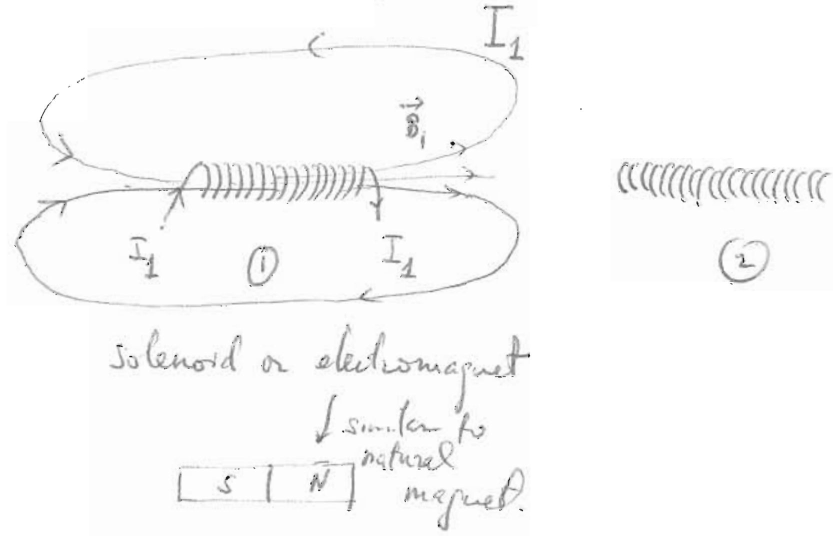
Ch 32 Inductance & Magnetic Energy

Capacitance:

$$C = \frac{Q}{V}$$

Mutual Inductance

$$M = \frac{\Phi_2}{I_1}$$



Φ_2 : magnetic flux through solenoid #2 due to the magnetic field created by solenoid #1 = $\vec{B}_1$.
("Phi")

$\Phi_2 = M I_1$ if I_1 is changing with time → Φ_2 would be changing with time → there will be a $\mathcal{E}_2$ inside solenoid #2 trying to compensate for any change in Φ_2 (Faraday's law)

$$\boxed{-\mathcal{E}_2 = \frac{d\Phi_2}{dt} = M\frac{dI_1}{dt}}$$

it is clear here why M is called "mutual inductance".

Question: $-\mathcal{E}_1 = M\dfrac{dI_2}{dt}$?

Yes! it is the same mutual inductance for a set of 2 solenoids or INDUCTORS

Now: $\vec{B}_1$ ~~also~~ also creates a magnetic flux through its solenoid (#1) $\rightarrow$ ϕ or the self-magnetic flux

$$L = \frac{\phi}{I} \quad : \quad \text{"self inductance"}$$

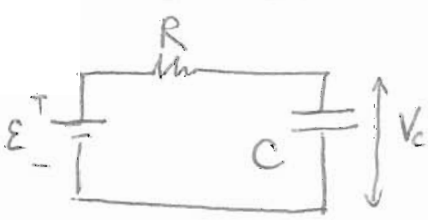
$$\Rightarrow \quad -\mathcal{E} = L\frac{dI}{dt}$$

self-induced voltage

self current

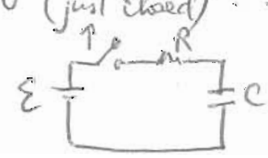
Units: L or M : $\dfrac{[\mathcal{E}]}{\frac{[I]}{[t]}} = \dfrac{V.s}{A} = H$ for Henry (SI system)

RC circuit:



V_c does not change instantaneously

t=0 (just closed): short circuit across C
$V_c = 0$

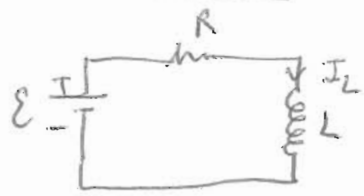


$t=\infty$ (long after): open-circuit across C
$V_c = \infty$

$$I_c = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$$

(current decreases exponentially from $\frac{\mathcal{E}}{R}$)
a max value

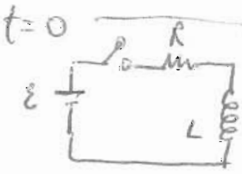
RL circuit:



(Helix for inductors or coils)

I_L cannot change instantaneously

t=0 $\rightarrow$ open circuit across L
$I_L = 0$
$V_L = \mathcal{E}$

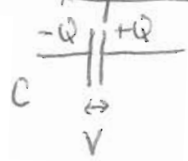


$t=\infty$ $\rightarrow$ short-circuit across L
$V_L = 0$

$$V_L = \mathcal{E}\, e^{-\frac{t}{(L/R)}}$$

(voltage decreases exponentially from $\mathcal{E}$)
a max value

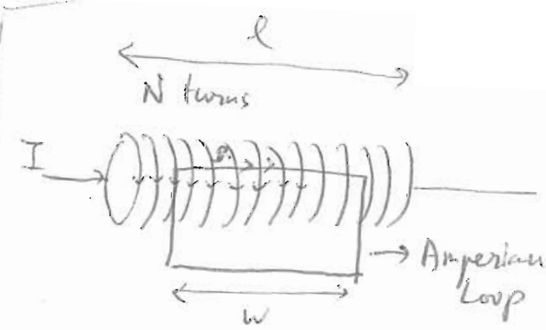
capacitor:



$$U = \frac{1}{2}CV^2 \quad (J)$$

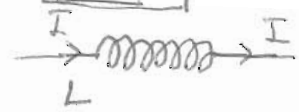
$$\boxed{u = \frac{U}{Al} = \frac{1}{2}\varepsilon_0 E^2} \quad \left(\frac{J}{m^3}\right)$$

volume



$$Bw = \mu_0 I \frac{N}{l} w \rightarrow B = \frac{\mu_0 N I}{l}$$

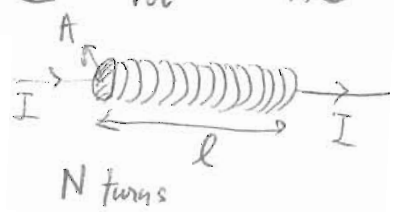
inductor



$$\varepsilon = -L\frac{dI}{dt}$$

$$\boxed{U_L = \int_0^t P_L\, dt = \int_0^t I\varepsilon\, dt = L\int_0^t I\frac{dI}{\cancel{dt}}\cancel{dt} = \frac{1}{2}L\left[I^2\right]_0^t = \frac{1}{2}LI^2}$$

$$\boxed{u_L = \frac{U_L}{Vol} = \frac{\frac{1}{2}LI^2}{Al} = \frac{1}{2}\frac{\mu_0 N^2 \cancel{A}}{\cancel{A}l^2}I^2 = \frac{1}{2\mu_0}B^2}$$



N turns

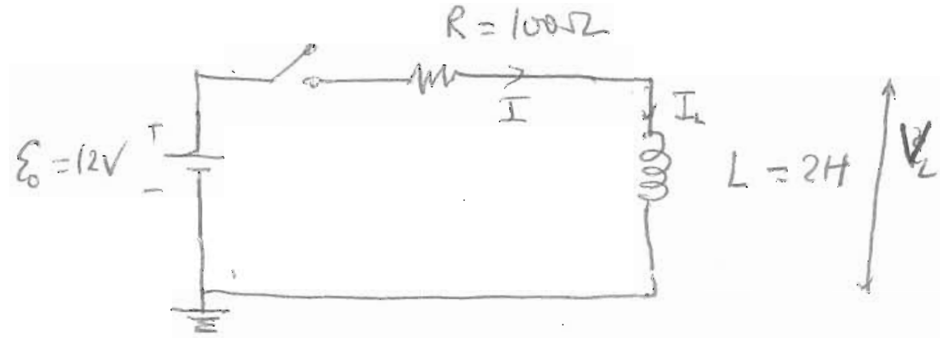
$$B = \frac{\mu_0 N I}{l} \rightarrow L = \frac{\Phi}{I} = \frac{BAN}{I} = \frac{\mu_0 N \cancel{I}}{l \cancel{I}} AN$$

$$L = \frac{\mu_0 N^2 A}{l}$$

$$\left(C = \frac{A\varepsilon_0}{d}\right)$$

$-||-$

32.31



t = 20ms after switch is closed:

a) Circuit current : $I_L = I = \dfrac{\mathcal{E}_0 - V_L}{R} = \dfrac{\mathcal{E}_0 - \mathcal{E}_0 e^{-\frac{t}{(\frac{L}{R})}}}{R}$

$V_L(t=0) = 12V$

$V_L(t=20ms) = 4.41V$

$$= \frac{12\left(1 - e^{-\frac{2\times10^{-2}}{\frac{2}{100}}}\right)}{100} = 75.9\,mA$$

b) $V_L(t=20ms) = \mathcal{E}_0\, e^{-\frac{t}{\frac{L}{R}}} = 12\, e^{-\frac{2\times10^{-2}}{\frac{2}{100}}} = 4.41V$

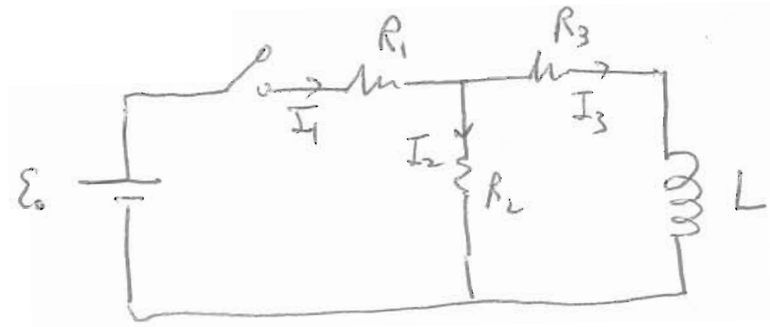
c) $V_R(t=20ms) = 12 - 4.41 = 7.59\ V$

d) $\dfrac{dI}{dt}(t=20ms) = \dfrac{dI_L}{dt}(t=20\ ms) = \dfrac{V_L(t=20ms)}{L} = \dfrac{4.41}{2} = 2.2\ \dfrac{A}{s}$

$$\left(\mathcal{E}_L = -L\frac{dI_L}{dt}\right)$$

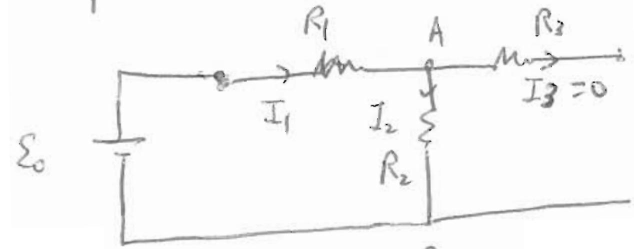
e) $P_R = IV_R = 75.9\times10^{-3} \times 7.59 = 0.576\,W$

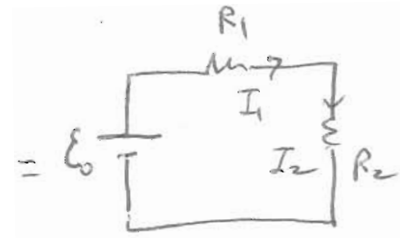
32-36



$\mathcal{E}_0 = 12V$
$R_1 = 4\Omega$; $R_2 = 8\Omega$;
$R_3 = 2\Omega$; $L = 2H$

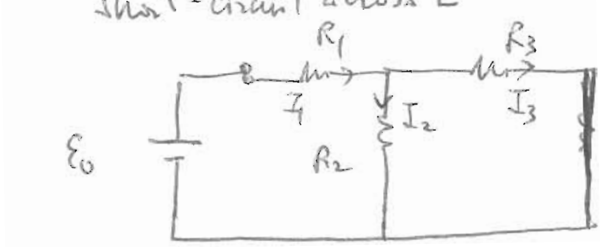
I_2? a) $t=0$ (right after switch is closed)
open circuit across inductor L





$$I_2 = I_1 = \frac{12V}{4\Omega + 8\Omega} = 1A$$

b) $t = \infty$ (long after switch is closed):
short-circuit across L



$$I_1 = I_2 + I_3$$

$$I_1 = \frac{\mathcal{E}_0}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} = \frac{12}{4 + \frac{16}{10}}$$

$$= \frac{12}{5.6} A = 2.14A$$

Can get I_2 or I_3 from I_1 by "current division":

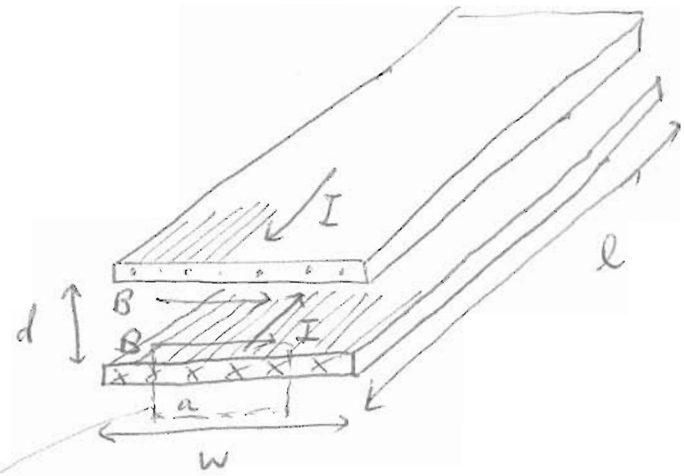
$$I_2 = I_1 \frac{R_3}{R_2 + R_3} = 2.14A \frac{2}{8+2} = 0.429A$$

$$I_3 = I_1 \frac{R_2}{R_2 + R_3} = 2.14A \frac{8}{8+2} = 1.71A$$

Note:
0.429A
+ 1.71A = 2.14A

c) Now switch is again opened: I_2?

32-66



Vol b/w bars = dwl

a) B b/w bars using Ampere's Law:

Amperian Loop $\oint \rightarrow B \cdot l = \mu_0 I_{enclosed}$

due to geometry of B = rectangle:

$$B \cancel{l} = \mu_0 \frac{I}{w} \cancel{l}$$

(assume field outside bars is zero)

$$\rightarrow \boxed{B = \frac{\mu_0 I}{w}}$$

b) $\frac{U}{l} = \frac{u \times Vol}{l} = \frac{u \times dw\cancel{l}}{\cancel{l}} = \frac{B^2}{2\mu_0} dw = \frac{\mu_0^{\cancel{2}} I^2}{2\cancel{\mu_0} w^{\cancel{2}}} d\cancel{w}$

$$\frac{U}{l} = \frac{\mu_0 I^2 d}{2w}$$

c) Compare this with $\frac{U}{l} = \frac{\frac{1}{2} L I^2}{l}$

$$\boxed{\frac{L}{l} = \frac{\mu_0 d}{w}}$$

These structures ~ parallel plate capacitors

Ch. 34 Maxwell's Equations & EM waves.

- Gauss Law : $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{enclosed}}{\varepsilon_0}$
- (") : $\oint \vec{B}\cdot d\vec{A} = 0$ (there are no magnetic monopoles discovered yet)
- Ampere's Law : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} = \mu_0 \underbrace{\int \vec{J}\cdot d\vec{A}}_{\text{current}} + \underbrace{}_{\text{Maxwell's contribution}}$
- Faraday's law : $\oint \vec{E}\cdot d\vec{l} = -\underbrace{\int \dfrac{\partial \vec{B}}{\partial t}\cdot d\vec{A}}_{\text{change of magnetic flux w.r.t. time due to } \vec{B}}$

$$\underbrace{\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}_{\text{"displacement current" (unit: A)}} = \mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

This term gives complete symmetry for $\vec{E}$ & $\vec{B}$ in the sense: from Ampere's Law: a time-varying $\vec{E}$ is the source of a $\vec{B}$. from Faraday's law: a time-varying $\vec{B}$ is the source of a $\vec{E}$. → EM waves

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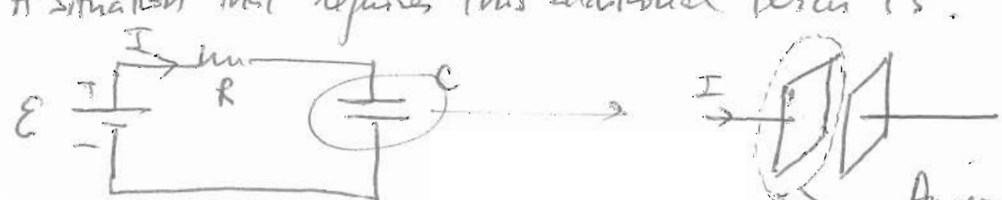
A situation that requires this additional term is:



Amperian loop in plane w/ left plate

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$.

Since I stops at center of this Amperian loop w/o crossing the loop → $I_{enclosed} = 0$

$$\rightarrow \oint \vec{B}\cdot d\vec{l} = 0$$

but there is a measurable magnetic field when a AC power source is applied.

→ Maxwell addition of $\varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$ "the displacement current".

EM waves (electromagnetic waves) in vacuum: { no materials: no charges, no currents, yes $\vec{E}$ & $\vec{B}$!

Gauss' law:

$$\oint \vec{E}\cdot d\vec{A} = 0$$
$$\oint \vec{B}\cdot d\vec{A} = 0$$

} provide no info.

Ampere's law: $\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}$

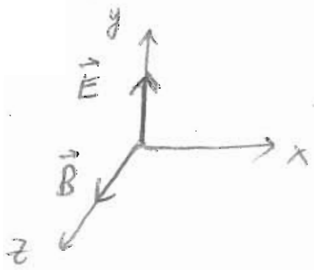
Faraday's law: $\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$

← complete symmetry b/w E & B in vacuum!

↳ $\vec{E}$ & $\vec{B}$ creates themselves: $E \underset{\text{time varying}}{\rightarrow} B \underset{\text{time varying}}{\rightarrow} E \rightarrow \dots$ = EM waves

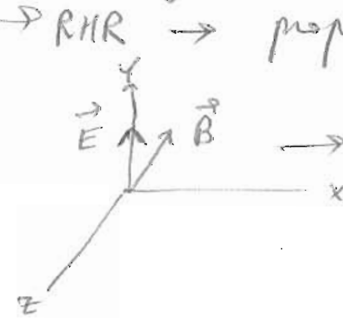
$$\vec{E} = E_p \sin(kx - \omega t)\,\hat{j}$$
$$\vec{B} = B_p \sin(kx - \omega t)\,\hat{k}$$

} they are perpendicular!



→ $\vec{E}\times\vec{B}$ gives direction of propagation

→ RHR → propagation along +x-axis



→ propagation along −x-axis

along direction of propagation (either x, y or z)

position time

linear freq.

$$\vec{E} = E_p \sin(kx - \omega t)\,\hat{j}$$

angular frequency : $2\pi f$

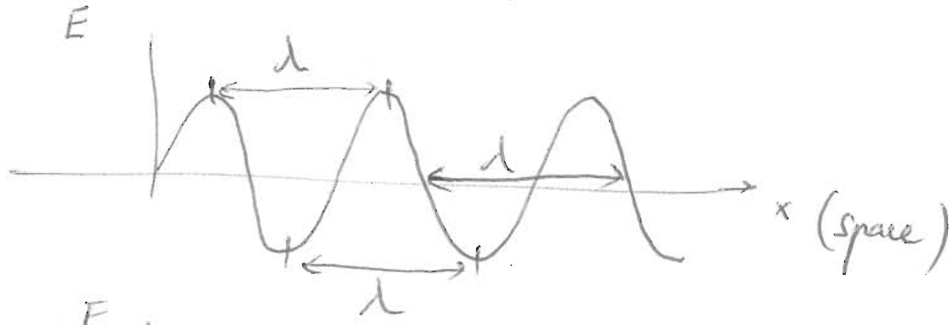
$= \dfrac{2\pi}{T}$

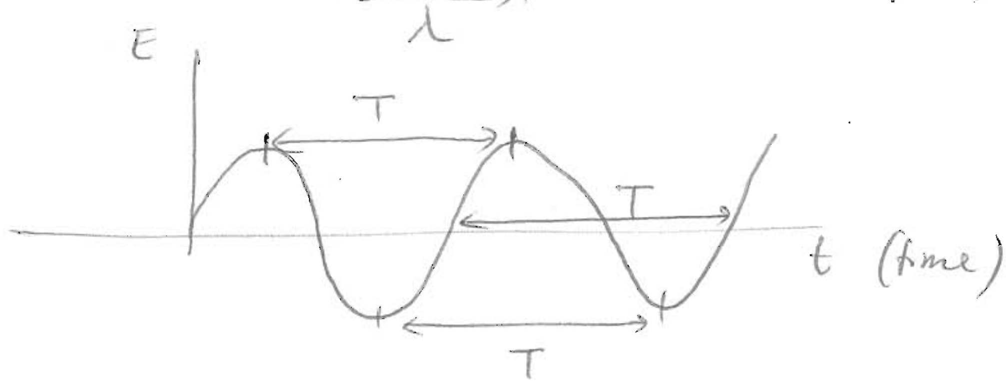
period

magnitude or amplitude of $\vec{E}$

wave number $= \dfrac{2\pi}{\lambda}$

wavelength.





$$\vec{E} = E_p \sin(kz - \omega t)\,\hat{k}$$

If propagation is along z → $\vec{E}$ cannot point along z as well.

(direction of propagation was given by $\vec{E} \times \vec{B}$ which is perpendicular to both $\vec{E}$ and $\vec{B}$)