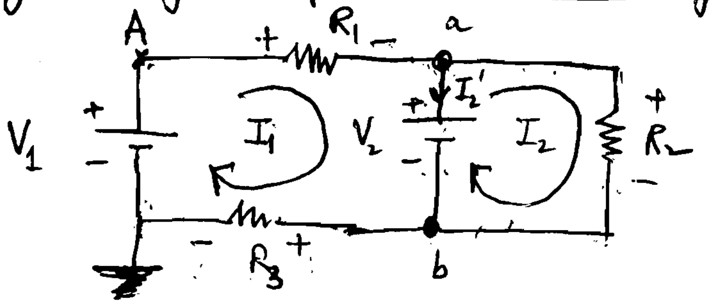
2) Resistors only, using loop & node analysis → Kirchoff's Laws



R_1 & R_2:
→ Not in parallel (not same voltage across)
→ Not in series (not same current thru)

Ground (zero potential)

Loop analysis

→ Total voltage difference across elements in a closed loop is 0.

→ Signs: assume a direction for current thru closed loop

1) Current thru battery − to + : voltage difference → positive
2) Current thru battery + to − : voltage difference → negative
3) Voltage difference is negative across a resistor (voltage drop across resistors)

1) $V_1 - I_1R_1 - V_2 - I_1R_3 = 0$

2) $V_2 - I_2R_2 = 0$

$$\begin{cases} I_1 = \dfrac{V_1 - V_2}{R_1 + R_3} & \text{(same current thru } R_3\text{)} \\ I_2 = \dfrac{V_2}{R_2} \end{cases}$$

Node analysis

→ Total current at any node is 0.

→ Signs:
1) Current into node → positive
2) Current leaving node → negative

ⓐ $I_1 - I_2 - I_2' = 0$

ⓑ $-I_1 + I_2 + I_2' = 0$ (same equation)

→ we have effectively one distinctive node

$$\frac{V_A - V_a}{R_1} - \frac{V_a - V_b}{R_2} - I_2' = 0$$

$$\frac{V_1 - (V_2 + I_1R_3)}{R_1} - \frac{V_2}{R_2} - I_2' = 0$$

(Ground as zero potential!)

$$\begin{cases} V_a = V_2 + I_1R_3 \\ \quad = V_1 - I_1R_1 \\ V_b = I_1R_3 \end{cases}$$

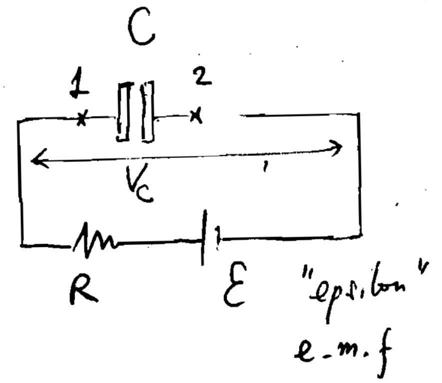
absolute voltages

$$\boxed{I_2' = \frac{V_1 - V_2}{R_1 + R_3} - \frac{V_2}{R_2}}$$ see next page

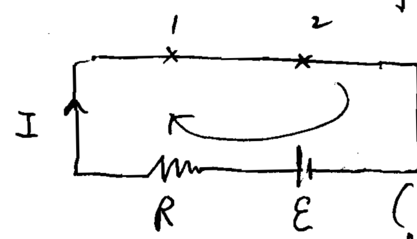
$$I_1 = \frac{V_1 - (V_2 + I_1R_3)}{R_1}$$

$$\rightarrow \boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

3) Circuits with resistors & capacitors



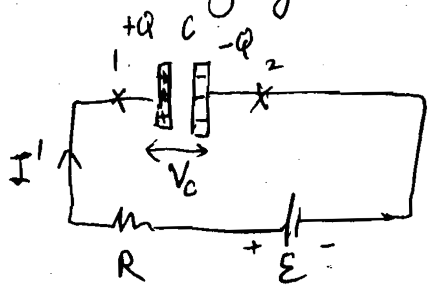
At $\boxed{t=0}$ we connect the uncharged capacitor: $Q=0$; $V_c=0$: this is like: a shortcircuit across 1 & 2 (like a wire is connecting 1 & 2)



$I = \dfrac{\mathcal{E}}{R}$

(I is current of + charges by convention)

At $\boxed{t>0}$: pos. charges move from right plate to left plate thru circuit: capacitor is now charging:



as charges are building up → current I' is getting smaller as V_c getting larger

loop equation: CW:

$$\mathcal{E} - I'R - V_c = 0$$

$$\boxed{I' = \frac{\mathcal{E} - V_c}{R}}$$

(as V_c gets larger: b/c more charges → higher σ → higher E & $V_c = E \cdot d$) → I' gets smaller. → at $t \to \infty$ → $I' = 0$

R & C circuits: $\begin{cases} t=0 & I = \dfrac{\mathcal{E}}{R} \\ t=\infty & I = 0 \end{cases}$

What $0<t<\infty$? : $\dfrac{d}{dt}\left(\mathcal{E} = I'R + V_C \right)$

$$0 = R\frac{dI'}{dt} + \frac{d}{dt}\left(\frac{Q}{C}\right) = R\frac{dI'}{dt} + \frac{1}{C}\underbrace{\frac{dQ}{dt}}_{I'}$$

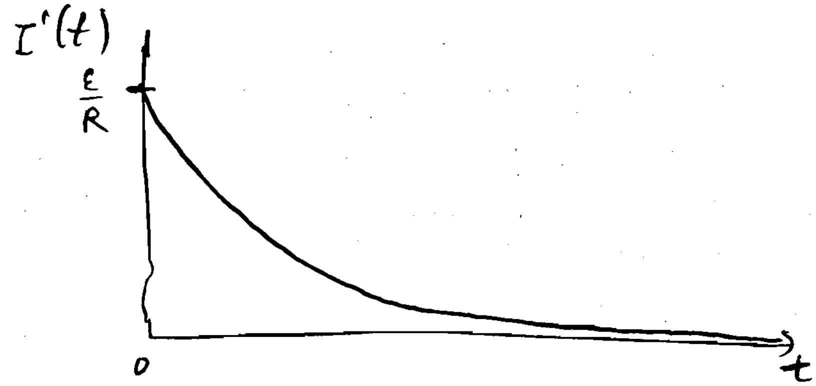
$\int \left(0 = R\dfrac{dI'}{dt} + \dfrac{1}{C} I' \right)$: differential equation : in I' w.r.t. t

$\int \left(\dfrac{dI'}{I'} = -\dfrac{1}{RC}\, dt \right) \rightarrow \qquad \ln I' = -\dfrac{t}{RC} + \text{const.}$

$$e^{\ln I'} = e^{-\frac{t}{RC}}\ \text{const.}$$

$$\boxed{I' = I'(0)\, e^{-\frac{t}{RC}}}$$

$$\boxed{I'(t) = \frac{\mathcal{E}}{R}\, e^{-\frac{t}{RC}}}$$



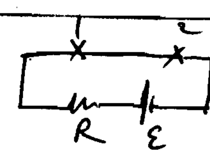
$RC = \tau$ = time constant.
(tau)

$t = \tau = RC \rightarrow I'(RC) = \dfrac{\mathcal{E}}{R}\dfrac{1}{e}$

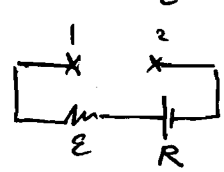
$e = 2.71\ldots$

RC circuits:
Summary

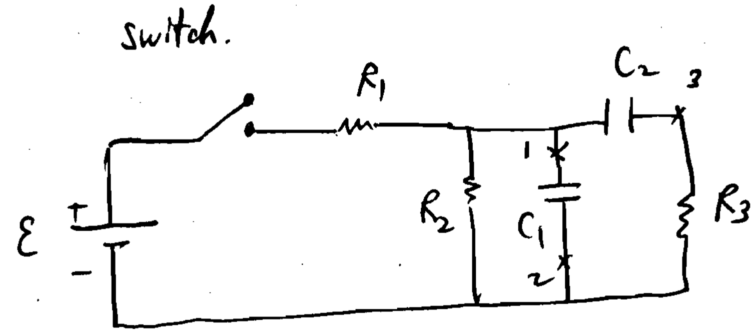
$t=0$ $\quad V_C = 0$ (shortcircuit):



$t=\infty$ $\quad I_C = 0$ (opencircuit)



25.64

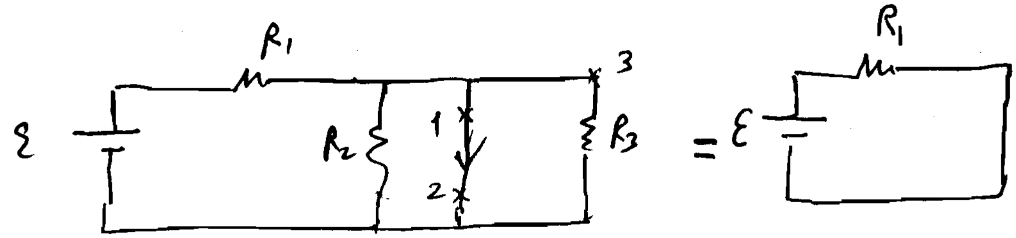


→ Switch initially open and C_1 & C_2 uncharged

$R_1 = R_2 = R_3 = R$

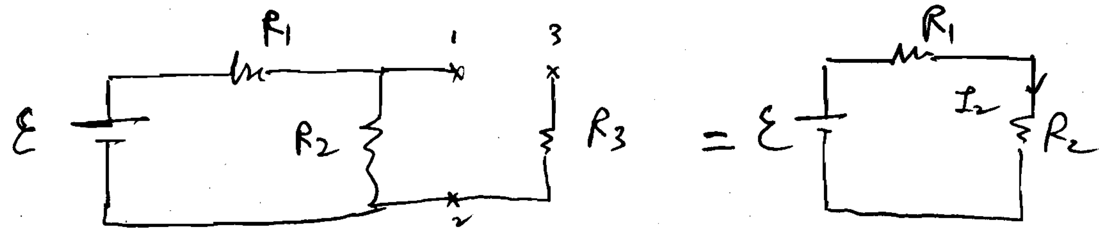
Find I_2 (thru R_2) a) just after switch is closed $t=0$:

→ $t=0$ short-circuit across C_1 & C_2



→ $I_2(R_2) = 0$

b) long after switch is closed: $t = \infty$ → $I_c = 0$ (open circuit).

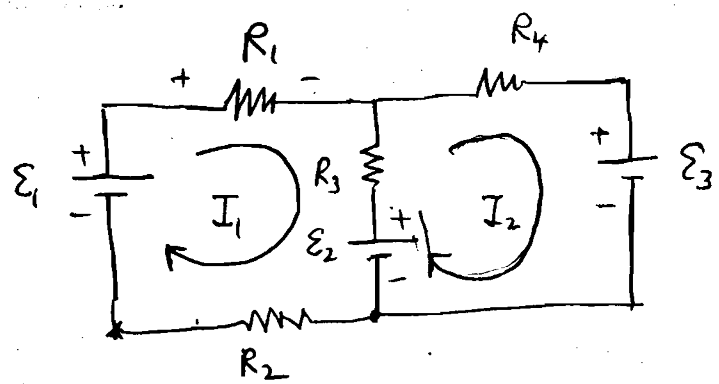


$$I_2 = \frac{\mathcal{E}}{R_1 + R_2}$$

c) Current in R_3 after switch is closed.

$\underset{t=0}{I_3(R_3) = 0}$ → value $\underset{t=\infty}{\longrightarrow 0}$.

25.53



$\mathcal{E}_1 = 6V$; $\mathcal{E}_2 = 1.5V$;
$\mathcal{E}_3 = 4.5V$
$R_1 = 270\Omega$; $R_2 = 150\Omega$;
$R_3 = 560\Omega$; $R_4 = 820\Omega$

Find current in R_3, give its direction:

Loop analysis: define directions for I_1 & I_2: CW

↳ Loop #1 (left): 1) $\mathcal{E}_1 - I_1R_1 - \underbrace{(I_1-I_2)}R_3 - \mathcal{E}_2 - I_1R_2 = 0$

↳ Loop #2 (right) 2) $+\ \mathcal{E}_2 - (I_2-I_1)R_3 - I_2R_4 - \mathcal{E}_3 = 0$

$$\mathcal{E}_1 - I_1(R_1+R_2) - I_2R_4 - \mathcal{E}_3 = 0$$

Current thru R_3 is $I_1 - I_2$, we need to find I_1 & I_2

$$I_1 = \frac{\mathcal{E}_1 - \mathcal{E}_3 - I_2R_4}{R_1+R_2} = \frac{1.5 - 820 I_2}{420}$$

2) $\mathcal{E}_2 - \mathcal{E}_3 - I_2(R_3+R_4) + I_1R_3 = 0$

$$-3 - 1380 I_2 + \underbrace{560\,\frac{1.5 - 820 I_2}{420}}_{2 - 1093.3 I_2} = 0$$

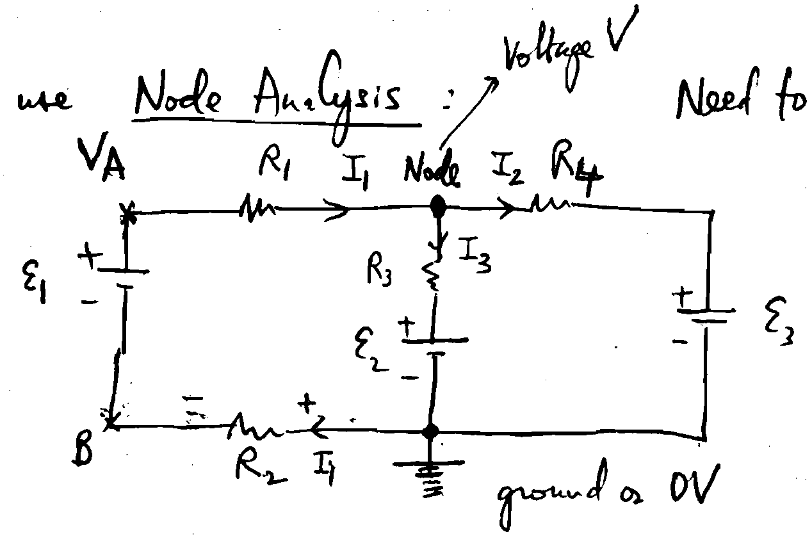
$$-1 - 2473.3 I_2 = 0 \rightarrow I_2 = -\frac{1}{2473.3} = -0.4\,mA$$

$$I_1 = \frac{1.5 - 820\,(0.4\times 10^{-3})}{420} = +\ 4.36\,mA$$

→ Current thru R_3 = $\underbrace{I_1 - I_2}$ = $4.36 - (-0.4) = \boxed{+4.76\,mA}$

downward.

Now use <u>Node Analysis</u>: Need to set <u>ground</u>!



What is current thru R_3?

<u>Node equation:</u> $I_1 - I_2 - I_3 = 0$

Write currents in terms of voltage: For absolute voltages don't use sign convention for closed loops.

$$I_1 = \frac{V_A - V}{R_1} = \frac{(\mathcal{E}_1 - I_1 R_2) - V}{R_1} \rightarrow I_1 R_1 = \mathcal{E}_1 - I_1 R_2 - V$$

$$\rightarrow \boxed{I_1 = \frac{\mathcal{E}_1 - V}{R_1 + R_2}}$$

$$\boxed{I_2 = \frac{V - \mathcal{E}_3}{R_4}} \; ; \; \boxed{I_3 = \frac{V - \mathcal{E}_2}{R_3}}$$

$$\frac{\mathcal{E}_1 - V}{R_1 + R_2} - \frac{V - \mathcal{E}_3}{R_4} - \frac{V - \mathcal{E}_2}{R_3} = 0$$

One equation & one unknown: V

Solve for V:

$$\frac{6 - V}{420} - \frac{V - 4.5}{820} - \frac{V - 1.5}{560} = 0$$

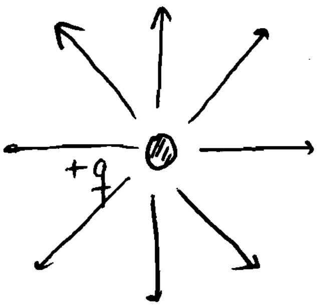
$\hookrightarrow V = 4.17V$

$$\hookrightarrow \underbrace{I_3}_{\text{downward}} = \frac{4.17 - 1.5}{560} = \boxed{4.76 mA}$$

If ground is placed instead at B → different equations, final answer should be the same!

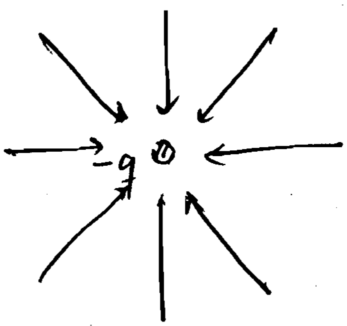
Ch. 26 Magnetic Field

Electric



+q ⊙ ⊙ +q repulsive

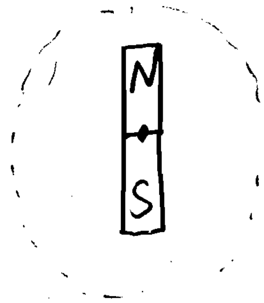
+q ⊙ ⊙ −q attractive



→ Electric monopoles are routine

→ Electric field lines are open.

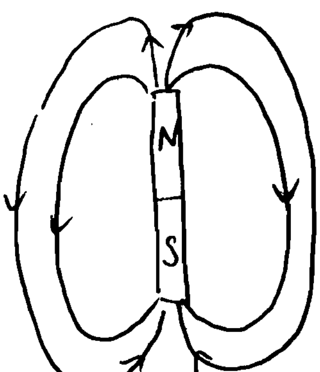
Magnetic



2 types of magnetic pole: North & South.

{ opposite poles attract each other
{ Like poles repel each other.

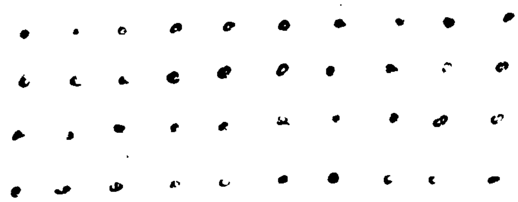
→ Magnetic monopoles have not been discovered.



→ Magnetic Field lines are closed lines

Effects of Magnetic Field $\vec{B}$

On a moving charge q with velocity $\vec{v}$ in a region with a magnetic field that is uniform and pointing out of the page:



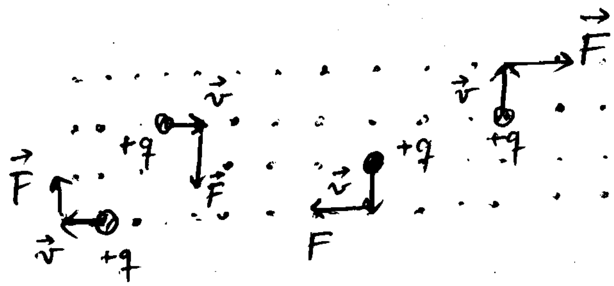
$\vec{B}$ out of page & uniform

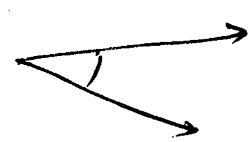
Experiments: 1) If charge q moves in or out of page → does not feel the magnetic field: $\vec{F} = 0$

2) If charge q moving on the page ($\perp \vec{B}$) → feels max. effect of the magnetic field.

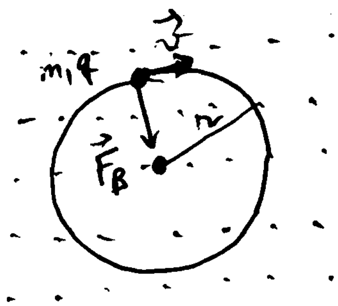
$$\vec{F} = q \underbrace{\vec{v} \times \vec{B}}_{\text{vector cross product b/w } \vec{v} \text{ and } \vec{B}}$$

it is a vector that is perpendicular to both $\vec{v}$ & $\vec{B}$, direction is given by the Right Hand Rule (RHR) → as RH fingers close from $\vec{v}$ to $\vec{B}$, thumb points in the direction of $\vec{v} \times \vec{B}$; magnitude given by $qvB\sin\theta$ (θ is the angle b/w $\vec{v}$ & $\vec{B}$)





So: the trajectory of a charged particle in a magnetic field: circular, since $\vec{F}_B$ is always perpendicular to the direction of motion, being the agent that provides the radial acceleration:



$\vec{B}$ is uniform out of page

$\vec{F}_B = q\vec{v} \times \vec{B}$ $\begin{cases} \text{direction: radial} \\ F_B = qvB \end{cases}$

r? $\rightarrow F_B = ma = m\dfrac{v^2}{r}$

$q\cancel{v}B = m\dfrac{v^{\cancel{2}}}{r} \rightarrow \boxed{r = \dfrac{mv}{qB}}$

→ Can confine charged particles into a small region using magnetic fields (fusion energy experiments)

How long would it take for q to complete one turn?

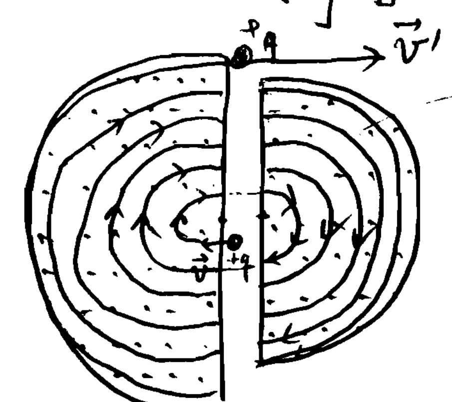
↳ Period: $\dfrac{2\pi r}{v} = \dfrac{2\pi \cancel{r}}{\frac{qB\cancel{r}}{m}} = \dfrac{2\pi m}{qB}$

Applications:

1) Cyclotron: (modern version synchrotron)

↳ Goal: accelerates charged particles to high speed: using $\vec{B}$ & $\vec{E}$.

Cyclotron radius is R



→ $\vec{B}$: uniform & pointing out of page in these D's.

$KE_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m\left(\dfrac{qBR}{m}\right)^2$

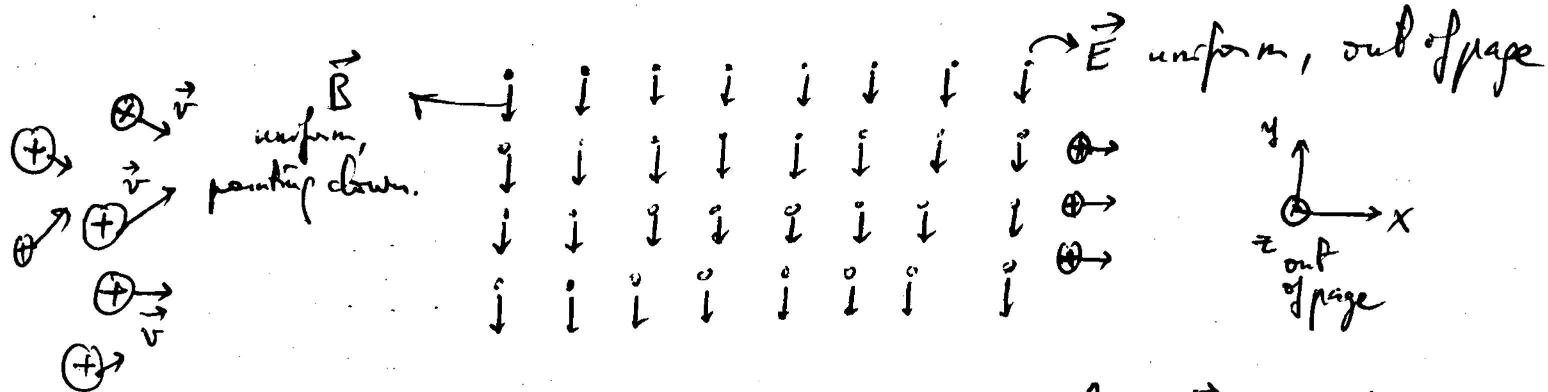
$= \dfrac{(qBR)^2}{2m}$

Tech. limitation
$v \to c = 3\times10^8$ m/s
↓ Synchrotron.

Apply $\vec{E}$ to give q a push, alternatively

2) Velocity selector:

ions (+) with different velocities, we can use a combination of $\vec{E}$ & $\vec{B}$ to select out the velocity of interest:



Effect of fields on ions: $\vec{E} = E\hat{k}$; $\vec{B} = B(-\hat{j})$

q with $\vec{v} = v\hat{\imath}$

$$\left\{ \begin{aligned} \vec{F}_E &= q\vec{E} = qE\hat{k} \\ \vec{F}_B &= q\vec{v}\times\vec{B} = qvB\,\underbrace{(\hat{\imath}\times(-\hat{j}))}_{-\hat{k}} \end{aligned} \right\}$$

at $\boxed{v = \dfrac{E}{B}}$ $\rightarrow$ $\vec{F}_E + \vec{F}_B = 0$

b/c: $qE - qvB = qE - q\frac{E}{\cancel{B}}\cancel{B} = 0$

At this speed, the ion going horizontally will go straight thru (it does not feel any field at all)

Calculation of the Magnetic Field or Source of the Magnetic Field.

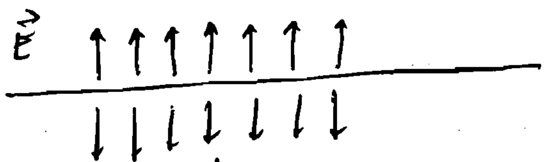
Electric: (source: charge) $\quad d\vec{E} = k \dfrac{dq}{r^2}\hat{r}$ inverse-square or Coulomb's law

Magnetic field: (source: current) $\quad d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\, d\vec{l}\times\hat{r}}{r^2}$ inverse-square or Biot-Savart Law

$\mu_0 = 4\pi\times10^{-7}\,\dfrac{N}{A^2}$ permeability in vacuum

Magnetic field due a line of current is wrapping around the current (that's why we need the cross product in Biot-Savart)

Line charge

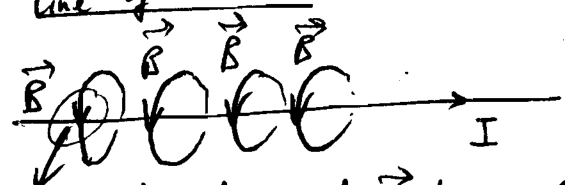


λ

$E = \dfrac{2K\lambda}{r}$

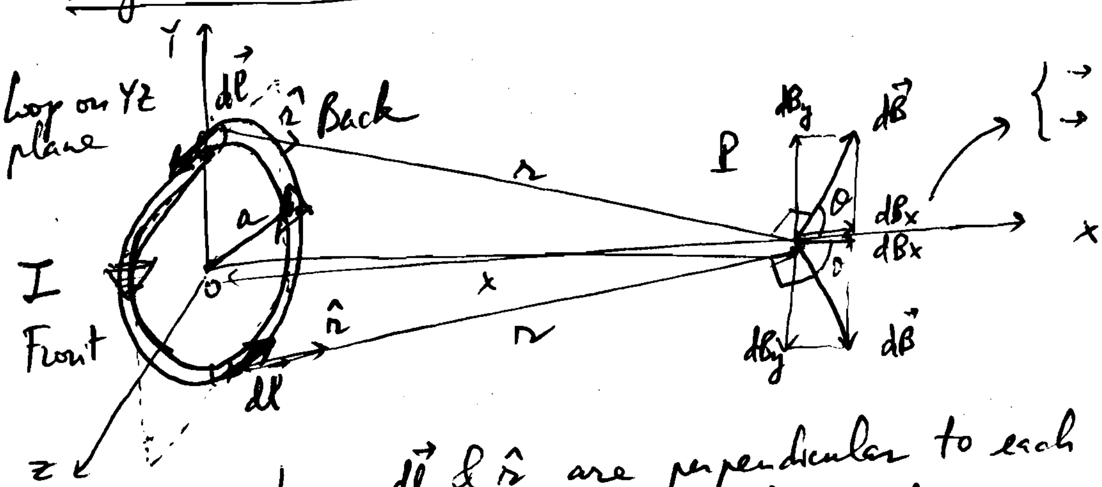
(r: sep. from line)

Line of current



Front, direction of $\vec{B}$ by RHR (thumb along I)

Magnetic field due to a loop of current.

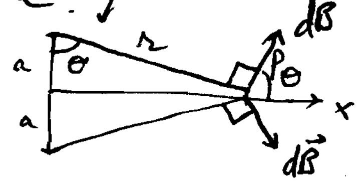


$\left\{\begin{array}{l}\rightarrow dB_y\text{'s are cancelled} \\ \rightarrow dB_x \text{ times } 2\end{array}\right.$

$d\vec{l}$ & $\hat{r}$ are perpendicular to each other, located on an inclined plane

$d\vec{l}$ in YZ plane

$\rightarrow$ $d\vec{l}\times\hat{r}$ will be $\perp$ to this plane!



From the top & bottom elements of currents:

$$dB_{Total} \; 2dB_x = 2dB\cos\theta = 2dB\frac{a}{r} = \frac{2a}{r}dB = \frac{2a}{r}\underbrace{\frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}}_{\text{Biot-Savart}}$$

$$\boxed{dB_{Total} = 2\frac{\mu_0}{4\pi}\frac{a\,I\,dl}{r^3}}$$

Top & bottom elements.

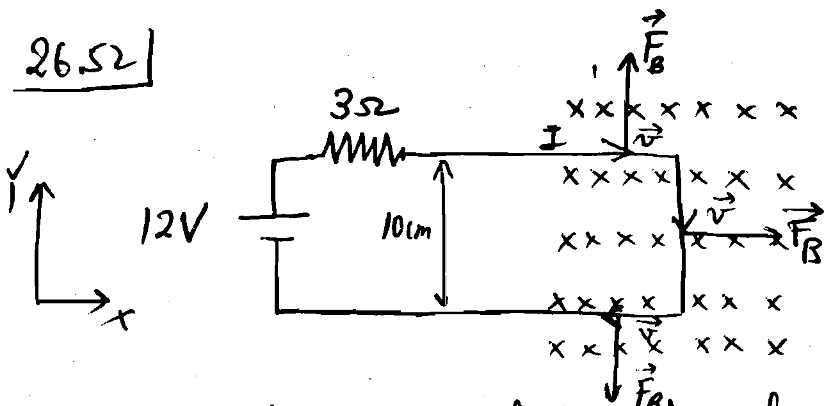
$$B_{Total} = \int_{\text{Half loop}} dB_{Total} = \frac{2\mu_0}{4\pi}\frac{a\,I}{r^3}\underbrace{\int_{\text{Half loop}} dl}_{\pi a} = \frac{\mu_0}{2}\frac{Ia^2}{(x^2+a^2)^{3/2}}$$

$\left(r = (x^2+a^2)^{1/2} \right)$

Magnetic field points along x axis due to a loop in the YZ plane of current I & radius a

→ Unit of B is Tesla or T (S.I.).

26.52



$\vec{B}$ into page, uniform.

$B = 38\,mT$

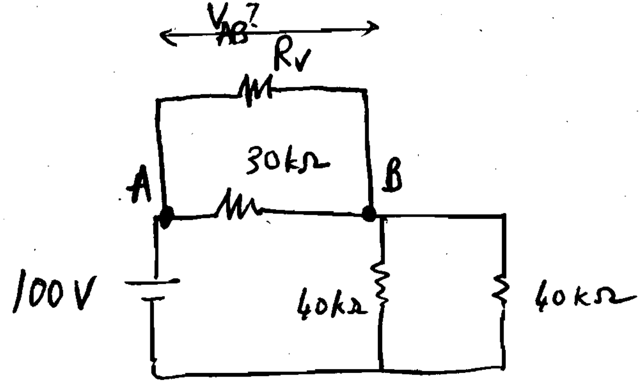
Net Force on circuit?

Current is composed of positive charges moving along the loop. The top & bottom force are equal & opposite b/c = same $\vec{v}$ & q (or same current) & same $\vec{B}$ b/c its uniform. Net force is due to $\vec{F}_B$ on vertical side immersed in the field region:

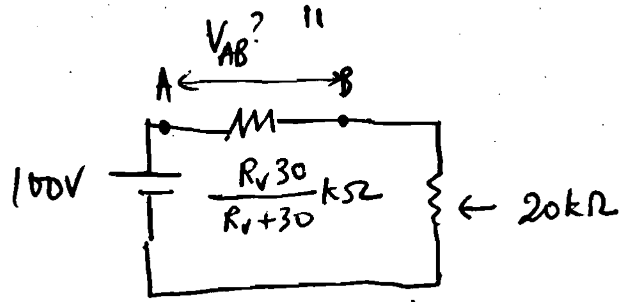
$$\vec{F}_B = q\vec{v}\times\vec{B} \rightarrow F_B = qvB = q\frac{\Delta y}{\Delta t}B = I\Delta y B$$

$$\vec{F}_B = \frac{12}{3}\,0.1\times 38\times 10^{-3}\,\hat{i} = 15.2\,mN\,\hat{i}$$

25.55



$$R_V = \begin{cases} a)\ 50k\Omega \\ b)\ 250\,k\Omega \\ c)\ 10^{7}\,\Omega = 10000\,k\Omega \end{cases}$$



(Parallel of $40k\Omega$ & $40k\Omega$

$$\frac{40\times40}{40+40}k\Omega = \frac{160\cancel{0}}{8\cancel{0}} = 20k\Omega$$

These two resistors are series:

V_{AB} by Voltage division:

$$V_{AB} = 100\,\frac{\frac{R_V 30}{R_V+30}}{\frac{R_V 30}{R_V+30}+20}\,V = 100\,\frac{R_V 30}{R_V 30 + 20(R_V+30)}$$

(R_V in $k\Omega$)

$$= \frac{R_V\,3000}{50R_V + 600}$$

a) $R_V = 50\ (k\Omega) \rightarrow V_{AB} = \dfrac{50\times3000}{50\times50+600} = \dfrac{15\times10^4}{3100}$

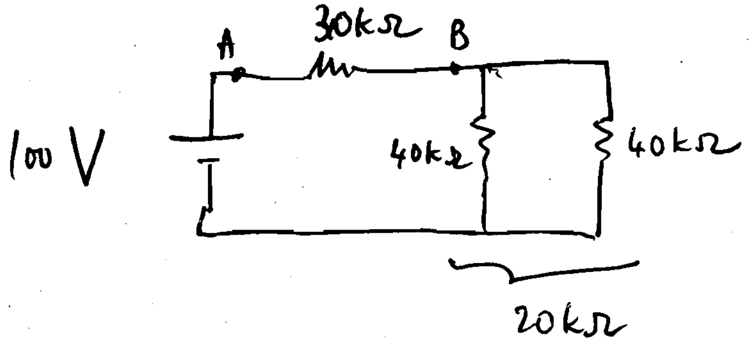
$= 48.39\,V$ ✓

b) $R_V = 250\ (k\Omega) \rightarrow V_{AB} = \dfrac{250\times3000}{250\times50+600} = \dfrac{75\times10^4}{12500+600}$

$= \dfrac{75\times10^4}{13100}\,V = 57.25$ ✓

c) $R_V = 10000\ (k\Omega) \rightarrow V_{AB} = \dfrac{10000\times3000}{10000\times50+600} = \dfrac{3\times10^7}{5\times10^5+600}$

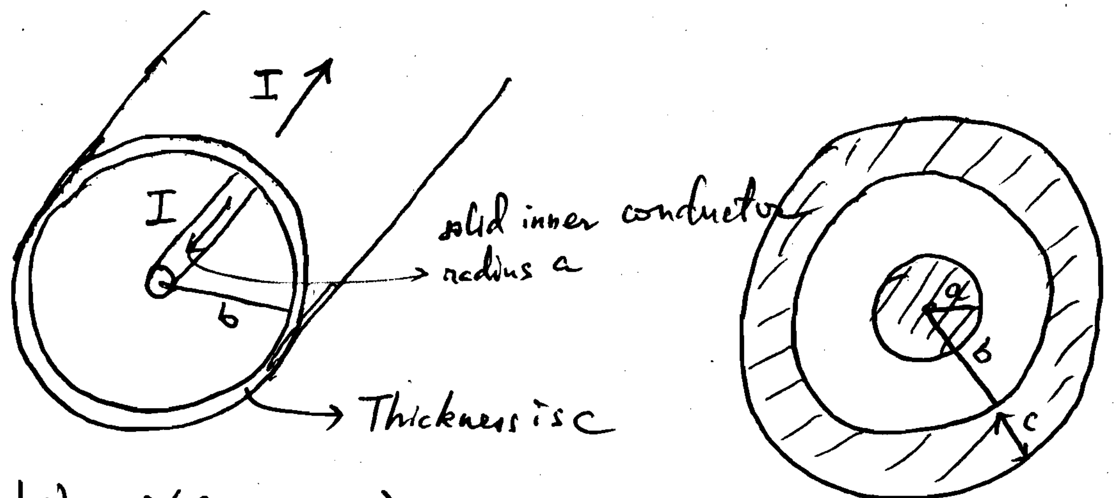
$= 59.93\ V$ (closest to theoretical value!)



We expect

$$V_{AB} = 100V \frac{30}{30+20} = 100\,\frac{3}{5} = 60\,V$$

26.68 Coaxial cable



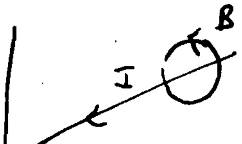
$$B(r) \begin{cases} a) & r<a \\ b) & a<r<b \\ c) & r>b+c \end{cases}$$

Using Ampere's Law

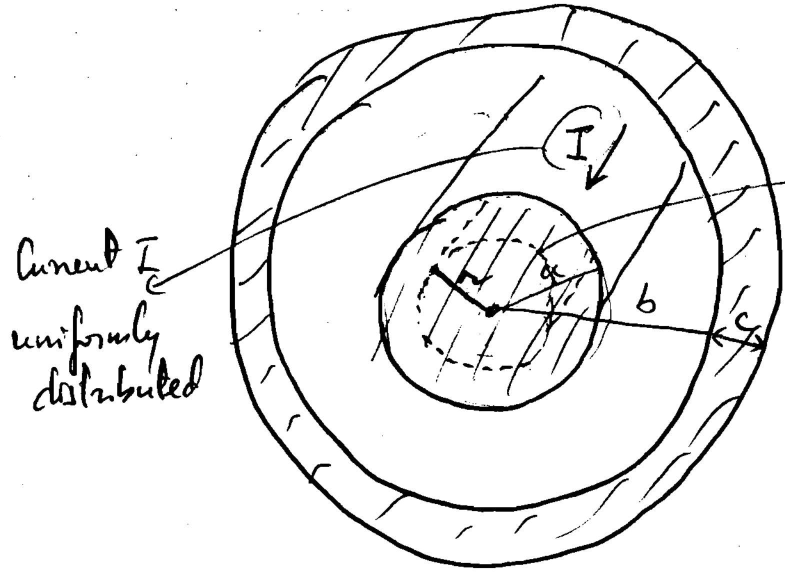
$$\oint_{\substack{\text{Closed}\\\text{loop}}} \vec{B}\cdot d\vec{l} = \mu_0\, I_{\substack{\text{enclosed}\\\text{by loop}}}$$

By choosing the correct Amperian loop we can factor out B in the left side.

→ Due to the long cylindrical symmetry → B is wrapping around the long line of current, with a fixed magnitude for certain separation r from the line.



↳ Amperian loop is just a circle of radius r:



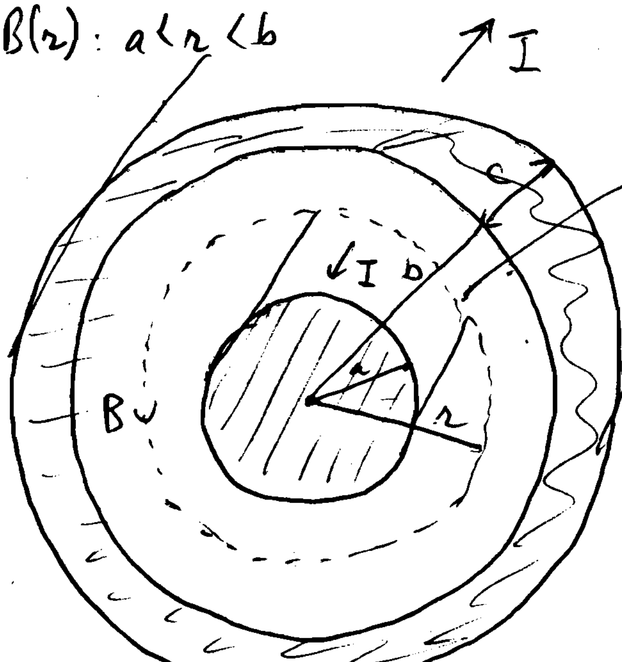
Amperian loop: since B is constant along this loop → we can factor it out of the left hand side integral (in Ampere's Law)

$$\underbrace{B\oint_{loop} dl}_{2\pi r} = \underbrace{\mu_0\left(\frac{I}{\pi a^2}\pi r^2\right)}_{I_{enclosed}}$$

$$\boxed{B = \frac{\mu_0 I \frac{r^2}{a^2}}{2\pi r} = \frac{\mu_0 I}{2\pi a^2} r}$$

$(r < a)$

b) $B(r): a < r < b$



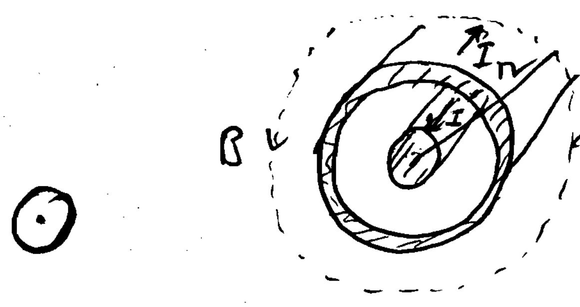
Amperian loop: can factor B out of the left hand side integral:

$$B\oint_{A.loop} dl = \mu_0 . I$$

$$B\, 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r} \quad (a < r < b)}$$

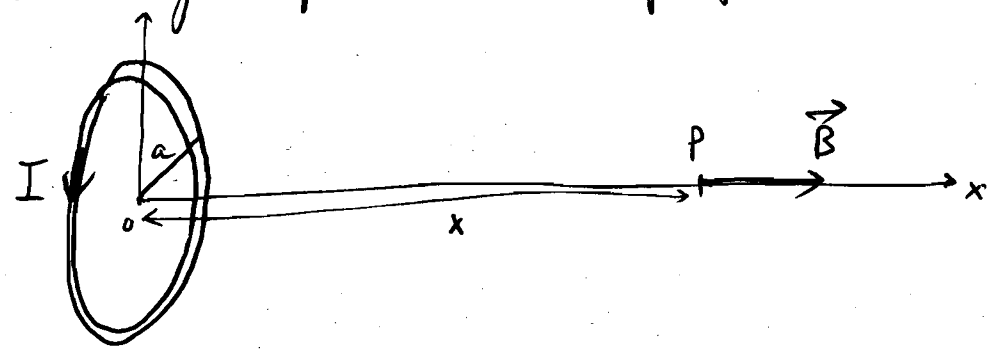
c) $B(r) \quad r > b+c$ →



Amperian Loop.

$$B . 2\pi r = \mu_0 . \underbrace{0} \rightarrow B = 0$$

Yesterday: Magnetic field due to a loop of current:



$\vec{B} = B(x)\hat{i}$; $B(x) = \dfrac{\mu_0}{2} \dfrac{I a^2}{(x^2+a^2)^{3/2}}$ (T for Tesla, S.I.)

If $x >> a$ (far away approx.) $\rightarrow$ $B \simeq \mu_0 \dfrac{I a^2}{x^3}$ inverse-cube law.

(a loop of current is the magnetic analog of an electric dipole, in a far away approximation)

Calculation

Electric field

→ Vector addition (Then integration)

→ Gauss Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

Gaussian surface 3D

static charges creates electric field

→ Electric potential V $\vec{E} = -\vec{\nabla} V$ (scalar addition, then derivative)

Magnetic field

→ Vector addition (Then integration)

→ Ampere Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

Amperian loop 2D

moving charges create magnetic field

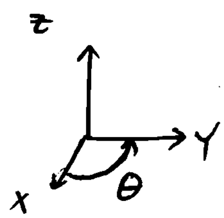
→ Vector Potential $\vec{A}$

$$\vec{B} = \underbrace{\vec{\nabla} \times}_{\text{rotational or curl}} \vec{A}$$

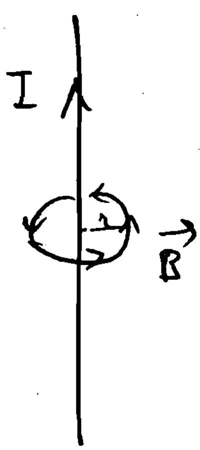
Ampere's law: $\displaystyle\oint_{\text{Amperian loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$

- Determine the correct Amperian loop that allows us to factor B out the left hand side integral.
- Current enclosed by that loop should be used in the right hand side.

Long straight wire:



I along z ; $\vec{B}$ on XY plane
Angle θ (spherical coordinate) goes from $\hat{x}$ on XY plane.



(RHR: thumb along I; fingers wrapping along $\vec{B}$)

$\vec{B} = B(r)\,\hat{\theta}$

Amperian loop → circle centered at the wire, at radius r → parallel with the field at all points along loop!

↳ $\cos\theta = 1$ → $\underbrace{\oint \vec{B}\cdot d\vec{l}}_{B(r)\times 2\pi r} = \mu_0 I$

$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$$

($E = \dfrac{2k\lambda}{r}$)

static line of charge

Ch. 27 Electromagnetic Induction

Faraday's Law :

$$\boxed{\mathcal{E} = - \frac{d\Phi_B}{dt}}$$

induced e.m.f or voltage

due to conservation of energy or Lenz's law

change of magnetic flux w.r.t. time

Magnetic flux : $\Phi_B = \int_{\text{surface 3D}} \vec{B} \cdot d\vec{A}$

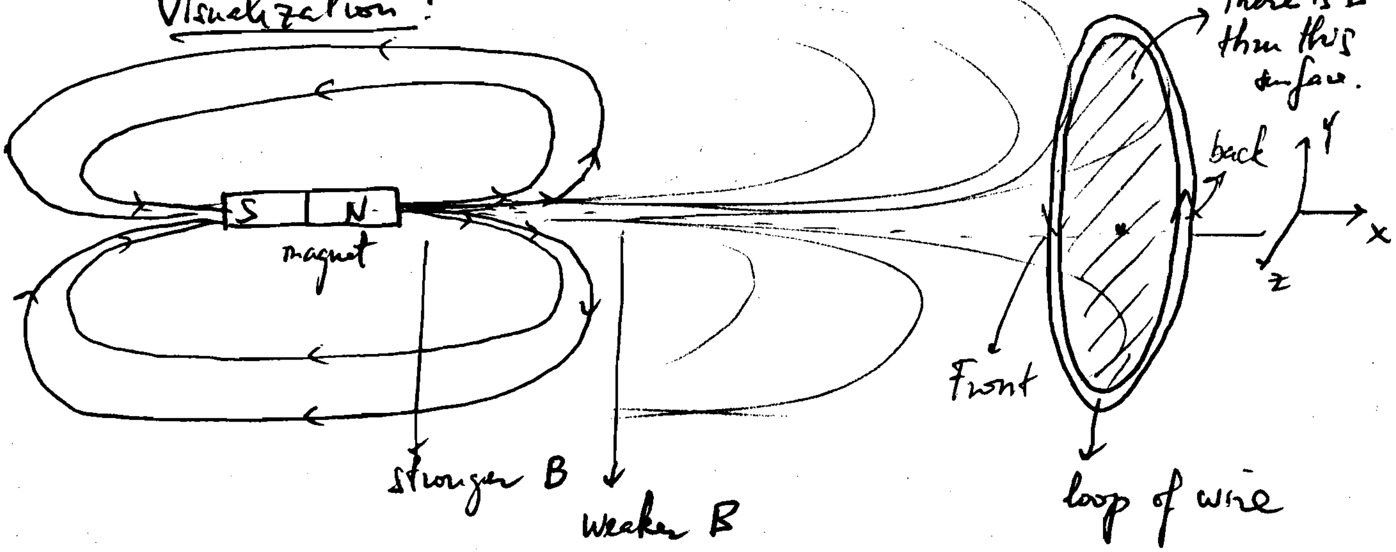
Φ_B "Phi"

$\vec{B}$ magn. field

$d\vec{A}$ element of area

If the magnetic flux thru some surface changes with time, it induces an electric potential on the loop enclosing that surface.

Visualization :



Φ_B : surface area of loop is const.
can change by moving the magnet. $\longrightarrow$ induce emf on loop wire

<u>Sign of the e.m.f</u>: tends to ~~cont~~ counteract the change in magnetic flux: if the Φ_B is increased with time due to a field pointing to the right $(+x)$ → the induced emf is such that it creates a counter magnetic field pointing to the left $(-x)$

↓

If the magnet moves along $+x$ → current goes up in front of loop & down in back (CW if seen from the right).

→ If $B: 2T \to 4T$ → same $\Delta t = 3s$ but direction of induced current is CCW

$\boxed{26.57}$ NMR (Nuclear Magnetic Resonance)

↳ measures energy to flip atomic nuclei in a given magnetic field.

$B = 7T$ How much energy needed to flip a proton from parallel to antiparallel

$\underset{\substack{\downarrow \\ \text{magnetic moment}}}{\mu} = 1.41 \times 10^{-26}\, A\, m^2$

$\vec{\mu} = I\vec{A}$: magn. moment of a loop carrying current I where the loop area is A.

Torque = (unit of energy) = for a loop of magn. mom. $\vec{\mu}$ in a magnetic field $\vec{B}$ $\quad \vec{\tau} = \vec{\mu} \times \vec{B} \longrightarrow \vec{\mu} \perp \vec{B} \to \tau = \mu B$

Flipping : $\underbrace{\Delta U}_{\text{energy needed}} = \Delta \tau = \mu B - (-\mu B) = 2\mu B$

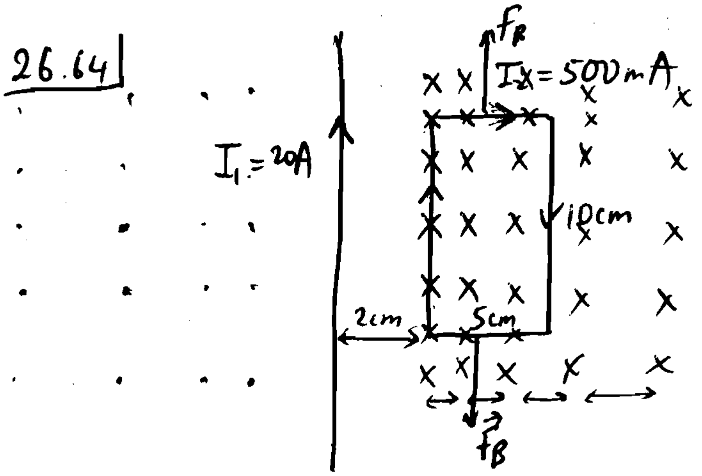
$= 2 \times 1.41 \times 10^{-26} \times 7$

$= 1.97 \times 10^{-25} J$

$\begin{cases} \text{Proton} \parallel B: & \mu B \\ \text{Proton antiparallel to } B: & -\mu B \end{cases}$

25.65 | P.P.

26.64 |



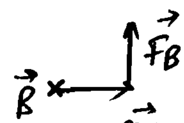
Net $\vec{F}_B$ on loop ?

$$\vec{F}_B = q\vec{v}\times\vec{B}$$
$$= \frac{q}{\Delta t}\vec{l}\times\vec{B} = I\vec{l}\times\vec{B}$$

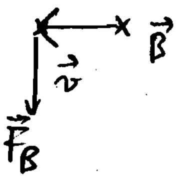
→ There is a magnetic field created by the ~~loop~~ straight wire:

↳ $\vec{B}(r) = \frac{\mu_0 I}{2\pi r}\hat{\Theta}$ (wrapping around the wire).

→ Top side of rectangle:

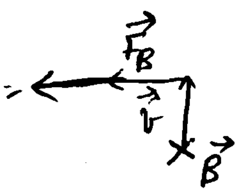


→ bottom side of rectangle:

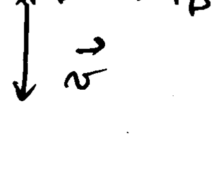


same r → same B → same F_B → No net force along vertical direction!

→ Left side of rectangle:



→ Right side of rectangle:



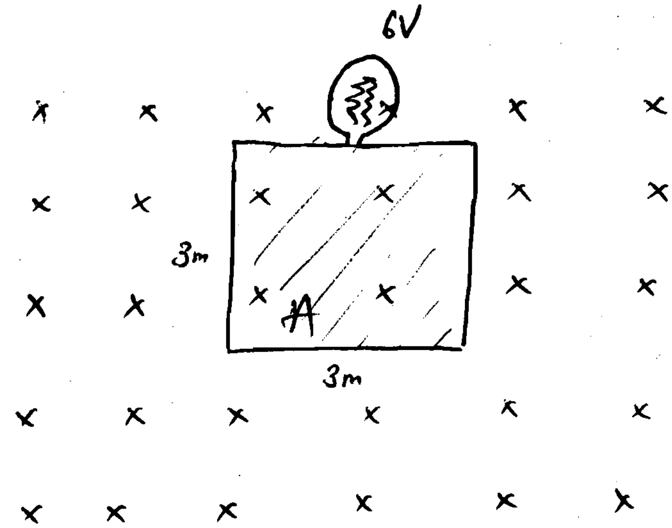
different r's for left & right sides → different F_B's → Net force along horizontal pointing toward wire!

Net Force (toward wire) $= F_{B_L} - F_{B_R}$

$$= I_2 l\left(\frac{\mu_0 I_1}{2\pi\, 0.02} - \frac{\mu_0 I_1}{2\pi\, 0.07}\right) = \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{0.02} - \frac{1}{0.07}\right)$$

$$= \frac{4\pi\times10^{-7}}{2\pi}\,0.5\times20\times0.1\left(\frac{1}{0.02} - \frac{1}{0.07}\right) = 7.14\times10^{-6}\,N$$

27.40



$B = 2T$
into page
→ B reduced to 0 steadily over time Δt

a) Δt for bulb to shine at full brightness.

To get an induced e.m.f. (to power the bulb) we need a change of the magnetic flux Φ_B thru the square loop.

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \underset{\substack{\downarrow \\ B \text{ is uniform}}}{=} B \cdot A$$

Since A is fixed → Φ_B will change if B will change.
(doesn't matter if increased or decreased!)

To get the answer we use Faraday's Law : $\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$

$$\underset{\substack{\downarrow \\ \text{This problem.}}}{\mathcal{E}} = -\frac{d(BA)}{dt} \underset{\substack{\downarrow \\ A \text{ is fixed.}}}{=} -\left(\frac{dB}{dt}\right) \cdot A = -\frac{\Delta B}{\Delta t} \cdot A$$

Need $6V = \mathcal{E}$ → $\boxed{\Delta t = \frac{\Delta B}{\mathcal{E}} A = \frac{2}{6} 3^2 = 3s}$

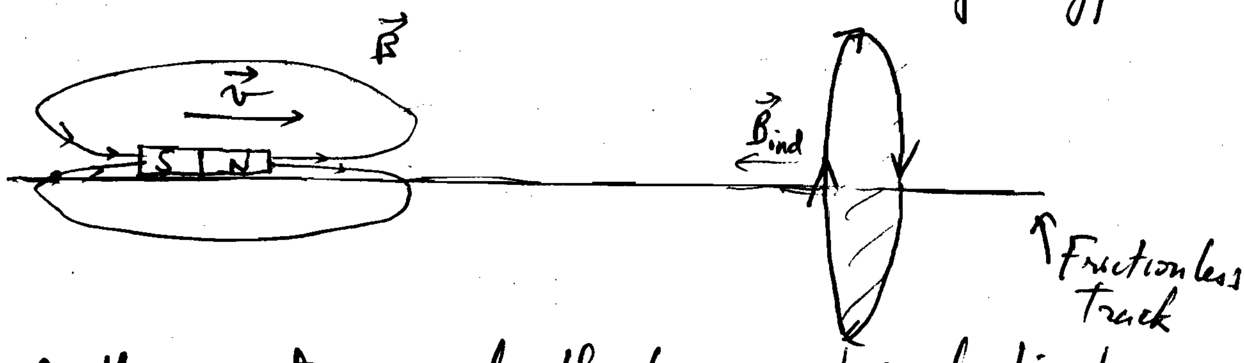
b) Which way will the loop current flow?
↳ sign : induced current is such that it creates a counter magnetic field to compensate for the change of flux.

When $B = 2T \to 0T$: lower m. flux into page
→ induced current make a counter B into the page to compensate for the decrease → current (induced) flows CW

Eletromagnetic Induction & conservation of energy:

↳ Faraday's Law: $\mathcal{E} = - \frac{d\Phi_B}{dt}$

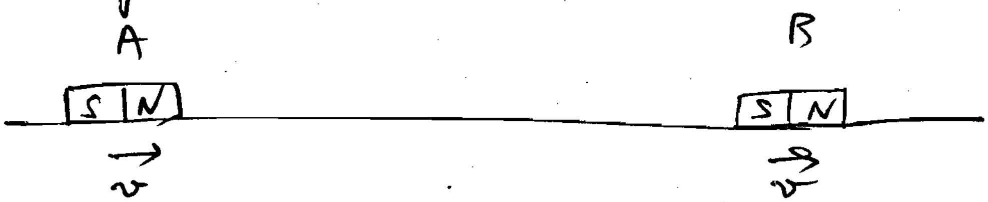
↳ opposite effect
Lenz's law or conservation of energy:

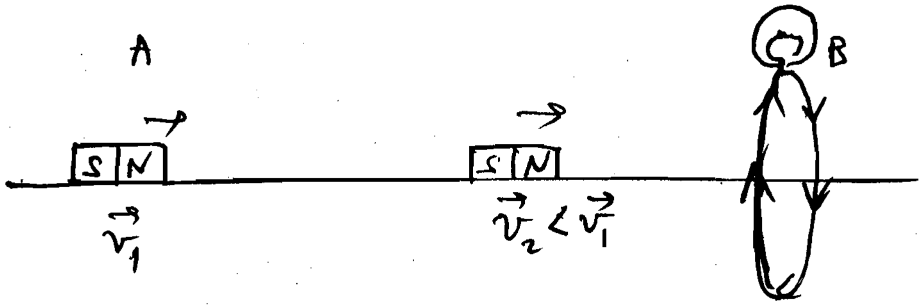


As the magnet approaches the loop the Φ_B (magn. flux) thru loop due to the magnet's field is increased.

The opposite effect is the induction of a voltage (so a current) in the loop such that the induced magnetic field opposes the change of flux caused by approaching magnet.

Now → Magnet can slide on a track without friction toward the loop. Would magnet's speed stay the same, get decreased / or increased as it approaches the loop?

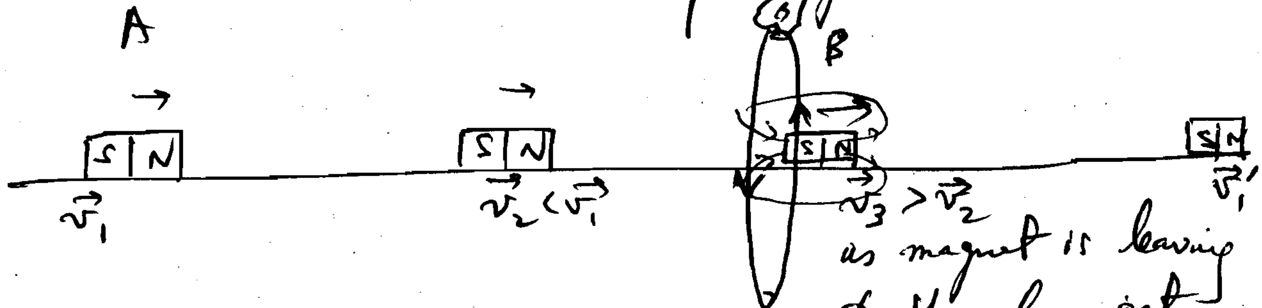




light turns on as magnet approaches.

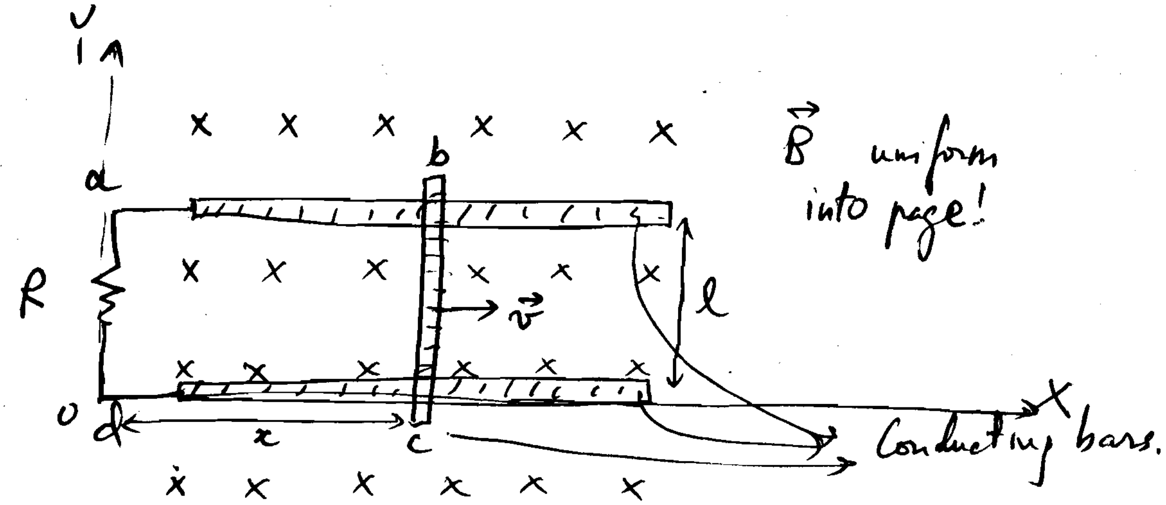
↓

Conserv. of energy: magnet's speed get decreased.



as magnet is leaving Φ_B thru loop get smaller → induced current will oppose this → creating induced magnetic field pointing right to add to the lower field due to leaving magnet.

27.47



a) Direction of current in R?

↓ induced current ← induced $\mathcal{E}$ ← change of Φ_B w.r.t. t.

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

$\vec{B}$: field; $d\vec{A}$: area enclosed by loop

→ loop: abcd → $\underline{\text{area}}$ $A = x\ell$

→ Flux: $\Phi_B = B \oint d\vec{A} = Bx\ell$

↳ increased since x increases b/c bar be going to the right

↳ induced $\mathcal{E}$

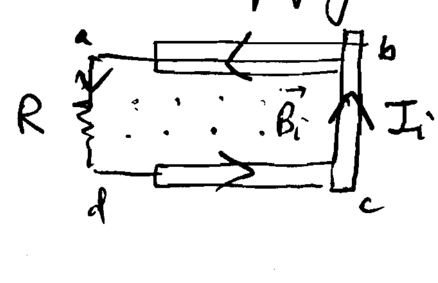
↳ induced I ⟶ induced B_i to <u>oppose</u> the [$\ominus$ in Faraday's law]

flux increase into the page.

↓

B_i out of page

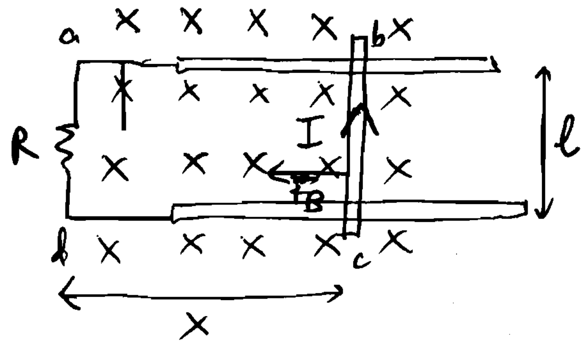
$$\boxed{\text{Induced current = downward at R}}$$



b) Since loop abcd opposes any change of flux → work is needed to move bar bc to the right!

$$\underline{P} = \frac{\text{Work}}{\text{time}} = \frac{F_B \, \Delta x}{\Delta t} = F_B v = I l B v$$

$$= \frac{\mathcal{E}}{R} l B v$$



$$= \frac{\frac{d}{dt}(B x l)}{R} l B v$$

$$= \frac{l B v}{R} l B v = \frac{(l B v)^2}{R}$$

Alternative:

$$P = I.V = I^2 R = \left(\frac{\mathcal{E}}{R}\right)^2 R = \left[\frac{\frac{d}{dt}(B x l)}{R}\right]^2 R$$

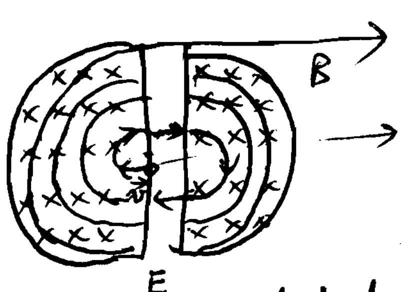
$$= \frac{(B v l)^2}{R^{\cancel{2}}} \cancel{R} = \frac{(B v l)^2}{R}$$

26.50 Cyclotron : accelerate deuterium nuclei (1p + 1n)

1p ↓ $q = +e$, $m \cong 2000 m_e$; 1n ↓ $q = 0$, $m \cong 2000 m_e$

$B = 2T$, at what f should the dee voltage be alternated.
↓

a)



→ deuterium $\begin{cases} q = +e \\ m = 2m_p \simeq 4000 \, m_e = 2 \times 1.67 \times 10^{-27} \text{kg} \end{cases}$

$m_e = 9.11 \times 10^{-31}$ kg.

$$q v B = F_B = m \frac{v^2}{r}$$

↳ needs to be alternating! What is this f?

$$f = \frac{1}{T}; T = \frac{2\pi r}{v} = \frac{2\pi \cancel{r}}{\frac{qB\cancel{r}}{m}} = \frac{2\pi m}{qB} \Rightarrow f = \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 2}{2\pi \times 2 \times 1.67 \times 10^{-27}}$$

$$= 15.2 \times 10^6 \text{ Hz}$$

b) $R = r_{max} = \frac{0.9\,m}{2}$ → what is KE_{max} for deuterons?

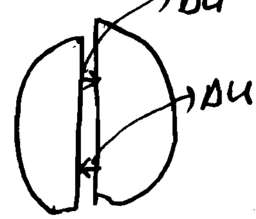
$$KE_{max} = \frac{1}{2} m_d v_{max}^2 = \frac{1}{2} m_d \left(\frac{qBr_{max}}{m_d}\right)^2$$

$$= \frac{q^2 B^2 r_{max}^2}{2m_d} = \frac{(1.6\times10^{-19} \times 2 \times 0.45)^2}{2\times\underbrace{(2\times1.67\times10^{-27})}_{m_d}}$$

$$= 3.1\times10^{-12}\,J$$

c) $\underbrace{\Delta V_{gap}}_{} = 1500\,V$ → how many orbits until max energy?

$$\Delta U = q\,\Delta V \longrightarrow \times 2 \text{ per orbit.}$$



$$\Delta U_{orbit} = 2q\,\Delta V = 2\times1.6\times10^{-19}\times 1500 = 3\times1.6\times10^{-16}\,J$$
$$= 4.8\times10^{-16}\,J$$

→ # orbits: $\frac{KE_{max}}{\Delta U_{orbit}} = \frac{3.1\times10^{-12}\,\cancel{J}}{4.8\times10^{-16}\,\cancel{J}} = 6.48\times10^{3}$ orbits.

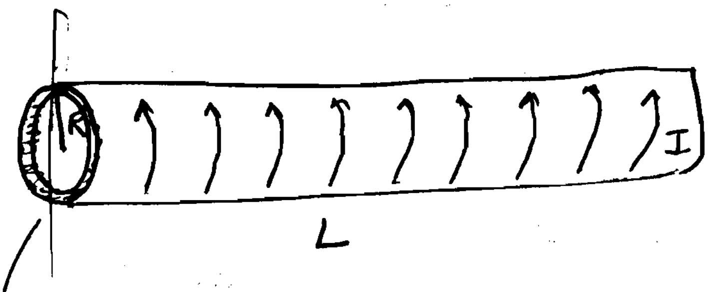
(We assume a small gap so time delay is negligible).

$m_p = 1.67\times10^{-27}\,kg$
$m_e = 9.11\times10^{-31}\,kg$ } $\frac{m_p}{m_e} = 1833$ or $m_p = 1833\,m_e \simeq 2000\,m_e$

$m_d = 2m_p = 2\times1.67\times10^{-27}\,kg$
↓
$(1p + 1n)$

26.76

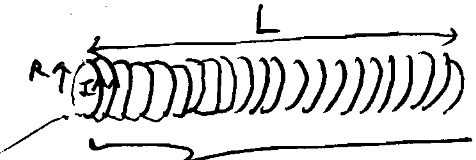
C

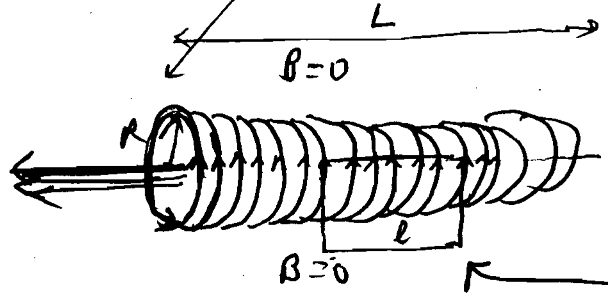


Hollow pipe

B ? $\begin{cases} r < R \\ r > R \end{cases}$

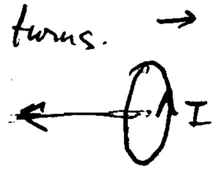
Resembles :





B

N turns. $\rightarrow \frac{N}{L} = n$ (# turns per unit length.)



Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}.$$

Find right Amperian loop so $\vec{B}$ can be factored out of the LHS integral.

$B \parallel$ top side ; $\perp$ vertical sides ; 0 bottom side

$$B \cdot l = \mu_0 \underbrace{nl}_{\text{\# turns enclosed by Amperian loop.}} I \longrightarrow \boxed{B = \mu_0 n I}$$

$$\boxed{B = \mu_0 \frac{N}{L} I}$$

$\rightarrow$ Pipe : I uniform over pipe $\rightarrow$ no turns as with solenoid $\rightarrow$

$$\begin{cases} B = \dfrac{\mu_0 I}{L} & (r < R) \\ B = 0 & (r > R) \end{cases}$$