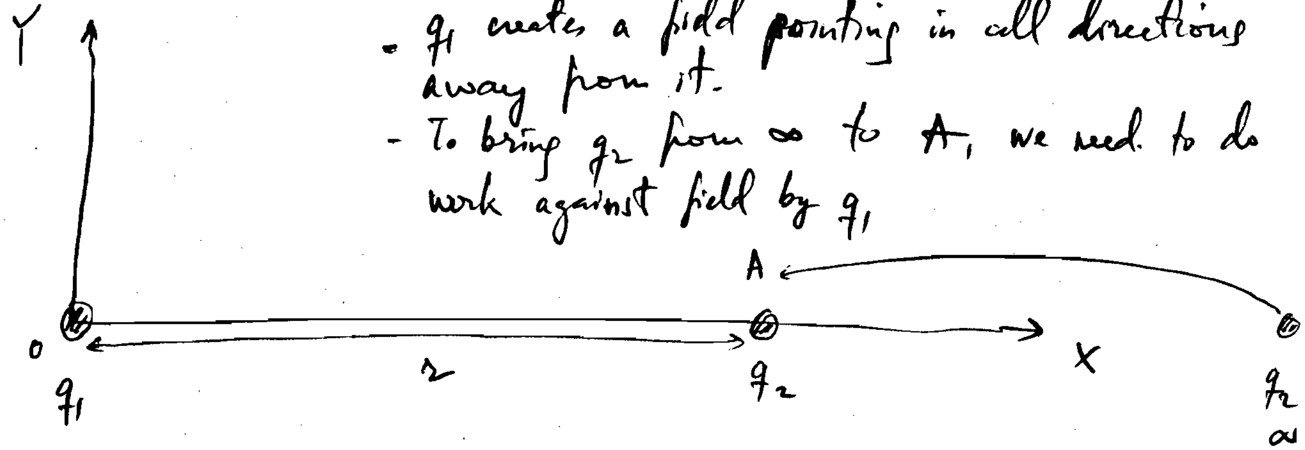
ch. 23 Electrostatic Energy & Capacitors:

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q'}$$ $q' \longrightarrow$ Test charge

U : electric potential energy (J)
V : electric potential (J/C = V for Volt)

$$\Delta U_{AB} = - W_{AB} = - \int_A^B F \cdot d\vec{l}$$

- q_1 creates a field pointing in all directions away from it.
- To bring q_2 from ∞ to A, we need to do work against field by q_1



→ Electric potential due to q_1 at point A is

$$\underbrace{V = \frac{k q_1}{r}}_{\infty \text{ as the reference point}} = \Delta V_{\infty A} = k q_1 \left(\frac{1}{r} - \frac{1}{\infty} \right)$$

∞ as the reference point. (it is the electric potential difference b/w ∞ and A)

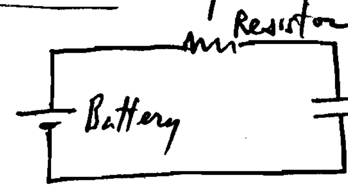
→ What is $\Delta U_{\infty A}$ if we move q_2 from ∞ to A?

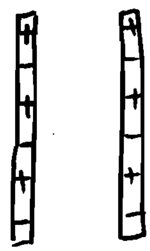
$$\Delta U_{\infty A} = q_2 \Delta V_{\infty A} = \frac{k q_1 q_2}{r}$$

We can store electric potential energy by bringing charges together!

Electrostatic Energy Storage Devices : capacitors : parallel-plate

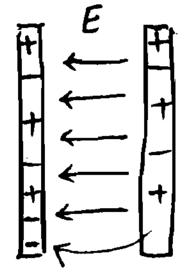
Symbol : || $\longrightarrow$





$Q=0$ $Q=0$

equal amount of charges of either type in each plate



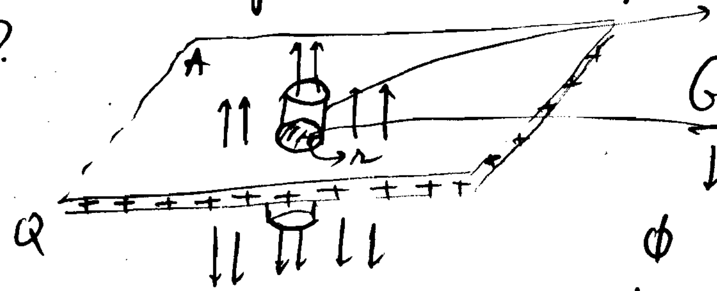
$Q=-e$ $Q=+e$

To store energy we bring charges against the field.

To continue bringing charges from one plate to the other I will need to do work (more and more we we progress). "Charging the capacitor"

The total energy stored = superposition of all work required to bring all e^- from right plate to left plate.

→ What is the electric field due to a plate of charge Q & area A?



Gauss Law :

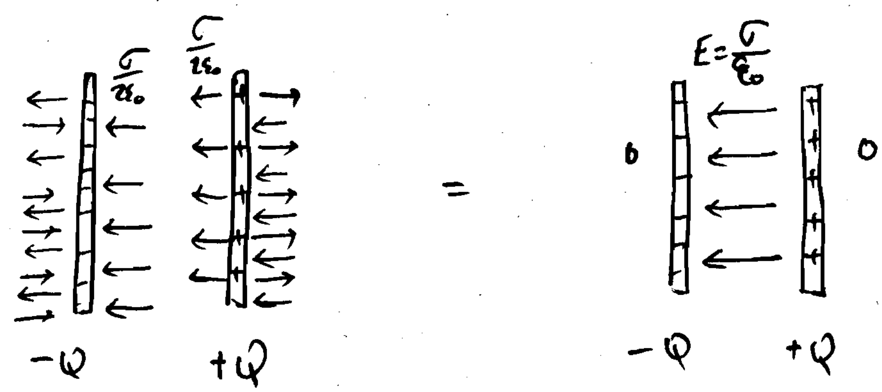
$$\Phi = \frac{q_{enclosed}}{\varepsilon_0}$$

$$E\,\pi r^2 \times 2 = \frac{\pi r^2 \frac{Q}{A}}{\varepsilon_0} = \frac{\pi r^2 \sigma}{\varepsilon_0}$$

$\frac{Q}{A} = \sigma$: charge density

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

What is the electric field b/w 2 parallel plates of charges $+Q$ & $-Q$



Total electric field b/w 2 parallel plate of charges $+Q$ & $-Q$ is $\frac{\sigma}{\varepsilon_0}$

$$dU = -dW = \underbrace{dq}_{\text{Test charge}}\, V = \underbrace{dq\, E\, l}_{} = dq\, \frac{\sigma}{\varepsilon_0}\, l = dq\, \frac{q}{A\,\varepsilon_0}\, l$$

l → separation b/w the plate.

$\frac{\sigma}{\varepsilon_0}$ → electric b/w plates.

$$V = -\int \vec{E}\cdot d\vec{l} = -E \cdot l$$

↓ constant electric field

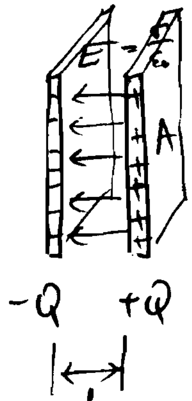
Total energy stored = $$U = \int dU = \frac{l}{A\varepsilon_0}\int_{0\,(\text{initial})}^{Q\,(\text{final charge})} q\, dq = \frac{l}{A\varepsilon_0}\,\frac{1}{2}Q^2$$

$$= \frac{1}{2}\,\underbrace{\varepsilon_0 A\, l}_{\substack{\text{vol. b/w plates} \\ (A \text{ surface of each plate})}}\ \underbrace{\frac{Q^2}{(\varepsilon_0 A)^2}}_{\left(\frac{\sigma}{\varepsilon_0}\right)^2 = E^2}$$

$$\boxed{\text{Total energy stored per unit vol} = \frac{U}{Al} = u = \frac{1}{2}\varepsilon_0 E^2} \rightarrow \frac{J}{m^3} \text{ (energy density)}$$

→ empty space b/w plate.

<u>Capacitance</u>: $C = \dfrac{Q}{V}$ $Q \rightarrow$ Total charge on each plate; $V \rightarrow$ potential b/w plates.



$$C = \frac{Q}{E \cdot l} = \frac{Q}{\frac{V}{\varepsilon_0} \cdot l} = \frac{\cancel{\sigma}}{\frac{\cancel{\sigma}}{\varepsilon_0 A} \cdot l} = \underbrace{\frac{A\varepsilon_0}{l}}$$

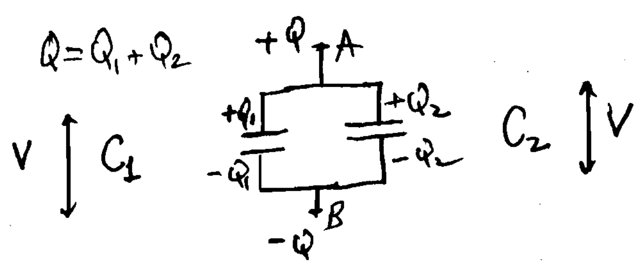
capacitance of a parallel plate of surface A, separation l

Total energy stored b/w parallel plates:

$$\boxed{U = \frac{1}{2}\varepsilon_0 A l E^2 = \frac{1}{2} \underbrace{\frac{\varepsilon_0 A}{l}}_{C} \underbrace{l^2 E^2}_{V^2} = \frac{1}{2} C V^2}$$

<u>Connecting capacitors</u>:

<u>Parallel connection</u>



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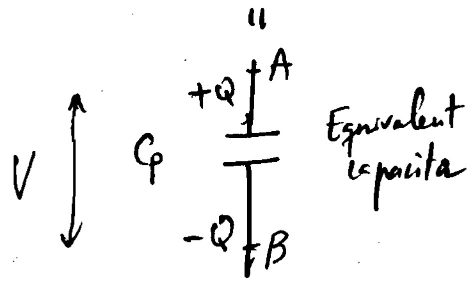
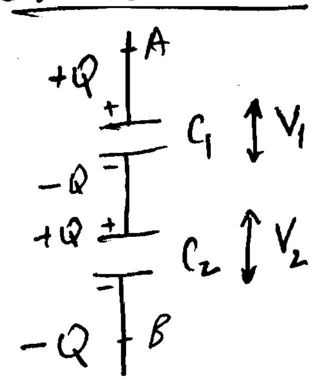


<u>Series connection</u>



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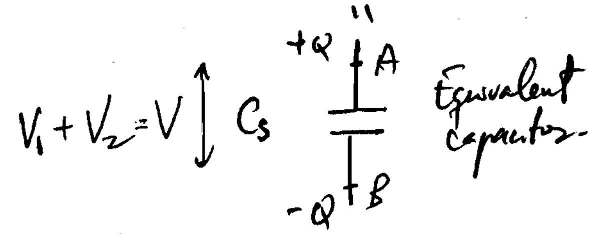


<u>Parallel (cont.)</u>

$V = \frac{Q}{C}$ (From the def. of C)

$$\begin{cases} V = \frac{Q_1}{C_1} \Rightarrow \frac{Q_1}{V} = C_1 \\ V = \frac{Q_2}{C_2} \Rightarrow \frac{Q_2}{V} = C_2 \\ V = \frac{Q}{C_p} = \frac{Q_1 + Q_2}{C_p} \end{cases}$$

$$C_p = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$$

$$\boxed{C_p = C_1 + C_2}$$

<u>Series (cont.)</u>

$$C_s = \frac{Q}{V}$$

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{V}{Q} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

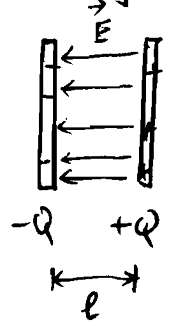
$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}}$$

or $\boxed{C_s = \frac{C_1 C_2}{C_1 + C_2}}$

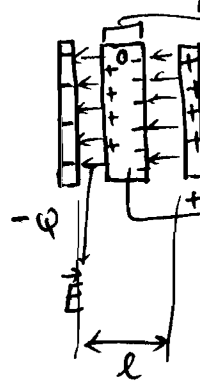
<u>Capacitance:</u> $C = \frac{Q}{V}$

How can we increase C?
↳ e.g. decrease V (electric potential difference b/w plates)

1) Inserting a conducting slab b/w plates.



$V = E.\ell$



→ width $\frac{\ell}{2}$

→ conducting slab: free e^- rearrange immediately as the slab is inserted
→ Total field inside conductor is 0!

$V' = E.\frac{\ell}{2}$

"Inserting a conducting slab of width $\frac{\ell}{2}$ = reducing the separation b/w plates by $\frac{\ell}{2}$"

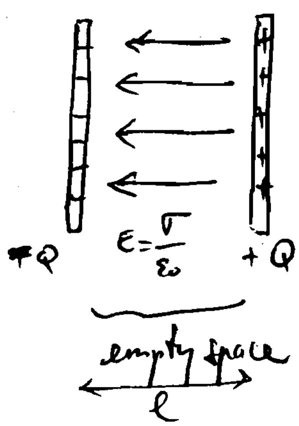
2) $C = \dfrac{Q}{V}$; $V = E \cdot l$; alternative is to reduce E: by inserting a dielectric slab filling the volume b/w plates:

$E_0 = \dfrac{\sigma}{\varepsilon_0}$ b/w plates. (empty space)

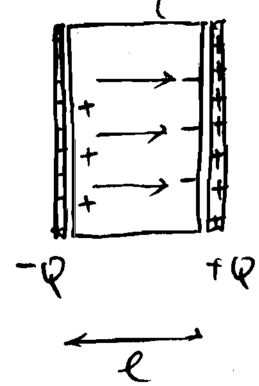
$E = \dfrac{\sigma}{\varepsilon}$ b/w plates (filled with dielectric)

$\varepsilon = K \varepsilon_0$; $K \geq 1$

$= \dfrac{\sigma}{K \varepsilon_0} = \dfrac{E_0}{K}$ Field is reduced!



$$C_0 = \frac{Q}{V_0} = \frac{Q}{E_0 l}$$



dielectric → get polarized, with some electric field in the opposite direction as that created by the plates, reducing it.

$$C = \frac{Q}{V} = \frac{Q}{E \cdot l} = \frac{Q}{\frac{E_0}{K} \cdot l}$$

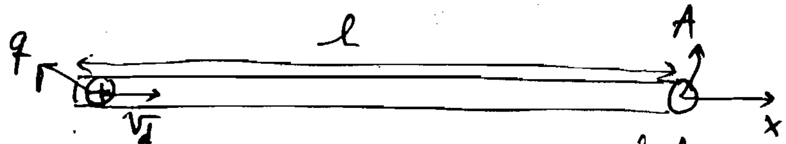
$$= K C_0$$

(Dielectric insert increases capacitance by a factor of K)

Ch. 24 Electric Current

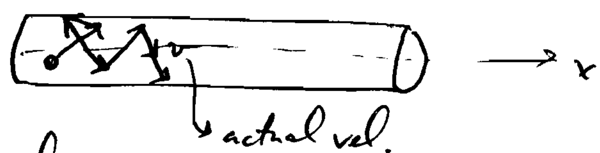
$$I \equiv \frac{\Delta q}{\Delta t} \; ; \qquad I = \frac{dq}{dt} \qquad \left(\frac{C}{s} \equiv A \;\; \text{Amp.} \right)$$

Microscopic expression:



$$I = \frac{\Delta q}{\Delta t} = \frac{n A \ell q}{\frac{\ell}{v_d}} = n q A v_d$$

v_d: drift speed or drift velocity: average velocity along x
actual velocity is much higher:



n = number of charge per unit volume

Copper wire: $A = 1\,mm^2$, $I = 5A$; each atom of copper contributes $1.3e$ of charge $\rightarrow v_d$?

$$v_d = \frac{I}{n q A}$$

We need to find number of atoms of Copper per unit volume.

Mass density $\rho_{copper} = 8920\,kg/m^3$: if we divide this
("Rho")
by the mass of one atom of copper $\rightarrow$ we will get n

$Cu = 63.55$ a.u. $= 63.55 \times 1.66 \times 10^{-27}$ kg
(atomic unit)

$$n = 8920 \frac{kg}{m^3} \; \frac{1 \text{ atom Cu}}{63.55 \times 1.66 \times 10^{-27}\,kg} = 8.5 \times 10^{28} \frac{\text{atoms Cu}}{m^3}$$

$$v_d = \frac{I}{nqA} = \frac{5A}{8.5\times10^{28}\times1.3\times1.6\times10^{-19}\times10^{-6}} \cong 0.283\ \frac{mm}{s}$$

Clearly this is not the actual speed of electrons:
just for curiosity: $\frac{1}{2}m\bar{v^2} = \frac{3}{2}kT$ (for gases)

if $v = 0.283\times10^{-3}\ \frac{m}{s} \rightarrow T = 6.1\times10^{-12}\ ^\circ K$!

Current density: $J = \frac{I}{Area} = \frac{nq\, Area\, v_d}{Area} = nqv_d \quad \left(\frac{A}{m^2}\right)$

↳ Normal or Ohmic materials: $\vec{J} = \sigma\vec{E}$

"sigma" conductivity.

(For conductors in electrostatic equilibrium: no current → no electric field)

$\rho \equiv \frac{1}{\sigma}$: resistivity

"rho"

Conductivity: $[\sigma] = \frac{[J]}{[E]} = \frac{\frac{A}{m^2}}{\frac{N}{C}} = \frac{A}{\underbrace{\frac{N}{C}m}_{V}\, m} = \frac{A}{V.m}$ (Volt)

$= \frac{1}{\frac{V}{A}.m} = \frac{1}{\Omega m}$

Ohm's law: $I = \frac{V}{R}$ (current = $\frac{\text{voltage}}{\text{resistance}}$)

$R = \frac{V}{I}$ ($\frac{V}{A} \equiv \underset{Ohm}{\Omega}$)

$[\rho] = \Omega m$ (resistivity)

$[R] = \Omega$ (resistance)

$$\underbrace{\boxed{J = \sigma E}}_{\text{Microscopic Ohm's Law}} \rightarrow \quad J = \frac{1}{\rho} E = \frac{1}{\rho}\frac{\Delta V}{\Delta x}$$

$$\downarrow$$

$$\frac{I}{\text{Area}} = \frac{1}{\rho}\frac{\Delta V}{\Delta x}$$

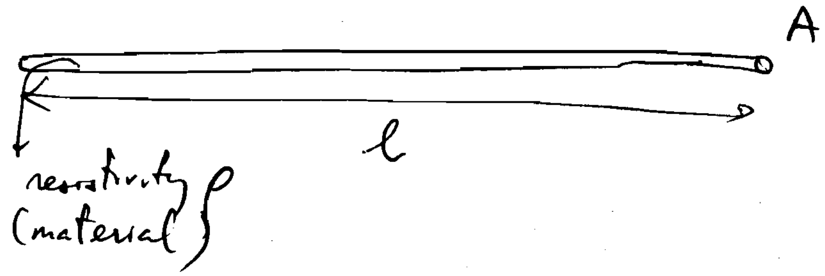
$$I = \frac{1}{\rho}\underbrace{\frac{\Delta V}{\Delta x}\,\text{Area}}_{\Delta V \cdot \ell} = \frac{\Delta V\,\ell}{\rho}$$

$$I = \frac{\Delta V}{\frac{\rho}{\ell}} = \frac{\Delta V}{R} \quad \text{or}$$

$$\boxed{I = \frac{V}{R}} \quad \text{Macroscopic Ohm's Law.}$$

- conductors : → high conductivity (low resistivity)
- semi-conductors : → become conductors when doped with impurities
- superconductors : → extremely high conductivity (almost zero resistivity)

<u>Resistance</u> : R (Ω)
omega "Ohm"



Resistance

$$R = \rho \frac{\ell}{A}$$

resistivity

material

geometry of the wire

It takes work to move charges along wires : mostly lost in the form of heat :

$$P = I \cdot V \begin{cases} = \frac{V}{R} V = \frac{V^2}{R} \\ = I \cdot IR = I^2 R \end{cases}$$

P → heat loss per unit time (unit is Watt)

$$I = \frac{V}{R} \quad \text{or} \quad V = IR$$

5a) 50 W / 100 W light bulbs at 120 V : which bulb has lower resistance ?

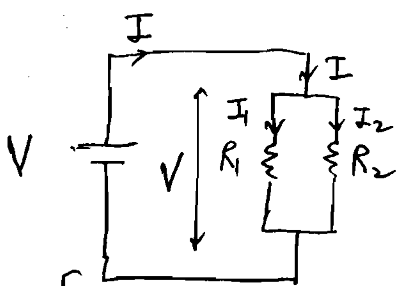
→ $P = \frac{V^2}{R}$ (same $V = 120V$ → higher P → lower resistance (100 W))

5b) Series & Parallel connection of resistors.

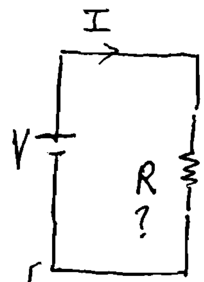
Resistors

(e.g. light bulb is a resistor)

Parallel



$= V$



"current division"

$I = I_1 + I_2$

$I = \frac{V}{R}$

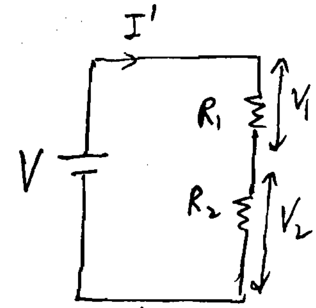
$= \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

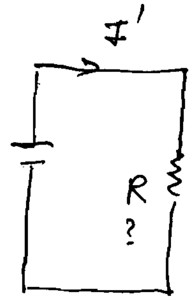
$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

(product over the sum)

Series



$= V$



"voltage division"

$V = V_1 + V_2 = I'R_1 + I'R_2$

$= I'(R_1 + R_2)$

$V = I'R$

$$\boxed{R = R_1 + R_2}$$

Voltage division:

$$V_1 = I'R_1 = \frac{V}{R_1 + R_2} R_1$$

$$V_1 = \underbrace{\frac{R_1}{R_1 + R_2}}_{<1} V \quad = \text{fraction of } V$$

$$V_2 = I'R_2 = \frac{R_2}{R_1 + R_2} V$$

$$V_1 + V_2 = \frac{R_1}{R_1 + R_2} V + \frac{R_2}{R_1 + R_2} V = V$$

Power Consumptions

$R_1 = R_2 = R$

Parallel

$R_{equiv} = \frac{R \cdot R}{R + R} = \frac{R}{2}$

one resistor $P_1 = I_1 V_1 = \frac{I}{2} V = \left(\frac{V}{\frac{R_{equiv}}{2}}\right) V = \frac{V^2}{\cancel{4}R} = \frac{\cancel{4}V^2}{R}$

Series

$R_{equiv} = 2R$

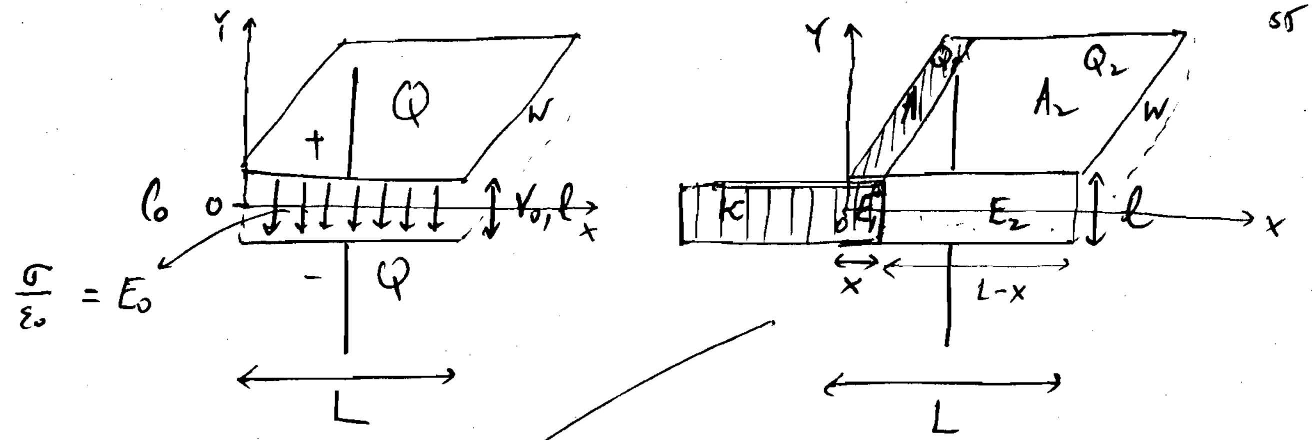
one resistor $P_1 = I' V_1 = \frac{V}{2R} \cdot \frac{V}{2} = \frac{V^2}{4R}$

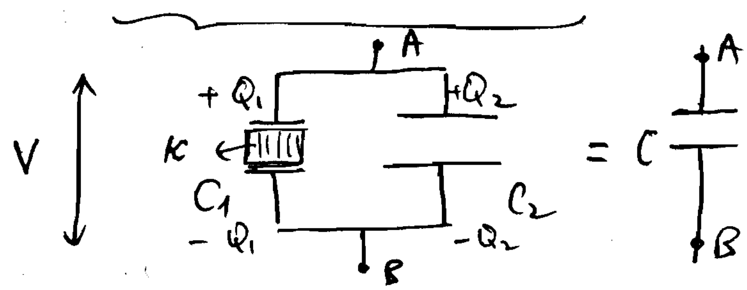
<u>Parallel connection:</u> $R_1 = R_2 = R$; $R_{equiv} = \frac{R}{2}$

$$P_1 = \underbrace{I_1 V_1}_{\text{current division}} = \left(\frac{I}{2}\right) V = \frac{\frac{V}{R_{equiv}}}{2} V = \frac{\frac{V}{\frac{R}{2}}}{2} V = \frac{\frac{2V}{R}}{2} V$$

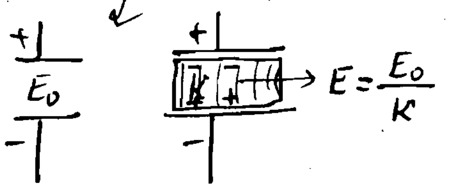
$$= \frac{V^2}{R} \checkmark$$

23.72





$$C = C_1 + C_2 = \frac{Q_1}{V} + \frac{Q_2}{V}$$



$\rightarrow$

1) $E_1 = \frac{E_0}{K}$ & $E_2 = E_0$

2) $E_1 = \frac{\sigma_1}{\varepsilon} = \frac{\sigma_1}{K\varepsilon_0} = \frac{\frac{Q_1}{A_1}}{K\varepsilon_0} = \frac{Q_1}{K\varepsilon_0 w x}$

$E_2 = \frac{\sigma_2}{\varepsilon_0} = \frac{Q_2}{\varepsilon_0 A_2} = \frac{Q_2}{\varepsilon_0 w(L-x)}$

$\boxed{E_1 = E_2}$ since discontinuity in the field is not possible

$$\frac{Q_1}{K\cancel{\varepsilon_0} w x} = \frac{Q_2}{\cancel{\varepsilon_0} w(L-x)} \rightarrow \boxed{Q_1 = Q_2 \frac{Kx}{L-x}}$$

$$C = C_1 + C_2 = \frac{Q_1 + Q_2}{V} = \frac{Q_2 \frac{Kx}{L-x} + Q_2}{V} = \frac{Q_2}{V}\left(\frac{Kx}{L-x} + 1\right)$$

$$\frac{Q_2}{V} = \frac{Q_2}{E_2 \ell} = \frac{\cancel{Q_2}}{\frac{Q_2}{\varepsilon_0 w(L-x)}\ell} = \frac{\varepsilon_0 w(L-x)}{\ell}$$

$$\rightarrow \boxed{C = \frac{\varepsilon_0 w(L-x)}{\ell}\left(\frac{Kx}{L-x} + 1\right) = \frac{w\varepsilon_0}{\ell}\left(Kx + L - x\right)}$$

Capacitance with dielectric slab inserted x into the spacing.

a) $$C\left(x=\frac{L}{2}\right)= \frac{w\varepsilon_0}{\ell}\Big(K\frac{L}{2}+\underbrace{L-\frac{L}{2}}_{\frac{L}{2}}\Big) = \frac{w\varepsilon_0 L}{2\ell}(K+1)$$

b) Stored energy: $$U(x) = \frac{1}{2}CV^2 = \frac{1}{2}\frac{w\varepsilon_0}{\ell}(Kx+L-x)V^2$$

$$V = E_1\ell = \frac{Q_1}{K\varepsilon_0 w x}\ell = Q\frac{\ell}{\varepsilon_0 w}\frac{1}{Kx+L-x}$$

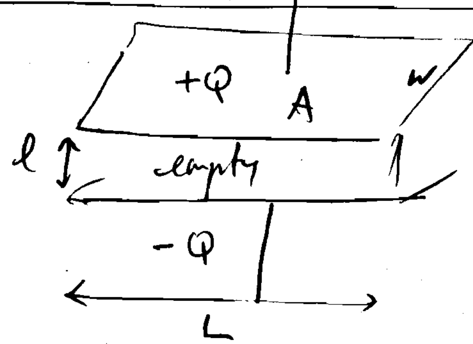
$$Q_1+Q_2 = Q \quad ; \quad Q_2 = Q_1\frac{L-x}{Kx}$$

$$\hookrightarrow Q_1 + Q_1\left(\frac{L-x}{Kx}\right) = Q \rightarrow Q_1\left[1+\frac{L-x}{Kx}\right] = Q$$

$$\boxed{Q_1 = Q\frac{Kx}{Kx+L-x}}$$

$$U(x) = \frac{1}{2}\frac{w\varepsilon_0}{\ell}(Kx+L-x)\frac{\ell^2}{(\varepsilon_0 w)^2}\frac{1}{(Kx+L-x)^2}Q^2$$

$$\boxed{U(x) = \frac{\ell}{2\varepsilon_0 w}\frac{Q^2}{Kx+L-x}} = \frac{\ell Q^2}{2\varepsilon_0 w L}\frac{L}{Kx+L-x} = \boxed{U_0\frac{L}{Kx+L-x}}$$



C_0

$$U_0 = \frac{1}{2}C_0V_0^2 = \frac{Q^2}{2C_0} = \frac{Q^2}{2\frac{Lw\varepsilon_0}{\ell}} = \frac{\ell Q^2}{2Lw\varepsilon_0}$$

(without insert)

$$C_0 = \frac{Q}{V_0} \rightarrow V_0 = \frac{Q}{C_0}$$

$$C_0 = \frac{A\varepsilon_0}{\ell} = \frac{Lw\varepsilon_0}{\ell}$$

$$U\left(x=\frac{L}{2}\right) = U_0\frac{L}{K\frac{L}{2}+L-\frac{L}{2}} = \frac{U_0}{\frac{1}{2}(K+1)} = \frac{2U_0}{K+1} = \frac{C_0V_0^2}{K+1}$$

c) Force on slab ?

$$\underset{\text{(magnitude)}}{\text{Force}} = -\frac{dU}{dx} \rightarrow \text{Why?} \qquad V = -\int \vec{E} \cdot d\vec{l}$$

$$\hookrightarrow \vec{E} = -\frac{dV}{dx}\,\hat{i}$$

$$\underbrace{q_{test}\,\vec{E}} = -\frac{d(q_{test}V)}{dx}\,\hat{i}$$

$$\text{Force} = -\frac{dU}{dx}\,\hat{i}$$

$$F(x) = \text{Force} = -\frac{d}{dx}U(x) = -\frac{d}{dx}\left(\frac{U_0 L}{Kx+L-x}\right) = -U_0 L\,(-1)\frac{K-1}{(Kx+L-x)^2}$$

$$F(x) = \frac{U_0 L(K-1)}{(Kx+L-x)^2}$$

$$F\left(x=\frac{L}{2}\right) = \frac{U_0 L(K-1)}{\left(K\frac{L}{2}+L-\frac{L}{2}\right)^2} = \frac{U_0 L(K-1)}{\frac{L^2}{4}(K+1)^2} = \frac{4U_0(K-1)}{L(K+1)^2}$$

$$\underset{U_0 = \frac{1}{2}C_0V_0^2}{=} \frac{2C_0V_0^2(K-1)}{L(K+1)^2}\ \checkmark$$

24.69

$m = 1500\,kg$

$26 \times 12V$ batteries in series $= 312V$ $\underbrace{(100\,A\text{-}h)}_{\text{charge } Q}$

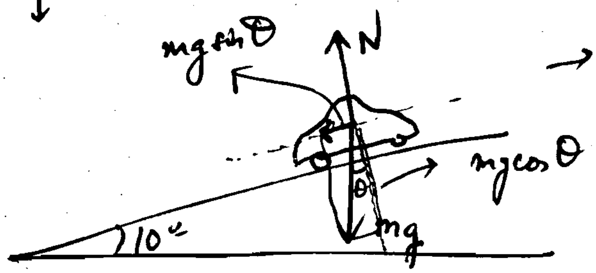
Motor: 85% efficient

Can climb $\theta = 10°$ slope at $45 \frac{km}{h} \cdot \frac{1h}{3600\,s} \cdot \frac{10^3 m}{1\,km}$

→ How long can it maintain this speed?

→ How much is needed? → How long can we last?

↓



We need to apply $F = mg\sin\theta$ → spend energy at a rate of $P = F \cdot v = mgv\sin\theta$: we need to spend this much (per unit time) to go up a slope of angle θ & at speed v.

→ How long we last → $\dfrac{\text{how much mech. energy we have}}{\text{needed energy rate}}$

$$= \frac{0.85 \times 100 \times 3600 \times 312}{mgv\sin\theta} \text{ (see next page)} = \frac{85 \times 3600 \times 312}{1500 \times 9.81 \times \frac{45}{3.6}\sin 10°}\,s$$

$= 2989\,s = 49.8\,min.$

Electrical circuits:

$$P = I.V = \frac{\text{energy}}{\text{time}}$$

$$\text{Electrical energy} = P.\Delta t = \underbrace{I\Delta t}_{Q}.V$$

$$\rightarrow \text{Mech energy} = 0.85\,QV$$

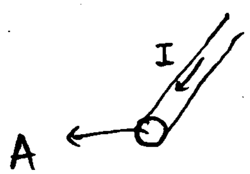
$$= 0.85 . \underbrace{100 \times 3600}_{C} . 312 \text{ J}$$

24.44) 12-gauge (2.1mm diameter) copper wire → 20 A

$d = 2.1$mm

a) J ?

$$J = \frac{I}{A} = \frac{20}{\pi\left(\frac{d}{2}\right)^2} = \frac{80}{\pi(2.1\times10^{-3})^2} = 5.77\times10^{6}\ \frac{A}{m^2}$$



b) E ?

$$E = \rho J$$

$$\left(J = \underset{\substack{\downarrow \\ \text{conductivity}}}{\sigma} E \right.$$

$$\left. E = \frac{1}{\sigma} J = \underset{\substack{\downarrow \\ \text{resistivity}}}{\rho} J \right)$$

↓

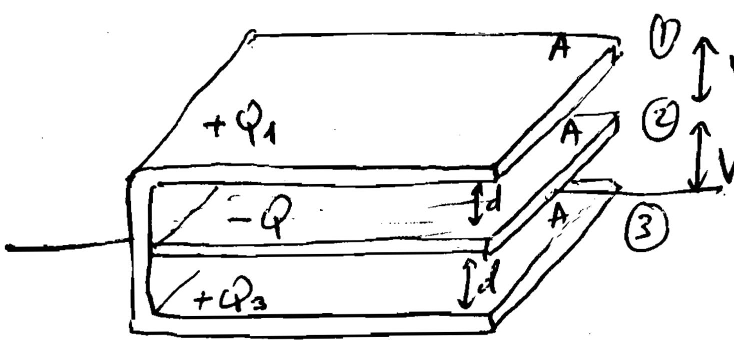
→ material specific.

→ Table: $1.68\times10^{-8}\ \Omega m$ copper

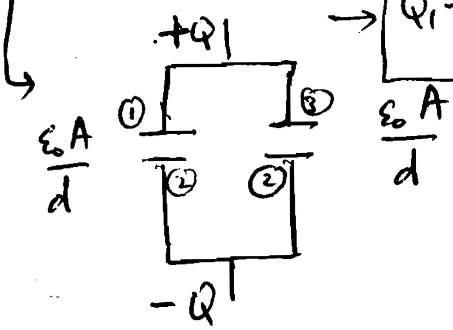
$$E = \rho J = 1.68\times10^{-8} \times 5.77\times10^{6}$$

$$= 0.097\ \frac{V}{m} \text{ or } \frac{N}{C}$$

23.43



$C = \frac{2\varepsilon_0 A}{d}$ prove!



$\boxed{\begin{aligned} Q_1 + Q_3 &= +Q \\ Q_2 &= -Q \end{aligned}}$

$=$

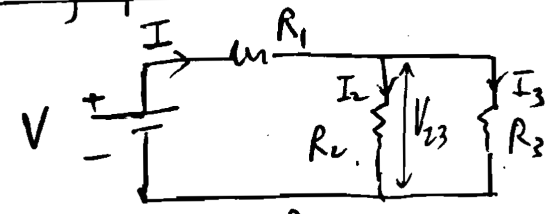
$\frac{2\varepsilon_0 A}{d}$

Ch 25: Electric Circuits

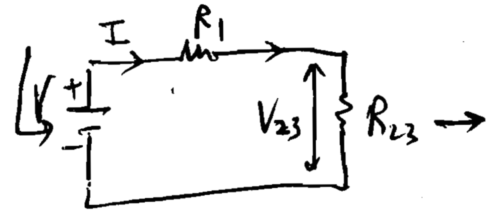
Linear (Linear relationship b/w Voltage V & current I)

2 types of circuits
- → Resistors only
  - 1) Reduce to  using series & parallel connection
  - 2) Using Loop or node analysis.
- → Resistors and capacitors

1) Using parallel & series connection:

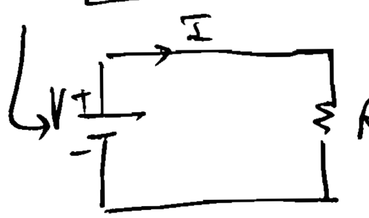


Solve for circuit → calculate I, I_2, I_3.



→ Parallel equivalent of R_2 & R_3

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3}$$



$$R_{123} = \text{Series equivalent of } R_1 \text{ \& } R_{23}$$
$$= R_1 + R_{23} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Now: Ohm's Law: $I = \frac{V}{R}$

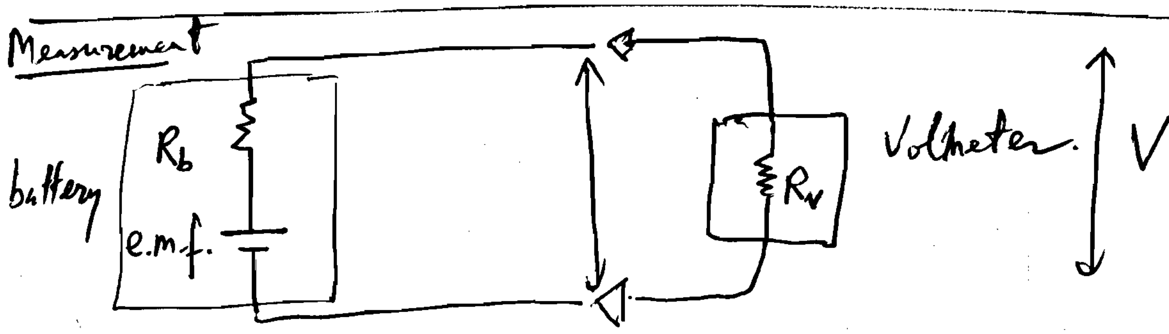
↳ in our circuit: $I = \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$

$$I_2 = \frac{V_{23}}{R_2} = \frac{I \cdot R_{23}}{R_2} = \frac{I \frac{\cancel{R_2} R_3}{R_2 + R_3}}{\cancel{R_2}} = I \underbrace{\frac{R_3}{R_2 + R_3}}_{<1}$$

$$I_3 = \frac{V_{23}}{R_3} = \frac{I R_{23}}{R_3} = \frac{I \frac{R_2 \cancel{R_3}}{R_2 + R_3}}{\cancel{R_3}} = I \underbrace{\frac{R_2}{R_2 + R_3}}_{<1}$$

Also: $I_3 = I - I_2$

Measurement



$R_V = 1000\Omega \rightarrow V = 4.5V$

$R_V = 1500\Omega \rightarrow V = 4.2V$

emf?