Differential forms of Maxwell's equations in vacuum:

$$\text{Ampere: } \oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \longrightarrow \frac{\partial B}{\partial x} = \mu_0\varepsilon_0 \frac{\partial E}{\partial t}$$

$$\text{Faraday: } \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \longrightarrow \boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

$$E = E_p \sin(kx - \omega t) \rightarrow \frac{\partial E}{\partial x} = k E_p \cos(kx - \omega t)$$

$$B = B_p \sin(kx - \omega t) \rightarrow -\frac{\partial B}{\partial t} = +\omega B_p \cos(kx - \omega t)$$

Ampere: $\mu_0\varepsilon_0 \omega E_p = k B_p \longrightarrow \frac{E_p}{B_p} = \frac{\frac{k}{\omega}}{\mu_0\varepsilon_0}$

Faraday: $k E_p = \omega B_p \longrightarrow \frac{E_p}{B_p} = \frac{\omega}{k}$ = wave speed = c

$$\frac{\mu_0\varepsilon_0 \omega}{k} = \frac{k}{\omega} \rightarrow \left(\frac{\omega}{k}\right)^2 = \frac{1}{\mu_0\varepsilon_0}$$

wave speed $= \frac{1}{\sqrt{4\pi\times10^{-7} \times 8.85\times10^{-12}}}$

$c = 3.10^8$ m/s.

EM Wave equation:

Start with differential form of Maxwell's Equations:

$$\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x} = \mu_0\varepsilon_0 \frac{\partial E}{\partial t}\right) \quad \text{(Ampere)} \rightarrow \frac{\partial^2 B}{\partial t \partial x} = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}\right) \quad \text{(Faraday)} \rightarrow \frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

Wave equation for electric field, speed: $\frac{1}{\ }$ = 3×10^8 m/s

Similarly:

$$\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right) \quad \text{(Ampere)} \rightarrow \frac{\partial^2 B}{\partial x^2} = -\mu_0\varepsilon_0 \frac{\partial^2 E}{\partial x\partial t}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}\right) \quad \text{(Faraday's)} \rightarrow \frac{\partial^2 E}{\partial t\,\cancel{\partial x}} = -\frac{\partial^2 B}{\partial t^2}$$

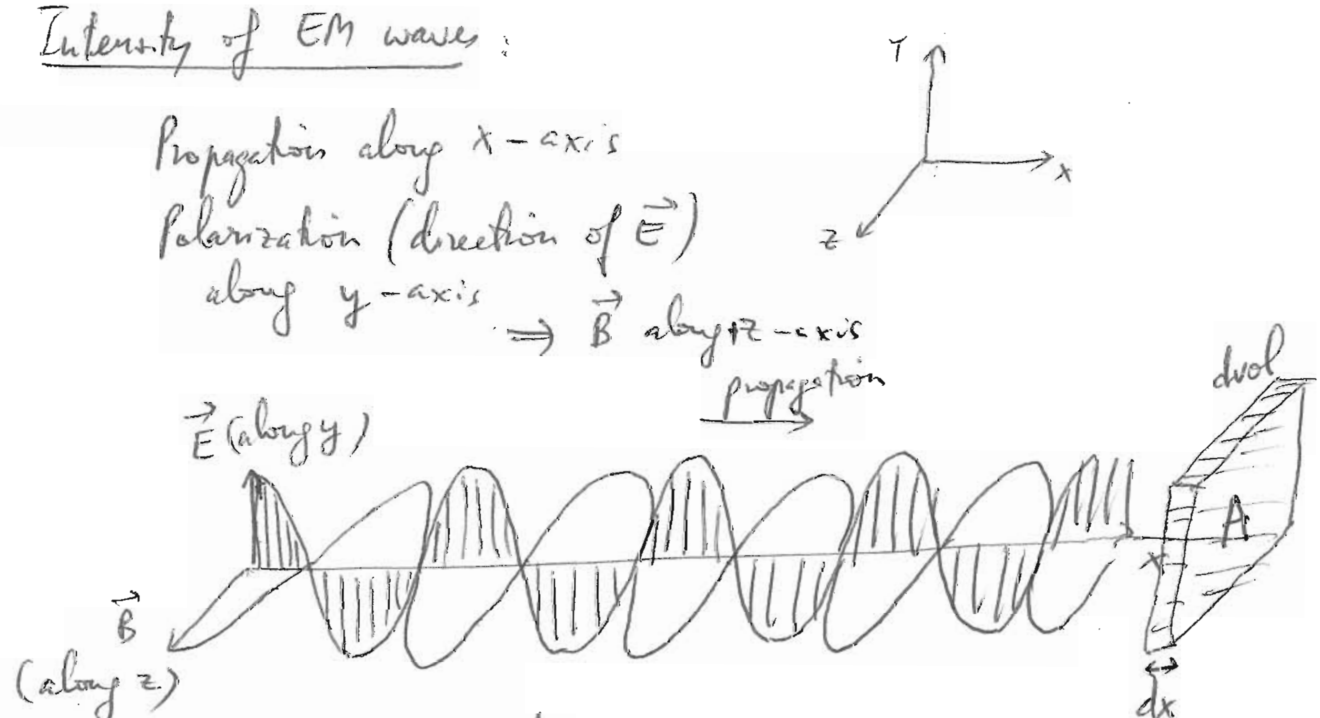
$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}}$$

Wave equation for magnetic field, speed: $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8\ m/s$

Intensity of EM wave:

Propagation along x-axis

Polarization (direction of $\vec{E}$) along y-axis $\Rightarrow \vec{B}$ along +z-axis



Wave Intensity: $S = \frac{\frac{dU}{dt}}{A}$ (? ~~inter~~ in terms of E & B?)

$$\underbrace{dU}_{\text{Total energy}} = \underbrace{u}_{\text{EM energy density}} \cdot dVol = u \cdot dxA \rightarrow \frac{dU}{dt} = u\underbrace{\frac{dx}{dt}}_{\text{wave speed } c}A = \left(\frac{1}{2}\varepsilon_0E^2 + \frac{1}{2\mu_0}B^2\right)cA$$

EM wave intensity: $S = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$

More compact form: $\frac{E}{B} = c$ or $B = \frac{E}{c}$

$$\boxed{S = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{E^2}{c^2 \mu_0}\right) = c\,\varepsilon_0 E^2}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c \rightarrow \frac{1}{\mu_0 \varepsilon_0} = c^2 \rightarrow \frac{1}{c^2 \mu_0} = \varepsilon_0$$

Also: $\boxed{S = c\,\varepsilon_0 E^2 = c^2 \varepsilon_0 E\left(\frac{E}{c}\right) = c^2 \varepsilon_0 E B = \frac{EB}{\mu_0}}$

$c^2 \varepsilon_0 = \frac{1}{\mu_0}$

Generally:

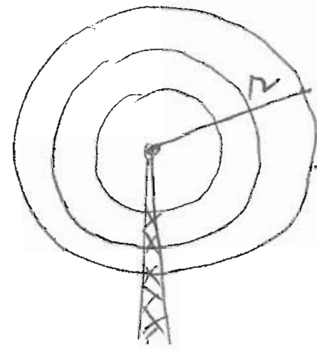
$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

(This why we said propagation direction was given by $\vec{E} \times \vec{B}$)

EM wave intensity & power: energy per unit time P

$$S = \frac{1}{A} \underbrace{\frac{dU}{dt}}_{P} = \frac{P}{A}$$

Spherical EM waves:



spheres $\rightarrow A = 4\pi r^2$

$$S = \frac{P}{4\pi r^2}$$

Intensity falls as inverse square law

EM waves

Radiation Pressure:

gases: transfer of momentum of gas molecules to wall

$\xrightarrow{\;\;}$ mv | wall.

$\xleftarrow{\;\;}$ $-mv$ | wall

$\Delta p = 2mv$ $\qquad$ $\Delta p = -2mv$

wall getting $2mv$ by conserv. of momentum → pressure it feels

Momentum of EM wave or radiation is $p = \frac{U}{c}$

Radiation pressure: average → $\frac{\overline{F}}{A} = \frac{\overline{\frac{dp}{dt}}}{A} = \frac{1}{c}\frac{\overline{\frac{du}{dt}}}{A} = \frac{\overline{S}}{c}$

$$\vec{S} = \frac{\overset{\sin}{\vec{E}} \times \overset{\sin}{\vec{B}}}{\mu_0} \rightarrow \overline{S} = \frac{EB}{2\mu_0} \rightarrow \boxed{\text{Rad. pressure} = \frac{EB}{2c\mu_0}}$$

$$\overline{\sin^2} = \frac{1}{2}$$

34.56

$m = 30\mu g$

Laser beam $\overline{P}$?

Assume foil reflects all lights

Radiation pressure $= \frac{\overline{S}}{c} \times 2$

Force $=$ Rad. Press. $\times A = mg$

$$\frac{\overline{S}}{c} 2A = mg$$

$$\frac{\overline{P}}{\cancel{A}c} 2\cancel{A} = mg \rightarrow \overline{P} = \frac{mgc}{2}$$

$$= \frac{30 \times 10^{-9} \times 9.81 \times 3 \times 10^{8}}{2}$$

$$= \frac{9 \times 9.81}{2} \text{ W}$$

$$= 44.1 \text{ W}$$

EM waves can exert pressure like gas molecules

→ They are also particles. (→ quantum physics)

photons

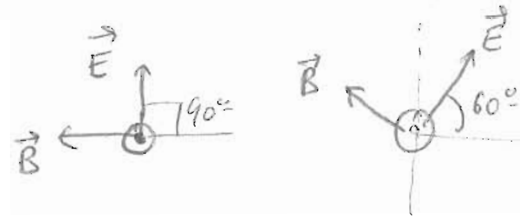
34.36 | Polarization in EM waves:

→ composed of $\vec{E}$ & $\vec{B}$ which are vectors.

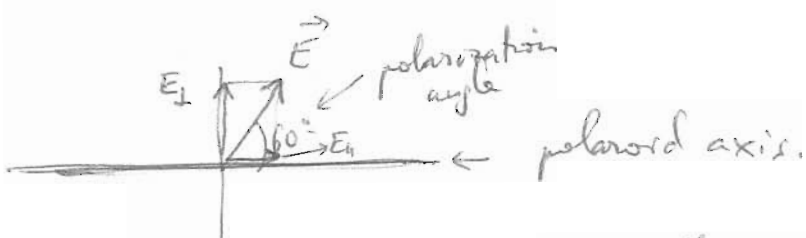
→ direction is important.

→ orientation of $\vec{E}$

Front view of some EM wave: (propagation toward us)



Polaroid materials: special axis such that only waves with polarization along that axis can propagate through.



$E_\parallel = E\cos 60°$ going thru.

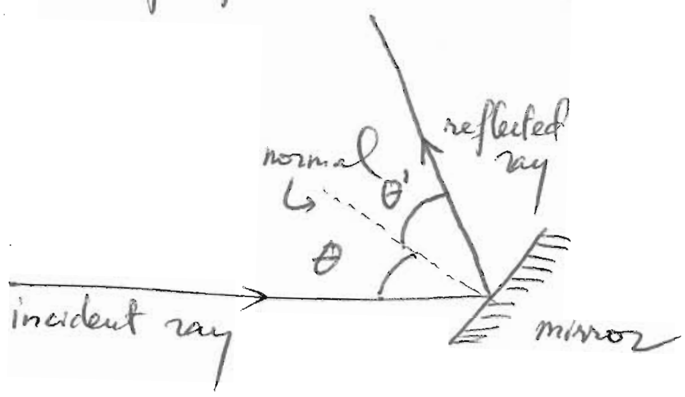
Intensity: $\sim E^2$ ⟶ intensity passing thru is $\cos^2 60°$ of original intensity

Unpolarized: $\vec{E}$ can point along any direction → $\frac{1}{2}$ along horizontal & $\frac{1}{2}$ along vertical.

Ch. 35 Reflection & Refraction:

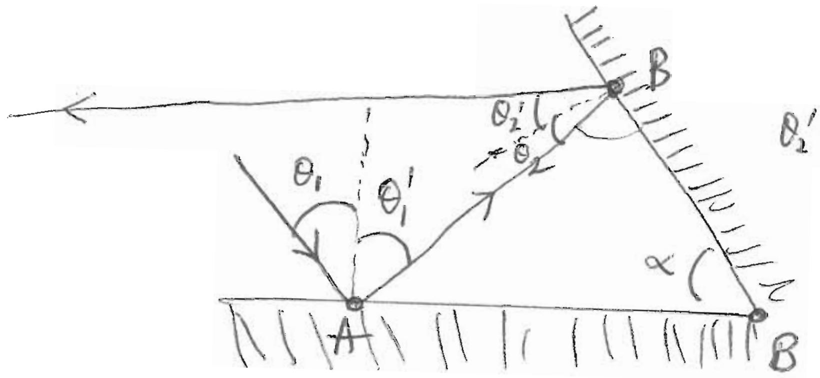
<u>Geometrical optics</u>: propagation of light using <u>light rays</u>
(no wave properties, just direction of propagation)

<u>Law of reflection</u>: $\theta' = \theta$



θ is incident angle
θ' is reflected angle

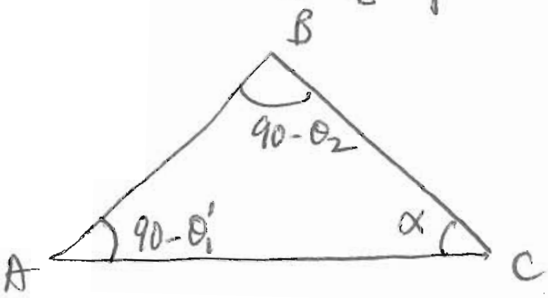
<u>Multiple reflections</u>:



$\theta_2' = \theta_2$

$\theta_1' = \theta_1$

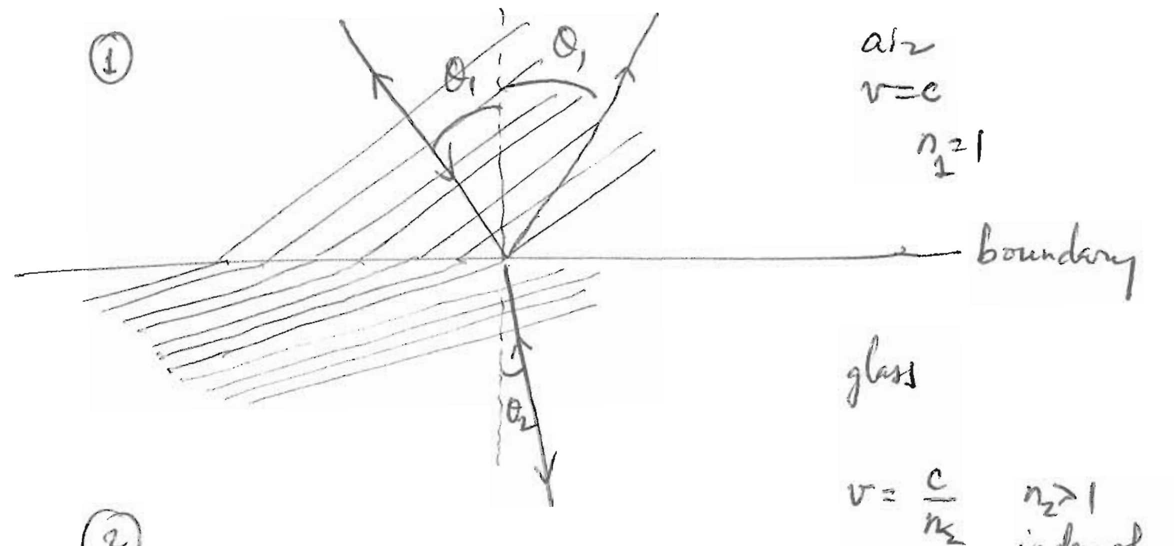
I can obtain θ_2' from θ_1 and α by using geometry:



$$\cancel{180°} = \cancel{90} - \underbrace{\theta_1'}_{=\theta_1} + \cancel{90} - \theta_2 + \alpha$$

$$\theta_2 = \boxed{\alpha - \theta_1 = \theta_2'}$$

Refraction:



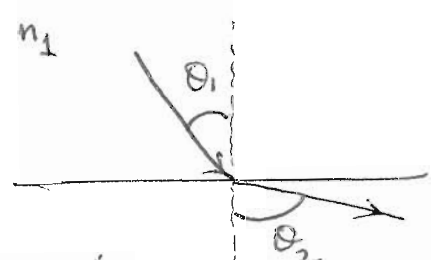
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Snell's Law or Law of refraction.

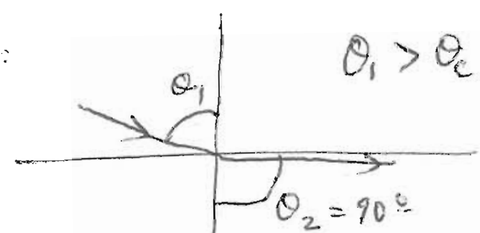
(angle of refraction, w.r.t. the normal, is smaller than incident angle when light ray travels from lower index to a higher index medium)

Usually higher density → higher index of refraction n or lower wave speed)

→ Light ray goes from higher n_1 to a lower n_2



Possible:



Refracted light travels parallel to boundary
→ there is no light in medium #2:
<u>Total internal reflection</u>

θ_c : is the first θ_1 such that $\theta_2 = 90°$:
(smallest)

$$n_1 \sin\theta_c = n_2 \underbrace{\sin 90°}_{1}$$

$$\rightarrow \boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

$n_2 < n_1$ or $\frac{n_2}{n_1} < 1$

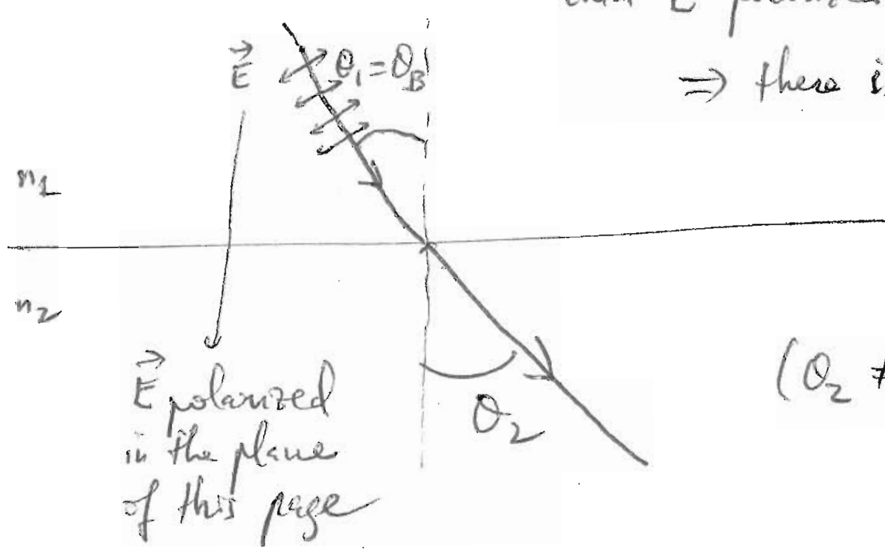
Smallest incident angle for total internal ~~reflexion~~ reflection

↓

all reflection, no refraction

→ Any situation with all refraction and no reflection?

Yes, at the Brewster's angle : $\theta_B = \tan^{-1}\frac{n_2}{n_1}$
and $\vec{E}$ polarized in plane of page.
⇒ there is no reflection.



$(\theta_2 \neq \theta_B)$

If $\vec{E}$ is polarized at an angle w.r.t. plane of this page then its polarization would have a component perpendicular to the page which is not affected by the Brewster's angle
→ there would be some reflection due to this component.