} = \text{along } \hat{j} \end{cases}$$

$\to$ not affecting motion of bar along x-direction.

$$\boxed{F - IlB = m\frac{dv}{dt}}$$

$$I_{induced} = \frac{\mathcal{E}}{R} = \frac{\frac{d\Phi_B}{dt}}{R} = \frac{B\frac{dA}{dt}}{R} = \frac{B\frac{d}{dt}(x\ell)}{R} = \frac{B\ell\frac{dx}{dt}}{R} = \frac{B\ell v}{R}$$

$$F - \frac{B^2\ell^2 v}{R} = m\frac{dv}{dt} \rightarrow \frac{dv}{dt} = \underbrace{\frac{F}{m} - \frac{B^2\ell^2}{mR}v}$$

in the form of:

$$\frac{d\rho}{dt} = A + B\rho$$

ρ would be an exponential function in time if $A=0$

We try $v = A - De^{-Ct}$ $\rightarrow$ $\boxed{v(t) = \frac{FR}{B^2\ell^2}\left(1 - e^{-\frac{B^2\ell^2}{Rm}t}\right)}$

coefficients

27.28

$L = 220\ mH$

$I = 350\ mA \longrightarrow 800\ mA$ } How much energy to be supplied?

$$U_L = \frac{1}{2}LI^2 \rightarrow \Delta U_L = U_2 - U_1 = \frac{1}{2}0.22\left[0.8^2 - 0.35^2\right]\ J$$
$$= 56.7\times10^{-3}\ J$$

27.56

$R = 3.3\Omega$, $L = 2.1 H$ } $\mathcal{E}_0 = 45V$

If $I_L = 9.5A$ how long has switch been closed?

Recall: $V_L = \mathcal{E}_0 e^{-\frac{t}{(L/R)}} \rightarrow I_L = \frac{\mathcal{E}_0 - V_L}{R} = \frac{\mathcal{E}_0\left(1 - e^{-t/(L/R)}\right)}{R}$

$$\rightarrow 1 - \frac{I_L R}{\mathcal{E}_0} = e^{-\frac{t}{L/R}} \xrightarrow{\ln} -\frac{t}{L/R} = \ln\left(1 - \frac{I_L R}{\mathcal{E}_0}\right)$$

$$t = -\frac{L}{R}\ln\left(1 - \frac{I_L R}{\mathcal{E}_0}\right) = -\frac{2.1}{3.3}\ln\left(1 - \frac{9.5\times3.3}{45}\right) = 0.76s$$

Ch 29 EM Waves (Cont.)

Maxwell's equations in vacuum : differential forms ;
involving derivatives instead of integrals

Ampere's (Modified) Law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A} \longrightarrow \frac{\partial B}{\partial x} = -\mu_0\varepsilon_0 \frac{\partial E}{\partial t} \quad (3)$$

Faraday's Law:

$$\underbrace{\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}}_{\text{integral forms}} \longrightarrow \underbrace{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}_{\text{differential forms}} \quad (4)$$

$$E = E_p \sin(kx-\omega t) \rightarrow \frac{\partial E}{\partial x} = kE_p \cos(kx-\omega t)$$

$$B = B_p \sin(kx-\omega t) \rightarrow -\frac{\partial B}{\partial t} = \omega B_p \cos(kx-\omega t)$$

(4) Faraday's Law in diff. form : $\rightarrow \; kE_p = \omega B_p$

or $\boxed{\dfrac{E_p}{B_p} = \dfrac{\omega}{k}} = \dfrac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \dfrac{\lambda}{T} = c$

(also in 27.70)

Similarly if we perform

$$\begin{cases} \dfrac{\partial B}{\partial x} = kB_p \cos(kx-\omega t) \\ \dfrac{\partial E}{\partial t} = -\omega E_p \cos(kx-\omega t) \end{cases}$$

(3) Modified Ampere's in diff-form $\rightarrow \; kB_p = \mu_0\varepsilon_0 \omega E_p$

$$\rightarrow \boxed{\frac{E_p}{B_p} = \frac{k}{\omega\mu_0\varepsilon_0}}$$

$$\Rightarrow \quad \frac{\omega}{k} = \frac{k}{\omega \mu_0 \varepsilon_0} \quad \rightarrow \quad \frac{\omega^2}{k^2} = \frac{1}{\mu_0 \varepsilon_0}$$

$$\downarrow$$

$$\left(\frac{\lambda}{T}\right)^2 = c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\rightarrow c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^8 \text{ m/s}.$$

$\Rightarrow$ Speed of EM wave $\left\{ \begin{array}{c} \text{light} \\ \text{radio} \\ \ldots \end{array} \right\}$ travel @ speed of light $c = 3 \times 10^8$ m/s

---

EM wave equation:

$$\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \right) \quad (3) \quad \rightarrow \quad \frac{\partial^2 B}{\partial x \partial t} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

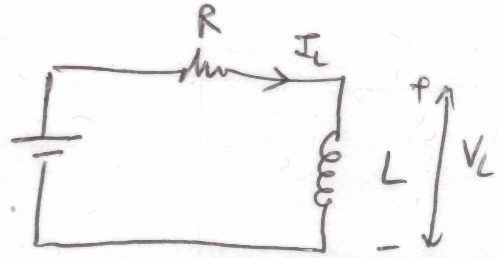
$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \right) \quad (4) \quad \rightarrow \quad \frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$$

$$\hookrightarrow \boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

wave equation for the electric field!

Note similarity with wave equation for a transverse wave in a string (perturbation was $\perp$ to direction of propagation). EM waves are a type of wave like the mechanical wave with the unique property that it can propagate in vacuum. (Thanks to Maxwell's Equations: $\vec{E} \rightarrow \vec{B} \rightarrow \vec{E} \rightarrow \vec{B} \rightarrow \ldots$)

27.59



$\mathcal{E}_0 = 60V$
$R = 22\Omega$
$L = 1.5H$

Rate of current change a) right after switch is closed.

$$I_L = \frac{\mathcal{E}_0}{R}\left(1 - e^{-\frac{t}{L/R}}\right) \rightarrow \frac{dI_L}{dt} = -\frac{\mathcal{E}_0}{R}\left(-\frac{1}{\frac{L}{R}}\right)e^{-\frac{t}{L/R}}$$

$$@\ t=0 \rightarrow \frac{dI_L}{dt} = \frac{\mathcal{E}_0}{\cancel{R}}\frac{\cancel{R}}{L} = \frac{60V}{1.5H} = 40\ \frac{A}{s}$$

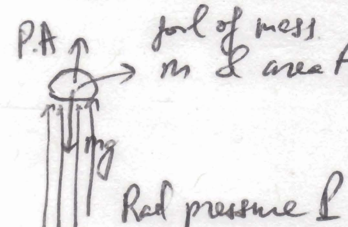
b) @ $t = 0.1s$

$$\frac{dI_L}{dt} = \frac{\mathcal{E}_0}{L}e^{-\frac{t}{L/R}} = \frac{60V}{1.5H}e^{-\frac{0.1}{1.5/22}} = 9.23\ \frac{A}{s}$$

29.56

Radiation pressure:

EM wave can apply a pressure ; laser beam to hold a piece of aluminum foil :



$m = 30\mu g$

What laser power is needed?

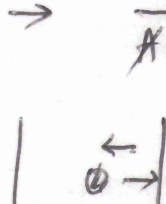
$$PA = mg$$

average rad. intensity $S = \frac{P}{A}$

$$\frac{\bar{S}}{c} \times \underbrace{2}_{\text{Foil is reflecting light.}} \times A = mg \rightarrow \frac{\bar{P}}{\cancel{A}c}2\cancel{A} = mg \rightarrow \boxed{\bar{P} = \frac{mgc}{2} = 44.1W}$$

speed of light $3\times10^8\ m/s$

$\sim$



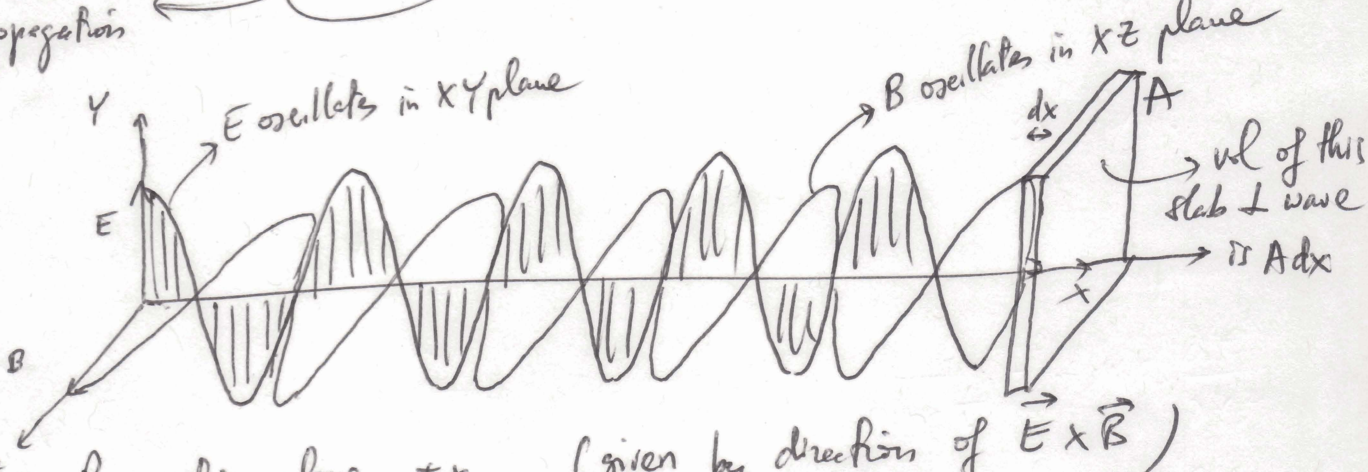
gas molecule reflected by wall → transferring $2mv$ to the wall.

Similarly :
$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}}$$
Wave equation for magnetic field.

Intensity of EM wave :

$\hookrightarrow$ $S = \dfrac{P}{\text{Area}} = \dfrac{\frac{dU}{dt}}{\text{Area}}$

perpendicular to propagation



→ Propagation along +x (given by direction of $\vec{E} \times \vec{B}$)
RHR

→ Polarization (direction of E) is along y-axis

Total energy : $dU = u\, d(\text{vol}) = uA\,dx \rightarrow \dfrac{dU}{dt} = uA\dfrac{dx}{dt}$

wave speed along x

$= uAc$

EM wave

$\rightarrow S = \dfrac{dU}{dt}\Big/\text{Area} = \dfrac{u\cancel{A}c}{\cancel{A}} = uc = \underbrace{\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}B^2\right)}_{\text{Total EM energy density}} c$

using $\dfrac{1}{\mu_0\varepsilon_0} = c^2$ or $\dfrac{1}{c^2\mu_0} = \varepsilon_0 \rightarrow S = \varepsilon_0 E^2 c = \varepsilon_0 c^2 E \underbrace{\left(\frac{E}{c}\right)}_{B} = \dfrac{EB}{\mu_0}$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} \quad \text{(with directions)}$$

intensity points in → direction of propagation.

29.44

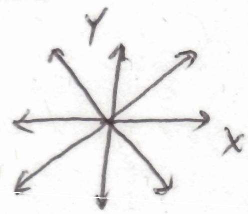
Unpolarized light of intensity S_0 incident on three polarizers

↓

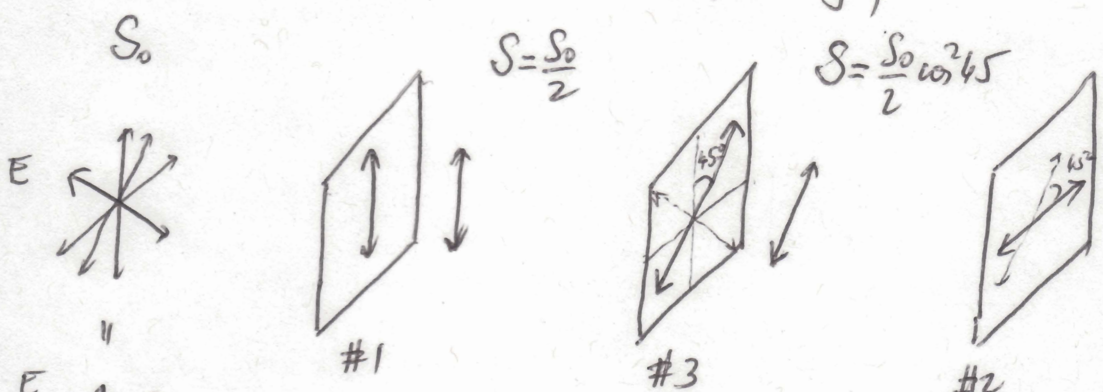
Electric field is not pointing along any particular direction. All directions (polarizations) are equally likely

a material only let light with $\vec{E}$ along its axis (of polarizer) to pass through

Front view:



Can decompose any direction into their x & y components → half along x & half along y

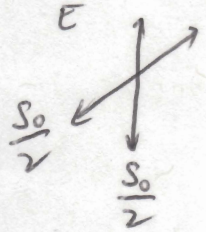


$S = ?$

$\frac{S_0}{2}\cos^4 45$

$= \frac{S_0}{2}\,\frac{1}{4}$

$= \frac{S_0}{8}$

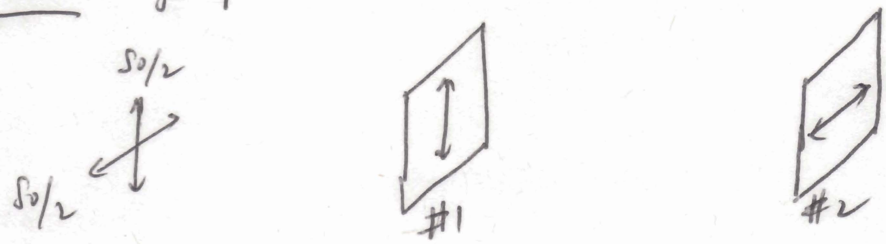


after polarizer #1 we only have light polarized along vertical axis

after #3: only $E\cos 45°$ (and so $B\cos 45°$) will pass through)

$S = EB$

Note: if polarizer # 3 is removed:



$S = 0$

Not happening in mechanical waves or sound waves
(EM waves are polarized due to vector nature of $\vec{E}$ & $\vec{B}$).

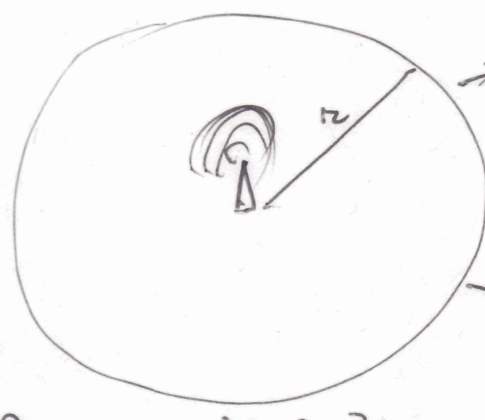
29.50. 1.5 km from transmitter: $E_p = 350 \frac{mV}{m}$

a) $P_{transmitter} \stackrel{?}{=}$

$$S = \frac{P}{\text{Area}} = \varepsilon_0 E_p^2 c \rightarrow P = \varepsilon_0 E_p^2 c A$$

Area: A; $\varepsilon_0 E_p^2 c$: EM wave (includes radio waves)

emits waves in all directions (3D)



→ sphere centered @ transmitter @ radius $r = 1.5$ km

→ $A = 4\pi r^2$

$$P = \varepsilon_0 c E_p^2 4\pi r^2 = 8.85\times10^{-12} \times 3\times10^{8} \times 4\pi \times 1500^2 \times 0.35^2 \text{ W}$$
$$= 9.2 \text{ kW}$$

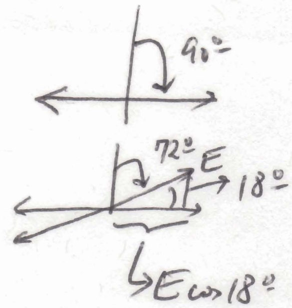
Average power: $\overline{P} = \left(\frac{1}{2}\right) \varepsilon_0 c E_p^2 4\pi r^2 = 4.59 \text{ kW}$

↓ average of $\cos^2$ or $\sin^2$ is $\frac{1}{2}$.

29.41

90° polarization → full : S_0

72° polarization →



since $S = \varepsilon_0 c E^2$ → → $S = S_0 \cos^2 18°$

$$\boxed{S = 0.905\, S_0}$$

29.59

Photon rocket emits beam of light

P of light source? to get a thrust of 35×10^6 N
force F

power

$$S = \frac{P}{A} = \frac{\frac{dU}{dt}}{A}$$

power
energy

intensity

Radiation pressure = $\frac{F}{A} = \frac{\frac{dp}{dt}}{A} = \frac{\frac{1}{c}\frac{dU}{dt}}{A} = \boxed{\frac{\bar{S}}{c}}$

force
momentum.
area

average intensity divided by the speed of light.

Radiation momentum

$$\boxed{p = \frac{U}{c}}$$

$$F = 35 \times 10^6 \text{ N} = \text{Rad Pressure} \times A = \frac{\bar{S}}{c} \times A$$

$$= \frac{\bar{P}}{\cancel{A} \cdot c} \times \cancel{A}$$

$$\rightarrow \bar{P} = cF = 3 \times 10^8 \times 35 \times 10^6 = 10^{16} \text{ W}$$

(Our total electric power output is 10^{12} W)