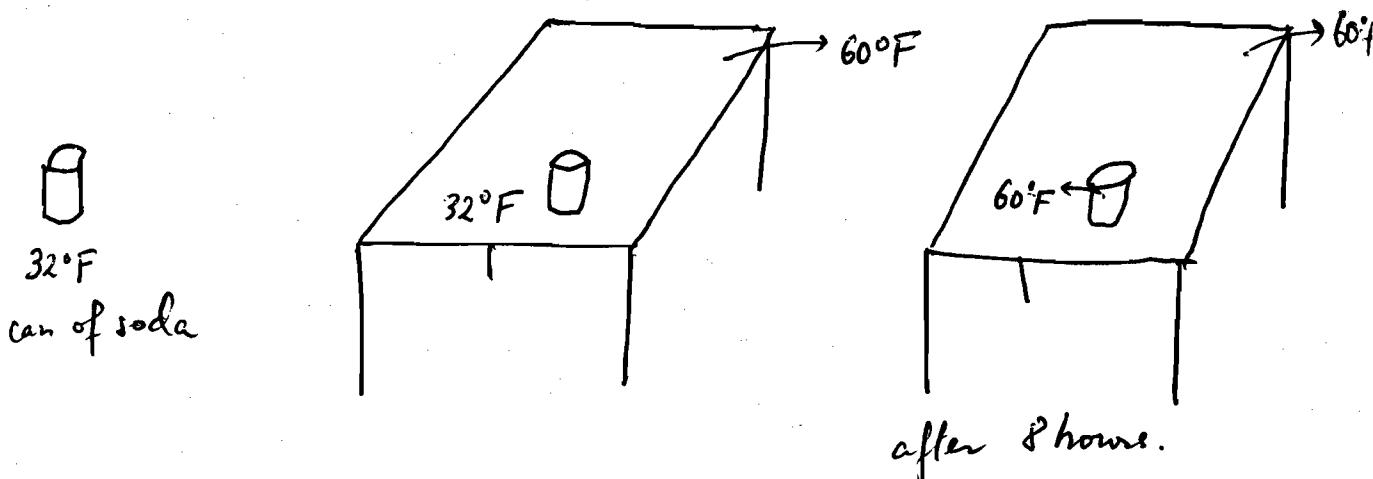


Ch 16 Temperature & Heat

1

Temperature: a macroscopic property, a consequence of the microscopic behavior



Two objects in thermal contact can exchange heat to arrive at a thermodynamic equilibrium (same temperature)

Temperature can be measured with a thermometer
(mercury; gas at constant volume; electronic, etc.)

$$\text{Fahrenheit } ^\circ\text{F} \rightarrow T_F = \frac{9}{5}T_c + 32$$

$$\text{ Celsius} \quad {}^{\circ}\text{C} \quad \rightarrow \quad T_c = \overline{T_k - 273.16} \text{ } {}^{\circ}\text{K}$$

Kelvin °K Triple point (water)

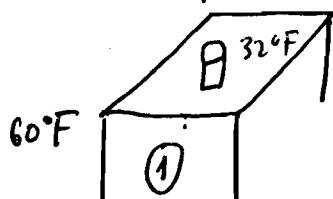
→ "Absolute zero" is 0°K .

Heat transfer (or exchange) rate:

(How fast heat can be transferred) \rightarrow (per unit time)

$$H = -k \frac{A}{x} \Delta T$$

$$\Delta T = 18^{\circ}\text{F}$$



Faster
Heat transfer.



Units: (S.I.)

$$[H] = [k][A] \frac{[\Delta T]}{[\Delta x]} \rightarrow ^{\circ K} m$$

\downarrow Energy per unit time: $\frac{J}{s}$

\downarrow m^2

length: m
mass: kg
time: s
temperature: $^{\circ K}$

k : thermal constant \rightarrow depends on type of materials
(metal conducts heat faster, etc.)

$$[k] = \frac{\frac{J}{s}}{m^2 \text{ } ^{\circ K}} = \frac{J}{m^2 \text{ } ^{\circ K} s} = \frac{W}{m^2 \text{ } ^{\circ K}} \quad (\text{SI})$$

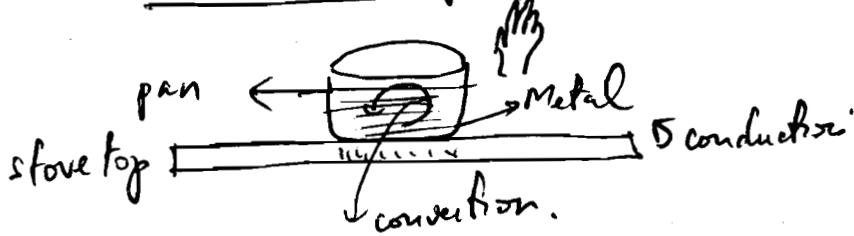
$$[k] = \frac{\text{Btu. in}}{\text{h. ft}^2 \text{ } ^{\circ F}} \quad (\text{British system})$$

$$H = -kA \frac{\Delta T}{\Delta x} \longrightarrow H = -\frac{A \cdot \Delta T}{R}$$

R-factor: $R = \frac{\Delta x}{k}$

$$\begin{cases} \text{SI: } \frac{m^2 \text{ } ^{\circ K}}{W} \\ \text{British: } \frac{\text{ft}^2 \text{ } ^{\circ F}}{\text{Btu/h}} \end{cases}$$

Mechanisms of heat transfer:



- conduction: (electrons, atoms, molecules)
- convection: (circulation)
- radiation

$$\text{Heat exchange: } -\Delta Q_{\text{Table}} + \Delta Q_{\text{soda}} = 0$$

$(\text{Soda } 32^{\circ}\text{F} = T_{\text{soda}}; \text{ Table: } T_{\text{Table}} = 60^{\circ}\text{F}; \text{ T.D. equilibrium} = 59.9^{\circ}\text{F})$

$$\Delta Q = C \Delta T$$

$$\downarrow \text{Heat Capacity } \left(\frac{\text{J}}{\text{kg} \cdot \text{K}} \right) \quad \left\{ \begin{array}{l} \xrightarrow{\text{material}} \\ \xrightarrow{\text{mass}} \end{array} \right. \quad \left| \begin{array}{l} C = c \cdot m \\ \downarrow \\ \text{Heat Capacity} \\ \downarrow \\ \text{Specific heat} \\ \downarrow \\ \text{mass} \end{array} \right.$$

$$= cm \Delta T$$

specific to different materials
(Table 16.1)

$$c_{\text{water}} = 4184 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$c_{\text{Copper}} = 386 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$\xrightarrow{\quad} c_{\text{Table}} m_{\text{Table}} \underbrace{(T_{\text{TD}} - T_{\text{Table}})}_{-} + c_{\text{soda}} m_{\text{soda}} \underbrace{(T_{\text{TD}} - T_{\text{soda}})}_{+} = 0$$

Heat loss rate (per unit time) by radiation of an object of area A & temperature T:

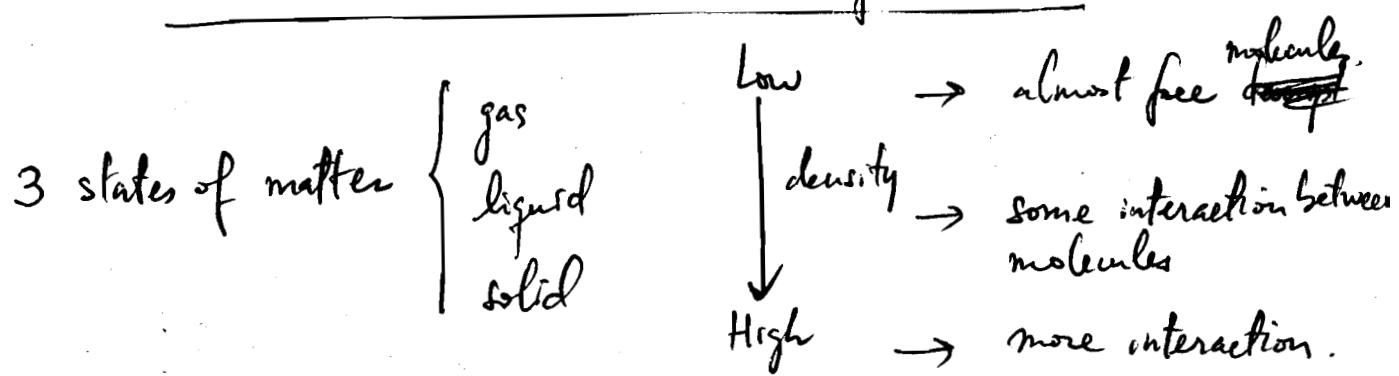
$$\text{Stefan-Boltzmann Law: } P = e \sigma A T^4 \leftarrow$$

emissivity

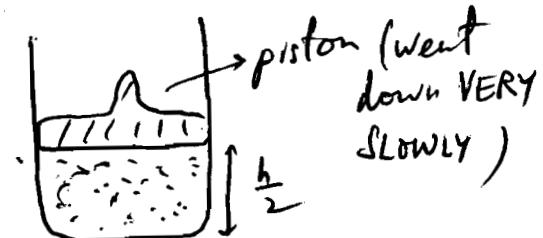
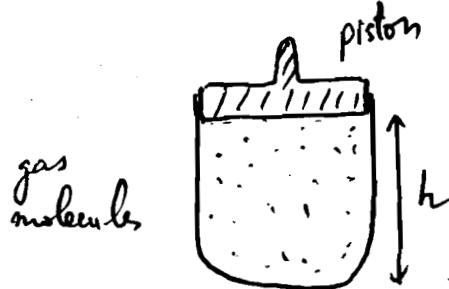
✓ no unit or dimensions

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$

Ch 17 Thermal Behavior of Matter:



Ideal gas = (completely free molecules)



Macroscopic properties

$\left\{ \begin{array}{l} N, V, P, T \\ \downarrow \\ \text{number of molecules} \end{array} \right.$	→	→	→
	volume	pressure	temp.

$$N, \frac{V}{2}, 2P, T$$

①

$$\Rightarrow PV = kNT$$

②

$$2P \frac{V}{2} = kNT$$

Ideal Gas Law

$$k = \text{Boltzmann constant} = 1.38 \times 10^{-23} \frac{\text{J}}{\text{°K}}$$

$$N = n N_A ; \quad N_A = 6.022 \times 10^{23}$$

of moles

$$PV = k n N_A T = n \frac{k N_A}{R} T \rightarrow PV = nRT$$

$R = 8.314 \frac{\text{J}}{\text{°K mol}}$

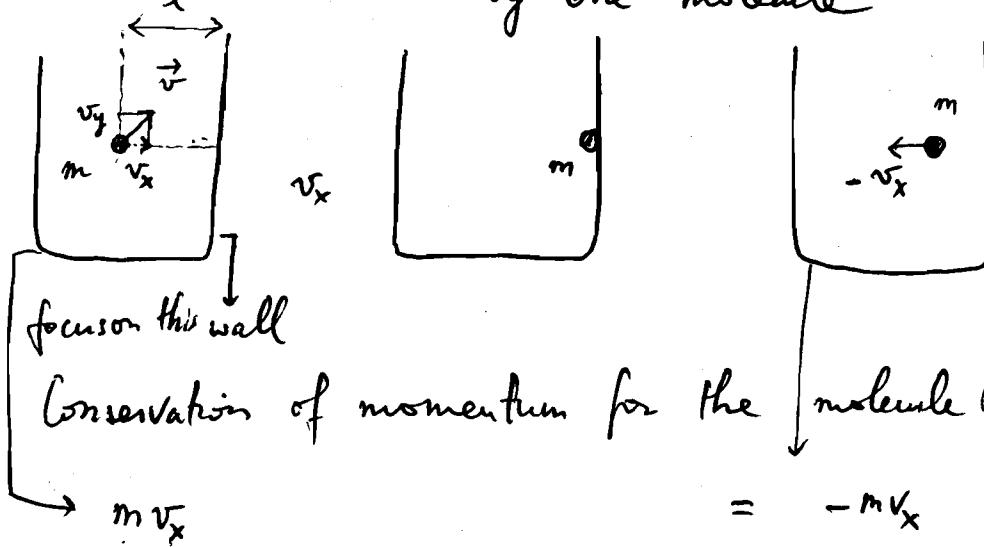
ideal gas

To take into consideration some interaction b/w gas molecules:

$$\left(P + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

Real gas law or van der waals law
 a & b are constants.

Ideal gas: mechanics of gas molecules : pressure exerted on the container wall by one molecule



Conservation of momentum for the molecule & wall system:

$$= -mv_x + \underbrace{2mv_x}_{\text{momentum received by the wall}}$$

$$P_i = \frac{F_i}{A} = \frac{\frac{\Delta p_i}{\Delta t}}{A} = \frac{\frac{2mv_{xi}}{\Delta t}}{A} = \frac{mv_{xi}^2}{\cancel{A}\cancel{\Delta t}}$$

Mv_{xi}

$$\text{By } N \text{ molecules: } P = \sum_{i=1}^N P_i = \frac{mN}{\text{Vol}} \left(\frac{\sum_{i=1}^N v_{xi}^2}{N} \right)$$

$\downarrow \text{volume} = \text{vol}$

$$P = \frac{mN}{\text{Vol}} \overline{v_x^2}$$

$\overline{v_x^2}$ (average of v_x^2)

$$\frac{\overline{v_x^2}}{\overline{v^2}} = \frac{1}{3}$$

$\overline{v_x^2}$ ↓
x-component
of total vel.

$\overline{v^2}$ ↓
Total velocity

$$\rightarrow P = \frac{m N}{3V} \cdot \frac{v^2}{V} \rightarrow PV = m N \frac{V^2}{3}$$

Derived from micro

Derived from microscopic mechanics of gas molecules.

Previously : $PV = kNT$

$$\rightarrow \text{Conclusion : } kT = m \frac{\overline{v^2}}{3} = \frac{2}{3} \frac{1}{2} m \overline{v^2}$$

Average K.E. of
a gas molecule

→ The average kinetic energy of a gas molecule is $\frac{3}{2}kT$

$$\frac{1}{2} m v^2 = \frac{3}{2} k T$$




More on this
in Statistical
Mechanics

Mechanics

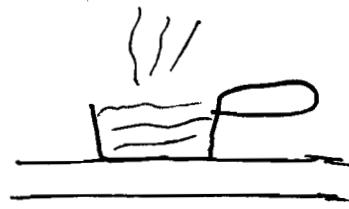
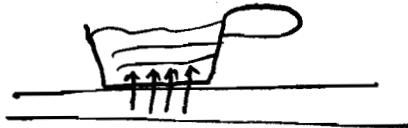
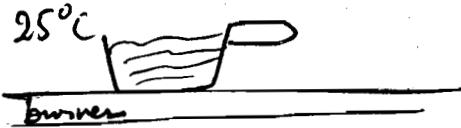
In 1D what is the average K.E. of a gas molecule?

$$\overline{\frac{1}{2}mv_x^2} = \frac{1}{2}kT.$$

Thermal behavior of matter:

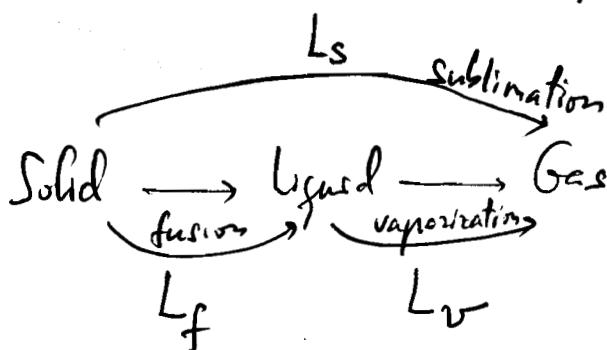
Heat is supplied, ΔQ ,
consequences are =

- 1) ΔT ($\Delta Q = cm \Delta T$)
 - 2) Change of phase ($\Delta Q = mL$)
 - 3) Expansion ΔL , or ΔV
- linear $\alpha = \frac{\Delta L}{L \Delta T}$;
 volume $\beta = \frac{\Delta V}{V \Delta T}$
- latent heat
 Transformation
 L_v (liq \rightarrow gas)
 L_f (solid \rightarrow liq)
 L_s (solid \rightarrow gas)



$$\Delta Q = cm \Delta T$$

Temperature increase



water starts
to boil at

$$100^\circ\text{C}$$

After 2 min

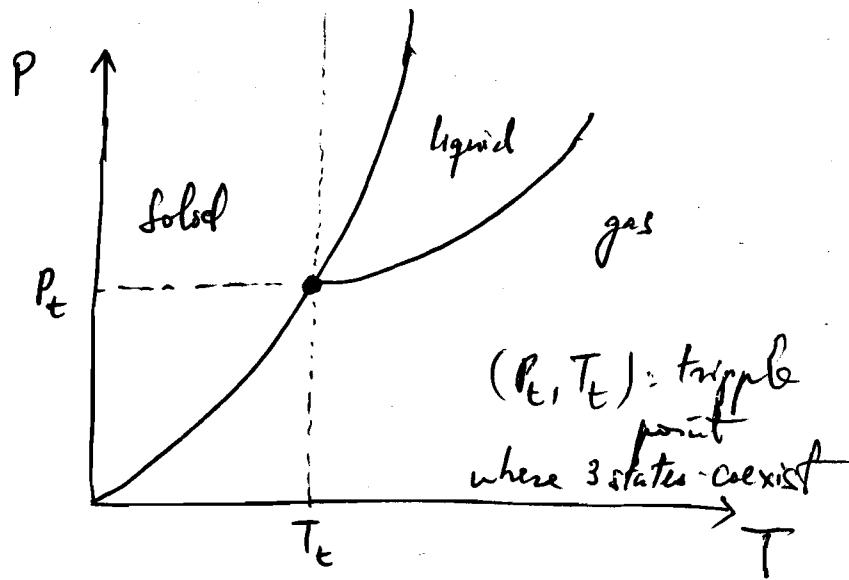
$$T = 100^\circ\text{C}$$

Change of phase
($\Delta T = 0$!)

Phase Diagram:

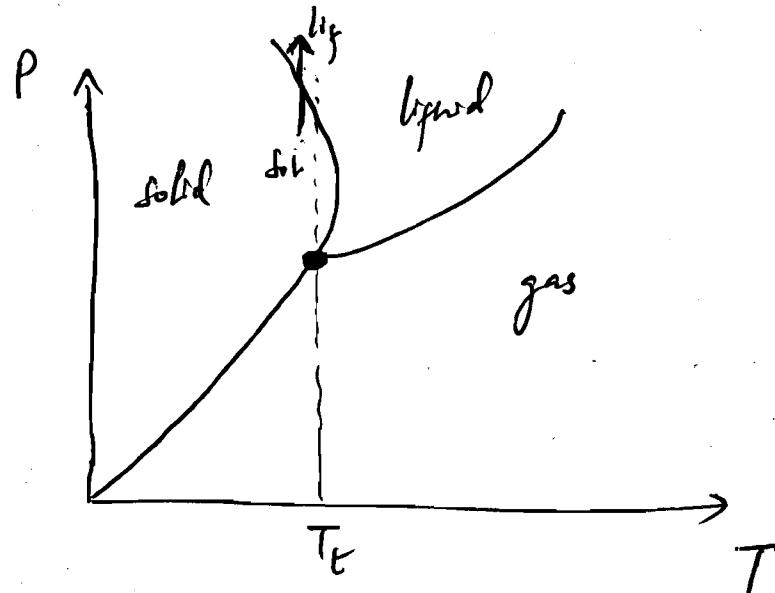
Normal
↓

$\text{Solid} \rightarrow \text{Liquid}$:
we need to
increase T



Water
T

$\text{Solid} \rightarrow \text{Liquid}$
 { → increase T
 { → increase P at
 same T



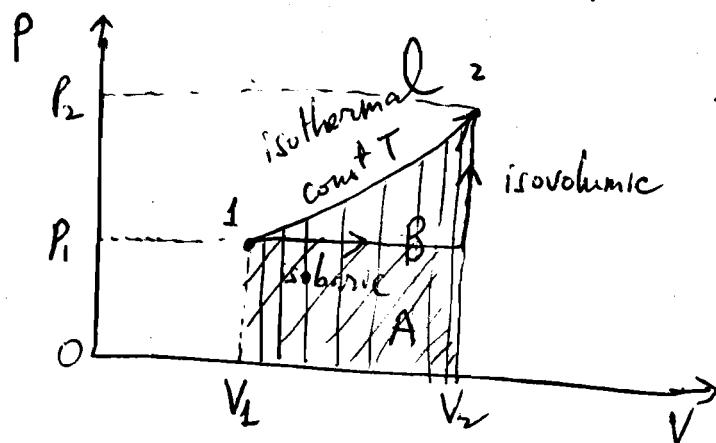
} By applying pressure on an ice cube, it melts.
 density $P_{\text{ice}} < P_{\text{water}}$

Ch 18 Heat, Work, and 1st Law of Thermodynamics

1st Law of Thermodynamics:

$$\left. \begin{array}{l} \text{Heat absorbed by a system } Q \\ \text{Work done by system } W \\ \text{Change of internal energy: } \Delta U \end{array} \right\} \Delta U = Q - W \quad (\text{conservation of energy})$$

$\Delta U, T, P, V$ are state variables (they do not depend on a particular process)
 Q, W are NOT state variables (they depend on a particular process)



1 & 2: states

State 1: P_1, V_1, T_1

State 2: P_2, V_2, T_2

In a PV diagram: the work done is the area under the curve. Work done from 1 to 2 via a isobaric + isovolumic process: $W = A$ is less than the work done if an isothermal process is followed ($W = B > A$)

Why the work done is the area under the curve in a PV diagram?

$$\text{Work done is } W = F \cdot \Delta x = \underbrace{P \cdot A \cdot \Delta x}_{\Delta V}$$

$$\rightarrow dW = PdV \rightarrow [W = \int dW = \int PdV]$$

area under the curve $P(V)$!

Isothermal process : (const. T) b/w state 1 and state 2

Work done in this process : $W = \int PdV$

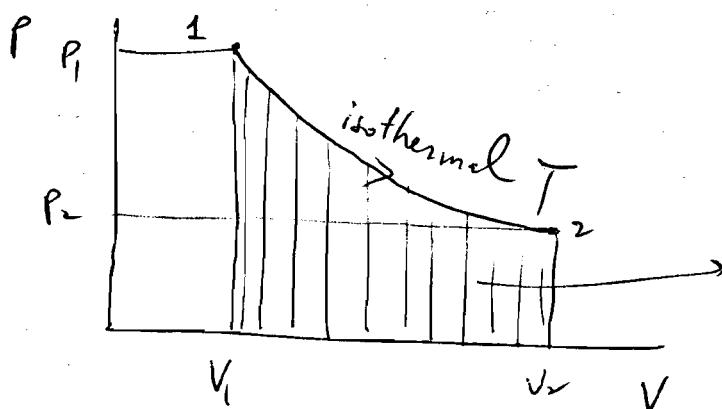
↪ Need $P(V)$

$$\rightarrow \text{Ideal gas} : PV = nRT \rightarrow P = \frac{nRT}{V}$$

Work done by an ideal gas in an isothermal process

$$W = \int_1^2 \frac{nRT}{V} dV = nRT \int_1^2 \frac{dV}{V} = nRT [\ln V_2 - \ln V_1]$$

$$= nRT \ln \left(\frac{V_2}{V_1} \right) \checkmark$$



For an ideal gas, this area is $nRT \ln \left(\frac{V_2}{V_1} \right)$

Ideal gas:

$\Delta U =$ 1) All internal energy comes from kinetic energy
 (since we assume for an ideal gas, there is no interaction)

$$\Delta U = \Delta K.E$$

$$2) \text{ Ave. k.E per molecule : } \frac{1}{2} m \bar{v^2} = \frac{3}{2} kT$$

$$3) \text{ Isothermal : } T_1 = T_2 \rightarrow$$

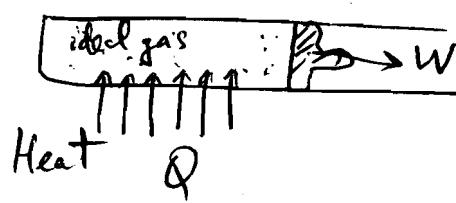
$$\boxed{\Delta U = 0}$$

isothermal, ideal gas

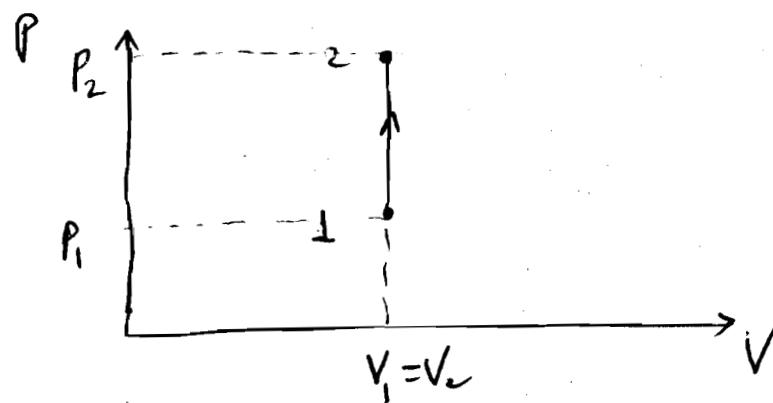
So then what is the heat absorbed by an ideal gas in an isothermal process?

$$\Delta U = Q - W = 0 \rightarrow Q = W = nRT \ln \frac{V_2}{V_1}$$

Suppose: $V_1 < V_2 \rightarrow \begin{cases} W > 0 \\ Q > 0 \end{cases}$



Isovolume Process (const. volume) b/w state 1 & state 2.



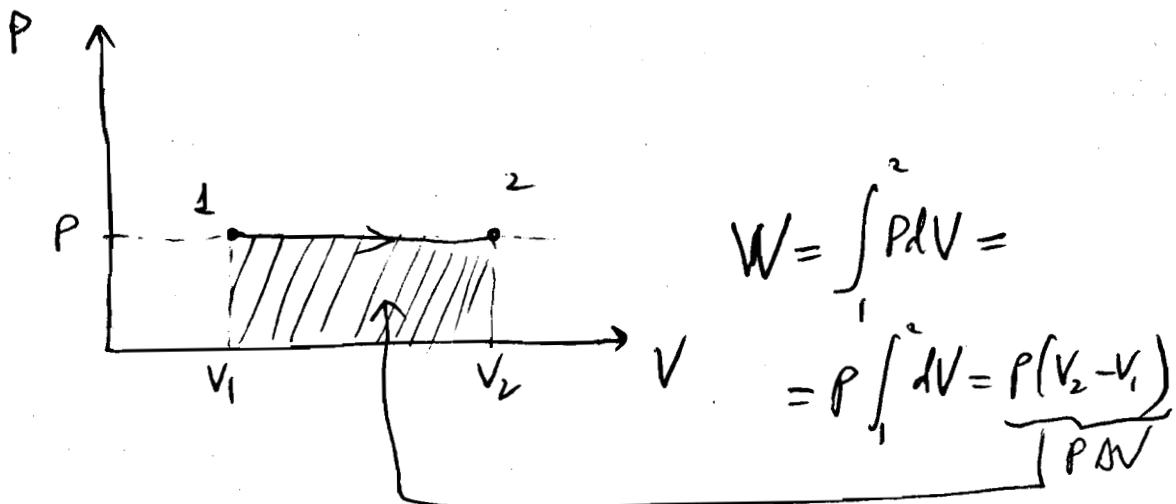
$$W = \int_1^2 P dV = 0$$

$$\Delta U = Q$$

Specific heat at constant V: $C_V = \frac{1}{n} \frac{Q}{\Delta T} = \frac{1}{n} \frac{\Delta U}{\Delta T}$

$$Q = n C_V \Delta T$$

Isobaric Process (const. pressure) b/w state 1 & state 2



Specific heat at constant P: $C_P = \frac{1}{n} \frac{Q}{\Delta T}$

$$Q = n C_P \Delta T$$

For an ideal gas: $\frac{PV}{W} = nRT =$

1st Law of T.D.: $\Delta U = Q - W = Q - PV$
 $\downarrow \quad \downarrow \quad \downarrow$ ideal gas.
 $nC_V\Delta T \quad nC_p\Delta T \quad nR\Delta T$

$$nC_V\Delta T = nC_p\Delta T - nR\Delta T$$

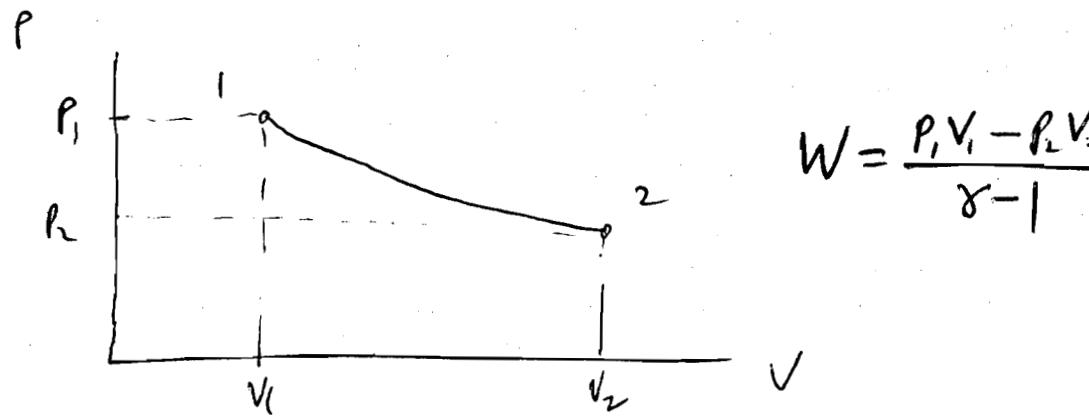
$$C_V = C_p - R \quad \text{or} \quad C_p = C_V + R$$

$$R = 8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}}$$

Ideal gas: $C_p > C_V$

Does this make sense? Yes $\left. \begin{array}{l} \text{const. } P = Q = \Delta U + W \rightarrow \text{can absorb} \\ \text{const. } V : Q = \Delta U + 0 \end{array} \right\}$ more heat

Adiabatic Process ($Q=0$) b/w state 1 & state 2



1st Law of T.D.: $\Delta U = -W$

ideal gas $\left[\begin{array}{l} P(V^\gamma) \text{ is constant} : P_1 V_1^\gamma = P_2 V_2^\gamma \\ \gamma = \frac{C_p}{C_V} > 1 \text{ no unit (or dimensionless)} \\ TV^{\gamma-1} \text{ is constant} : T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \end{array} \right]$

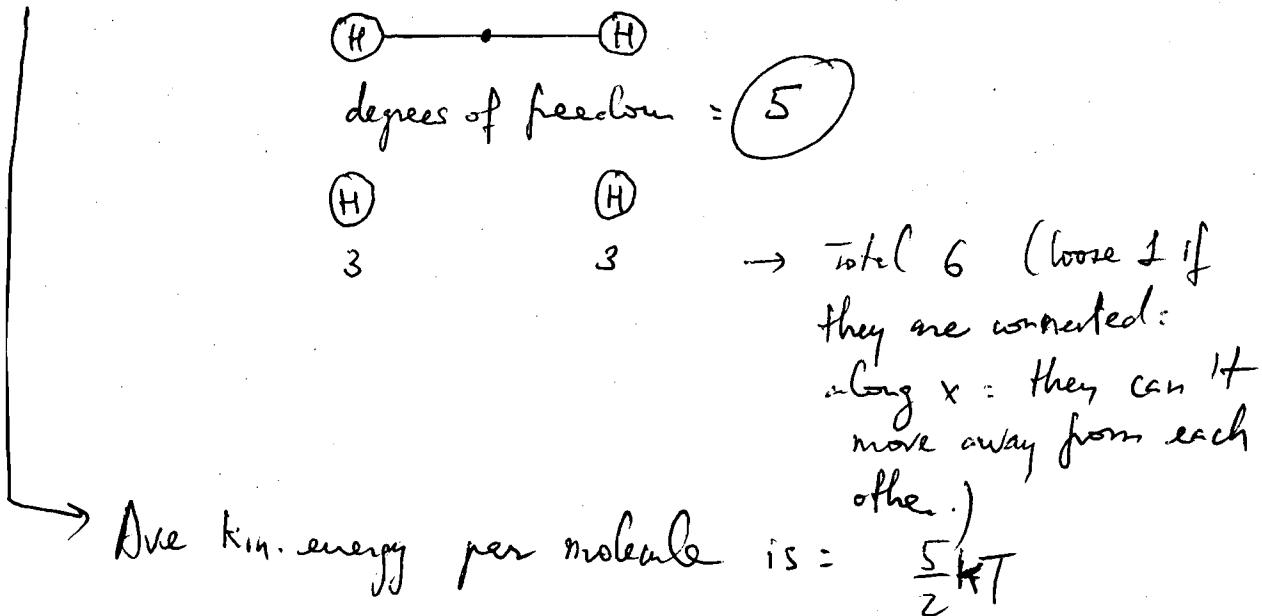
→ Ideal gas: what are possible values for γ ?

$$\text{Ave. k.E per molecule: } \frac{1}{2}mv^2 = \frac{3}{2}kT \quad (3D)$$

↳ Ave. KE per molecule per degree of freedom: $\boxed{\frac{1}{2}kT}$

→ Monatomic gases ($\text{Ne}, \text{Ar}, \text{etc.}$): molecules are single atoms \longrightarrow Ave. kE per molecule is $\frac{3}{2}kT$

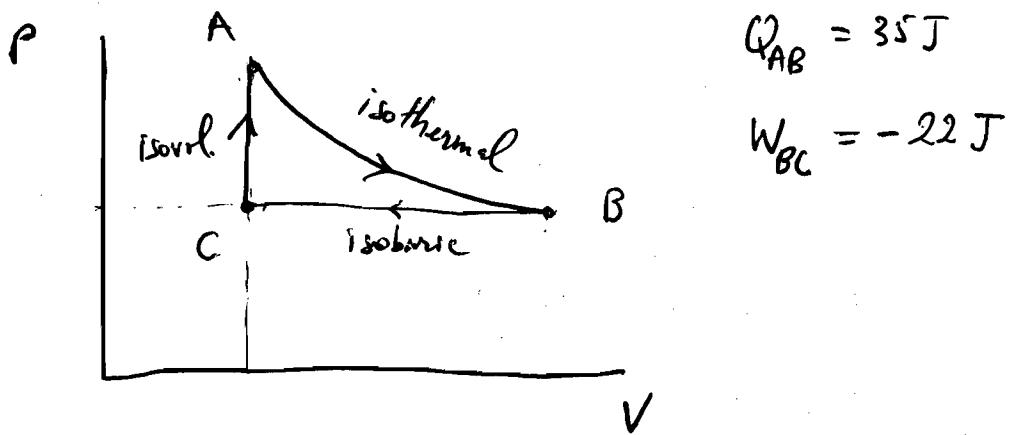
→ Diatomic gases ($\text{H}_2, \text{O}_2, \text{etc.}$): molecules are groups of 2 atoms



$$C_V = \frac{1}{n} \frac{\partial U}{\partial T} \left\{ \begin{array}{l} \text{Monatomic: } \Delta U = N \frac{3}{2}kT \longrightarrow C_V = \frac{3}{2}R \\ n = \frac{N}{NA} \\ \text{Diatomic: } \Delta U = N \frac{5}{2}kT \longrightarrow C_V = \frac{5}{2}R \end{array} \right.$$

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} \left\{ \begin{array}{l} \text{Monatomic: } \gamma = 1.67 \\ \text{Diatomic: } \gamma = 1.4 \end{array} \right. \overbrace{\text{ideal gas.}}$$

18.42



$$Q_{AB} = 35 \text{ J}$$

$$W_{BC} = -22 \text{ J}$$

a)

$$b) W_{ABC} = W_{AB} + W_{BC} = 35 - 22 = 13 \text{ J}$$

AB : isothermal (ideal gas : $\Delta U = 0 \rightarrow Q = W$)

$$W_{AB} = Q_{AB} = 35 \text{ J}$$

$$c) Q_{BCA} = Q_{BC} + Q_{CA} = n c_p \Delta T_{BC} + n c_v \Delta T_{CA}$$

$$c_p = \frac{1}{n} \frac{Q_{BC}}{\Delta T_{BC}}$$

$$c_v = \frac{1}{n} \frac{Q_{CA}}{\Delta T_{CA}}$$

$$\rightarrow \Delta U = Q - W \rightarrow \Delta U_{ABC} = 0$$

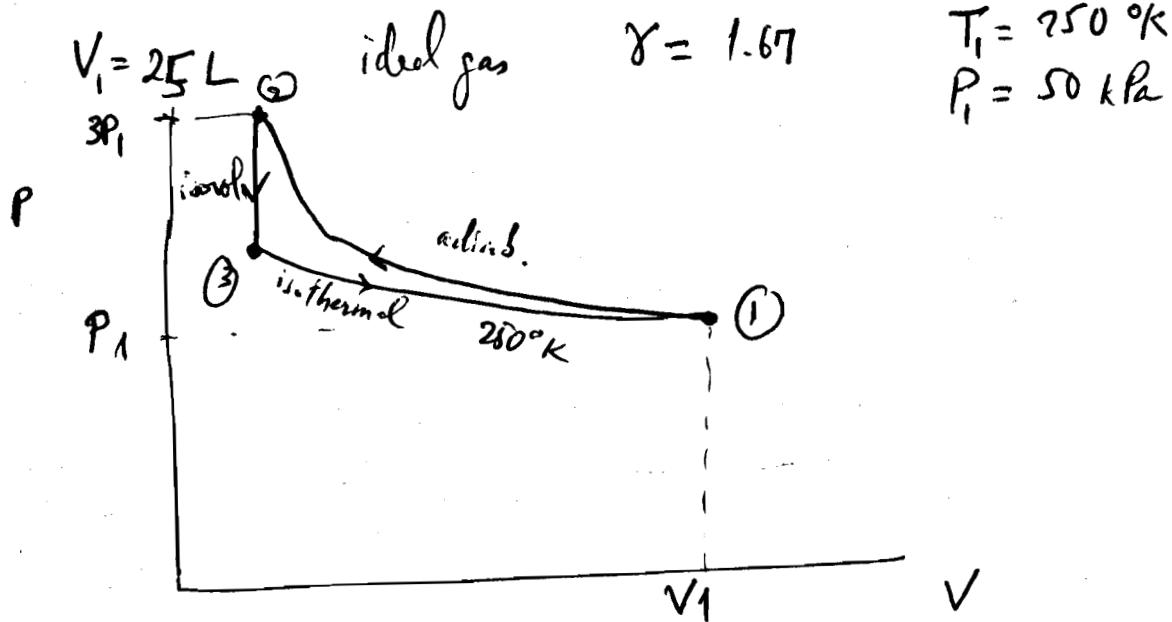
$$\rightarrow Q_{ABC} = W_{ABC} = 13 \text{ J}$$

$$Q_{AB} + Q_{BCA} = 13 \text{ J}$$

$$Q_{BCA} = 13 \text{ J} - Q_{AB} = 13 - 35 = -22 \text{ J}$$

(From B to C to A gas delivers 22 J)

18.49



a) Work done on gas?

We can find work done by the gas W_{1231}

$$W_{1231} = W_{12} + 0 + W_{31}$$

$$W_{31} = nRT \ln \frac{V_1}{V_3} = nRT \ln \frac{V_1}{V_2} \rightarrow \text{need}$$

$$\textcircled{2} \text{ from } \textcircled{1} \text{ via adiabatic: } P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$V_1^\gamma = 3 V_2^\gamma$$

$$\left(\frac{V_1}{V_2}\right)^\gamma = 3 \rightarrow \frac{V_1}{V_2} = 3^{\frac{1}{\gamma}}$$

$$\text{Also: } P_1 V_1 = nRT$$

$$\hookrightarrow W_{31} = P_1 V_1 \ln \frac{V_1}{V_2} = P_1 V_1 \ln 3^{\frac{1}{\gamma}} = \frac{P_1 V_1}{\gamma} \ln 3$$

$$= \frac{50 \times 10^3 \times 25 \times 10^{-3}}{1.67} \ln 3 \text{ J} = 822 \text{ J}$$

$$W_{12} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{P_1 V_1 - 3P_1 V_1 \frac{1}{3^{1/8}}}{\gamma - 1}$$

(1) \rightarrow (2) adiabatic.

$$= P_1 V_1 \frac{1 - 3^{(1-\frac{1}{8})}}{\gamma - 1}$$

$$= 50 \text{ kPa} \times 25 \times 10^{-3} \frac{1 - 3^{\frac{0.67}{1.67}}}{0.67}$$

$$= -1033 \text{ J}$$

$$W_{1231} = -1033 + 822 = -211 \text{ J}$$

\rightarrow Work done on gas is 211 J

(17.46) Refrigerator extracts energy from its inside at $95 \frac{\text{J}}{\text{s}}$
How long to freeze $m_w = 0.75 \text{ kg}$ at $T_w = 0^\circ\text{C}$?

\downarrow
 $Q = \text{how much heat we need to extract from the water to freeze it?} \rightarrow \text{Heat of transformation:}$

$$Q = m_w L_f = 0.75 \text{ kg} \times 334 \frac{\text{kJ}}{\text{kg}} = 250.5 \text{ kJ}$$

$$\text{Table 17.1 : } 334 \frac{\text{kJ}}{\text{kg}}$$

$$t = \frac{Q}{95 \frac{\text{J}}{\text{s}}} = \frac{250500 \text{ J}}{95 \frac{\text{J}}{\text{s}}} = \frac{1 \text{ min}}{60 \text{ s}} = 43.95 \text{ min.}$$

18.33; 18.57;

18.33

$$P_H = 125 \text{ mm Hg} \rightarrow \text{gas } d_H ?$$

a) gauge pressure $P_L = 80 \text{ mm Hg}$ \rightarrow  $d_L = 1.52 \text{ mm}$

(not including atmospheric pressure)

$PV = \text{constant}$, Why? b/c $PV = nRT$

↳ body temp.
is const.

$$P_L V_L = P_H V_H$$

$$P_L \cdot \frac{4}{3} \pi \left(\frac{d_L}{2} \right)^3 = P_H \cdot \frac{4}{3} \pi \left(\frac{d_H}{2} \right)^3 \rightarrow P_L d_L^3 = P_H d_H^3$$

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

$$1 \text{ mm Hg} = 133.3 \text{ Pa}$$

→ We can use mmHg instead of Pa in this problem b/c conversion factor will cancel up & down.

What is Latm in mm Hg?

$$\frac{1 \text{ atm}}{101300 \text{ Pa}} = \frac{133.3 \text{ Pa}}{1 \text{ mm Hg}} = \frac{133.3 \text{ atm}}{101300 \text{ mm Hg}} = \frac{1 \text{ atm}}{\frac{101300}{133.3} \text{ mm Hg}}$$

$$= \frac{1 \text{ atm}}{760 \text{ mm Hg}}$$

1) Work done by heart?

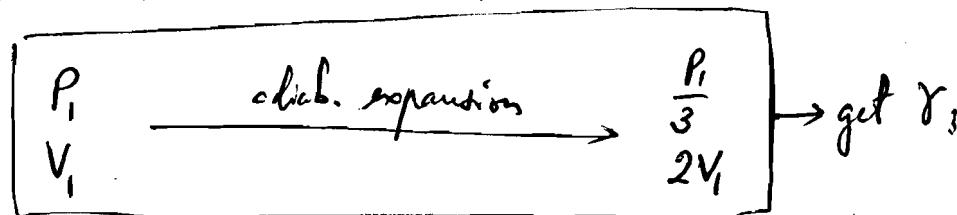
Work done by gas = isothermal process (const. temp at the body temp.)

$$\begin{aligned}
 W_{LH} &= nRT \ln\left(\frac{V_H}{V_L}\right) = P_L V_L \ln\left(\frac{d_H^3}{d_L^2}\right) = 3P_L V_L \ln\left(\frac{d_H}{d_L}\right) \\
 &= 3(80+760) \cdot \frac{133.3}{1} \cdot \frac{4\pi (0.76 \times 10^{-3})^3}{3} \ln\left(\frac{1.49}{1.52}\right) \\
 &= 617675 \times 10^{-9} \ln\left(\frac{1.49}{1.52}\right) = -12312 \times 10^{-9} \text{ J} \\
 &= -12 \times 10^{-6} \text{ J} = -12 \mu\text{J}
 \end{aligned}$$

$12 \mu\text{J}$

18.57

Ar (monatomic) & O₂ (diatomic)

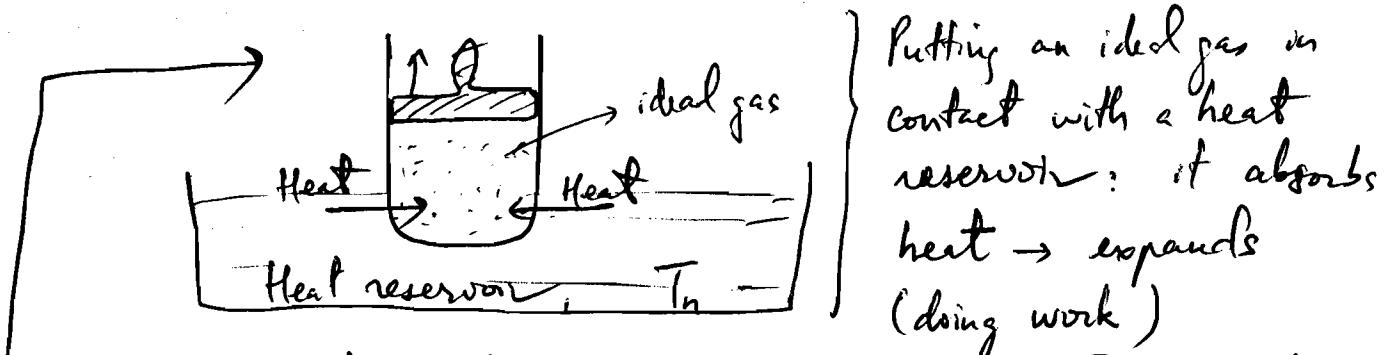


What fraction of molecules are Ar?



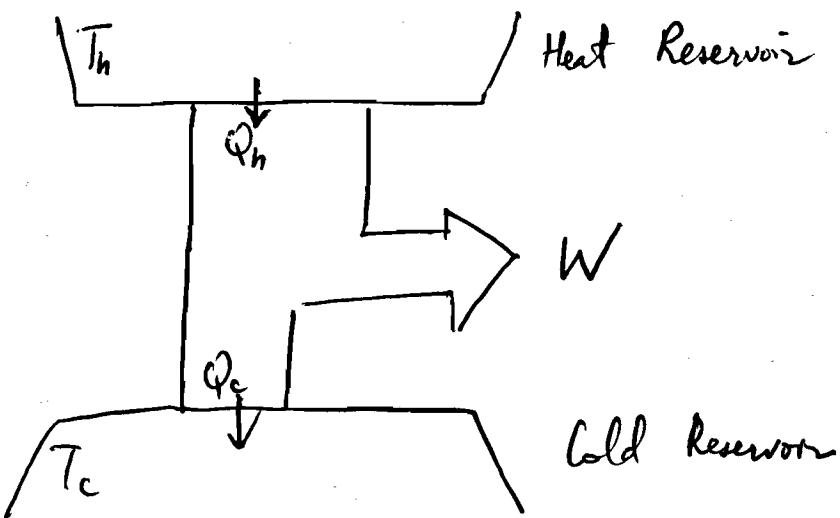
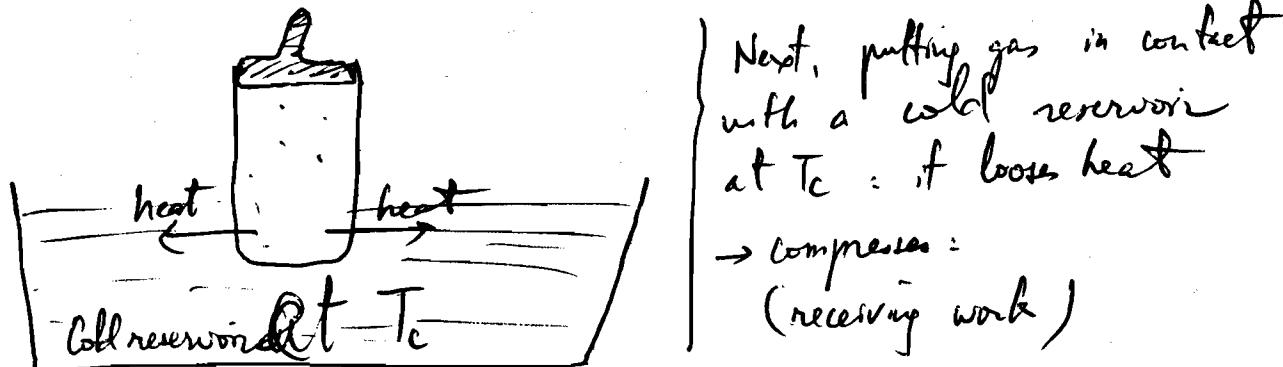
Ch 19 2nd Law of Thermodynamics

Heat reservoir: source of heat, at constant temperature



Repeating
in
cycs:
↓
Heat
Engine

How to continue to get work from the gas? : Repeat the process: bring the piston back to its original position by putting the heated gas in contact with cold reservoir



$$\Delta U_{\text{engine}} = Q_{\text{net}} - W = Q_h - Q_c - W$$

↓
1st Law of T.D.

$Q_h - Q_c = W$

→ Ideal, isothermal process = 0
(const. T.)

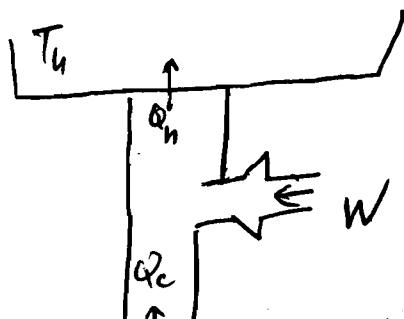
efficiency of an ideal gas heat engine: $\epsilon = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h}$

$$= 1 - \frac{|Q_c|}{|Q_h|} < 1$$

\sim
 $(Q_c < Q_h)$

2nd Law of T.D.: it is impossible to build a heat engine operating in cycles that extracts heat from a hot reservoir (and returning some of it to a cold reservoir) that can deliver a 100% of work.

Refrigerators: reversed heat engines



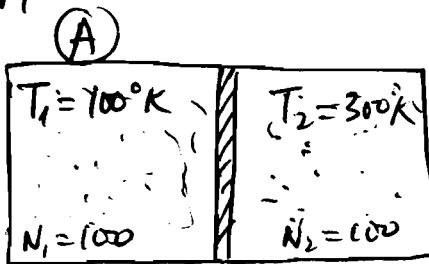
2nd of T.D.: it is impossible to transfer heat from a cold reservoir to a hot reservoir without requiring any work.

Coefficient of Performance: $C.O.P. = \frac{Q_c}{W}$

$$\text{Entropy: } \Delta S = \int_1^2 \frac{dQ}{T}$$

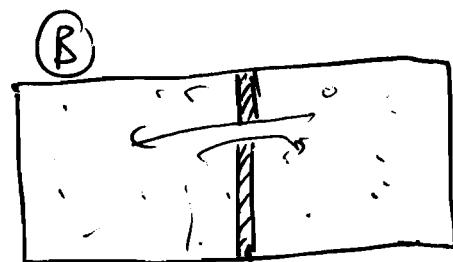
3rd Law of T.D.: entropy of a closed system can never decrease $\Delta S \geq 0$

(entropy is related to order).



$$V_1 = V_2$$

$$P_2 > P_1 \text{ why?}$$



mixed together.

$$\xrightarrow{\quad} \left\{ \begin{array}{l} PV = nRT \text{ (macroscopic)} \\ v_{th} = \sqrt{\frac{3kT}{m}} \text{ (microscopic)} \end{array} \right.$$

: higher thermal speed,
higher pressure on the wall

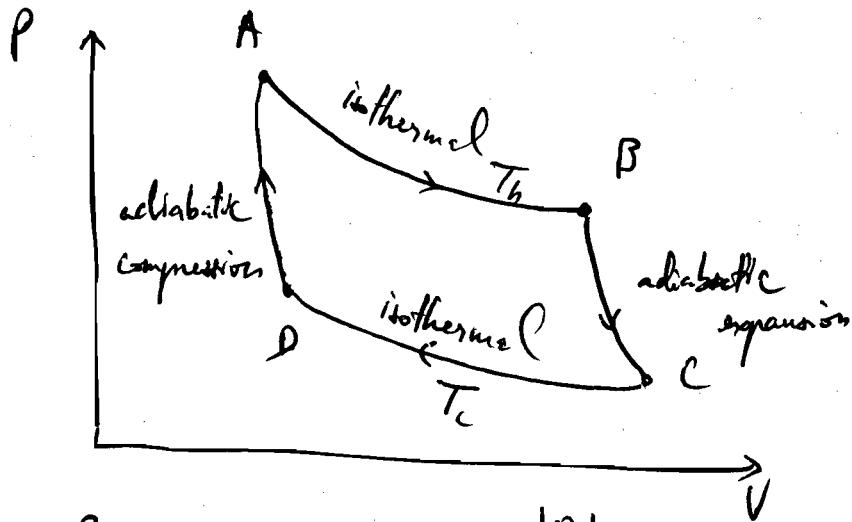
$\textcircled{A} \rightarrow \textcircled{B}$ spontaneously mixed together:

order has decreased. (ΔS increased).

\rightarrow Natural process is always increasing entropy.

Carnot Engines: 4 reversible processes $\begin{cases} 2 \text{ isothermal} \\ 2 \text{ adiabatic} \end{cases}$

ideal gas, max. achievable efficiency = ϵ_{\max}



$$\epsilon_{\max} = ? ; \quad \epsilon = 1 - \frac{|Q_c|}{|Q_h|}$$

$$Q_h = \text{heat absorbed from hot reservoir during isothermal exp. } A \rightarrow B = nRT_h \ln \frac{V_B}{V_A}$$

$$Q_c = (\text{isothermal } C \rightarrow D) = nRT_c \ln \frac{V_D}{V_C}$$

There is a connection b/w the volumes =

$$B \rightarrow C \text{ adiabatic} : T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$$

$$\left. \left\{ \begin{aligned} \frac{V_B}{V_C} \cdot \frac{V_B}{V_A} &= 1 \\ \frac{V_D}{V_C} &= \frac{V_A}{V_B} \end{aligned} \right. \right\} \rightarrow \left(\frac{V_B}{V_C} \right)^{\gamma-1} = \frac{T_C}{T_B} = \frac{T_{\text{cold}}}{T_h} = \frac{T_c}{T_h}$$

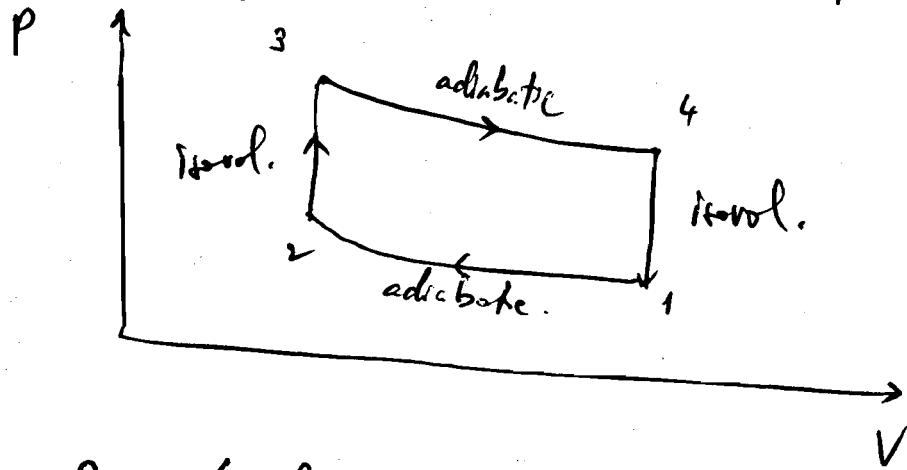
$$D \rightarrow A \text{ adiabatic} : T_D V_D^{\gamma-1} = T_A V_A^{\gamma-1}$$

$$\left. \left\{ \begin{aligned} \left(\frac{V_D}{V_A} \right)^{\gamma-1} &= \frac{T_A}{T_D} = \frac{T_c}{T_h} \end{aligned} \right. \right\}$$

$$\epsilon = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{\left| n R T_c \ln \frac{V_D}{V_C} \right|}{\left| n R T_h \ln \frac{V_B}{V_A} \right|} = 1 - \frac{n R T_c \ln \frac{V_C}{V_D}}{n R T_h \ln \frac{V_B}{V_A}}$$

$$\boxed{e_{\max} = 1 - \frac{T_c}{T_h} < 1}$$

Otto Cycle Engines : 4 reversible processes $\begin{cases} 2 \text{ adiabatic} \\ 2 \text{ isochoric} \end{cases}$



$$e_{\text{Otto}} < e_{\max}$$

$$\Delta S = \int_1^2 \frac{dQ}{T} \quad \left\{ \begin{array}{l} \text{isovol: } dQ = n c_v dT \quad (c_v = \frac{1}{n} \frac{\Delta Q}{\Delta T} \Rightarrow \frac{1}{n} \frac{dQ}{dT}) \\ \Delta S = \int_1^2 \frac{n c_v dT}{T} = n c_v \int_1^2 \frac{dT}{T} = n c_v [\ln T]_1^2 \\ \text{isothermal: } \Delta S = \frac{1}{T} \int_1^2 dQ = \frac{Q_2 - Q_1}{T} = \frac{\Delta Q}{T} \end{array} \right.$$