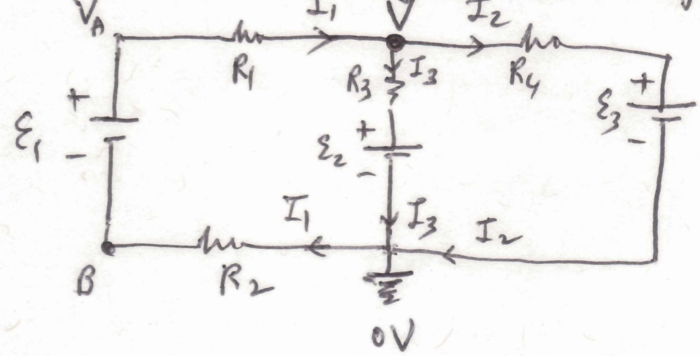
25.53 (cont.) Now using Node Analysis: (Looking for current thru R_3)



1) Set the ground potential (zero)

2) Identify node

3) Assume directions for currents I_1; I_2; I_3 as shown

4) Node equation: $I_1 - I_2 - I_3 = 0$

5) Write these currents in term of the voltages:

a) $I_1 = \dfrac{V_A - V}{R_1} = \dfrac{(\mathcal{E}_1 - I_1 R_2) - V}{R_1} \rightarrow I_1 R_1 = \mathcal{E}_1 - I_1 R_2 - V$

Ohm's law @ R_1

$V_A = \mathcal{E}_1 - I_1 R_2$

$$\boxed{I_1 = \frac{\mathcal{E}_1 - V}{R_1 + R_2}}$$

b) $I_2 = \dfrac{V - \mathcal{E}_3}{R_4}$

Ohm's law @ R_4

c) $I_3 = \dfrac{V - \mathcal{E}_2}{R_3} \longrightarrow I_3 = \dfrac{4.17 - 1.5}{560} = 4.76\,mA$ downward

Ohm's law @ R_3

$\longrightarrow$ Node equation: $\dfrac{\mathcal{E}_1 - V}{R_1 + R_2} - \dfrac{V - \mathcal{E}_3}{R_4} - \dfrac{V - \mathcal{E}_2}{R_3} = 0$

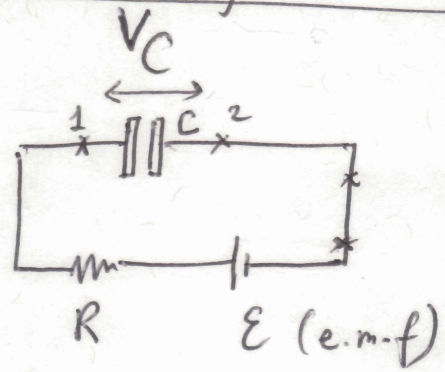
(one unknown: V → solve:

$$\frac{6 - V}{420} - \frac{V - 4.5}{820} - \frac{V - 1.5}{560} = 0$$

$\rightarrow \boxed{V = 4.17V}$

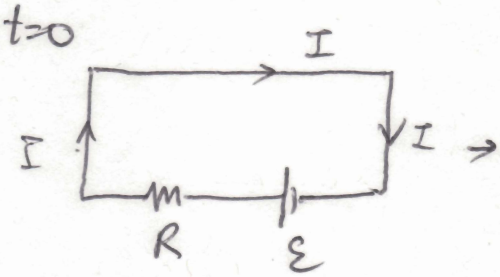
* If ground is placed @ B → different equations, but same final answers.

3) Circuits involving resistors & capacitors:



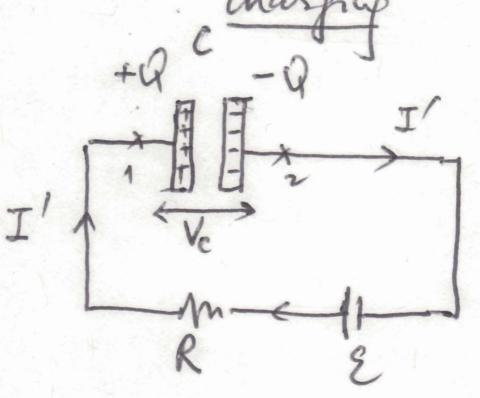
@ time 0 ($t=0$) we connect the uncharged capacitor with the rest of the circuit $\boxed{Q=0}$, $\boxed{V_C=0}$ (no Electric field since $Q=0$) → the capacitor @ $t=0$ (circuit connected) behaves like a piece of wire:

@ $t=0$



→ @ $t=0$ → $\boxed{I = \frac{\mathcal{E}}{R}}$ (I: current of positive charges)

@ $t>0$: positive charges move from right plate to the left plate through circuit: <u>the capacitor is charging</u>



as charges are building up on the plates → I' gets smaller & smaller (harder to move charges at a later time since we need to go against the larger electric field created by the built-up charges) as V_c gets larger:

@ $t>0$ $\begin{cases} I \text{ decreases from } \frac{\mathcal{E}}{R} \\ \underbrace{V_c \text{ increases from } 0}_{V_c = E \cdot d} \end{cases}$

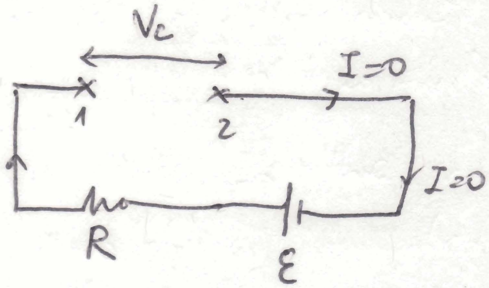
d ↑ sep. b/w plates

@ $t>0$ $\mathcal{E} - I'R - V_c = 0$

$$I' = \frac{\mathcal{E} - V_c}{R}$$

@ $t=\infty$ (very long after the circuit is closed)

$\begin{cases} I = 0 & \text{(capacitor is fully charged} \to \text{no further charge transfer);} \\ V_c = \text{max voltage} \end{cases}$



RC circuits: $\begin{cases} @\ t=0 & : I = \frac{\varepsilon}{R} \\ @\ t=\infty & : I = 0 \end{cases}$ $\downarrow$ How or what is $I(t)$? $0<t<\infty$

$$\frac{d}{dt}\Big(\varepsilon - I'R - \underbrace{V_c}_{V_c = \frac{Q}{C}} = 0 \Big) \xrightarrow[\frac{d\varepsilon}{dt}=0]{} - R\frac{dI'}{dt} - \frac{d}{dt}\left(\frac{Q}{C}\right) = 0$$

$V_c = \frac{Q}{C}$

C = constant

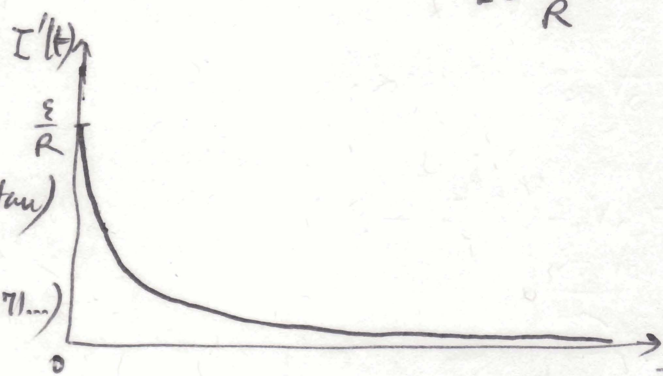
$$R\frac{dI'}{dt} + \frac{1}{C}\underbrace{\frac{dQ}{dt}}_{I'} = 0$$

$$R\frac{dI'}{dt} = -\frac{1}{C}I' \rightarrow \frac{dI'}{I'} = -\frac{1}{RC}dt \xrightarrow{\text{integrate}}$$

$$\ln I' = -\frac{t}{RC} + \text{const.} \xrightarrow[e^{(\ )}]{} I' = \underbrace{I'(0)}_{\text{constant}} e^{-\frac{t}{RC}}$$

$I'(0)$: constant $\downarrow$ $I = \frac{\varepsilon}{R}$

$$\boxed{I'(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}}$$

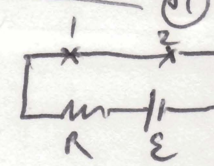


RC: time constant $= \tau$ (tau)
(unit : s)

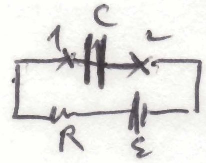
@ $t=\tau$ $\rightarrow$ $I' = \frac{\varepsilon}{R}\frac{1}{e}$ $(e = 2.71\ldots)$

RC circuit

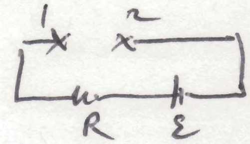
- @ $t=0$ $\quad I = \frac{\mathcal{E}}{R}$; $V_C = 0 \rightarrow C$ = in short circuit:



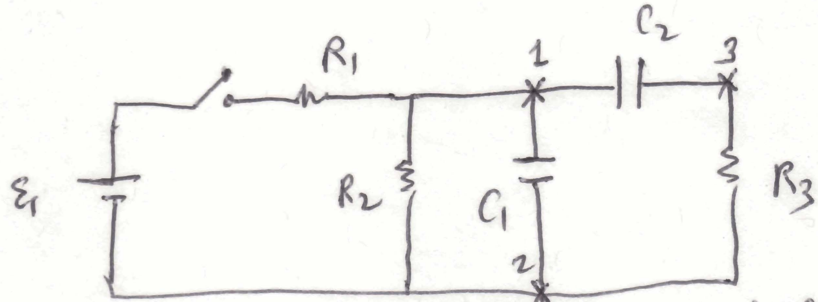
- @ $0 < t < \infty$: $I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$



- @ $t = \infty$, $I = 0 \rightarrow C$: in open circuit;



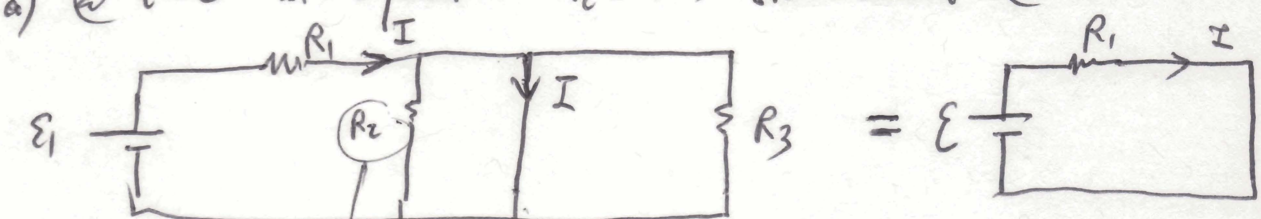
25.64



- switch initially open
- C_1 & C_2 are uncharged
- $R_1 = R_2 = R_3 = R$

Find current in R_2
- a) @ $t=0$ (just after switch is closed)
- b) @ $t=\infty$ (long " " " ")
- c) current in R_3 qualitatively

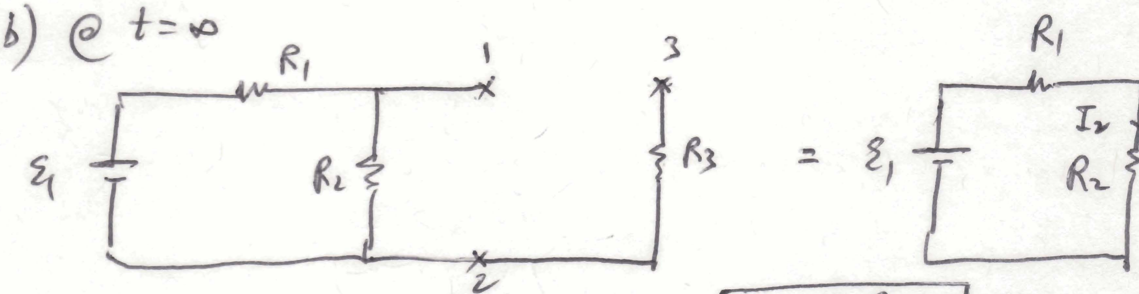
a) @ $t=0$ at capacitors $V_c = 0 \rightarrow$ short circuit @ C_1 & C_2



$\boxed{\text{current thru } R_2 = 0}$

(also current thru $R_3 = 0$)

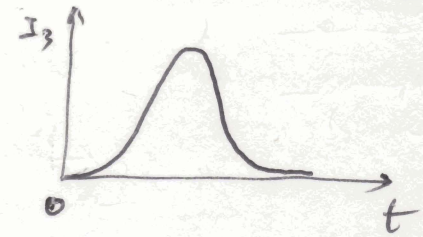
b) @ $t = \infty$



$$\boxed{I_2 = \frac{\mathcal{E}_1}{R_1 + R_2}}$$

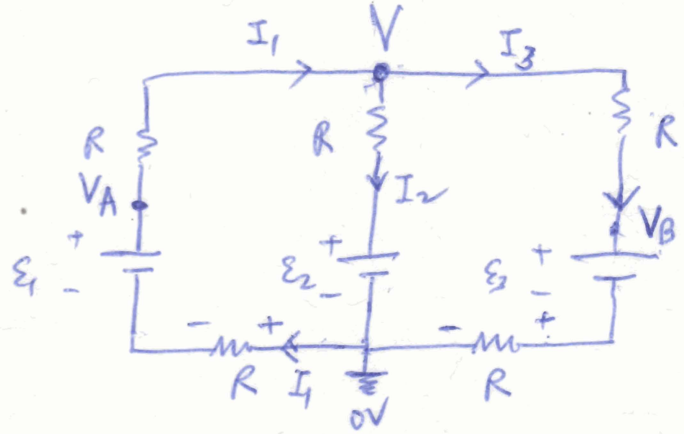
Ohm's law

c) Current in R_3 : $\equiv I_3$
$$\begin{cases} @t=0 \rightarrow I_3=0 \\ @t=\infty \rightarrow I_3=0 \end{cases}$$



25.55

$\boxed{25.75}$



$R = 1.5 M\Omega$

$\varepsilon_1 = 75 mV$

$\varepsilon_2 = 45 mV$

$\varepsilon_3 = 20 mV$

Node analysis:

1) Determine the node:
2) Set zero potential (ground)
3) Assume directions for currents: $I_1; I_2; I_3$
or assign
4) Node equation: $\boxed{I_1 - I_2 - I_3 = 0}$

$\boxed{\text{Find } I_3}$

5) Write currents in term of potentials

$$I_1 = \frac{V_A - V}{R} = \frac{\varepsilon_1 - I_1R - V}{R} \rightarrow 2I_1R = \varepsilon_1 - V \rightarrow \boxed{I_1 = \frac{\varepsilon_1 - V}{2R}}$$

Ohm's law

$$V_A = \varepsilon_1 - I_1R$$

$$I_2 = \frac{V - \varepsilon_2}{R}$$

$$I_3 = \frac{V - V_B}{R} = \frac{V - \varepsilon_3 - I_3R}{R}$$

$$V_B = \varepsilon_3 + I_3R$$

$$2I_3R = V - \varepsilon_3$$

$$\boxed{I_3 = \frac{V - \varepsilon_3}{2R}}$$

$$\frac{\varepsilon_1 - V}{2R} - \frac{V - \varepsilon_2}{R} - \frac{V - \varepsilon_3}{2R} = 0$$

$$\frac{\varepsilon_1 - V}{2} - (V - \varepsilon_2) - \frac{(V - \varepsilon_3)}{2} = 0$$

$$\frac{\varepsilon_1}{2} + \varepsilon_2 + \frac{\varepsilon_3}{2} - 2V = 0$$

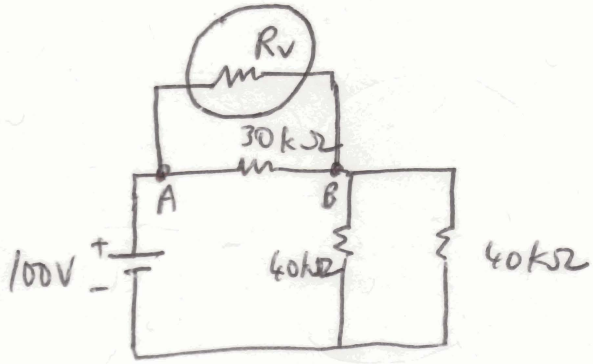
$$V = \frac{1}{2}\left(\frac{\varepsilon_1}{2} + \varepsilon_2 + \frac{\varepsilon_3}{2}\right) = \frac{1}{2}\left(\frac{75}{2} + 45 + \frac{20}{2}\right) mV$$

$\boxed{\cancel{V = 41 mV}}$ (~~down~~) $V = 46.25 mV$

Then $I_3 = \dfrac{V - \mathcal{E}_3}{2R} = \dfrac{46.25\,mV - 20\,mV}{2\times 1.5\,M\Omega} = \dfrac{26.25}{2\times1.5}\dfrac{10^{-3}}{10^{6}} = 8.75\times10^{-9}\,A$

$= \cancel{8.75\,nA} = 8.75\,nA$ (down)

25.55



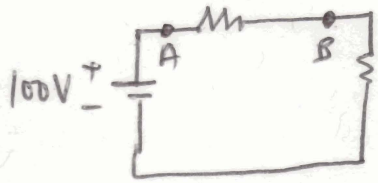
Voltmeter connected across AB

R_V : internal resistance of a voltmeter

a) $R_V = 50\,k\Omega$ b) $R_V = 250\,k\Omega$ c) $R_V = 10\,M\Omega$

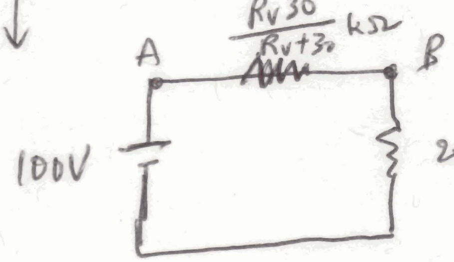
V_{AB} ? using parallel & series combination:

$R_V \| 30k\Omega = \dfrac{R_V 30}{R_V + 30}\,k\Omega$ (keeping R_V in kΩ)



$40k\Omega \| 40k\Omega = \dfrac{40^2}{80}\,k\Omega = 20\,k\Omega$

$R_1 \| R_2 \Rightarrow \dfrac{R_1 R_2}{R_1 + R_2}$



$\rightarrow V_{AB} = 100V \dfrac{\frac{R_V 30}{R_V+30}}{\frac{R_V 30}{R_V+30} + 20}$

$= 100V \dfrac{R_V 30}{R_V 30 + 20(R_V + 30)}$

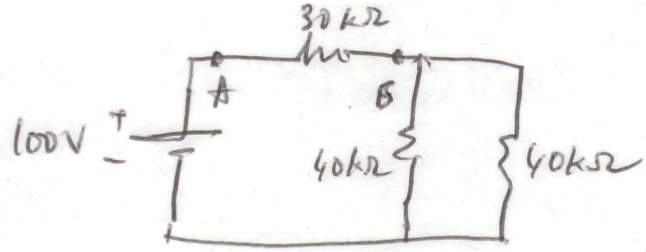
$V_{AB} = \dfrac{R_V\,3000}{50 R_V + 600}\,V$

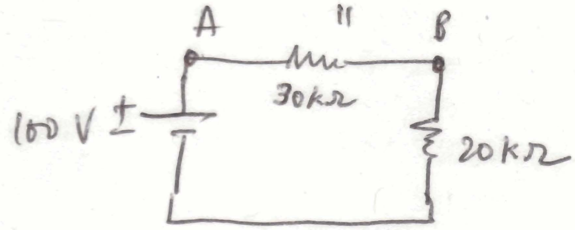
a) $R_V = 50k\Omega \rightarrow V_{AB} = \dfrac{50\times3000}{50\times50 + 600} = 48.39\,V$

b) $R_V = 250k\Omega \rightarrow V_{AB} = \dfrac{250\times3000}{250\times50 + 600} = 57.25\,V$

c) $R_V = 10M\Omega = 10000k\Omega \rightarrow V_{AB} = \dfrac{10000\times3000}{\ldots} = 59.93\,V$

What value for V_{AB} we would like to read? The circuit w/o voltmeter was:





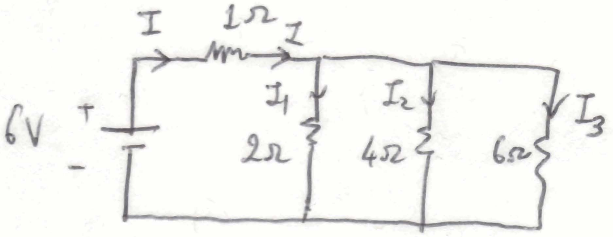
$$V_{AB} = 100\,V \frac{30\ \cancel{k\Omega}}{(30+20)\cancel{k\Omega}}$$

$$= 100\ \frac{3}{5} = 60\,V$$

↓

Best voltmeter is that with R_V very large: $10 M\Omega$

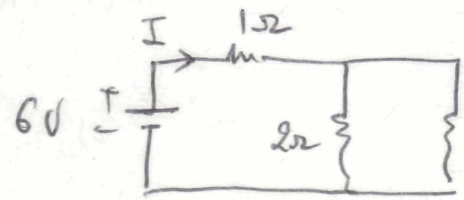
25.48



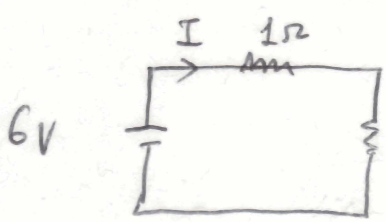
a) I? b) I_3?

a) One battery & 4 resistors in series or parallel connection

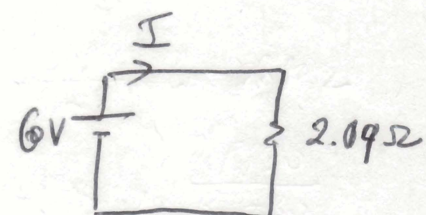
↳ reduce to one battery and one resistor → use Ohm's law solve for I.



$\frac{4.6}{10}\Omega = 2.4\Omega$



$\frac{2\times2.4}{4.4} = 1.09\Omega$ =



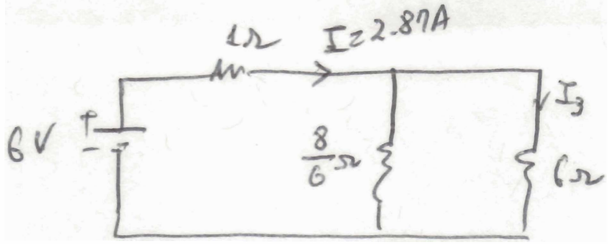
$\rightarrow I = \frac{6}{2.09} = 2.87A$

$$R_1 \parallel R_2 \rightarrow \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12} \rightarrow R = \frac{12}{11}\Omega = 1.09\Omega$$

b)

82d

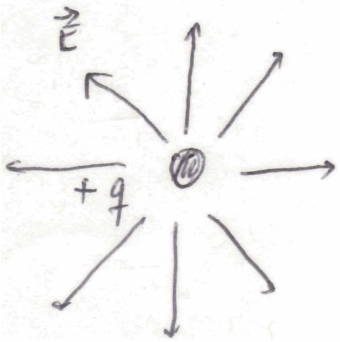


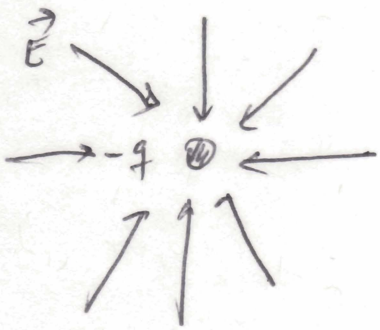
$$\rightarrow I_3 = I \frac{\frac{8}{6}}{\underbrace{\frac{8}{6}+6}_{<1}} = 2.87 \frac{8}{8+36}$$

$$= 0.522A$$

Ch 26 Magnetic Field

Electric





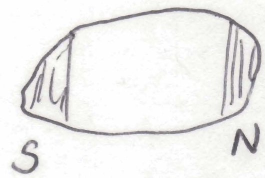
2 types of charge $\left\{ \begin{array}{l} + \\ - \end{array} \right.$

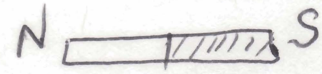
Equal charges repel
Opposite charges attract

Electric monopoles are routine

Electric field lines are open
(not ~~connected~~ closed)

Magnetic

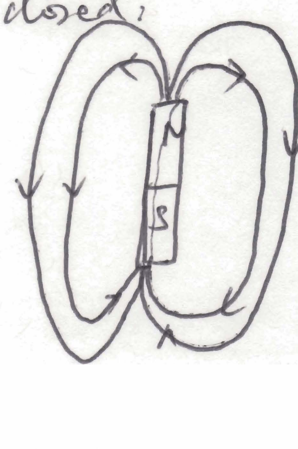




2 types of magnetic poles
North & South, but
always in one piece

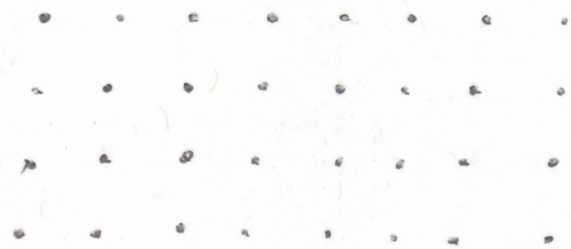
Equal poles repel
Opposite poles attract
Magnetic monopoles are not
found.

Magnetic field lines are
closed:



Effects of a Magnetic Field $\vec{B}$:

On a moving charge q with velocity $\vec{v}$ in a region with a magnetic field that is uniform & pointing out of page:



$\vec{B}$ out of page & uniform.

Experiments:

1) charge q moves in or out of page (or parallel to $\vec{B}$) does not feel any effect

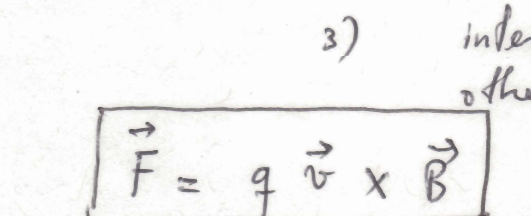
2) charge q moving on page ($\perp \vec{B}$): feels max. effect of the magnetic field

3) intermediate effects if $\vec{v}$ forms any other angle with $\vec{B}$

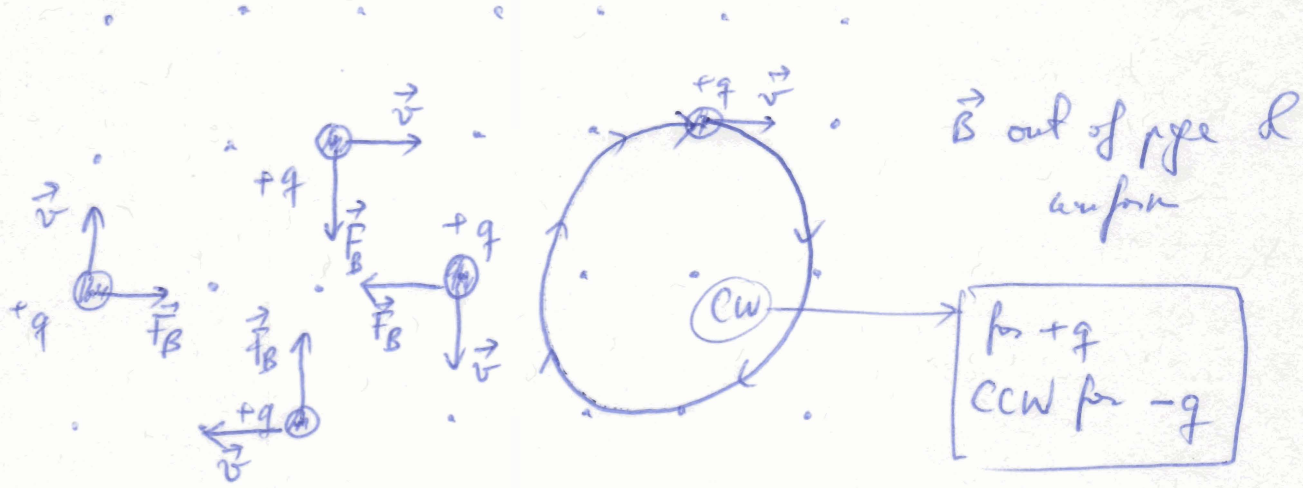
$$\boxed{\vec{F} = q\,\vec{v} \times \vec{B}}$$

↳ vector cross product b/w $\vec{v}$ & $\vec{B}$ } is another vector that is ~~perpendicu~~ perpendicular to both $\vec{v}$ & $\vec{B}$, with direction given by the right hand rule (RHR): as you turn the right hand fingers from $\vec{v}$ to $\vec{B}$, thumb points in direction of $\vec{v} \times \vec{B}$.

→ magnitude is $vB\sin\theta = v\sin\theta\, B$ (θ is the angle b/w $\vec{v}$ & $\vec{B}$)

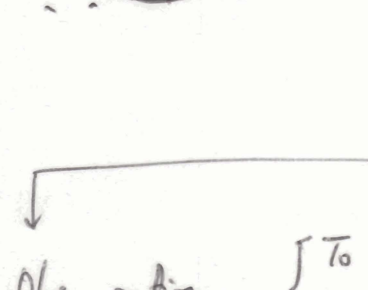


} Only the perpendicular component (to $\vec{B}$) of $\vec{v}$ contributes to the magnetic force on the cha



Effect of the magnetic field on a moving charge (cont.)

The trajectory of a charged particle in a magnetic field is circular: $\vec{F}_B = q\vec{v} \times \vec{B}$ is always perpendicular to the direction of motion (a vector cross product $\vec{a} \times \vec{b}$ is always $\perp \vec{a}$ & $\perp \vec{b}$), this magnetic force will provide the radial acceleration for the charge to follow a circular orbit



$\vec{B}$ uniform & out of page

(No tangential acceleration)

r: orbital radius

Uniform circular motion

2nd Newton's Law: $F_{net} = m \cdot a = m\frac{v^2}{r}$

Magnetic force $\longrightarrow$ $q\cancel{v}B = m\frac{v^{\cancel{2}}}{r}$

$\vec{v}$ on page $\perp \vec{B}$

$$\boxed{r = \frac{mv}{qB}}$$

Observation: { To achieve small r: → very high B → under research in plasma fusion experiments

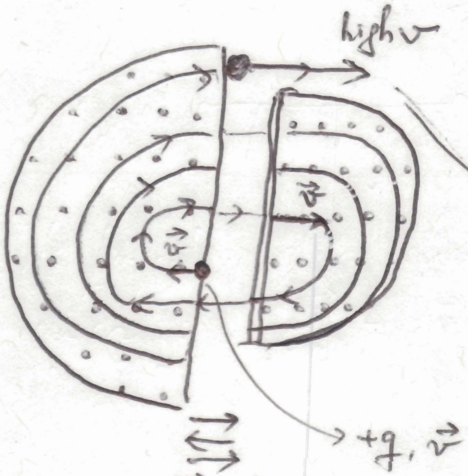
Orbital period? time for +q to complete one cycle: $T = \frac{2\pi r}{v}$

Orbital period: $\boxed{T = \frac{2\pi r}{v} = \frac{2\pi \frac{m\cancel{v}}{qB}}{\cancel{v}} = \frac{2\pi m}{qB}}$

Applications

1) Cyclotron : (modern version is synchrotron)

↳ Goal: accelerate charged particle to very high speed using $\vec{E}$ & $\vec{B}$.



$\vec{B}$ = uniform & out of page inside the dee's

Cyclotron radius is R

$\vec{E}$: to give $+q$ a push (acceleration) to the other side.

$$KE_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} m \left(\frac{qBR}{m}\right)^2$$

$$r = \frac{mv}{qB} \rightarrow v_{max} = \frac{qBR}{m}$$

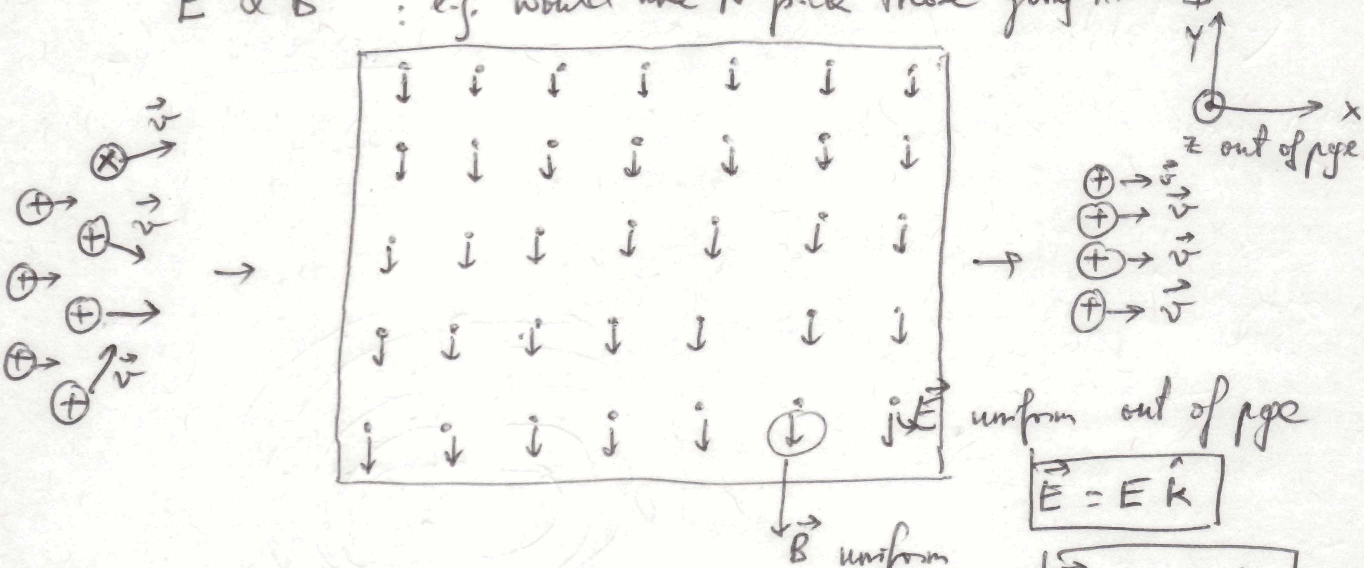
$$\boxed{KE_{max} = \frac{qBR^2}{2m}}$$

Current $v_{max} \rightarrow c = 3 \times 10^8$ m/s $\rightarrow$ relativistic corrections

↳ Synchrotron

2) <u>Velocity selector</u>:

ions (+) with different velocities: can pick out those with the desired velocity by using a combination of the $\vec{E}$ & $\vec{B}$: e.g. would like to pick those going $\parallel$ x-axis.



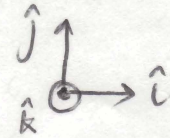
$\boxed{\vec{E} = E\hat{k}}$

$\boxed{\vec{B} = B(-\hat{j})}$

What special about those ions w/ $\vec{v} = v\hat{\imath}$?

$$\begin{cases} \vec{F}_E = q\vec{E} = qE\hat{k} \\ \vec{F}_B = q\vec{v}\times\vec{B} = q\,v\hat{\imath}\times B(-\hat{j}) = -qvB\,\underbrace{\overbrace{(\hat{\imath}\times\hat{j})}^{\hat{k}}}_{RHR} \end{cases}$$

$$= -qvB\hat{k}$$



$$\text{at } v = \frac{E}{B} \rightarrow \quad \vec{F}_E + \vec{F}_B = (qE - qvB)\hat{k}$$

$$= q(E - vB)\hat{k}$$

$$= 0$$

→ Those with $\vec{v}$ with additional components will get deviated by this machine

Calculation of the Magnetic field or Sources of the Magnetic Field

Electric : $d\vec{E} = \dfrac{k\,dq}{r^2}\,\hat{r}$: inverse-square law or Coulomb's Law

↳(source : charge)

Magnetic : $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,(d\vec{l}\times\hat{r})}{r^2}$: Inverse-square law Biot-Savart Law

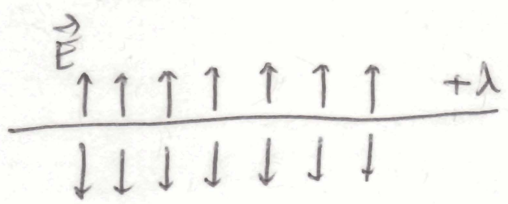
↳(source : current)

Need moving charges to create a magnetic field.

$\mu_0 = 4\pi\times10^{-7}\,\dfrac{N}{A^2}$: permeability in vacuum

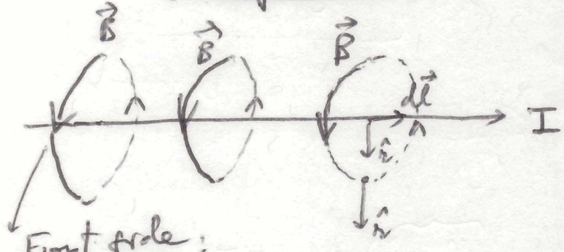
Magnetic field due to a line of current is wrapping around the current

line of charge



$E = \dfrac{2k\lambda}{r}$ → sep. from line of charge

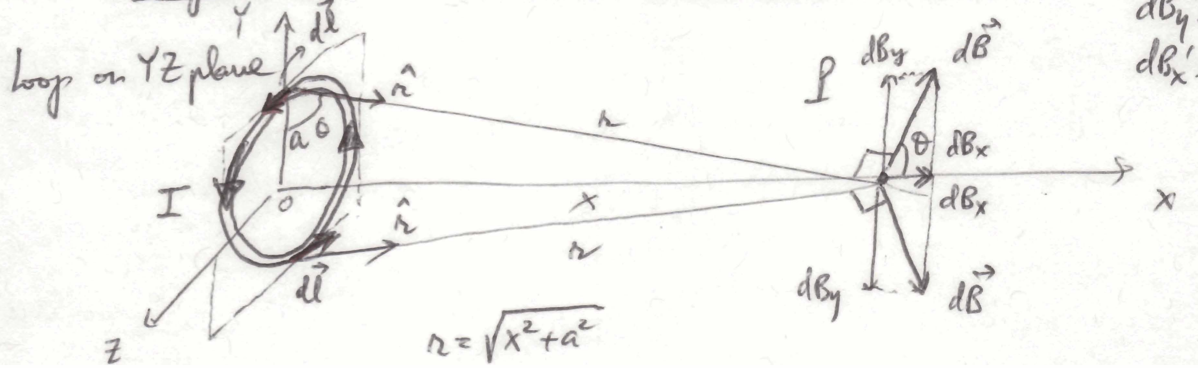
line of current:



Front side:

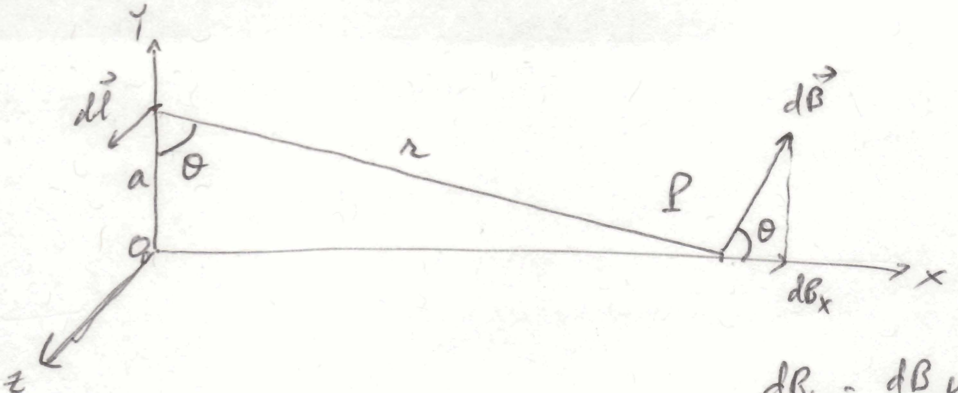
RHR : RH fingers turn as $\vec{B}$ → thumb points in direction of current I

Magnetic field due to a loop of current :



$r = \sqrt{x^2+a^2}$

dB_y's are cancelled
dB_x's are adding



$$dB_x = dB\cos\theta = \frac{dB\,a}{r}$$

From top & bottom elements of currents:

$$dB_{Total} = 2\,dB_x = \frac{2\,dB\,a}{r} = 2\frac{\mu_0}{4\pi}\frac{a\,I\,dl}{r^3}$$

$\downarrow$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

$\downarrow$

$d\vec{l}\times\hat{r} = dl$

in this example

$$B_{total} = \int_{\text{Half loop}} dB_{Total} = \frac{2\mu_0}{4\pi}\frac{a\,I}{r^3}\boxed{\int_{\text{Half loop}} dl} = \frac{\mu_0}{2}\frac{I a^2}{(x^2+a^2)^{3/2}}$$

$\boxed{\int_{\text{Half loop}} dl} \rightarrow \pi a$

r: all points on loop are @ r from P

Direction along $\hat{i}$.

$$\rightarrow \boxed{\vec{B} = \frac{\mu_0}{2}\frac{I a^2}{(x^2+a^2)^{3/2}}\,\hat{i}} \quad (\text{T for Tesla})$$

Magnetic field created by a loop of current I (radius a) @ a point P along its axis

<u>Observation</u>: $x \gg a \rightarrow (x^2+a^2)^{3/2} \simeq x^{2\cdot\frac{3}{2}} = x^3$

$\hookrightarrow B \simeq \frac{\mu_0}{2}\frac{I a^2}{x^3}$ (inverse-cube law very far away from loop of current)

Electric $\rightarrow$ dipole $\rightarrow$ inverse-cube law:

$\rightarrow$ loop of current is the magnetic analog of the electric dipole

Calculation of fields

Electric

→ Vector addition (using Coulomb's law)

→ Gauss Law

$$\underbrace{\oint_{\text{Gaussian surface (3D)}} \vec{E}\cdot d\vec{A}}_{\phi} = \frac{q_{enclosed}}{\varepsilon_0}$$

→ Electric Potential V (scalar addition)

$$\vec{E} = -\vec{\nabla} V$$

↓ gradient or derivative.

Magnetic

→ Vector addition (using Biot-Savart law)

→ Ampere Law:

$$\boxed{\oint_{\substack{\text{Amperian}\\\text{Loop (2D)}}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\substack{\text{enclosed}\\\text{by Amperian}\\\text{loop}}}}$$

→ Vector Potential $\vec{A}$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

~~rotational~~

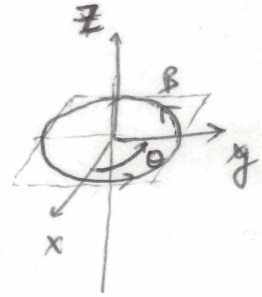
rotational or curl of $\vec{A}$

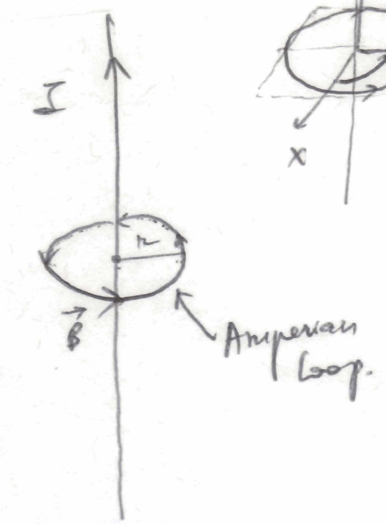
Ampere's Law:

$$\oint_{\substack{\text{Amperian}\\\text{loop.}}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\substack{\text{enclosed}\\\text{by Amperian}\\\text{loop}}}$$

1) Determine the Amperian loop based on symmetry such that $\oint \vec{B}\cdot d\vec{l} = \vec{B}\cdot\oint d\vec{l}$ ($\vec{B}$ is constant on that loop!)
2) Current enclosed by <u>that</u> loop

Calculate B due to a long line of current.





I along z → B on xy plane
↓
$\vec{B} = B\,\hat{\theta}$
↳ unit vector for angle θ (starts from x axis)

$$\boxed{\vec{B}(r) = B(r)\,\hat{\theta}}$$

Ampere Law:
↓
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

1) Amperian loop: B is const along this loop.
@ a fixed r → B is constant since there is no preferred direction around wire along a circle of radius r →
Amperian loop: circle centered at the long current of radius r → $\vec{B}\cdot\oint_{Amp.\,Loop} d\vec{l}$

2) Determine current enclosed by this loop: I

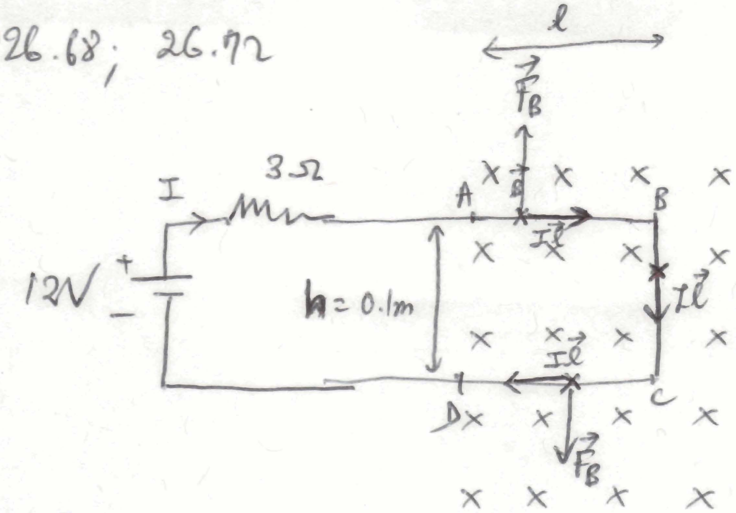
$$\vec{B}\cdot\underbrace{\oint d\vec{l}}_{2\pi r} = \mu_0 I \;\rightarrow\; B\,2\pi r = \mu_0 I$$

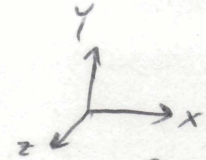
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$\boxed{\vec{B}(r) = \frac{\mu_0 I}{2\pi r}\,\hat{\theta}}$$

26.52; 26.68; 26.72

26.52





B uniform & into the page : $\vec{B} = B(-\hat{k})$

$B = 38 \times 10^{-3}$ T

$\vec{F}_{net}$ on circuit :→ Why is there a force on circuit? Magnetic field would apply a magnetic force on moving charges, since there is a current in the circuit : $\vec{F}_B = q\vec{v} \times \vec{B} = \frac{q}{\Delta t} \vec{\ell} \times \vec{B} = I\vec{\ell} \times \vec{B}$

$I = \frac{12V}{3\Omega} = 4A$

q → charge; $\vec{v}$ → charge velocity $= \frac{\vec{\ell}}{\Delta t}$; $\vec{B}$ → field

Intersections of circuit w/ field region : A, B, C, D → look for magnetic forces on AB ; BC ; DC

On : AB : $I\vec{\ell} \times \vec{B}$: RHR → $\vec{F}_{B_{AB}} = I\ell B\,\hat{j}$

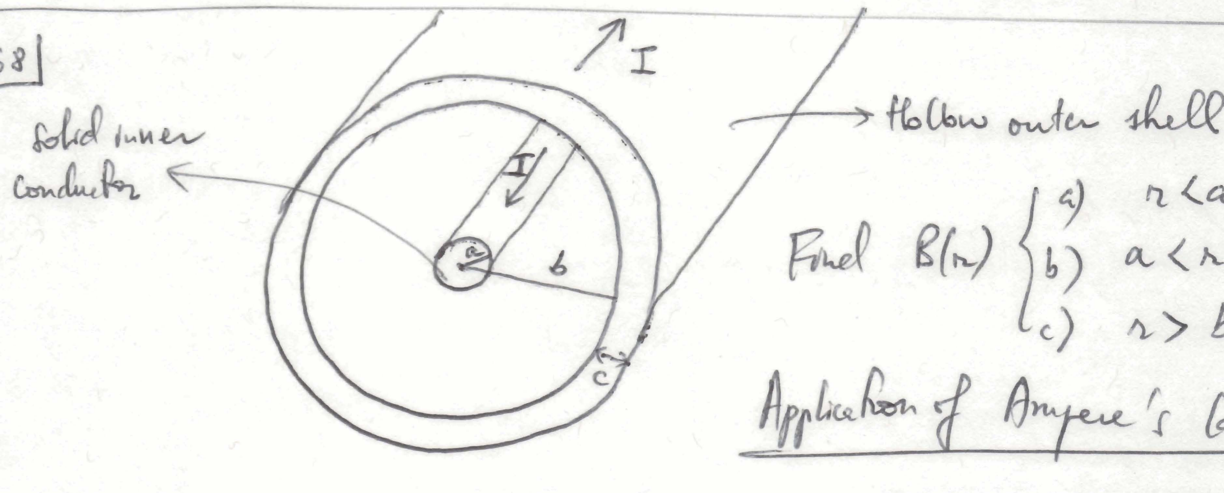
on CD : → $\vec{F}_{B_{CD}} = I\ell B\,(-\hat{j})$

} $\vec{F}_{net_y} = 0$

on BC : → $\vec{F}_{B_{BC}} = IhB\,\hat{i}$ → $\boxed{\vec{F}_{net_x} = IhB\,\hat{i}}$

→ $4 \times 0.1 \times 38 \times 10^{-3}\,\hat{i}$ (N) $= 15.2 \times 10^{-3}$ N $\hat{i}$

26.68



Find $B(r)$ $\begin{cases} a) & r < a \\ b) & a < r < b \\ c) & r > b+c \end{cases}$

Application of Ampere's law

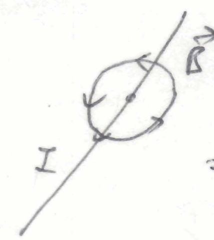
Application of Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

1) Determine the Amperian loop such that:

$$\oint_{A\text{-loop}} \vec{B} \cdot d\vec{l} = \vec{B} \cdot \oint_{A\text{-loop}} d\vec{l} = B \cdot 2\pi r$$

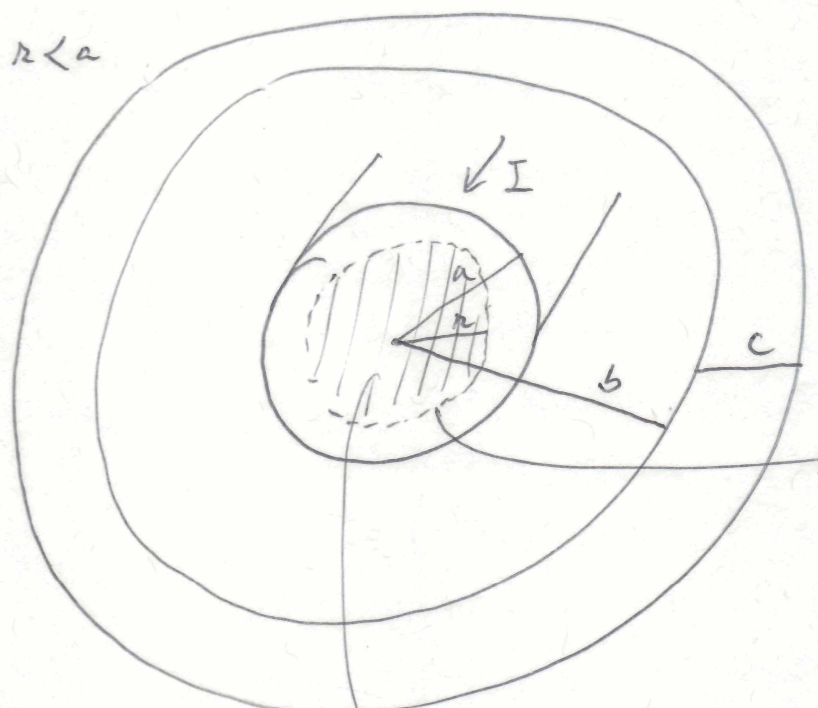
$\vec{B}$ is constant along the A-loop.



$\Rightarrow$ A-loop is a circle centered @ the center of the coaxial cable, with radius r

2) I enclosed by Amperian loop:

a) $r < a$



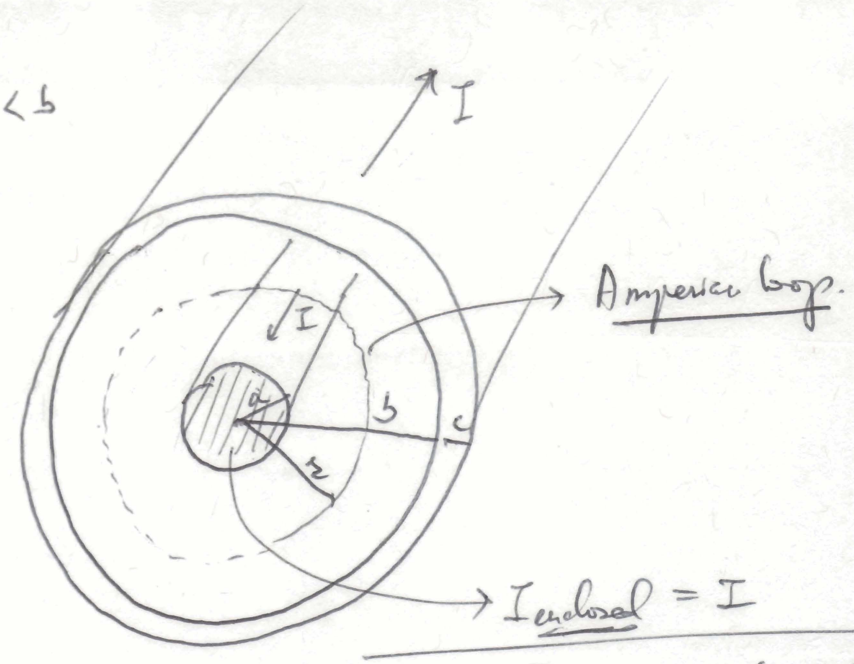
Amperian loop.

Current enclosed: $I_{enclosed} = I \frac{\pi r^2}{\pi a^2}$

$$B\, 2\pi r = \mu_0 I \frac{r^2}{a^2} \rightarrow \boxed{B = \frac{\mu_0 I}{2\pi a^2} r \; ; \; r < a}$$

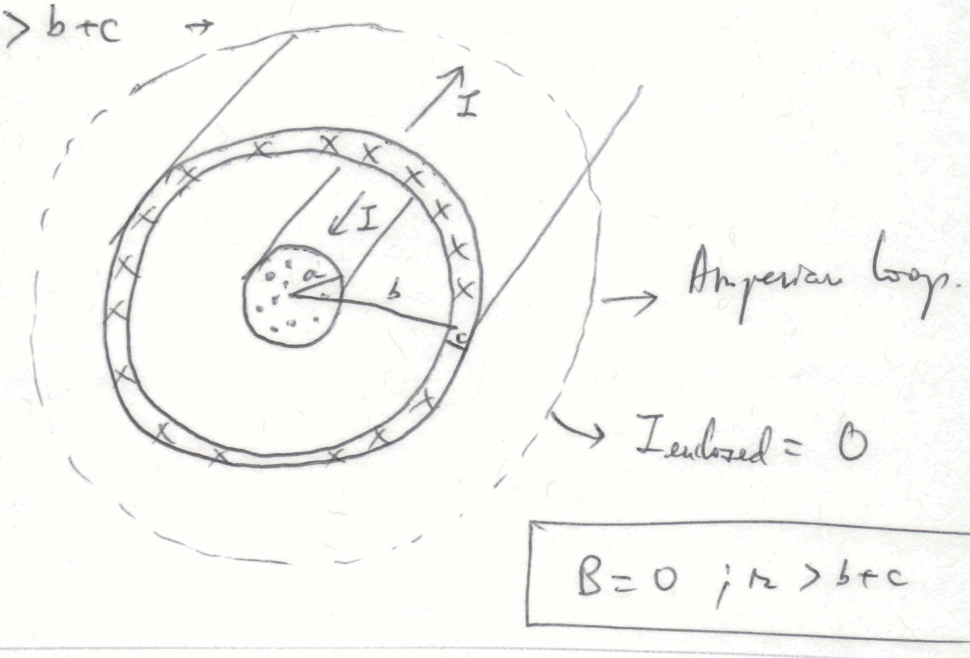
inside inner conductor.

b) $a < r < b$



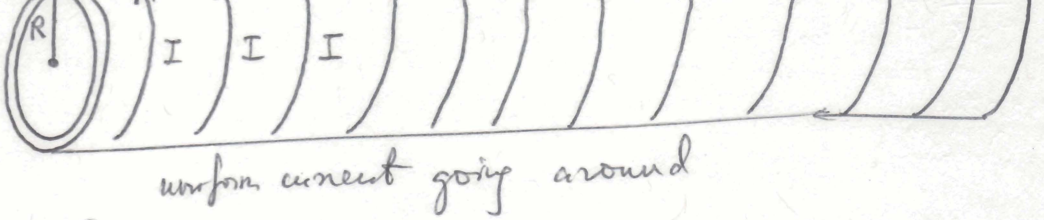
$B 2\pi r = \mu_0 I \rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \;;\quad a < r < b}$

c) $r > b + c \rightarrow$



$\boxed{B = 0 \;;\ r > b + c}$

26.76



uniform current going around

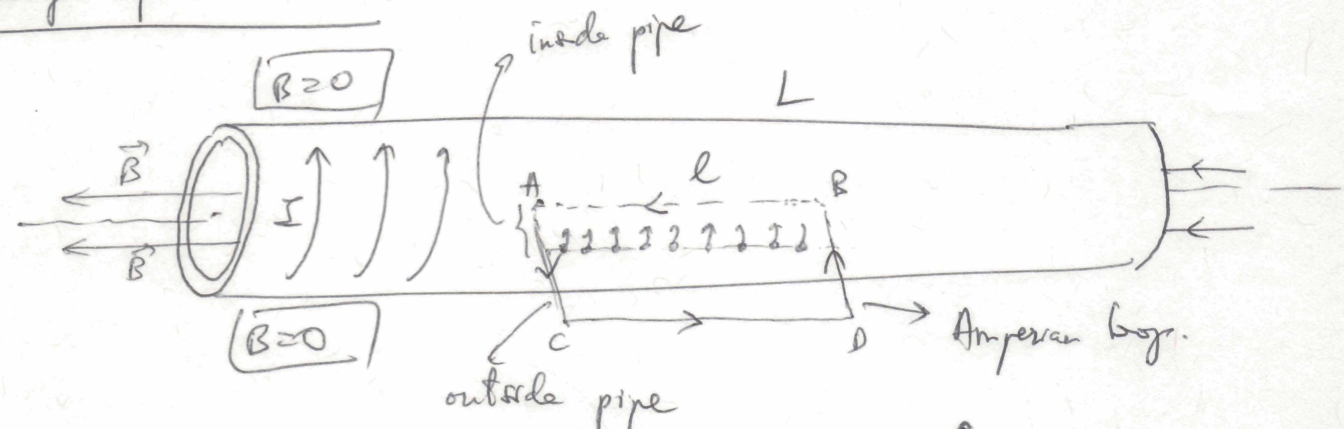
$B(r) \begin{cases} r < R \\ r > R \end{cases}$

Pipe = ∞ number of rings with current $\frac{I}{L}$

$r < R \rightarrow \vec{B}$ points along pipe axis

Yesterday: $\vec{B} = \frac{\mu_0 I R^2}{2\underbrace{(R^2)}_{x=0}^{3/2}} = \frac{\mu_0 I}{2R}$

Application of Ampere's law:



1) $\vec{B} \parallel$ pipe axis. $\oint_{BACD} \vec{B} \cdot d\vec{\ell} = B \int_{BA\cancel{CD}} d\ell = B\ell$

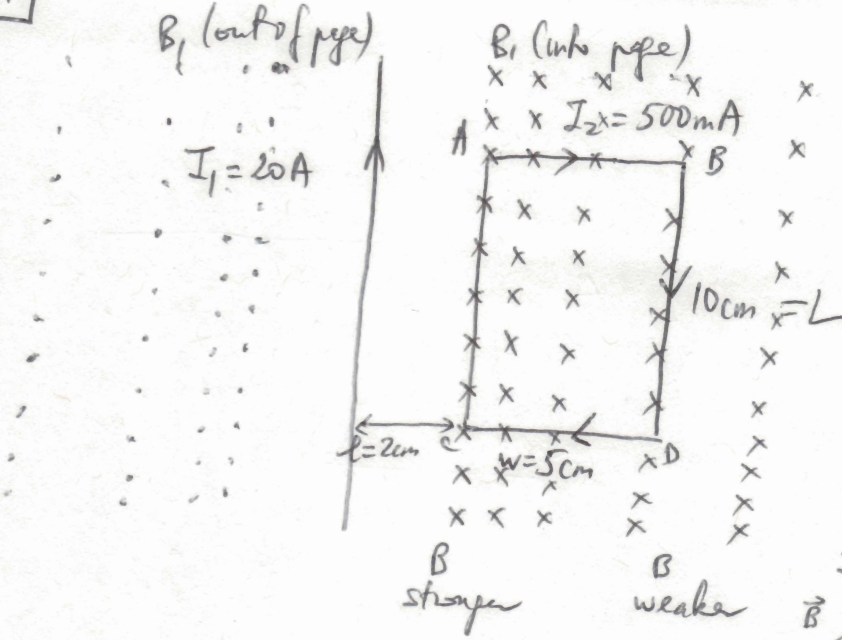
No contribution AC & DB since $d\vec{\ell} \perp \vec{B}$ along those side.

2) $I_{enclosed} = I \frac{\ell}{L}$

$\rightarrow$ Ampere's Law: $B\cancel{\ell} = \mu_0 I \frac{\cancel{\ell}}{L} \rightarrow \boxed{B = \frac{\mu_0 I}{L}}$

inside pipe.

26.64

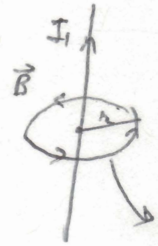


Net magnetic force on loop.

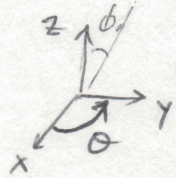
$\vec F_B = q\vec v \times \vec B = I\vec\ell \times \vec B$

$I\vec\ell$ → I_2 $\quad$ $\vec B$ → by I_1

For a line of current:



$\vec B = B(r)\,\hat\theta$

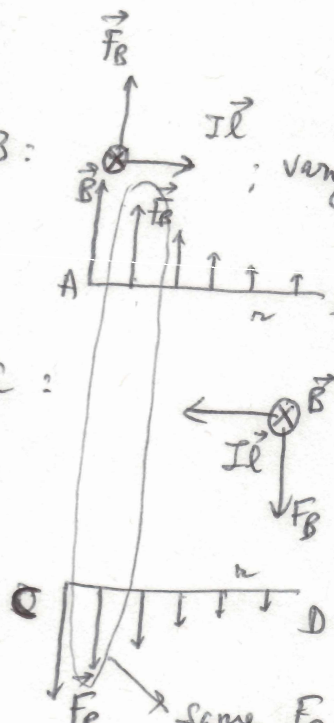


Amperian loop $\begin{cases} B\,2\pi r = \oint \vec B \cdot d\vec\ell \\ \mu_0 I_{enclosed} = \mu_0 I_1 \end{cases}$

$B\,2\pi r = \mu_0 I_1$

$$\boxed{B(r) = \frac{\mu_0 I_1}{2\pi r}}$$

F_B on AB :

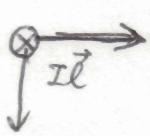


; varying from A to B

on DC :

same F_B, opposite direction → cancel out

on BD :



$\vec F_B(r = 0.07m) = F_B(\hat j)$

on CA :

$\vec F_B(r = 0.02m) = F_B(-\hat j)$

$\vec F_{net} = \left(-F_B(r = 0.02m) + F_B(r = 0.07)\right)\hat j$

$$\vec{F}_{net} = \left[- I_2 L \frac{\mu_0 I_1}{2\pi l} + I_2 L \frac{\mu_0 I_1}{2\pi (l+w)} \right] \hat{j}$$

$$= - \frac{0.5 \times 0.1 \times \overset{2}{\cancel{4\pi}} \times 10^{-7} \times 20}{\cancel{2\pi} \times \cancel{}} \left(\frac{1}{0.02} + \frac{1}{0.07} \right) \hat{j}$$

$$= -2 \times 10^{-7} \left(\frac{1}{0.02} + \frac{1}{0.07} \right) \hat{j} = -7.14 \times 10^{-6} N \, \hat{j}$$

Ch 27 Electromagnetic Induction

Faraday's Law:
$$\boxed{\mathcal{E} = - \frac{d\phi_B}{dt}}$$

$\mathcal{E}$: induced e.m.f. or induced voltage

$-$: conservation of energy or Lenz's law

$\frac{d\phi_B}{dt}$: change of magnetic flux w.r.t time

Magnetic flux: $\phi_B = \int_{\text{3D surface}} \vec{B} \cdot d\vec{A}$

ϕ_B: "Phi"

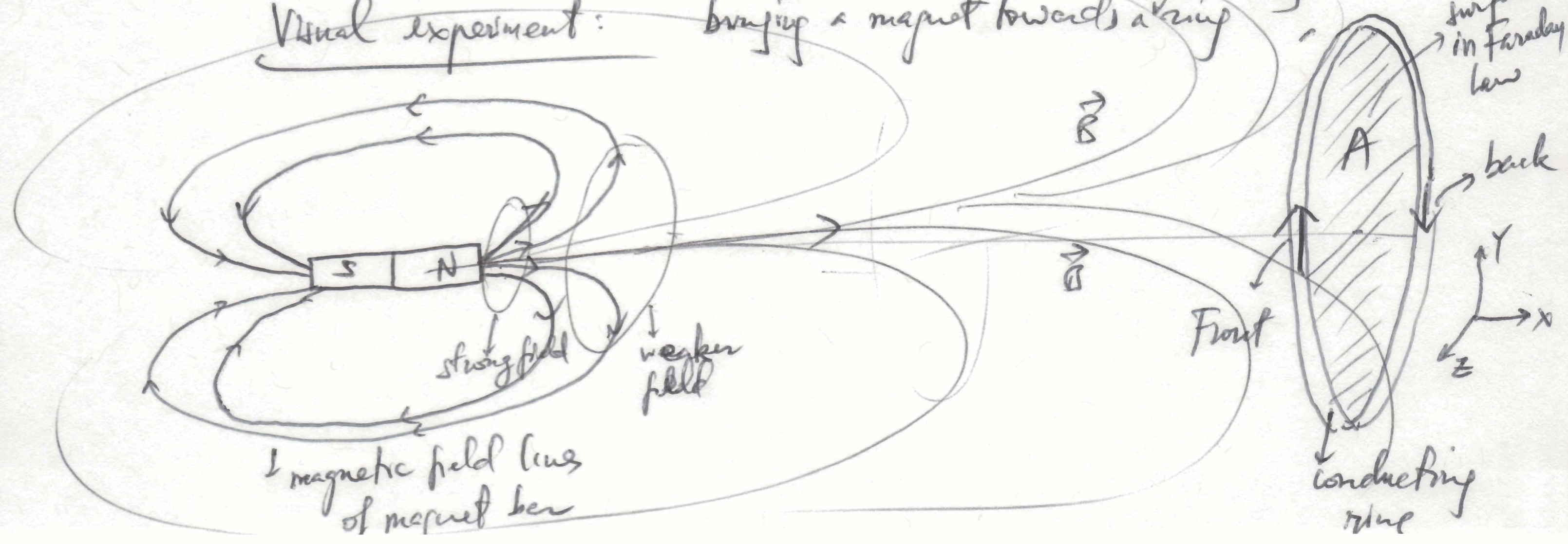
$\vec{B}$: magnetic field through surface

$\cdot$: scalar product: ϕ_B comes from that field that is $\perp$ to the surface.

$d\vec{A}$: element area of that surface

→ If the magnetic flux through some surface area change with time, that would induce an electric potential (or electric field) on the loop enclosing the surface

Visual experiment: bringing a magnet towards a conducting ring



Field ($\vec{B}$) from magnet creates Φ_B thru surface A enclosed by the conducting ring.

When magnet comes closer to the ring: Φ_B increases over time

→ Faraday's law: induces an electric potential (so electric field) on loop enclosing surface A, which is our conducting ring. The potential creates a current (induced current). But what is its direction? → Lenz's law: such that it counters the increase in Φ_B: by creating a counter magnetic field to that of the magnet ($\vec{B}_{magnet} = B_m \hat{\imath}$)

↳ $\vec{B}_{counter} = B_c (-\hat{\imath})$

↓ $I \begin{cases} \text{up in front of ring} \\ \text{down in back of ring} \end{cases}$

(If you look from the right: current is in CW direction on ring).

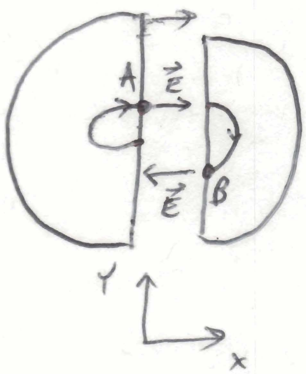
100

26-50 Cyclotron to accelerate deuterium nuclei $(1p + 1n) \rightarrow \begin{cases} q = +e \\ m \simeq 4000\, m_e \end{cases}$

$(m_p = m_n \simeq 2000\, m_e)$

a) $B = 2T$: what frequency should Dee voltage be switched/alternated?

Recall: how the charge was accelerated: by an electric field in the gap b/w the two dee's



When particle arrives at gap from the left dee (A): $\vec{E} = E\hat{i}$; when it later arrives at the gap from the right (B) $\rightarrow \vec{E} = E(-\hat{i})$

$\rightarrow$ Continued acceleration $\Rightarrow \vec{E}$ needs to be switched ($180°$) @ certain frequency.

$$f = \frac{1}{T}\ ; \quad T = \frac{2\pi r}{v} = \frac{2\pi \cancel{r}}{\frac{qB\cancel{r}}{m}} = \frac{2\pi m}{qB} \rightarrow f = \frac{qB}{2\pi m}$$

$$F_B = qvB = m\frac{v^2}{r}$$

$$\boxed{v = \frac{qBr}{m}} \rightarrow \text{velocity of deuteron in orbit of radius } r$$

charge of deuterium

$$f = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times \cancel{2}}{2\pi \times \underbrace{\cancel{2}\times 1.67\times10^{-27}}_{}} = 15.2\times10^{6}\ Hz = 15.2\ MHz.$$

mass of deuterium

mass of a proton or a neutron

$\rightarrow$ Dee voltage should be switched @ $2f$

b) if Dee's have $R = \frac{0.9}{2}m$ $\rightarrow$ what is KE_{max} for these deuterons?

$$KE_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2}\cancel{m}\frac{q^2B^2R^2}{m^{\cancel{2}}} = \frac{1}{2}\frac{(1.6\times10^{-19}\times 2\times 0.45)^2}{2\times 1.67\times10^{-27}}$$

when $r = R$

$$= 3.1\times10^{-12}\ J$$

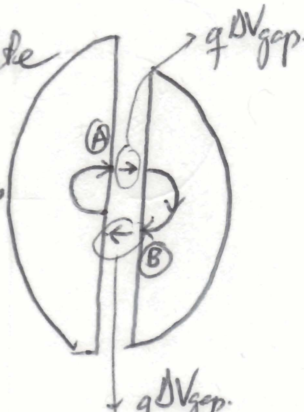
c) If $\Delta V_{gap} = 1500V$ → how many orbits to achieve KE_{max}?

→ deuterons receive energy only @ the gap (acceleration)

$$\# \text{orbits} = \frac{KE_{max}}{\underbrace{\Delta U_{\text{per orbit}}}_{\text{energy received by a deuteron per orbit}}} = \frac{KE_{max}}{2q\,\Delta V_{gap}} = \frac{3.1\times10^{-12}\,J}{2\times1.6\times10^{-19}\times1500\ \frac{J}{\text{orbit}}}$$

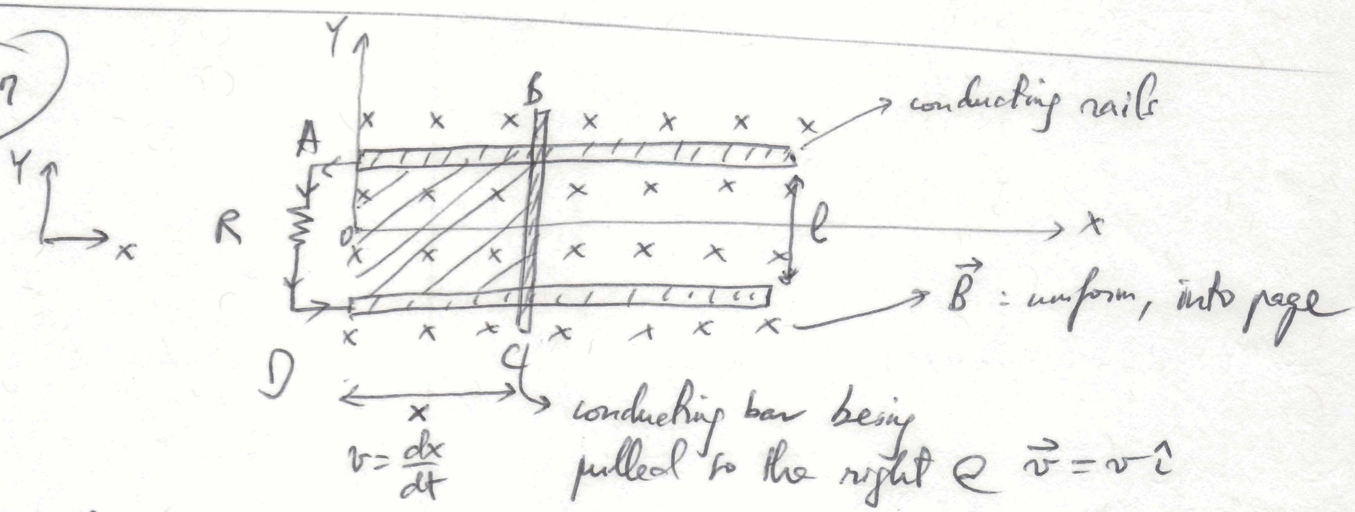
$= 6.48\times10^{3}$ orbits

For each complete orbit: a deuteron receives $2q\,\Delta V_{gap}$ of energy



(We assume small gap b/w dees so time delay by gap is negligible)

27.47



a) Direction of current in R (resistor)?

Why is there a current when there is no battery? → Electromagnetic induction: $\mathcal{E} = -\underbrace{\frac{d\Phi_B}{dt}}_{}$

change of magnetic flux over time thru the surface enclosed by the circuit ABCD

Why Φ_B change with time if B is not? b/c $\Phi_B = \oint \vec{B}\cdot d\vec{A}$ $= \vec{B}\cdot\vec{A} = BA$ → as A increases (A surface area enclosed by circuit ABCD)

→ Makes sense there is a current in the resistor!

What direction?

As bar moves to the right Φ_B into page increase → current induced in the circuit will try to decrease this Φ_B =
How can it decrease Φ_B if A increases anyway?

→ Current goes such that it creates a magnetic field in the opposite direction as the existing one → out of page

What direction of I would produce an out of page magnetic field? CCW (RHR : RH fingers turn as current → thumb gives direction of magnetic field)

→ Current downward at resistor.

b) Agent pulling bar does work @ what rate?

Why ~~work~~ is needed for the bar to move?

- Our loop ABCD opposes any change of Φ_B through it.
- Induced current: some energy is needed to power this current!

$$P = \text{power} = \frac{\text{Work}}{\text{time}} = \frac{F_B \cdot \Delta x}{\text{time}} = F_B\, v$$

v ↓ velocity of the bar

F_B ↓ magnetic force on moving bar (there is a current I in the bar)

$$F_B = I \ell B$$

B → existing field

I ↓ induced current

ℓ ↓ length of moving bar.

From induced voltage $\mathcal{E}$: $I = \frac{\mathcal{E}}{R}$ ↓ Ohm's law.

From Faraday's law: $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B \cdot A)$ ↓ area of ABCD loop.

$$= -B\frac{dA}{dt} = -B\frac{d(x\ell)}{dt} = -B\ell\frac{dx}{dt} = -B\ell v$$

$$P = F_B v = \underbrace{IlB}_{F_B} v = \underbrace{\frac{\varepsilon}{R}}_{I} lB v = \frac{Blv}{R} lB v = \frac{(Blv)^2}{R}$$

Work rate needed for bar to move to the right at velocity v

Alternative solution: $P = I.V = I^2 R = \frac{\varepsilon^2}{R^{\cancel{2}}} \cancel{R} = \frac{1}{R}\left(\frac{d\Phi_B}{dt}\right)^2$

↳ energy needed to power the induced current

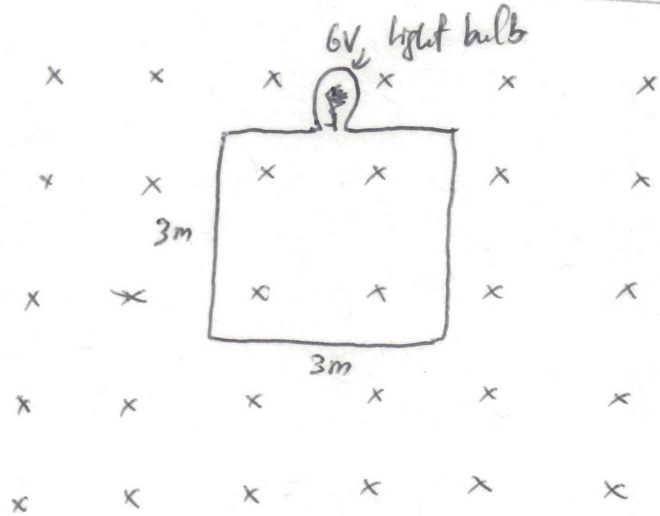
I ↓ induced current ↓ $\frac{\varepsilon}{R}$

V ↓ Ohm's law $V = \frac{I}{R}$

$$= \frac{1}{R}\left[\frac{d}{dt}(Blx)\right]^2$$

$$= \frac{1}{R}(Blv)^2 \checkmark$$

27.40



$B = 2T$ uniform, into page

reduced steadily to 0 over Δt

a) Δt? for full brightness during this time.

↳ EM induced current I → need $\boxed{\frac{d\Phi_B}{dt}}$; area enclosed by circuit is fixed, equal to A $(\Phi_B = BA)$ — B ↓ reduced, A ↓ fixed

↳ If $\frac{d\Phi_B}{dt}$ is low (Δt is too large) → ε may not reach 6V to bulb to shine @ full brightness.

$$\varepsilon = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} = -A\frac{\Delta B}{\Delta t} \rightarrow 6V = -9m^2\frac{(0-2)T}{\Delta t}$$

↓ at least $\varepsilon = 6V$

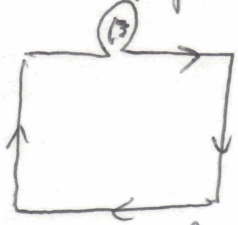
$\rightarrow \Delta t = \frac{18}{6} s = 3s.$

b) Direction of induced current in loop:

As B goes lower $2T \rightarrow 0T$ ∴ ϕ_B into page is decreased

→ induced current will try to oppose this decrease by creating a magnetic field into the page!

RHR →



RH fingers should turn in such a way that its thumb points into page: CW.

Electromagnetic induction & conservation of energy:

Visual experiment: The interaction b/w magnet & conducting ring is only via electromagnetic induction (Faraday's law)

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Magnet on frictionless track:

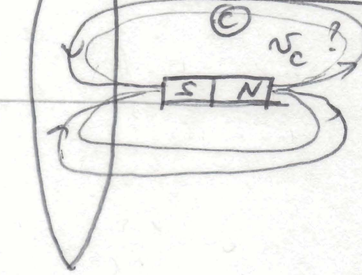
Induced to counteract the increase of Φ_B to the right as the magnet approaches.

1) As magnet goes from A to B
Will this magnet have @ B $\begin{cases} \text{same } v \\ \text{higher } v \\ \boxed{\text{smaller } v} \end{cases}$

What powers this Induced?

A $v_A = v$

B $v_B < v$



2) Will the magnet loose some of its KE forever?

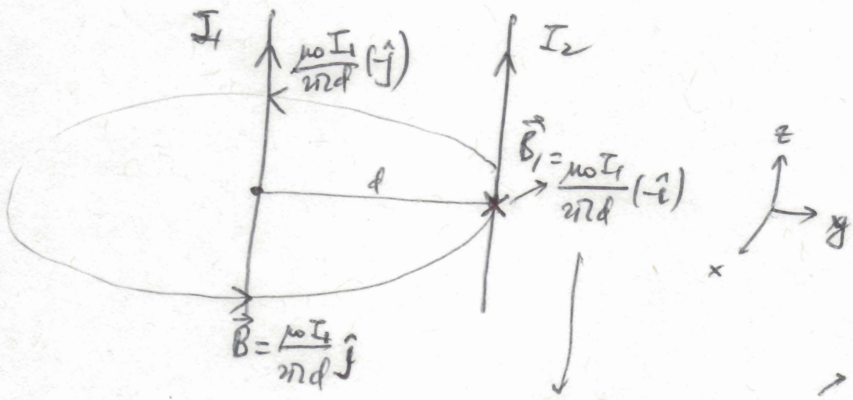
Yes $\begin{cases} v_C = v_B < v \\ v_C < v_B \end{cases}$

$\boxed{\text{No} \; \{ \boxed{v_C > v_B}}$ ✓

As the magnet goes to the other side of the ring: Φ_B to the right now decreases: what happens to $I_{induced}$ in the ring?
→ will change direction to CCW (if seen from the right of the sketch)
→ will transfer energy back to magnet.

Magnet will recover all of initial KE if there is 0 dissipation (superconducting ring)

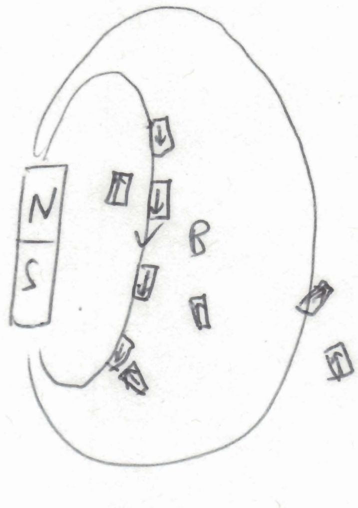
26.44



$I_2 \vec{l} \times \vec{B}_1$

Field by I_1 → $F_{12} = I_2 l \frac{\mu_0 I_1}{2\pi d}(-\hat{j})$
(by I_1 on I_2)

2 parallel currents are attractive to each other.

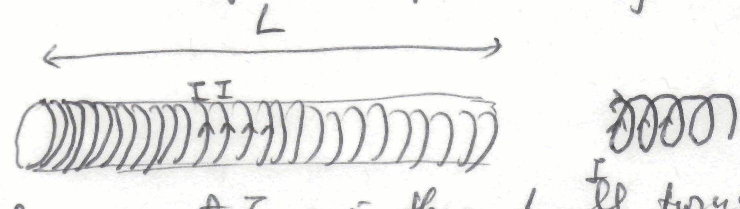


alignment

energy: $\vec{\mu} \times \vec{B}$ → min.
↓
magn. moment
filling

26.44 $\quad n = \frac{N}{L}$: number of turns per unit length $= 3300 \frac{\text{turns}}{m}$

Solenoid:



Same current I going through all turns.

N turns of superconducting wires $\rightarrow$ $n = \frac{N}{L}$

$I = 4100\,A$

Find B inside solenoid: Apply Ampere's law

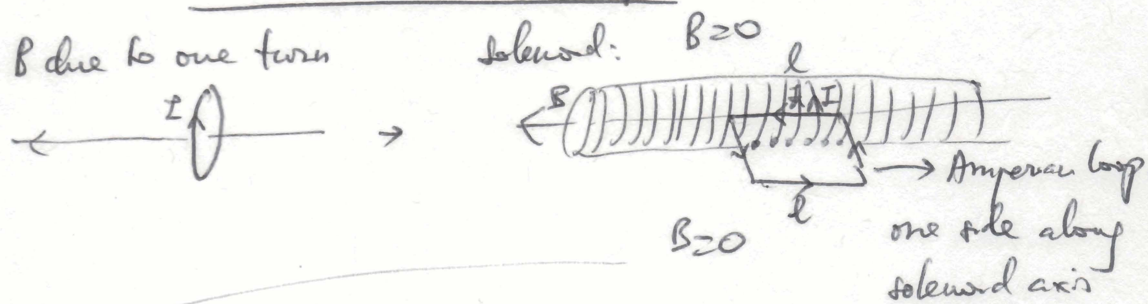
$$\boxed{\oint_{\text{Amp loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\substack{\text{enclosed}\\ \text{by}\\ \text{A-loop}}}}$$

1) Determine the Amperian loop: such that

$$LHS = \vec{B}\cdot \oint_{\text{A-loop}} d\vec{l}$$

$\downarrow$

B constant on that loop!



$$LHS = B\,l$$

2) $I_{\substack{\text{enclosed}\\ \text{by the A-loop.}}} = [\underbrace{\#\text{ turns within } l}_{n\,l}]\, I = I n l$

$$\rightarrow \quad B\cancel{l} = \mu_0 I n \cancel{l} \rightarrow \boxed{B = \mu_0 I n}$$

$$B = 4\pi \times 10^{-7} \times 3300 \times 4.1 \times 10^{3}\ T = 17\,T.$$