If in general $E$ is polarized not in plane with page but at some angle \( \theta \), it has a component $\parallel$ page and a component $\perp$ page. The component $\parallel$ page, at Brewster's incident angle, gets no reflection, but that $\perp$ page will get some reflection.

Ch 31: Images & Optical Instruments

Image formation by a mirror:
- Use at least 2 rays.

How tall a mirror should we use to see our whole body? (a) Same as height (body) (b) $\frac{2}{3}$ height (c) $\frac{2}{5}$ height (d) $\frac{2}{7}$ height (e) $\frac{2}{9}$ height

\[ h \]

\[ \theta = \theta' \]

Object

Image (formed by extension rays)

Real rays

Virtual image: image formed by extension rays, no light is converging at its location actually.

Mirror: reflects all lights (metal coating the other side of glass)
Curved mirror: e.g. concave mirror

Focal point:
1) Incident ray || axis will reflect through F
2) Incident ray through F will reflect || axis

Image formed by extension rays → virtual image, no real light rays converge at that the image location.
if you put a screen at the image location (behind the mirror) there is no light on it!

Mirror equation: \( \frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \) (based on geometry)

Magnitude: \( M = \frac{h'}{h} = -\frac{l'}{l} \)

Signs: Mirrors \( f: \) 
\begin{align*}
+ & \text{ concave mirror} \\
- & \text{ convex mirror}
\end{align*}

\( l': \) 
\begin{align*}
+ & \text{ if image on same side of object} \\
- & \text{ if image on the other side of mirror (virtual image)}
\end{align*}
When do we get a real image with concave mirror?

Real image:

Lenses:

\[
\begin{align*}
\text{Lenses:} & \quad \rightarrow \\
\text{converging lens} & \quad \left(\text{convex lens}\right) \\
\text{diverging lens} & \quad \left(\text{concave lens}\right)
\end{align*}
\]

Image formation with lens:

Lense equation:

\[
\frac{1}{e} + \frac{1}{e'} = \frac{1}{f}
\]

(same as mirror eq.)

Rules:
1) ray will emerge through F
2) ray hitting center C goes straight through
Eyes:

Near-sighted (myopic):
- Converging lens (convex)
- Diverging lens (concave)
- Focal length \( f \) is negative
- Diopters: \( \frac{1}{f} \)
- Blurred image: Image of a dot is a spot

Far-sighted (hyperopic):
- Converging lens (convex)
- Focal length \( f \) is positive
- Diopters: \( \frac{1}{f} \)
- Blurred image: Image of a dot is sharp

Muscles to focus lens to faraway objects or close by
Ch32: Interference & Diffraction:

waves

Physical Optics: using wave properties in addition to geometry

\[ \text{Superposition:} \]

\[ \begin{align*}
\text{constructive} & \quad (\text{wave} + \text{wave} = 0) \\
\text{destructive} & \quad \text{out of phase}
\end{align*} \]

Double-slit interference:

\[ L \gg A \Rightarrow 18 \parallel 2B \]

1 wave $\rightarrow$ 2 waves after slits $\ (a \ll \lambda) \]

\[ \text{identical} \]

How did this bright spot?

- wave #1 & wave #2 travel parallel paths $\ (L \gg d) \rightarrow B$, with a difference in distance travelled $\Delta \text{path} = d \sin \theta$.

Since #1 & #2 are identical waves but travelled different distances $\rightarrow$ they arrive at B in different phase: 2 extreme situations:

- in phase $\rightarrow$ constructive interference
- out of phase $\rightarrow$ destructive interference
A) Constructive interference = waves 1 & 2 arrive at B in phase, producing a bright spot.

\[ \text{Superposition} \]

\[ \text{wave } 1 \]
\[ \text{at } B \]

\[ + \]

\[ \text{wave } 2 \]
\[ \text{at } B \]

\[ \text{this inphase situation.} \]
\[ \text{will happen if} \]
\[ \text{Path} = m\lambda \quad (m = 0, 1, 2, \ldots) \]
\[ \text{point C} \]

\[ d \sin \theta_m = m\lambda \quad (m = 0, 1, 2, \ldots) \]
\[ \theta_m \text{ corresponds to different bright spots on the screen.} \]

\[ y_m = L \tan \theta_m \]
\[ = L \tan \left( \sin^{-1} \left( \frac{m\lambda}{d} \right) \right) \]
\[ \text{length} \]
\[ \Delta \text{path} = m\lambda \quad (m = 0, 1, 2, \ldots) \]
\[ \Delta \text{phase} = m \pi \quad (m = 0, 1, 2, \ldots) \]
\[ \text{angle} \]
8) **Interference:** dark spots. Waves 1 and 2 arrive at D out of phase.

- Wave #1 at D
- Wave #2 at D
- Superposition

This out-of-phase situation happens if

\[ \text{Path} = (2m+1) \frac{\lambda}{2} \]

\( m = 0, 1, 2, \ldots \)

\( \text{(odd multiple of half wavelength)} \)

\[ \sin \Theta_m = \frac{(2m+1) \lambda}{2d} \]

\( \Theta_m \) corresponds to different dark spots on the screen.

\[ g_m = L \tan \left[ \sin^{-1} \left( \frac{(2m+1) \lambda}{2d} \right) \right] \]

\( m = 0, 1, 2, \ldots \)

\[ \text{Path} = (2m+1) \frac{\lambda}{2} \quad \text{for path length} \]

\[ \text{Phase} = (2m+1) \pi \quad \text{for phase angle} \]
Three-slit interference:

Three identical waves after the slits form one wave after another, but different distances travelled → waves arrive at different places at B. Those in phase (constructive interference) → out of phase (destructive interference).

A) Constructive interference:

1 & 2: \( d \sin \theta_n = n \lambda \)
2 & 3: \( d \sin \theta_n = m \lambda \)
1 & 3: \( 2d \sin \theta_n = 2m \lambda \)

B) Destructive interference:

\( \rightarrow 2 \text{ wave} \rightarrow \) out of phase by \( \frac{\lambda}{2} = \pi \) or \( 180^\circ \)
\( \rightarrow 3 \text{ wave} \rightarrow \) out of phase by \( \frac{\lambda}{3} = 2\pi / 3 \) or \( 120^\circ \)

N = # of slits

Loc of dark spots: \( d \sin \theta_n = (n + \frac{1}{3}) \lambda \rightarrow d \sin \theta_n = \frac{n}{N} \lambda \) for more than 2.
Interference in thin films:

1. Parallel like in double slit experiment.
2. Interference in thin films

Particular to thin film interference:

A) When wave gets reflected from low n to higher n it gets inverted. Get a phase of \( \varphi = \pi \) (\( \text{path} = \frac{\lambda}{2} \))

B) \( \text{path}_{12} = 2d \)
   (small \( \theta \) in thin films)

Interference by wave 1 & wave 2:

- In phase \( \pi \) constructive intef.: \( 2d = n\lambda + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \) (\( n = 0, 1, 2, \ldots \))
- Out of phase \( \varphi \) destructive intef.: \( \text{Path} = 2d = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2} = 2n\frac{\lambda}{2} + \varphi = (n+1)\lambda \) (\( n = 0, 1, 2, \ldots \))
Diffraction: Superposition of waves in one slit.

$L \gg a$

Path (at top & middle point): \( \frac{a}{2} \sin \Theta \)

Path = \( \frac{(2n+1)}{2} \lambda \) (n = 0, 1, 2, ...)

Top & middle: \( \sqrt{a \lambda} \sin \Theta = \frac{(2n+1)\lambda}{2} \) (n = 0, 1, 2, ...)

Huygens principle: each point on a wave front is a source of waves.

- Each point in the slit is a source of waves: e.g., dots in above slit are sources.

\[ a \sin \Theta = (2n+1)\lambda \quad (n = 0, 1, 2, ...) \]

Loc of dark spots in a diffraction

Diffraction limit: \( \Theta_{\text{min}} = \frac{1.22\lambda}{D} \)

\( \rightarrow \) diameter of slit.
double-slit using laser of \( \lambda = 633 \text{nm} \); \( d = 6.5 \mu \text{m} \);

\( L = 1.7 \text{m} \)

\[ \lambda = 633 \text{nm} \]

\[ L = 1.7 \text{m} \gg 6.5 \mu \text{m} \ (\text{very good approx.}) \]

**bright fringes**: 

\[ \text{constructive interference: } \frac{\text{path}}{d \sin \theta_n} = n \lambda \]

\[ \text{1st bright: } n = 1 \rightarrow \theta_1 = \frac{\lambda}{d} \]

\[ y_1 = L \tan \theta_1 \]

\[ \text{2nd bright: } n = 2 \rightarrow \theta_2 = \frac{2 \lambda}{d} \]

\[ y_2 = L \tan \theta_2 \]

\[ y_2 - y_1 = L \left( \tan \theta_2 - \tan \theta_1 \right) \]

\[ = L \left( \tan \left[ \sin^{-1} \left( \frac{2 \times 633 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] - \tan \left[ \sin^{-1} \left( \frac{633 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] \right) \]

\[ = 1.7 \left( \tan \left[ \sin^{-1} \left( \frac{2 \times 633 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] - \tan \left[ \sin^{-1} \left( \frac{633 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] \right) \]

\[ = 17.17 \text{ cm} \]

5) \[ y_4 - y_3 = L \left( \tan \theta_4 - \tan \theta_3 \right) \]

\[ = L \left( \tan \left[ \sin^{-1} \left( \frac{4 \lambda}{d} \right) \right] - \tan \left[ \sin^{-1} \left( \frac{3 \lambda}{d} \right) \right] \right) \]

\[ = 1.7 \left( \tan \left[ \sin^{-1} \left( \frac{4 \times 633 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] - \tan \left[ \sin^{-1} \left( \frac{3 \times 633 \times 10^{-9}}{6.5 \times 10^{-6}} \right) \right] \right) \]

\[ = 20 \text{ cm} \]
\[ n_{\text{air}} = 1.582 \quad n_{\text{violet}} = 1.633 \]

**Snell's Law:**
- Left boundary
  - Violet: \( \sin \theta' = n_{\text{violet}} \cos \theta_i \)
  - Right boundary
  - Violet: \( \sin \theta'' = n_{\text{violet}} \cos \theta_i \)

**Angular Dispersion**
\[ \chi = \theta'' - \theta' \]

**Geometry:**
- Equilateral triangle
  - Angle at center: 120°
  - Angle at sides: 60°

**Snell's Law:**
- Right boundary
  - Violet: \( \sin \theta'' = n_{i} \sin \theta'' \)
  - Red: \( \sin \theta'' = n_{i} \sin \theta'' \)

**Repeat for red ray:**
- Shell's Law at left boundary
  - Violet: \( n_{i} \sin \theta_i = \sin \theta'' \)
  - Red: \( n_{i} \sin \theta_i = \sin \theta'' \)

**Calculations:**
\[ \theta'' = 67.7° \]
\[ \theta' = 26.5° \]
\[ \chi = 60.8° - 26.5° = 34.3° \]
a) How many reflections?
Two reflections

1) With x-axis: Incident at $0^\circ \rightarrow$ exit at ?

\[ \theta_2 \]
\[ \theta_1 = 90 - \alpha = 90 - 37.5 = 52.5^\circ \]
\[ \theta_1 + \theta_2 + 105^\circ = 180^\circ \rightarrow \theta_2 = 180 - 105 - \theta_1 = 75 - 52.5^\circ = 22.5^\circ \]

**Geometry:**

Perpendicular lines to the sides of an angle form the same angle.

exit direction:

\[ \theta_2 + 90 + \alpha = 22.5 + 90 + 37.5 = 150^\circ \text{ CCW} \]

\[ 360 - 150^\circ = 210^\circ \text{ CW} \]
\[ \Delta y = L \left\{ \tan^{-1} \left( \frac{(2n+3)\lambda}{2d} \right) - \tan^{-1} \left( \frac{(2n+1)\lambda}{2d} \right) \right\} \]

\[ \text{Small angle approximation: for fading not too far from central spot (origin of XY ends) compare } L = 6.5 \text{ km}. \]

\[ L = 6.5 \text{ km} \]

\[ 0 = \text{small angle} \]

\[ \tan \approx \sin \]

\[ \text{Notice that with small angle approx (spots close origin XY) separation b/w consecutive fading spots does not depend on order } n. \]

\[ \Delta y = \frac{6500 \times \lambda}{400} \quad \therefore \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{103.4 \times 10^6} = \frac{3}{1.039} \text{ m} \]

\[ \therefore \quad \Delta y = \frac{6500 \times \frac{3}{1.039}}{400} = 46.9 \text{ m} \]

\[ \text{Time sep. b/w fades: } \frac{\Delta y}{v} = \frac{46.9 \text{ m}}{60 \text{ km} \cdot 1 \text{ km} \cdot 1000 \text{ m}} = 2.82 \text{ s} \]

Every 2.82s you would hear a fade in the radio signal.

\[ T = 2.82 \text{ s} \quad \Rightarrow \quad \text{How often: } \frac{1}{T} = \frac{1}{2.82 \text{ s}} = \]
$\Delta t = \frac{d}{c}$

How often does the signal appear to fade?

- Due to interference by the two identical waves, the signal fades as the wavelength is
- one of the buildings.

- Since there are dark and bright spots along the path.

- 103.9 MHz FM radio
- 40 W
- L = 6.5 km
- 100 µs
Visible spectrum:

\[ \lambda_v = 400 \text{ nm} \rightarrow \lambda_r = 700 \text{ nm} \]

(violet) \rightarrow (red) (Higher f) \rightarrow (Lower f)

Lowest pair of consecutive orders for overlap/bly overlaps spectra as dispersed by a grating

\[ n_v > n_r \]

For a same order \( n \) spot for red light are further out than spot for violet light.

Overlap of spot (violet) of order \( n+1 \) coincide with spot (red) of order \( n \).

\[ \sin \Theta_n \text{red} = \frac{n \lambda \text{red}}{a} = \sin \Theta_{n+1} \text{violet} = \frac{(n+1) \lambda \text{violet}}{a} \]

\[ n \lambda \text{red} = (n+1) \lambda \text{violet} \]

\[ n (\lambda \text{red} - \lambda \text{violet}) = \lambda \text{violet} \rightarrow n = \frac{\lambda \text{violet}}{\lambda \text{red} - \lambda \text{violet}} \]

\[ z = \frac{4}{7-4} = \frac{4}{3} = 1.33 \]

\( n \): integer only \( \rightarrow n = 2 \) (red)

\( n+1 = 3 \) (violet)
\( f = 35 \text{ cm} \)

a) \( l = 40 \text{ cm} \) → sep. both \( i \) & \( i' \)?

\( l' \) is + (Image: the other side of lens) \( l + l' \)

\[
\frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \rightarrow \frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = \frac{l - f}{lf} \Rightarrow l' = \frac{lf}{l-f}
\]

\[
\frac{40 \times 35}{40 - 35} = 280 \text{ cm}
\]

sep. \( o & i' \) is \( 40 \text{ cm} + 280 \text{ cm} = 320 \text{ cm} \).

b) \( f = 30 \text{ cm} \)

upright \( i' \) virtual image

\( l'(-) \) → sep. both \( i' \) & \( i' \):

\[
l'(-) = l + (l' + l)
\]

\[
\rightarrow | -210 + 20 | = 180 \text{ cm}.
\]
What is smallest $n_1$ for no further total reflection:

1) at (A) & (C) same ray paths regardless $n_2 = 1$ or larger (liquid)

2) at (B) $\theta > \theta_c$ since there was a total internal reflection when $n_2 = 1$ ($n_2 \sin \theta_c = 1$  
   $\Rightarrow \theta_c = \sin^{-1} \left( \frac{1}{n_2} \right)$)

When prism is in a liquid: $n_2 > 1 \Rightarrow \theta'_c = \sin^{-1} \left( \frac{n_1}{n_2} \right)$

What should be $n_2$ (liquid) so $[\theta < \theta'_c]$? $\Rightarrow$ no longer total internal reflection at boundary (B).

From geometry: $\theta = 45^\circ$  
$45^\circ \leq \sin^{-1} \left( \frac{n_1}{n_2} \right)$

$\sin 45^\circ \leq \sin \left( \sin^{-1} \frac{n_1}{1.52} \right)$

$1.52 \sin 45^\circ \leq n_1$

$n_{\text{min}} = 1.52 \sin 45^\circ = 1.07$
When prism is in a liquid, $n_i > 1 \Rightarrow \theta_i'$ is larger than $\theta \Rightarrow (\theta < \theta_i') \Rightarrow$ no longer total internal reflection at 6.
\( \lambda = 633 \text{nm} \)

\[ a = 2.5 \mu m \]

- **Diffraction**
  - **Width of central peak?** \( \Rightarrow 2\theta_1 \): \( \theta_1 \) angle for 1st dark spot.

- **Dark spot:** \( a \sin \theta_m = m\lambda \) \( \Rightarrow \theta_1 = \sin^{-1} \left( \frac{\lambda}{a} \right) \)

\[ = \sin^{-1} \left( \frac{633 \times 10^{-7}}{2.5 \times 10^{-6}} \right) \]

\[ = 14.7^\circ \]

\[ \Rightarrow 2 \times 14.7^\circ = 29.4^\circ . \]