Ch. 23 Electric Charge, Force, and Field

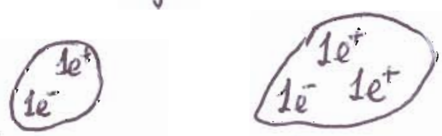
Q_1 Q_2

Interaction b/w these 2 distribution of $\boxed{\text{charges}}$: via the electric field : to calculate the electric $\boxed{\text{force}}$ of Q_1 on Q_2 we calculate the electric $\boxed{\text{field}}$ created by Q_1 : $\vec{E}_1$, then the force on Q_2 is $\underset{\text{(by } Q_1 \text{ on } Q_2)}{\vec{F}_{12}} = Q_2 \vec{E}_1$

<u>Charges</u>: • types : 2 $\begin{cases} + & \text{(a proton has } 1e^+ \text{)} \\ - & \text{(elementary charge : that of electron } \bar{e} \\ & \quad 1e^- = -1.6 \times 10^{-19} C \text{)} \end{cases}$

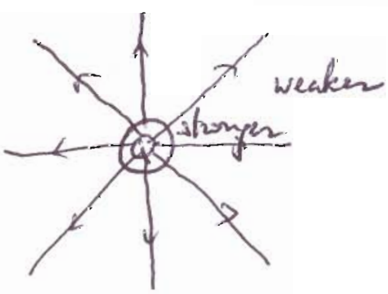
C ↓ Coulomb SI unit of charge.

• superposition of charge :



$Q_{Total} = 0\,C$ $Q_{Total} = 1e^+ = +1.6 \times 10^{-19} C$

• Electric field : $\vec{E} = \frac{kQ}{r^2}\hat{r}$



k : electric constant $= 9 \times 10^9 \frac{Nm^2}{C^2}$

Q : total charge

r : separation from the charge

$\hat{r}$: unit radial vector, pointing away from the charge.

$q_e = 1e^- = -1.6\times10^{-19}\,C$

$m_e = 9.11\times10^{-31}\,kg$

$\vec{E} = k\frac{Q}{r^2}\hat{r}$; $\vec{g} = -G\frac{M}{r^2}\hat{r}$

$k = 9\times10^{9}\ \frac{Nm^2}{C^2}$ $G = 6.67\times10^{-11}\ \frac{Nm^2}{kg^2}$

huge strength compared to gravitational field (can't learn as we learn to walk).

$$\frac{\text{Electric force}}{\text{Grav. force}} = \frac{q_p\, k\frac{q_e}{\cancel{r^2}}}{m_p\, G\frac{m_e}{\cancel{r^2}}} \stackrel{?}{=} 2.27\times10^{39}$$

b/w a proton of mass m_p charge $1e^+$ and an electron of mass m_e and charge $1e^-$

$q_e = 1e^-$; $q_p = 1e^+$

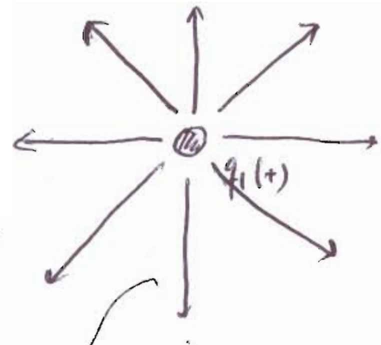
m_e

$m_p \cong 2000\, m_e = 1.67\times10^{-27}\,kg$

Why at the macroscopic level, under normal conditions we feel the grav. force but not the electric force? b/c of superposition & balance of + and - charges.

Electric field due to one charge :

Repulsive

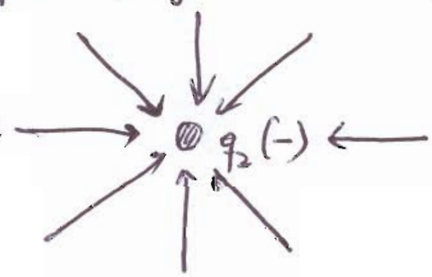


$$\vec{E}_1 = \frac{k\, q_1}{r^2}\,\hat{r}$$

→ Radial unit vector $\hat{r}$ is pointing away from the charge (regardless of the type of charge)

Electric field lines : indicate the directions & strengths of the electric field (higher density of lines = stronger field)

Attractive



$$\vec{E}_2 = -\frac{k|q_2|}{r^2}\,\hat{r}$$

→ q_2 is "test charge" in the field created by q_1 : force by q_1 on q_2 :

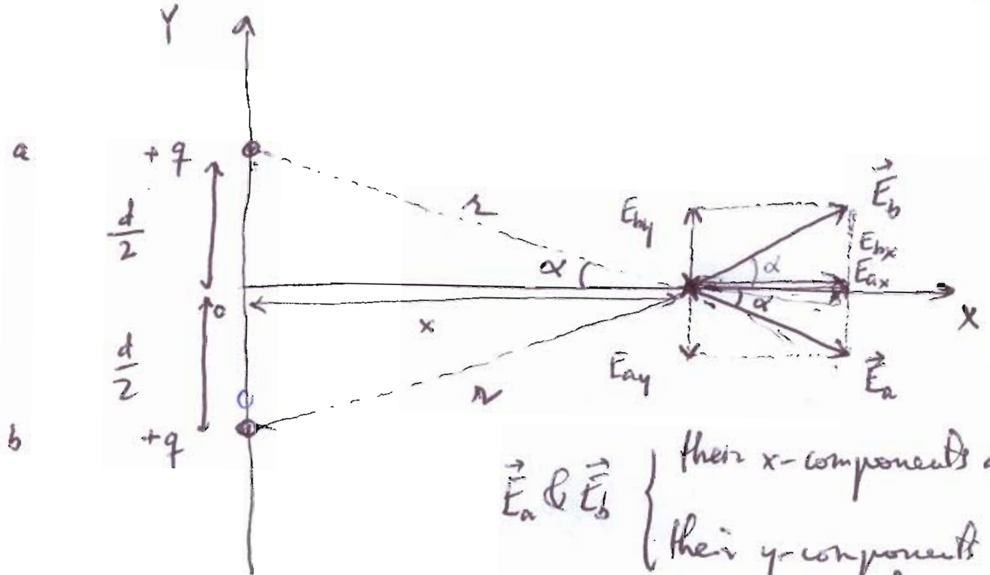
$$\underset{(q_1 \text{ on } q_2)}{\vec{F}_{12}} = \underbrace{\underset{\ominus}{q_2}\ \underset{\text{repulsive}}{\vec{E}_1}}_{\text{attractive}}$$

→ q_1 is "test charge" in the field created by q_2 : force by q_2 on q_1 is

$$\underset{(q_2 \text{ on } q_1)}{\vec{F}_{21}} = \underbrace{\underset{\oplus}{q_1}\ \underset{\text{attractive}}{\vec{E}_2}}_{\text{attractive.}}$$

Electric forces b/w a + & a − are attractive ; likewise, b/w <u>two +'s</u> or two −'s are <u>repulsive</u>.

Electric field by two positive charges: by superposition of fields created by one charge



$\vec{E}_a$ & $\vec{E}_b$ $\left\{\begin{array}{l}\text{their x-components are } \overset{\text{equal}}{\text{parallel}} \\ \text{their y-components are equal and opposite}\end{array}\right\} \rightarrow \vec{E} = \overset{2E_{ax}}{\overset{\uparrow}{E}}\,\hat{x}$ repulsive

What is E?

$\rightarrow$ We know $E_a = E_b = k\frac{q}{r^2}$; $r = \left(x^2 + \frac{d^2}{4}\right)^{1/2}$

$$= \frac{kq}{x^2 + \frac{d^2}{4}}$$

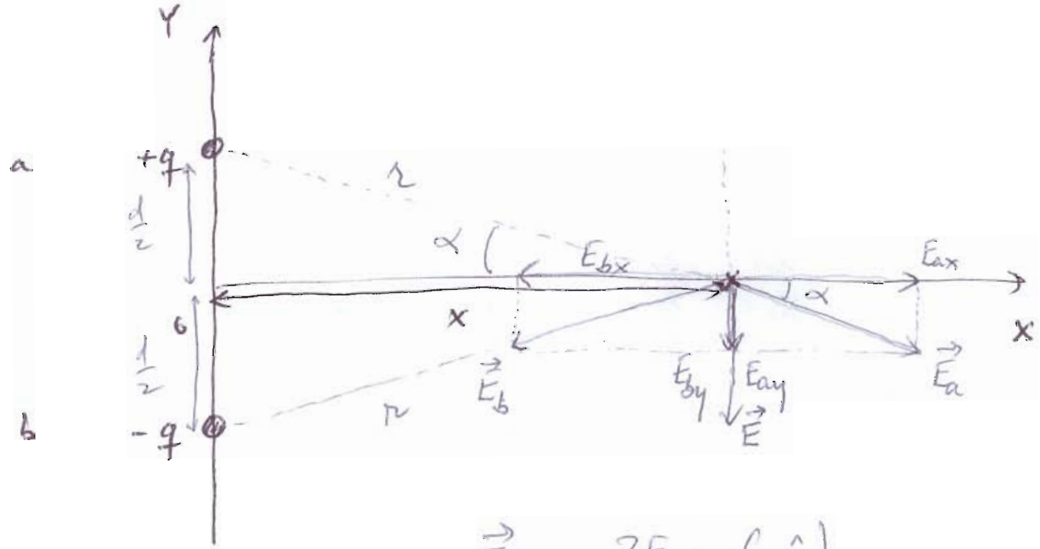
$$\rightarrow E_{ax} = E_a \cos\alpha = E_a \cdot \frac{x}{r} = \underbrace{\frac{kq}{\left(x^2 + \frac{d^2}{4}\right)}}_{E_a} \cdot \frac{x}{\left(x^2+\frac{d^2}{4}\right)^{1/2}} = \frac{kq\,x}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}$$

$$\rightarrow E = 2E_{ax} = \frac{2kqx}{\left(x^2+\frac{d^2}{4}\right)^{3/2}} \Rightarrow \boxed{\vec{E} = \frac{2kqx}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}\,\hat{x}}$$

unit: $\frac{N}{C}$ (Newton) (Coulomb)

Electric field by one positive and one negative charge:

Dipole configuration (far away field satisfies inverse cube law)



$$\vec{E} = 2E_{ay}\,(-\hat{y})$$

or $(-\hat{j})$ unit vector along negative Y direction.

$$= 2E_a \sin\alpha \;(-\hat{j})$$

$$= \cancel{2}\,\underbrace{\frac{kq}{r^2}}_{E_a}\;\frac{\frac{d}{\cancel{2}}}{r}\,(-\hat{j}) = \frac{kqd}{r^3}\,(-\hat{j})$$

$$\boxed{\vec{E} = \frac{kqd}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}\,(-\hat{j})}$$

→ Far away from the dipole: $x \gg \frac{d}{2}$ → $x^2 + \left(\frac{d}{2}\right)^2 \simeq x^2$

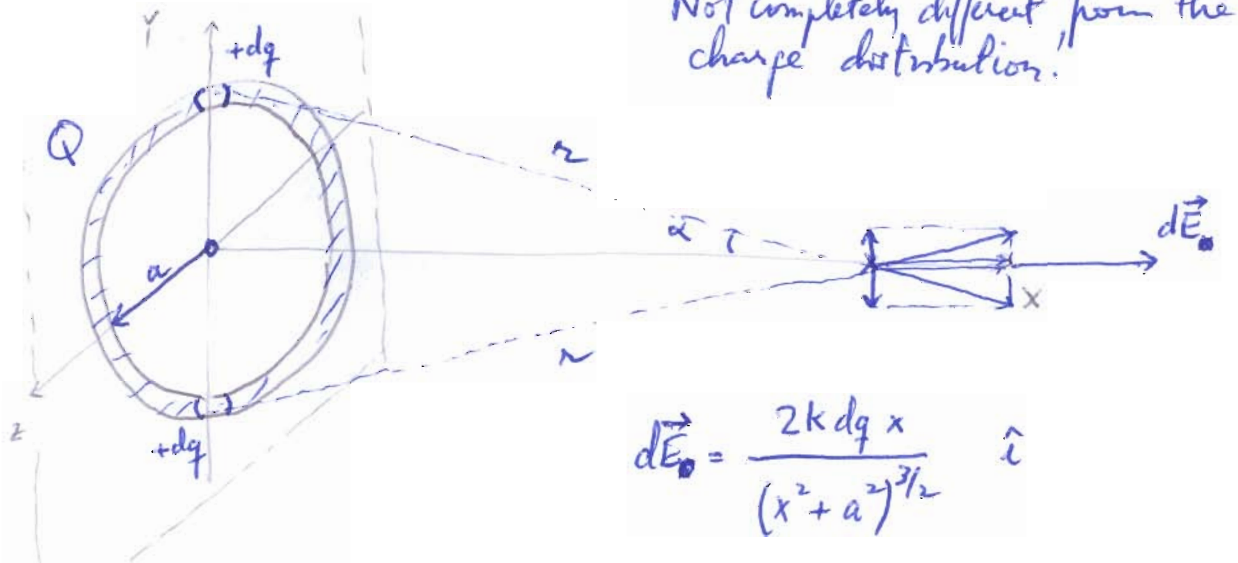
or $\left(x^2+\left(\frac{d}{2}\right)^2\right)^{3/2} \simeq x^{2\cdot\frac{3}{2}} = x^3$

$$\vec{E}_{Dipole} \simeq \frac{kqd}{x^3}\,(-\hat{j})$$

inverse-cube law.

Electric field for a continuous ring of charge: for a point on its axis

Not completely different from the two charge distribution!



$$d\vec{E} = \frac{2k\,dq\,x}{(x^2+a^2)^{3/2}}\,\hat{\imath}$$

ring on YZ plane

$$\vec{E} = \int_{\substack{\text{half ring}\\ \text{(I am doing pairs)}}} d\vec{E} = \hat{\imath}\int_{\substack{\text{half ring}\\ \text{within YZ plane}}} dE = \hat{\imath}\,\frac{2kx}{(x^2+a^2)^{3/2}}\int_{\text{half ring}} dq$$

separation r to point x is the same for all points along the ring.

$$\boxed{\vec{E} = \hat{\imath}\,\frac{\cancel{2}kQx}{\cancel{2}\,(x^2+a^2)^{3/2}} = \frac{kQx}{(x^2+a^2)^{3/2}}\,\hat{\imath}}$$

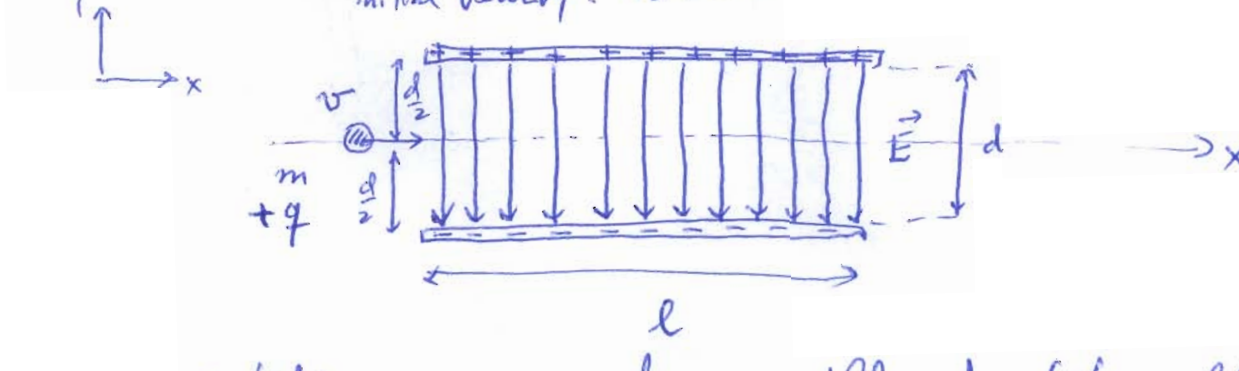
Observation: far away $x \gg a \rightarrow \vec{E} = \frac{kQ\cancel{x}}{x^{\cancel{3}2}}\,\hat{\imath}$

$$\boxed{\vec{E} = \frac{kQ}{x^2}\,\hat{\imath}}$$

Eletric field in matter :

23.59 ink-jet printers.

initial velocity : $\vec{v}_0 = v\hat{\imath}$



What is v_{min} so drop would not hit a plate

$\vec{F}_{on\,drop} = +q\vec{E} = \underbrace{+qE}(-\hat{\jmath})$ (pointing down; would be pointing up for a negatively charged drop)

Need to make sure drop would not hit the bottom plate.

By 2nd Newton's Law : drop will be accelerating downward due to the eletric force :

$$\vec{F}_{on\,drop} = m\vec{a} \rightarrow \vec{a} = \underbrace{\frac{qE}{m}}(-\hat{\jmath})$$

Trajectory should be such that by the time $x = l$, $t = \frac{l}{v}$ (const. speed along x - direction)

$y < \frac{d}{2}$

$\downarrow$ const. acceleration $(y = y_0 + v_{0y}t + \frac{1}{2}at^2)$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}\frac{l^2}{v^2} < \frac{d}{2}$$

$$\frac{qEl^2}{md} < v^2 \quad \text{or} \quad v > \underbrace{\sqrt{\frac{qEl^2}{md}}}_{v_{min}}$$