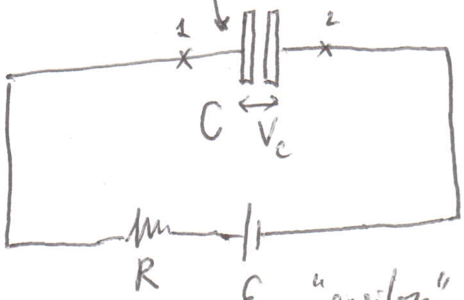
Circuits with capacitors:

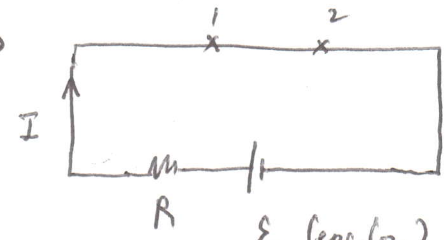


"epsilon"
(Battery's voltage)
e.m.f.

$\boxed{t=0}$ we connect the uncharged capacitor

$Q=0 \rightarrow E=0 \rightarrow V_c=0$

=

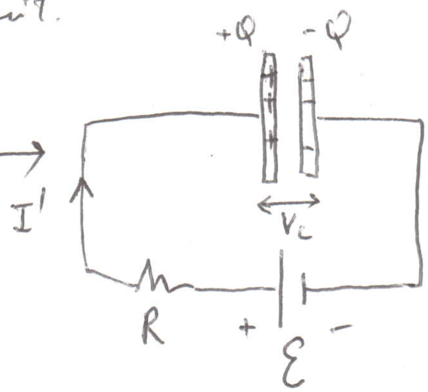


$$I = \frac{\mathcal{E}}{R}$$

No potential difference b/w 1 & 2
→ shortcircuit (like a wire connecting 1 & 2)

→ I is current of positive charges (by convention)

$\boxed{t>0}$: Positive charges move from right plate to left plate through circuit.



"The capacitor is charging"
$V_c \neq 0$

Loop equation: CCW:

$$\mathcal{E} - I'R - V_c = 0$$

$$I' = \frac{\mathcal{E} - V_c}{R} < I$$

(current gets smaller b/c it is harder to move charges from one plate to the other as the electric field is $\frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$)

Loop equation : $\frac{d}{dt}\left(\mathcal{E} = I'R + \underbrace{V_c}_{\frac{Q}{C}} \right)$

$$0 = R\frac{dI'}{dt} + \frac{1}{C}\underbrace{\frac{dQ}{dt}}_{I'}$$

differential equation

$$R\frac{dI'}{dt} = -\frac{1}{C}I' \rightarrow \int \left[\frac{dI'}{I'} = -\frac{1}{RC}dt \right]$$

$$e^{\left[\ln I' = -\frac{1}{RC}t + \text{const} \right]}$$

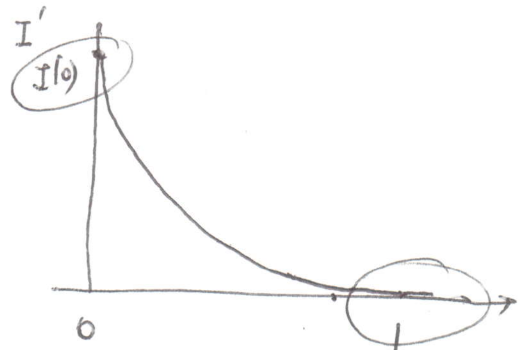
$$I' = e^{-\frac{1}{RC}t} \cdot \text{const}$$

$$I'(t) = \text{const}\; e^{-\frac{t}{RC}}$$

$$= I(0)\, e^{-\frac{t}{RC}}$$

$$\boxed{I'(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}}$$

Current over time in an RC circuit

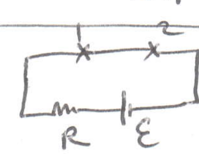


$t \to \infty : I'(\infty) = 0$

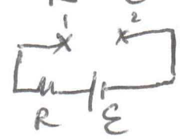
$RC \equiv \tau$ (tau) : time constant : $t = \tau = RC \rightarrow I' = \frac{I(0)}{e} = \frac{I(0)}{2.71}$

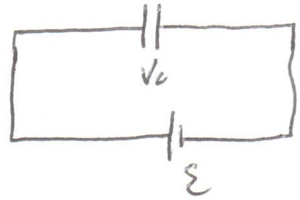
Summary 2 extreme situations

$t = 0 \rightarrow V_c = 0$ (short circuit)



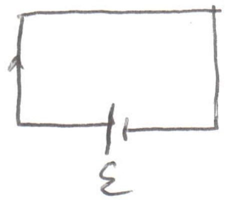
$t = \infty \rightarrow I_C = 0$ (open circuit)





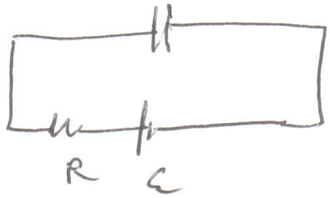
$t=0 \rightarrow V_c = 0$

$\simeq$



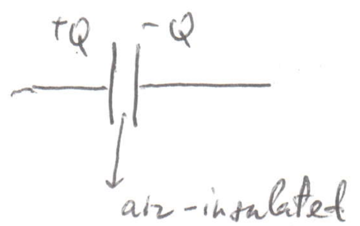
$I = \frac{\varepsilon}{0} = \infty$

- The role of R in series was to limit the max. current at $t=0$



$\rightarrow t=0 \quad = I = \frac{\varepsilon}{R}$

- $\underbrace{\text{Capacitor Break down voltage}}_{V_B}$:

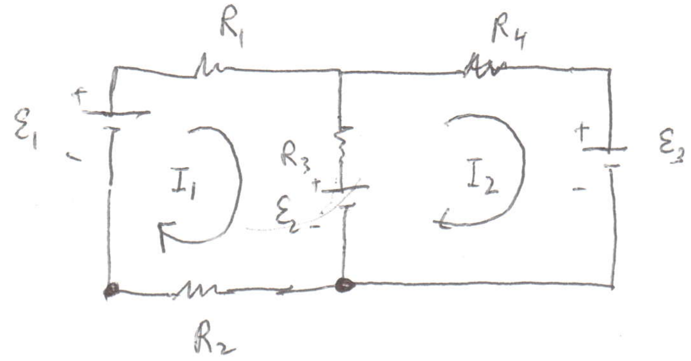


$\longleftrightarrow$

$V_c > V_B$

→ In the previous consideration ε is such that $V_c < V_B$

28.34



$\boxed{I_1 - I_2 = 4.76\,mA}$

↑ down thru R_3

$\boxed{\text{Current through } R_3?}$

$\mathcal{E}_1 = 6V;\ \mathcal{E}_2 = 1.5V;\ \mathcal{E}_3 = 4.5V$

$R_1 = 270\Omega;\ R_2 = 150\Omega;$

$R_3 = 560\Omega;\ R_4 = 820\Omega$

Loop analysis: assume I_1 & I_2 in C.W.

Signs: current − to + at $\mathcal{E}$ → positive contribution; " + to − at $\mathcal{E}$ → negative contribution; across a resistor → negative contribution — to loop equation

1) $+\mathcal{E}_1 - I_1R_1 - (I_1 - I_2)R_3 - \mathcal{E}_2 - I_1R_2 = 0$

+ 2) $+\mathcal{E}_2 - (I_2 - I_1)R_3 - I_2R_4 - \mathcal{E}_3 = 0$

$\mathcal{E}_1 - I_1(R_1 + R_2) - I_2R_4 - \mathcal{E}_3 = 0$

$I_1 = \dfrac{\mathcal{E}_1 - \mathcal{E}_3 - I_2R_4}{R_1 + R_2}$

$= \dfrac{6 - 4.5 - I_2\,820}{420}$

(a) $\boxed{I_1 = \dfrac{1.5 - 820\,I_2}{420}}$

$-3V - I_2(R_3 + R_4) + I_1R_3 = 0$

(b) $\boxed{-3 - 1380\,I_2 + 560\,I_1 = 0}$

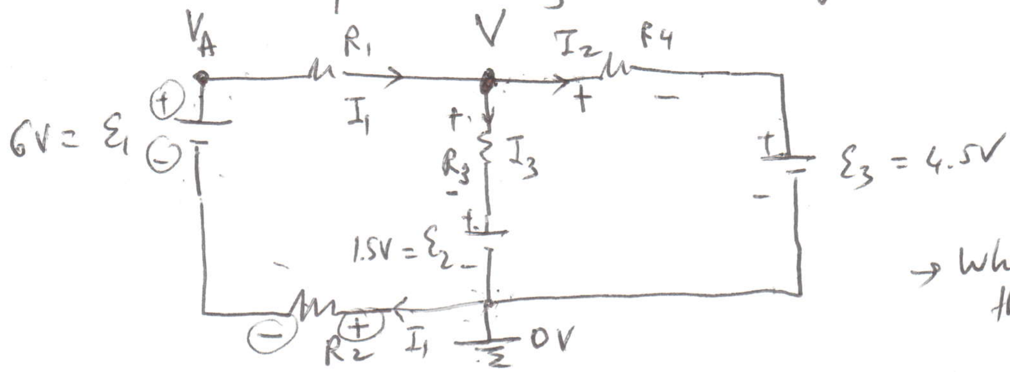
$-3 - 1380\,I_2 + \underbrace{560 \times \dfrac{1.5 - 820\,I_2}{420}}_{2 - 1093.3\,I_2} = 0$

$-1 - 2473.3\,I_2 = 0 \rightarrow \boxed{I_2 = -0.4\,mA} \rightarrow \boxed{I_1 = +4.36\,mA}$

C.W.

I_2 ... in C.C.W

Let's do same question using node analysis:



→ What's the current thru R_3?

$\boxed{\text{Need to set the ground}}$

Assume I_1; I_2; I_3 in those directions

Node equation: $+I_1 - I_2 - I_3 = 0$.

Write currents in term of voltages:

$$I_1 = \frac{\text{Voltage across } R_1}{R_1} = \frac{V_A - V}{R_1} = \frac{\mathcal{E}_1 - I_1R_2 - V}{R_1}$$

$$V_A = \mathcal{E}_1 - I_1R_2$$

$$I_1R_1 = \mathcal{E}_1 - I_1R_2 - V$$

$$I_1(R_1+R_2) = \mathcal{E}_1 - V$$

$$\boxed{I_1 = \frac{\mathcal{E}_1 - V}{R_1+R_2}}$$

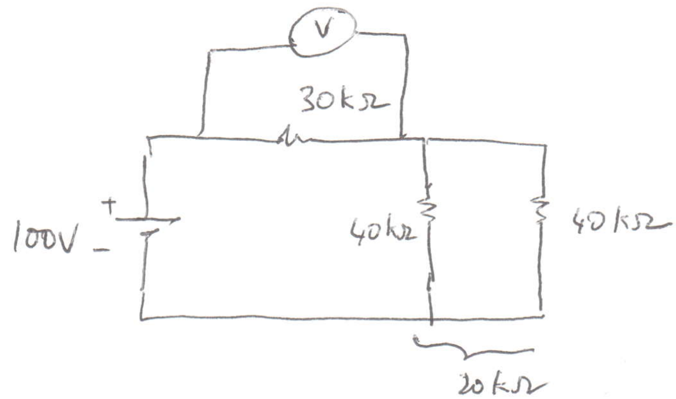
I_2: $V = I_2R_4 + \mathcal{E}_3 \rightarrow \boxed{I_2 = \frac{V-\mathcal{E}_3}{R_4}}$

I_3: $V = I_3R_3 + \mathcal{E}_2 \rightarrow \boxed{I_3 = \frac{V-\mathcal{E}_2}{R_3}} \rightarrow \frac{4.17-1.5}{560} = 4.76mA$

$$\frac{\mathcal{E}_1 - \cancel{I_1R_2} - V}{R_1+R_2} - \frac{V-\mathcal{E}_3}{R_4} - \frac{V-\mathcal{E}_2}{R_3} = 0$$

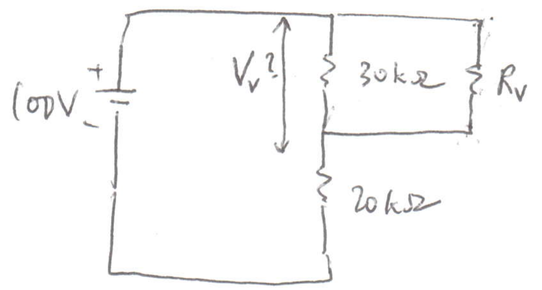
$$\frac{3-V}{420} - \frac{V-4.5}{820} - \frac{V-1.5}{560} = 0 \rightarrow \boxed{V = 4.17V}$$

28.42



a) $R_v = 50\,k\Omega$ b) $R_v = 250\,k\Omega$ c) $R_v = 10\,M\Omega$

What is the voltage reading across the $30\,k\Omega$ in each case?



$$V_v = 100V \frac{\frac{30 R_v}{30+R_v}}{\frac{30R_v}{30+R_v} + 20}$$

$$= 100V \frac{30 R_v}{30 R_v + 20(30+R_v)}$$

$$= 100V \frac{30 R_v}{50 R_v + 600}$$

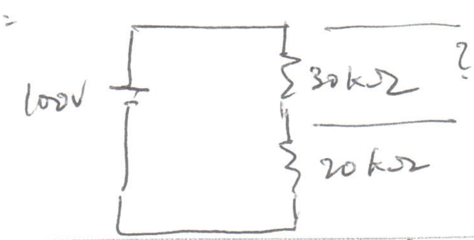
a) $R_v = 50\,k\Omega \rightarrow V_v = 100V \dfrac{1500}{2500 + 600} = \dfrac{15}{31} 100V = 48.4\,V$

b) $R_v = 250\,k\Omega \rightarrow V_v = 100V \dfrac{7500}{12500 + 600} = 57.3\,V$

c) $R_v = 10^6\,\Omega = 1000\,k\Omega \rightarrow V_v = 100V \dfrac{30000}{50000 + 600} = 59.9\,V$

good

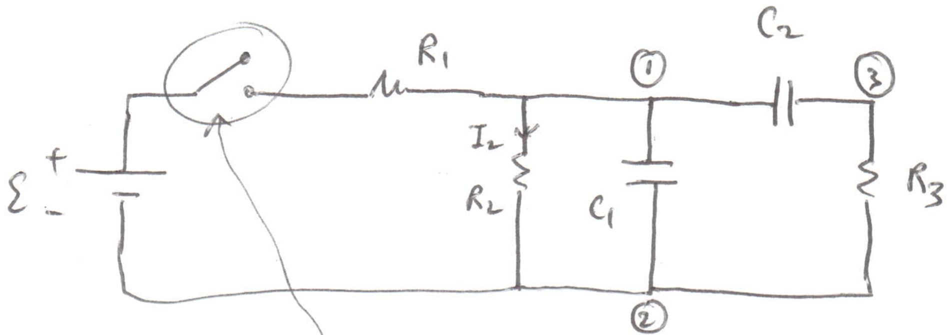
w/o Voltmeter:



? $V_v = 100V \dfrac{30}{30+20} = \dfrac{3}{5} 100V = 60V$

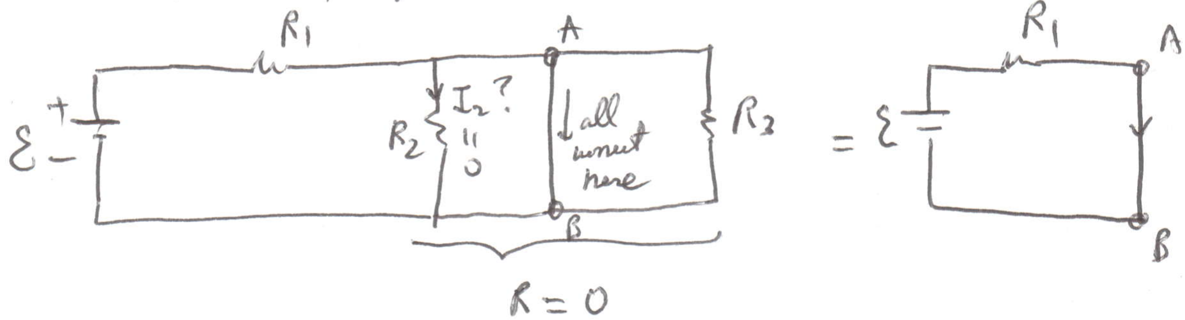
Elements (meters) to connect in parallel → largest $R_{internal}$

28.56



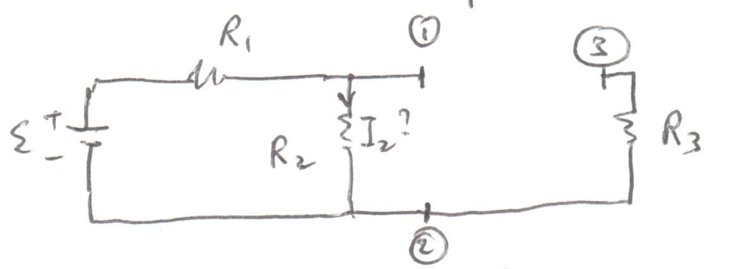
What I_2
a) switch is just closed → $(t=0)$
b) long after switch is closed → $(t=\infty)$

a) $t=0$: C's are like shortcircuited (~~connected~~ can be replaced by a piece of wire)

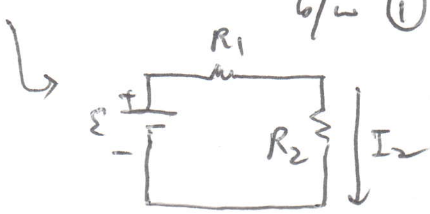


$\boxed{I_2=0}$

b) $t=\infty$: C's are like open circuit (can be opened)



No current b/w 2 & 3 neither b/w 1 & 3



$$\boxed{I_2 = \frac{\mathcal{E}}{R_1+R_2}}$$

Ch 29 Introduction to Magnetic Field

Effects of a magnetic field $\vec{B}$ on a moving charge q velocity $\vec{v}$

$\vec{B}$ uniform out of page:

If charge q also moves out of page → does not feel the magnetic field : $\vec{F} = 0$. Same if charge goes into the page. $\vec{F} = 0$ (if $\vec{v}$ is parallel to $\vec{B}$). If $\vec{v}$ is on the page → will feel the maximum effect of $\vec{B}$, ~~max if along x or y~~ :

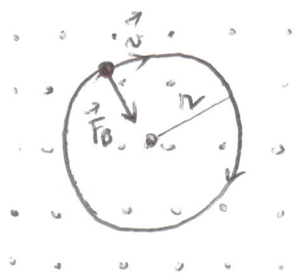
$$\boxed{\vec{F} = q\,\vec{v} \times \vec{B}}$$

"cross product" b/w $\vec{v}$ & $\vec{B}$ is a vector that is <u>perpendicular</u> to <u>both</u> $\vec{v}$ and $\vec{B}$, direction given by RHR (as fingers close from $\vec{v}$ to $\vec{B}$, thumb indicates direction of $\vec{v} \times \vec{B}$), magnitude by

$qvB\sin\theta$ (θ angle b/w v & B)

Ch 29 Intro to Magnetic Field (Cont.)

Trajectory of a charged particle in a magnetic field: a circular motion since $\vec{F}_B$ is always perpendicular to the direction of motion, being the agent to provide the radial acceleration:



$\vec{B}$ uniform and out of page

r?

$\vec{F}_B = q\vec{v} \times \vec{B}$ $\begin{cases} \text{direction: radial (RHR)} \\ F_B = qvB \end{cases}$

$$qvB = m\underbrace{\frac{v^2}{r}}_{\text{radial acceleration}}$$

$$\Rightarrow \boxed{r = \frac{mv}{qB}}$$

Period: time to complete one turn: $\dfrac{2\pi r}{v} = \dfrac{2\pi \cancel{r}}{\left(\dfrac{qB\cancel{r}}{m}\right)} = \dfrac{2\pi m}{qB}$

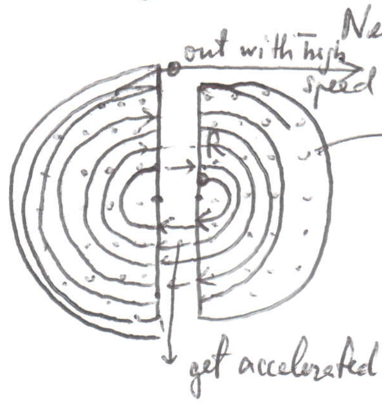
Applications:

1) Cyclotron: (modern version: synchrotron)

Goal: accelerate the charge particle to high speed.

Need to use the electric field $F_E = qE = ma$

Cyclotron radius is R



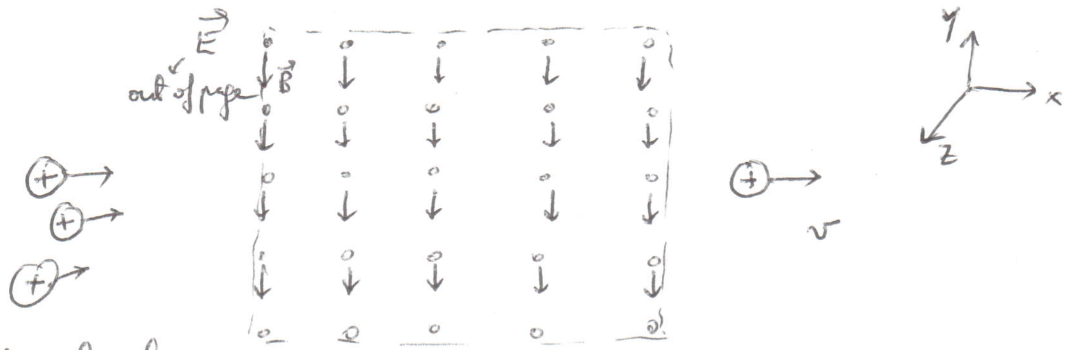
$\vec{B}$ uniform, out of page to keep charged particle in circular motion within the two half-circles

What is $K.E._{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m\left(\frac{RqB}{m}\right)^2 = \frac{1}{2m}(RqB)^2$

Synchrotron: $v \sim c = 3\times10^8\,m/s$

2) <u>Velocity selector</u> :

ions (+) with velocity $\vec{v}$: all possible values



different values of v

1) q, $\vec{v}$

$\vec{v} = v\hat{i}$

$\vec{B} = B(-\hat{j})$

$$\begin{cases} \vec{F}_E = q\vec{E} = qE\hat{k} & (\vec{E} \text{ is out of page}) \\ \vec{F}_B = q\vec{v}\times\vec{B} = qvB(-\hat{k}) \end{cases}$$

at $v = \dfrac{E}{B} \rightarrow \vec{F}_E + \vec{F}_B = 0 \Rightarrow$ the particle (~~and~~ regardless of what charge) will suffer no deflection.

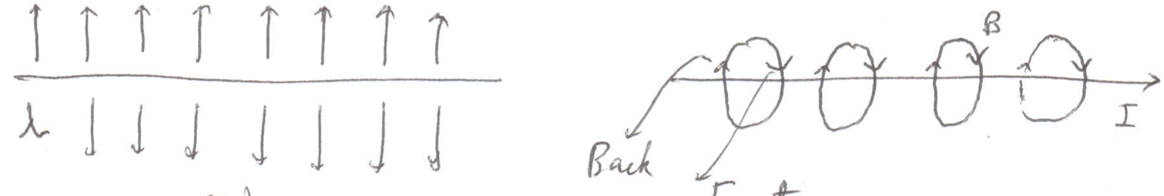
Ch 30 Sources of the Magnetic Field
or Calculation of the Magnetic Field

Electric field: $d\vec{E} = k \frac{dq}{r^2} \hat{r}$ inverse-square law or Coulomb's law

Magnetic field: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$ inverse-square law or Biot-Savart law

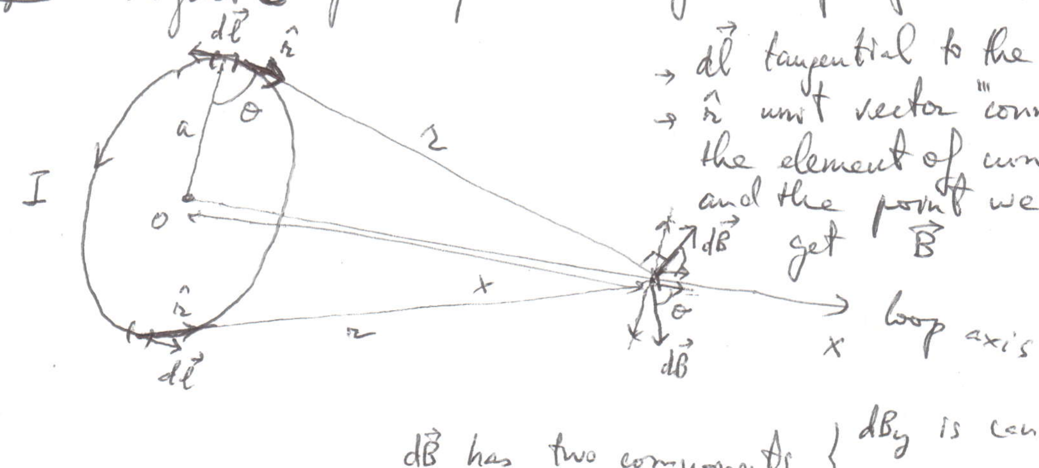
$\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$ permeability in vacuum

(magnetic field due to a line of current is wrapping around the current ⇒ hence we need the cross-product)



$E = \frac{2k\lambda}{r}$

Example: Magnetic field produced by a loop of current:



→ $d\vec{l}$ tangential to the loop

→ $\hat{r}$ unit vector "connecting" the element of current and the point we want to get $\vec{B}$

$d\vec{B}$ has two components $\begin{cases} dB_y \text{ is cancelled} \\ dB_x \text{ is doubled} \end{cases}$

$$dB = 2\,dB_x = 2\,dB\cos\theta = 2\,dB\,\frac{a}{r} = \frac{2a}{r}\,\frac{\mu_0}{4\pi}\,\frac{I\,dl}{r^2} = 2\,\frac{\mu_0}{4\pi}\,\frac{a\,I\,dl}{r^3}$$

$\vec{dl} \perp \hat{r} \Rightarrow dl \times \hat{r} = dl \cdot 1 \cdot \sin 90 = dl$

$$\boxed{B(x) = \int_{\substack{\text{half}\\ \text{circle}}} dB = 2\,\frac{\mu_0}{4\pi}\,\frac{a I}{r^3} \underbrace{\int_{\substack{\text{half}\\ \text{circle}}} dl}_{\pi a} = \frac{\mu_0}{2}\,\frac{I a^2}{r^3} = \frac{\mu_0}{2}\,\frac{I a^2}{(x^2+a^2)^{3/2}}}$$

Magnetic field at a point x along the axis of a loop of current I of radius a

→ B → unit in S.I. : T. for Tesla

→ If point x is very far from the loop : $x \gg a$

$$B(x) \simeq \frac{\mu_0}{2}\,\frac{I a^2}{x^3} \quad \text{inverse - cube law}$$

($E = \dfrac{kqd}{x^3}$: inverse cube-law : far away from the dipole)

∼ the loop of current is the analog of electric dipole !

Electric field calculation :
- → vector superposition
- → Gauss' Law $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{enclosed}}{\varepsilon_0}$
- → Electric potential V : $\vec{E} = -\vec{\nabla} V$

Magnetic field calculation
- → vector superposition
- → Ampere's Law : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$
- → Vector potential $\vec{A}$

→ Calculation of $\vec{B}$ due to a long straight current using Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$.

Gauss law: $\underbrace{\oint \vec{E} \cdot d\vec{A}}$ $= \dfrac{q_{enclosed}}{\varepsilon_0}$

Find the Gaussian surface such that $\vec{E}$ is constant on that surface

$$E \cdot A = \frac{q_{enclose}}{\varepsilon_0} \Rightarrow E = \frac{q_{enclosed}}{\varepsilon_0 A}$$

area of Gaussian surface

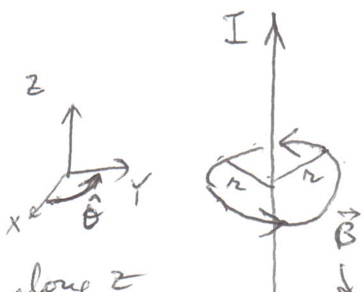
Ampere's law: $\underbrace{\oint_{loop} \vec{B} \cdot d\vec{l}}$ $= \mu_0 I_{enclosed}$

Find the Amperian loop such that $\vec{B}$ is constant along that loop:

$$B \cdot L = \mu_0 I_{enclosed} \Rightarrow B = \frac{\mu_0 I_{enclosed}}{L}$$

length of Amperian loop

Application 1) for a long straight current I



I along z
$\vec{B}$ circling on XY plane

$$B \cdot L = \mu_0 I_{enclosed} = \mu_0 I$$

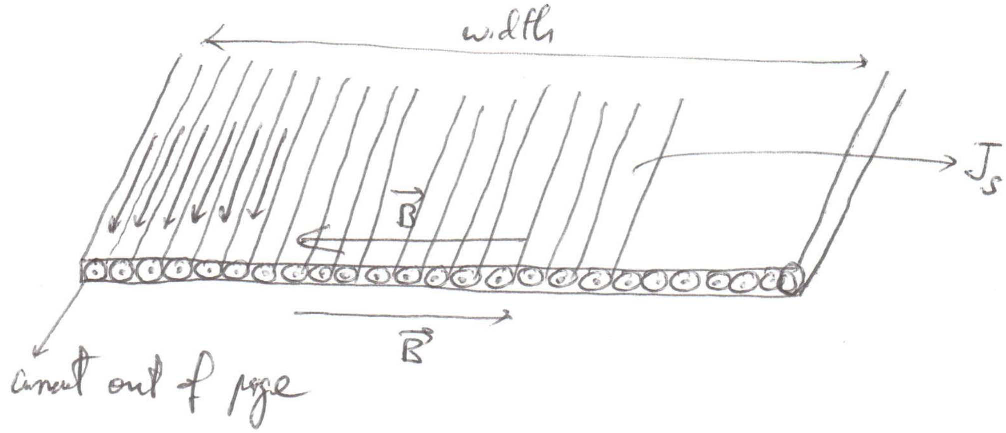
$$\rightarrow B = \frac{\mu_0 I}{2\pi r}$$

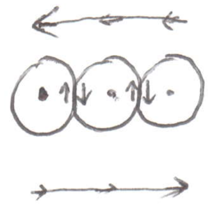
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$$

By symmetry is the same along this loop centered at the current

→ Amperian loop of radius r

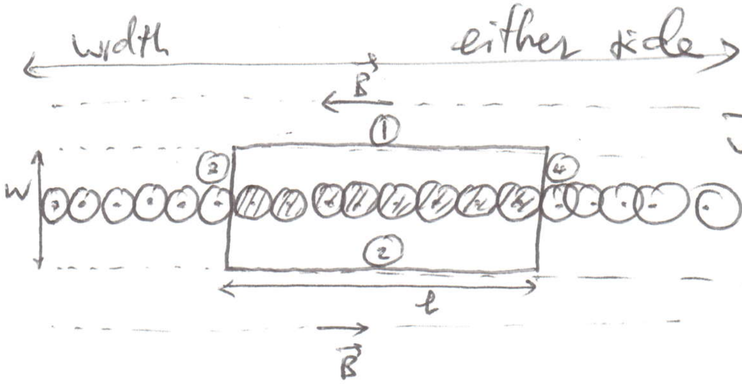
2) for a sheet of current with current per unit width J_s





$\vec{B}$ parallel to the plane, and constant at a fixed separation from the plane (∞ plane: same amount of current flow to either side)

J_s = current per unit width



Front view

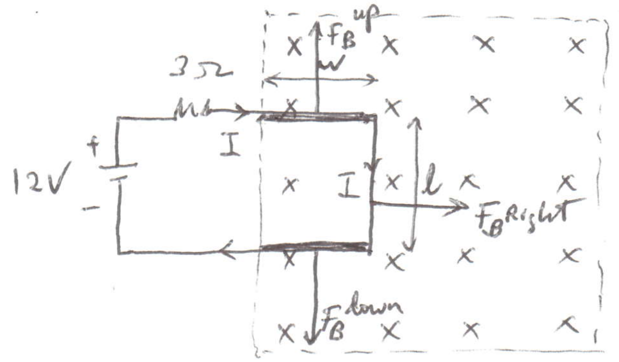
$$\oint \vec{B}\cdot d\vec{l} = \underset{1}{B\cdot l} + \underset{2}{Bl} + \underset{3}{0w} + \underset{4}{0w}$$

$$= 2B\cancel{l} \qquad = \mu_0 \underbrace{I_{enclosed}}_{J_s \cancel{l}}$$

$$\rightarrow \boxed{B = \frac{\mu_0 J_s}{2}}$$

$$\left(E = \frac{\sigma}{2\varepsilon_0} \right)$$

29.36



uniform & into the page

$B = 38\,mT$

$l = 0.1\,m$

Y, x

Magnetic force on a charge inside a magnetic field:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Magnetic force on a current I of length l by a magnetic field $\vec{B}$:

$$d\vec{F}_B = dq \frac{d\vec{l}}{dt} \times \vec{B} \rightarrow \boxed{F_B = I l B}$$

$dq \frac{d\vec{l}}{dt}$ = I

$$\vec{F}_B = I \int (d\vec{l} \times \vec{B})$$

$\boxed{d\vec{l} \perp \vec{B} \text{ and } \vec{B} \text{ const}}$

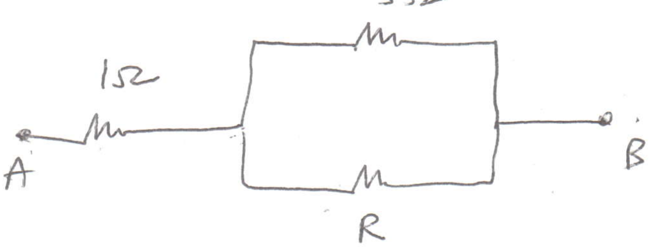
$(0 \text{ if } d\vec{l} \parallel \vec{B})$

$$F_B^{up} + F_B^{down} = 0$$

$$F_{Bnet} = F_B^{Right} = I l B = \frac{12V}{3\Omega}\, 0.1\,m\ \ 38\,mT = 15.2\,mN$$

$$F_{Bnet} = 15.2\,mN\ \hat{i}$$

28.60



=

$$R = 1 + \frac{3R}{\underbrace{3+R}}$$

$$R - 1 = \frac{3R}{3+R} \rightarrow (R-1)(3+R) = 3R$$

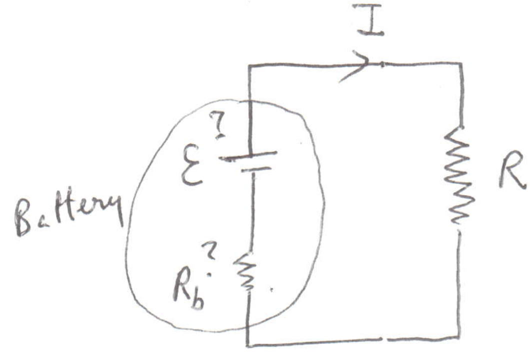
$$\cancel{3R} - R + R^2 - 3 = \cancel{3R}$$

$$R^2 - R - 3 = 0$$

$$R = \frac{1 \pm \sqrt{1+12}}{2}$$

$$\boxed{R = \frac{1+\sqrt{13}}{2} = 2.3\,\Omega}$$

28.28



R	I
50Ω	26 mA
22Ω	43 mA

$$I = \frac{\mathcal{E}}{R + R_b} \rightarrow \begin{cases} 26\,mA = \dfrac{\mathcal{E}}{50 + R_b} & (1) \\ 43\,mA = \dfrac{\mathcal{E}}{22 + R_b} & (2) \end{cases}$$

$$\frac{26}{43} = \frac{22 + R_b}{50 + R_b}$$

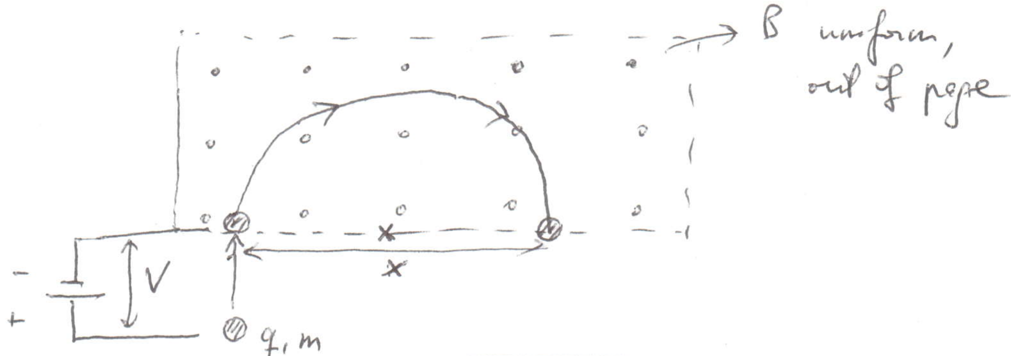
$$\frac{26}{43} 50 + \frac{26}{43} R_b = \frac{22}{\cancel{50}} + \frac{1}{\cancel{50}} R_b$$

$$\left(\frac{26}{43} - \frac{1}{\cancel{50}}\right) R_b = \frac{22}{\cancel{50}} - \frac{26}{43} 50$$

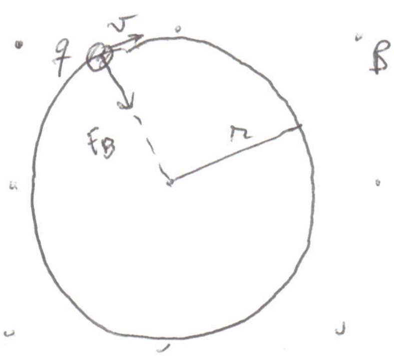
$$R_b = 20.8\ \Omega$$

$$(1) \rightarrow \mathcal{E} = 26\,mA\,(50 + 20.8)\,\Omega = 1840\,mV$$

29.27 **Mass spectrometer:**



$$\boxed{x = \frac{2}{B}\sqrt{\frac{2V}{q/m}}}$$



$$\underbrace{F_B = q\cancel{v}B = m\frac{v^{\cancel{2}}}{r}} \to \boxed{r = \frac{mv}{qB}}$$

$$x = 2r = \frac{2mv}{qB}$$

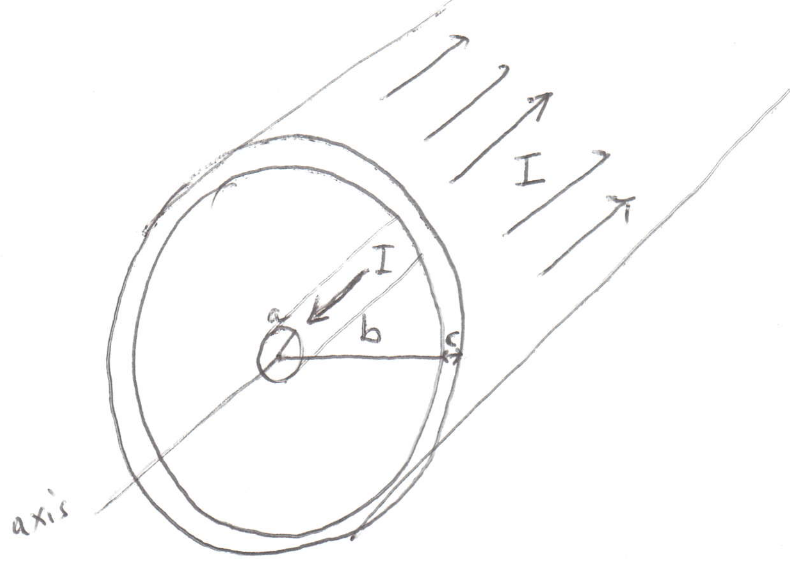
The B (magnetic) region does not change v → initial speed:

Electric potential energy supplied to charge q by V is $\Delta U = qV = \frac{1}{2}mv^2 \to v = \sqrt{\frac{2qV}{m}}$

$$x = 2\frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{2}{B}\sqrt{\frac{2V}{\frac{m}{q}}\left(\frac{m}{q}\right)^2} = \frac{2}{B}\sqrt{2V\frac{m}{q}}$$

$$= \frac{2}{B}\sqrt{\frac{2V}{\frac{q}{m}}}$$

30.40



$$B(r) = ? \quad \begin{cases} r < a \\ a < r < b \\ r > b+c \end{cases}$$

Application of Ampere's Law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$.

→ Find the Amperian loop such that $\vec{B}$ is constant:

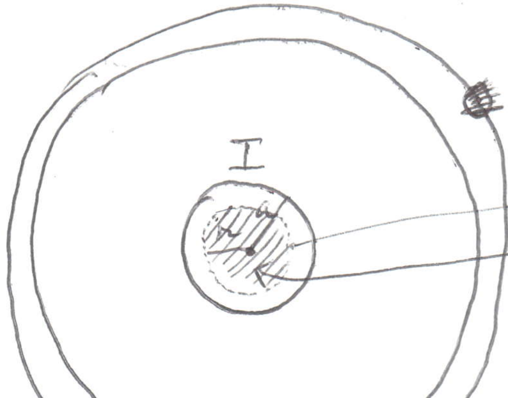
(then $BL = \mu_0 I_{enclosed}$

$B = \dfrac{\mu_0 I_{enclosed}}{L}$)

Length of Amperian loop

→ Due to symmetry: B will be wrapping around the loop line of current, with constant value at a fixed r (distance from the axis)

Front view:

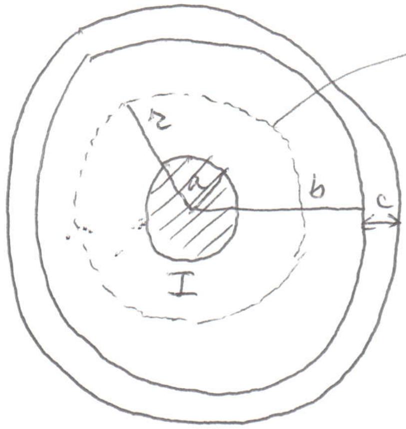


→ Amperian Loop : B constant.

$$B = \frac{\mu_0 I_{enclosed}}{2\pi r}$$

$$\boxed{B = \frac{\mu_0}{2\pi \cancel{r}} I \frac{\cancel{\pi} r^{\cancel{2}}}{\cancel{\pi} a^2} = \frac{\mu_0 I}{2\pi a^2} r \quad r < a}$$

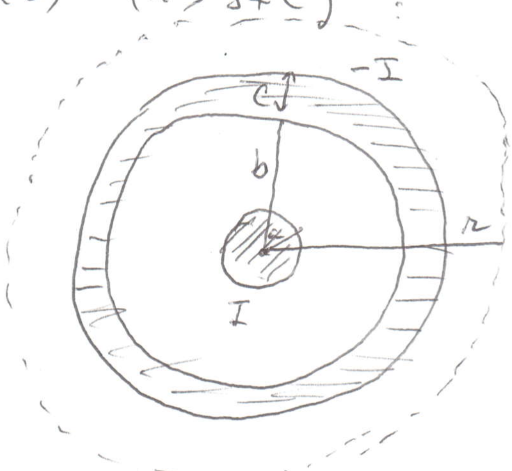
For $B(r)$ $(a<r<b)$ we place our Amperian loop accordingly:



$$B = \frac{\mu_0 I_{enclosed}}{L}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r} \quad (a<r<b)}$$

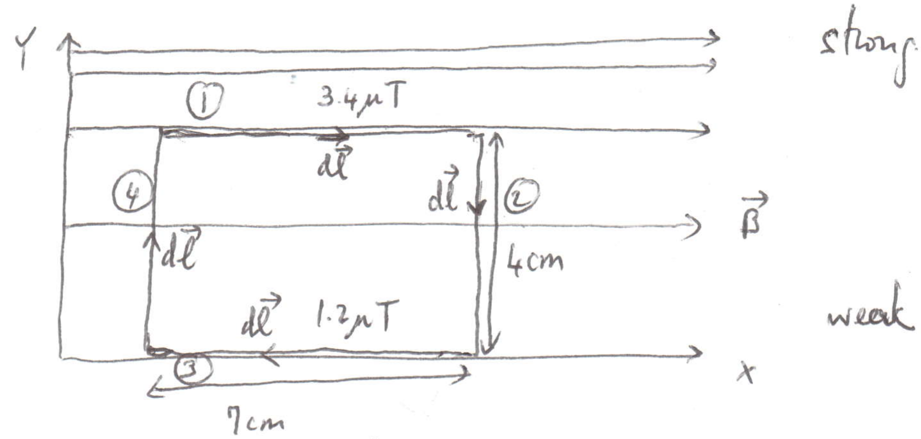
For $B(r)$ $(r > b+c)$



$$\left[B = \frac{\mu_0 \overbrace{I_{enclosed}}^{I-I=0}}{2\pi r} = 0 \right]$$

$r > b+c$

30.31



How much current flows through the area encircled by the loop?

$\downarrow$

$I_{enclosed}$

$\downarrow$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

We need to do $\underbrace{\oint \vec{B} \cdot d\vec{l}}$ for a rectangular loop.

$$= \int_{1} \vec{B} \cdot d\vec{l} + \int_{2} \vec{B} \cdot d\vec{l} + \int_{3} \vec{B} \cdot d\vec{l} + \int_{4} \vec{B} \cdot d\vec{l}$$

$B_{1} \times 0.07m$ $B_{3} \times 0.07m$

$\downarrow$ B is NOT constant.

but $\underbrace{\vec{B} \cdot d\vec{l}}_{B\,dl \cos\theta} = 0$ b/c $\vec{B} \perp d\vec{l}$

$B\,dl \cos\theta = 0$, $\theta = 90°$

$$\rightarrow I_{enclosed} = \frac{B_1 \times 0.07 + B_{3} \times 0.07}{\mu_0} = \frac{(3.4\times10^{-6} + 1.2\times10^{-6})0.07}{4\pi\times10^{-7}}$$

$$= 0.12\,A$$