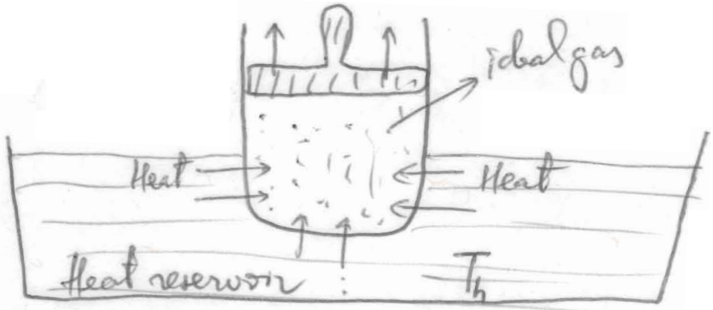
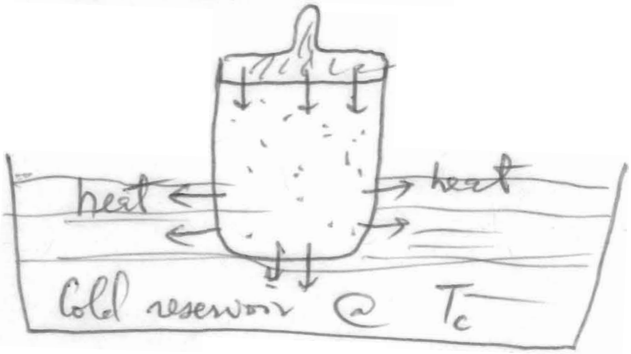
Ch 19 2nd Law of Thermodynamics:

Heat reservoir: source of heat, at constant temperature



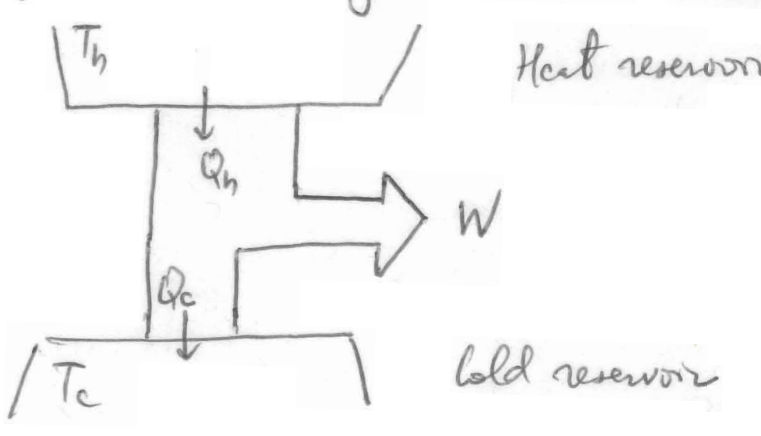
→ Putting an ideal gas in contact with a heat reservoir it will absorb heat → expands (doing work)

The expansion will stop at some point. How to continue getting work from the gas? → Bring the piston back to its original position by putting the gas (heated) in contact with cold reservoir



→ Putting gas in contact with a cold reservoir @ T_c. It loses heat → compresses (receiving work)

Repeating this cycle → ideal gas serves as a heat engine



$$\Delta U_{engine} = Q_{Net} - W = Q_h - Q_c - W$$

1^{st} Law of T.D.

ideal gas as heat engine → isothermal processes @ constant T_h or T_c → $\Delta U_{engine} = 0$

$$\rightarrow \quad Q_h - Q_c = W$$

Efficiency of an ideal gas heat engine: $e = \dfrac{W}{Q_h} = \dfrac{Q_h - Q_c}{Q_h}$

$$\Rightarrow e = 1 - \underbrace{\frac{|Q_c|}{|Q_h|}}_{<1} < 1$$

<1 since $|Q_c| < |Q_h|$ (gas did some work)

<u>2^{nd} Law of T.D.</u>: it is impossible to build a heat engine operating in cycles that extracts heat from a hot reservoir (an returning some of it to a cold reservoir) that can deliver a 100% of work

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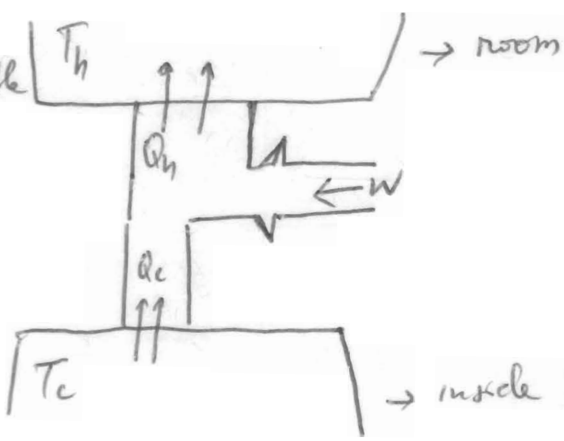
<u>Refrigerators</u>: reversed heat engines

<u>2^{nd} Law T.D</u>: it is impossible to transfer heat from a cold reservoir to a hot reservoir without requiring any work.



Coefficient of Performance

$$C.O.P. = \frac{Q_c}{W}$$

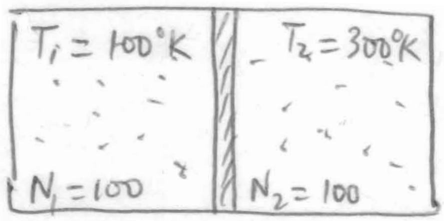
Entropy: $\Delta S \equiv \int_1^2 \frac{dQ}{T}$

→ The entropy of a closed system can never decrease

↳ $\Delta S \geq 0$

→ Entropy ~ level of disorder

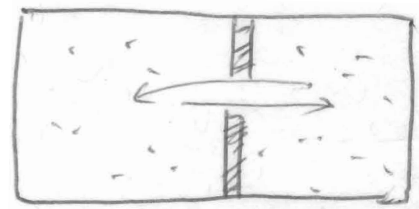
A



$V_1 = V_2$

$P_2 > P_1$

B



mixed together.

spontaneously mix together:
order is decreased → ~~ΔS~~
disorder is increased → ΔS has increased

$PV = nRT \rightarrow P = \frac{nRT}{V}$

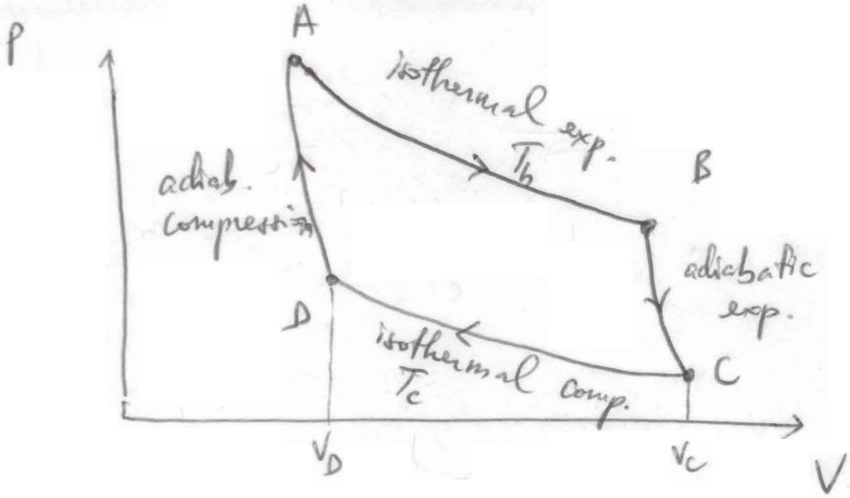
$v_{th} = \sqrt{\frac{3kT}{m}}$: higher speed → higher momentum transfer to walls → higher P on walls.

Natural process : → entropy increases.

Carnot Engines : → heat engines that follow 4 reversible processes (2 isothermal; 2 adiabatic)

↳ $e < 1$. Efficiency achieved w/ Carnot Engine are the max. achievable efficiency

$\boxed{e_{max}}$?



Carnot Engine: cycle
$A \to B \to C \to D \to A$

$$e_{max} = 1 - \frac{|Q_c|}{|Q_h|}$$

Q_h = heat absorbed from hot reservoir during isothermal expansion $A \to B$ $= nRT_h \ln\left(\frac{V_B}{V_A}\right)$

$$\Delta U = 0 = Q - W$$

Q_c = isothermal process $C \to D$ $= nRT_C \ln\left(\frac{V_D}{V_C}\right)$

Connection b/w the volumes:

$B \to C$ adiab. expansion: $T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$

$$\left(\frac{V_B}{V_C}\right)^{\gamma-1} = \frac{T_C}{T_B} = \frac{T_c}{T_h}$$

$D \to A$: adiab. compression: $T_D V_D^{\gamma-1} = T_A V_A^{\gamma-1}$

$$\left(\frac{V_D}{V_A}\right)^{\gamma-1} = \frac{T_A}{T_D} = \frac{T_h}{T_c}$$

$$\frac{V_B}{V_C} = \frac{V_A}{V_D} \to \boxed{\frac{V_B}{V_A} = \frac{V_C}{V_D}}$$

$$\boxed{e_{max} = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{\left| nRT_c \ln\left(\frac{V_D}{V_C}\right)\right|}{\left| nRT_h \ln\left(\frac{V_B}{V_A}\right)\right|}}$$

$$= 1 - \frac{\cancel{nR}T_c \cancel{\ln\left(\frac{V_C}{V_D}\right)}}{\cancel{nR}T_h \cancel{\ln\left(\frac{V_B}{V_A}\right)}} = \boxed{1 - \frac{T_c}{T_h}}$$

make sure your $\ln$ is >0

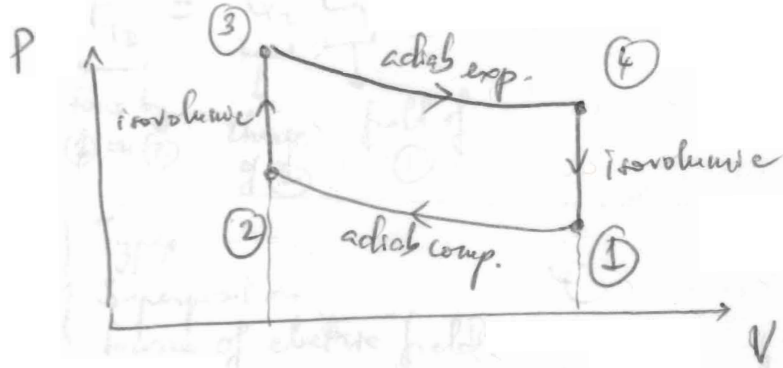
$(nRT > 0)$

its argument should be >1

$$\frac{V_B}{V_A} = \frac{V_C}{V_D}$$

(adiabatic $B \to C$ & $D \to A$)

Otto Cycle Engines: follow 4 reversible processes (2 adiabatic; 2 isovolumic)



$$e_{Otto} < e_{Carnot} = e_{max}$$

$$\Delta S = \int_1^2 \frac{dQ}{T}$$

isothermal: $\Delta S = \frac{1}{T}\int_1^2 dQ = \frac{Q_2 - Q_1}{T} = \frac{\Delta Q}{T}$

isovolumic: $dQ = n c_v \, dT \quad \left(c_v \equiv \frac{1}{n}\frac{dQ}{dT}\right)$

$$\Delta S = \int_1^2 \frac{dQ}{T} = n c_v \int_1^2 \frac{dT}{T} = n c_v \ln\left(\frac{T_2}{T_1}\right)$$

Ch 20: Electric Charge, Force, Field

Charge distribution Q_1 & Q_2

Interaction b/w these two $\boxed{\text{charge}}$ distributions : via the electric $\boxed{\text{fields}}$ $\vec{E}_1$ (created by Q_1) & $\vec{E}_2$ (created by Q_2)

To calculate the electric $\boxed{\text{force}}$ exerted by Q_1 on Q_2 :

$$\underbrace{\vec{F}_{12}}_{\substack{\text{force by}\\ ① \text{ on } ②}} = \underbrace{Q_2}_{\substack{\text{charge}\\ \text{of } ②}} \underbrace{\vec{E}_1}_{\substack{\text{field of}\\ ①}}$$

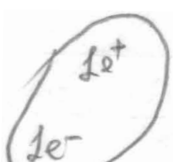
<u>Charges</u>: $\begin{cases} \text{Types} \\ \text{Superposition} \\ \text{Source of electric field.} \end{cases}$

<u>Types of charges</u>: 2 $\begin{cases} + & \text{(a proton has charge } +1e) \\ - & \text{(an electron has charge } -1e) \end{cases}$

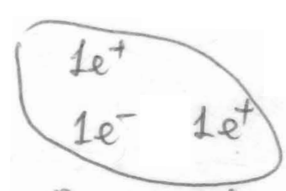
$e = 1.6 \times 10^{-19}\ C$ ↳ Coulomb (SI unit for charge)

↳ $\begin{cases} +1e = +1.6 \times 10^{-19} C \\ -1e = -1.6 \times 10^{-19} C \end{cases}$

<u>Superposition</u>:



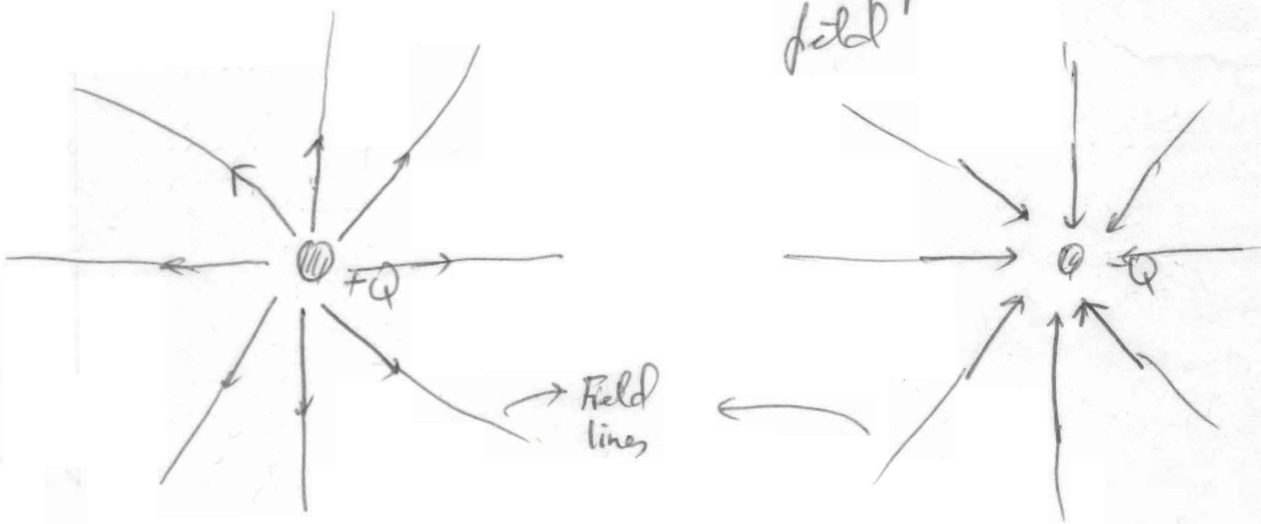
$Q_{net} = 0$



$Q_{net} = 1e^+ = +1.6 \times 10^{-19} C$

Charge as source of electric field:

$$\vec{E} = k \frac{Q}{r^2} \hat{r} \quad \begin{cases} k = & \text{electric constant: } 9 \times 10^9 \frac{Nm^2}{C^2} \\ Q = & \text{net charge} \\ r = & \text{separation from the charge to the field point} \\ \hat{r} = & \text{unit radial vector, points away from the source of the field} \end{cases}$$



Electric field

$$\vec{E} = \frac{kQ}{r^2} \hat{r}$$

can be $\begin{cases} \text{attractive } (-Q) \\ \text{or} \\ \text{repulsive } (+Q) \end{cases}$

$$k = 9 \times 10^9 \frac{Nm^2}{C^2}$$

Gravitational field

$$\vec{g} = -\frac{G\overset{\text{dump.}}{M}}{r^2} \hat{r}$$

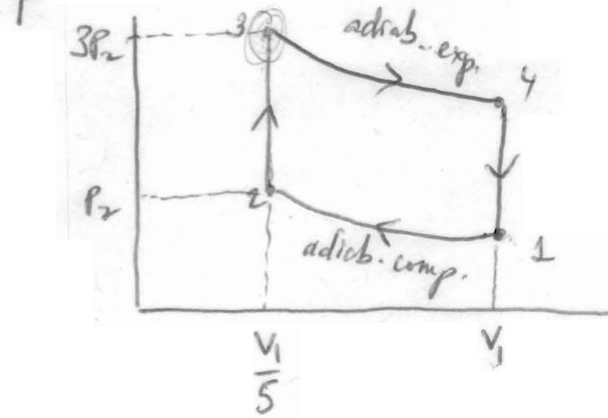
always attractive

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

19.28 ; 19.35

19.54 Gasoline engine on Otto Cycle $\left\{ \begin{array}{l} 2 \text{ adiabatic} \\ 2 \text{ isovolume} \end{array} \right\}$ reversible

a)



V

Find e in term of γ

$$\boxed{\begin{array}{l} P_3 = 3P_2 \\ V_2 = V_3 = \frac{V_1}{5} = \frac{V_4}{5} \end{array}} \rightarrow \frac{V_3}{V_4} = \frac{1}{5}$$

$$V_3 = \frac{V_4}{5}$$

$$e \equiv \left(\frac{Q_h}{W}\right)^{-1} = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{|Q_c|}{|Q_h|} < 1$$

By def: no heat transfer in adiabatic processes $\rightarrow$

$$Q_h = Q_{23} \underset{\text{isovolumic}}{=} nC_V(T_3 - T_2) \quad ; \quad Q_c = Q_{41} \underset{\text{isovolume}}{=} nC_V(T_1 - T_4)$$

$\rightarrow$ We need the temperatures :

<u>Adiabatic processes</u>: $1 \rightarrow 2$: $\left\{ \begin{array}{ll} P_1 V_1^{\gamma} = P_2 V_2^{\gamma} & (1) \\ T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} & (2) \end{array} \right.$

$3 \rightarrow 4$: $\left\{ \begin{array}{ll} T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} & (3) \\ P_3 V_3^{\gamma} = P_4 V_4^{\gamma} & (4) \end{array} \right.$

$$e = 1 - \frac{|T_1 - T_4|}{|T_3 - T_2|}$$

$$= 1 - \frac{\left|T_4\left(\frac{T_1}{T_4} - 1\right)\right|}{\left|T_3\left(1 - \frac{T_2}{T_3}\right)\right|}$$

$$= 1 - \frac{T_4}{T_3} \frac{\cancel{\left|\frac{T_1}{T_4} - 1\right|}}{\cancel{\left|1 - \frac{T_2}{T_3}\right|}}$$

$$\left\{ \begin{array}{ll} T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} & (2) \\ T_4 V_4^{\gamma-1} = T_3 V_3^{\gamma-1} & (3) \end{array} \right.$$

$$\rightarrow \boxed{\frac{T_1}{T_4} = \frac{T_2}{T_3}} \rightarrow$$

$$\left\{ \begin{array}{ll} P_1 V_1^{\gamma} = P_2 V_2^{\gamma} & (1) \\ P_4 V_4^{\gamma} = P_3 V_3^{\gamma} & (4) \end{array} \right.$$

$$\boxed{\frac{P_1}{P_4} = \frac{P_2}{P_3} = \frac{1}{3}}$$

$$\boxed{e = 1 - \frac{T_4}{T_3} = 1 - 5^{1-\gamma}}$$

(3) $\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{5}\right)^{\gamma-1} = 5^{1-\gamma}$

data given.

~~(4) $\left(\frac{V_3}{V_4}\right)^{\gamma} = \frac{P_4}{P_3} \rightarrow$~~

b) Find T_{max} in term of T_{min}

$$PV = nRT \quad \begin{cases} (1) \\ (2) \\ (3) \quad P_3 V_3 = 3P_2 V_2 \rightarrow \boxed{T_3 = 3T_2} \\ (4) \quad P_4 V_4 = \end{cases}$$

$T_{max} = T_3 \rightarrow T_{min} = T_1$

(3) Adiab. $3 \rightarrow 4$: $T_3 = T_4 \left(\frac{V_4}{V_3}\right)^{\gamma-1} = T_4\, 5^{\gamma-1}$

Data : $V_3 = \frac{V_4}{5} \rightarrow \frac{V_4}{V_3} = 5$

From a) $\frac{T_1}{T_4} = \frac{T_2}{T_3} = \frac{1}{3} \rightarrow \boxed{T_3 = T_4\, 5^{\gamma-1} = 3 \times 5^{\gamma-1}\, T_1}$

$\hookrightarrow T_4 = 3T_1$

c) Carnot engine b/w $T_h = T_3$ & $T_c = T_1$:

$$e_{Carnot} = e_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_1}{T_3} = 1 - \frac{1}{3 \times 5^{\gamma-1}}$$

$$\boxed{e_{otto} = 1 - 5^{1-\gamma}} \qquad \boxed{e_{max} = 1 - \frac{5^{1-\gamma}}{3}}$$

($e_{otto} < e_{max}$ → recall: max. efficiency is that of a Carnot

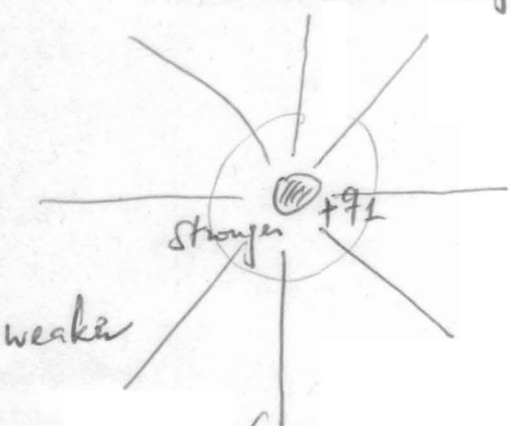
Ch 20 (cont.)

How to calculate the electric field: (can calculate forces if we know the field)

Due to one charge: q_1:

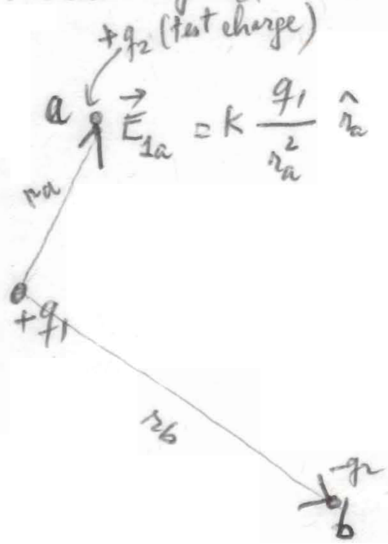
$$\vec{E}_1 = k\frac{q_1}{r^2}\hat{r}$$

$\hat{r}$: unit vector, in radial directions away from charge
r: separation from the source q_1



→ Electric field lines: used to show direction & $\boxed{\text{strength}}$ of the electric field)

density of lines. higher density → stronger field.



$$\vec{E}_{1a} = k\frac{q_1}{r_a^2}\hat{r}_a$$

$$\vec{E}_{1b} = k\frac{q_1}{r_b^2}\hat{r}_b$$

→ If we place a test charge at a → electric force by q_1 on the test charge $\vec{F}_1 = q_2\vec{E}_1$ → $\vec{F}_{1a} = q_2\vec{E}_{1a} = k\dfrac{q_1 q_2}{r_a^2}\hat{r}_a$

since $q_1 > 0$ & $q_2 > 0$ → direction given by $\hat{r}_a$ → repulsive force.

→ A negative test charge at b feels an attractive force toward $+q_1$: $\vec{F}_{1b} = -q_2\vec{E}_{1b} = -k\dfrac{q_1 q_2}{r_b^2}\hat{r}_b$

Summary: electric forces b/w two charges of same types are repulsive; b/w charges of opposite types are attractive

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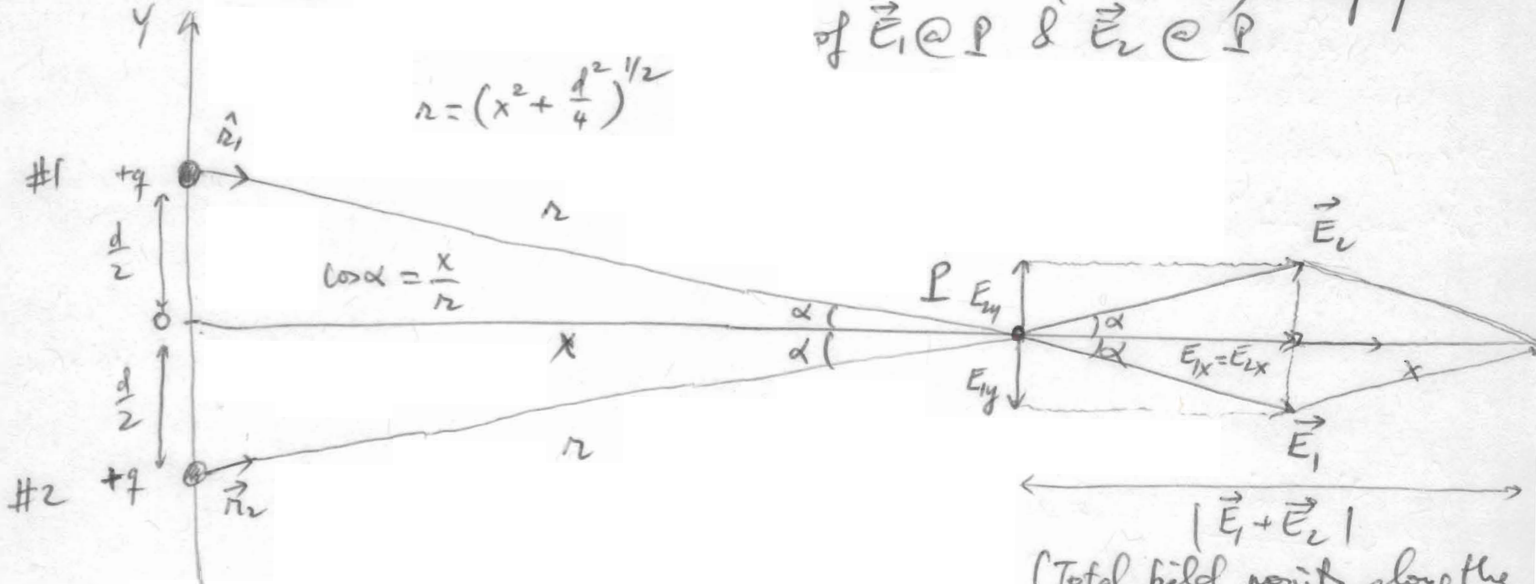
Due to 2 positive charges: Total electric field $\vec{E}$ @ P along the x-axis (mid-line) → superposition of $\vec{E}_1$@P & $\vec{E}_2$ @ P



(Total field points along the x-axis)

$$E_{1x} = E_1 \cos\alpha$$

$$E_{2x} = E_2 \cos\alpha = E_1 \cos\alpha$$

$\left(E_1 = \frac{kq}{r^2} = E_2 = \frac{kq}{r^2} \right.$: lengths, not as vectors)

$$\vec{E}_1 = \left(k \frac{q}{r^2}\right) \hat{r}_1 = E_{1x}\,\hat{i} + E_{1y}\,\hat{j} = E_1 \cos\alpha\,\hat{i} - E_1 \sin\alpha\,\hat{j}$$

$$\vec{E}_2 = \left(k \frac{q}{r^2}\right) \hat{r}_2 = E_{2x}\,\hat{i} + E_{2y}\,\hat{j} = E_1 \cos\alpha\,\hat{i} + E_1 \sin\alpha\,\hat{j}$$

$$\rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2 = 2E_1 \cos\alpha\,\hat{i}$$

$$\vec{E} = \frac{2kq}{r^2} \frac{x}{r}\,\hat{i} = \frac{2kq\,x}{r^3}\,\hat{i} = \frac{2kq\,x}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}\,\hat{i}$$

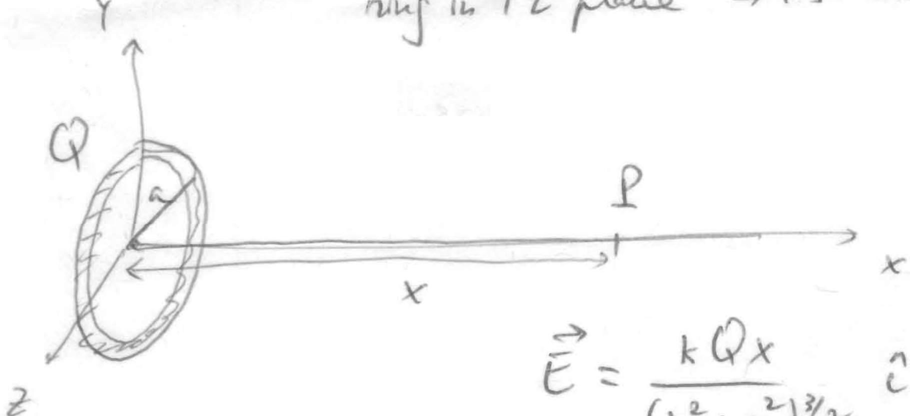
Unit: $\frac{N}{C}$ for electric field in SI.

$$\vec{F} = q\vec{E}$$

N $\quad$ C

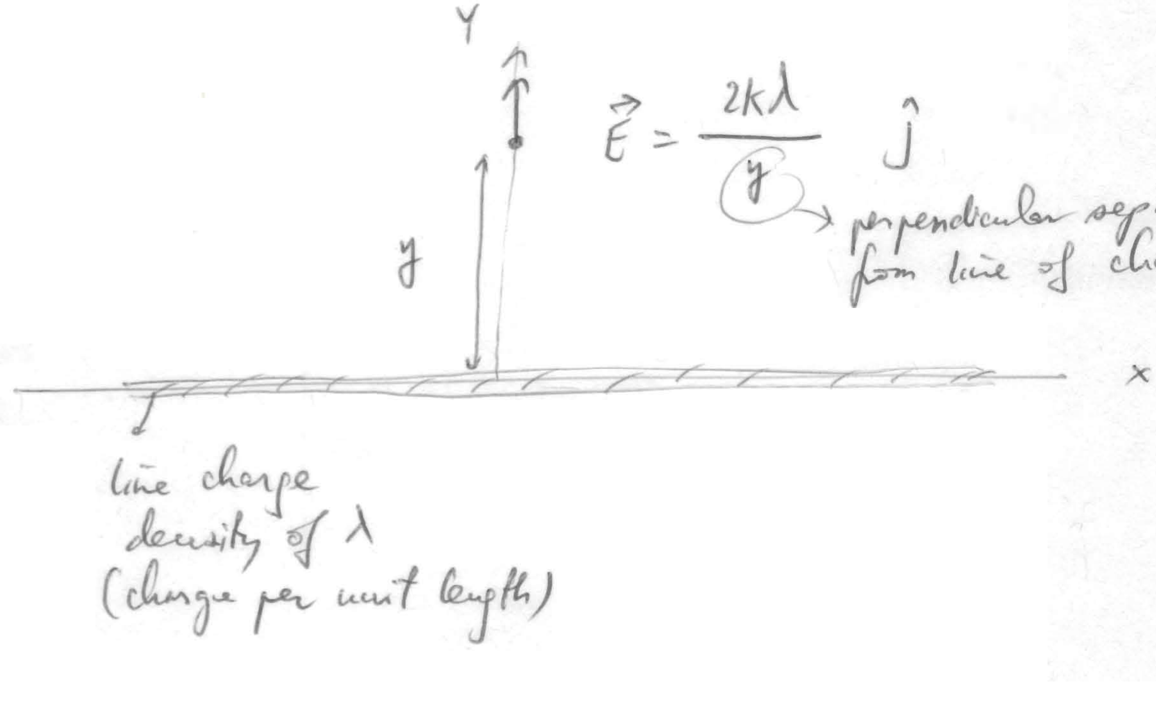
Due to a continuous ring of charge @ a point along its axis.

ring in YZ plane → its axis = x-axis



$$\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{i}$$

Due to an ∞ long line of charge along x-axis:

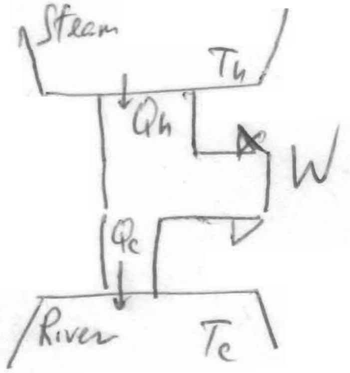


$$\vec{E} = \frac{2k\lambda}{y}\,\hat{j}$$

y → perpendicular separation from line of charge.

line charge density of λ (charge per unit length)

19.28 Power plant → Heat engine $T_h = 250\ °C$ $T_c = 30\ °C$ delivers Power $= \frac{W}{t} = 800\,MW$



$e = 0.28$

a) $e_{max} = e_{Carnot} = 1 - \frac{T_c}{T_h}$

$= 1 - \frac{303.16\,°K}{523.16\,°K} = 0.42$

b) Rate waste heat to River? → $\frac{Q_c}{time}$

$\Delta U = 0$ → $Q_h - Q_c = W$
↓
Heat exchange via isothermal process

$e \equiv \frac{W}{Q_h}$

$Q_c = Q_h - W$
$= \frac{W}{e} - W$
$= W\left(\frac{1}{e} - 1\right)$

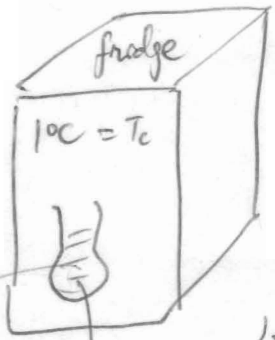
$$\rightarrow \frac{Q_c}{time} = \frac{W}{time} \underbrace{\left(\frac{1}{e} - 1\right)}_{\text{no dimension (no unit)}} = 800\,MW \left(\frac{1}{0.28} - 1\right) = 2057\,MW$$

c) $\frac{18\,kW}{house}$ → # houses could be heated with this waste heat

$$\frac{2057 \times 10^3\ kW}{\frac{18\ kW}{house}} = \frac{2057 \times 10^3}{18}\ houses = 114\,000\ houses.$$

19.35

room:
$T_h = 25°C$



H_2O ←
4L → 4kg
$\rho_w = 1 kg/L$

↓ T = 9°C

- 130W motor
- Takes 4 min (240 s) to cool H_2O from 9°C to 1°C

a) $COP = \dfrac{Q_c}{W}$

$$W = 130 \frac{J}{s} \times 240 s = 31200 J$$

$$Q_c = m_w c_w \Delta T = 4 \times 4184 \times 8°K = 134000 J$$

$$\rightarrow COP = \frac{134000}{31200} = 4.29$$

b) $$COP_{max} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{T_c}{T_h - T_c} = \frac{274.16°K}{24°K} = 11$$

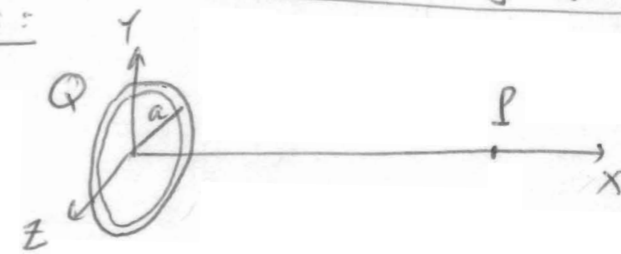
Reversed Carnot Engine

How to calculate the Electric field?

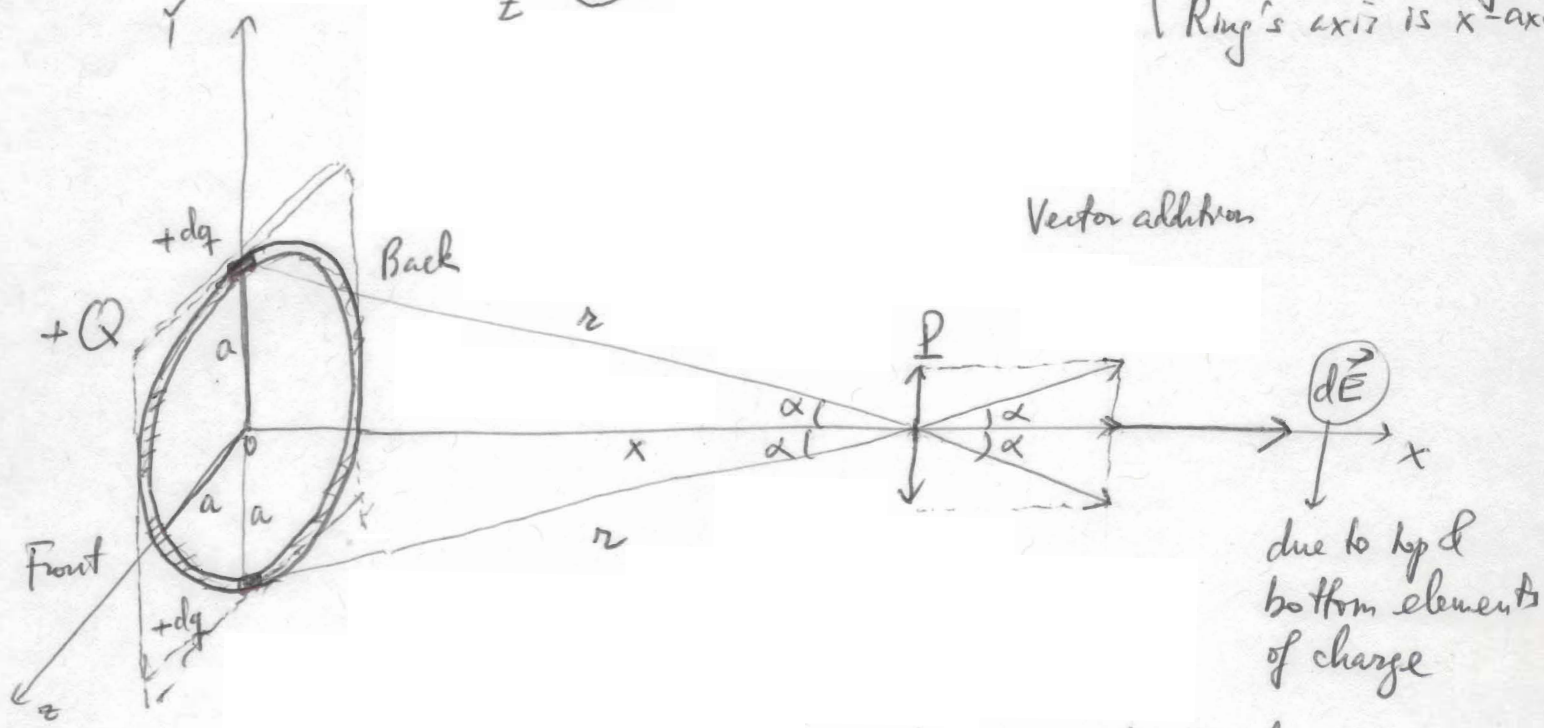
1) Vector addition (Ch. 20)
2) Gauss Law (Ch. 21) ↳ using symmetry
3) Electric Potential (Ch. 22) ↳ using scalars & derivative

Example of Method #1:

Electric field due to a continuous ring of charge, at a point along its axis:



Ring in YZ plane; origin of coordinates at center of ring. Ring's axis is x-axis



↳ Using results for field due to 2 positive charges:

$$d\vec{E} = \frac{2k\,dq\,x}{(x^2+a^2)^{3/2}}\,\hat{\imath}$$

For total field due to whole ring: $\vec{E} = \int_{\text{half ring}} d\vec{E} = \frac{\cancel{2}kx}{(x^2+a^2)^{3/2}}\,\hat{i}\,\underbrace{\int_{\text{half ring}} dq}_{Q/\cancel{2}}$

$$\boxed{\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{i}}$$

Observations: approximation if P is far from the ring: $x \gg a$

$\rightarrow x^2+a^2 \simeq x^2 \rightarrow \boxed{\vec{E}_{far} = \underbrace{\frac{kQ}{x^2}}_{\text{due to a "point" charge of value } Q}\,\hat{i}}$

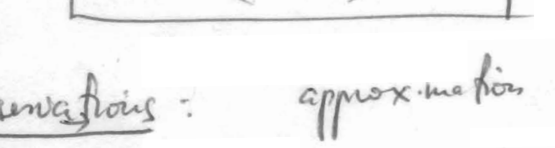
this makes sense

Electric field due to a long (or ∞) line of charge:

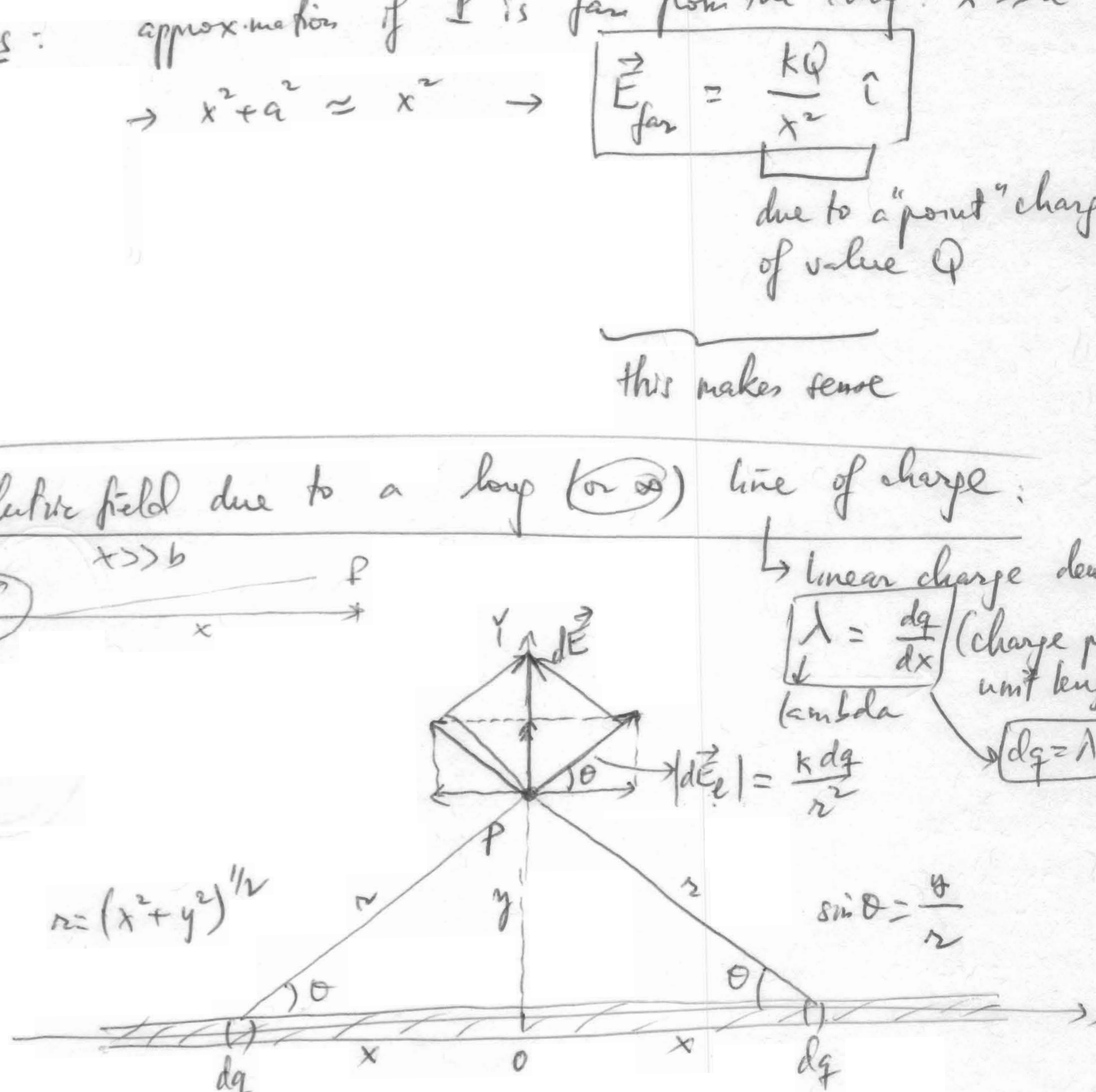
↳ linear charge density $\lambda = \frac{dq}{dx}$ (charge per unit length)

λ: lambda → $\boxed{dq = \lambda dx}$

$x = 10m$
$b = 0.1m$
$x \gg b$



$b = 1m$
$b = 5m$



$d\vec{E} = 2\,dE_\ell \sin\theta\,\hat{j}$ (vector addition)

$= 2\frac{k\,dq}{r^2}\frac{y}{r}\hat{j} = 2ky\frac{\lambda dx}{r^3}\hat{j} \rightarrow \vec{E}_{\text{whole line}} = \int_{\text{half line}} d\vec{E} = 2ky\lambda\hat{j}\int_{\text{half}} \frac{dx}{(x^2+y^2)^{3/2}}$

From table: $$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

$$\rightarrow \boxed{\vec{E}_{\infty\,\text{line}} = 2Ky\lambda\,\hat{j}\,\underbrace{\left[\frac{x}{y^2(x^2+y^2)^{1/2}}\right]_{x=0}^{x=\infty}}_{\left[\frac{1}{y^2}-0\right]} = \frac{2k\lambda}{y}\,\hat{j}}$$

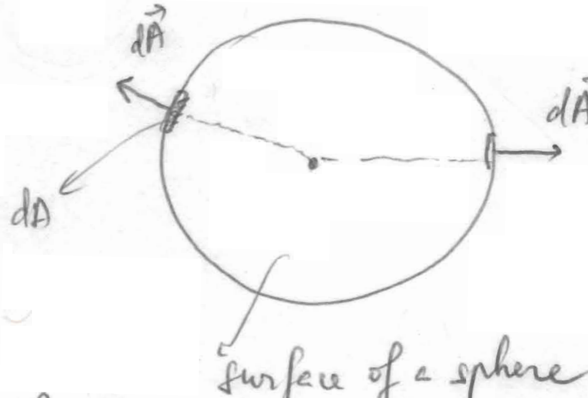
Method #2: Gauss law → Ch. 21

Electric flux: $$\boxed{\underset{\text{"Phi"}}{\phi} \equiv \oint_{\text{closed surface}} \vec{E} \cdot d\vec{A}}$$

Scalar product b/w two vectors
$\vec{A} \cdot \vec{B} = AB\cos\theta$

A: $|\vec{A}|$, B: $|\vec{B}|$, θ: angle b/w $\vec{A}$ & $\vec{B}$

$d\vec{A}$: element of area vector on the closed surface.

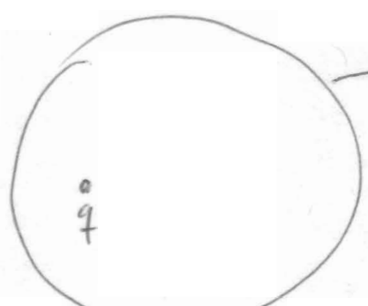


* ϕ can be calculated easily for simple surfaces: sphere, cylinder, rectangular box, etc.

Direction is given by a unit vector perpendicular to the area, away from the surface

Gauss Law: $\boxed{\Phi_{\substack{\text{closed}\\ \text{surface}}} = \dfrac{q_{\text{enclosed by that surface}}}{\varepsilon_0}}$

ε_0: dielectric constant in vacuum $= \dfrac{1}{4\pi k} = 8.85 \times 10^{-12} \dfrac{C^2}{Nm^2}$

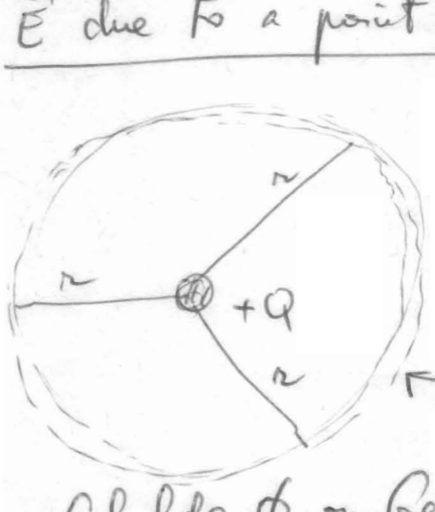


$\rightarrow \Phi = \dfrac{q}{\varepsilon_0}$

* How to calculate $\vec{E}$ if we know the flux Φ?
If we can write the flux Φ in term of $\vec{E}$ $\rightarrow$ can get E from Φ. $\rightarrow$ simple geometries: sphere, cylinder, etc...

Example for Method #2;

1) $\vec{E}$ due to a point charge: spherical symmetry: all points @ a separation r from Q will feel same field.



Gauss Law: $\rightarrow$ determine the $\boxed{\text{Gaussian surface}}$ $\rightarrow$ spherical symmetry $\rightarrow$ spherical surface centered at charge +Q

$\rightarrow$ Calculate Φ on Gaussian surface:

$\Phi = \oint \vec{E} \cdot d\vec{A} = \int \frac{kQ}{r^2}\hat{r} \cdot dA\,\hat{r} = \frac{kQ}{r^2} \underbrace{\oint dA}_{\text{surface of sphere of radius } r \;\; 4\pi r^2} = \frac{kQ 4\pi \cancel{r^2}}{\cancel{r^2}}$

$\hat{r} \cdot \hat{r} = 1.1 \cos(0^\circ) = 1$

in summary $\left\{ \begin{array}{l} \phi = 4\pi k Q \\ \\ \underset{\text{Gauss law}}{\phi = \dfrac{Q}{\varepsilon_0}} \end{array} \right\} \rightarrow 4\pi k \cancel{Q} = \dfrac{\cancel{Q}}{\varepsilon_0}$

$$\boxed{\varepsilon_0 = \frac{1}{4\pi k}}$$

or $k = \dfrac{1}{4\pi\varepsilon_0}$

$\vec{E}$ due to a point charge using Gauss law:

$$\vec{E} = E\hat{r}$$

Gauss law

$$\phi = \oint \vec{E} \cdot d\vec{A} = \oint E\hat{r} \cdot dA\,\hat{r} = \oint E\,dA = E\underbrace{\oint dA}_{4\pi r^2}$$

Gaussian spherical surface. E @ same r is constant.

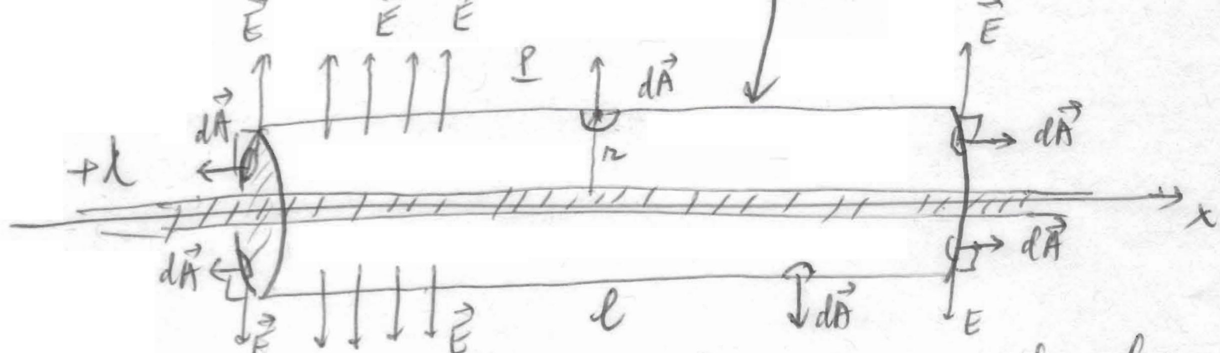
$$\phi = \frac{Q}{\varepsilon_0}$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_0} \rightarrow \boxed{E = \frac{Q}{4\pi\varepsilon_0\, r^2} = \frac{kQ}{r^2}}$$

---

2) $\vec{E}$ due to a ∞ line of charge (charge density λ) using Gauss law → symmetry: cylindrical ↳ Gaussian surface

↳ $\lambda = \dfrac{dq}{dx}$



Gaussian surface: cylinder of length l. radius = sep. from line of charge to point we want to calculate E : r

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \oint_{Body} E\,dA = E \underbrace{\int_{Body} dA}_{2\pi r l} = E 2\pi r l$$

→ no contribution from left & right sides of the Gaussian cylinder.
→ for body of cylinder $\vec{E} \parallel d\vec{A}$

E is fixed on body of cylinder (same sep r to line of charge)

$$\Phi = \frac{\text{charge enclosed by Gaussian surface}}{\varepsilon_0}$$

$$= \frac{\lambda l}{\varepsilon_0}$$

$$E 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0} \rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}}$$

$\varepsilon_0 = \frac{1}{4\pi k}$

same result as with using vector addition.

Method #3: Electric Potential (Ch. 22)

Potential energy difference b/w points A & B

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}$$

Electrical interaction: $\vec{F} = q'\vec{E}$

q': Test charge (not the source field)

Electric potential energy difference b/w A & B?

$$\Delta U_{AB} = -q' \int_A^B \vec{E} \cdot d\vec{l}$$

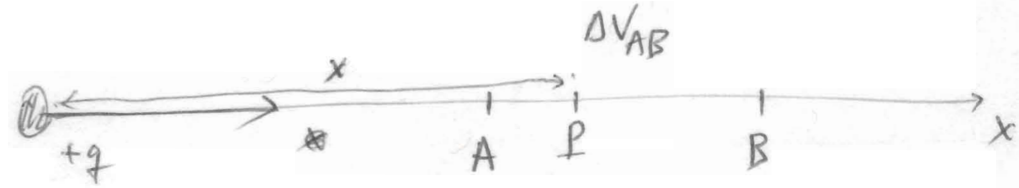
Electric potential (not energy!) difference b/w A & B =

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q'} = -\int_A^B \underbrace{\vec{E}}_{\text{electric field.}} \cdot \vec{dl}$$

unit: SI $= \frac{J}{C} = V$ for Volt

Example # 1: Electric potential difference due to a point charge

~~Field due to a point charge using electric potential:~~



$$\Delta V_{AB} = -\int_A^B \frac{kq}{x^2}\hat{\imath} \cdot dx\,\hat{\imath} = -kq\underbrace{\int_A^B \frac{dx}{x^2}}_{\left[-\frac{1}{x}\right]_A^B} = kq\left(\frac{1}{x_B} - \frac{1}{x_A}\right)$$

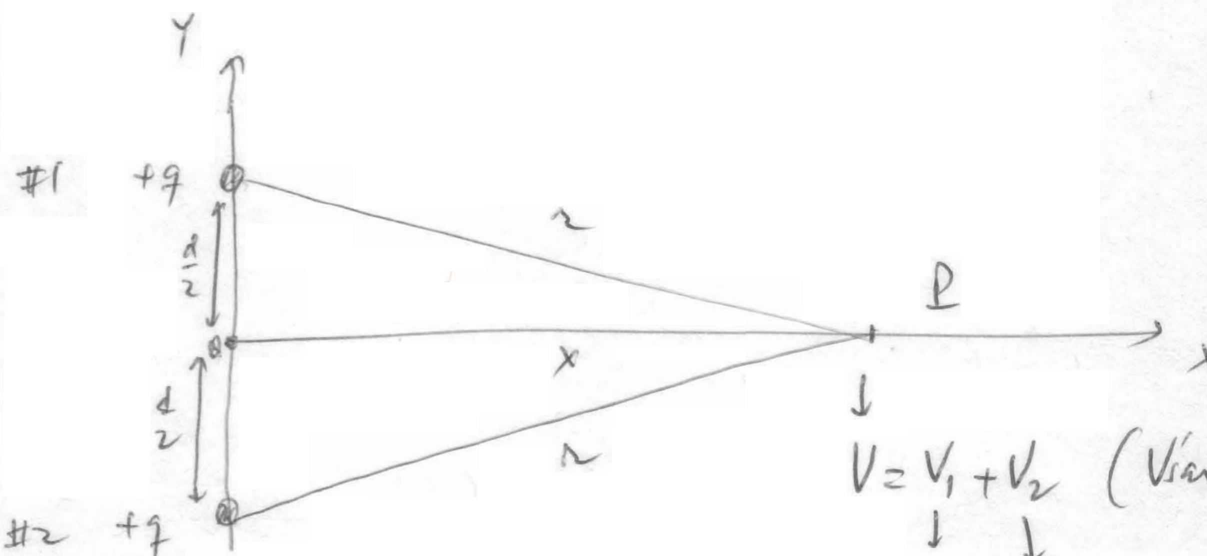
$\hat{\imath}\cdot\hat{\imath} = 1$

Ref point for electric potential: $A \to \infty$:

$$\Delta V_{\infty B} = V_B = \frac{kq}{x_B} \text{ or } \frac{kq}{r_B}$$

$$\boxed{V(r) = \frac{kq}{r}}$$ → V is a scalar, not a vector.

Electric potential due 2 point charges at @ P:



$V = V_1 + V_2$ (V's are scalars!)

$V_1 \to \frac{kq}{r}$, $V_2 \to \frac{kq}{r}$

$$V = \frac{2kq}{r} = \frac{2kq}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}} = V(x)$$

$$V = -\int \vec{E} \cdot d\vec{l} \Rightarrow \vec{E} = -\vec{\nabla} V$$

$\vec{\nabla}$ → gradient (derivative vector)

$$\vec{\nabla} \equiv \left(\frac{d}{dx} \hat{i} + \frac{d}{dy} \hat{j} + \frac{d}{dz} \hat{k} \right)$$

$$\vec{E} = -\vec{\nabla} V = (-)\left(\frac{dV}{dx} \hat{i} + \underbrace{\frac{dV}{dy}}_{0} \hat{j} + \underbrace{\frac{dV}{dz}}_{0} \hat{k} \right)$$

$$= -2kq \frac{d}{dx}\left(\frac{1}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}} \right) = -2kq \left(-\frac{1}{\cancel{2}}\right) \cancel{2} \frac{x}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

$$\boxed{\vec{E} = \frac{2kqx}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}} \hat{i}}$$

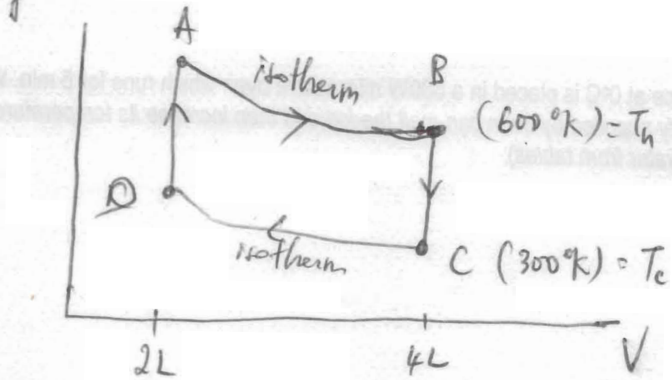
same as with vector addition.

19.40 ; 19.46

20.50 ; 20.78 ; 21.56 ; 22.67

19.40

ideal monoatomic gas @ 600 K V = 2L n = 0.2



a) Net heat absorbed during $A \to B \to C \to D \to A$ & W.

$$\boxed{Q_{AB} = W_{AB}} = nRT_h \ln \frac{V_B}{V_A} = 0.2 \times 8.314 \times 600 \ln 2 \ \text{J} = +691.5 \ \text{J}$$

(Heat absorbed $A \to B$)

$W_{BC} = 0 \to Q_{BC}$ cooling $= nC_v \Delta T = n \frac{3}{2} R (T_C - T_B) = 0.2 \times \frac{3}{2} \times 8.314 \times (-300) = -748.3$ J

$C_v = \frac{3}{2} R$

(Heat ejected $B \to C$)

$$\boxed{Q_{CD} = W_{CD}} = nRT_c \ln \frac{V_D}{V_C} = -0.2 \times 8.314 \times 300 \ln 2 = -345.8 \ \text{J}$$

$\ln\left(\frac{1}{2}\right) = -\ln 2$

(Heat ejected $C \to D$)

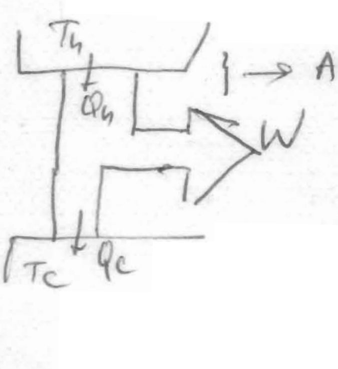
$W_{DA} = 0 \to Q_{DA}$ heating $= nC_v \Delta T = n \frac{3}{2} R (T_A - T_D) = 0.2 \times \frac{3}{2} \times 8.314 \times 300 = +748.3$ J

(Heat absorbed)

$$\to Q_{net\ ABCDA} = 691.5 - 345.8 = 345.7 \ \text{J}$$

$$W_{ABCDA} = Q_{ABCDA} = 345.7 \ \text{J}.$$

b) $e = \frac{W}{Q_{absorbed}} = \frac{345.7 \ \text{J}}{(691.5 + 748.3) \ \text{J}} = \frac{345.7}{1440} = 0.24 \to \boxed{24\%}$



$\} \to A \to B$ & $D \to A \to Q_{absorb} = 1440$ J

$\to e = \frac{W}{Q_{absorb}}.$

48

(49)

(19.46) $n = 5$ ideal diatomic ($\gamma = 1.67$) $P = 1\,\text{Atm}$

$C_v = \frac{5}{2}R$ $T_i = 300\,°K$

a) ΔS if $T_f = 500\,K$ @ constant volume $dQ = nc_v\,dT$

$$\hookrightarrow \Delta S = \int_i^f \frac{dQ}{T} = nc_v \int_i^f \frac{dT}{T} = nc_v \ln\left(\frac{T_f}{T_i}\right) = 5 \times \frac{5}{2} \times 8.314 \times \ln\frac{5}{3}\ \frac{J}{°K}$$

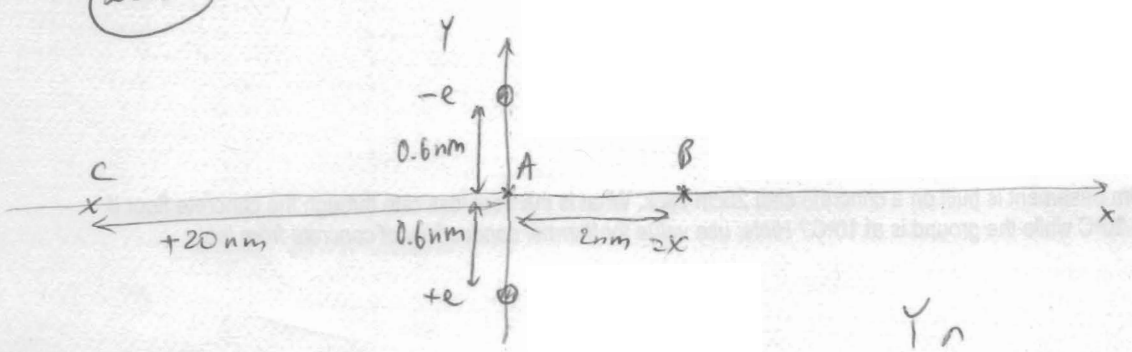
$$= 53.1\ J/°K$$

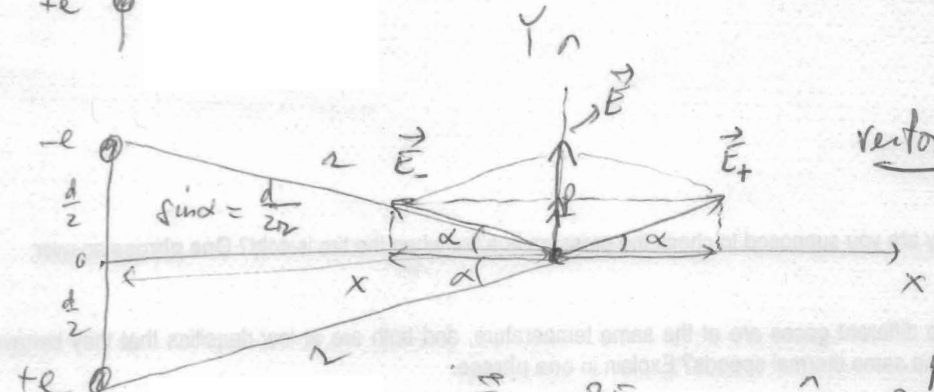
b) @ constant P: $\rightarrow \Delta S = nC_p \ln\left(\frac{T_f}{T_i}\right) = 5 \times \frac{7}{2} \times 8.314 \times \ln\frac{5}{3} = 74.3\ \frac{J}{°K}$

$C_p \hookrightarrow \frac{7}{2}R$.

c) adiabatic, $dQ = 0 \rightarrow \Delta S = 0$

(20.50)





vector addition

$$\vec{E} = 2E_{+y}\,\hat{j} = 2\underbrace{\frac{ke}{r^2}}_{E_+}\sin\alpha\,\hat{j}$$

$$\underbrace{\qquad\qquad}_{E_{+y}}$$

$$\vec{E} = \cancel{2}\frac{ke}{r^2}\frac{d}{\cancel{2}r}\,\hat{j} = \frac{kde}{r^3}\,\hat{j}$$

a) $\vec{E}_A = \frac{8kde}{d^3}\,\hat{j} = \frac{8 \times 9\times10^9 \times \cancel{1.2\times10^{-9}} \times 1.6\times10^{-19}}{(1.2\times10^{-9})^{\cancel{3}\,2}}\,\hat{j}\ \left(\frac{N}{C}\right) = 8\times10^9\ \frac{N}{C}\,\hat{j}$

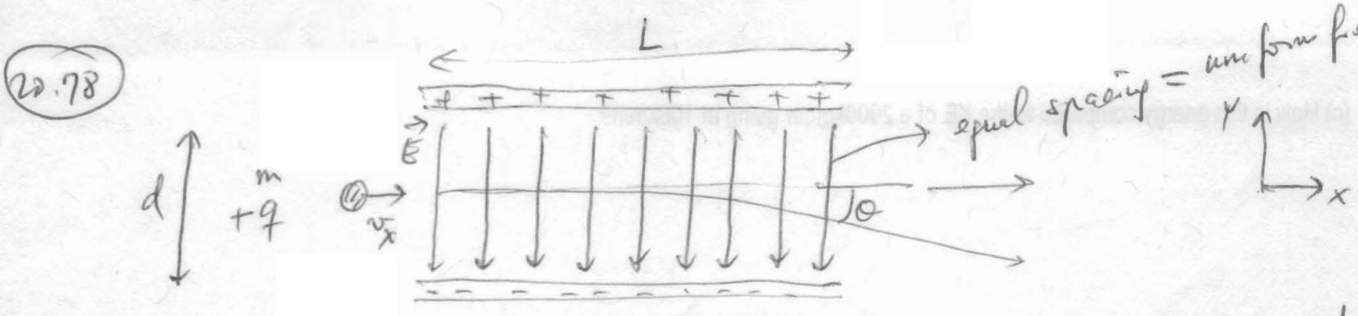
$r = \frac{d}{2}$

b) $\vec{E}_B = \frac{kde}{(x^2 + \frac{d^2}{4})^{3/2}}\,\hat{j} = \frac{9\times10^9 \times 1.2\times10^{-9} \times 1.6\times10^{-19}}{\left[(2\times10^{-9})^2 + (0.6\times10^{-9})^2\right]^{3/2}}\,\hat{j} = 1.9\times10^8\ \frac{N}{C}\,\hat{j}$

c) $\vec{E}_C = 2.16\times10^5\ \frac{N}{C}\,\hat{j}$

(50)

20.78



$v_{x\,min}$ = so drop makes thru L without going more than $\frac{d}{2}$

in the vertical direction.

→ time to spent b/w plates should be such that

$$y = \frac{1}{2} a_y t^2 = \frac{1}{2}\frac{F_y}{m} t^2 = \frac{1}{2}\frac{qE}{m} t^2 \quad \left(y < \frac{d}{2}\right)$$

Newton's law

$F_y = qE$

$$\frac{F_e}{F_g} = 10^{40}$$

$$t = \sqrt{\frac{2my}{qE}} \underset{y = \frac{d}{2}}{=} \sqrt{\frac{md}{qE}} \rightarrow t < \sqrt{\frac{md}{qE}}.$$

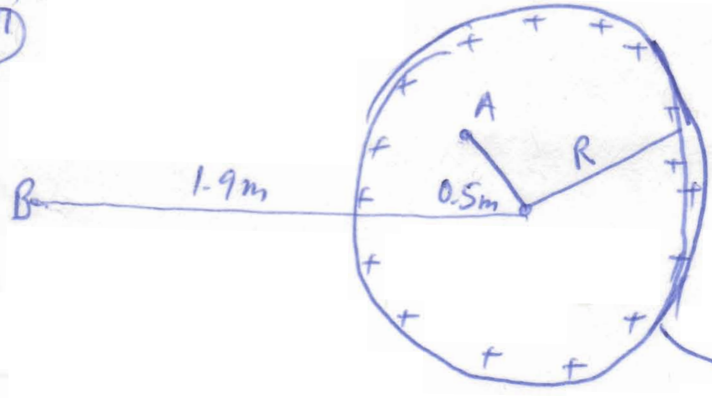
$$\text{Also} \quad \frac{L}{v_x} = t \rightarrow \frac{L}{v_x} < \sqrt{\frac{md}{qE}} \rightarrow \boxed{v_x > L\sqrt{\frac{qE}{md}}}$$

$$\boxed{v_{x\,min} = L\sqrt{\frac{qE}{md}}}$$

$$L\sqrt{\frac{qE}{md}} < v_x$$

21.47; 21.56; 21.70; 19.42; 22.31
22.53; 22-67; 22.70

21.47



$R = 0.7m$

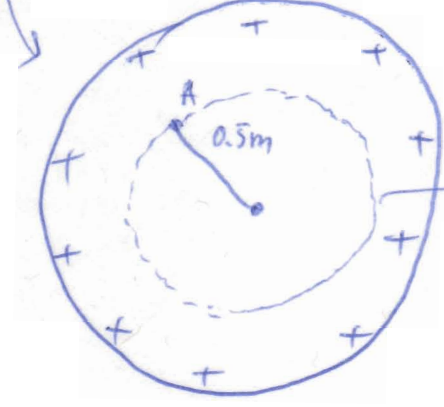
$E = 26\frac{kN}{C}$ @ surface.

a) E_A? b) $\vec{E}_B$? c) $Q_{balloon}$

Application of Gauss Law:

1) Determine Gaussian surface based on symmetry
2) $\Phi = \int \vec{E} \cdot d\vec{A}$ on that surface
3) Compare this w/ $\Phi = \frac{q_{enclosed}}{\varepsilon_0}$

a)



1) spherical Gaussian surface.

2) Spherical symmetry: $\vec{E} \parallel d\vec{A}$

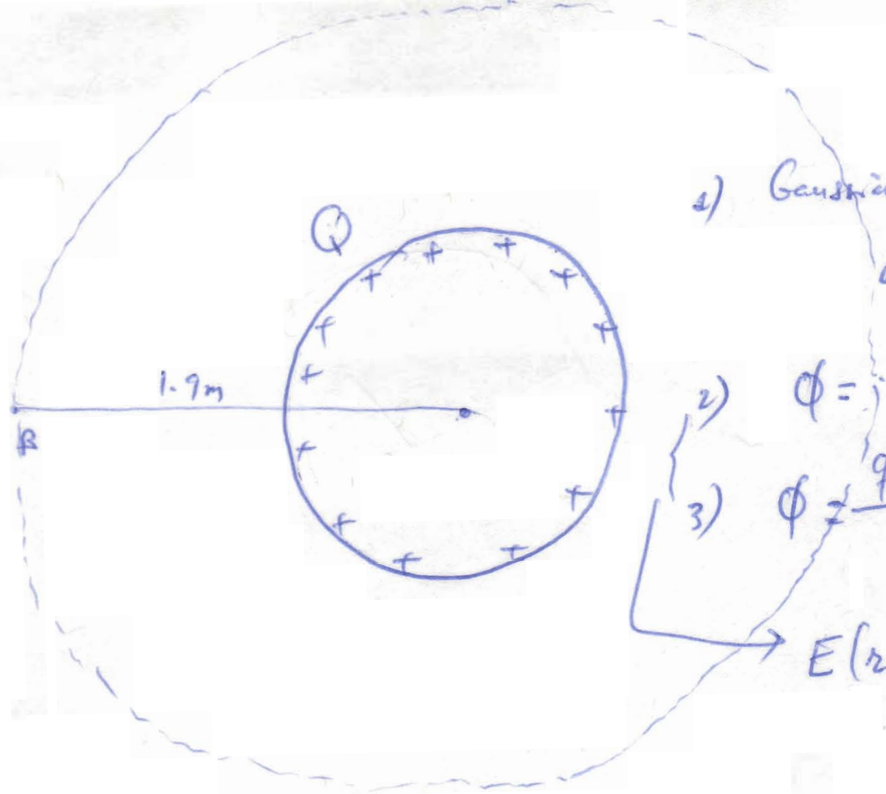
$\rightarrow \vec{E} \cdot d\vec{A} = E\,dA$ being E constant at same r

$\Phi = E \underbrace{\oint dA}_{\text{Area of Gaussian surface}} = E \cdot 4\pi r^2$ $(r = 0.5m)$

3) $\Phi = \frac{0}{\varepsilon_0}$

$\Big\} E_A = 0$

b)



1) Gaussian surface.

2) $\Phi = E\, 4\pi r^2 \quad (r = 1.9\,m)$

3) $\Phi = \dfrac{q_{enclosed}}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0}$

$$E(r \geq 0.7m) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\boxed{= K \frac{Q}{r^2}}$$

Outside, balloon behaves like a point charge in term of the field its creates.

$$E_R(r = 0.7m) = \frac{k\,Q}{0.7^2}$$

$$E_B(r = 1.9m) = \frac{k\,Q}{1.9^2}$$

$$\frac{E_B}{E_R} = \frac{0.7^2}{1.9^2}$$

$$E_B = E_R \frac{0.7^2}{1.9^2}$$

$$= 26\frac{kN}{C}\,\frac{0.7^2}{1.9^2} = \boxed{3.53\frac{kN}{C}}$$

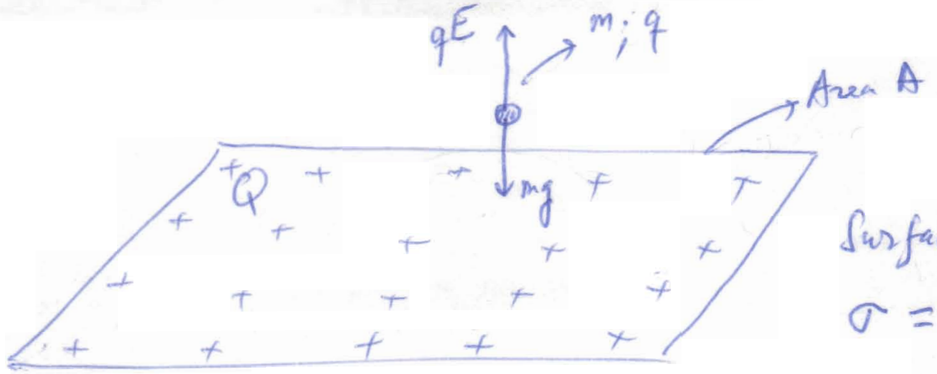
c) $E(r = 0.7m) = E_R = \dfrac{kQ}{R^2} \rightarrow Q = \dfrac{E_R R^2}{k} = \dfrac{26000 \times 0.7^2}{9 \times 10^9}\,C$

$$= 1.42 \times 10^{-6}\,C$$

$$= 1.42\,\mu C$$

micro

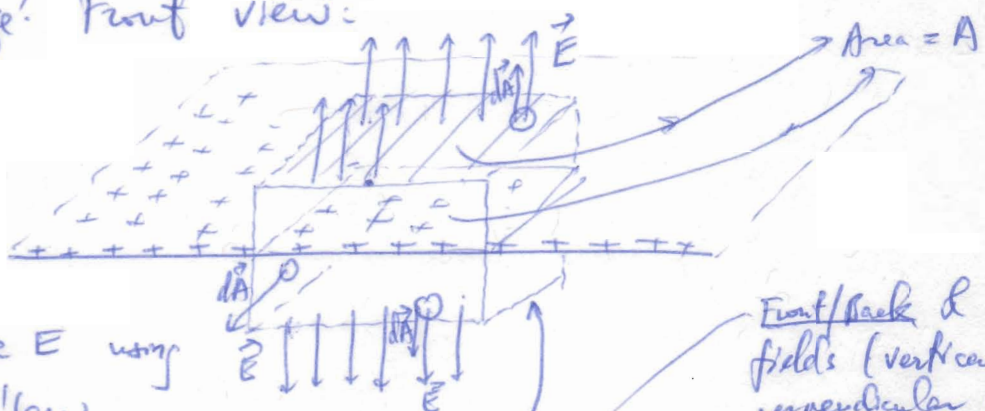
21.56



$m = 5\times10^{-3}$ kg
$q = 15\times10^{-6}$ C

Surface charge density
$\sigma = \frac{Q}{A}$?

What is E: or electric field created by a large or ∞ ~~surface~~ plane of charge? Front view:



Calculate E using Gauss' law

Front/Back & Left/Right fields (vertical) are perpendicular to $d\vec{A}$

1) Gaussian surface → rectangular box thru point

2) $\phi = \oint \vec{E}\cdot d\vec{A} = E\int_{\text{top \& bottom surface}} dA = E.2A$

3) Compare w/ $\phi = \frac{q_{enclosed}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$

$E2A = \frac{\sigma A}{\varepsilon_0}$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Field due to a ∞ plane of charge of density σ

Particle suspending in air: $qE = mg$

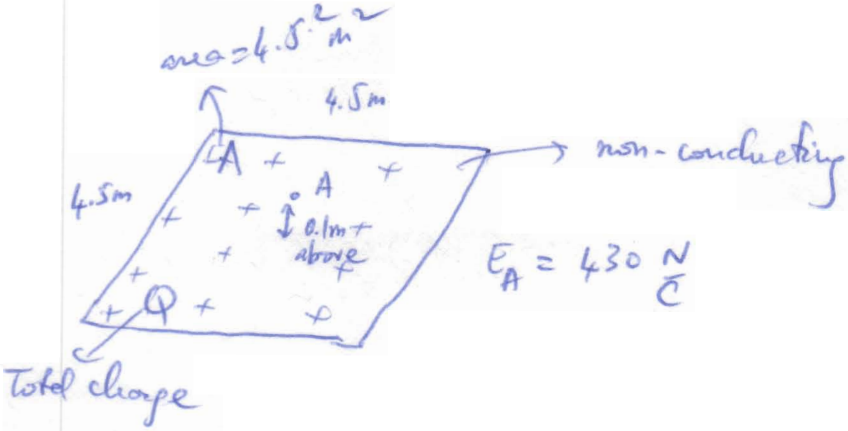
$\frac{q\sigma}{2\varepsilon_0} = mg \rightarrow \sigma = \frac{mg\,2\varepsilon_0}{q}$

$= \frac{5\times10^{-3}\times 9.81\times2\times 8.85\times10^{-12}}{15\times10^{-6}}$

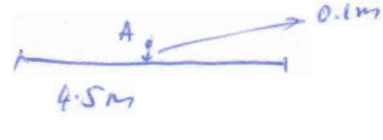
$= 57.8\times10^{-9}\ \frac{C}{m^2}$

$= 57.8\ \frac{nC}{m^2}$

21.70



Front view:



@ this sep. the plane looks like an ∞ plane of charge,
↳ from 21.56:

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

$$\sigma = \frac{Q}{A}$$

$$E = \frac{Q}{2A\varepsilon_0} \rightarrow Q = 2A\varepsilon_0 E$$

$$= 2 \times 4.5^2 \times 8.85 \times 10^{-12} \times 430 = 0.154 \times 10^{-6}\,C$$

$$\boxed{Q = 0.154\,\mu C}$$

22.31 $V(x,y,z) = 2xy - 3zx + 5y^2$ (Volts) ; xyz in meters

a) $P(1m, 1m, 1m) \rightarrow V(1m, 1m, 1m) = 2 - 3 + 5 = 4V$ ↓ Volt

b) $\vec{E}(1m,1m,1m) = -\left[(2y - 3z)\hat{\imath} + (2x + 10y)\hat{\jmath} + (-3x)\hat{k}\right]$

$$\vec{E} = -\vec{\nabla} V = \left(-\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}\right)$$

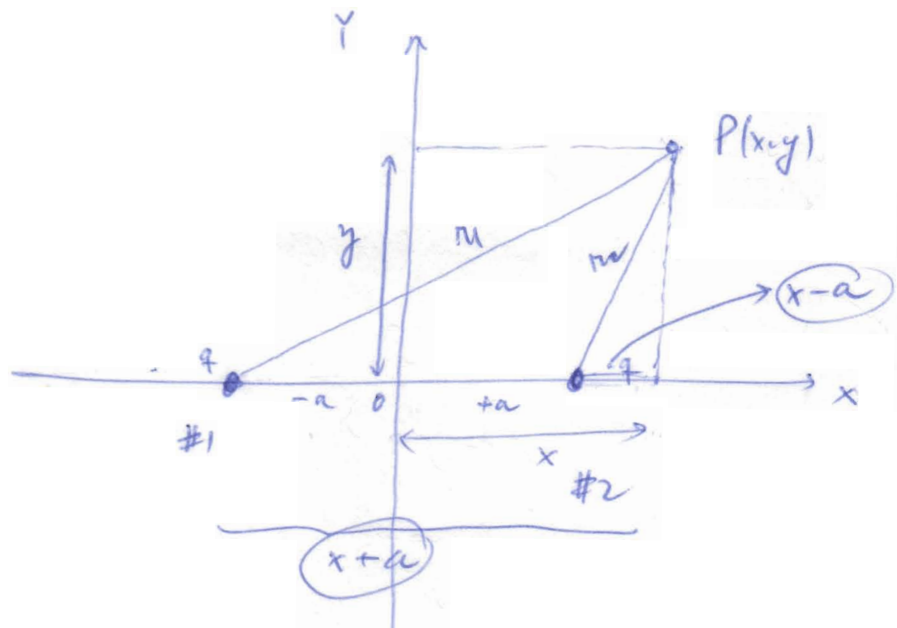
$$= -\left[-\hat{\imath} + 12\hat{\jmath} - 3\hat{k}\right]\frac{N}{C} = \underbrace{(\hat{\imath} - 12\hat{\jmath} + 3\hat{k})\frac{N}{C}}$$

x = 1m ; y = 1m ; z = 1m.

$E = \frac{F}{q} \rightarrow \frac{N}{C}$ also $\vec{E} = \vec{\nabla} V \rightarrow \frac{V}{m}$

$$\left\{\begin{array}{l} E_x = 1\,\frac{N}{C} \text{ or } \frac{V}{m} \\ E_y = -12\,\frac{N}{C} \text{ or } \frac{V}{m} \\ E_z = 3\,\frac{N}{C} \text{ or } \frac{V}{m} \end{array}\right.$$

22-53



a) $V(x,y)$ = scalar superposition (algebraic addition) of that by each charge. → $V = \frac{kq}{r}$

↳ $\frac{kq}{r_1} + \frac{kq}{r_2} = \frac{kq}{\left[(x+a)^2+y^2\right]^{1/2}} + \frac{kq}{\left[(x-a)^2+y^2\right]^{1/2}}$

b) $V(x \gg a,\ y \gg a) \simeq \frac{k(2q)}{(x^2+y^2)^{1/2}}$ → electric potential @ P due to a point charge of value 2q

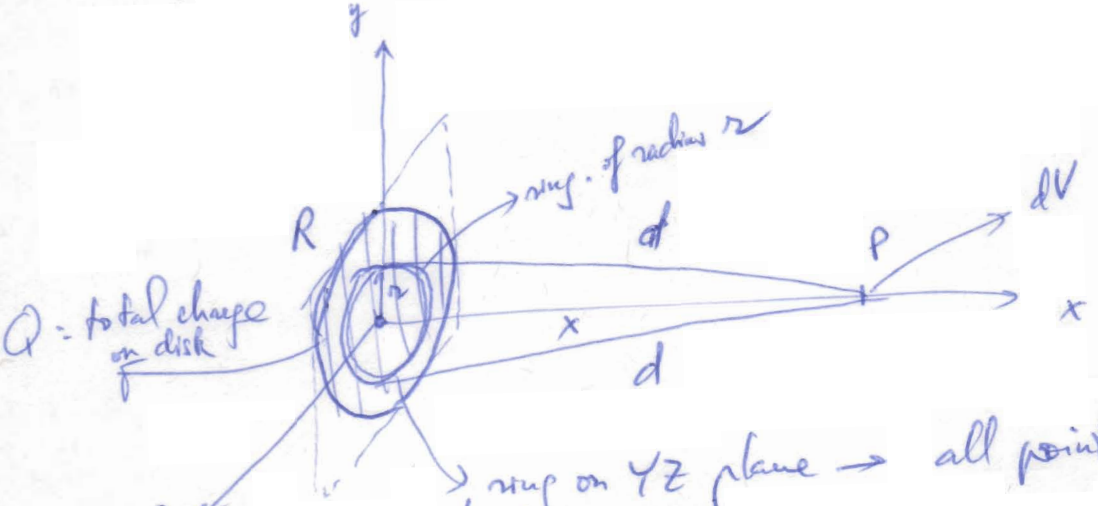
$x+a \simeq x$

$x-a \simeq x$

↳ as a check on your derivation in a)

Another example of Method #3: calculate $\vec{E}$ from V:

Electric potential due to a uniformly charged circular disk at a point along its axis:



ring on YZ plane → all points @ sep d to ~~from~~ P

charge on ring: $dq = \sigma \underbrace{dA}_{\text{of ring}} = \boxed{\sigma 2\pi r\, dr}$

Element of potential dV due to this ring @ P: $dV = \frac{k\,dq}{d}$

$$dV = \frac{k\,dq}{(x^2+r^2)^{1/2}} \rightarrow V = \int_{r=0}^{r=R} dV = k\int_{r=0}^{r=R} \frac{dq}{(x^2+r^2)^{1/2}}$$

$$= k\int_0^R \frac{\sigma 2\pi r\, dr}{(x^2+r^2)^{1/2}} = 2\pi\sigma k \underbrace{\int_0^R \frac{r\,dr}{(x^2+r^2)^{1/2}}}$$

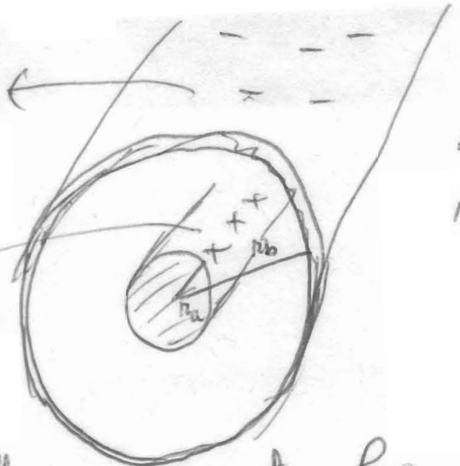
$$V = 2\pi\sigma k \left(\sqrt{R^2+x^2} - \sqrt{x^2}\right).$$

$$\rightarrow \vec{E} = -\frac{\partial V}{\partial x}\hat{i} \quad \left(\text{no } E_y \text{ nor } E_z \text{ since } \frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0\right)$$

$$= -2\pi\sigma k\left(\frac{1}{\cancel{2}}\frac{\cancel{2}x}{\sqrt{R^2+x^2}} - \frac{1x}{\sqrt{x^2}}\right)$$

22-67

$-\lambda = -75\,\frac{nC}{m}$ ← charge density → $\lambda = 75\,\frac{nC}{m}$



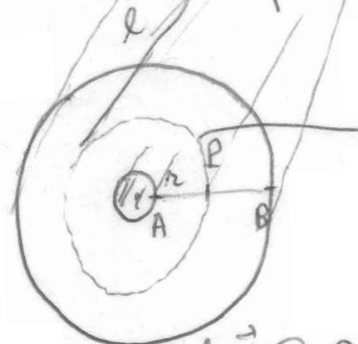
Coaxial cable.

$r_a = 0.002\,m$

$r_b = 0.01\,m$

a) Potential difference b/w outer & inner conductors

Front view:



→ Gaussian Cylinder

$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{r}$$: Need $\vec{E}$ @ P $(r_a < r < r_b)$ → $\underline{\text{Gauss law}}$:

1) $\underline{\text{Gaussian surface}}$: cylinder of radius r

2) $\phi = \oint \vec{E}\cdot d\vec{A}$
$\vec{E}$: radial
$d\vec{A}$ { body : radial $\parallel \vec{E}$; front : out of page $\perp \vec{E}$; back : into page $\perp \vec{E}$ }
$\phi = \oint E\,dA = E\int_{Body} dA$ (E const @ fixed r)

$\rightarrow \phi = E\, 2\pi r\, l$ (l: length of Gaussian cylinder)

3) $\phi = \frac{q_{enclosed}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

$E\, 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0} \rightarrow E = \frac{\lambda}{2\pi r \varepsilon_0} = \frac{2\lambda}{4\pi\varepsilon_0 r}$

$k = \frac{1}{4\pi\varepsilon_0}$

$$\boxed{E = \frac{2\lambda k}{r}}$$

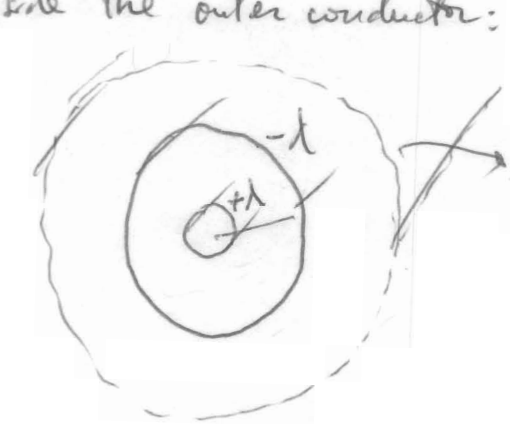
b/w A & B. radial →

B/w A & B: $\vec{E} = \frac{2\lambda}{kr}\hat{r}$

$$\Delta V_{AB} = -\int_A^B \frac{2\lambda k}{\cancel{kr}} \underbrace{\hat{r}\cdot\hat{r}}_{1}\, \underbrace{dr}_{d\vec{r}} = -\frac{2\lambda k}{\cancel{k}}\underbrace{\int_A^B \frac{dr}{r}}_{[\ln r]_A^B} = -\frac{2\lambda k}{\cancel{k}}\ln\left(\frac{r_b}{r_a}\right)$$

$$= +\frac{2\lambda k}{\cancel{k}}\ln\left(\frac{r_a}{r_b}\right) = 2\times 75\times 10^{-9} \times 9\times 10^{9}\,\ln\left(\frac{2}{10}\right) = -2170\,V$$

Observation: outside the outer conductor:



→ Gaussian cylinder encloses both conductors.

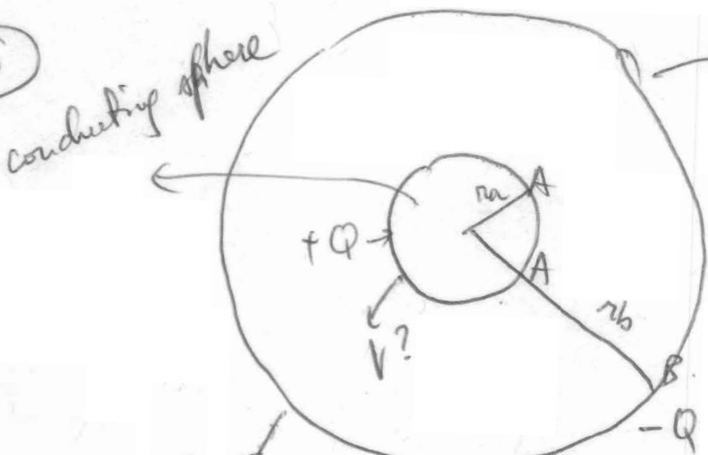
↓

$E_{outside} = 0$

---

b) Any change on ΔV_{AB} if outer conductor is charged to $+150 \frac{nC}{m}$? → No → since the Gaussian cylinder b/w A & B does not enclose the outer conductor.

(22.70)



→ conducting shells.

$r_a = 5 \text{ cm}$

$Q = 60 \text{ nC}$

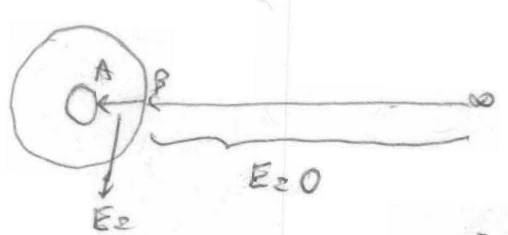
$r_b = 15 \text{ cm}$

$$\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{r}$$

$$V_{@ \text{ sphere surface}} = \Delta V_{\infty A} = -\int_\infty^A \vec{E} \cdot d\vec{r}$$

$$= -\int_\infty^B \vec{E} \cdot d\vec{r} - \int_B^A \vec{E} \cdot d\vec{r}$$

$$\int_\infty^B \vec{E} \cdot d\vec{r} \to 0$$

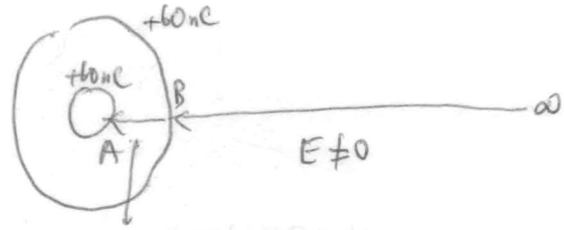


↳ Gauss law: $E \, 4\pi r^2 = \frac{Q}{\varepsilon_0} \to E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$ b/w A & B

$$\Delta V_{\infty A} = -\int_B^A \frac{kQ}{r^2} dr = -kQ \int_B^A \frac{dr}{r^2} = kQ\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = 9\times10^9 \times 60\times10^{-9}\left(\frac{1}{0.05} - \frac{1}{0.15}\right)$$

$$= 7200 \text{ V}$$

5)



(E same as in a))

$$\Delta V_{\infty A} = \Delta V_{AB} + \Delta V_{\infty B} = 7200\,V - \underbrace{\int_{\infty}^{B} \vec{E}\cdot d\vec{r}}_{-k2Q\left[-\frac{1}{r}\right]_{\infty}^{B}} = k2Q\,\frac{1}{r_B}$$

$$= 7200\,V + 2\times 9\times 10^{9}\times 60\times 10^{-9}\,\frac{1}{0.15}$$

$$= 14400\,V$$