Ch23 Electrostatic Energy & Capacitors

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q'} \leftarrow \text{test charge}$$

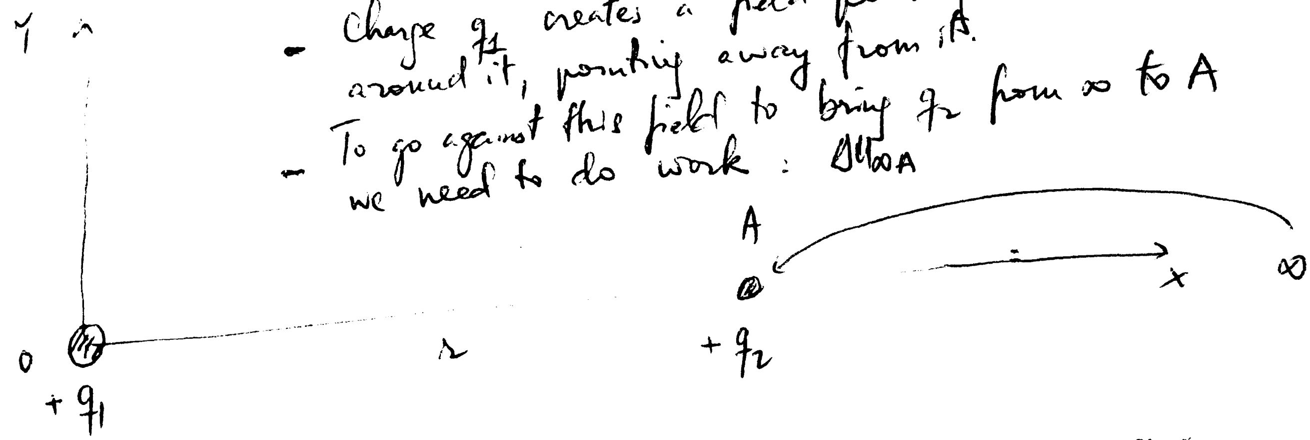
U : electric potential energy (J)

V : electric potential ($\frac{J}{C}$ = V for volt)

$$\Delta U_{AB} = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}$$

Store electrostatic energy by separating charges of opposite type or bringing charge of same type together. For example:

- Charge q_1 creates a field pointing in all direction around it, pointing away from it.
- To go against this field to bring q_2 from ∞ to A we need to do work : $\Delta U_{\infty A}$

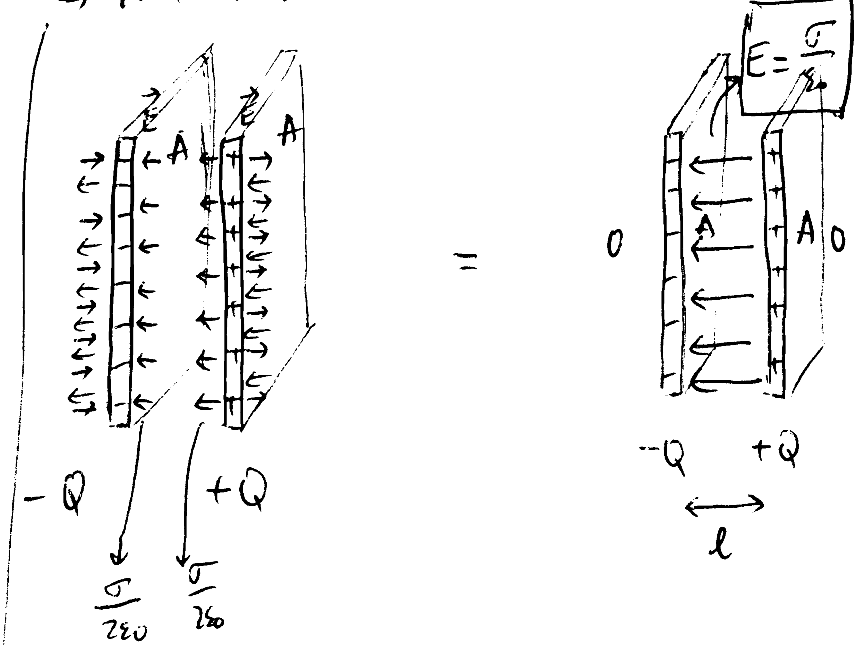


$$\Delta U_{\infty A} = q_2 \Delta V_{\infty A} = q_2 \, k q_1 \left(\frac{1}{r} - \underbrace{\frac{1}{\infty}}_{0} \right) = k \frac{q_1 q_2}{r}$$

Observation :
- a) Smaller r $\rightarrow$ larger energy stored.
- b) Larger q_2 & q_1 $\rightarrow$ more energy stored.

What is the electrostatic energy stored b/w the plates?

→ Find U :



Total electric field b/w 2 parallel plates of charges $-Q$ & $+Q$ is $\frac{\sigma}{\varepsilon_0}$ $\left(\sigma = \frac{Q}{A}\right)$

$$dU = -dW = -\underbrace{dq}_{\text{infinitesimal test charge}} V = dq\, E l = dq \frac{\sigma}{\varepsilon_0} l = \boxed{dq \frac{q}{A} \frac{l}{\varepsilon_0}}$$

$$V = -\int \vec{E} \cdot d\vec{l} = -E \int dl = -El$$

& $\boxed{\text{const field}}$

$$U = \int dU = \frac{l}{A\varepsilon_0} \int_0^Q q\, dq = \frac{l}{A\varepsilon_0} \frac{1}{2} Q^2 = \frac{1}{2} \frac{A\varepsilon_0 l}{(A\varepsilon_0)^2} Q^2$$

$$U = \frac{1}{2} \varepsilon_0 \underbrace{Al}_{\substack{\text{volume} \\ \text{b/w plates} \\ \text{Vol.}}} \underbrace{\frac{Q^2}{A^2 \varepsilon_0^2}}_{\frac{\sigma^2}{\varepsilon_0^2} = E^2} = \frac{1}{2} \varepsilon_0 \,\text{vol}\, E^2 = \frac{1}{2} \varepsilon_0 E^2 (\text{vol})$$

Technological issue for hybrid cars.

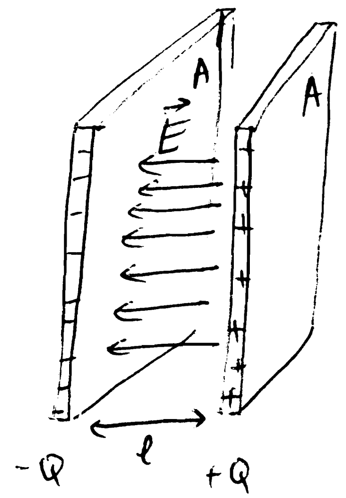
$$\boxed{u = \text{electrostatic energy density} = \frac{U}{\text{vol}} = \frac{1}{2} \varepsilon_0 E^2} \rightarrow \frac{J}{m^3}$$

↳ dielectric constant in vacuum.

↳ replace by ε if there is a dielectric material b/w plates

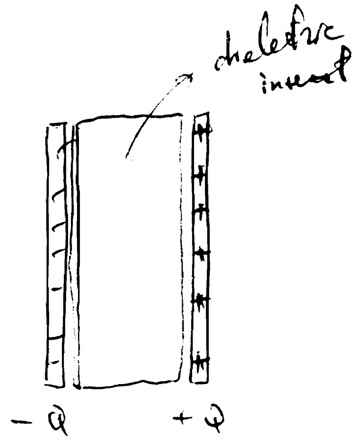
$\varepsilon = K\varepsilon_0$ (K: Kappa > 1: dielectric without)

Capacitance : $C = \dfrac{Q}{V}$ $Q \rightarrow$ Total charge on either plate
$V \rightarrow$ electric potential b/w plates.



$$C = \frac{Q}{E.l} = \frac{Q}{\frac{\sigma}{\varepsilon_0}\, l} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0}\, l} = \frac{A\varepsilon_0}{l}$$

Capacitance for a parallel plate capacitor of surface A & separation l w/o any dielectric insert b/w the plates.



Total energy stored in a capacitor (parallel-plate):

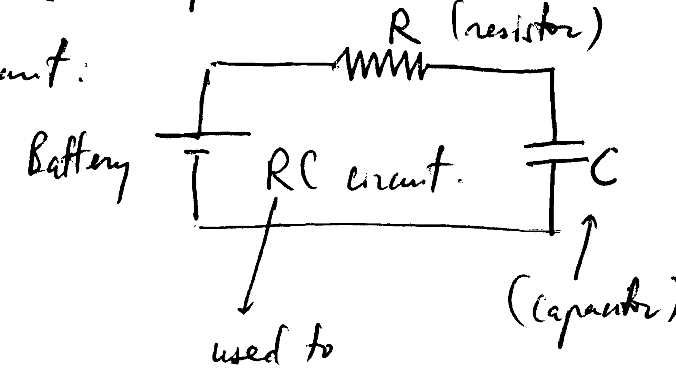
$$U = \frac{1}{2}\varepsilon_0 E^2\, \mathrm{vol} = \frac{1}{2}\varepsilon_0 E^2\, Al\,\frac{l}{l} = \frac{1}{2}\underbrace{\frac{A\varepsilon_0}{l}}_{C}\;\underbrace{E^2 l^2}_{V^2}$$

$V^2 \rightarrow$ electric potential

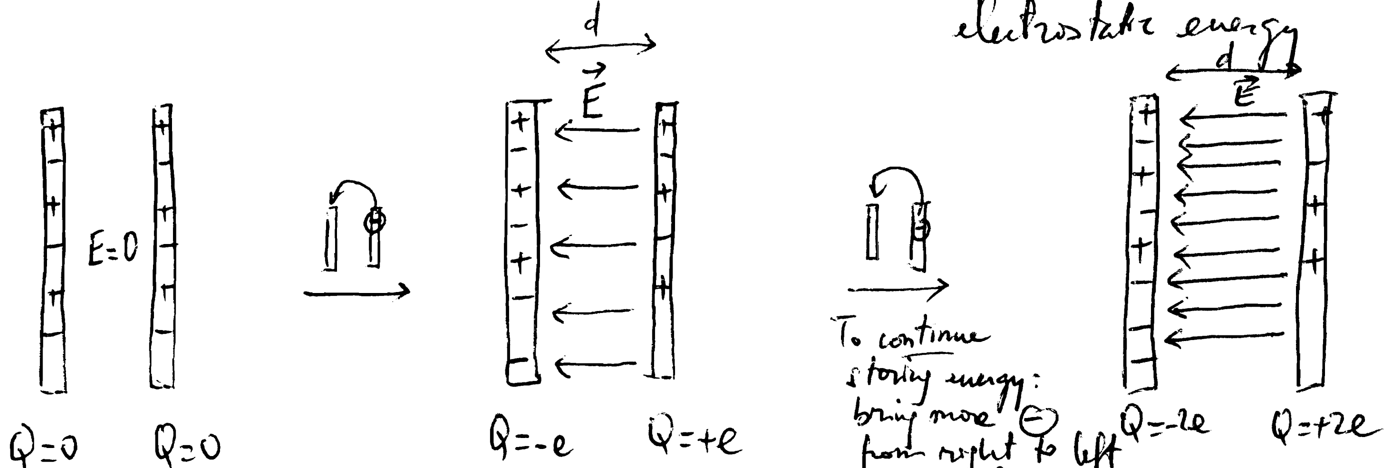
$$\boxed{U = \frac{1}{2} C V^2}$$

Electrostatic Energy Storage Devices: Capacitors – Parallel Plate

Symbol: $||$; in a circuit:



used to charge the capacitor or store electrostatic energy



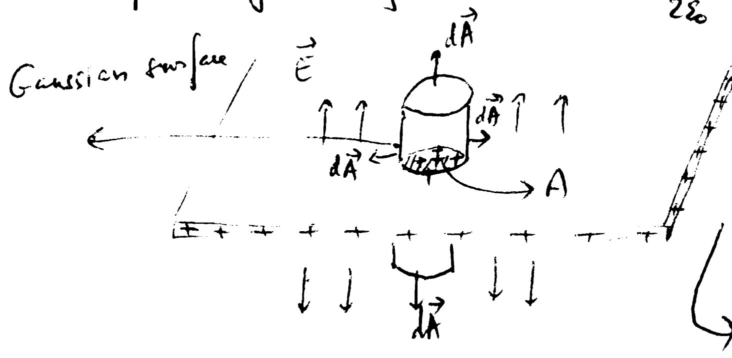
Equal amount of charges of either type in EACH plate

Assume $d \ll$ dimension of the plates $\rightarrow$ Electric field is that of an ∞ plane of charge

To continue storing energy: bring more $\ominus$ from right to left plate $\downarrow$ Need to work against the field. $\rightarrow$ harder than the moving the 1st $\ominus$

"Charging a capacitor"

E-field due to ∞ plane of charge: $E = \frac{\sigma}{2\varepsilon_0}$ (σ: surface charge density of the plane)



$$\phi = \oint \vec{E}\cdot d\vec{A} = E\int_{\substack{\text{top \& bottom}\\ \text{of cylinder}}} dA = E2A$$

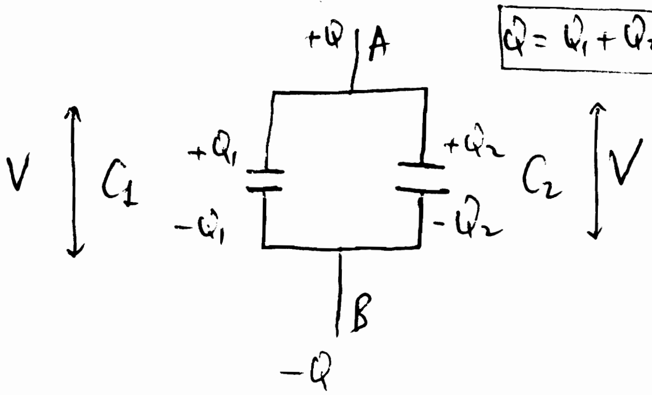
$$\phi = \frac{q_{enclosed}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\rightarrow E2\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

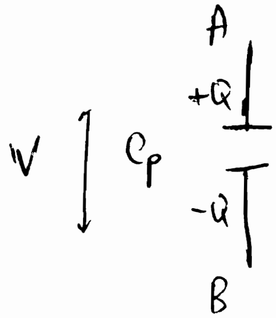
Connecting capacitors

Parallel

$\boxed{Q = Q_1 + Q_2}$



$\Downarrow$



Equivalent capacitor

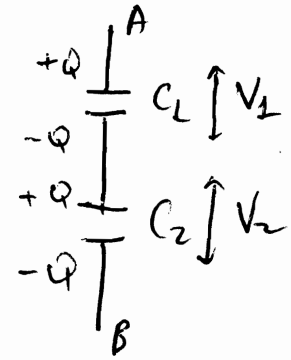
$$V = \frac{Q}{C} \left\{ \begin{array}{l} V = \dfrac{Q_1}{C_1} \Rightarrow C_1 = \dfrac{Q_1}{V} \\ V = \dfrac{Q_2}{C_2} \Rightarrow C_2 = \dfrac{Q_2}{V} \\ V = \dfrac{Q}{C_p} \Rightarrow C_p = \dfrac{Q}{V} \end{array} \right.$$

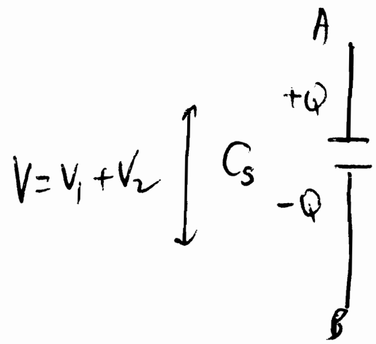
$$C_p = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$$

$$\boxed{C_p = C_1 + C_2}$$

(to increase capacitance connect capacitors in parallel)

Series





Equivalent capacitor

$$C_s = \frac{Q}{V}$$

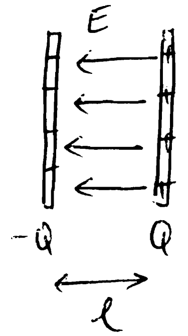
$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

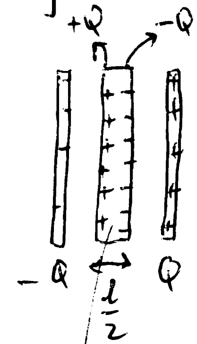
$$C_s = \frac{\cancel{Q}}{\cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)} \rightarrow \boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}}$$

or $\boxed{C_s = \dfrac{C_1 C_2}{C_1 + C_2}}$

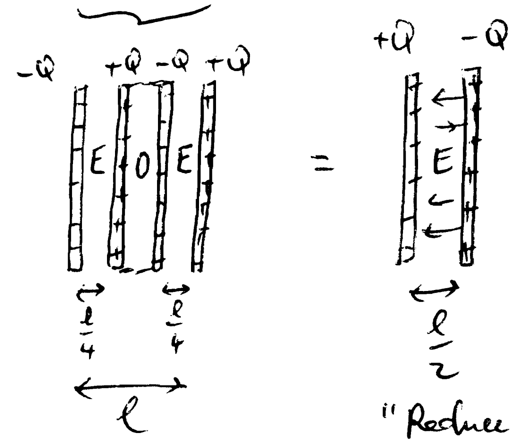
To increase capacitance:

1) Connecting capacitors in parallel
2) Inserting a conducting slab b/w plates





conducting slab
i
conductor:

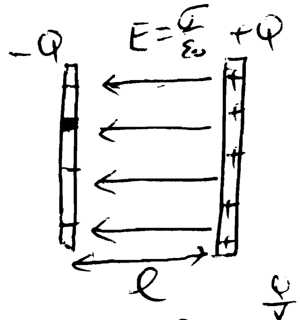


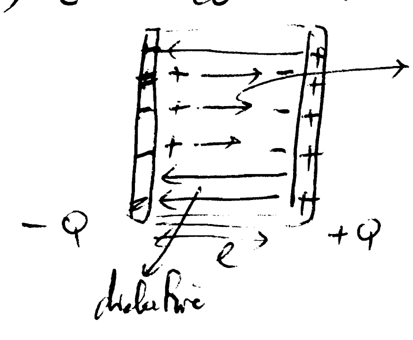
"Reduce spacing b/w plates by a factor of 2"

$$V \xrightarrow{E \cdot l} \frac{V}{2} \rightarrow C \xrightarrow{\frac{Q}{V}} 2C$$

3) Insert a dielectric slab b/w plates

↳ $\varepsilon = K\varepsilon_0 \qquad K > 1$





Field is reduce by a factor of $K \rightarrow$ also V

$$V \rightarrow \frac{V}{K} \longrightarrow C \xrightarrow{\frac{Q}{V}} KC$$

23.68

Lightning flash : $\begin{cases} Q = 30\,C \\ V = 30\,MV \ (\text{mega volt} = 10^6\,V) \end{cases}$

↳ every 5 s

How long storm will continue if energy is not replenished

→ Example 23.4 → energy in storm is 140 GJ

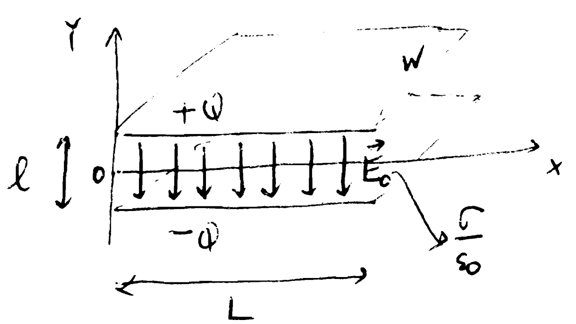
giga → 10^9

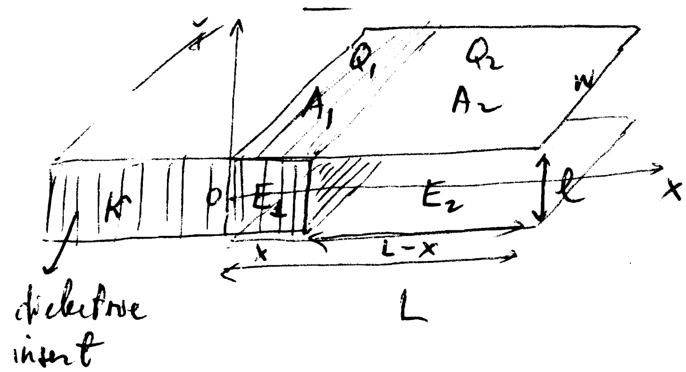
→ How many flashes? $\dfrac{140\,GJ}{\text{energy per flash}} = \dfrac{140 \times 10^9\,J}{QV}$

$$U = qV \qquad = \frac{140 \times 10^9\,\cancel{J}}{30 \times 30 \times 10^6\,\cancel{J}} = \frac{140 \times 10^3}{900} = 156 \text{ flashes.}$$

→ How long? $156 \text{ flashes} \times 5\,s/\text{flash} \times \dfrac{1\,\text{min}}{60\,s} \approx 13\,\text{min}.$

23.72





$C_0;\ V_0$

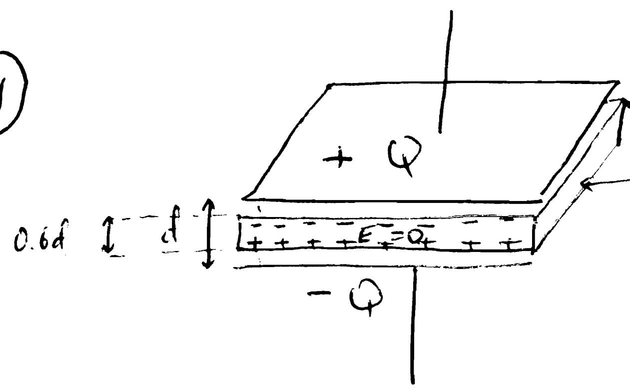
a) $$C(x) = \frac{w\varepsilon_0}{l}\left(Kx + L - x\right)$$

will prove.

b) $$U = \frac{1}{2} C V^2 = U_0 \frac{L}{Kx + L - x}$$

c) $$F = -\frac{dU}{dx} = \frac{U_0 L (K-1)}{(Kx + L - x)^2} \rightarrow F\left(x = \frac{L}{2}\right) = \frac{4 U_0 (K-1)}{L (K+1)^2} = \frac{2 C_0 V_0^2 (K-1)}{L (K+1)^2}$$

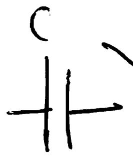
23.51



Parallel-plate capacitor w/ inserted conducting slab.
→ electrons free to move.
& E = 0 inside conductor

a) What is the new capacitance?
Since $E=0$ inside conducting slab → field region b/w plates is reduced to a width of $\underline{\underline{0.4d}}$

Since $V = E.\ell$ →
$$\begin{cases} V_0 = E \times d & \text{(w/o conducting slab)} \\ V = E \times 0.4d = 0.4V_0 & \text{(Electric potential is reduced by a factor of 0.4)} \end{cases}$$



$C = \frac{Q}{V}$ Q → charge on either plate
V → electric potential b/w plates.

$$\begin{cases} C_0 = \frac{Q}{V_0} \text{ (w/o slab)} \\ C = \frac{Q}{V} = \frac{Q}{0.4V_0} = \frac{C_0}{0.4} \end{cases}$$
$= 2.5\, C_0$
(Capacitance increased by a factor of 2.5).

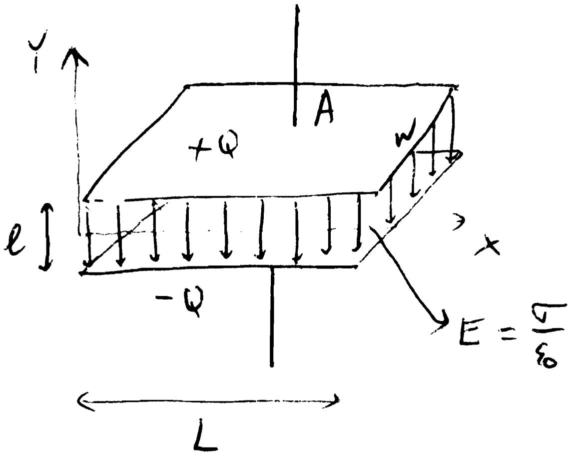
b) Capacitor not connected to anything → same charge Q with or without inserted conducting slab.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C\frac{Q^2}{C^2} = \frac{Q^2}{2C}$$
$$C = \frac{Q}{V} \rightarrow V = \frac{Q}{C}$$

$$\begin{cases} U_0 = \frac{Q^2}{2C_0} \\ U = \frac{Q^2}{2C} = \frac{Q^2}{2 \times 2.5C_0} = \frac{U_0}{2.5} \end{cases}$$
$= 0.4U_0$

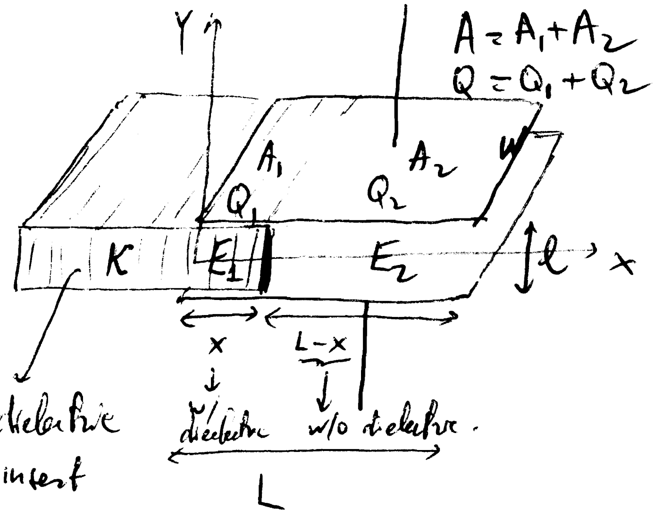
(Stored energy reduced by a factor of 0.4)

23.72)



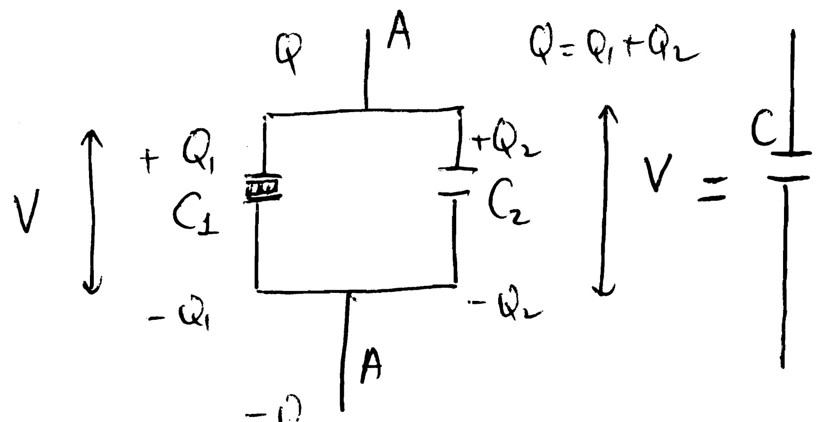
C_0 ; V_0 ; U_0

$A = A_1 + A_2$
$Q = Q_1 + Q_2$



What is C when dielectric slab is inserted a distance x?

2 capacitors connected in parallel:



Parallel connection : $C = C_1 + C_2 = \dfrac{Q_1}{V} + \dfrac{Q_2}{V}$

Find Q_1 & Q_2

Q_1 → charge on top plate in region w/ dielectric

Q_2 → charge in region w/o dielectric

1) $E_2 = E_0$

$E_1 = \dfrac{E_0}{K}$ ← $\boxed{E_0 = \dfrac{\sigma}{\varepsilon_0} \qquad E_1 = \dfrac{\sigma}{K\varepsilon_0} = \dfrac{E_0}{K}}$

2) $E_1 = \dfrac{\sigma_1}{K\varepsilon_0} = \dfrac{Q_1/A_1}{K\varepsilon_0} = \dfrac{Q_1}{xwK\varepsilon_0}$

$E_2 = \dfrac{\sigma_2}{\varepsilon_0} = \dfrac{Q_2/A_2}{\varepsilon_0} = \dfrac{Q_2}{(L-x)w\varepsilon_0}$

3) $\boxed{E_1 = E_2}$ (discontinuity in the field is not observed)

→ $\dfrac{Q_1}{xwK\varepsilon_0} = \dfrac{Q_2}{(L-x)w\varepsilon_0}$ → $\boxed{Q_1 = Q_2 \dfrac{Kx}{L-x}}$

$$C = \frac{Q_1 + Q_2}{V} = \frac{Q_2 \frac{Kx}{L-x} + Q_2}{V} = \frac{Q_2}{V}\left(\frac{Kx}{L-x} + 1\right)$$

$$\frac{Q_2}{V} = \frac{Q_2}{E_2 l} = \frac{\cancel{Q_2}}{\frac{\cancel{Q_2}}{(L-x)w\varepsilon_0} l} = \frac{(L-x)w\varepsilon_0}{l}$$

$$C(x) = \frac{(L-x)w\varepsilon_0}{l}\left(\frac{Kx}{L-x} + 1\right) = \frac{w\varepsilon_0}{l}\left[Kx + L - x\right]$$

$$\boxed{C(x) = \frac{w\varepsilon_0}{l}\left[\underbrace{x(K-1)}_{\text{positive}} + L\right]}$$

capacitance w/ dielectric insert a distance x into the spacing b/w plates.

a) $x = \frac{L}{2} \rightarrow$ $C\left(x = \frac{L}{2}\right) = \frac{w\varepsilon_0}{l}\left[\frac{L}{2}(K-1) + L\right] = \frac{w\varepsilon_0 L}{2l}\left[K - 1 + 2\right]$

$$C\left(x = \frac{L}{2}\right) = \frac{w\varepsilon_0 L}{2l}(K+1)$$

b) $U = \frac{1}{2}CV^2$

$$C(x) = \frac{w\varepsilon_0}{l}\left[x(K-1) + L\right]$$

$$V = E_1 l = \frac{Q_1}{K\varepsilon_0 w x} l = \frac{\frac{\cancel{Kx}Q}{Kx + L - x} l}{\cancel{Kx}\varepsilon_0 w} = \frac{Ql}{\varepsilon_0 w(Kx + L - x)}$$
$$= E_2 l$$

$$Q_1 + Q_2 = Q$$
$$Q_1 = Q_2 \frac{Kx}{L-x} \rightarrow Q_2 = \frac{L-x}{Kx} Q_1$$

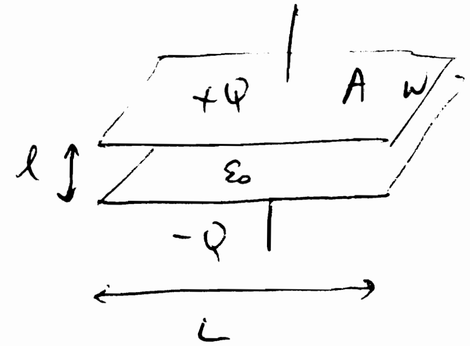
$$Q_1 + \frac{L-x}{Kx} Q_1 = Q$$

$$Q_1 = \frac{Q}{1 + \frac{L-x}{Kx}} = \frac{Kx\,Q}{Kx + L - x}$$

$$U(x) = \frac{1}{2}\frac{\cancel{w\varepsilon_0}}{\cancel{l}}\cancel{\left[x(K-1)+L\right]}\frac{Q^2 l^{\cancel{2}}}{w^{\cancel{2}}\varepsilon_0^{\cancel{2}}\left[x(K-1)+L\right]^{\cancel{2}}} = \frac{1}{2}\frac{l}{w\varepsilon_0}\frac{Q^2}{\left[x(K-1)+L\right]}$$

$$= \underbrace{\frac{1}{2}\frac{lQ^2}{w\varepsilon_0 L}}_{U_0}\frac{L}{\left[x(K-1)+L\right]} \rightarrow \boxed{U(x) = U_0\frac{L}{x(K-1)+L}}$$

Why $U_0 = \frac{1}{2}\frac{\ell Q^2}{w\varepsilon_0 L}$?



$$\boxed{U_0 = \frac{1}{2}C_0V_0^2 = \frac{Q^2}{2C_0} = \frac{Q^2}{2\frac{Lw\varepsilon_0}{\ell}} = \frac{Q^2\ell}{2Lw\varepsilon_0}}$$

$$V_0 = \frac{Q}{C_0}$$

$$C_0 = \frac{Q}{V_0} = \frac{\cancel{E_0}A\varepsilon_0}{\cancel{E_0}\,\ell} = \frac{Lw\varepsilon_0}{\ell}$$

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{\frac{Q}{A}}{\varepsilon_0} \rightarrow Q = E_0 A \varepsilon_0$$

$$x = \frac{L}{2} \rightarrow U\left(x=\frac{L}{2}\right) = U_0 \frac{\cancel{L}}{\frac{\cancel{L}}{2}(K-1)+\cancel{L}} = U_0\frac{1}{\frac{K-1}{2}+1}$$

$$= U_0\frac{2}{K+1} = \frac{C_0V_0^2}{K+1}$$

$$\downarrow$$

$$\frac{1}{2}C_0V_0^2$$

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c) Force on slab?

$$F = -\frac{dU}{dx} \quad \text{b/c} \quad V = -\int \vec{E}\cdot d\vec{\ell}$$

$$\Rightarrow \vec{E} = -\frac{dV}{dx}\hat{i}$$

$$\Rightarrow \vec{F} = q\vec{E} = -q\frac{dV}{dx}\hat{i} = -\frac{d(qV)}{dx}\hat{i} = -\frac{d(U)}{dx}\hat{i}$$

$$U = U_0\frac{L}{x(K-1)+L}$$

$$\rightarrow F = -\frac{dU}{dx} = -U_0L\frac{d}{dx}\left[x(K-1)+L\right]^{-1}$$

$$\boxed{F(x) = U_0L\frac{K-1}{\left[x(K-1)+L\right]^2}}$$

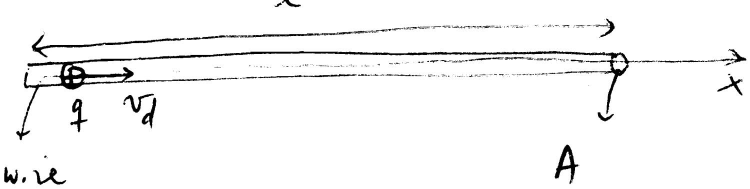
$$F\left(x=\frac{L}{2}\right) = \frac{U_0L(K-1)}{\left[\frac{L}{2}(K-1)+L\right]^2} = \frac{U_0(K-1)}{\left[\frac{K-1}{2}+1\right]^2} = \frac{4U_0(K-1)}{L[K+1]^2} = \frac{2C_0V_0^2(K-1)}{L(K+1)^2}$$

Ch 24 Electric Current:

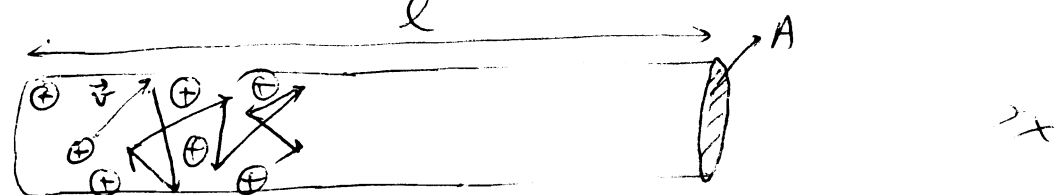
$$I = \frac{\Delta q}{\Delta t} \quad ; \quad I = \frac{dq}{dt} \qquad \left(\text{unit } \frac{C}{s} \equiv A \text{ for Amp}\right)$$

Motion of charges:

Microscopic expression:



v_d: drift velocity: average velocity along x-direction.



$\vec{v}$ very high, since its direction is random, the average value is small $\rightarrow v_d$

n: number of charge per unit volume

$$I = \frac{\Delta q}{\Delta t} = \frac{nAl\,q}{\frac{l}{v_d}} = nqAv_d \quad (\text{microscopic current})$$

Copper wire $A = 1\,mm^2$; $I = 5A$; each atom of Copper contributes $1.3e$ of charge, what is the v_d?

$$v_d = \frac{I}{nqA}$$

Need # atoms of copper per unit volume

↳ Mass density of Copper $\rho = 8920\,kg/m^3$

$(\rho_w = 1000\,kg/m^3)$

↳ Mass of one atom of Copper = Cu → 63.55 a.u. (atomic unit)

$\rightarrow$ Mass of atom of copper $= 63.55\ \text{au}\ \boxed{\dfrac{1.66\times 10^{-27}\,\text{kg}}{\text{au}}}$

Table : conversion.

$=$

$\rightarrow$ #atoms of copper per unit volume $= \dfrac{\rho_{Cu}\ (\text{kg}/\text{m}^3)}{63.55\times 1.66\times 10^{-27}\,(\text{kg})}$

$= 8.5\times 10^{28}\ \dfrac{\text{atoms Cu}}{\text{m}^3}$

$$v_d = \frac{I}{nqA} = \frac{5\,A}{8.5\times10^{28}\times 1.3\times 1.6\times 10^{-19}\times 10^{-6}}$$

$= 0.283\ \dfrac{\text{mm}}{\text{s}}$

$\downarrow$

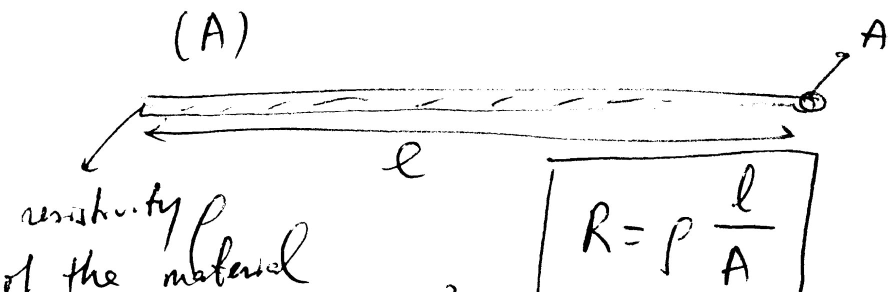
This is not the actual velocity : $\frac{1}{2}m\bar{v^2} = \frac{3}{2}kT$

$\rightarrow \underbrace{T = 6.1\times10^{-12}\ {}^{\circ}K}_{\text{unrealistic.}}$

<u>Ohm's law</u>. $I = \dfrac{V}{R}$ $\leftarrow$ electric potential (V)

R $\leftarrow$ resistance (Ω for Ohm)

I $\downarrow$ current (A)



$$\boxed{R = \rho\,\frac{l}{A}}$$

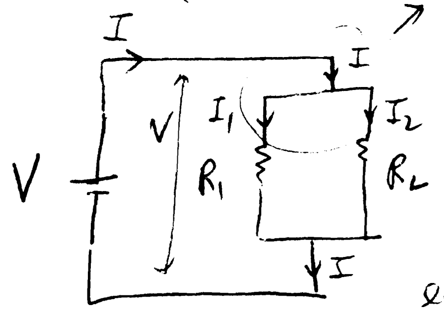
Power $= P = I.V \begin{cases} = \dfrac{V^2}{R} \\ = I^2R \end{cases}$

P $\downarrow$ heat loss per unit time

Resistors

(light bulb is a resistor)

Parallel ⟷ Current division



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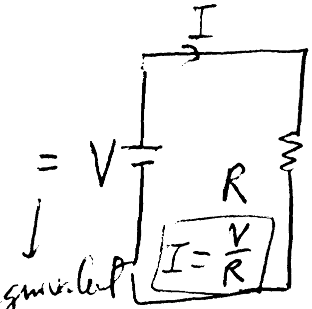


↓ equivalent circuit

↓ same total current I

$$I = I_1 + I_2$$
$$= \frac{V}{R_1} + \frac{V}{R_2}$$
$$\underbrace{= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

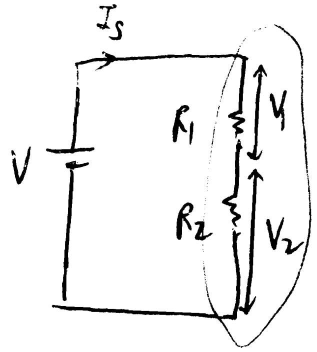
$$I = \frac{V}{R}$$

$$\cancel{V}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\cancel{V}}{R}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$ Parallel connection

or $$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Series ⟷ Voltage division



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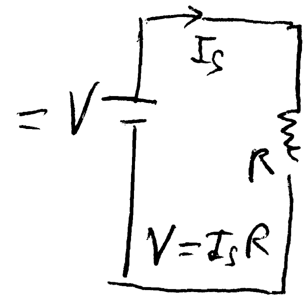


$$V = V_1 + V_2 = I_s R_1 + I_s R_2$$
$$\underbrace{= I_s (R_1 + R_2)}$$

↓

$$V = I_s R$$

$$\boxed{R = R_1 + R_2}$$ series connection.

Voltage division:

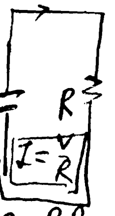
$$V_1 = I_s R_1 = \frac{V}{R_1 + R_2} R_1 = \underbrace{\frac{R_1}{R_1 + R_2}}_{<1} V$$

$$V_2 = I_s R_2 = \underbrace{\frac{R_2}{R_1 + R_2}}_{<1} V$$

$$V_1 + V_2 = \left(\frac{R_1}{R_1 + R_2} + \frac{R_2}{R_1 + R_2}\right) V = V$$

Power consumption (needed so e⁻ can go thru the resistors)

$$P = IV$$



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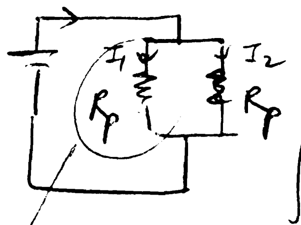
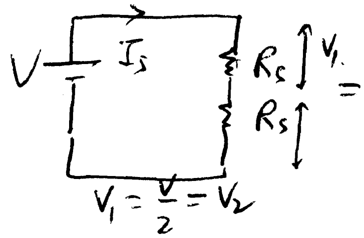


$$\boxed{I_1 = \frac{I}{2} = I_2}$$
$$R_1 = R_2 = R_p$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_p}{2}$$

$$P = I_1 V = \frac{I}{2} V = \frac{\frac{V}{R}}{2} V = \frac{V^2}{2R}$$
$$= \frac{V^2}{2 R_p} = \frac{V^2}{R_p}$$



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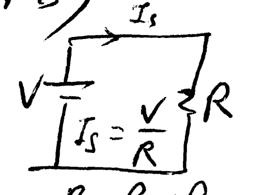


$$V_1 = \frac{V}{2} = V_2$$

$$R = R_1 + R_2$$
$$= 2R_s$$

$$P = I_s V_1 = \frac{V}{R} V_1 = \frac{V}{2R_s} \frac{V}{2} = \frac{V^2}{4R_s}$$

24-69

$m = 1500\,kg$ powered by $26 \times 12V$ battery in series.

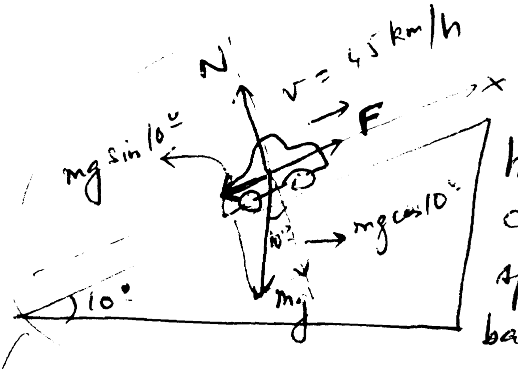
$\rightarrow 312V$

100 A.h

same unit for charge

→ charge Q provided by each battery.

Total charge = charge Q from one battery.



how long can keep this speed w/ the batteries?

Motor: 85% efficient:

$$1J \rightarrow \boxed{\text{motor}} \rightarrow 0.85J$$

Engine applies a force F such that $F_{net_x} = 0$ $(a=0)$

$$F - mg\sin 10° = 0$$

$$\boxed{F = mg\sin 10°}$$

@ speed v $\Rightarrow$ $P = F \cdot v$ power needed to go up 10° slope @ speed v

$$= mg\sin 10°\, v$$

$$\text{How long} = \frac{\boxed{\text{total energy available}}}{\text{energy consumption rate}} = \frac{0.85QV}{mg\sin 10°\, v} = \frac{0.85 \times 100 \times 3600 \times 312}{1500 \times 9.81 \times \sin 10° \times \frac{45}{3.6}}$$

$= 2984s$

$= 49.8\,min.$

Energy from batteries: $P \cdot \Delta t = I \cdot V \cdot \Delta t = \underbrace{I\Delta t}_{Q}\, V$

↳ Energy available from engine $= 0.85QV$

24.61.

$A = \pi(1.05\times10^{-3})^2\ m^2$

$$v_d = \frac{I}{n\,q\,A}$$

A → cross sectional area

q → charge contribution per atom: 3.5e

n → # of Al atoms per unit volume

↓

Need this data: → Start with mass density of Al $\underbrace{\rho = 2702\ \frac{kg}{m^3}}_{\text{Table or google}}$

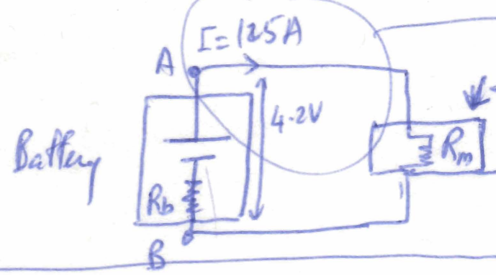
→ # atoms per m³ → $\dfrac{\rho}{\text{mass of one atom Al}}$

→ mass of one atom Al: $26.98\ \cancel{a.u.}\ \dfrac{1.66\times10^{-27}\,kg}{\cancel{a.u}}$

$$n = \frac{2702\ \frac{\cancel{kg}}{m^3}}{26.98\times1.66\times10^{-27}\ \cancel{kg}}$$

$$v_d = \frac{20\ A}{\frac{2702}{26.98\times1.66\times10^{-27}}\,3.5\times1.6\times10^{-19}\times\pi(1.05\times10^{-3})^2} = \underbrace{0.171\ mm/s}_{\text{low.}}$$

24.51

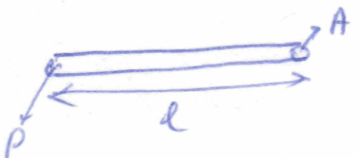


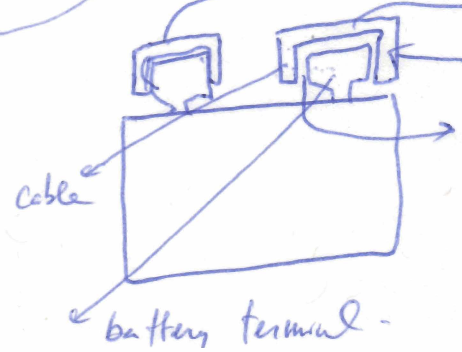
$$R = \frac{V}{I} = \frac{4.2V}{125A} = 33.6\,m\Omega$$

Data ~~cable resistance is 0.6 mΩ)~~

Starter motor.

Resistance = $R = \rho\dfrac{l}{A}$





corrosion will increase resistance

↓

33mΩ increase.

Ch25 Electrical Circuits

Linear (linear relationship b/w voltage V & current I)

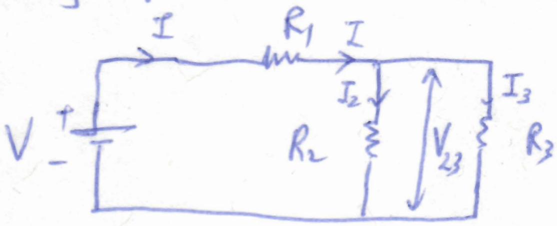
R, C, L

$\underbrace{R, C}_{\text{electric}}\ \underbrace{L}_{\text{magnetic}}$

2 types of electric circuits

- → Resistors only
  1) Reduce to  using parallel & series connections
  2) Use loop or Node analysis
- → Resistors & capacitors { will discuss

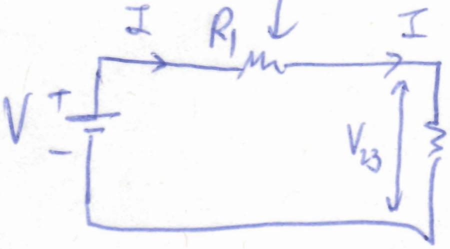
1) Using parallel & series connections:



Solve for this circuit → calculate I^V_1, I_2, I_3

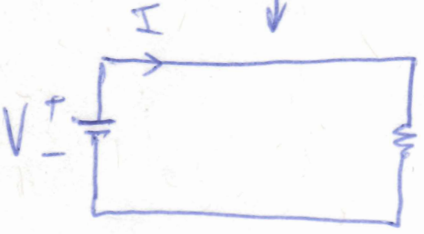
→ using series & parallel.
R_1 R_2 & R_3

(Note: R_1 is NOT in series w/ R_2 neither R_3 but with their combination)



$$R_{23} = \frac{R_2 R_3}{R_2 + R_3}$$

R_1 is in series w/ R_{23}



$$R_{123} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Can solve for I using Ohm's law: $I = \frac{V}{R}$

$$I = \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

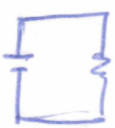
$$I_2 = \frac{V_{23}}{R_2} = \frac{I R_{23}}{R_2} = \frac{I \frac{\cancel{R_2} R_3}{R_2+R_3}}{\cancel{R_2}} = \underbrace{\frac{R_3}{R_2+R_3}}_{<1} I$$

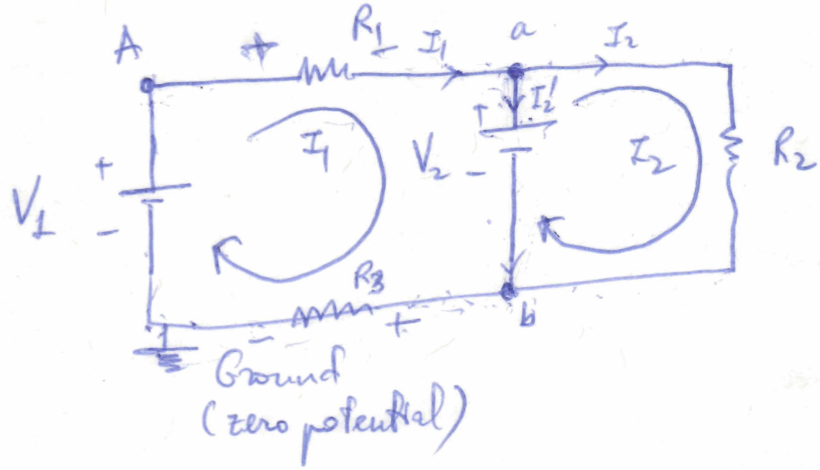
(current division)

$$I_2 + I_3 = I$$

$$I_3 = \frac{V_{23}}{R_3} = \frac{I R_{23}}{R_3} = \underbrace{\frac{R_2}{R_2+R_3}}_{<1} I$$

---

2) Reduction to  is not possible (happens where there are two or more batteries in the circuit) → $\underbrace{\text{Use Loop or Node Analysis}}_{\text{Kirchhoff's laws}}$



Observations:

1) R_1 & R_2 are not in parallel (their terminals are not directly connected) neither series (not same current going through both)

Loop Analysis

Total voltage difference across elements in a closed loop is zero

→ Signs: assume a direction for the current flowing thru the closed loop.

1) Current thru battery: − to +: → positive voltage @ battery
2) Current thru battery: + to − → negative voltage @ battery
3) Voltage difference always negative

Node Analysis.

Total current @ any node is 0

Signs:

1) Into node → positive current
2) Leaving node → negative current

visual inspection

Loop Analysis

$\downarrow$

$\rightarrow$ 2 loops in this example: I_1 & I_2 in CW direction

1) $+V_1 - I_1R_1 - V_2 - I_1R_3 = 0$

2) $+V_2 - I_2R_2 = 0$

$$\begin{cases} I_1 = \dfrac{V_1 - V_2}{R_1 + R_3} \\ I_2 = \dfrac{V_2}{R_2} \end{cases}$$

Node Analysis

$\rightarrow$ Visual inspection: 2 nodes @ a & b,

1) $I_1 - I_2 - I_2' = 0$

2) $-I_1 + I_2 + I_2' = 0$ Same as 1)

$\rightarrow$ One independent node!

$$\boxed{\frac{V_A - V_a}{R_1} - \frac{V_a - V_b}{R_2} - I_2' = 0}$$

$$\frac{V_1 - (V_2 + I_1R_3)^{\circledast}}{R_1} - \frac{V_2}{R_2} - I_2' = 0 \checkmark$$

(Ground is zero potential)

$$I_2' = \frac{V_1 - V_2}{R_1 + R_3} - \frac{V_2}{R_2}$$

$$I_1 = \frac{V_1 - (V_2 + I_1R_3)}{R_1} = \boxed{\frac{V_1 - V_2}{R_1 + R_3}}$$

$\circledast$ $V_a = V_2 + I_1R_3$ $(a \to b \to 0)$

$V_b = I_1R_3$

Also $V_a = -I_1R_1 + V_1$ $(a \to A \to 0)$

$V_2 + I_1R_3 = -I_1R_1 + V_1$

$\downarrow$

$I_1(R_1 + R_3) = V_1 - V_2$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

23.35

Car battery: Energy stored : $U = 4 \times 10^6 J$

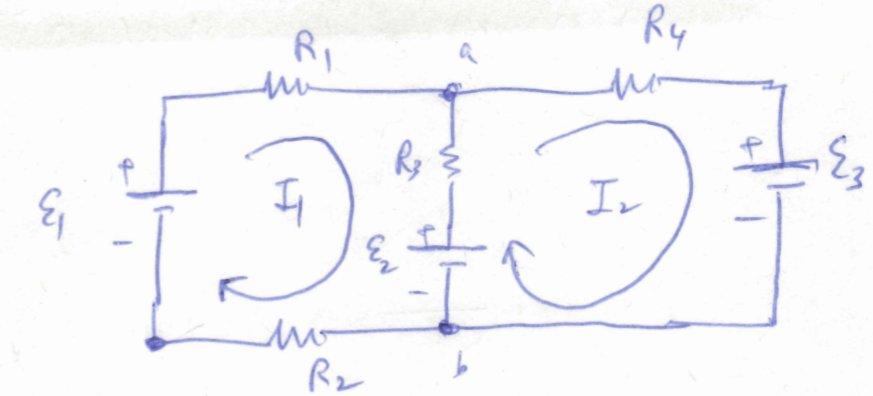
Electric field : $E = 30 \times 10^3 \frac{V}{m} \approx \frac{N}{C}$

<u>Volume</u> ?

We know : $u = \frac{1}{2} \varepsilon_0 E^2 \equiv \frac{U}{Vol} \rightarrow Vol = \frac{U}{u} = \frac{U}{\frac{1}{2} \varepsilon_0 E^2}$

$$Vol = \frac{4 \times 10^6}{\frac{1}{2} 8.85 \times 10^{-12} \times (3 \times 10^4)^2} = 1 \times 10^9 m^3 = 1 (km)^3$$

25.53



$\mathcal{E}_1 = 6V$; $\mathcal{E}_2 = 1.5V$

$\mathcal{E}_3 = 4.5V$

$R_1 = 270\Omega$; $R_2 = 150\Omega$

$R_3 = 560\Omega$; $R_4 = 820\Omega$

Find current in R_3 including direction (up or down).

<u>Loop Analysis</u>: 2 loops (visual inspection): assume direction for I_1 & I_2 → CW

<u>Loop #1</u>: (left) 1) $+\mathcal{E}_1 - I_1R_1 - (I_1 - I_2)R_3 - \mathcal{E}_2 - I_1R_2 = 0$

<u>Loop #2</u>: (Right) 2) $+\mathcal{E}_2 - (I_2 - I_1)R_3 - I_2R_4 - \mathcal{E}_3 = 0$

$$\mathcal{E}_1 - \mathcal{E}_3 - I_1(R_1 + R_2) - I_2R_4 = 0$$

$$\to I_1 = \frac{\mathcal{E}_1 - \mathcal{E}_3 - I_2R_4}{R_1 + R_2} = \frac{1.5 - 820 I_2}{420} \quad (\mathrm{I})$$

(Observation: current through R_3 is $I_1 - I_2$)

Plug (I) in equation 2) $\quad \mathcal{E}_2 - \mathcal{E}_3 - I_2(R_3 + R_4) + I_1R_3 = 0$

$$-3 - 1380 I_2 + 560 I_1 = 0$$

$$\to -3 - 1380 I_2 + \underbrace{\frac{560}{420}(1.5 - 820 I_2)}_{2 - 1093.3 I_2} = 0$$

$$\to -1 - 2473.2 I_2 = 0 \to \boxed{I_2 = -0.4\,mA}$$

down.

$$\to I_1 = \frac{1.5 - 820 \times (-0.4 \times 10^{-3})}{420} = +4.36\,mA$$

→ Current thru $R_3 = \underset{(\text{down})}{I_1} - \underset{\text{down}}{I_2} = 4.36\,mA - (-0.4\,mA) = +4.76\,mA$

<u>downward</u>

N.D. Analysis → same result