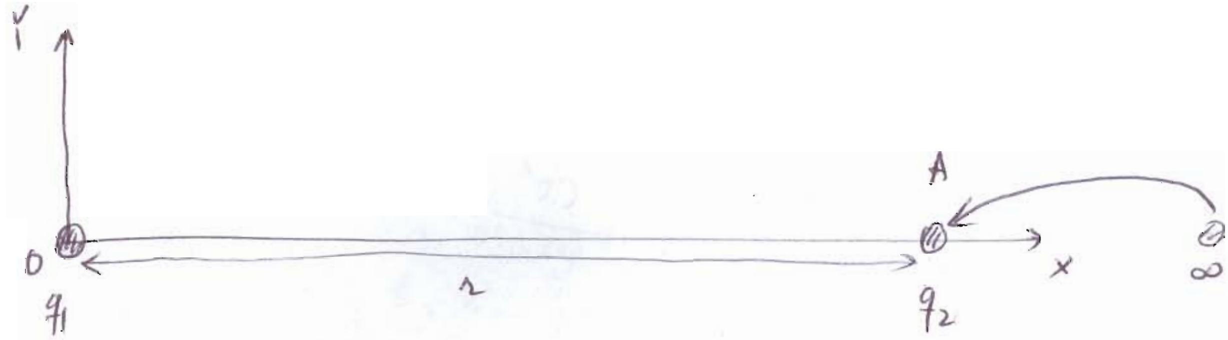
# Ch. 26: Electrostatic Energy and Capacitors

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{\underbrace{q'}_{\text{Test charge}}}$$

U: electric potential energy
V: electric potential

$$\Delta U_{AB} = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}$$




→ $q_2$ is in the field created by $q_1$: [electric potential due to $q_1$ at point A is $\underbrace{V = \frac{kq_1}{r}}_{\text{Ref. point at } \infty} = \Delta V_{\infty A} = kq_1\left(\frac{1}{r} - \frac{1}{\infty}\right)$

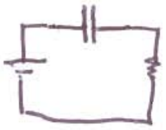
it is also the electric potential difference b/w ∞ & A.] → No ref. about $q_2$

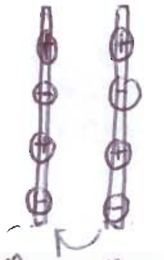
→ What is the $\Delta U_{\infty A}$ if we move charge $q_2$ into the picture?

$$\Delta U_{\infty A} = q_2 \, \Delta V_{\infty A} = \frac{k q_1 q_2}{r}$$

⇒ We can store electric potential energy by bringing charges together

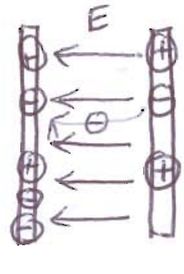
## Electrostatic energy storage devices : capacitors = parallel-plate capacitors





$Q=0 \quad Q=0$

equal amount of charges of each type in each plate.



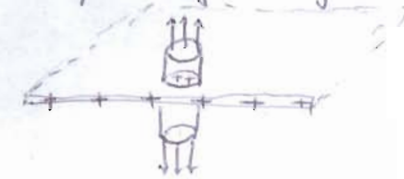

$Q=-e \quad Q=+e$

To store energy we need a field to bring charges against it.

$F = \ominus E$ force on this electron is pointing toward the + plate
→ To bring it to the left plate : I need to do some work = "charging the capacitor"
↓
$\boxed{\text{Total energy stored = superposition of all work required to bring all } e^- \text{ from right plate to left plate.}}$

What is $E'$ ? due to a plate of charge $Q$ and area $A$

↳ Gauss Law :



$$E'A = \frac{q_{enclosed}}{\varepsilon_0}$$

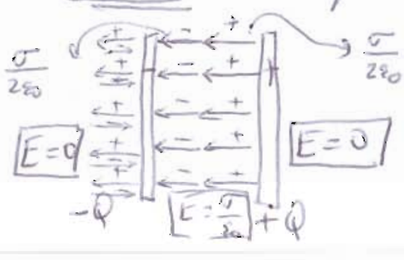
$$E' = \frac{q_{enclosed}}{\varepsilon_0 A}$$

→ contained in one cross section of cylinder $\pi r^2$

$A = 2\pi r^2$ → top & bottom cross sections

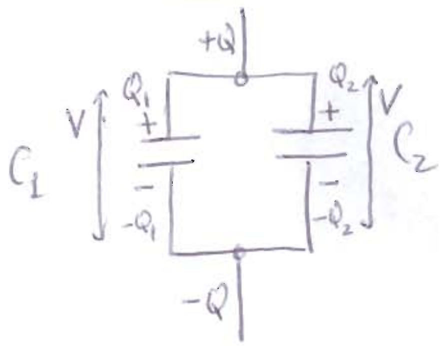
$$\boxed{E' = \frac{\sigma \cancel{\pi r^2}}{\varepsilon_0 \, 2\cancel{\pi r^2}} = \frac{\sigma}{2\varepsilon_0}}$$

What is $E$ b/w 2 plates of charges $Q$ & $-Q$




$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

## Parallel




computing

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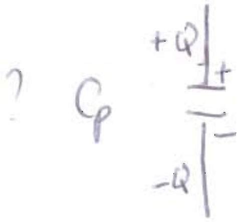
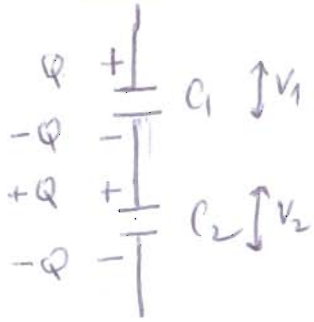



$$V = \frac{Q}{C} \quad \begin{cases} V = \frac{Q_1}{C_1} \to \frac{Q_1}{V} = C_1 \\ V = \frac{Q_2}{C_2} \to \frac{Q_2}{V} = C_2 \\ V = \frac{Q}{C_p} = \frac{Q_1 + Q_2}{C_p} \end{cases}$$

$$\boxed{C_p = \frac{Q_1 + Q_2}{V} = C_1 + C_2}$$

## Series

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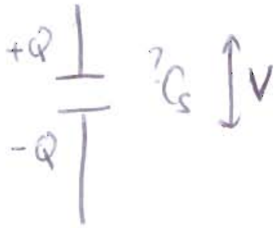


$$\left.\begin{aligned} V_1 &= \frac{Q}{C_1} \\ V_2 &= \frac{Q}{C_2} \end{aligned}\right\} \quad C_s = \frac{Q}{V} \quad \text{or} \quad \frac{1}{C_s} = \frac{V}{Q}$$

$$V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{V}{Q} = \boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{C_s = \frac{C_1 C_2}{C_1 + C_2}}$$

$$C = \frac{Q}{V}$$

→ How to increase the capacitance of a parallel plate capacitor?

1) Inserting a conducting slab b/w plates
(reduce $V \rightarrow \uparrow C$ )
E was still the same (same charge Q)

2) $V = E \cdot \ell$
↓
separation.

Now we can also reduce V by reducing E for same Q:
can do this by inserting a dielectrics b/w the plates:

$$\varepsilon_0 \rightarrow \varepsilon = K\varepsilon_0$$

↳ dielectric coefficient kappa, $>1$

b/w the two plates $\left\{ E_0 = \frac{\sigma}{\varepsilon_0} \right.$ vacuum $\longrightarrow$ $E = \frac{\sigma}{K\varepsilon_0} = \frac{E_0}{K} < E_0$ dielectrics

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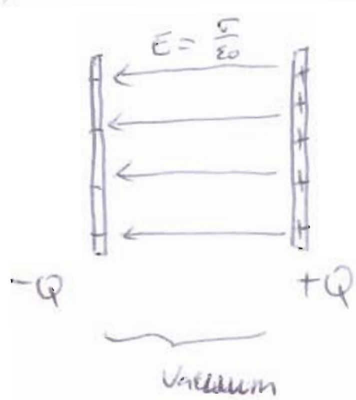
Microscopically why E is reduced in the presence of a dielectric?





polarized,
dielectrics can carry an electric field within, unlike conductors.
↳ of reversed direction ⇒ total field b/w plates is <u>reduced</u>

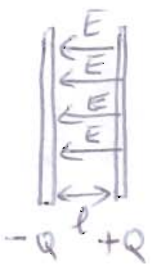
$$dU = -dW = dq\,V = dq\,E.\underbrace{\ell}_{\text{separation b/w plates}} = dq\,\underbrace{\frac{\sigma}{\varepsilon_0}}_{\text{electric field b/w plates}}\ell = dq\,\frac{q}{\varepsilon_0 A}\,\ell$$

$$V \equiv -\int \vec{E}\cdot d\vec{l}$$

Total energy stored: $U = \frac{\ell}{\varepsilon_0 A}\underbrace{\int_0^Q q\,dq}_{\frac{1}{2}Q^2} = \frac{1}{2}\underbrace{\varepsilon_0 A\ell}_{\text{vol. b/w plate}}\frac{Q^2}{\varepsilon_0^2 A^2}$ $\rightarrow \frac{\sigma^2}{\varepsilon^2} = E^2$

$$\boxed{\text{Energy stored b/w plates per unit volume} \quad u = \frac{U}{A\ell} = \frac{1}{2}\varepsilon_0 E^2 \quad \left(\frac{J}{m^3}\right) \text{ S.I.}}$$

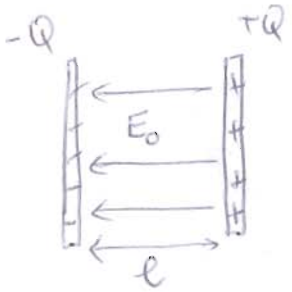
<u>Capacitance</u> $C \equiv \frac{Q}{V}$ $\begin{matrix}\rightarrow \text{charge of one plate} \\ \rightarrow \text{potential b/w plates.}\end{matrix}$

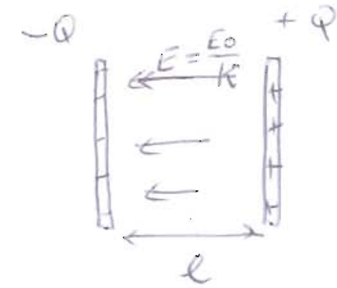



$$\boxed{C = \frac{Q}{E.\ell} = \frac{Q}{\frac{\sigma}{\varepsilon_0}\ell} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0}\ell} = \frac{A\varepsilon_0}{\ell}}$$

Total energy stored b/w parallel plates:

$$\boxed{U = \frac{1}{2}\varepsilon_0 A\ell\,E^2 = \frac{1}{2}\underbrace{\frac{\varepsilon_0 A}{\ell}}_{C}\ell^2E^2 = \frac{1}{2}CV^2}$$



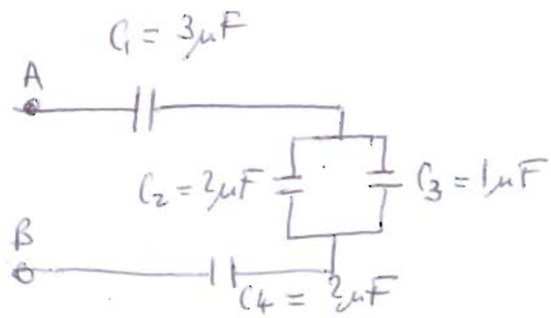

$$C_0 = \frac{Q}{V_0} = \frac{Q}{E_0 \ell}$$

$$C = \frac{Q}{V} = \frac{Q}{\underbrace{\frac{E_0 \cdot \ell}{K}}_{V_0}} = K \frac{Q}{E_0 \ell} = K C_0$$

$$V = \frac{V_0}{K} \longrightarrow$$

$$U = \frac{1}{2} C V^2 = \frac{1}{2}(K C_0)\left(\frac{V_0}{K}\right)^2 = \underbrace{\left[\frac{1}{2} C_0 V_0^2\right]}_{U_0} \frac{1}{K} = \frac{U_0}{K}$$

26.57 |

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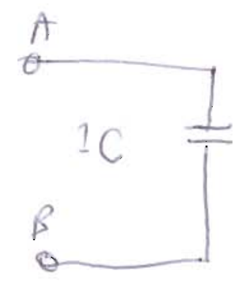



$C = 0.86 \mu F$
Please check it yourself -

Capacitance =

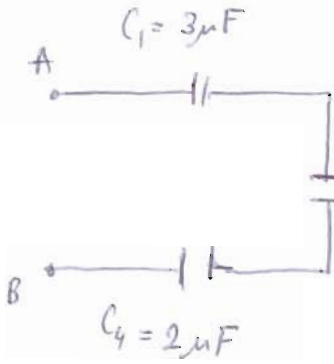
$$C = \frac{Q}{V} \quad \left(\frac{C}{\frac{N}{C} m} = \frac{C^2}{Nm} = \frac{C^2}{J} = \underbrace{F \text{ for Farad}}_{SI.}\right)$$

$\mu F = 10^{-6} F$ or $mF = 10^{-3} F$

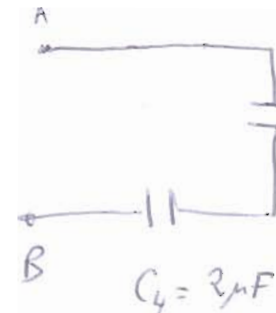
"micro Farady" "mili Farad"

26.47 |

**26.57**

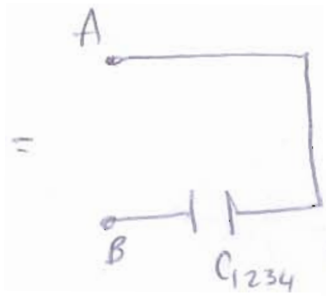



$C_{23} = C_2 + C_3 = 3\mu F$ =



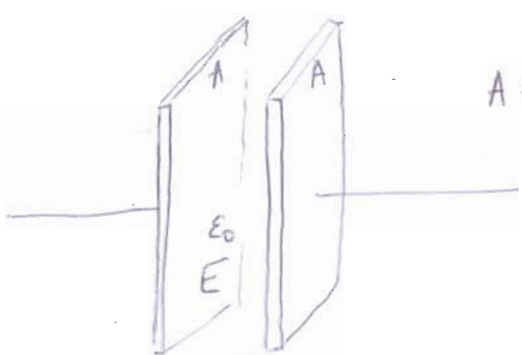

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{3 \cdot 3}{6} = 1.5\mu F$$

=

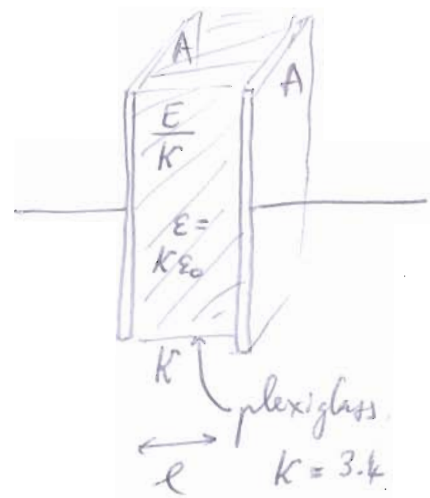



$$C_{1234} = \frac{C_{123} \cdot C_4}{C_{123} + C_4} = \frac{1.5 \times 2}{1.5 + 2} = \frac{3}{3.5} = 0.86\mu F$$

**26.68**




$A = 76\,cm^2$




$$C_0 = \frac{Q}{V_0} = \frac{A\varepsilon_0}{l} = \frac{76\times10^{-4} \times 8.85\times10^{-12}}{1.2\times10^{-3}} = 56\times10^{-12} F = 56\,pF$$ "pico Farad"

$V_0 = 900V$

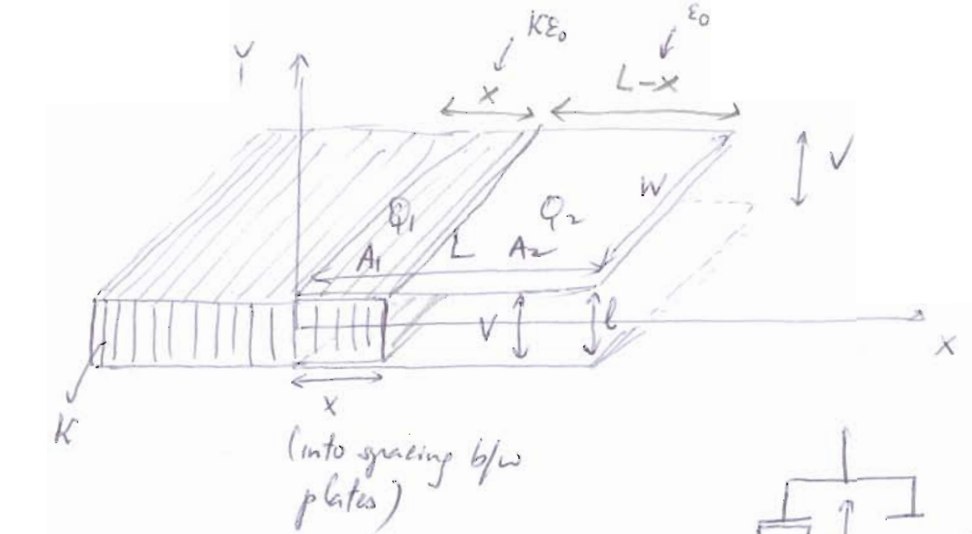
$$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} 56\times10^{-12} \times 900^2 = 22.7\mu J$$

$$C = \frac{A\varepsilon}{l} = K C_0 = 3.4 \times 56pF = 191pF$$

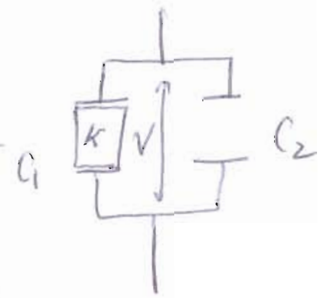
$$V = \frac{V_0}{K} = \frac{900}{3.4} V = 265V$$

$$U = \frac{1}{2K} C_0 U_0^2 = \frac{U_0}{K} = \frac{22.7}{3.4}\mu J = 6.68\mu J$$

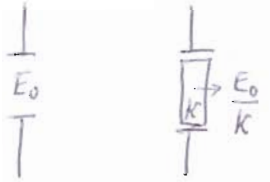
26.81



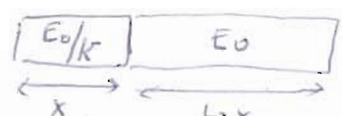

What is $C$?



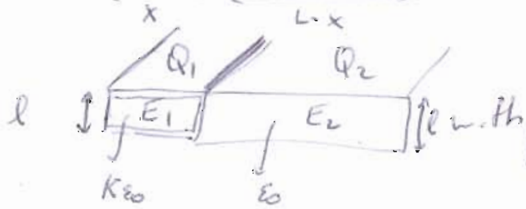

$$\hookrightarrow C_1 + C_2 = \frac{Q_1}{V} + \frac{Q_2}{V}$$




Here we can't have




, but:




with $\boxed{E_1 = E_2}$

$$\left.\begin{aligned} E_1 &= \frac{Q_1}{K\varepsilon_0 \; A_1} = \frac{Q_1}{K\varepsilon_0 \, xw} \\ E_2 &= \frac{Q_2}{\varepsilon_0 \; A_2} = \frac{Q_2}{\varepsilon_0 \, (L-x) w} \end{aligned}\right\} \rightarrow \frac{Q_1}{K\cancel{\varepsilon_0}\, x\cancel{w}} = \frac{Q_2}{\cancel{\varepsilon_0}(L-x)\cancel{w}}$$

$$\boxed{Q_1 = Q_2 \frac{Kx}{L-x}}$$

$$C = C_1 + C_2 = \frac{Q_1 + Q_2}{V} = \frac{Q_2 \frac{Kx}{L-x} + Q_2}{V} = \underbrace{\frac{Q_2}{V}}\left(\frac{xK}{L-x} + 1\right)$$

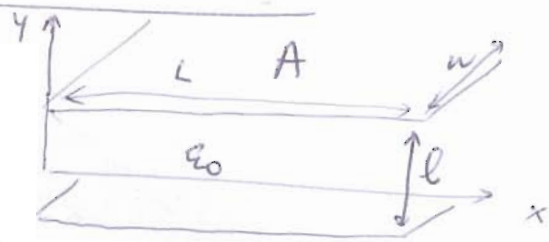
$$\frac{Q_2}{V} = \frac{Q_2}{E_2 \ell} = \frac{\cancel{Q_2}}{\frac{\cancel{Q_2}}{A_2 \varepsilon_0} \ell} = \frac{A_2 \varepsilon_0}{\ell} = \frac{(L-x) w \varepsilon_0}{\ell}$$

$$\rightarrow \boxed{C = \frac{(L-x) w\varepsilon_0}{\ell}\left(\frac{xK}{L-x} + 1\right) = \frac{w\varepsilon_0}{\ell}\left(xK + L - x\right)}$$

If dielectric is inserted halfway into spacing b/w plates: $x=\frac{L}{2}$

$$C_{x=\frac{L}{2}} = \frac{w\varepsilon_0}{\ell}\left(\frac{L}{2}K + \underbrace{L-\frac{L}{2}}_{\frac{L}{2}}\right) = \frac{w\varepsilon_0 L}{2\ell}(K+1)$$

$$C_0 = \frac{A\varepsilon_0}{\ell} = \frac{Lw\varepsilon_0}{\ell}$$




$$\boxed{C = C_0\,\frac{K+1}{2}}$$

$$U_{x=\frac{L}{2}} = \frac{1}{2} C V_0^2$$

Force on slab: $\rightarrow \frac{dU}{dx}$

$$\frac{d}{dx}\left[\frac{1}{2} C(x) V_0^2\right]$$

$$\frac{d}{dx}\left[\frac{1}{2}\frac{w\varepsilon_0}{\ell}(xK+L-x)V_0^2\right]$$

$$\boxed{\text{Force} = \frac{1}{2}\frac{w\varepsilon_0 V_0^2}{\ell}(K-1)}$$

$$V = -\int \vec{E}\cdot d\vec{l}$$

$$\hookrightarrow \vec{E} = -\frac{dV}{dx}\hat{i}$$

$$\underbrace{q\vec{E}}_{\text{Force}} = -\frac{d(\underbrace{qV}_{U})}{dx}\hat{i} = -\frac{dU}{dx}\hat{i}$$