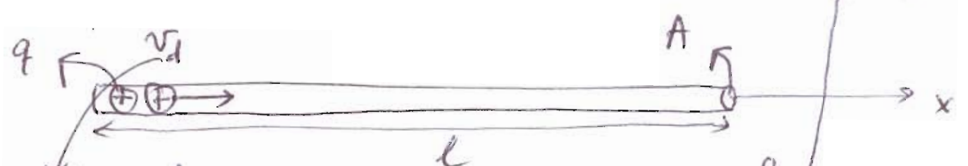
Ch. 27 Electric Current:

$$I \equiv \frac{\Delta q}{\Delta t} \quad ; \quad I = \frac{dq}{dt} \quad \left(\frac{C}{s} \equiv A \text{ for Amp.} \right)$$

Microscopic expression: $\quad I = \dfrac{\Delta q}{\Delta t} = \dfrac{nAlq}{\frac{l}{v_d}} = nqAv_d$



drift velocity or average velocity along x

$$\Delta t = \frac{l}{v_d}$$

n = # of charge per unit volume

Copper wire ; $A = 1\,mm^2$; $I = 5A$

each atom of copper contribute $1.3e$ of charge

$\rightarrow v_d$? $\quad v_d = \dfrac{I}{nqA}$

need → # atoms of copper per unit volume

Mass density: $\rho_{copper} = 8920\ kg/m^3$: if we divide this by the mass of one atom of copper → we get n

→ Cu : 63.55 a.u $= 63.55 \times 1.66 \times 10^{-27}$ kg

$$n = 8920 \frac{kg}{m^3} \; \frac{1 \text{ atom Cu}}{63.55 \times 1.66 \times 10^{-27}\ kg} = 8.5 \times 10^{28} \frac{\text{atoms Cu}}{m^3}$$

$$\rightarrow v_d = \frac{5A}{8.5 \times 10^{28} \times 1.3 \times 1.6 \times 10^{-19} \times 10^{-6}} \frac{m}{s} = 0.283 \frac{mm}{s}$$

$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$ only for gas!

$v_d = 283 \times 10^{-6} \frac{m}{s} \longrightarrow T = 6.1 \times 10^{-12}\ °K$!

Current density : $J = \frac{I}{Area} = \frac{nq\, Area\, v_d}{Area} = nqv_d \left(\frac{A}{m^2}\right)$

Normal materials : $\boxed{\vec{J} = \sigma \vec{E}}$

"sigma" conductivity

(this agrees with what we said about conductors in equilibrium : $E = 0 \Rightarrow J = 0$ or $I = 0$)

$\rho \equiv \frac{1}{\sigma}$: resistivity

"rho"

Conductivity : $[\sigma] = \frac{[J]}{[E]} = \frac{\frac{A}{m^2}}{\frac{N}{C}} = \frac{A \cdot \cancel{m}}{\underbrace{\frac{N}{C} \cdot m}_{\text{Volt or } V} \cdot m} = \frac{1}{\Omega m}$

$\boxed{\text{Ohm's law} : I = \frac{V}{R}}$ or $R = \frac{V}{I}$ $\left(\frac{V}{A} \equiv \Omega\right)$

resistance "omega" Ohm

Resistivity : $[\rho] = \Omega m$

$$\underbrace{J = \sigma E}_{\text{Microscopic Ohm's Law}} \quad \rightarrow \quad J = \frac{1}{\rho} E = \frac{1}{\rho}\frac{dV}{dx}$$

$$\downarrow$$

$$\frac{I}{\text{Area}} = \frac{1}{\rho}\frac{dV}{dx}$$

$$I = \frac{1}{\rho}\underbrace{\frac{dV}{dx}\text{Area}}_{\hookrightarrow V.\, l} = \frac{V}{\underbrace{\left(\frac{\rho}{l}\right)}_{R}}$$

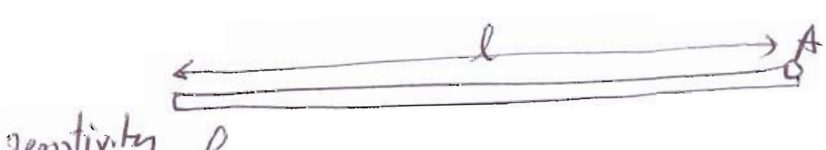
$$\boxed{I = \frac{V}{R}}$$

→ Conductors: high conductivity

→ semiconductors: become conductors when doped with impurities

→ superconductors: will have zero resistivity at certain T

<u>Ohm's law</u>: $I = \frac{V}{R}$ ← resistance (Ω)



$$\boxed{R = \rho \frac{l}{A}}$$

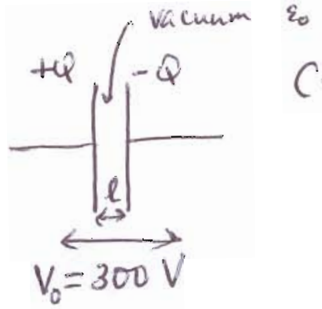
ρ ↓ material-specific

<u>Energy dissipation</u>: lost energy when charged particles are moving thru the wire, in the form of heat loss:

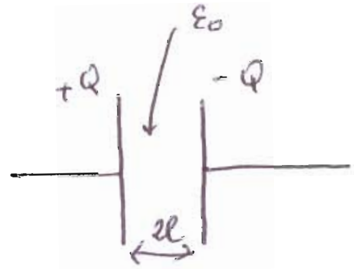
$$P = IV \begin{cases} = \dfrac{V}{R} V = \dfrac{V^2}{R} \\ = I\,IR = \boxed{I^2 R} \end{cases}$$

power or heat loss per unit time

26.73



$C = 20\mu F$



$$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2}(20\times10^{-6})\, 300^2 = 0.9\text{J}$$

Total energy stored = U_0

$$U = \frac{1}{2} C V^2 = \frac{1}{2}\frac{C_0}{2}(2V_0)^2 = \frac{1}{2} C_0 V_0^2\, 2 = 2U_0$$

→ same charges since it is disconnected from the charging battery → $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$ (same)

→ $C = \frac{Q}{V}$, $V \to E.d = E.2\ell = 2V_0$

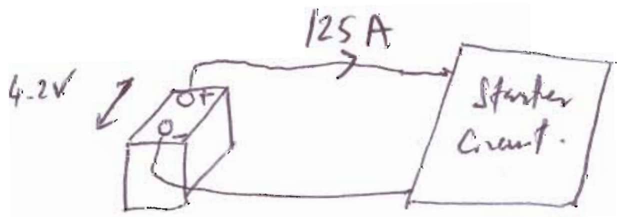
$$= \frac{Q}{2V_0} = \frac{C_0}{2}$$

Total energy stored is $2U_0$

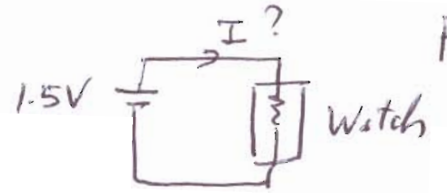
Where did the extra U_0 come from? → by pulling the plates apart.

27.41

$$\left.\begin{array}{l} V = 4.2\text{V} \\ I = 125\text{ A} \end{array}\right\} R \overset{?}{=} \frac{V}{I} = \frac{4.2\text{ V}}{125\text{ A}} = 0.0336\ \Omega$$



27.45



$P = 240\,\mu W$

$$P \overset{(0)}{=} IV = \begin{cases} = \dfrac{V^2}{R} & (1) \\ = I^2 R & (2) \end{cases}$$

$$I = \frac{P}{V} = \frac{240 \times 10^{-6}}{1.5}\,A = 160 \times 10^{-6}\,A = 160\,\mu A$$

27.58

Potassium (K): $J = 470 \frac{kA}{m^2}$

$v_d = 0.2 \; mm/s$

Mass density $\rho_K = 860 \frac{kg}{m^3}$

$N_e \equiv$ # of free electrons contributed by each atom of potassium?

$$I = nqAv_d \Rightarrow J = nqv_d = n N_e e v_d$$

$e = 1.6 \times 10^{-19}\,C$

$$N_e = \frac{J}{n e v_d} = \frac{470 \times 10^3}{1.32 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.2 \times 10^{-3}} = 1.1$$

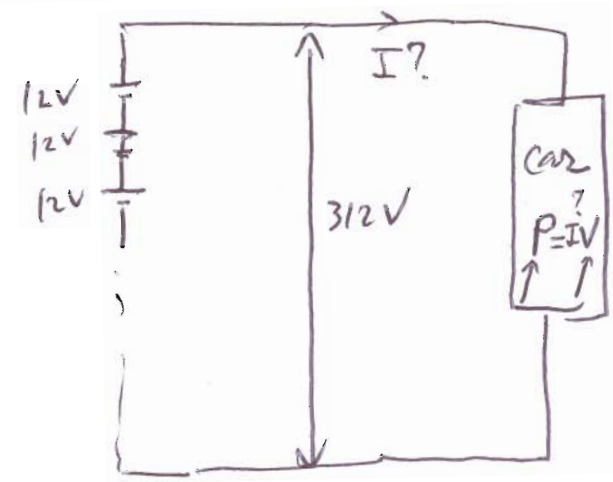
We need to find n or # of atom of potassium per unit volume from the mass density for potassium:

$$n = \underbrace{\rho_K = 860 \frac{kg}{m^3}}_{\rho_K} \cdot \frac{1 \text{ atom K}}{39.10 \times 1.66 \times 10^{-27}\,kg} = 1.32 \times 10^{28} \frac{\text{atoms K}}{m^3}$$

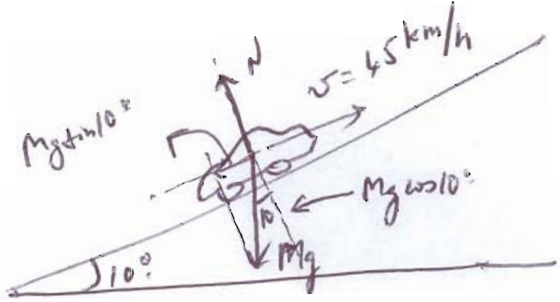
27.67 GM's EV1 (electric vehicle)

$M = 1500\,kg$

26 12-V batteries in series.



85% of E. energy → Mech energy



No friction, no air resistance.

We need to overcome $\underbrace{Mg \sin 10°}_{\text{Force}}$ to bring car up the ramp at constant speed.

$$P = F \cdot \frac{d}{t} = F \cdot v = Mg \sin 10° \times v$$

$$= 1500 \times 9.81 \times \sin 10° \times 45 \frac{km}{h} \cdot \frac{1h}{3600\,s} \frac{1000\,m}{1\,km}$$

$$= \cancel{46k\ \ W}$$

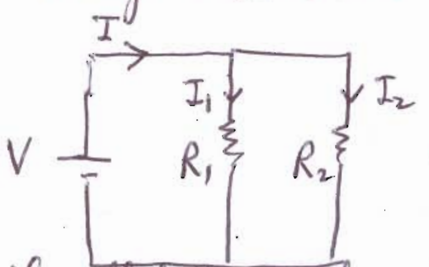
$31940\,W$ → Mechanical power needed.

$$\rightarrow I = \frac{P_{electrical}}{V} = \frac{\left(\frac{31940}{0.85}\right)}{312} = 120\,A$$

Series and parallell combination of resistors

Parallell

Voltage across each is the same: V



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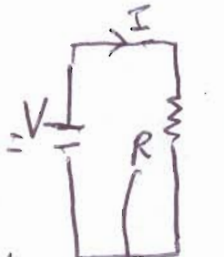


equiv. resistance

$I = \frac{V}{R}$

Also called "current division"

$I = I_1 + I_2$

(If $R_1 > R_2$: $I_2 > I_1$)

Ohm's law : relating

$\underbrace{V, I_1, R_1}$ $\qquad$ $\underbrace{V, I_2, R_2}$

$I_1 = \frac{V}{R_1}$ $\qquad$ $I_2 = \frac{V}{R_2}$

$$I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{V}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}$$

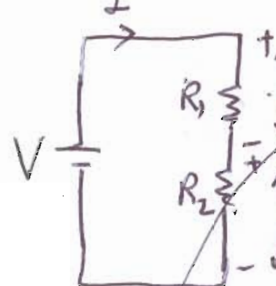
$$\Rightarrow R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$= \left(\frac{R_2 + R_1}{R_1 R_2}\right)^{-1}$$

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Series

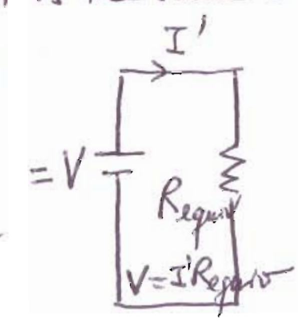
Current thru each is the same I'



$V_1 = I' R_1$

$V_2 = I' R_2$

$=$



$$\left.\begin{array}{l} V = V_1 + V_2 = I'(R_1 + R_2) \\ V = I' R_{equiv} \end{array}\right\}$$

$$\boxed{R_{equiv} = R_1 + R_2}$$

Consequence: $I' = \frac{V}{R_{equiv}} = \frac{V}{R_1 + R_2}$

$$\left\{\begin{array}{l} V_1 = \frac{V}{R_1 + R_2} R_1 = V \frac{R_1}{R_1 + R_2} \\ V_2 = V \frac{R_2}{R_1 + R_2} \end{array}\right.$$

Also called "voltage division"

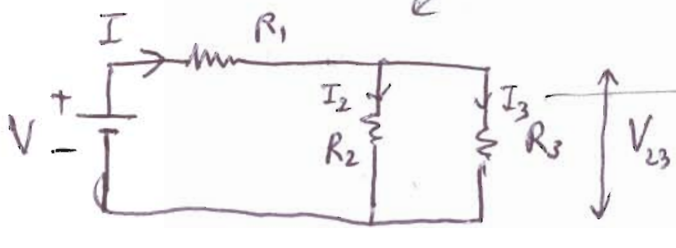
Power consumption: $R_1 = R_2 = R$

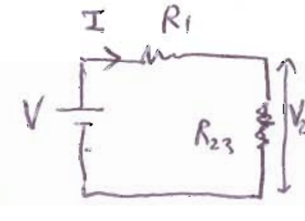
Parallelle	Series
$R_{equiv} = \frac{R}{2}$	$R_{equiv} = 2R$
$P = I_1 V_1 = \frac{I}{2} V = \frac{\frac{V}{R/2}}{2} V = \frac{4V^2}{R}$	$P' = I' V_1 = I' \frac{V}{2} = \frac{V}{2R} \frac{V}{2} = \frac{V^2}{4R}$

Ch. 28 Electric Circuits

Linear (relation b/w V & I)

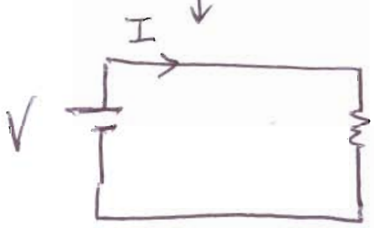
2 types { - with resistors only : { 1) reduce to with series & parallel combinations; 2) can reduce to using loop or node analysis
- resistors and capacitors





Given: V, R_1, R_2, R_3

Solve for this circuits = providing I ①, I_2 ③, I_3 ④, V_{23} ②



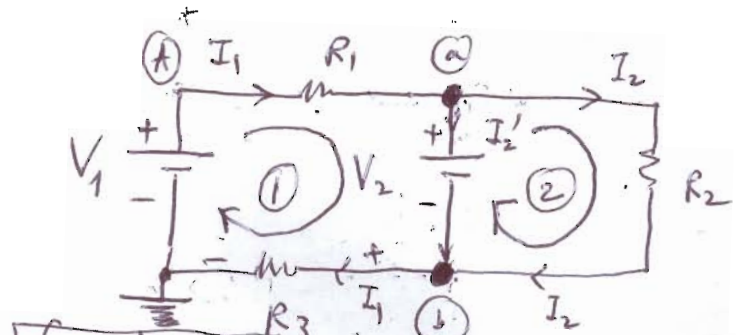
$$R_{Total} = R_1 + R_{23} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

① $$I = \frac{V}{R_{Total}} = \frac{V}{R_1 + \frac{R_2 R_3}{R_2+R_3}} \quad \checkmark$$

③ $$I_2 = \frac{V_{23}}{R_2} = \frac{I \frac{\cancel{R_2} R_3}{R_2+R_3}}{\cancel{R_2}} = I \frac{R_3}{R_2+R_3} \quad \checkmark$$

② $$\left(V_{23} = I \frac{R_2 R_3}{R_2 + R_3} \right)$$

④ $$I_3 = I - I_2 \quad \checkmark$$



$\rightarrow$

Ground (ref. point for potential)

Kirchoff's Laws.

Loop

Total voltage difference across elements in a closed loop = 0

→ Signs: assume a direction for current thru closed loops

1) Current thru a battery from − to + → positive sign
2) Current thru battery from + to − → negative sign.
3) Negative across any resistor (voltage drop)

Loop equations.

① $+V_1 - I_1R_1 - V_2 - I_1R_3 = 0$

② $+V_2 - I_2R_2 = 0$

$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3} \qquad I_2 = \frac{V_2}{R_2}}$$

Nodal

Total current at any node = 0

→ Signs:

1) Current into node → positive sign
2) Current leaving node → negative sign

ⓐ $\boxed{+I_1 - I_2' - I_2 = 0}$

ⓑ $I_2' + I_2 - I_1 = 0$ (repeated)

$$\frac{V_A - V_a}{R_1} - I_2' - \frac{V_a - V_b}{R_2} = 0$$

or

$$\frac{V_1 - (V_2 + I_1R_3)}{R_1} - I_2' - \frac{V_2}{R_2} = 0$$

$$\boxed{I_2' = \frac{V_1 - V_2}{R_1} - \frac{V_2}{R_2}}$$

Find I_1 & I_2

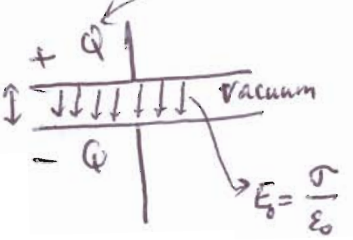
$$I_1 = \frac{V_1 - (V_2 + I_1R_3)}{R_1} \rightarrow I_1R_1 + I_1R_3 = V_1 - V_2$$

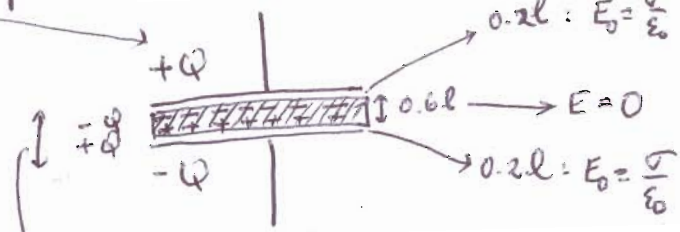
$$\boxed{I_1 = \frac{V_1 - V_2}{R_1 + R_3}}$$

isolated capacitor

26.47

$V_0 = E_0 l$





$C_0 = \frac{Q}{V_0}$

$U_0 = \frac{1}{2} C_0 V_0^2$

$V = E_0 \times 0.2l + 0 \times 0.6l + E_0 \times 0.2l$

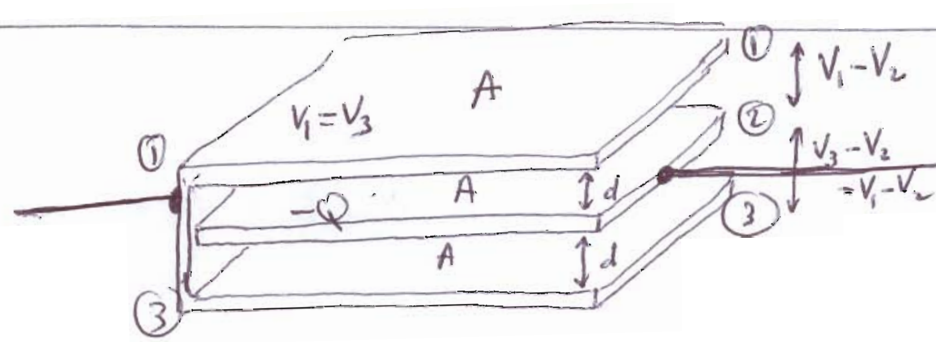
$= E_0 \times 0.4 l = 0.4 V_0$

$\rightarrow C = \frac{Q}{V} = \frac{Q}{0.4 V_0} = 2.5\, C_0$

$\rightarrow U = \frac{1}{2} C V^2 = \frac{1}{2} 2.5 C_0 (0.4 V_0)^2$

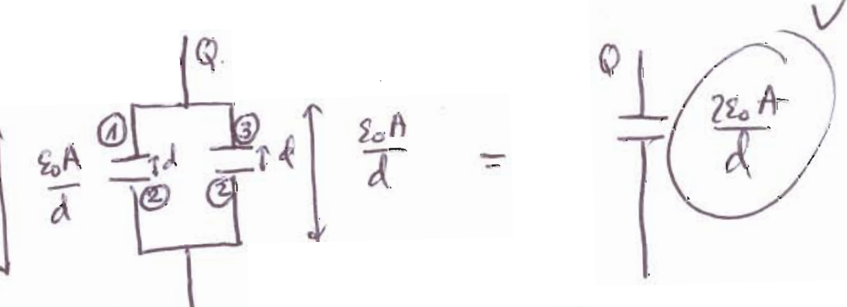
$$\boxed{U = \underbrace{\frac{1}{2} C_0 V_0^2}_{U_0} \, \frac{2.5 \times 0.4^2}{0.4} = 0.4 U_0}$$

26.37



Prove: $C = \frac{2\varepsilon_0 A}{d}$

<u>Neglect edge effects:</u>



→ Same potential across ~~P~~ 1 & 2 as 3 & 2 ✓

→ $Q_1 + Q_3 = Q$? | Assume ② is $-Q$ ✓

$Q_2 = -Q$

27.22

12-gauge copper wire → 20A

d = 0.21cm

a) $J = \frac{I}{\text{Area}} = \frac{20A}{\pi\left(\frac{0.0021}{2}\right)^2} = 5.77\times10^{6} \frac{A}{m^2}$

b) $E = \frac{J}{\sigma} = \rho J = 1.68\times10^{-8} \times 5.77\times10^{6} \frac{N}{C} = 0.097 \frac{N}{C}$ or $0.097 \frac{V}{m}$

(Microscopic Ohm Law: $J = \sigma E$)

sigma or conductivity $\left(\frac{1}{\Omega m}\right)$

Ohm

rho or resistivity $\frac{1}{\sigma}$

(Ωm)

Ohm

Table 27.1 :

$\rho_{copper} = 1.68\times10^{-8}\ \Omega m$

$\left[V = \frac{N}{C} m \rightarrow \frac{N}{C} = \frac{V}{m} \right]$