

Ch 27: Inductance & Magnetic Energy

- Capacitors as storage devices for electric energy

$$\text{Capacitance } C = \frac{Q}{V}$$

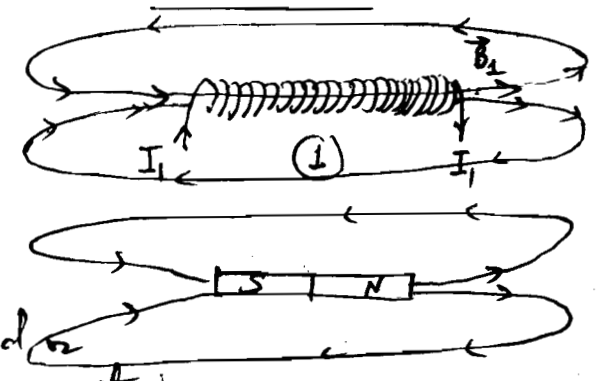
- Inductors as storage devices for magnetic energy

$$\text{Mutual inductance } M = \frac{\Phi_2}{I_1}$$

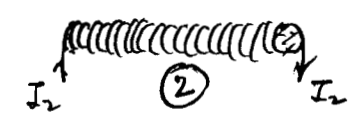
2 solenoids:

$$B_1 = \mu_0 n_1 I_1$$

turns per unit length.



Solenoid #2 electromagnet is similar to a natural magnet bar.



Solenoid #2 is within the magnetic field B_1 created by #1

Φ_2 = magnetic flux through solenoid #2 due to the B_1 created by solenoid #1

$\Phi_2 = M I_1$, if I_1 depends on time $\rightarrow \Phi_2$ also depends on time \rightarrow Faraday's law: there will be an induced emf in solenoid #2

$$[-\mathcal{E}_2 = \frac{d\Phi_2}{dt} = M \frac{dI_1}{dt}]$$

$\rightarrow M$ "mutual inductance" = since it relates the induced voltage in #2 with a changing current in #1

1) Question: how about voltage induced in #1 due to changing current in #2?

$$-\varepsilon_1 = M \frac{dI_2}{dt}$$

↳ same mutual inductance!

2) Question: would \vec{B}_1 created by solenoid #1, produce a magnetic flux through itself? Yes! ϕ or self-magnetic flux.

$$L = \frac{\phi}{I}$$

self-flux
self inductance
self current

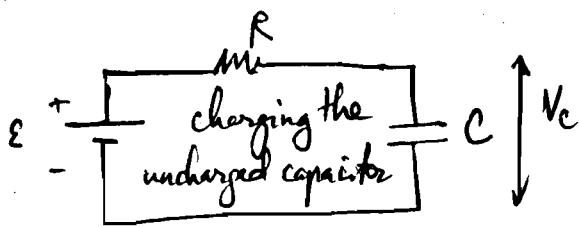
$\phi = LI \rightarrow$ Faraday's law: if a self current depends on time
 \rightarrow self flux depends on time
 \rightarrow self induced emf:

$$-\varepsilon = L \frac{dI}{dt}$$

↑ inductance
↓ self induced voltage
self current

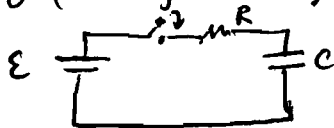
Units: L & M $\Rightarrow [L] = \frac{[\varepsilon]}{\frac{[I]}{[t]}} = \frac{V}{\frac{A}{s}} = H \text{ (Henry)}$
 SI.

RC circuit



V_c does not change instantaneously
 ↓
 electric inertia

$t=0$ (circuit just closed): $V_c=0$
 short-circuit across C

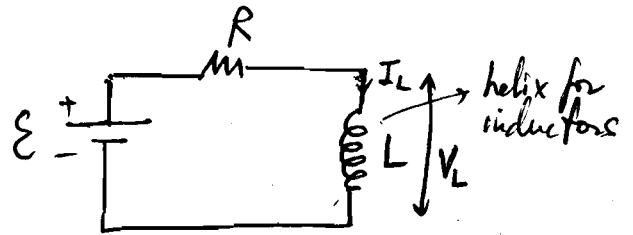


$t=\infty$ (long after): $I_c=0$
 open-circuit across C

$$I_c = \frac{\epsilon}{R} e^{-\frac{t}{RC}}$$

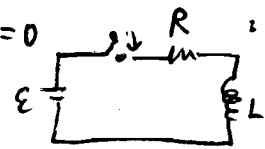
(current decreases exponentially from its max. $\frac{\epsilon}{R}$)

RL circuit



I_L does not change instantaneously
 ↓
 magnetic inertia

$t=0$: $I_L=0, V_L=\epsilon$
 open circuit across L



$t=\infty$ (long after) : short-circuit across L
 $V_L=0$

$$V_L = \epsilon e^{-\frac{t}{L/R}}$$

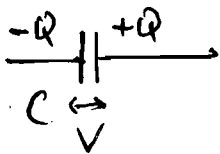
(voltage decreases exponentially from its max ϵ)

$$U_c = \frac{1}{2} CV^2$$

$$U_L = \frac{1}{2} LI^2$$

Magnetic energy

Electric energy & capacitors



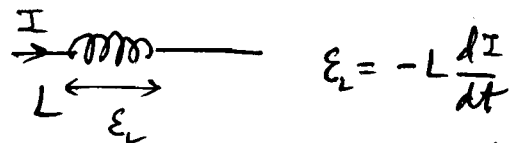
Energy $U_c = \frac{1}{2} CV^2$ (J)

Energy density $u_c = \frac{U_c}{Al} = \frac{1}{2} \epsilon_0 E^2$ ($\frac{J}{m^3}$)

cross sect. area of plates
 separation l/w plates

Parallel plates : $C = \frac{A\epsilon_0}{l}$

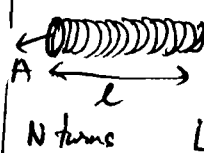
Magnetic energy & inductors



$$\epsilon_L = -L \frac{dI}{dt}$$

$$U_L = \int_0^t P_L dt = \int_0^t I |\epsilon_L| dt = L \int_0^t I \frac{dI}{dt} dt = \frac{1}{2} L [I^2]_0^t = \frac{1}{2} LI^2$$
 (J)

$$u_L = \frac{U_L}{Al} = \frac{\frac{1}{2} LI^2}{Al} = \frac{1}{2} \frac{\mu_0 N^2}{Al^2} I^2 = \frac{1}{2} \frac{B^2}{\mu_0}$$

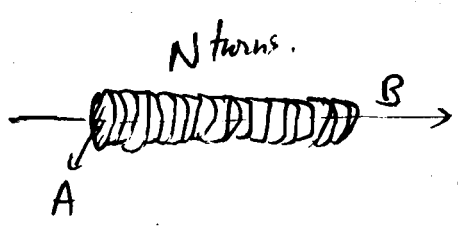


What is L for a solenoid?

$$L = \frac{\Phi}{I} = \frac{BAN}{I} = \frac{\mu_0 \frac{N}{l} I AN}{I} = \mu_0 \frac{N^2 A}{l}$$

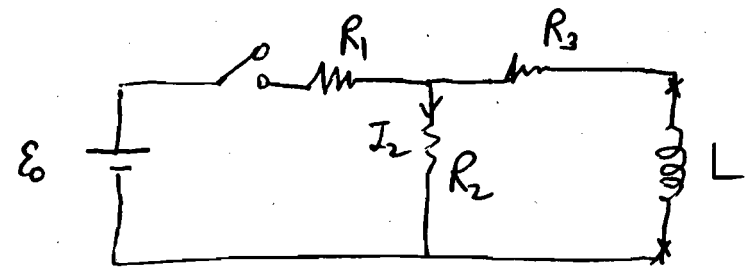
Magnetic flux: $\Phi = \oint \mathbf{B} \cdot d\mathbf{A} = B \cdot A$

\downarrow
B uniform



Flux through one turn is BA
Flux through solenoid is NBA

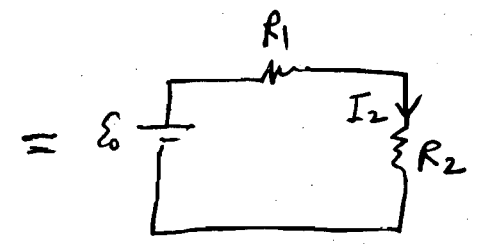
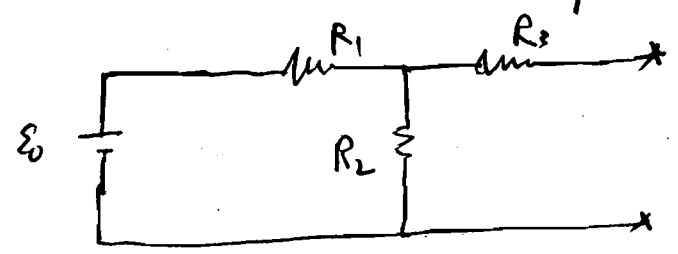
27.62



- $\epsilon_0 = 12V$
- $R_1 = 4\Omega$
- $R_2 = 8\Omega$
- $R_3 = 2\Omega$
- $L = 2H$

c) Find I_2 right after switch is closed.

$t=0 \rightarrow$ inductor: $I_L = 0$: open circuit across L



$$I_2 = \frac{\epsilon_0}{R_1 + R_2} = \frac{12V}{12\Omega}$$

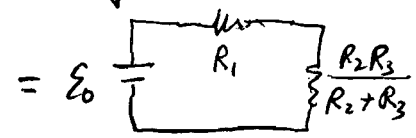
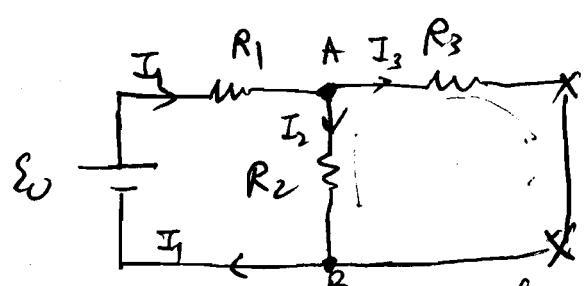
$$I_2 = 1A$$

b) Find I_2 long after current is closed

$t = \infty \rightarrow$ inductor: $V_L = 0 \rightarrow$ short-circuit: or connected by a piece of wire of zero resistance



shock at low body resistance an example of shortcircuit.



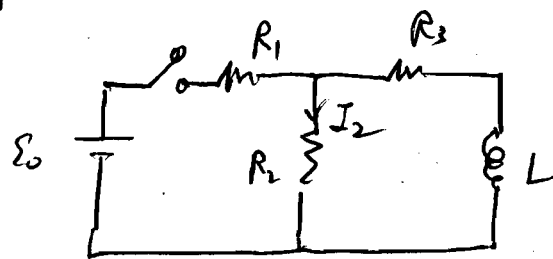
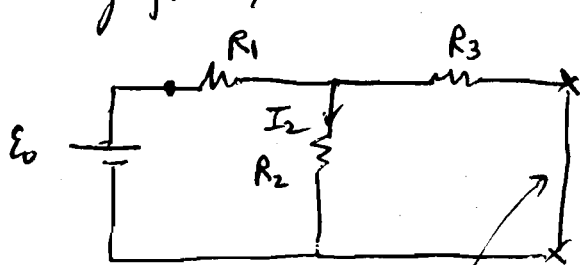
First find I_1 , then I_2 by current division

$$I_1 = \frac{\epsilon_0}{R_1 + (R_2 \parallel R_3)} = \frac{\epsilon_0}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{12V}{4 + \frac{8 \times 2}{8 + 2}} = \frac{12}{4 + 1.6} = \frac{12}{5.6} = 2.14A$$

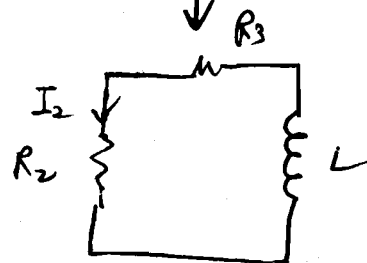
$$I_1 \text{ divides into } I_2 \text{ \& } I_3 : \begin{cases} I_2 = I_1 \frac{R_3}{R_2 + R_3} = 2.14 \frac{2}{8 + 2} = 0.429A \\ I_3 = I_1 \frac{R_2}{R_2 + R_3} = 2.14 \frac{8}{8 + 2} = 1.71A \end{cases}$$

$$I_2 + I_3 = 0.429 + 1.71 = 2.14A \checkmark$$

c) Long after, now switch is reopened: I_2 ?



$t = \infty$
Inductor acts like a short circuit but it is still there!



$$I_2 = 0.429A$$

$\left\{ \begin{array}{l} \rightarrow L \text{ stored energy} \\ \rightarrow L = \text{opposes changes in } I \\ \quad \hookrightarrow \text{inertia} \end{array} \right.$

$\left\{ \begin{array}{l} \frac{1}{2} m v^2 = \text{mass inertia for speed (opposes change of speed)} \\ \frac{1}{2} C V^2 = \text{Capacitors oppose change in } V_c ! \\ \frac{1}{2} L I^2 = \text{inductors oppose change in } I_L ! \end{array} \right.$

27.70

$$u_e = \frac{1}{2} \epsilon_0 E^2 \quad ; \quad u_m = \frac{1}{2\mu_0} B^2$$

$$u_e = u_m \rightarrow \frac{E}{B} ?$$

$$\frac{u_e}{u_m} = 1 = \frac{\frac{1}{2} \epsilon_0 E^2}{\frac{1}{2\mu_0} B^2} = \epsilon_0 \mu_0 \frac{E^2}{B^2} = 1$$

$$\rightarrow \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{8.85 \times 10^{-12} \times 4\pi \times 10^{-7}}} = 2.99 \times 10^8 \frac{m}{s} = c$$

speed of light
↑

$$= \frac{1}{\sqrt{\frac{C^2}{Nm^2} \cdot \frac{N}{A^2}}} = \frac{1}{\frac{C}{A \cdot m}} = \frac{A \cdot m}{C}$$

$$= \frac{\frac{C}{s} \cdot m}{C} = \frac{m}{s}$$

Ch 29 Maxwell's equations & EM waves.

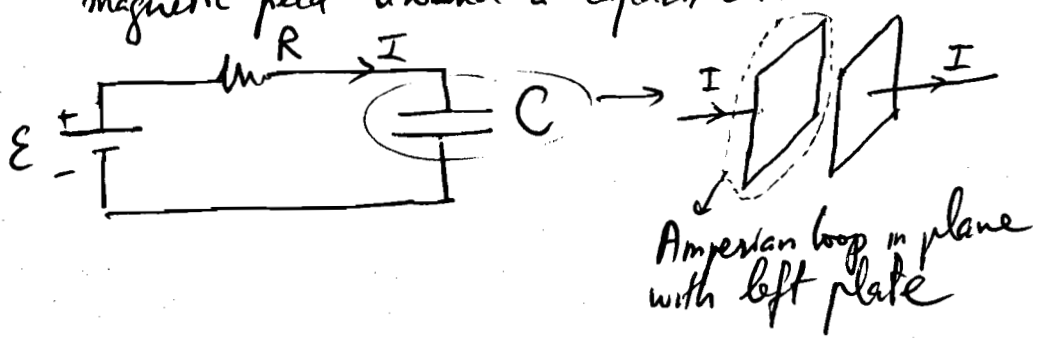
- 1) Gauss's law $\oint_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$
- 2) Analog of Gauss's law for magnetic field $\oint \vec{B} \cdot d\vec{A} = 0$ (N & S goes together)
- 3) Ampere's law $\oint_{\text{closed loop}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} + \mu_0 I_{\text{displacement}}$
Maxwell's contribution
- 4) Faraday's law $\oint \vec{E} \cdot d\vec{l} = - \frac{d\phi_B}{dt} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$
induced voltage

a time-varying magnetic field creates an electric field

$$\mu_0 \epsilon_0 \frac{d\phi_E}{dt} = \mu_0 \epsilon_0 \underbrace{\int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}}_{I_{\text{displacement}}}$$

Hint: 1) This would complete a similarity b/w Ampere & Faraday's law \rightarrow we can then say that also a time-varying electric field creates a magnetic field \rightarrow Electromagnetic waves which can propagate in vacuum.

2) Provide an explanation for the measured magnetic field around a capacitor in a RC circuit

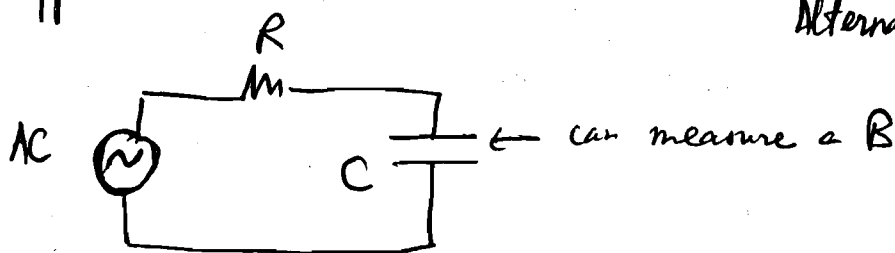


→ Without the Maxwell's term: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$
 Amperian loop

Note: Does I cross the Amperian loop? No
 → current enclosed by this loop is 0

$$\oint \vec{B} \cdot d\vec{l} = 0 \rightarrow \vec{B} = 0$$

However there is a measured \vec{B} when AC power is applied
 ↓
 Alternating Current



→ With $\mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$: we can explain the presence of \vec{B} due a time-varying \vec{E} b/w plates.

$$\mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A} = \mu_0 I_{\text{displacement}}$$

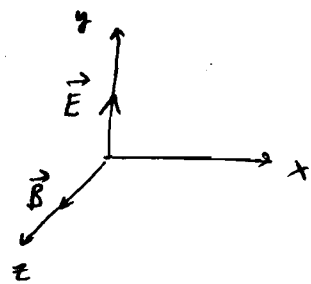
$I_{\text{displacement}}$ (they have dimension of current)

Electromagnetic waves in vacuum → no materials :
no charges, no currents
but yes \vec{E} , yes \vec{B}

- Maxwell's equations provide no info in vacuum
- 1) Gauss's law : $\oint \vec{E} \cdot d\vec{A} = 0$
 - 2) $\oint \vec{B} \cdot d\vec{A} = 0$
 - 3) Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$
 - 4) Faraday's law $\oint \vec{E} \cdot d\vec{l} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$
- $\vec{E} \rightarrow \vec{B} \rightarrow \vec{E} \rightarrow \vec{B} \rightarrow \dots$: EM waves.
time-varying fields

$$\vec{E} = E_p \sin(kx - \omega t) \hat{j}$$

$$\vec{B} = B_p \sin(kx - \omega t) \hat{k}$$

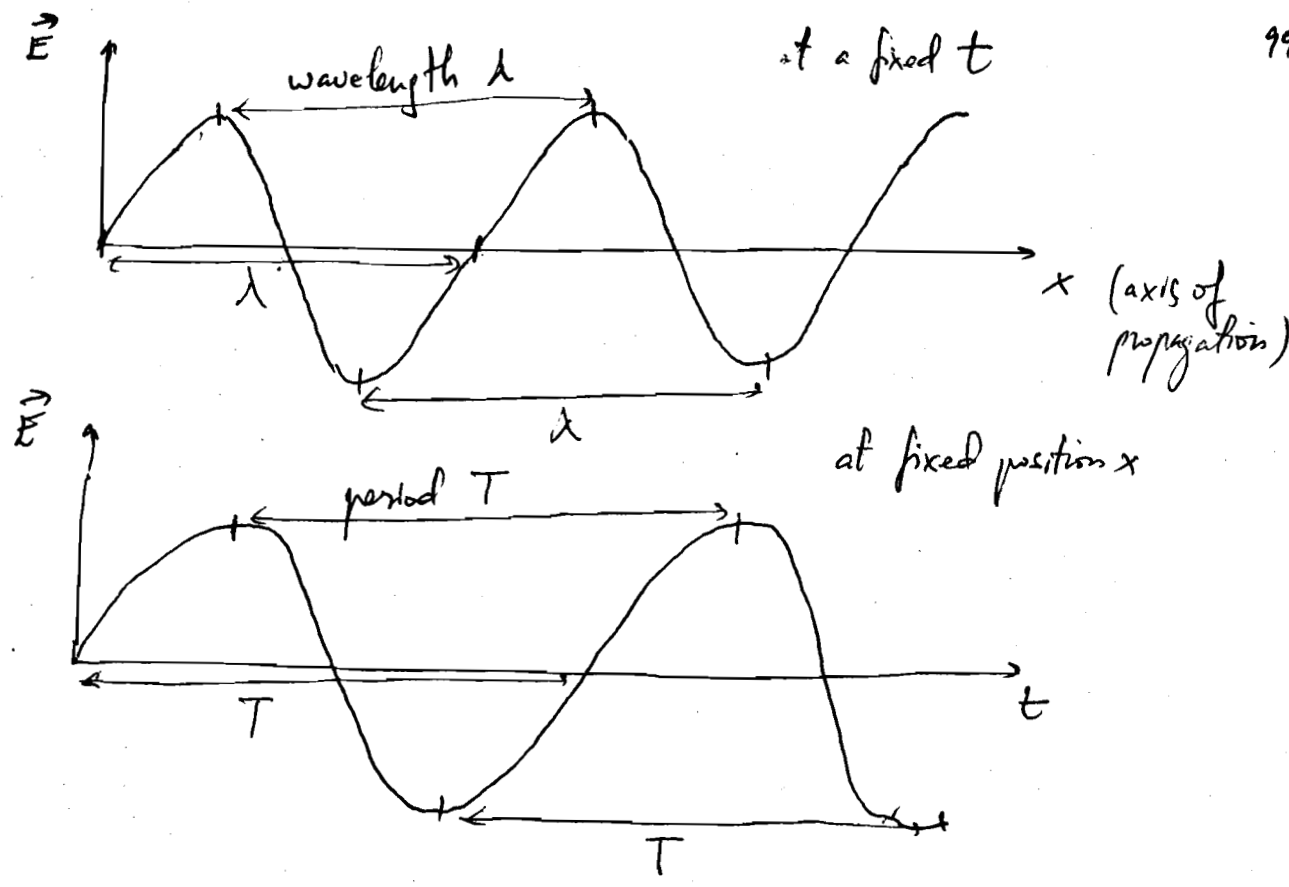


Direction of propagation is +x
given by $\vec{E} \times \vec{B}$ (RHR)

(line fingers of right hand along \vec{E} , turn them toward \vec{B} , thumb is direction of propagation)

$$\vec{E} = E_p \sin(kx - \omega t)$$

E_p → magnitude or amplitude
 kx → position
 ωt → time
 k → wave number = $\frac{2\pi}{\lambda}$ → wave length.
 ω → angular frequency = $\frac{2\pi}{T}$



Maxwell's equations in vacuum = differential forms
 (involving derivatives instead of integrals)

Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A} \rightarrow \frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

Faraday's law: $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} \rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$

integral forms differential forms.

$E = E_p \sin(kx - \omega t) \rightarrow \frac{\partial E}{\partial x} = k E_p \cos(kx - \omega t)$
 $B = B_p \sin(kx - \omega t) \rightarrow -\frac{\partial B}{\partial t} = \omega B_p \cos(kx - \omega t)$
 Faraday's law in differential form = $k E_p \cos(kx - \omega t) = \omega B_p \cos(kx - \omega t)$

(a) $\frac{E_p}{B_p} = \frac{\omega}{k} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} = c$

Ampere's law in differential form: $k B_p = \mu_0 \epsilon_0 \omega E_p$

(b) $\rightarrow \frac{E_p}{B_p} = \frac{k}{\omega \mu_0 \epsilon_0}$

(a) & (b) : $\frac{\omega}{k} = \frac{k}{\omega \mu_0 \epsilon_0} \rightarrow \frac{\omega^2}{k^2} = \frac{1}{\mu_0 \epsilon_0}$
 $c^2 = \frac{1}{\mu_0 \epsilon_0}$

$\rightarrow c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^8 \text{ m/s}$

EM wave speed is $c = 3 \times 10^8 \text{ m/s}$

EM wave equation :

$\frac{\partial}{\partial t} \left(\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \right) \rightarrow \frac{\partial^2 B}{\partial t \partial x} = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$

$\frac{\partial}{\partial x} \left(\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \right) \rightarrow \frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$

$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$

wave equation for the electric field.

ultimately dictates the wave expression: $E = E_p \sin(kx - \omega t)$

Wave equation is similar to that for a transverse wave in a string or sound waves, etc. (mechanical wave).

The unique property of EM waves (can propagate in vacuum) is describe by Maxwell's equations ($\vec{E} \rightarrow \vec{B} \rightarrow \vec{E}$, etc.).

$$\frac{\partial}{\partial x} \left(\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \right) \rightarrow \frac{\partial^2 B}{\partial x^2} = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial x \partial t}$$

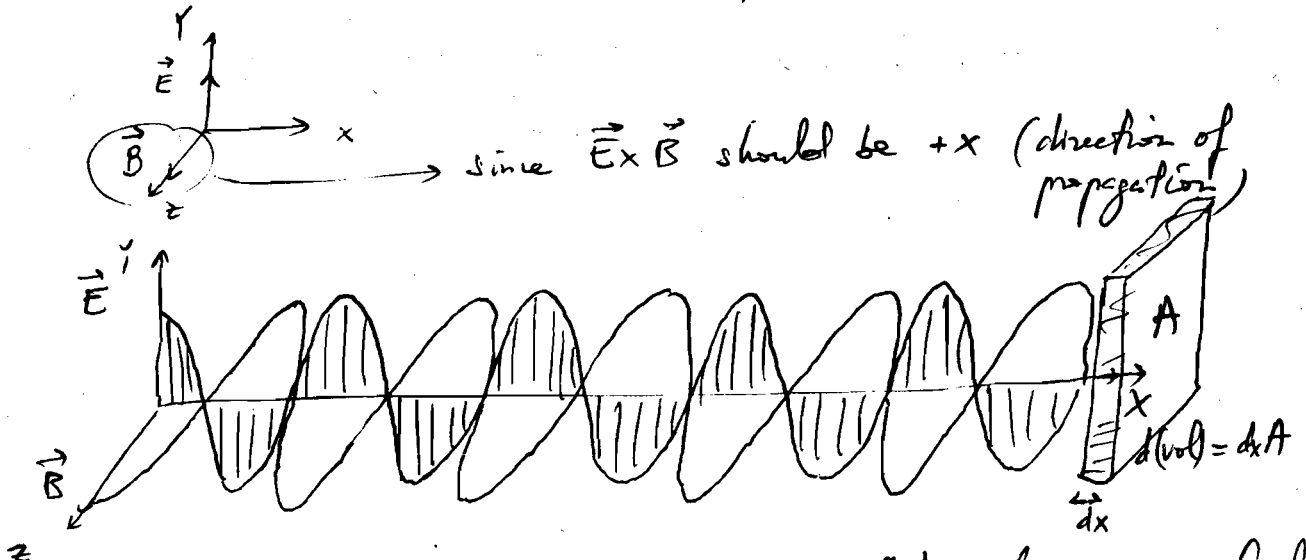
$$\frac{\partial}{\partial t} \left(\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \right) \rightarrow \frac{\partial^2 E}{\partial t \partial x} = -\frac{\partial^2 B}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

wave equation for the magnetic field

Intensity of EM waves:

- Propagation along x ; polarization along y
 (direction of \vec{E})



E & B propagate along x but oscillate along perpendicular directions

$$\text{Wave intensity } S = \frac{\frac{dU}{dt}}{\text{Area}} = \frac{uA/c}{A} = uc = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) c$$

$$dU = u d(\text{vol}) = u A dx \rightarrow \frac{dU}{dt} = u A \frac{dx}{dt}$$

wave speed
c

→ Wave intensity $S = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) c$

$$\frac{E}{B} = c \rightarrow B = \frac{E}{c}$$

$$S = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{E^2}{c^2 \mu_0} \right) c = \frac{1}{2} (\epsilon_0 E^2 + \epsilon_0 E^2) c$$

$$\left. \begin{array}{l} \frac{1}{\mu_0 \epsilon_0} = c^2 \\ \frac{1}{c^2 \mu_0} = \epsilon_0 \end{array} \right\} \Rightarrow \boxed{S = \epsilon_0 E^2 c}$$

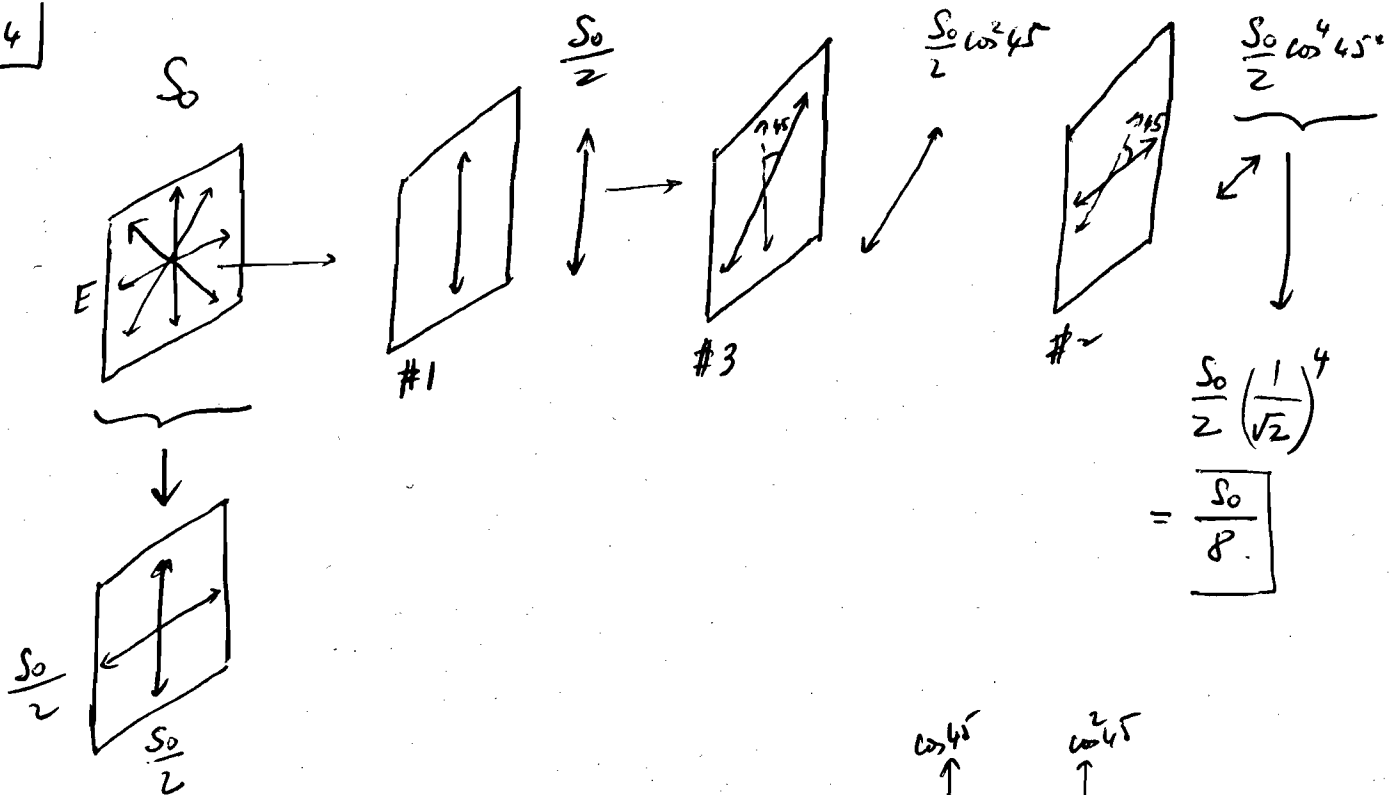
→ wave intensity: $S = \epsilon_0 E^2 c = \epsilon_0 c^2 E \underbrace{\left(\frac{E}{c} \right)}_B = \frac{EB}{\mu_0}$

$$\boxed{S = \frac{EB}{\mu_0}}$$

More generally: $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

(intensity points in the direction of propagation!)

29.44



Vectors E are doing components

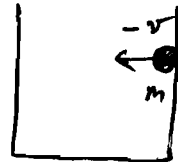
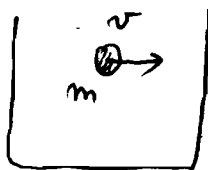
$$S = \frac{EB}{\mu_0} = E^2 c \epsilon_0$$

Polarization property of EM wave.

29.56

Radiation Pressure:

→ Air pressure:

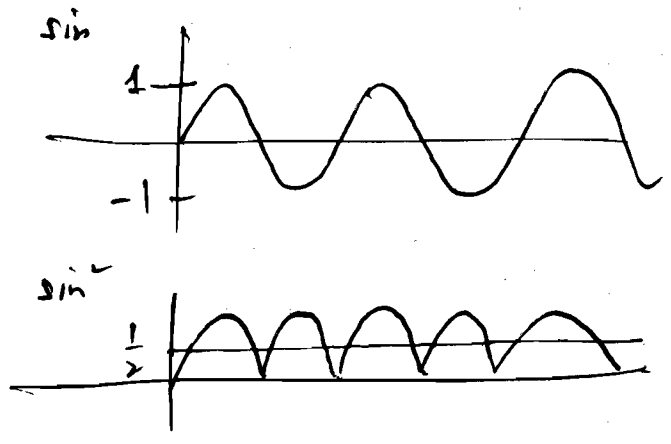
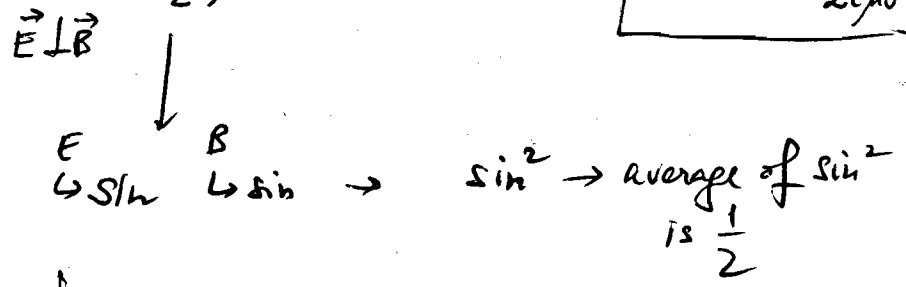


↓
 $\Delta p = 2mv$
 Momentum transfer to well
 $\hookrightarrow P = \frac{F}{A} = \frac{\frac{\Delta p}{\Delta t}}{A}$

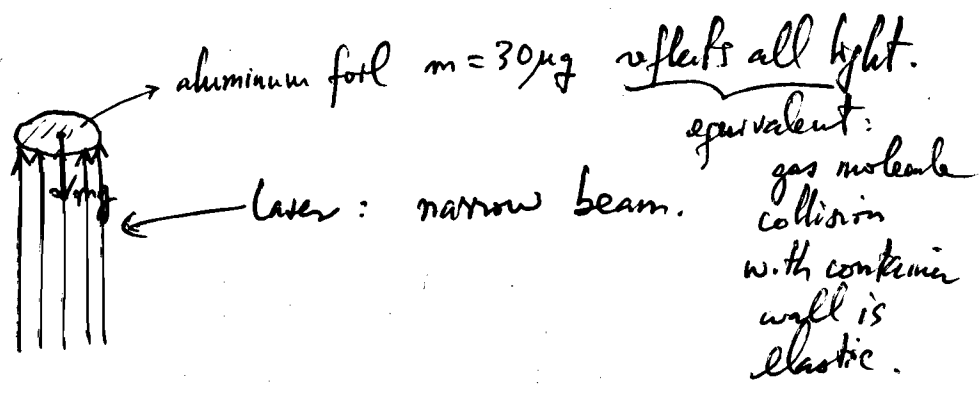
→ Momentum of EM wave or radiation is $p = \frac{U}{c}$

→ Radiation pressure = $\frac{\overline{F}}{A} = \frac{\overline{\frac{dp}{dt}}}{A} = \frac{\frac{1}{c} \overline{\frac{du}{dt}}}{A} = \frac{\overline{S}}{c}$

Since $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \rightarrow \overline{S} = \frac{1}{2} \frac{EB}{\mu_0}$ → Rad. Press = $\frac{EB}{2c\mu_0}$



29.56



Laser Power?

Rad. pressure = $\frac{\overline{S}}{c}$

Radiation force = Rad. press × A = mg

$\frac{\overline{S}}{c} \times 2 \times A = mg$

$S = \frac{P}{A}$ $\frac{P}{Ac} \times 2 \times A = mg \rightarrow \overline{P} = \frac{mgc}{2}$

$\overline{P} = \frac{30 \times 10^{-6} \times 9.81 \times 3 \times 10^8}{2} = 44.1 \text{ W} \rightarrow \text{EM waves or radiation can exert pressure!}$

Rad. momentum is $p = \frac{U}{c}$

Rad. pressure is $\frac{\bar{F}}{A} = \frac{\frac{dp}{dt}}{A} = \frac{\frac{1}{c} \frac{dU}{dt}}{A} = \frac{\bar{S}}{c}$

→ For transfer (momentum) to something that reflects all the light (~ elastic collision) → need extra factor of 2!

→ Rad pressure = $\frac{\bar{S}}{c} \times 2$

(→ Reflect all light or elastic collision of photons
light "particles"
or light quanta)

29.59

"Photon rocket"

Rocket: gas ejected for rocket to get thrust.

Thrust = $F = 35 \times 10^6 \text{ N}$ → Power of light source?

↳ Radiation force = F

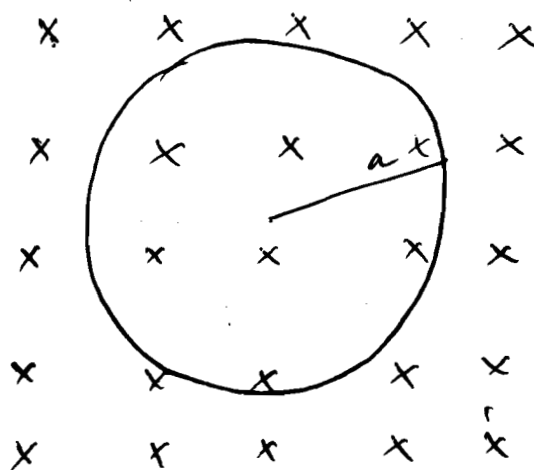
$\frac{\bar{S}}{c} \times A$

R.press.

$\frac{\bar{P}}{A \cdot c} \times A \rightarrow \bar{P} = Fc = 35 \times 10^6 \text{ N} \times 3 \times 10^8 \frac{\text{m}}{\text{s}}$
 $= 105 \times 10^{14} \frac{\text{J}}{\text{s}}$

$\bar{P} \approx 10^{16} \text{ W}$

27.53

Resistance R

B uniform, into page

Magnetic flux:

$$\Phi_B = B \cdot A$$

B changes in time
from $B_1 \longrightarrow B_2$

$$\int I_{\text{induced}} \cdot dt = \int \frac{dq}{dt} dt = Q$$

Show $Q = \frac{\pi a^2}{R} (B_2 - B_1)$

$$Q = \int I_{\text{induced}} \cdot dt = \int \frac{\mathcal{E}}{R} dt = \int \frac{\left| -\frac{d\Phi_B}{dt} \right|}{R} dt$$

$$= \left| -\frac{1}{R} [\Phi_B]_1 \right| = \left| -\frac{1}{R} [B_2 A - B_1 A] \right|$$

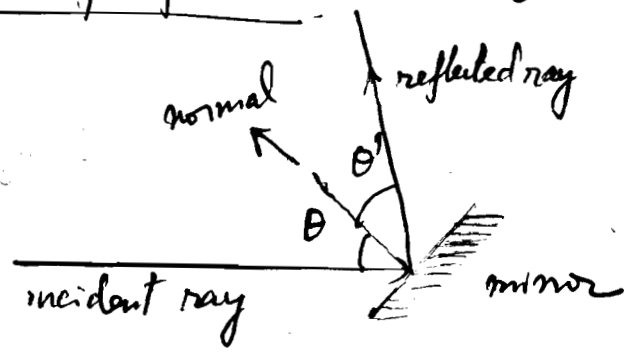
$$= \left| \frac{A}{R} (B_1 - B_2) \right| = \left| \frac{\pi a^2}{R} (B_1 - B_2) \right| = \frac{\pi a^2}{R} (B_2 - B_1)$$

Ch. 30 Reflection & Refraction

Geometrical optics : propagation of light using light rays
 (ignore wave properties such as interference, diffraction, polarization)
 ↳ Physical Optics

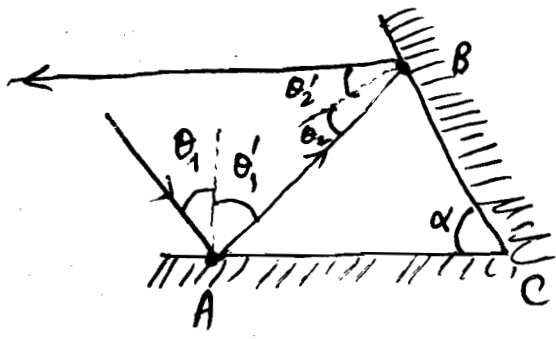
Rays propagate in straight lines

Law of reflection : $\theta' = \theta$



inc. ray forms angle θ w.r.t. normal to the reflecting surface.

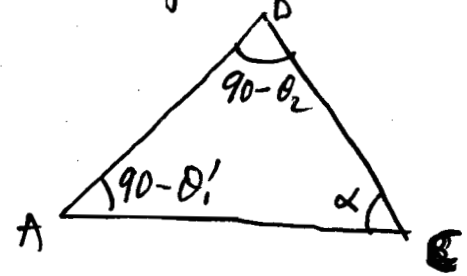
Multiple reflections (see movies of on optical paths in fibers)



$\theta_1' = \theta_1$
 $\theta_2' = \theta_2$

Can get θ_2' from θ_1 :

Triangle ABC

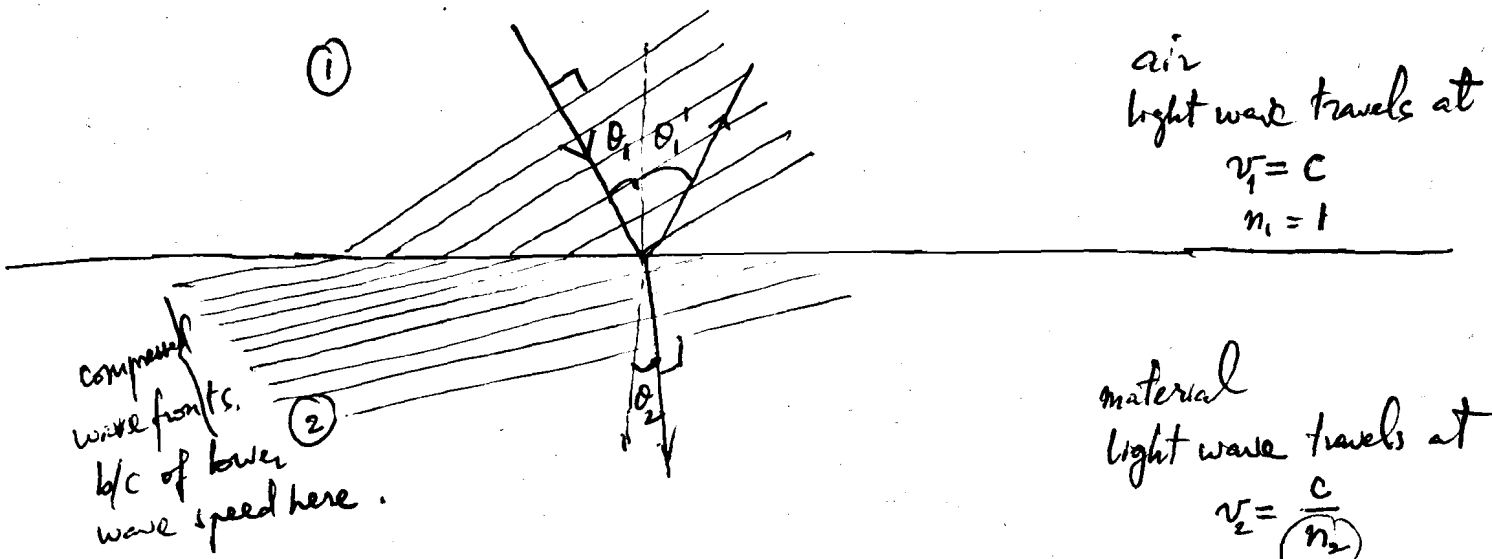
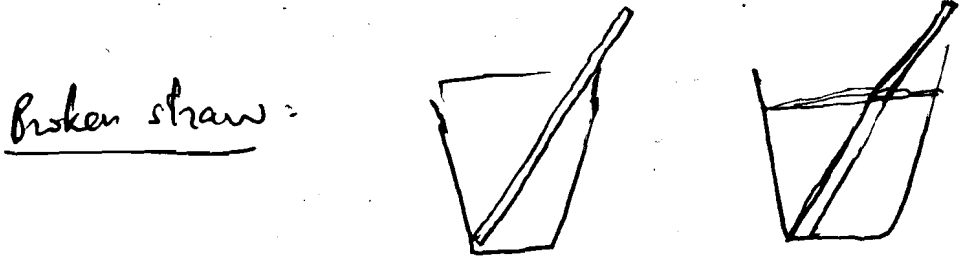


$180^\circ = 90 - \theta_1' + 90 - \theta_2 + \alpha$
 $0 = -\theta_1' - \theta_2 + \alpha$

$\theta_2 = \alpha - \theta_1' = \alpha - \theta_1$

$\theta_2' = \alpha - \theta_1$

Refraction : when light rays travel from one material to a different material

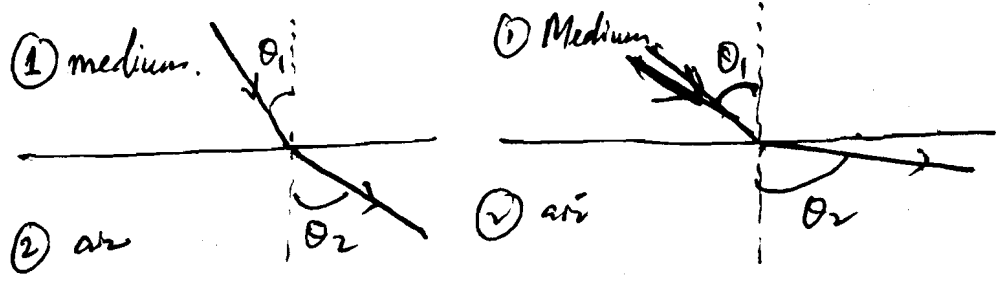


Snell's law or law of refraction :

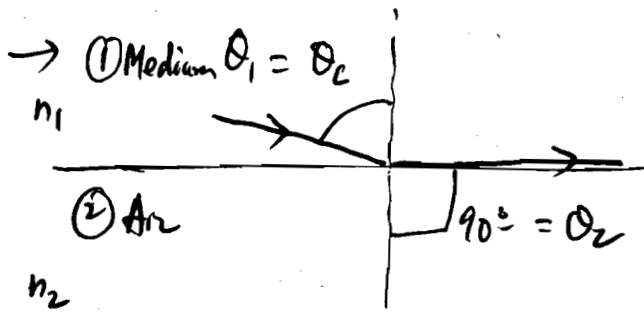
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

\downarrow inc. angle \downarrow refracted angle

Air \rightarrow medium : θ_2 (refracted angle) $<$ θ_1 (incident angle)
 Medium \rightarrow air : θ_2 (refracted angle) $>$ θ_1 (incident angle)



When $\theta_1 = \theta_c$ (critical angle) $\rightarrow \theta_2 = 90^\circ$



at $\theta_1 = \theta_c$ we have total internal reflection (going from higher index n_1 to a lower index n_2)

$$\hookrightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2 \rightarrow \left\{ \begin{array}{l} \theta_1 = \theta_c \\ \theta_2 = 90^\circ \end{array} \right\} \rightarrow n_1 \sin \theta_c = n_2 \frac{\sin 90^\circ}{1}$$

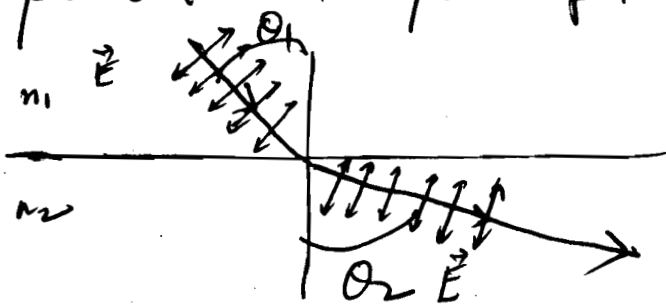
$$\theta_c = \sin^{-1} \frac{n_2}{n_1} < 1$$

\rightarrow This was a situation where we have all reflection, & no refraction!

\rightarrow Any situation with all refraction and no reflection? Yes. It happens at the polarizing angle or Brewster's angle:

$$\theta_p \text{ or } \theta_B = \tan^{-1} \frac{n_2}{n_1}$$

\downarrow This polarizing angle only affects the electric field that is polarized in the plane of the page:



When $\theta_1 = \theta_p$ or θ_B and if \vec{E} oscillates only in the plane of page \rightarrow there is no reflection!

\rightarrow Taking picture of something behind glass

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If in general \vec{E} is polarized not in plane with page but at some angle \rightarrow it has a component \parallel page & a component \perp page. The component \parallel page, at Brewster incident angle gets no reflection, but that \perp page will get some reflection.