Differential forms of Maxwell’s equations in vacuum:

**Amperes:** \( \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \varepsilon_0 \frac{d\mathbf{E}}{dt} \) \[ \Rightarrow \frac{\partial B}{\partial x} = \mu_0 \frac{\partial E}{\partial t} \]

**Faraday:** \( \oint \mathbf{E} \cdot d\mathbf{A} = -\frac{\partial \mathbf{B}}{\partial t} \) \[ \Rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \]

\[ E = E_0 \sin(kx - \omega t) \Rightarrow \frac{\partial E}{\partial x} = k E_0 \cos(kx - \omega t) \]

\[ B = B_0 \sin(kx - \omega t) \Rightarrow -\frac{\partial B}{\partial t} = -\omega B_0 \cos(kx - \omega t) \]

**Amperes:** \( \mu_0 E_0 \omega B_0 = k B_0 \quad \Rightarrow \quad \frac{E_0}{B_0} = \frac{k}{\omega} \mu_0 \)

**Faraday:** \( k E_0 = \omega B_0 \quad \Rightarrow \quad \frac{E_0}{B_0} = \frac{\omega}{k} \mu_0 \)

\[ \frac{\mu_0 \omega}{k} = \frac{k}{\omega} \quad \Rightarrow \quad \left( \frac{\omega}{k} \right)^2 = \frac{1}{\mu_0 \varepsilon_0} \]

\[ \text{wave speed} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \times 8.85 \times 10^{-12}}} \]

\[ c = 3 \times 10^8 \text{ m/s} \]

**EM Wave equation:**

Start with differential form of Maxwell’s Equations:

\( \frac{\partial}{\partial t} \left( \frac{\partial B}{\partial x} - \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \right) \) (Amperes) \[ \Rightarrow \frac{\partial B}{\partial x} - \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \]

\( \frac{\partial}{\partial x} \left( \frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} \right) \) (Faraday) \[ \Rightarrow \frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} \]

Wave equation for electric field strength: \( c = 3 \times 10^8 \text{ m/s} \)
Similarly:
\[
\frac{\delta}{\delta x} \left( \frac{\partial B}{\partial x} \right) = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t},
\]
(Empirical)
\[
\frac{\delta}{\delta t} \left( \frac{\partial B}{\partial x} \right) = -\frac{\partial B}{\partial t},
\]
(Faraday's)
\[
\frac{\delta}{\delta x} \left( \frac{\partial E}{\partial x} \right) = -\frac{\partial E}{\partial t}.
\]

Wave equation for magnetic field, speed:
\[
\frac{1}{\mu_0} = \frac{c}{\lambda_0} = \text{wave speed}.
\]

Intensity of EM wave:

- Projection along \(x\)-axis:
- Polarization (direction of \(E\)) along \(y\)-axis:
\[
\vec{B} \perp \text{projection},
\]

\[
\vec{E} \ | \ (\text{along} \ y)
\]
\[
\vec{B} \ | \ (\text{along} \ z)
\]

Wave Intensity:
\[
S = \frac{\delta u}{\delta t} \quad (?) \text{rate in terms of } E \& B?
\]
\[
du = u \, \delta Vol = u \, \delta x \delta A \quad \Rightarrow \quad \frac{\delta u}{\delta t} = u \, \frac{\delta x}{\delta t} \, \delta A = (\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} B^2) c \delta A
\]
\[
\text{Total Energy Density}
\]

\[
\text{Wave speed } c
\]
EM wave intensity:

\[ S = \frac{c}{2} \left( \frac{\varepsilon_0 E^2 + \frac{B^2}{\mu_0}}{c^2} \right) \]

More compact form:

\[ \frac{E}{B} = c \Rightarrow B = E \frac{c}{c^2} \]

\[ S = \frac{c}{2} \left( \varepsilon_0 E^2 + \frac{E^2}{c^2 \mu_0} \right) = \frac{c \varepsilon_0 E^2}{c^2 \mu_0} \]

Also:

\[ \varepsilon_0 = \frac{1}{c^2 \mu_0} \Rightarrow \frac{1}{\varepsilon_0} = c^2 \mu_0 \]

\[ S = c \varepsilon_0 E^2 = c^2 \varepsilon_0 E \frac{E}{c} = c^2 \varepsilon_0 E B \]

Generally:

\[ \vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \]

(This why we used propagation direction to be given by \( \vec{E} \times \vec{B} \)).

EM wave intensity & power:

\[ \text{Energy per unit time} \ P \]

\[ S = \frac{1}{A \frac{dA}{dt}} = \frac{P}{A} \]

Spherical EM wave:

\[ S = \frac{P}{4\pi r^2} \]

Intensity falls as inverse square law.
Radiation pressure:

Gas: transfer of momentum of gas molecules to wall

\[ p = \frac{U}{c} \]

Momentum of EM wave or radiation is \[ p = \frac{U}{c} \]

Radiation pressure:

\[ F = \frac{\partial p}{\partial t} = \frac{1}{c} \frac{\partial U}{\partial t} = \frac{S}{c} \]

\( S = \frac{E \times B}{\mu_0} \rightarrow S = \frac{EB}{2\mu_0} \) →

\[ \text{Rad. pressure} = \frac{EB}{2\mu_0} \]
34.56

\[ m = 30 \mu g \]

 Beam beam \( P \)?

Assume foil reflects all light

Radiation pressure \( \frac{S}{c} \times 2 \)

Force \( = \) Real. Press. \( \times A \) \( = mg \)

\[ \frac{S}{c} \times 2A = mg \]

\[ \frac{P}{\pi c} \times 2A = mg \Rightarrow \overline{P} = \frac{mgc}{2} \]

\[ = \frac{3 \times 10^{-9} \times 9.81 \times 3 \times 10^8}{2} \]

\[ = \frac{9 \times 9.81}{2} W \]

\[ = 44.1 W \]

Electromagnetic waves exert pressure like gas molecules.

\[ \Rightarrow \] They are also particles. (\( \rightarrow \) quantum physics)
Polarization in EM waves:

- Composed of $\mathbf{E}$ & $\mathbf{B}$ which are vectors.
- Orientation of $\mathbf{E}$ is important.

Front view of some EM wave: (propagation toward us)

Polaroid materials: special axis such that only waves with polarization along that axis can propagate through.

$E_x = E_0 \cos \theta$ going thru.

Intensity: $\sim E^2$ → intensity passing thru is $\cos^2 60^\circ$ of original intensity.

Unpolarized: $\mathbf{E}$ can point along any direction → $\mathbf{I}$ always horizontal & $\mathbf{I}$ always vertical.
Chapter 35: Reflection & Refraction

Geometrical optics: propagation of light using light rays
(no wave properties, just direction of propagation)

Law of reflection: \( \theta' = \theta \)

\( \theta \) is incident angle
\( \theta' \) is reflected angle

Multiple reflections:

I can obtain \( \theta'' \) from \( \theta \) and \( \alpha \) by using geometry:

\[
180^\circ = \theta' + \theta'' + \alpha
\]

\( \theta'' = (\alpha - \theta) = \theta_{\text{reflected}}' \)
Reflection:

1. \( \frac{1}{n_1 \sin \theta_1} = \frac{1}{n_2 \sin \theta_2} \)

Snell's Law or Law of Refraction:

- Angle of refraction w.r.t. the normal is smaller than incident angle when light ray travels from lower index to a higher index medium.
- Usually higher density → higher index of refraction → lower wave speed.
- Light ray goes from higher \( n_1 \) to a lower \( n_2 \).

2. Reflected light travels parallel to boundary.

- There is no light in medium #2.
- Total internal reflection.
$\theta_c$ is the first $\theta_1$ such that $\theta_2 = 90^\circ$.

\[
n_l \sin \theta_c = n_2 \sin 90^\circ
\]

\[
\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)
\]

$n_2 < n_1$ or $\frac{n_2}{n_1} < 1$

Smallest incident angle for total internal reflection

All reflection, no refraction

Any situation with all refraction and no reflection?

Yes, at the Brewster's angle: $\theta_B = \tan^{-1} \frac{n_2}{n_1}$

and $E$ polarized in plane of page.

$\Rightarrow$ there is no reflection.

If $E$ is polarized at an angle w.r.t. plane of this page
then its polarization would have a component perpendicular to the page which is not affected by the Brewster's angle.

$\Rightarrow$ there would be some reflection due to this component.