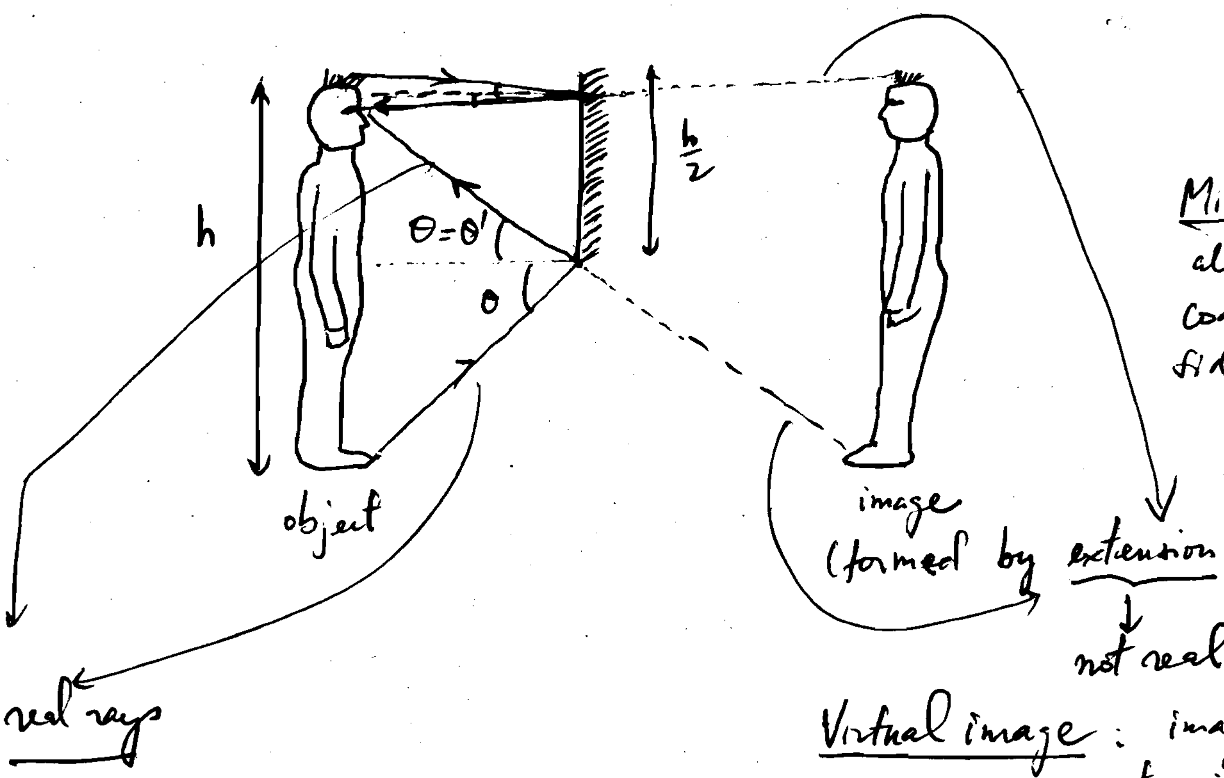
If in general $\vec{E}$ is polarized not in plane with page but at some angle → it has a component ∥ page & a component ⊥ page. The component ∥ page, at Brewster incident angle gets no reflection, but that ⊥ page will get some reflection.

Ch 31 Images & Optical Instruments

Image formation by a mirror:

↳ use at least 2 rays.

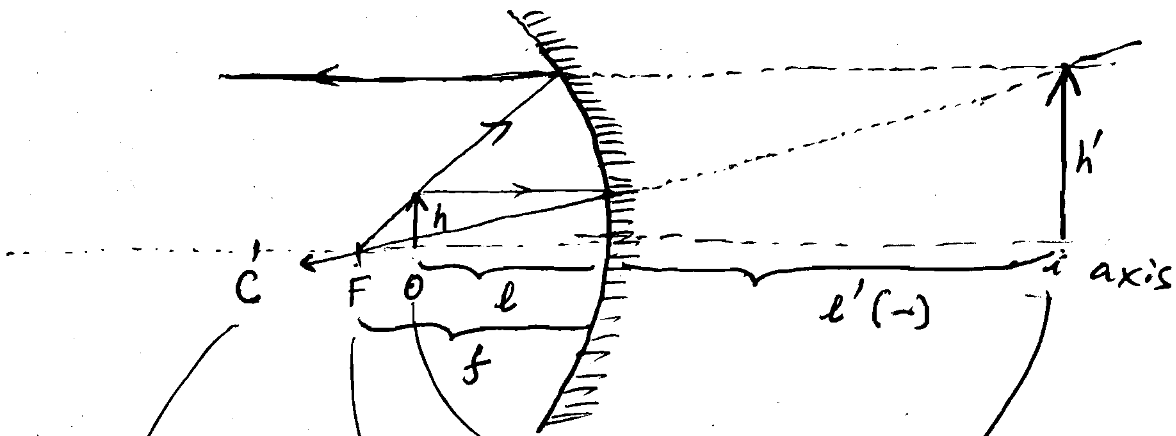
How tall a mirror should we use to see our whole body? a) same as height (body) b) 2/3 height c) 1/2 height ✓



Mirror: reflects all lights (metal coating the other side of glass)

Virtual image: image formed by extension rays, no light is converging at its location actually.

Curved mirror : e.g. concave mirror



Object.

image

center of the spherical mirror

Focal point: 1) incident rays ∥ axis will reflect through F
2) incident rays through F will reflect ∥ axis

Image formed by extension rays. → virtual image: no real light rays converge at ~~that~~ the image location: if you put a screen at the image location (behind the mirror) there is no light on it!

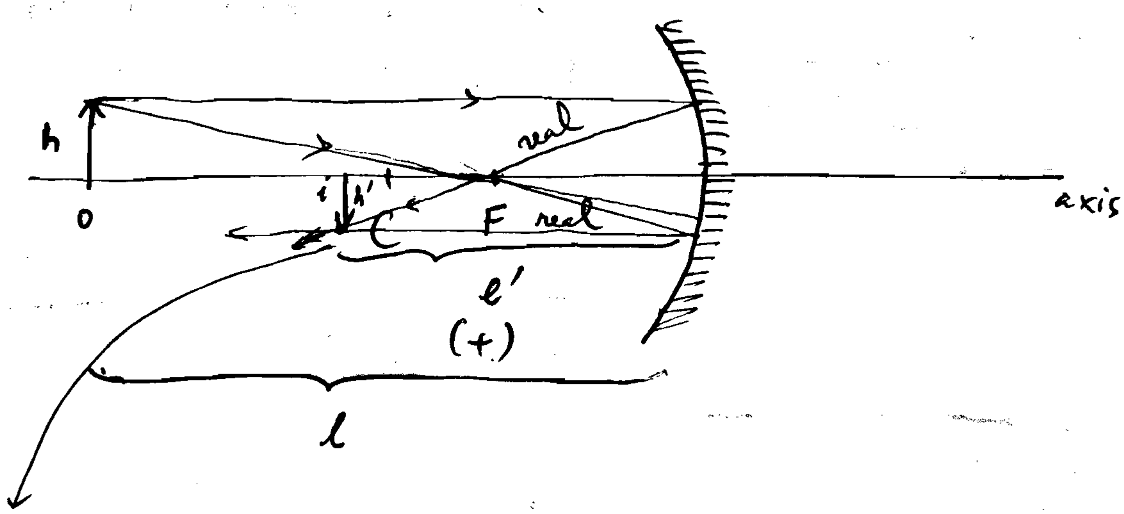
Mirror equation : $\boxed{\frac{1}{l} + \frac{1}{l'} = \frac{1}{f}}$ (based on geometry)

Magnification: $M = \frac{h'}{h} = -\frac{l'}{l}$

Signs: Mirrors f : + concave mirror ; − convex mirror

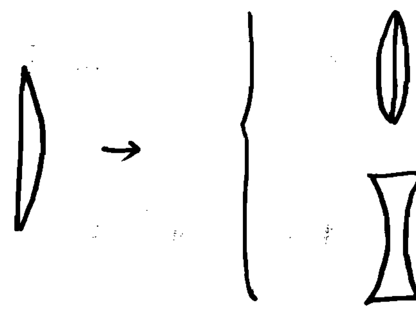
l' : + if image on same side of object (real image)
− if image on the other side of mirror (virtual image)

When do we get a real image with concave mirror?



Real image:

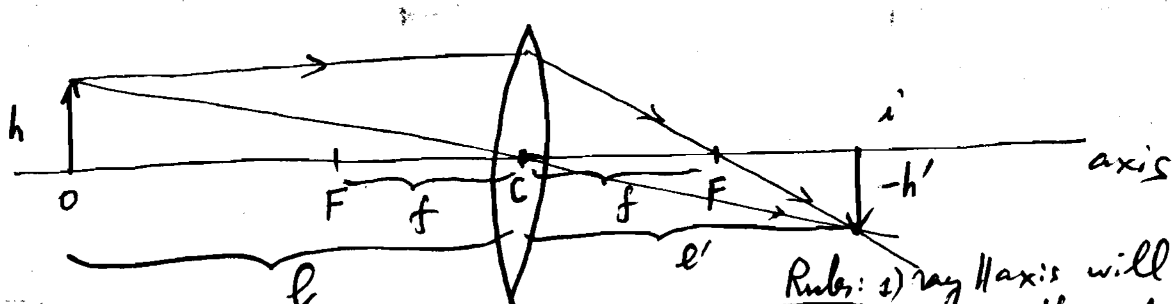
Lenses:



converging lense (convex lense)

diverging lense (concave lense)

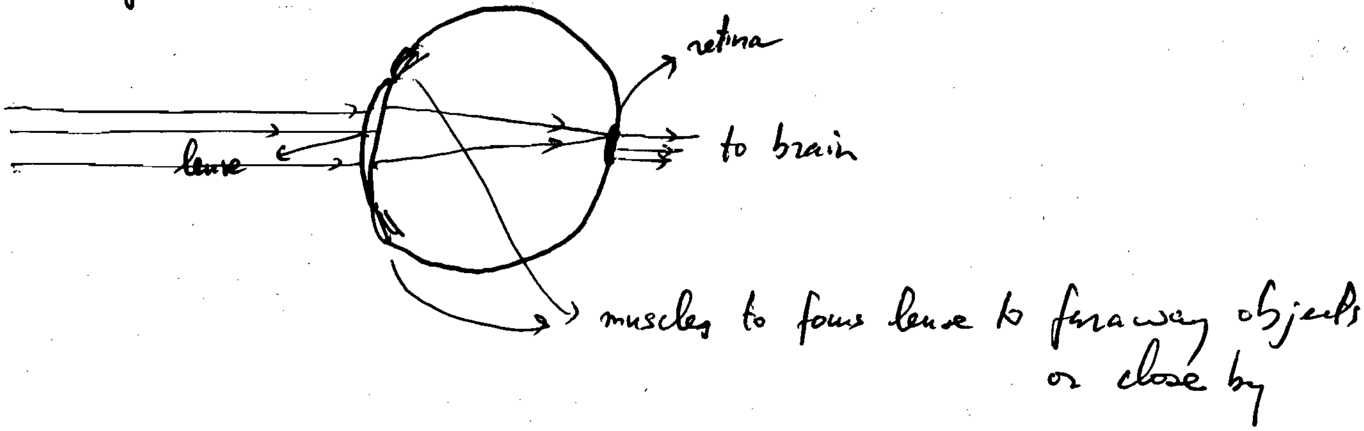
Image formation with lenses:



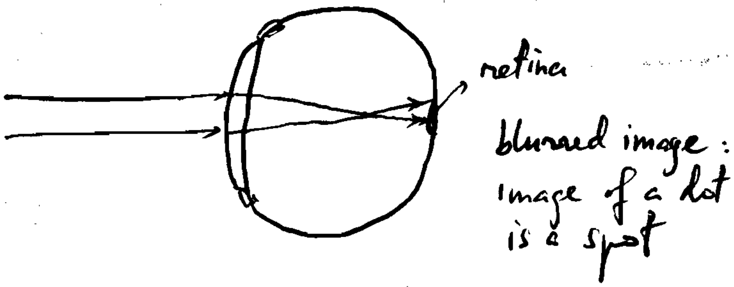
Rules: 1) ray ∥ axis will emerge through F
2) ray hitting center C goes straight thru;

Lense equation: $\boxed{\frac{1}{l} + \frac{1}{l'} = \frac{1}{f}}$ (same as mirror eq.)

Eyes:



Near sighted (myopic)



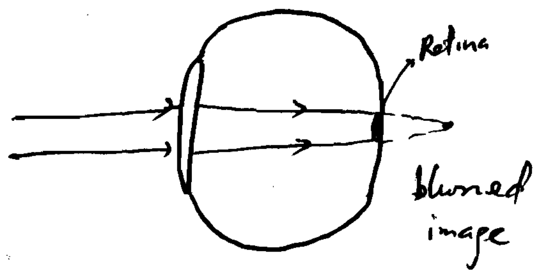
corrective lenses:

diverging lense
(concave)

↳ focal length f (−)

↳ diopters = $\frac{1}{f(m)}$

Far sighted (hyperopic)



corrective lense:

converging lense
(convex)

$f(+)$

Ch32: Interference & Diffraction:

$\downarrow$ waves

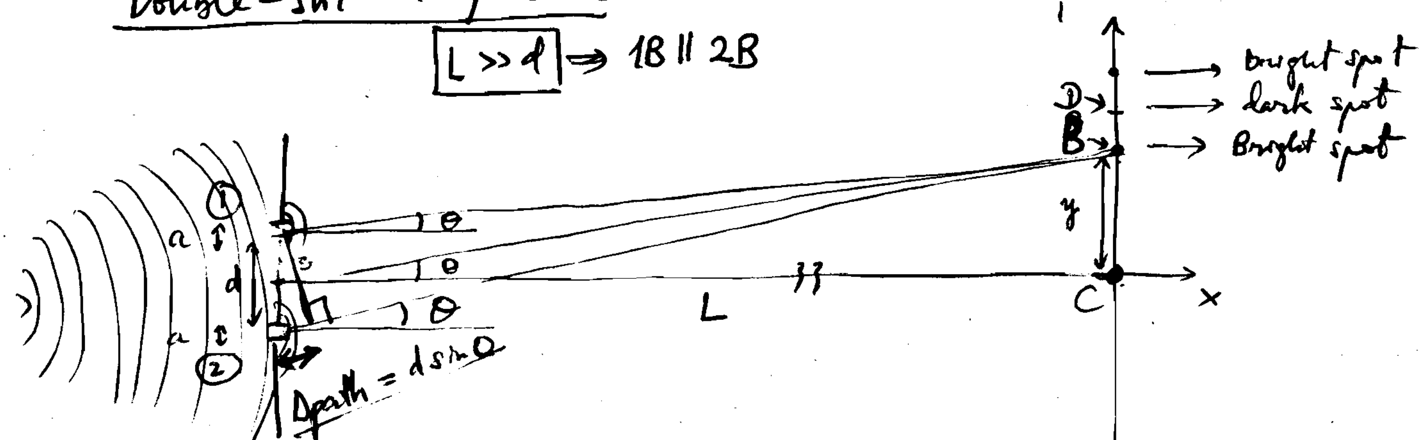
Physical Optics: using wave properties in addition to geometry

$\hookrightarrow$ superposition $\begin{cases} \text{constructive} \\ \text{destructive } (\text{1wave} + \text{1wave} = 0) \end{cases}$

out of phase

Double-slit interference:

$\boxed{L \gg d} \Rightarrow 1B \parallel 2B$

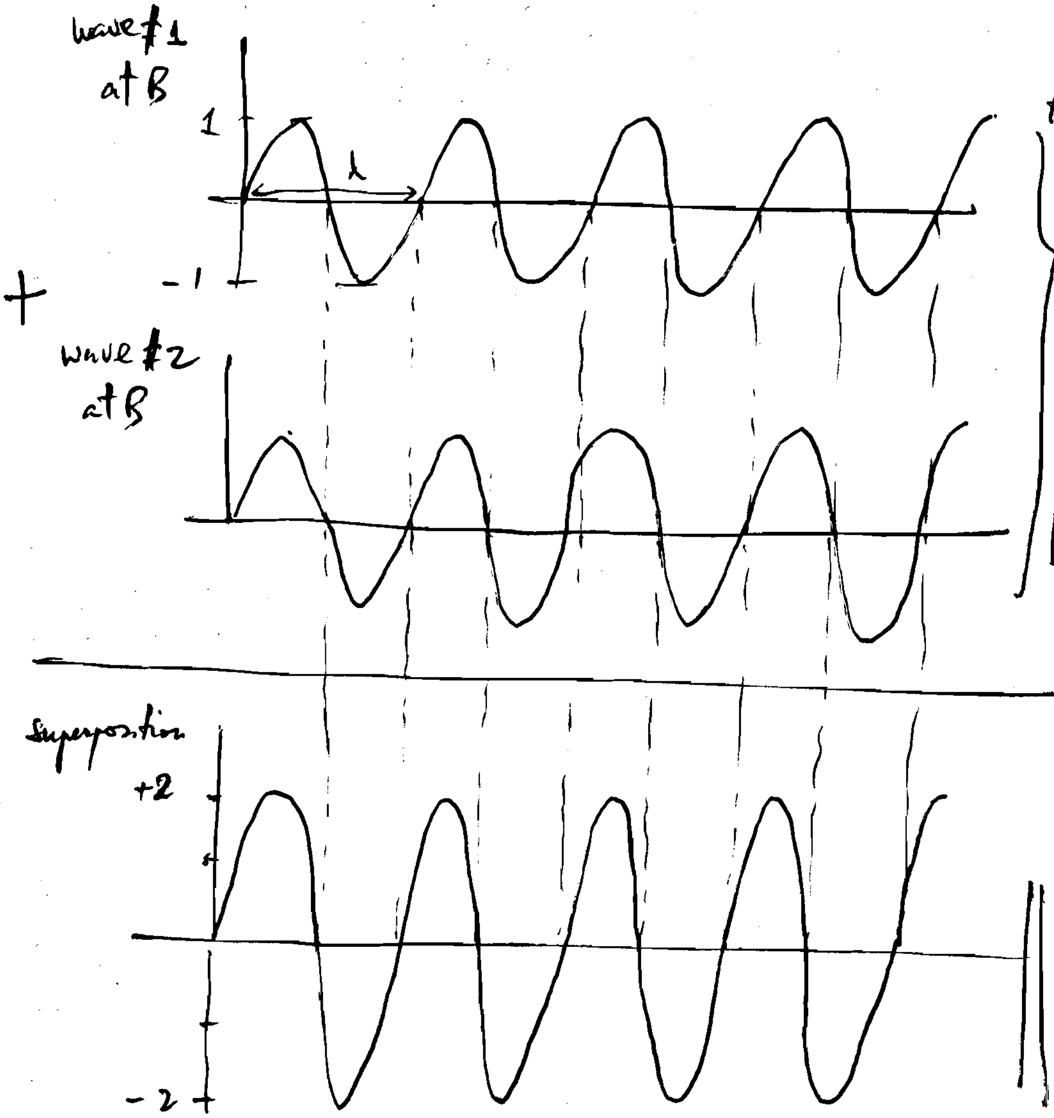


1 wave $\rightarrow$ 2 waves after slits ($a \ll \lambda$)
identical.

Screen: result of superposition of 2 waves: pattern of dark & bright fringes.

How did we get this bright spot?
wave #1 & wave #2 travels parallel paths ($L \gg d$) to B, with a difference in distance travelled of $\Delta path = d \sin\theta$: Since #1 & #2 are identical waves but travelled different distances $\rightarrow$ they arrive at B in different phase: 2 extreme situations: $\begin{cases} \rightarrow \text{in phase} \rightarrow \text{const. interf.} \\ \rightarrow \text{out of phase} \rightarrow \text{destructive interference} \end{cases}$

A) Constructive interference = waves 1 & 2 arrive at B in phase, producing a bright spot.



this inphase situation will happen if

$\Delta path = m\lambda \quad (m = 0, 1, 2, \ldots)$ point C

$\boxed{d \sin\theta_m = m\lambda}$ $(m = 0, 1, 2, \ldots)$

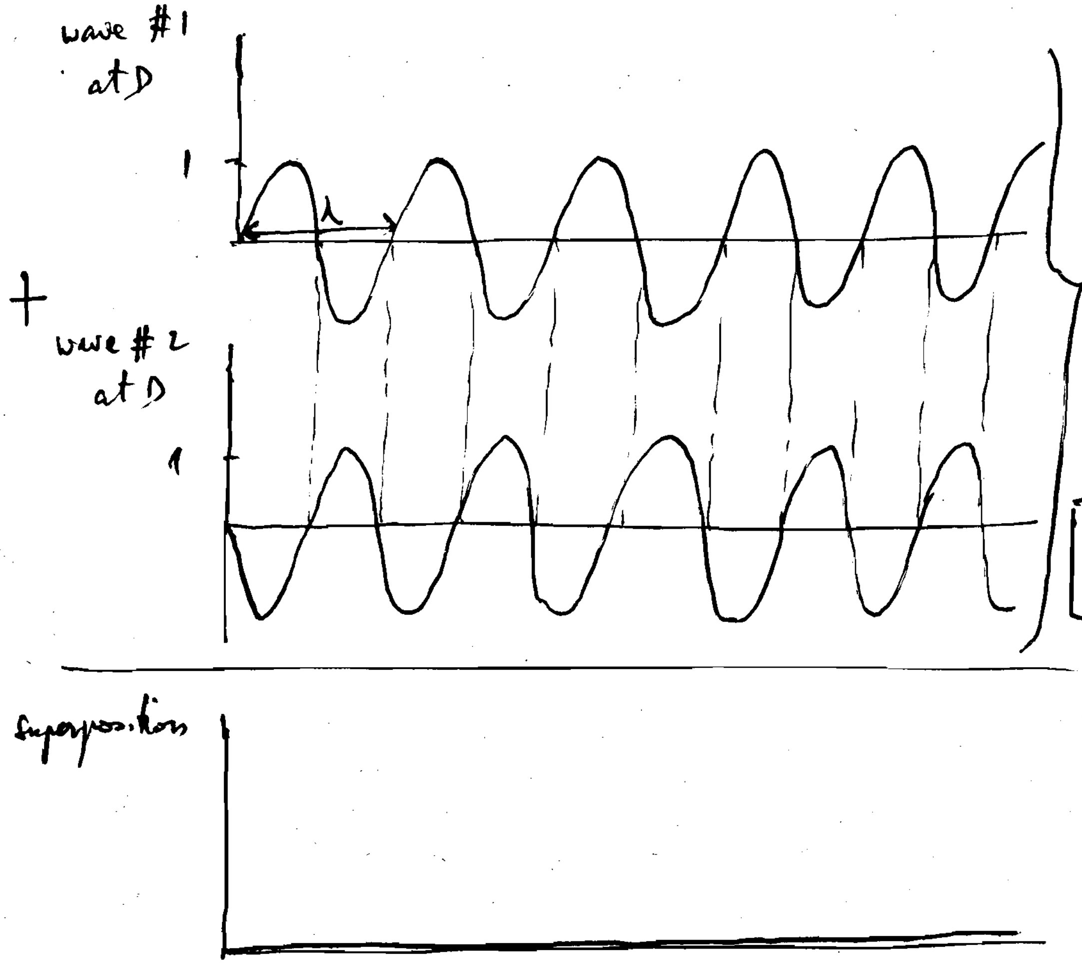
θ_m corresponds to different bright spots on the screen.

$$y_m = L \tan\theta_m = L \tan\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right)$$

$\Delta path = m\lambda \quad (m = 0, 1, 2, \ldots)$ length

$\Delta phase = m 2\pi \quad (m = 0, 1, 2, \ldots)$ angle

B) Destructive Interference : dark spots : wave 1 & 2 arrive at D out of phase :



this out-of-phase situation happens if

$$\Delta path = (2m+1)\frac{\lambda}{2}$$

$(m = 0, 1, 2, \ldots)$

(odd multiple of half wavelengths)

$$\boxed{d \sin\theta_m = (2m+1)\frac{\lambda}{2} \quad (m = 0, 1, 2, \ldots)}$$

θ_m corresponds to different dark spots on the screen.

$$y_m = L \tan\left[\sin^{-1}\left(\frac{(2m+1)\lambda}{2d}\right)\right]$$

$m = 0, 1, 2, \ldots$

$\Delta path = (2m+1)\frac{\lambda}{2}$ $(m = 0, 1, 2, \ldots)$

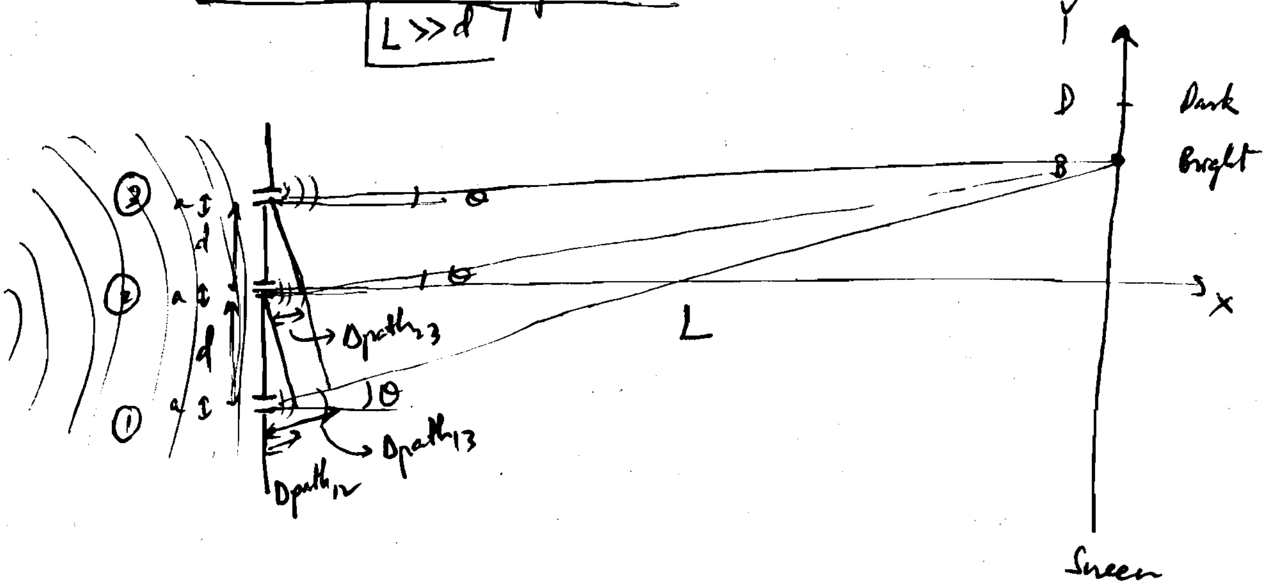
→ length

$\Delta phase = (2m+1)\pi$ $(m = 0, 1, 2, \ldots)$

→ angle

Three-slit interference:

$\boxed{L \gg d}$



Three identical waves after the slits from one wave $a \ll \lambda$
Again different distances travelled $\longrightarrow$ waves arrive at different phases at B. $\longrightarrow$ extreme situations $\begin{cases} \text{in phase (constructive interference)} \\ \text{out of phase (dest. interf.)} \end{cases}$

A) Constructive interference:

$$\left.\begin{array}{ll} 1 \,\&\, 2: & d\sin\theta_m = m\lambda \\ 2 \,\&\, 3: & d\sin\theta_m = m\lambda \\ 1 \,\&\, 3: & 2d\sin\theta_m = 2m\lambda \end{array}\right\} \boxed{d\sin\theta_m = m\lambda}$$

B) Destructive interference:

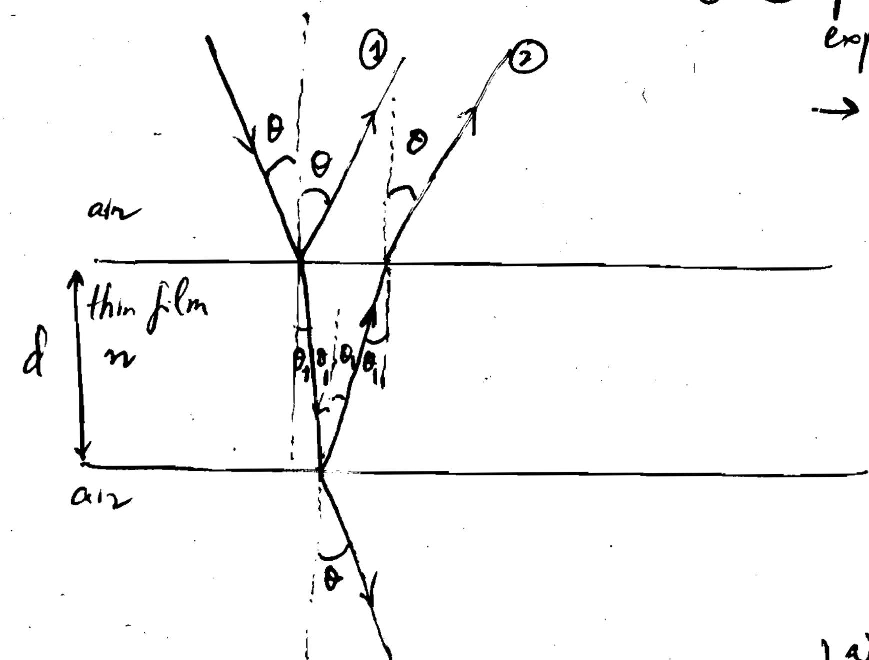
- 2 wave $\longrightarrow$ out of phase by $\frac{\lambda}{2}$ or π or $180°$ $\quad \uparrow\downarrow = 0$
- 3 wave $\longrightarrow$ out of phase by $\frac{\lambda}{3}$ or $\frac{2\pi}{3}$ or $120°$ $\quad$ 120° $= 0$

N = # of slits.

Loc of dark spots: $d\sin\theta_n = (n + \frac{1}{3})\lambda \longrightarrow \boxed{d\sin\theta_n = \frac{n}{N}\lambda}$

more than 2

Interference in thin films :

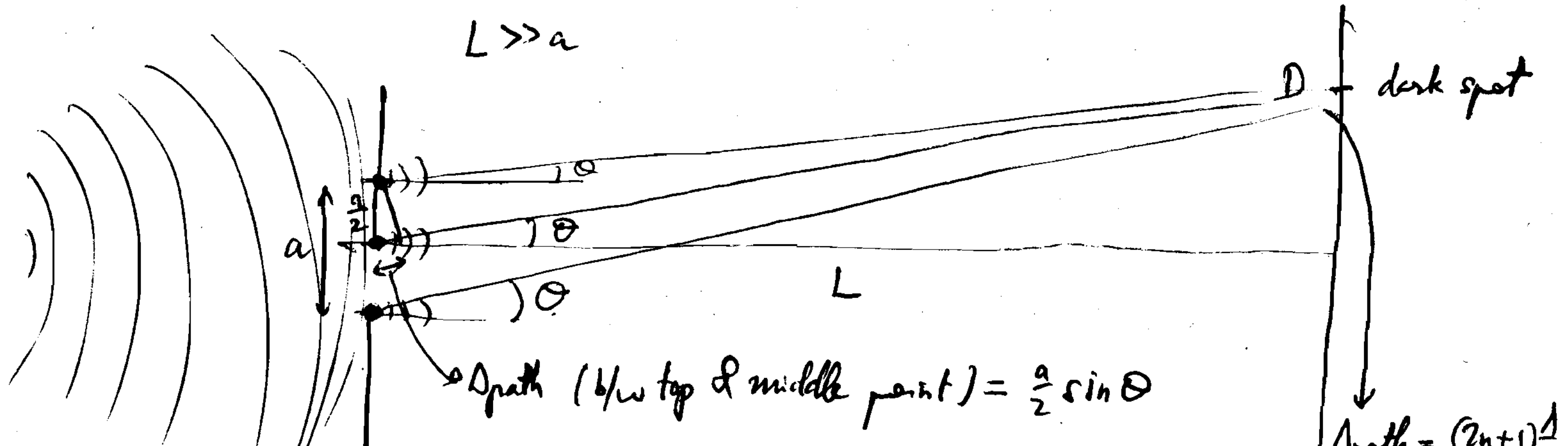


① & ② parallel like in double-slit experiment.
→ interference in thin films

→ Particular to this thin film interference
a) when wave #1 gets reflected from low n to higher n → it gets inverted → get a Δphase $= \pi$ (a Δpath $= \frac{\lambda}{2}$)
b) $\Delta path_{12} = 2d$ (small θ_1 in thin films)

→ Interference b/w wave #1 & wave #2 : extreme situations:
- in phase or constructive interf. : $\underbrace{2d}_{\Delta path} = n\lambda + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$ $(n = 0, 1, 2, \dots)$
- out-of-phase or destructive interf.: $\Delta path = 2d = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$
$= 2n\frac{\lambda}{2} + \lambda$
$= (n+1)\lambda$
$(n = 0, 1, 2, \dots)$

Diffraction: Superposition of waves in one slit.



Top & middle: → $\boxed{\Delta path = \frac{a}{2}\sin\theta = (2n+1)\frac{\lambda}{2} \ (n=0,1,2\ldots)}$ Destructive.

Huyghens principle: each point on a wave front is a source of waves.
→ each point in the slit is a source of waves: e.g. dots in above slit are source.

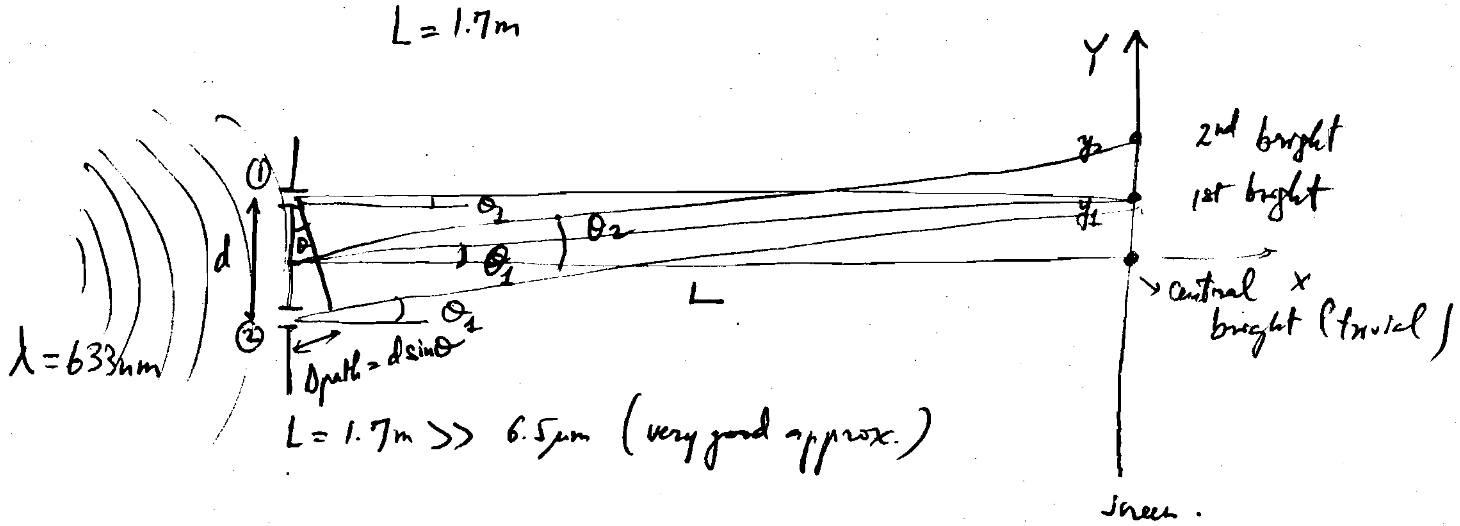
$$\boxed{a\sin\theta_n = (2n+1)\lambda \ , (n=0,1,2\ldots)}$$

loc of dark spots in a diffraction

Diffraction limit: $\theta_{min} = \dfrac{1.22\lambda}{D}$

D → diameter of slit.

32.38 double-slit using laser of $\lambda = 633\,nm$; $d = 6.5\,\mu m$;
$L = 1.7\,m$



bright fringes: const. interf: $\Delta path = n\lambda \longrightarrow$ 1st bright : $n = 1 \to \boxed{\theta_1 = \sin^{-1}\frac{\lambda}{d}}$

$d\sin\theta_n = n\lambda$ $\boxed{y_1 = L\tan\theta_1}$

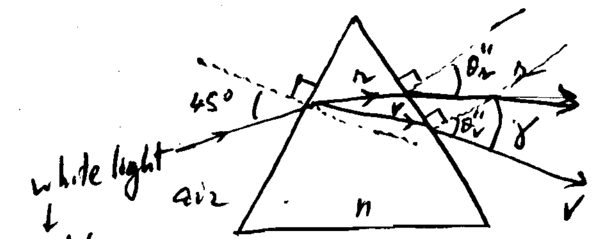
2nd bright : $n = 2 \to \boxed{\theta_2 = \sin^{-1}\frac{2\lambda}{d}}$

$\boxed{y_2 = L\tan\theta_2}$

a) $$y_2 - y_1 = L\left(\tan\theta_2 - \tan\theta_1\right)$$
$$= L\left(\tan\left[\sin^{-1}\frac{2\lambda}{d}\right] - \tan\left[\sin^{-1}\frac{\lambda}{d}\right]\right)$$
$$= 1.7\left\{\tan\left[\sin^{-1}\left(\frac{2\times 633\times 10^{-9}}{6.5\times 10^{-6}}\right)\right] - \tan\left[\sin^{-1}\left(\frac{633\times 10^{-9}}{6.5\times 10^{-6}}\right)\right]\right\}$$
$$= 17.17\ cm$$

b) $$y_4 - y_3 = L\left\{\tan\theta_4 - \tan\theta_3\right\}$$
$$= L\left\{\tan\left[\sin^{-1}\left(\frac{4\lambda}{d}\right)\right] - \tan\left[\sin^{-1}\left(\frac{3\lambda}{d}\right)\right]\right\}$$
$$= 1.7\left\{\tan\left[\sin^{-1}\left(\frac{4\times 633\times 10^{-9}}{6.5\times 10^{-6}}\right)\right] - \tan\left[\sin^{-1}\left(\frac{3\times 633\times 10^{-9}}{6.5\times 10^{-6}}\right)\right]\right\}$$
$$= 20\ cm$$

30.28



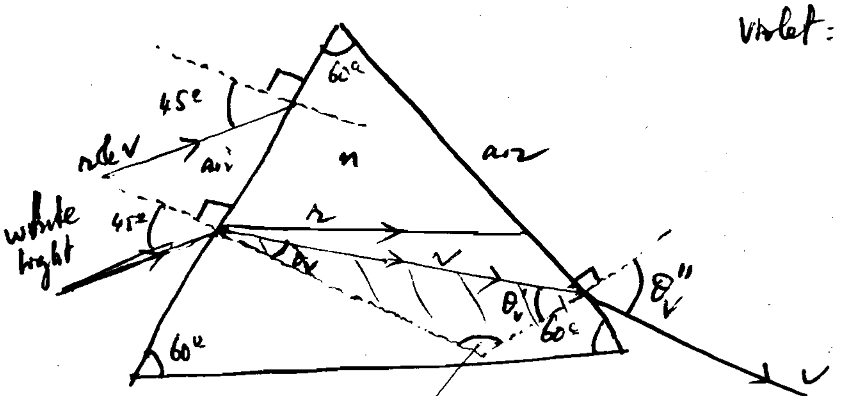
γ: angular dispersion

$\gamma = \theta_r'' - \theta_v''$

white light $\downarrow$ all λ's:

$\lambda_{red} \leftrightarrow \lambda_{violet}$

Prism: $n_{red} = 1.582 \quad n_{violet} = 1.633$



Snell's Law: left boundary.

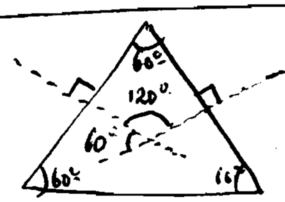
violet: $1 \sin 45° = n_v \sin\theta_v \rightarrow \theta_v = \sin^{-1}\left(\frac{\sin 45}{1.633}\right) = 25.5°$

$\theta_v + \theta_v' + 120° = 180° \rightarrow \boxed{\theta_v' = 60 - \theta_v} \rightarrow \theta_v' = 60 - 25.5° = 34.5°$

incident angle on right boundary

refracted angle on left boundary

geometry: equilateral triangle.



Snell's law: Right boundary

Violet: $n_v \sin\theta_v' = 1 \sin\theta_v''$

$\rightarrow \theta_v'' = \sin^{-1}(1.633 \times \sin 34.5)$

$\boxed{\theta_v'' = 67.7°}$

Repeat for red ray:

Snell's law at left boundary: $1 \sin 45° = 1.582 \sin\theta_r \rightarrow \theta_r = \sin^{-1}\left(\frac{\sin 45°}{1.582}\right) = 26.5°$

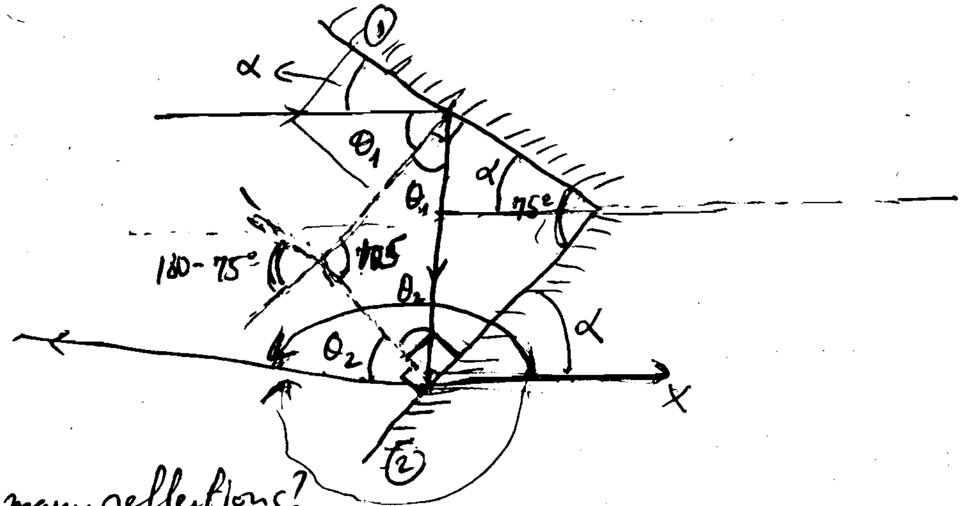
$\theta_r' = 60 - \theta_r = 33.5°$

Snell's law at right boundary: $n_r \sin\theta_r' = 1 \sin\theta_r'' \rightarrow \theta_r'' = \sin^{-1}(1.582 \sin 33.5)$

$\boxed{\theta_r'' = 60.8°}$

$\rightarrow \boxed{\gamma = \theta_v'' - \theta_r'' = 67.7 - 60.8 = 6.85°}$

30.29



a) How many reflections?

Two reflections

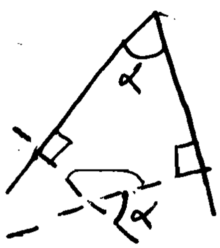
b) W.r.t x axis: incident at 0° → exit at ?

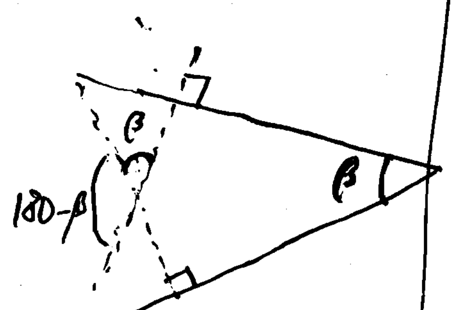
θ_2?

$\theta_1 = 90 - \alpha = 90 - 37.5 = 52.5°$

$\theta_1 + \theta_2 + 105° = 180° \rightarrow \theta_2 = 180 - 105 - \theta_1 = 75 - 52.5° = 22.5°$

Geometry:





Perpendicular lines to the sides of an angle form the same angle.

$\theta_2 + 90 + \alpha =$

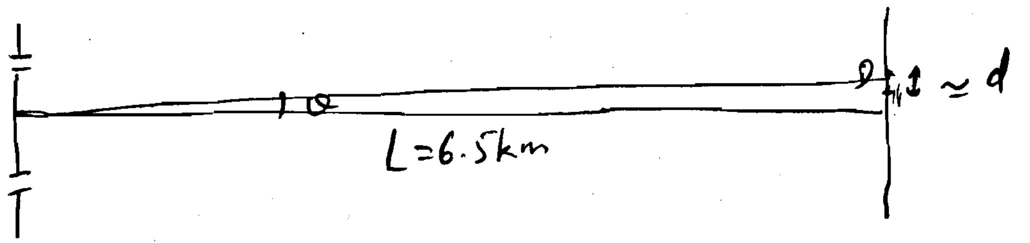
$22.5 + 90 + 37.5$

$= \boxed{150° \text{ CCW}}$

exit direction { or $360 - 150° = 210°$ CW

$$\Delta y = L\left\{ \tan\left[\sin^{-1}\left(\frac{(2n+3)\lambda}{2d}\right)\right] - \tan\left[\sin^{-1}\left(\frac{(2n+1)\lambda}{2d}\right)\right]\right\}$$

↳ small angle approximation: for fading not two far from central spot (origin of XY coords) compared $L = 6.5$ km.



$\theta \simeq$ small angle.

↳ $\tan \sim \sin$

$$\Delta y = L\left\{ \frac{(2n+3)\lambda}{2d} - \frac{(2n+1)\lambda}{2d} \right\} = L\left\{ \frac{3\lambda}{2d} - \frac{\lambda}{2d} \right\}$$

$$\boxed{\Delta y = \frac{L\lambda}{d}}$$

$y_{n+1} - y_n$

Notice that with small angle approx (spots close origin XY) separations b/w consecutive fading spots does not depend on order n.)

$$\Delta y = \frac{6500 \times \lambda}{400} \quad ; \quad \lambda = \frac{c}{f} = \frac{3\times10^8}{103.9\times10^6} = \frac{3}{1.039}\,m$$

$$\hookrightarrow \Delta y = \frac{6500 \times \frac{3}{1.039}}{400} = 46.9\,m$$

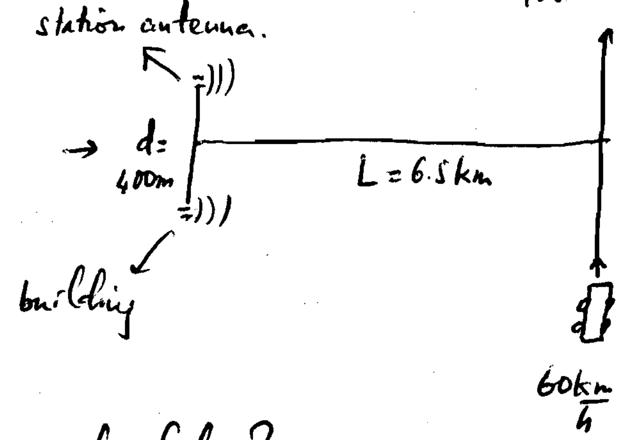
→ Time sep. b/w fades: $\dfrac{\Delta y}{v} = \dfrac{46.9\,m}{60\,\frac{km}{h}\cdot\frac{1h}{3600s}\cdot\frac{1000m}{1km}} = 2.82s$

Every 2.82s you would hear a fade in the radio signal

$T = 2.82s$ → How often: $\dfrac{1}{T} = \dfrac{1}{2.82s} =$

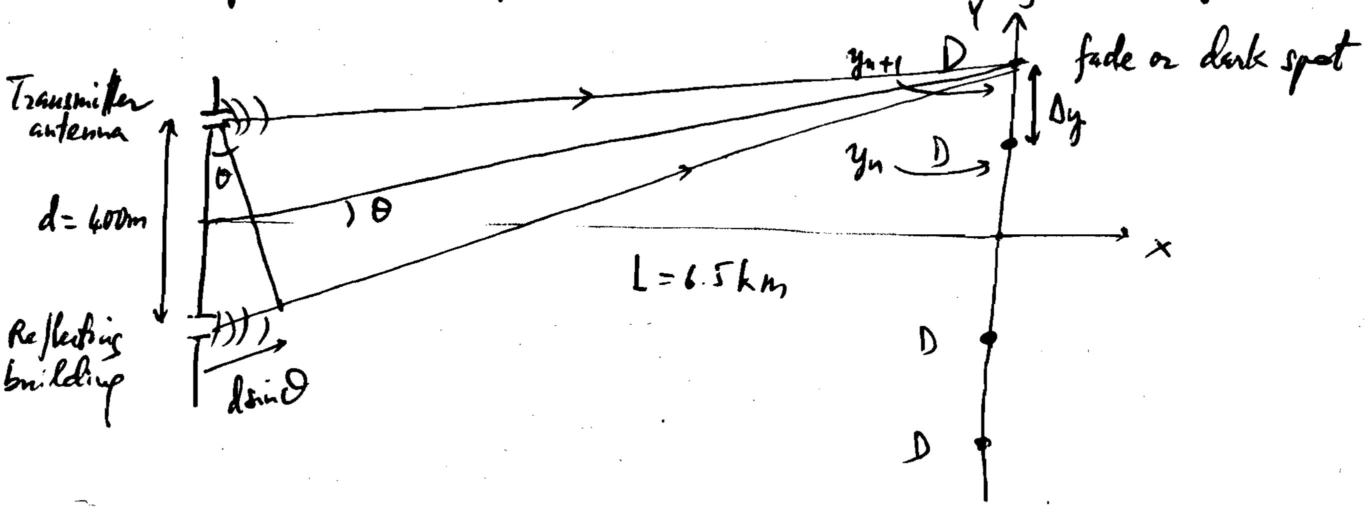
32.70

103.9 MHz FM radio →



How ~~many times~~ often the signal appears to fade?

→ since there are "dark" and "bright" spots along that road due to interference b/w the two identical waves from antenna of station & the reflecting building.



$$\frac{\Delta y}{v} = \text{~~how many~~ time b/w fades while driving at speed } v$$

$$\Delta y = y_{n+1} - y_n = L\tan\theta_{n+1} - L\tan\theta_n$$

"Dark" spots or destructive interference : $d\sin\theta_n = (2n+1)\frac{\lambda}{2}$

$$\theta_n = \sin^{-1}\left[\frac{(2n+1)\lambda}{2d}\right]$$

$$\theta_{n+1} = \sin^{-1}\left[\frac{[2(n+1)+1]\lambda}{2d}\right]$$

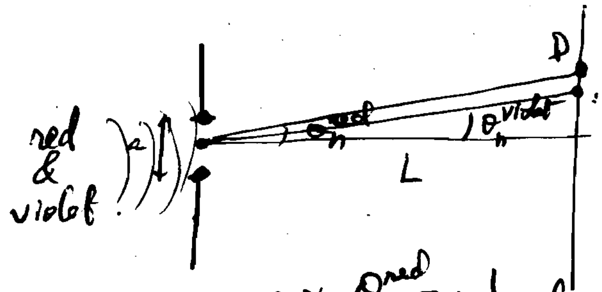
$$= \sin^{-1}\left[\frac{(2n+3)\lambda}{2d}\right]$$

32.42 Visible spectrum: $\lambda_v = 400\,nm \longrightarrow \lambda_r = 700\,nm$

violet (Higher f) red (lower f)

Lowest pair of consecutive orders for overlap b/w visible spectra as dispersed by a grating?

$n_v > n_r$



$a \sin\theta_n^{red} = n\lambda_{red}$

$\sin\theta_n^{red} = \frac{n\lambda_{red}}{a}$

For a same order n spots for red light are further out than spot for violet light

↓

overlap if spot (violet) of order n+1 coincide with spot (red) of order n.

$$\sin\theta_n^{red} = \frac{n\lambda_{red}}{a} = \sin\theta_{n+1}^{violet} = \frac{(n+1)\lambda_{violet}}{a}$$

$$\rightarrow \quad n\lambda_{red} = (n+1)\lambda_{violet}$$

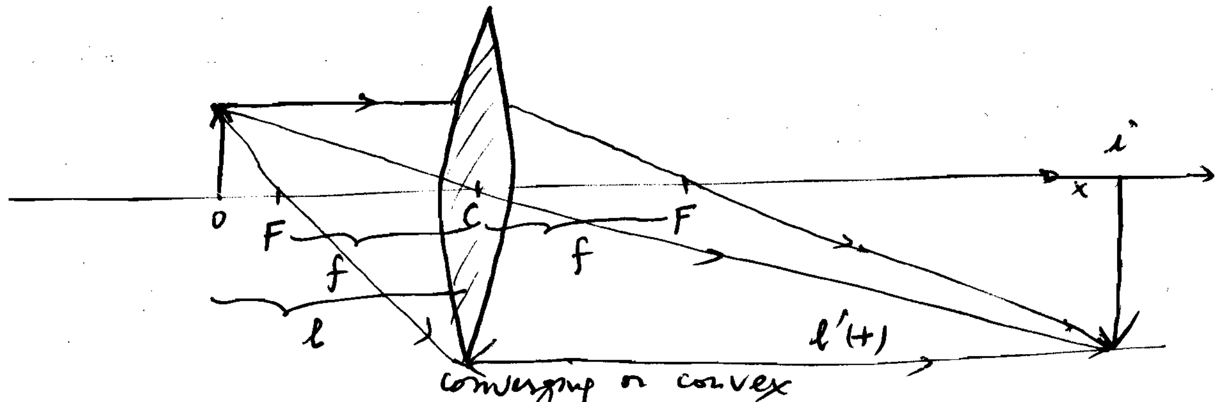
$$n(\lambda_{red} - \lambda_{viol}) = \lambda_{viol} \rightarrow n = \frac{\lambda_{violet}}{\lambda_{red} - \lambda_{violet}}$$

$$= \frac{4}{7-4} = \frac{4}{3} = 1.33.$$

n: integer only $\rightarrow$ $n = 2$ (red)

$n+1 = 3$ (violet)

31.50



$f = 35$ cm

a) $l = 40$ cm $\rightarrow$ sep. b/w o & i ?

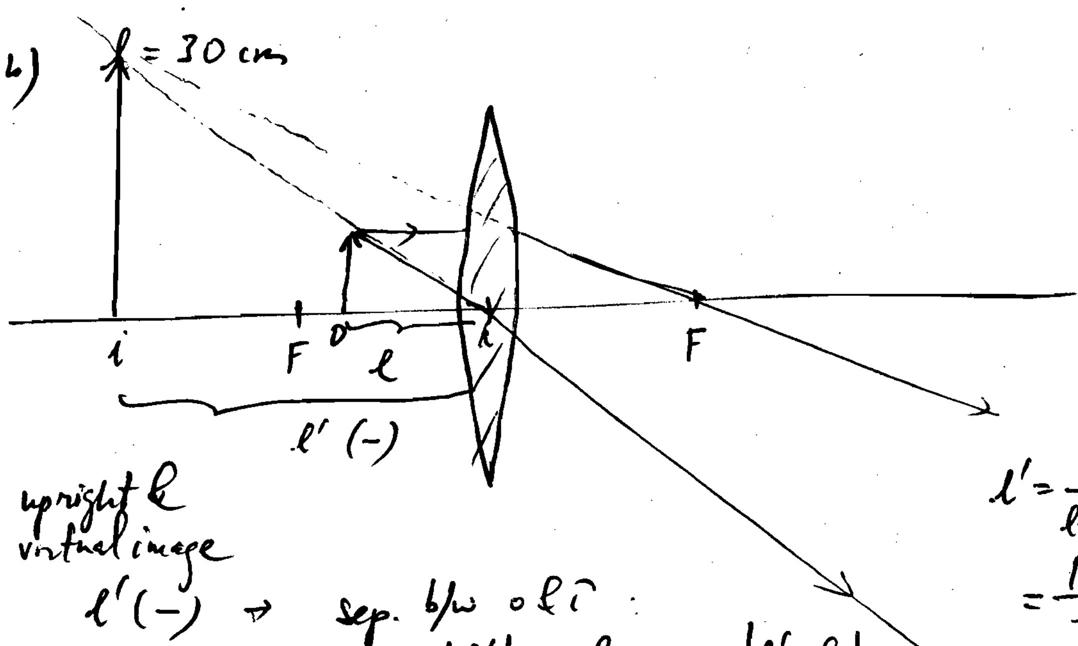
l' is + (Image: the other side of lense) } sep o & i is $l + l'$

$$\frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \rightarrow \frac{1}{l'} = \frac{1}{f} - \frac{1}{l} = \frac{l-f}{lf} \text{ or } l' = \frac{lf}{l-f}$$

$$= \frac{40 \times 35}{40 - 35} = 280 \text{cm}$$

sep. o & i is 40 cm $+ 280$ cm $= 320$ cm.

b) $l = 30$ cm



upright & virtual image

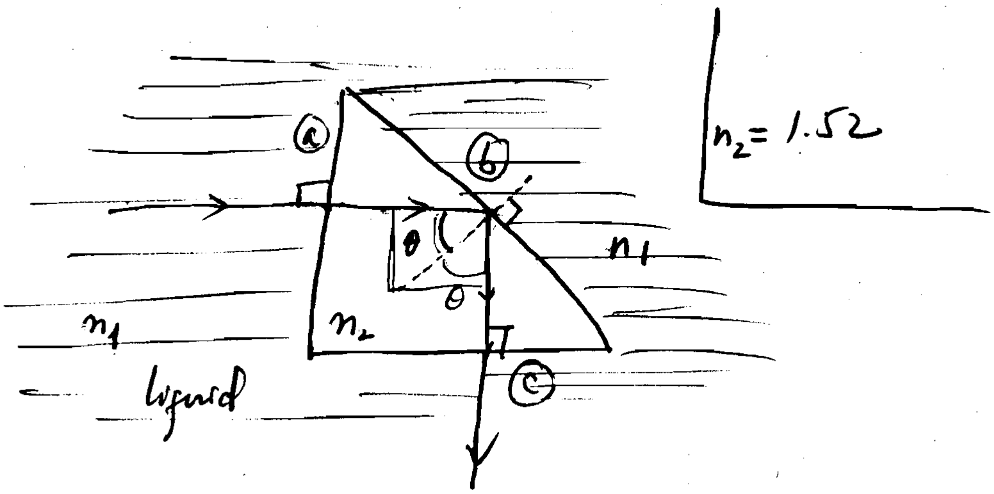
$l'(-) \rightarrow$ sep. b/w o & i :

$|l'| - l$ or $|l' + l|$

$\hookrightarrow |-210 + 30| = 180$cm.

$$l' = \frac{lf}{l-f} = \frac{30 \times 35}{30 - 35} = \frac{1050}{-5} = -210 \text{ cm}$$

30.44



What is smallest n_1 for no further total reflection:

1) at ⓐ & ⓒ same ray paths regardless $n_1 = 1$ or larger (liquid)

2) at ⓑ $\theta \geq \theta_c$ since there was a total internal reflection when $n_1 = 1$ ($n_2 \sin\theta_c = 1$
$\rightarrow \theta_c = \sin^{-1}\left(\frac{1}{n_2}\right)$)

When prism is in a liquid: $n_1 > 1 \rightarrow \theta_c' = \sin^{-1}\left(\frac{n_1}{n_2}\right)$

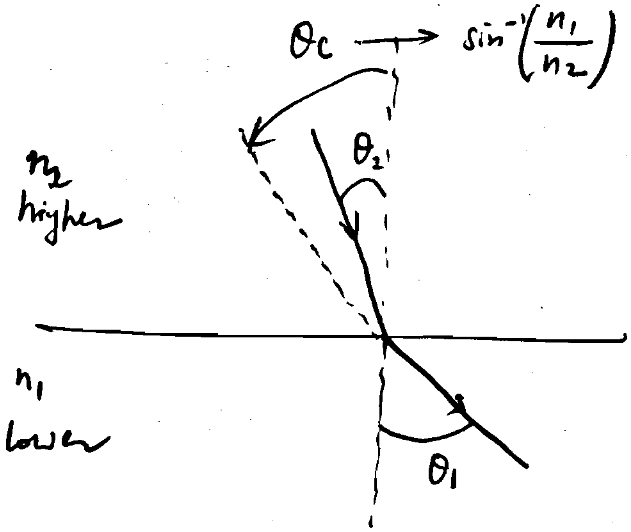
What should be n_1 (liquid) so $\boxed{\theta \leq \theta_c'}$? $\rightarrow$ no longer total internal reflection at boundary ⓑ.

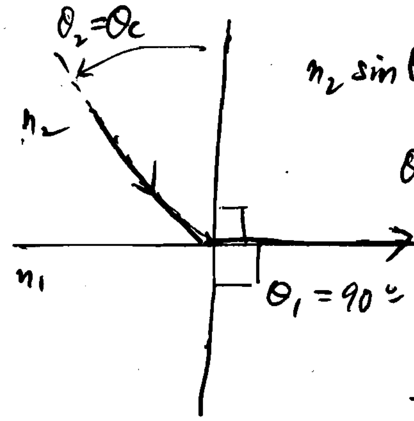
From geometry: $\theta = 45°$ $\quad 45° \leq \sin^{-1}\left(\frac{n_1}{n_2}\right)$

$$\sin 45° \leq \sin\left(\sin^{-1}\frac{n_1}{1.52}\right)$$

$$1.52 \sin 45° \leq n_1$$

$$n_{1\,min} = 1.52 \sin 45° = 1.07$$

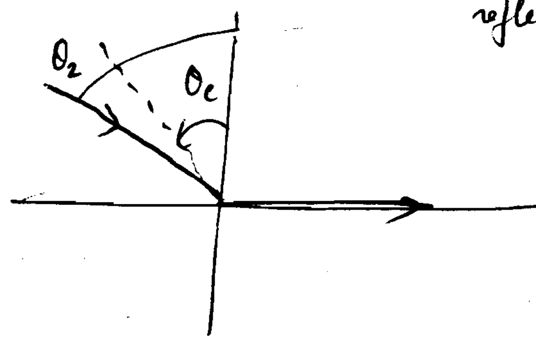


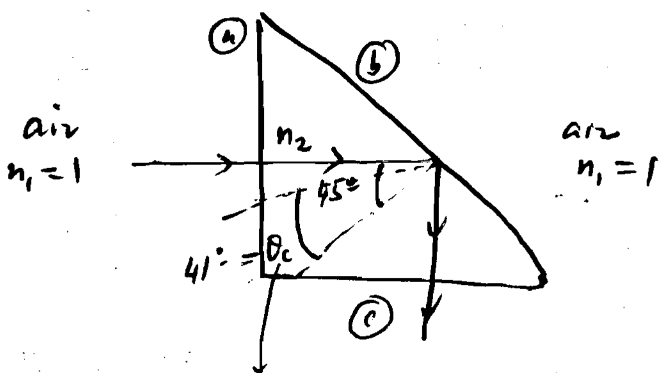


$n_2 \sin\theta_c = n_1 \underbrace{\sin 90°}_{1}$

$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$

Critical angle for Total internal reflection.





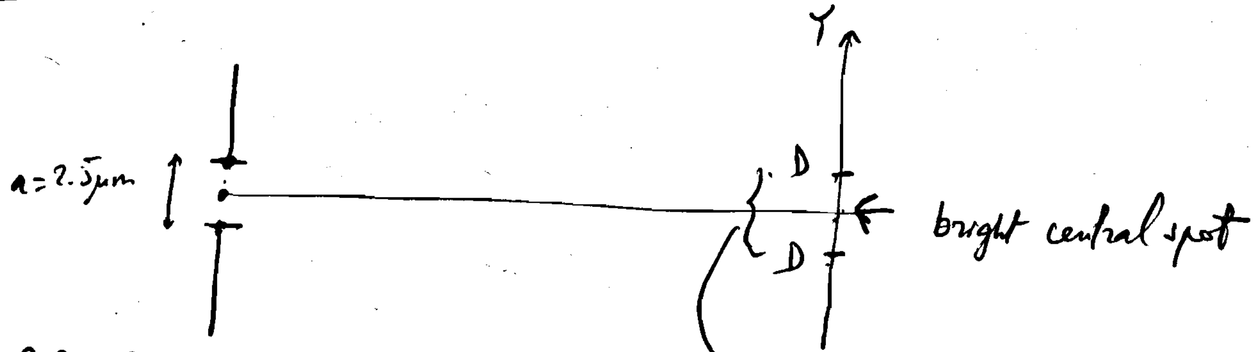
$\theta_c = \sin^{-1}\left(\frac{1}{1.52}\right) = 41°$ since $\theta = 45° \geq \theta_c = 41°$

→ Total internal reflection at b

When prism is in a liquid $n_1 > 1$ → θ_c' is larger than θ → $(\theta < \theta_c')$ ⇒ no longer total internal reflection at b

32.27

$\lambda = 633\,nm$



Diffraction

Width of central peak ? $\rightarrow$ $2\theta_1$: θ_1 angle for 1st dark spot.

$\hookrightarrow$ Dark spot : $a \sin\theta_m = m\lambda$ $\rightarrow$ $\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right)$

m: order.

$= \sin^{-1}\left(\frac{633 \times 10^{-9}}{2.5 \times 10^{-6}}\right)$

$= 14.7°$.

$\hookrightarrow 2 \times 14.7° = 29.3°$.