Capacitance:
\[ C = \frac{Q}{V} \]

Mutual Inductance
\[ M = \frac{\phi_2}{I_1} \]

Solenoid or electromagnet
similar to
induced magnet.

\[ \phi_2 : \text{magnetic flux through solenoid #2} \]
\[ \phi_1 : \text{due to the magnetic field created by solenoid #1} \]

\[ \phi_2 = M I_1 ; \text{If } I_1 \text{ is changing with time} \rightarrow \phi_2 \text{ would be changing with time} \rightarrow \text{there will be a } E_2 \text{ inside solenoid #2 trying to compensate for any change in } \phi_2 \] (Faraday's law)

\[ -E_2 = \frac{d\phi_2}{dt} = M \frac{dI_1}{dt} \] it is clear here why it is called mutual inductance.

Question: \[ -E_1 = M \frac{dI_2}{dt} \]

Yes! It is the same mutual inductance for a set of 2 solenoids or inductors.
Now: $\phi$ also creates a magnetic flux though its solenoid \( (#1) \rightarrow \phi \) on the self-magnetic flux.

$$L = \frac{\phi}{I} = \text{"self inductance"}$$

$$\Rightarrow \quad -\mathcal{E} = L \frac{dI}{dt}$$

self-induced voltage

Units: \( L \in \text{M} \quad \frac{[\mathcal{E}]}{[I]} = \frac{[V.S]}{[A]} = \text{H for Henry} \) (SI system)

$\text{RC Circuit:}$

$RL$ Circuit:

$V_c$ does not change instantaneously.

$t=0$ (just closed): short-circuit across $C$ \( V_c = 0 \)

$t=\infty$ (long after): open-circuit across $C$ \( V_c = \frac{E}{2} \)

\( I_c = \frac{E}{R} - \frac{t}{RC} \) (current decays exponentially from \( \frac{E}{R} \))

\( V_c = \mathcal{E} - \frac{V}{L(R)} \) (voltage decays exponentially for \( \mathcal{E} \))

$E = \mathcal{E} + V_c$
\[ U = \frac{1}{2} CV^2 \text{ (J)} \]
\[ n = \frac{1}{2} \frac{L}{Ae} \left( \frac{J}{m^2} \right) \]

\[ E = -L \frac{dI}{dt} \]
\[ L = \int_0^t P \, dt = \int_0^t I \frac{dI}{dt} \, dt = \int_0^t I \frac{dI}{dt} \, dt = \frac{1}{2} L \left( \frac{dI}{dt} \right)^2 \]
\[ \frac{d}{dt} \left[ \frac{1}{2} I^2 \right]_0 = \frac{1}{2} L \frac{dI}{dt} \]
\[ n = \frac{1}{2} \frac{I^2}{L} \frac{d}{dt} \]

\[ B = \frac{\mu_0 N I}{\ell} \rightarrow L = \frac{\mu_0 N^2 A}{
\]
t = 20 ms after switch is closed:

a) Circuit current: \( I_c = \frac{V_c - V_L}{R} = \frac{V_c - (\frac{V_c}{L} \cdot \frac{t}{L})}{R} \)

\[ V_L(t=0) = 12 \, \text{V} \]
\[ V_L(t=20 \, \text{ms}) = 4.41 \, \text{V} \]

\[ I_c = \frac{12 (1 - e^{-\frac{2 \times 10^{-2}}{100}})}{100} = 75.9 \, \text{mA} \]

b) \( V_L(t=20 \, \text{ms}) = E_0 e^{-\frac{t}{\frac{L}{R}}} = 12 e^{-\frac{2 \times 10^{-2}}{100}} = 4.41 \, \text{V} \)

c) \( V_R(t=20 \, \text{ms}) = 12 - 4.41 = 7.59 \, \text{V} \)

d) \( \frac{dI}{dt}(t=20 \, \text{ms}) = \frac{dI_c}{dt}(20 \, \text{ms}) = \frac{V_L(t=20 \, \text{ms})}{L} = \frac{4.41}{2} = 2.2 \, \text{A/s} \)

\( (E_L = -L \frac{dI_c}{dt}) \)

e) \( P = I V_R = 75.9 \times 10^{-3} \times 7.59 = 0.576 \, \text{W} \)
I_e:

a) \( t = 0 \) (right after switch is closed):
   - Open circuit across inductor \( L \)
   - \( I_e = I_1 = \frac{12V}{4 \mu F + 8 \mu F} = 1A \)

b) \( t = \infty \) (long after switch is closed):
   - Short circuit across \( L \)
   - \( I_t = I_1 + I_3 \)
   - \( I_1 = \frac{12}{R_1 + \frac{R_2R_3}{R_2 + R_3}} = \frac{12}{4 + \frac{16}{10}} = \frac{12}{5.6} = 2.14A \)

Can find \( I_2 \) or \( I_3 \) from \( I_t \) by "current division":

\[
I_2 = I_t \frac{R_2}{R_2 + R_3} = 2.14A \frac{2}{8 + 2} = 0.429A \]

\[
I_3 = I_t \frac{R_2}{R_2 + R_3} = 2.14A \frac{8}{8 + 2} = 1.71A
\]

Note:
\[
\begin{align*}
0.429A + 1.71A &= 2.14A
\end{align*}
\]

c) Now switch is again opened: \( I_e \)
B by w bars using Ampere's law:
Amperian loop $B \ell = \mu_0 I$ enclosed

due to geometry of $B = \text{rectangle}$:

$B \ell = \frac{\mu_0 I}{w}$

(assume field outside bars is zero)

$B(x) = \frac{\mu_0 I}{w}$

b) \[
\frac{U}{l} = \int \frac{B^2}{2\mu_0} dl = \frac{\mu_0 I^2}{2w} \int \frac{1}{w} dl
\]

$\frac{U}{l} = \frac{\mu_0 I^2}{2w}$

c) Compare this with $\frac{U}{l} = \frac{1}{2} \frac{LI^2}{L}$

These structures ~ parallel plate capacitor
- Gauss law: \( \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \)
- Gauss law: \( \oint \mathbf{B} \cdot d\mathbf{A} = 0 \) (there are no magnetic monopoles discovered yet)
- Ampère's law: \( \oint \mathbf{B} \cdot d\mathbf{A} = \mu_0 I_{\text{enclosed}} + \mu_0 \oint \mathbf{J} \cdot d\mathbf{A} \)
- Faraday's law: \( \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\mathbf{F}}{dt} \cdot d\mathbf{A} \)

\[ \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{d\mathbf{B}}{dt} \]

\[ \mu_0 \varepsilon_0 \frac{d\mathbf{B}}{dt} = \mu_0 \varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} \]

"displacement current" (unit: \( \text{A} \))

This term gives complete symmetry for \( \mathbf{E} \) & \( \mathbf{B} \) in the sense:
- from Ampère's law: a time-varying \( \mathbf{E} \) is the source of a \( \mathbf{B} \)
- from Faraday's law: a time-varying \( \mathbf{B} \) is the source of a \( \mathbf{E} \)  \( \Rightarrow \) EM waves

A situation that requires this additional term is:

\[ \oint \mathbf{E} \cdot d\mathbf{l} = 0 \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

American loop in plane of left plate
Since I stop at order of this imaginary loop w/o knowing the
loop \( \Rightarrow \text{ It cancels out.} \)
\[
\int \vec{B} \cdot d\vec{l} = 0
\]
but there is a measurable magnetic field, when a AC
power source is applied.

\[ \Rightarrow \text{Maxwell addition of } \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A} \text{ "the displacement current"} \]

\[ \text{EM waves (electromagnetic waves) in vacuum: } \game \text{ no change, no}\]
\[
\text{Gauss' law: } \int \vec{E} \cdot d\vec{A} = 0 \]
\[
\int \vec{B} \cdot d\vec{A} = 0 \quad \text{provide an info.}
\]

\[ \text{Amper's law: } \int \vec{B} \cdot d\vec{l} = \mu \varepsilon_0 \frac{dL}{dt} \]
\[ \mu \varepsilon_0 \int \vec{B} \cdot d\vec{A} \quad \text{complete symmetry}
\]

\[ \text{Faraday's law: } \int \vec{E} \cdot d\vec{l} = - \int \frac{d\vec{B}}{dt} \cdot d\vec{A} \quad \text{in vacuum!}
\]

\[ \overleftrightarrow{E} \& \overleftrightarrow{B} \text{ create themselves: } \overleftrightarrow{E} \Rightarrow \overleftrightarrow{B} \Rightarrow \overleftrightarrow{E} \Rightarrow \ldots \Rightarrow \text{EM waves}
\]

\[ \overleftrightarrow{E} = E_0 \sin (kx - wt) \hat{i} \quad \text{they are perpendicular!}
\]

\[ \overleftrightarrow{B} = B_0 \sin (kx - wt) \hat{k} \]

\[ \vec{E} \times \vec{B} \text{ gives direction of propagation}
\]
\[ \Rightarrow \text{PAR } \Rightarrow \text{ propagation along } +x \Rightarrow \text{new}
\]

\[ \overleftrightarrow{E} \text{ & } \overleftrightarrow{B} \Rightarrow \text{ propagation along } -x \Rightarrow \text{evident}
\]
\[ E = E_0 \sin \left( \frac{2\pi}{\lambda} x - wt \right) \]

along directions of propagation (either x, y or z)

\[ \lambda \text{ wave number} = \frac{2\pi}{\lambda} \]

\[ T \text{ period} = \frac{2\pi}{f} \]

magnitude or amplitude of \( E \)

\[ E \]

\[ \lambda \]

\[ T \]

along x - \( E \) cannot point along y or z as well.,

\[ E = E_0 \sin \left( \frac{2\pi}{\lambda} x - wt \right) \hat{\mathbf{r}} \] (direction of propagation was given by \( \mathbf{E} \times \mathbf{B} \) which is perpendicular to both \( \mathbf{E} \) and \( \mathbf{B} \))