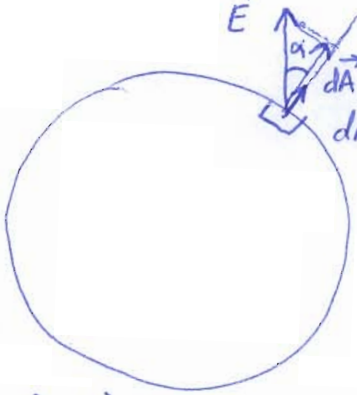
Ch 24 Gauss's Law:

So far in Ch. 23 we calulate the electric field by superpositi[on]
here we learn an alternative.

Electric flux: ϕ "Phi" $= \oint \vec{E} \cdot d\vec{A}$ $\left(\frac{N}{C} m^2\right)$

$\oint$ → integral is along a closed surface (separates inside from outside)

$\vec{E} \cdot d\vec{A}$ → scalar or dot product



dA (element of area on closed surface)

$d\vec{A}$ → has direction perpendicul[ar] to dA, pointing away from the surface

$\vec{E} \cdot d\vec{A}$ is $E\,dA \cos\alpha$ is the product of the projection of $\vec{E}$ along the direction of $d\vec{A}$ with dA.

Gauss Law:

$$\boxed{\phi_{\text{Through a closed surface}} = \frac{q_{\text{enclosed}}}{\varepsilon_0}}$$

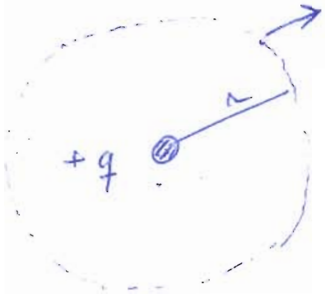
ε_0 "Epsilon sub zero" = dielectric constant in vacuum $= 8.85 \times 10^{-12} \frac{C^2}{N m^2}$

This allows calculation of $\vec{E}$ when we can identify a simple surface such that $\vec{E}$ is constant on that surface.

$$\phi = \oint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \underbrace{\oint d\vec{A}}_{\vec{A}} = EA = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$\boxed{E = \frac{q_{\text{enclosed}}}{\varepsilon_0 A}}$$

Application for a single charge:



→ Gaussian surface : sphere centered at the charge

A sphere has constant separation to a charge located at its center → $E = \frac{kq}{r^2}$ will be constant on this surface. $\phi = EA \rightarrow E = \frac{\phi}{A} = \frac{q_{enclosed}}{\varepsilon_0 A}$

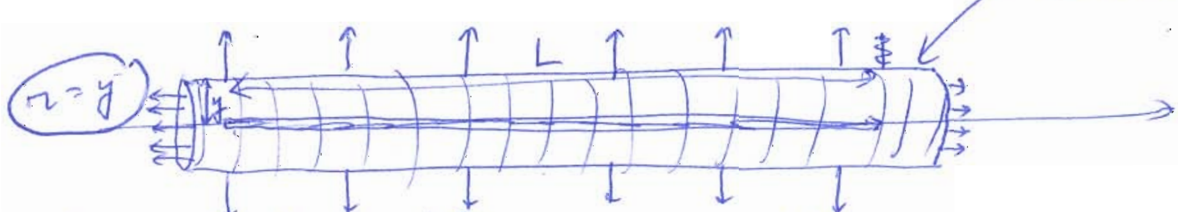
$$E = \frac{q}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 4\pi r^2} = \frac{kq}{r^2}$$

$$\downarrow$$

$\boxed{k = \frac{1}{4\pi\varepsilon_0}}$ (please check!)

Application for a long line of charge
∞

Gaussian surface



Charge density $\lambda = \frac{+Q}{L}$ → E ?

$$E = \frac{\phi}{A} = \frac{q_{enclosed}}{\varepsilon_0 A} = \frac{\lambda \cancel{L}}{\varepsilon_0 \, 2\pi r \cancel{L}}$$

$$= \frac{\lambda 2}{4\pi\varepsilon_0 \, y} = \frac{2k\lambda}{y}$$

Ch. 25 Electric Potential

Electricity $\begin{cases} \text{fields \& forces} \\ \quad \text{inverse square} \\ \text{- electric potential energy} \end{cases}$

Gravitation $\begin{cases} \text{- field \& forces} \\ \quad \text{inverse square} \\ \text{- grav. potential energy} \end{cases}$

↳ change of ~~gr.~~ potential energy:

$$\Delta U_{AB} = - W_{AB} = - \int_A^B \vec{F} \cdot d\vec{\ell}$$

scalar or dot product

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$

angle b/w $\vec{a}$ & $\vec{b}$

For grav. potential energy $\vec{F} = m\vec{g}$

For electric potential energy $\vec{F} = q\vec{E}$

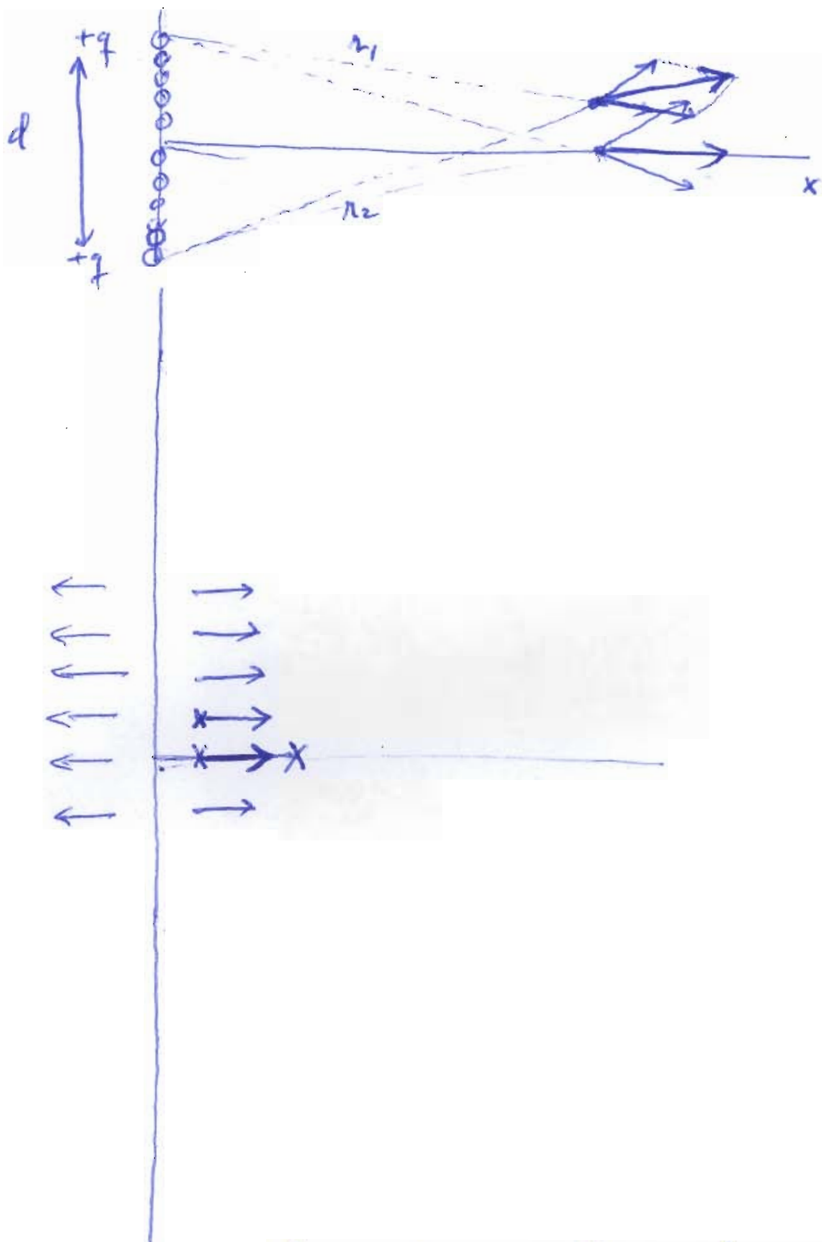
$$\Delta U_{AB} = -q \boxed{\int_A^B \vec{E} \cdot d\vec{\ell}} = -q \, \Delta V_{AB}$$

Electric potential

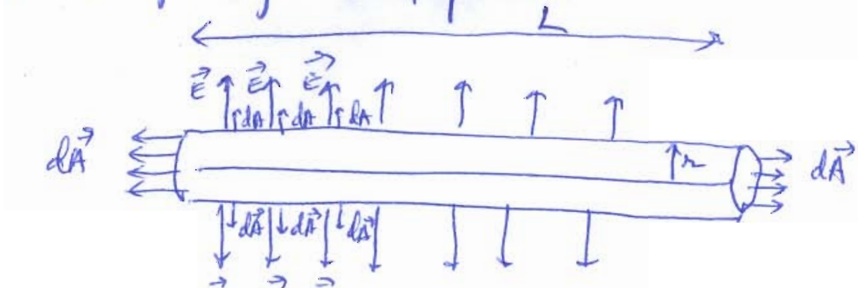
$$\Delta V_{AB} = - \int_A^B \vec{E} \cdot d\vec{\ell} \qquad \left(\frac{Nm}{C} = \frac{J}{C} \right)$$

↳
- Is a scalar (no direction)
- If known then $\vec{E} = - \vec{\nabla} V$

  "gradient" or derivative operator
- Superposition = = numeric addition



∞ line of charge. (any point can be considered as a mid point!

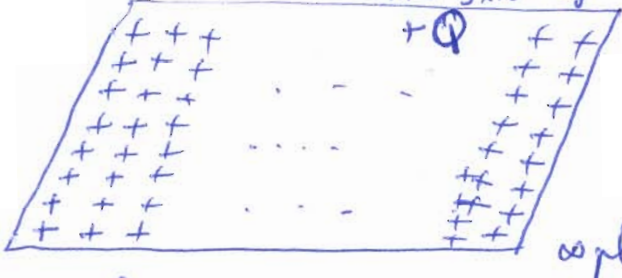


$$\phi = \oint \vec{E}\cdot d\vec{A} = \overbrace{\int_{\substack{\text{Body}\\\text{of}\\\text{cylinder}}} \vec{E}\cdot d\vec{A}}^{E\cdot 2\pi r L} + \overbrace{\int_{\substack{\text{Top}\\\&\\\text{bottom}\\\text{cross section}}} \vec{E}\cdot d\vec{A}}^{0}$$

Application of Gauss' Law to an infinite plane of charge

24.35

$q = 15\times10^{-6}\,C$
$m = 5\times10^{-3}\,kg$



∞ plane (horizontal)

$\sigma = \dfrac{Q}{A}$? is need to suspend particle q, m in the air ?

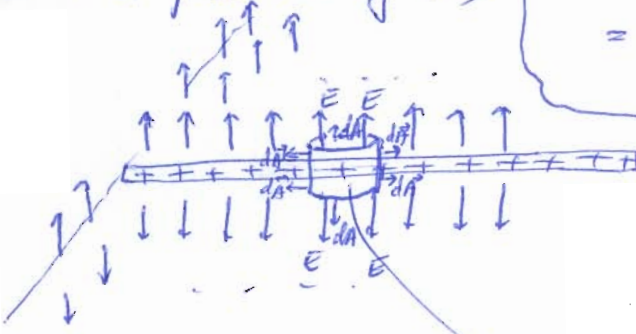
$$mg = qE = q\frac{\sigma}{2\varepsilon_0} \rightarrow \sigma = \frac{2\varepsilon_0 mg}{q}$$

$$= \frac{2\times8.85\times10^{-12}\times5\times10^{-3}\times9.81}{15\times10^{-6}C}$$

$$= 57.8\,\frac{nC}{m^2}$$

What is E? (should be a function of σ)

View on front edge:



Gaussian cylinder with radius a

$$\phi = \frac{q_{enclosed}}{\varepsilon_0}$$

$\downarrow$

$E.A$

$\downarrow$

area of Gaussian surface where E is uniform all over the surface.

just top & bottom crosssections.

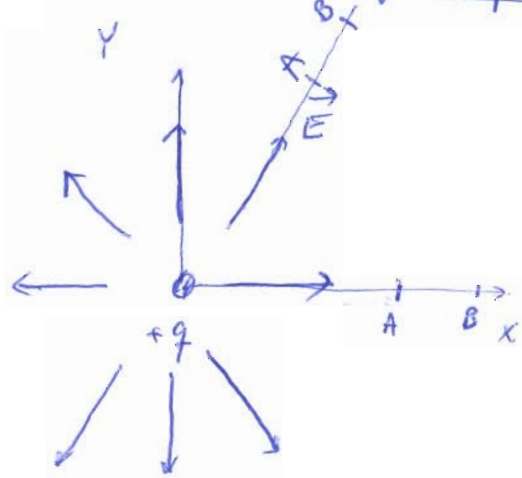
Total charge enclosed.

$\pi a^2 \sigma$

$$E.2\pi a^2 = \frac{\pi a^2 \sigma}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$ due an ∞ plane of charge.

Electric potential for a point charge:



$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot\underline{\vec{dl}}$$

$$= -\int_A^B \underbrace{\frac{kq}{r^2}\hat{r}}_{\frac{kq}{x^2}\hat{i}} \cdot \hat{i}\,dx$$

$$= -\int_A^B kq\,\frac{dx}{x^2} = kq\left[\frac{1}{x}\right]_A^B$$

$$= kq\left[\frac{1}{x_B} - \frac{1}{x_A}\right]$$

In general: $\Delta V_{AB} = kq\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$ (along any other direction)

change or increment?

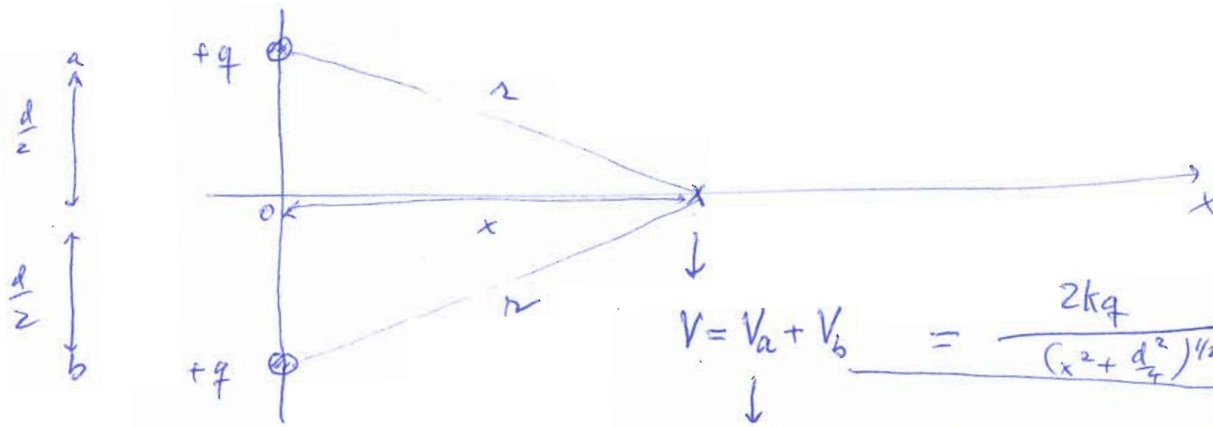
If we agree that $r_A = \infty$ is the reference point

then $\Delta V_{\infty B} = kq\left[\frac{1}{r_B} - \frac{1}{\infty}\right] = kq\frac{1}{r_B}$

$$\frac{1}{\infty} \rightarrow 0$$

$\longrightarrow$ $\boxed{V(r) = kq\frac{1}{r}}$ (Electric potential, ref point at ∞)

Electric potential for 2 charges. :



$$V = V_a + V_b = \frac{2kq}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}} = V(x)$$

$$V_a = \frac{kq}{r} = \frac{kq}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}} = V_b$$

How do I get $\vec{E}$ from V ?

$$\vec{E} = -\vec{\nabla} V = -\left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right) V$$

$$\vec{E} = -\hat{i}\frac{d}{dx}\left(\frac{2kq}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}}\right) = -\hat{i}\, 2kq \frac{d}{dx}\frac{1}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}}$$

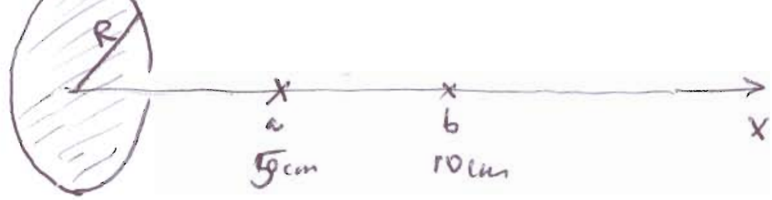
$$= -\hat{i}\, 2kq \frac{d}{dx}\left(x^2 + \frac{d^2}{4}\right)^{-1/2} = \hat{i}\, kq \frac{2x}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

$$\frac{d\, y^n}{dy} = n\, y^{n-1}$$

From the electric potential: electric field due to 2 + charges at a point along the x-axis is

$$\boxed{\vec{E} = \frac{2kqx}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}\,\hat{i}}$$

25.71

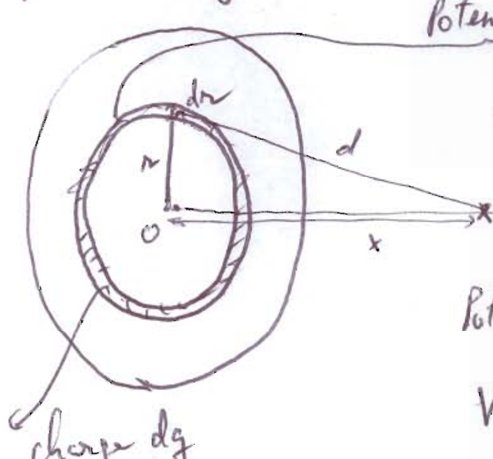


Q

$$V(a) = 150V, \quad V(b) = 110V \quad \Big\} \quad R? \quad Q?$$

$$V(x) = \frac{2kQ}{R^2}\left(\sqrt{R^2+x^2} - \sqrt{x^2}\right)$$

Electric potential for a uniformly charged circular disk at a point along its axis:



Potential due to ring:

$$dV = \frac{k\,dq}{d} = \frac{k\,dq}{(r^2+x^2)^{1/2}}$$

Potential due to whole disk: superposition.

$$V(x) = \int_{r=0}^{r=R} \frac{k\,dq}{(r^2+x^2)^{1/2}} = \frac{2\pi\sigma k}{2}\underbrace{\int_{r=0}^{r=R}\frac{2r\,dr}{(r^2+x^2)^{1/2}}}$$

$$dq = \underset{\substack{\downarrow \\ \text{charge density}}}{\sigma}\, dA = \sigma \underbrace{2\pi r}_{\text{circumference}} dr$$

$$2r\,dr = d(r^2) = d(r^2+x^2)$$

$\downarrow$ x is constant for the ring

$$r^2+x^2 \equiv u$$

$$V(x) = \pi\sigma k\int_{x^2}^{R^2+x^2}\frac{du}{u^{1/2}} = 2\pi\sigma k\left(\sqrt{R^2+x^2} - \sqrt{x^2}\right) = \frac{2kQ}{R^2}\left(\sqrt{R^2+x^2} - \sqrt{x^2}\right)$$

$$\int\frac{du}{u^{1/2}} = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} = 2u^{1/2} = 2\sqrt{u} \quad \Big| \quad \sigma = \frac{Q}{\pi R^2}$$

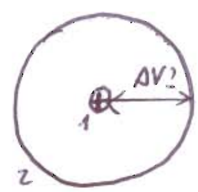
$$150 = \frac{2kQ}{R^2}\left(\sqrt{R^2 + 0.05^2} - 0.05\right)$$

$$110 = \frac{2kQ}{R^2}\left(\sqrt{R^2 + 0.1^2} - 0.1\right)$$

$$\frac{150}{110} = \frac{\sqrt{R^2 + 0.05^2} - 0.05}{\sqrt{R^2 + 0.1^2} - 0.1} \quad \rightarrow R = 0.14\ m$$

$$\rightarrow Q = 1.67 \times 10^{-9}\ C$$

25.64) Front view:



$R_1 = 2mm$

$\lambda_1 = 75 \frac{nC}{m}$

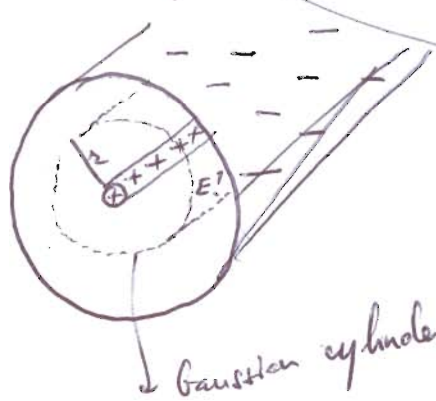
$R_2 = 10\ mm$

$\lambda_2 = -75 \frac{nC}{m}$

a) $\Delta V = -\int_2^1 \vec{E} \cdot d\vec{r} = -\int_2^1 \frac{2k\lambda\, dr}{r} = -2k\lambda \ln\left(\frac{2mm}{10mm}\right) = 2k\lambda \ln\left(\frac{10mm}{2mm}\right)$

$= 2.17kV$

What is $\vec{E}$ b/w inner & outer conductor?



Gaussian cylinder : $E =$ by a long line (inner conductor) of charge.

$= \frac{2k\lambda_1}{r}$

? b) What is ΔV if $\lambda_2 = +150\, nC/m$

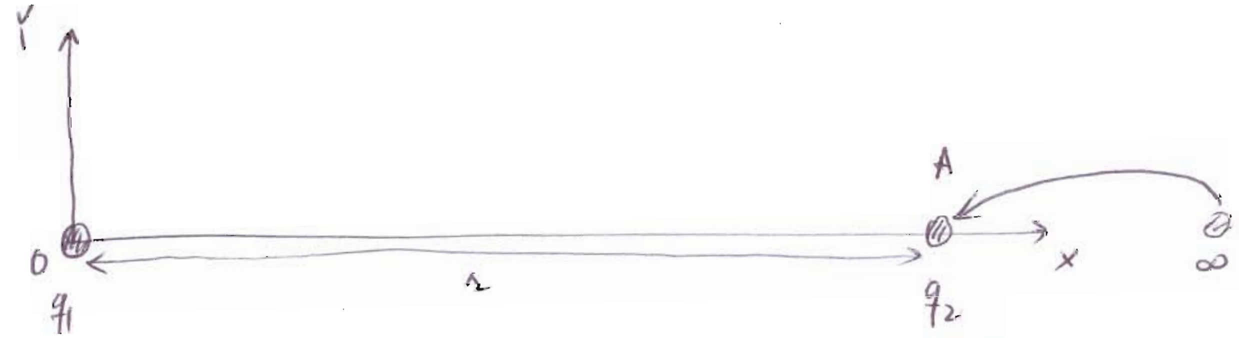
$V = 2.17kV$

Ch. 26: Electrostatic Energy and Capacitors

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q'} \quad \begin{cases} U: \text{ electric potential energy} \\ V: \text{ electric potential} \end{cases}$$

$q' \rightarrow$ Test charge

$$\Delta U_{AB} = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}$$



$\rightarrow$ q_2 is in the field created by q_1 : [electric potential due to q_1 at point A is $\underbrace{V = \frac{kq_1}{r}}_{\text{Ref. point at } \infty} = \Delta V_{\infty A} = kq_1\left(\frac{1}{r} - \cancel{\frac{1}{\infty}}\right)$

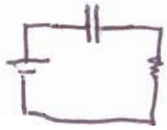
it is also the electric potential difference b/w ∞ & A.] $\rightarrow$ No info about q_2

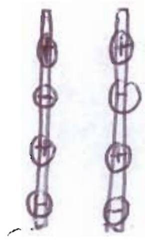
$\rightarrow$ What is the $\Delta U_{\infty A}$ if we move charge q_2 into the picture?

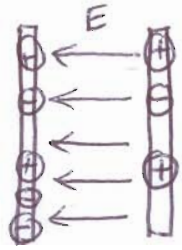
$$\Delta U_{\infty A} = q_2 \Delta V_{\infty A} = \frac{k q_1 q_2}{r}$$

$\Rightarrow$ We can store electric potential energy by bringing charges together

Electrostatic energy storage devices: capacitors = parallel-plate capacitors







$\underbrace{Q=0 \quad Q=0}$ equal amount of charges of each type in each plate.

$Q=-e \quad Q=+e$

To store energy, we need a field to bring charges against it.