Ch. 31 Electromagnetic Induction:

Magnetic flux: $\Phi_B = \int_{\text{Surface}} \vec{B} \cdot d\vec{A}$

("Phi")

$d\vec{A}$ → element of area

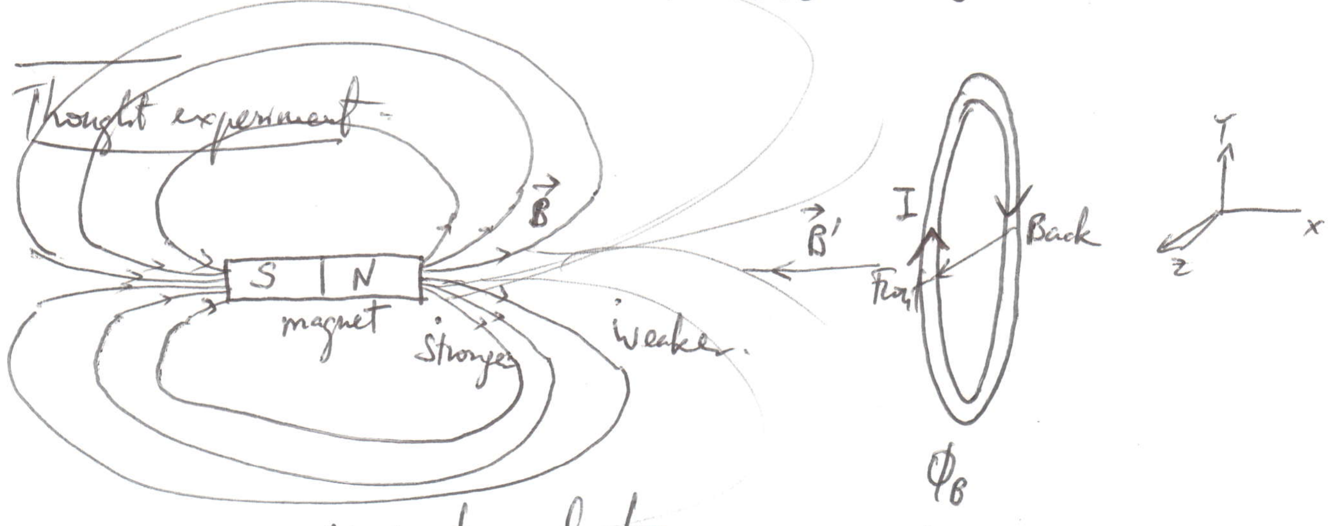
Relevant when $\vec{B}$ is crossing some loop; the integral above is over the area enclosed by loop.

Faraday's law: if Φ_B changes with time: it ~~induce~~ produces a magnetic induction in the loop in the form of an electric potential: e.m.f or $\mathcal{E}$ (electromotive force)

$$\boxed{\mathcal{E} = - \frac{d\Phi_B}{dt}}$$

$-$ → Lenz's law

Thought experiment:



Moves toward loop ⟶ Φ_B increases b/c of larger B passing through.

according to Faraday's law → $\mathcal{E}$ in loop

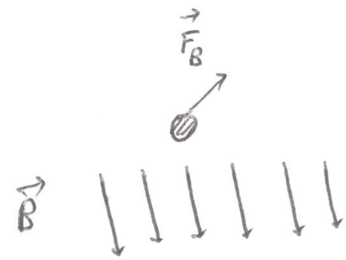
→ current in loop: I

Direction of I : sign in Faraday's law or Lenz's law

→ It is a negative induction : it tends to compensate for any change in Φ_B :

If there is an increase in Φ_B : $\mathcal{E}$ & I will tend to reduce Φ_B by creating a $\vec{B}'$ pointing in the opposite direction.
→ direction of I is related to this $\vec{B}'$ (I should be such that it creates this $\vec{B}'$)

29.11

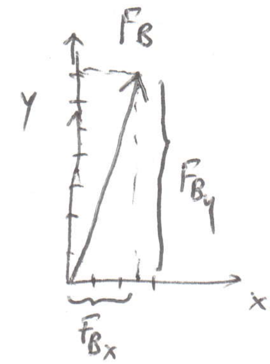


$q = 1.4\,\mu C$

$v = 185\ m/s$

$\vec{F}_B = (2.5\,\hat{i} + 7\,\hat{j})\ \mu N$

$\vec{B} = (4.2\,\hat{i} - 15\,\hat{j})\ mT$



$\vec{F}_B = q\,\vec{v} \times \vec{B}$

Magnitude : $F_B = q\,v\,B\sin\theta$?

Angle b/w $\vec{v}$ & $\vec{B}$

$$\sin\theta = \frac{F_B}{q\,v\,B} = \frac{\sqrt{2.5^2 + 7^2}}{1.4\times10^{-6} \times 185 \times \sqrt{4.2^2 + 15^2}} = 0.644$$

$$\theta = \sin^{-1}(0.644) = \begin{cases} 40.1^\circ \\ 139.9^\circ \end{cases}$$

29.53

NMR

Magnetic Resonance Imaging

$B = 7T$

Energy to flip a proton ($\mu = 1.41\times10^{-26}\ Am^2$) parallel to antiparallel ?

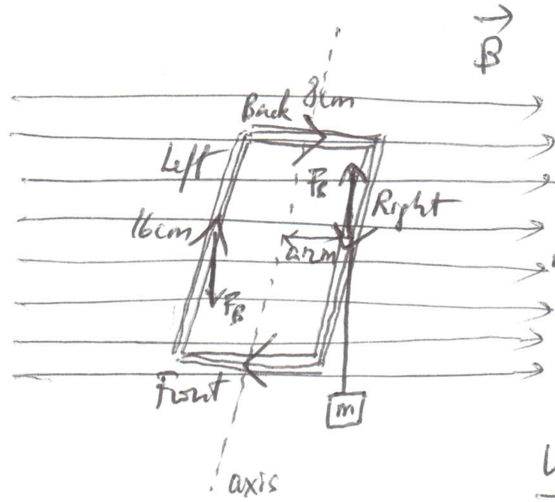
$\underline{\text{Magnetic moment } \vec{\mu} = I\vec{A}}$

Torque : $\vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \tau = \mu B$

$\Delta U = 2\mu B = 2 \times 1.41\times10^{-26} \times 7 = 1.97\times10^{-25}\ J$

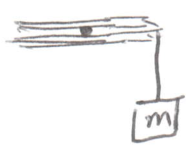
$$\left.\begin{array}{l}\text{Proton parallel to } B : \mu B \\ \text{Proton antiparallel to } B : -\mu B\end{array}\right\} \quad \mu B - (-\mu B) = 2\mu B$$

29.69



→ 10 turn wire loop
$I = 2A$ (per turn)
$m = 50g$.
$\vec{B}$ uniform, horizontal
What B to hold loop as is?

Front view:



B need to create a torque to cancel that by the mass m.

How? What are the magnetic force on loop by $\vec{B}$?

$\vec{F}_B = I \vec{l} \times \vec{B}$:
- → upward on right side of loop for indicated current (C.C.W from above)
- → downward on left side
- → zero on front
- → zero on back.

→ What are the magnetic torques on loop? $\vec{\tau} = \vec{r} \times \vec{F}$

or product of F_B with "the arm": separation b/w application point and axis of rotation perpendicular:

$$\tau_B = 0.04 \times \underbrace{10 \times 2}_{\text{Total current}} \times 0.16 \times B \quad \text{due } F_B \text{ on Right side}$$

same sign as τ_B due to F_B on left side

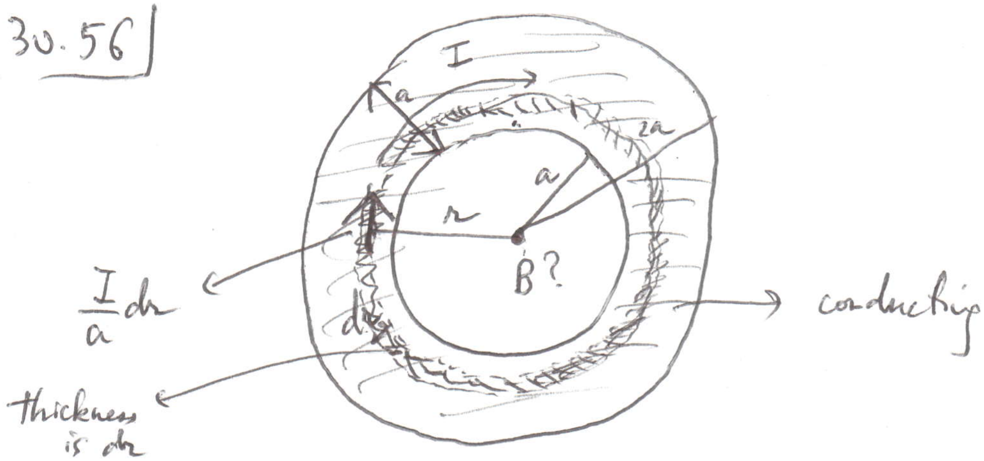
$$\rightarrow \tau_{B_{Total}} = 2\tau_B = 0.08 \times 3.2 \times B$$

$$\tau_{mg} = \text{arm} \times mg = 0.04 \times m \times g$$

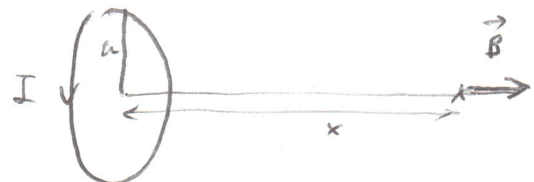
equilibrium: $\tau_{B_{Total}} = \tau_{mg}$

$$B = \frac{0.04 \times 0.05 \times 9.81}{0.08 \times 3.2} = 0.077\,T$$

30.56



Previous derivation:



$$\tilde{B}(x) = \frac{\mu_0}{2}\frac{Ia^2}{(x^2+a^2)^{3/2}} \rightarrow \text{at center of loop } x=0$$

$$\tilde{B} = \frac{\mu_0}{2}\frac{I\cancel{a^2}}{a^{\cancel{3}}} = \frac{\mu_0 I}{2a} \quad (a = \text{radius of loop of current}).$$

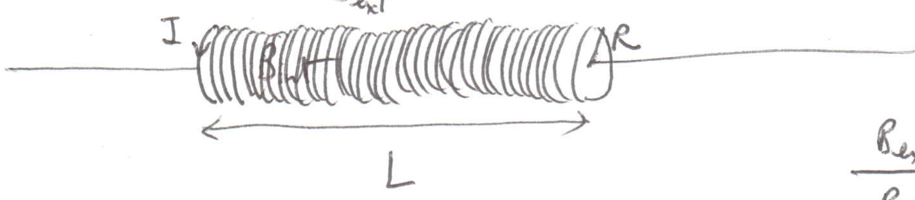
What's new? need to integrate this for r from a to 2a

$$\boxed{d\tilde{B} = \frac{\mu_0 \frac{I}{a}dr}{2r}}$$

$$\rightarrow B = \int_{r=a}^{r=2a} d\tilde{B} = \int_{r=a}^{r=2a} \frac{\mu_0 I}{2ar}dr = \frac{\mu_0 I}{2a}\underbrace{\int_{r=a}^{r=2a}\frac{dr}{r}}_{\ln\left(\frac{2\cancel{a}}{\cancel{a}}\right)} = \frac{\mu_0 I}{2a}\ln 2$$

30.49

Solenoid:



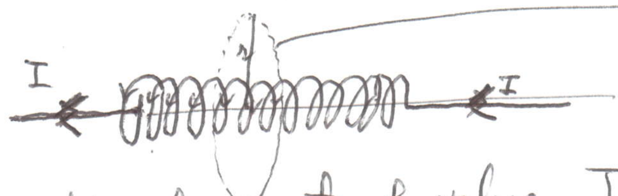
$n = \frac{N}{L}$ (# of turns per unit length)

$\frac{B_{ext}}{B_{int}} \xrightarrow[n\to\infty]{} 0$

Solenoid: to obtain large B along its axis (and zero B outside solenoids)

$\longrightarrow B = \frac{\mu_0 I}{2R}$

B_{ext}:



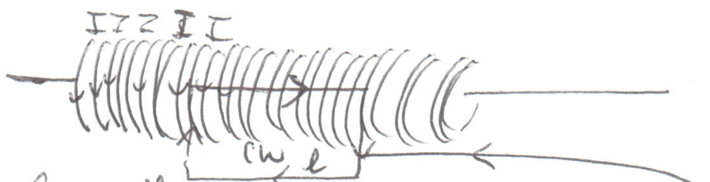
Amperian loop of length $2\pi r$

↳ long line of current of value I : Ampere's law :

$$\boxed{B_{ext} = \mu_0 \frac{I}{2\pi r} \quad (r > R)}$$

This is just due to one single current!

B_{int}:



will be along the axis → Amperian loop : it's

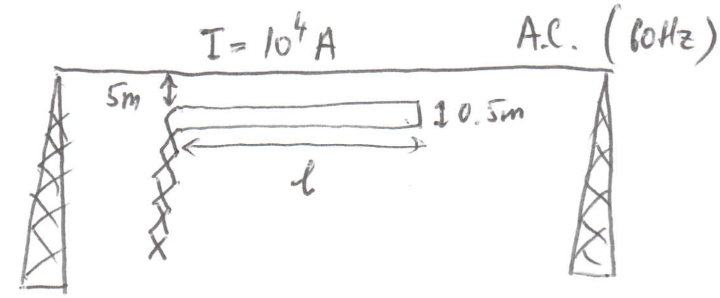
$B_{int} L = \mu_0 I_{enclosed}$

$B_{int} \cancel{l} = \mu_0 I n \cancel{l}$

$$\boxed{B_{int} = \mu_0 I n \quad (r < R)}$$

$$\rightarrow \frac{B_{ext}}{B_{int}} = \frac{\cancel{\mu_0} \frac{I}{2\pi r}}{\cancel{\mu_0} I n} = \frac{1}{2\pi r n} \xrightarrow[n\to\infty]{} 0 \ \checkmark$$

31.57



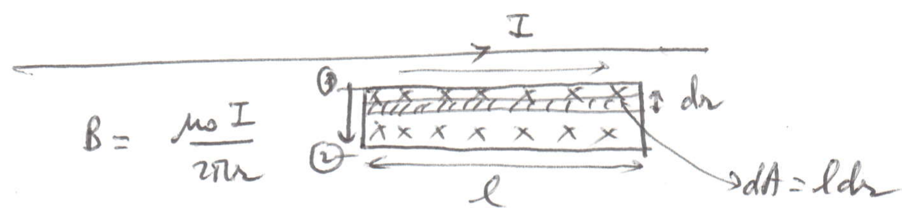
$I = I_0 \sin \omega t \quad ; \quad I_0 = 10^4 A \quad ; \quad \omega = 2\pi f$

a) $V_0 = 170V \rightarrow$ what is l?

As the current in the power line is switching 60 times per second, the B it creates will change accordingly
$\rightarrow \Phi_B$ thru the farmer's loop will change / inducing a $\mathcal{E}$ in it to counter this change

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$B = \frac{\mu_0 I}{2\pi r}$



$= \frac{\mu_0 I}{2\pi} \int_{①}^{②} \frac{1}{r} l\, dr$

$= \frac{\mu_0 I l}{2\pi} \int_{5}^{5.5} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{5.5}{5}\right)$

$\Phi_B(t) = \frac{\mu_0 l I_0 \sin \omega t}{2\pi} \ln\left(\frac{5.5}{5}\right)$

$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 l \omega I_0}{2\pi} \ln\left(\frac{5.5}{5}\right) \cos \omega t$

$V = V_0 \cos \omega t$

$V_0 = \frac{\mu_0 l \omega I_0}{2\pi} \ln\left(\frac{5.5}{5}\right)$

170V

$\rightarrow l = \frac{2\pi \times V_0}{\mu_0 \omega I_0 \ln\left(\frac{5.5}{5}\right)} = \frac{\cancel{2\pi} \times 170}{4\pi \times 10^{-7} \times \cancel{2\pi} \times 60 \times 10^4 \ln\left(\frac{5.5}{5}\right)} = 2.37 km$

b) Average Power Consumption if $R = 5\Omega$

$P = IV$

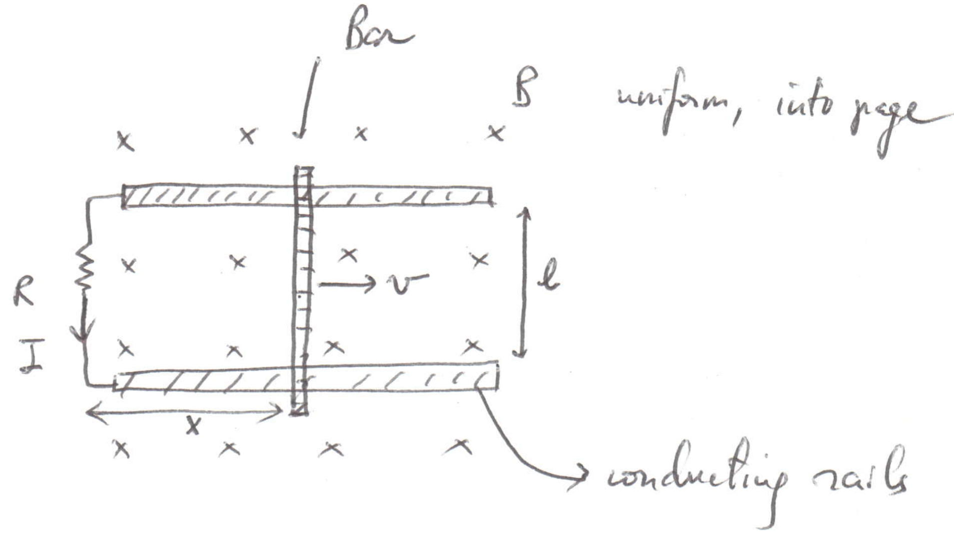
$P_{av.} = \frac{1}{2} I_0 V_0 = \frac{1}{2} \frac{V_0}{R} V_0 = \frac{1}{2} \frac{V_0^2}{R} = \frac{1}{2} \frac{170^2}{5} = 2.89\,kW$

c) $10¢/kWh$ (How much stolen per day?)

One day: $2.89\,\cancel{kW} \times 24\,\cancel{h} \times \frac{\$0.10}{\cancel{kWh}} = \underline{\underline{\$6.94}}$

d) check accounting: power delivered versus money collected.

31.27]



a) Direction of current in resistor :
CCW to reduce the increase in Φ_B as bar moves right.
→ down at R.

b) What rate of work for agent pulling bar = power consumed by circuit.

Conservation of energy

$$P = IV$$
$$= I(IR) = I^2R$$

$$I = \frac{\mathcal{E}}{R} = \frac{-\frac{d\Phi_B}{dt}}{R} = \frac{-\frac{d}{dt}(B.A)}{R} = \frac{-\frac{d}{dt}(Blx)}{R}$$

of loop

$$= -\frac{Bl}{R}\underbrace{\frac{dx}{dt}}_{v}$$

$$P = I^2R = \left(\frac{-Blv}{R}\right)^2 R = \frac{(Blv)^2}{R}$$

27.74

$J = 75 \frac{mA}{cm^2}$ $15°C \longrightarrow 20°C$ How long

$c_w = 4184 \frac{J}{kg\,°K}$

$\rho_w = 0.22\,\Omega m$

From 27.73: power per unit volume dissipated : $J^2\rho$

energy supplied per unit volume after time t is $J^2\rho t = \frac{m c_w \Delta T}{Vol}$

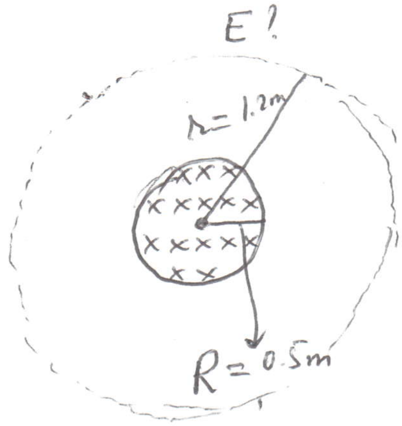
$$J^2 \underset{\substack{\downarrow \\ \text{resistivity}}}{\rho}\, t = \underset{\substack{\downarrow \\ \text{mass density}}}{\rho_{mass}}\, c_w\, \Delta T$$

$$t = \frac{\rho_{mass} c_w \Delta T}{J^2 \rho} = \frac{1000 \frac{\cancel{kg}}{m^3}\; 4184 \frac{J}{\cancel{kg}\,\cancel{°K}}\; 5\cancel{°K}}{\left(75 \cdot 10^{-3} \frac{A}{\cancel{cm^2}} \frac{\cancel{cm^2}}{10^{-4} m^2}\right)^2 0.22\,\Omega.m}$$

$= 169\,s$

31.38

Tokamak



$$\frac{dB}{dt} = 5.1 \frac{T}{ms}$$

(during a pulse)

$E(r = 1.2m) = ?$

a) Faraday's law → $\mathcal{E}$ an electric potential : $\mathcal{E} = \oint \vec{E} \cdot d\vec{l}$

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

Now $\vec{E}$ is the same at fixed r (along the dotted circle) b/c symmetry:

$$-\frac{d\phi_B}{dt} = E(r)\, 2\pi r$$

$$-\frac{d}{dt}(B \cdot \text{Area})$$

$$-\frac{d}{dt}(B \cdot \pi R^2) = E \cdot 2\pi r$$

$$\left(-\frac{dB}{dt}\right)\cancel{\pi} R^2 = E\, 2\cancel{\pi} r \quad \rightarrow \quad E = \frac{\left(-\frac{dB}{dt}\right) R^2}{2r}$$

$$|E| = \frac{5.1 \frac{T}{\cancel{ms}}\ 0.5^2\ \frac{1\cancel{ms}}{10^{-3} s}}{2 \times 1.2}$$

$$= 531\ \frac{V}{m}$$

b) What is the direction of E?
is the direction of I which is CCW to counter the increase of ϕ_B b/c of the pulse.

c) Energy gained by a proton at $r = 1.2m$ per turn

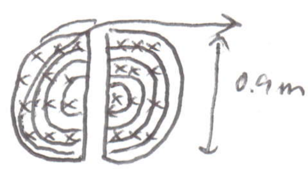
$$\underset{\downarrow}{U} = q\underset{\underset{\mathcal{E}}{\downarrow}}{V} = q\oint \vec{E}\cdot d\vec{l} = q\cdot E\cdot 2\pi r$$

$$= 1.6\times10^{-19} \times 531 \times 2\pi \times 1.2$$

$$= 6.41\times10^{-19}\,J$$

29.26

29.25



deuterium $(1p + 1n)$

$q = +e$, $m \cong 2000 m_e$; $q = 0$, $m \cong 2000 m_e$

$B = 2T$

What freq. for alternating voltage?

$$f_d = \frac{1}{T} = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times 2}{2\pi\times 4000\times 9.11\times10^{-31}} = 1.4\times10^{7}$$

$$= 14\ MHz.$$

a) protons instead of deuterium: $f_p = 2 f_d$

b) α particles i.o. deuterium: $f_\alpha = f_d$ $\left(\begin{matrix} q \to 2e \\ m \to 2m_p \end{matrix}\right)$
$(2p + 2n)$

Max. achievable KE: $\dfrac{(RqB)^2}{2m}$ (Note $\dfrac{q^2}{m}$) v.s. $\dfrac{q}{m}$ in the frequency

$$KE_{max\,d} = \frac{(0.45\times 1.6\times10^{-19}\times 2)^2}{2\times 4000\times 9.11\times10^{-31}}\,J$$

a) protons i.o. deuterium: $KE_{max\,p} = 2\,KE_{max\,d}$ $\left(m_p = \frac{m_d}{2}\right)$

b) α particles i.o. deuterium: $KE_{max\,\alpha} = 2\,KE_{max\,d}$ $\begin{cases} m_\alpha = 2m_d \\ q_\alpha = 2q_d \end{cases}$
$(2p + 2n)$