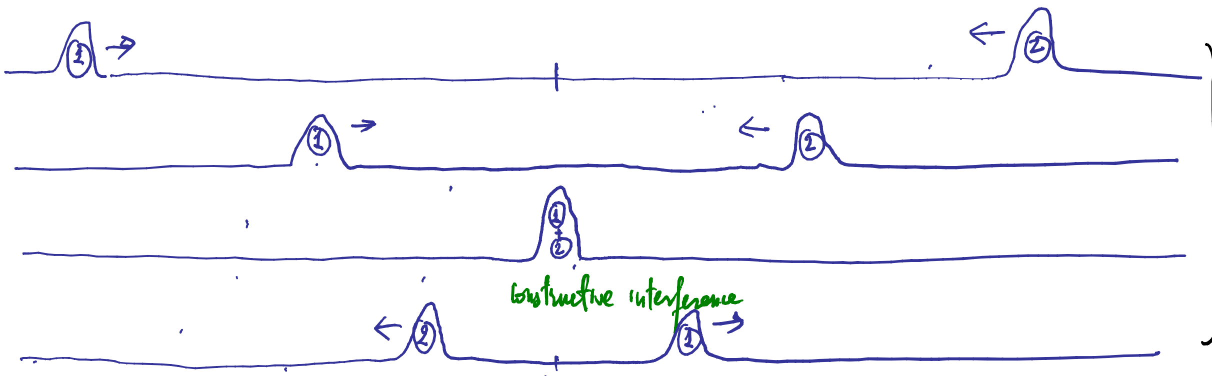
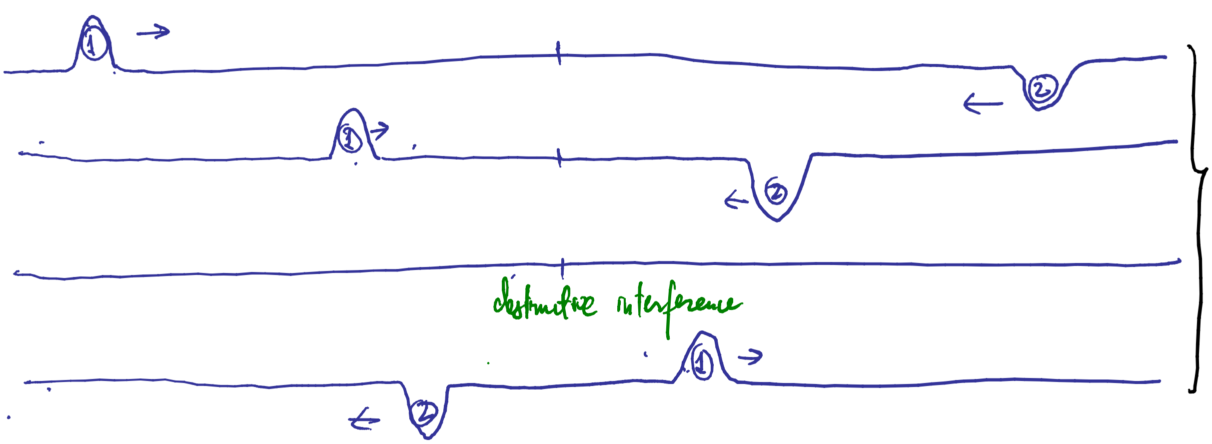
Wave properties
- Wave superposition
  - (i) Beats : tuning of string instruments, tuning of airplane bimotor engines, etc.
  - (ii) Standing waves : wind instruments (pipes, flutes, etc.)
  - (iii) Wave interference :
    - constructive
    - $\boxed{\text{destructive}}$ $1+1=0$
- Doppler effect: when the wave source is moving, LiDAR (speed detector)

Wave superposition:



① & ② same amplitude & same phase



① & ② same amplitude & opposite phase

1) Beat phenomena:

↳ Math description: Two transverse waves traveling in the same direction { same amplitudes A, different frequencies ω_1 & ω_2 & wave numbers k_1 & k_2

$$y_1(x,t) = A\sin(k_1 x - \omega_1 t)$$
$$y_2(x,t) = A\sin(k_2 x - \omega_2 t)$$

} Transverse waves: propagation in $+x$ ⊥ oscillation in y

Wave superposition: we will look, for simplicity, @ $x=0$, for all time t:

$$y(0,t) = y_1(0,t) + y_2(0,t) = A\sin(-\omega_1 t) + A\sin(-\omega_2 t)$$

$$= -A\left(\sin\omega_1 t + \sin\omega_2 t\right)$$

Trigonometry: $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

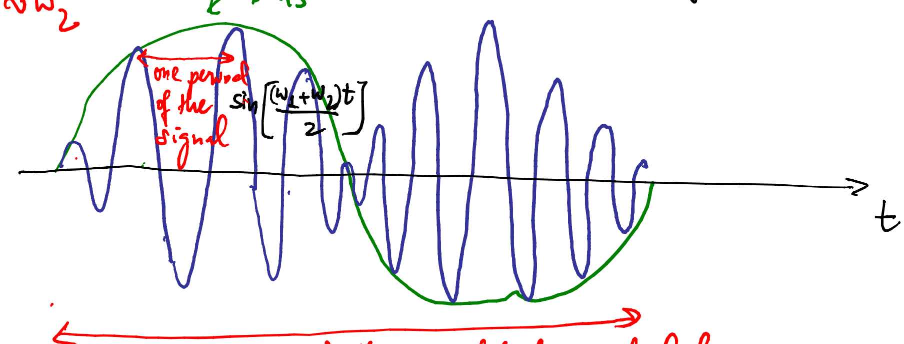
Total wave: $y(0,t) = -2A\sin\left[\frac{(\omega_1+\omega_2)t}{2}\right]\boxed{\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right]} \rightarrow$ Beats

Special case: $\omega_1 \sim \omega_2$ $\left\{ \begin{array}{l} \frac{\omega_1+\omega_2}{2} \sim \omega_1 \\ \frac{\omega_1-\omega_2}{2} \text{ very small compared } \omega_1, \omega_2 \end{array}\right.$

↳ For example when ω_1 & ω_2 are sufficient large for our hearing abilities our drum & stick can't translate fast enough sound waves into electric signals for our brain to "hear". We can only hear a flat sound when they are played separately. But when the waves are combined we can hear the beats b/w the two waves!

$\omega_1 \sim \omega_2$

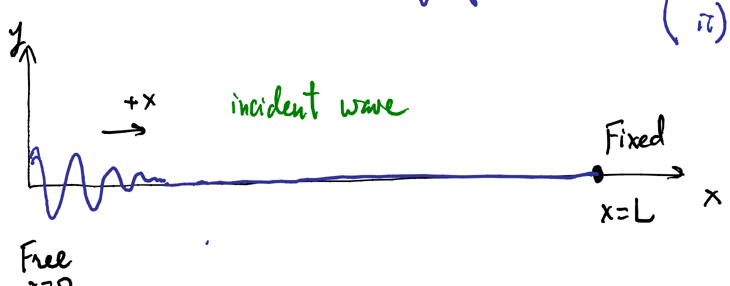
Beats: low-frequency → can be heard easily



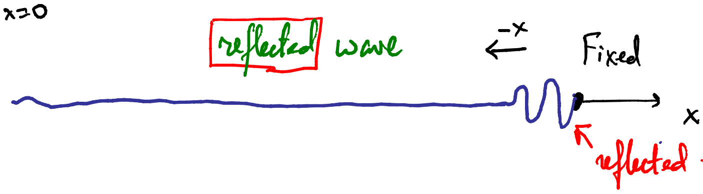
one period of the amplitude modulation $-2A\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right]$

2) Standing waves: another wave superposition of $\left\{ \begin{array}{l} \text{i) a wave propagating in } +x \\ \text{and} \\ \text{ii) its reflection propagating in } -x \end{array}\right.$

- String along x-axis
- Transverse wave



$y_1(x,t) = A\cos(kx - \omega t)$



Reflected wave propagates in $-x$

$y_2(x,t) = -A\cos(kx + \omega t)$

If we continue to send incident waves they will superimpose on the reflected waves:

↳ Wave superposition: $y(x,t) = y_1(x,t) + y_2(x,t) = A\cos(\underbrace{kx-\omega t}_{\alpha}) - A\cos(\underbrace{kx+\omega t}_{\beta})$

Trig: $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

↳ $y(x,t) = -2A\sin kx\sin(-\omega t) = 2A\sin kx\sin\omega t$ ← Total wave: incident + reflected

At $x=L$: fixed end $\Leftrightarrow$ $y(L,t)=0$ This is a condition for the total wave!

$$y(L,t) = 2A\sin kL \underbrace{\sin\omega t}_{\text{can be any number b/w } -1 \,\&\, 1} = 0 \Rightarrow \sin kL = 0 \Leftrightarrow kL = n\pi \quad (\underbrace{n=1,2,3,\text{etc.}}_{\text{standing wave mode!}})$$

String of length L: $kL = n\pi \quad (n=1,2,3,\text{ etc.})$

$$\frac{2\pi}{\lambda_n}L = n\pi$$

$$\lambda_n = \frac{2\pi}{n\pi}L = \frac{2L}{n} \rightarrow \lambda_n = \{\overset{n=1}{2L}, \overset{n=2}{L}, \overset{n=3}{\tfrac{2L}{3}}, \overset{n=4}{\tfrac{L}{2}}, \ldots\}$$

wave lengths for standing wave modes in string of length L

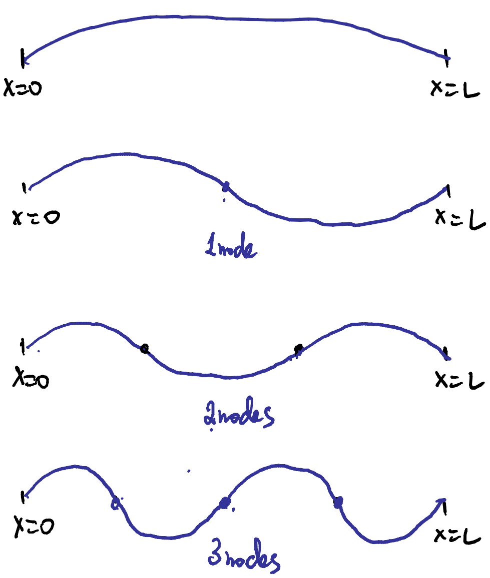
Modes:

longest: $\lambda_1 = 2L$

$\lambda_2 = L$

$\lambda_3 = \frac{2L}{3}$

$\lambda_4 = \frac{L}{2}$



As λ decreases, # osc. in the standing wave increases

shorter λ, higher f

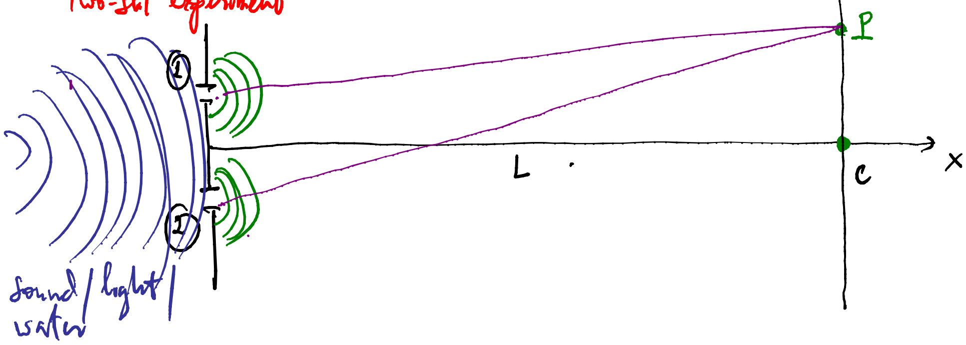
$\downarrow$

$$v = \frac{\lambda}{T} = \lambda \cdot f$$

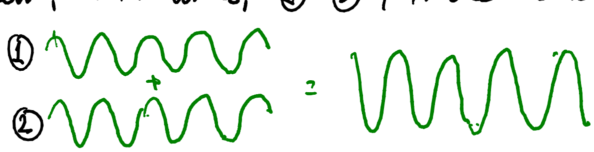
3) Wave superposition : interference :

$\hookrightarrow$ Two identical waves travelling different paths and arriving at a same point in space.

Two-slit experiment



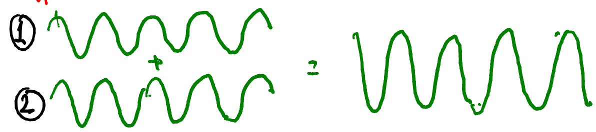
i) @ C on screen, both waves, ① & ②, traveled same path length, → arrive in phase



Constructive interference
amplitude is doubled.

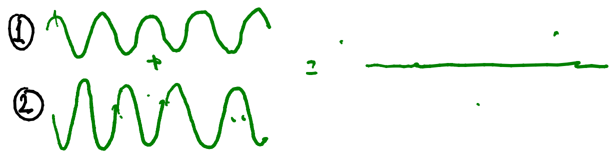
ii) @ P on screen, ① & ② arrive out of phase, in principle, since they traveled different path lengths to P : three possible outcomes for wave superposition:

1) Phase difference is a multiple of the wavelength : $\Delta path = n\lambda$ $(n=1,2,3,\dots)$
or path difference



Constructive interference
amplitude is doubled.

2) Phase difference is an odd multiple of half wavelength : $\Delta path = (2n+1)\frac{\lambda}{2}$
$(n=0,1,2,3\dots)$

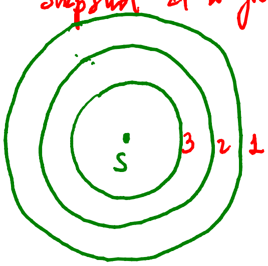


destructive interference

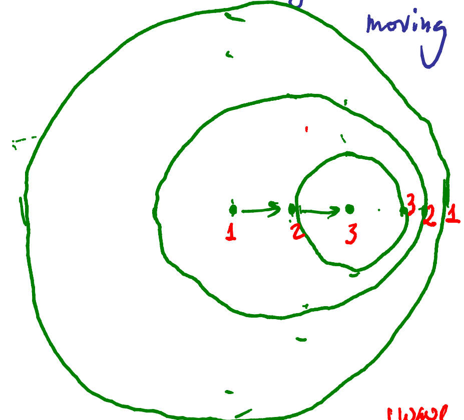
3) Phase difference is some value in between : some light / sound / water wave rest

4) <u>Doppler Effect</u>: when source of wave is moving

Source of wave is at rest
snapshot at a given time



Source of wave is moving to the right



When source was @ 1, it emitted wave 1 etc...

<u>Consequence</u>: Doppler effect
- wave is compressed in front — shorter λ
- wave is spaced out in back — longer λ

→ change of frequency if source is moving

Source approaching:

$$\lambda' = \lambda - uT$$

- λ': new wavelength
- λ: original wavelength
- u: source speed
- T: wave period

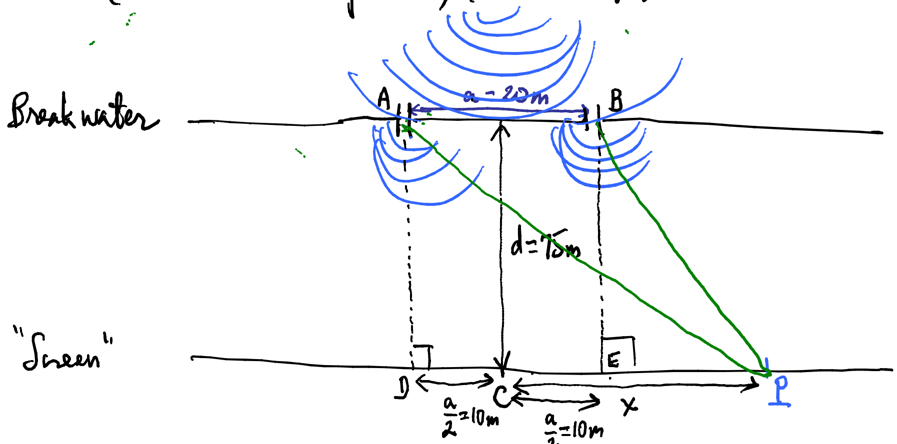
$$f' = \frac{f}{1-\frac{u}{v}}$$

- f': new frequency
- f: original frequency
- u: source speed
- v = wave speed $= \frac{\lambda}{T} = \lambda \cdot f$

Source receding : $- \longrightarrow +$ in the previous equations for λ' & f' !

Wave interference example:

Swimming pool with 2 breakwater openings on long side, separated by a distance $a = 20m$. We observe water wave interference the other side of the pool at distance $d = 75m$. Find locations for maxima (constructive interference) & minima (destructive interference); $\lambda_{water} = 16m$



Typical two-slit experiment

@ P waves A & B arrived after traveling different paths: phase difference at superposition is given by $\Delta path = AP - BP = \underbrace{\sqrt{75^2 + (x+10)^2}}_{\Delta ADP} - \underbrace{\sqrt{75^2 + (x-10)^2}}_{\Delta BEP}$

1) $\Delta path$ depends on x (location of P)
2) When $x = 0$: @ C
$\Delta path = 0 \rightarrow$ in phase
$\rightarrow$ maxima @ C ✓

@ P

Maxima if $\Delta path = n\lambda$ $(n = 1, 2, 3, etc...)$

↳ 1st: $\sqrt{75^2+(x+10)^2} - \sqrt{75^2+(x-10)^2} = \lambda \rightarrow$ solve for x_1 (location for 1st maxima from C) → Right and left of C

↳ 2nd: [id.] $= 2\lambda \rightarrow$ solve for x_2

etc.

Minima if $\Delta path = (2n+1)\frac{\lambda}{2}$ $(n = 0, 1, 2, 3, etc...)$

↳ 1st: $\sqrt{75^2+(x+10)^2} - \sqrt{75^2+(x-10)^2} = \frac{\lambda}{2} \rightarrow$ solve for $x_{1,min}$ (location of 1st minima right & left of C)

↳ 2nd: [id] $= \frac{3\lambda}{2} \rightarrow$ solve for $x_{2,min}$

etc.

Manual calculation for location of 1st max. x_1:

$$\sqrt{75^2+(x+10)^2} - \sqrt{75^2+(x-10)^2} = 16$$

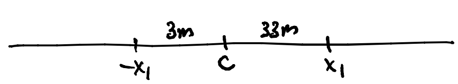
$$75^2 + (x+10)^2 + 75^2 + (x-10)^2 - 2\sqrt{75^2+(x+10)^2}\sqrt{75^2+(x-10)^2} = 256$$

$$\sqrt{75^2+(x+10)^2}\sqrt{75^2+(x-10)^2} = 5597 + x^2$$

$$[75^2+(x+10)^2]\cdot[75^2+(x-10)^2] = x^4 + 11194x + 5597^2$$

$$5625^2 + 5625\underbrace{[(10-x)^2 + (10+x)^2]}_{2x^2+100} + \underbrace{[(10-x)(10+x)]^2}_{[x^2-100]^2} = x^4 + 11194x + 5597^2$$

→ Quadratic equation: $11050x^2 - 11194x + 1449216 = 0 \rightarrow x_1 = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \pm 33m$



location of 1st max:

14.72] 2.25 m long pipe, one end open

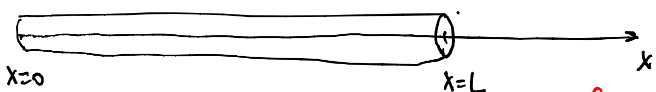
Standing wave: $f_n = 345\,Hz$; $f_{n+1} = 483\,Hz$

Find a) Fundamental frequency b) Sound speed or wave speed

↳ Wave superposition: $y(x,t) = y_1(x,t) + y_2(x,t) = A\cos(\underbrace{kx-\omega t}_{\alpha}) - A\cos(\underbrace{kx+\omega t}_{\beta})$

Trig: $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

↳ $y(x,t) = -2A\sin kx \, \sin(-\omega t) = \boxed{2A\sin kx \, \sin \omega t}$ ← Total wave: incident + reflected



open → total wave has max amplitude here

↓

$\sin kL = \pm 1$ → $kL = (2n+1)\frac{\pi}{2}$ ($n = 0, 1, 2, 3,$ etc.)

Def. of wave number: $k_n = \frac{2\pi}{\lambda_n}$ → $\frac{\cancel{2}\pi}{\lambda_n} L = (2n+1)\frac{\pi}{\cancel{2}}$ → $\lambda_n = \frac{4L}{2n+1}$ wavelength for mode n in a pipe w/ an open end

Wave speed: $v = \lambda \cdot f$ → $f = \frac{v}{\lambda}$ → $f_n = \frac{v}{\lambda_n}$ → frequency for mode n in a pipe w/ an open end

Given: $\boxed{\frac{f_n}{f_{n+1}} = \frac{\frac{\cancel{v}}{\lambda_n}}{\frac{\cancel{v}}{\lambda_{n+1}}} = \frac{\lambda_{n+1}}{\lambda_n} = \frac{\frac{\cancel{4L}}{2(n+1)+1}}{\frac{\cancel{4L}}{2n+1}} = \frac{2n+1}{2n+3}}$

$\frac{345}{483} = \frac{115}{161} = \frac{2n+1}{2n+3}$ → $230n + 345 = 322n + 161$

$184 = 92n$ → $\boxed{n=2}$ & $f_2 = 345\,Hz$.

a) For our formula (odd number $(2n+1)$, $n = 0, 1, 2, 3,$ etc...) lowest mode is $n=0$

↳ $f_0 = \frac{v}{\lambda_0}$

↳ $\frac{f_2}{f_3} = \frac{5}{7}$; $\underbrace{\frac{f_1}{f_2} = \frac{3}{5}; \quad \frac{f_0}{f_1} = \frac{1}{3}}$

$\frac{f_0}{f_2} = \frac{1}{5}$ → $f_0 = \frac{f_2}{5} = \frac{345}{5} = \boxed{69\,Hz = f_0}$

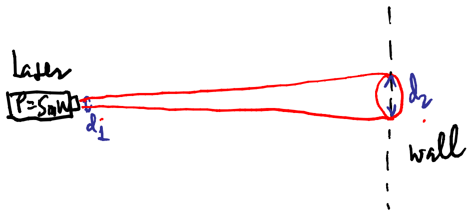
b) $f_0 = \frac{v}{\lambda_0}$ → $v = \lambda_0 \cdot f_0 =$

$= 4L \cdot 69 = 4 \cdot 2.25 \cdot 69 = 621\,\frac{m}{s}$ in the pipe.

$\boxed{14.52}$

5mW Laser beam → beam's intensity a) @ laser   b) at wall

$\underbrace{d_1 = 1\cdot 10^{-3}\,m \qquad d_2 = 36\cdot 10^{-3}\,m}_{\text{beam diameters}}$



Intensity decreases b/c of the beam spreading!

$$I_1 = \frac{P}{A_1} = \frac{P}{\pi R_1^2} = \frac{4P}{\pi d_1^2} = \frac{4\cdot 5\cdot 10^{-3}}{\pi\cdot(10^{-3})^2} = \frac{20\cdot 10^{3}}{\pi}\,\frac{W}{m^2} = 6.37\,\frac{kW}{m^2}$$

$$I_2 = \frac{P}{A_2} = \frac{P}{\pi R_2^2} = \frac{4P}{\pi d_2^2} = \frac{4\cdot 5\cdot 10^{-3}}{\pi\cdot(36\cdot 10^{-3})^2} = \frac{I_1}{36^2} = 4.91\cdot 10^{-3}\,\frac{kW}{m^2}$$

Ch 15 Fluid Motion

Fluid
- Gas : density ρ (rho) is variable $\rightarrow$ gas is compressible
- Liquid : density ρ is constant, $\rightarrow$ a liquid is non-compressible

Describing fluid motion
- Density ρ : by def, mass per unit volume $\rho \equiv \frac{dM}{dV}$ or $\frac{M}{V}$ SI unit $\left(\frac{kg}{m^3}\right)$
  - $\rho_{air} = 1 kg/m^3$; $\rho_{water} = 1000\ kg/m^3$ $\rho_{liquid} > \rho_{gas}$
- Pressure P : by def, normal force per unit area $P = \frac{dF}{dA}$ or $\frac{F}{A}$ $\left(\frac{N}{m^2}\right)$
  - SI unit : $\frac{N}{m^2} = Pa$ (Pascal)
  - Atm (Atmosphere) : $1 Atm = 1.013 \cdot 10^5\ Pa$

Fluid equation of motion
1) Hydrostatic equilibrium $\frac{dP}{dh} = g\rho$
   - P: pressure
   - h: height or vertical distance
   - $g = 9.81 m/s^2$
   - ρ = fluid density

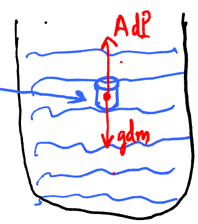
   If g & ρ are constant $\frac{dP}{dh}$ = constant $\Rightarrow$ Pressure increases linearly with depth h
2) Conservation of mass $v \cdot A$ = constant
   - v = fluid speed
   - A = cross-sectional area of the pipe

   fluid goes faster through narrower pipes
3) Conservation of energy or Bernoulli's equation
   $$\frac{1}{2}\rho v^2 + \rho g y + P = \text{constant}$$

1) Hydrostatic equilibrium: why $\frac{dP}{dh} = g\rho$?

i) 2nd Newton's Law : net force on an element of fluid in hydrostatic equilibrium is 0

Force on a cylindrical element of fluid
- height dh
- cross-sectional area A
- infitessimal mass dm
  - $dm = dV\rho = A dh \rho$

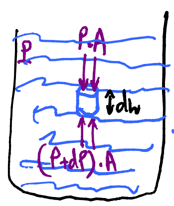


$F_{net} = m \cdot a = 0$

$AdP - gdm = 0 \rightarrow \cancel{A}dP = gdm = g\cancel{A}\rho\, dh$

$\rightarrow \frac{dP}{dh} = g\rho$ ✓

ii) Why buoyant force on this element of fluid is AdP ?

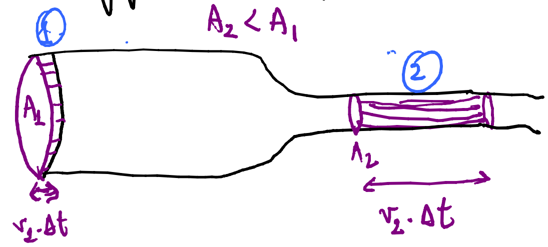


Net pressure of fluid of this element of fluid is pointing up : $(P+dP)A - P \cdot A = dP \cdot A$

This is the buoyant force on the element fluid

2) Conservation of mass: no leaking $\Leftrightarrow$ mass in = mass out

Fluid moving in a pipe with different cross-sectional areas



$$m_1 = m_2$$

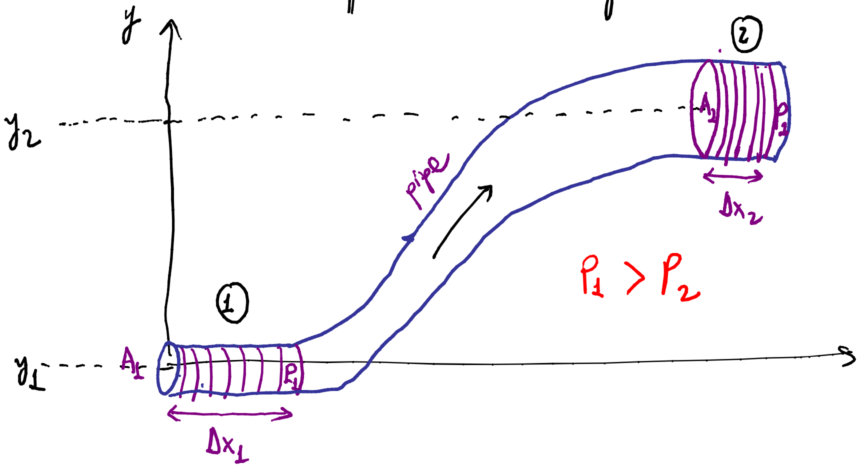
$$\rho V_1 = \rho V_2$$

$$\cancel{\rho} A_1 \cdot v_1 \cdot \cancel{\Delta t} = \cancel{\rho} A_2 v_2 \cdot \cancel{\Delta t} \rightarrow v_1 A_1 = v_2 A_2 = \text{constant}$$

V_1: volume of disk @ 1
V_2 volume of cylinder @ 2

vA = constant in fluid motion. (when A smaller, v is higher!)

3) Conservation of energy: aplies to a pipe w/ different cross-sectional areas at different vertical positions:



Can we use conservation of energy to describe the motion of this fluid

$$\Delta W = \Delta(KE + PE)$$

$$F_1 \cdot \Delta x_1 - F_2 \cdot \Delta x_2 = KE_2 - KE_1 + PE_2 - PE_1$$

$P = \frac{F}{A}$

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

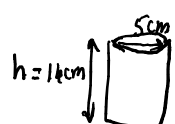
$$\frac{1}{Vol}\left[P_1 A_1 \Delta x_1 + \frac{1}{2} m v_1^2 + mgy_1\right] = \frac{1}{Vol}\left[P_2 A_2 \Delta x_2 + \frac{1}{2} m v_2^2 + mgy_2\right]$$

Vol of fluid $\begin{cases} V = A\Delta x \\ V_1 = A_1 \Delta x_1 = A_2 \Delta x_2 = V_2 \equiv V \end{cases}$

$$\Rightarrow \boxed{P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}}$$

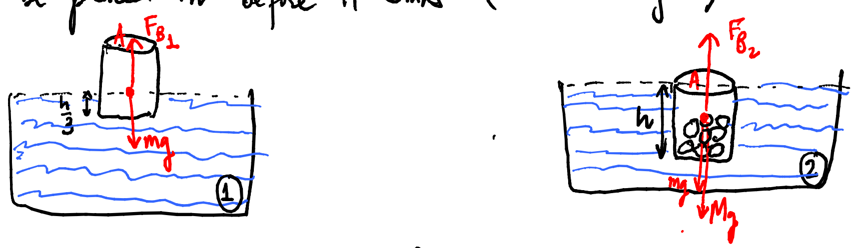
Bernoulli's equation in terms of P & ρ
v: fluid velocity
y: fluid vertical position
g: acceleration of gravity

15.48 Glass beaker:



empty it floats with $\frac{h}{3}$ submerged, how many 12-g rocks can be placed in before it sinks (h submerged)

Step 1:



o: 12-g each rock

Step 2: physics: difference in bouyant force b/w ① & ② compensates for the weight of rocks

Bouyant force: $\frac{dP}{dh} = g\rho \rightarrow dP = g\rho\, dh \rightarrow [P = g\rho h] \cdot A \rightarrow \overset{P \cdot A}{F_B} = g\rho \underbrace{hA}_{V \text{ (vol.)}}$

$\rightarrow F_B = g\rho_w V_w$ { Bouyant force by water is proportional to volume of water displaced

There is a difference in volume of water displaced in ① & ②:

$$F_{B_2} - F_{B_1} = g\rho_w\left(h \cdot A - \frac{h}{3} \cdot A\right) = g\rho_w \frac{2}{3} h \cdot A$$

This difference in buoyant force compensates the weight of rocks: $Nm_r g$ $(m_r = 12g)$

$$\rightarrow \cancel{g}\rho_w \frac{2}{3}h \cdot A = Nm_r \cancel{g} \rightarrow N = \frac{\rho_w \frac{2}{3}h \cdot A}{m_r} \overset{\text{SI units}}{=} \frac{1000 \cdot \frac{2}{3} \cdot 0.14 \cdot \pi(2.5 \cdot 10^{-2})^2}{12 \cdot 10^{-3}}$$

$$\rho_w = 10^3 \frac{kg}{m^3}$$

$N = 15.27 \rightarrow N = 15$ rocks before it sinks.

15.58 | Balloon $m = 1.6g$ empty, inflated w/ helium $(\rho_{He} = 0.18 \frac{kg}{m^3})$ to a sphere 28 cm diameter. How many 0.63g paper clips can we hang from balloon before it loses buoyancy?

Step 1: Fluid motion:

$\frac{dP}{dh} = g\rho$

$\downarrow$

$P = g\rho h$

$\downarrow$

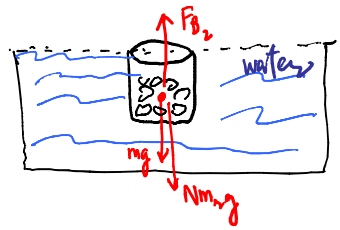
$F_B = g\rho V$

15.48 (Fluid = water)

Buoyant force = $F_B = g\rho_w V_w$

V_w volume of water displaced by beaker

$$F_{B_2} - F_{B_1} = Nm_{rock}\, g$$



Fluid Motion

15.58 (Fluid = air)

Buoyant force: $F_B = g\rho_{air} V_{air}$

V_{air}: volume of air displaced by balloon

$$F_B \overset{\text{Hydrostatic equilibrium}}{=} mg + m_{He}g + Nm_{clip}\,g$$

$$\cancel{g}\rho_{air}V_{air} - \cancel{g}\rho_{He}V_{He} - m\cancel{g} = Nm_{clip}\cancel{g}$$

$V_{air} = V_{Balloon} = V_{He}$

vol. air displaced is volume of balloon

balloon is filled with Helium

Step 2: physics equation:

$$N = \frac{(\rho_{air} - \rho_{He})V_{Balloon} - m}{m_{clip}}$$

$\rho_{air} = 1 kg/m^3$

$\rho_{He} = 0.18\, kg/m^3$

$V_{Balloon} = \frac{4}{3}\pi R^3$ $(R = 14 \cdot 10^{-2}m)$

$m = 1.6 \cdot 10^{-3} kg$

$m_{clip} = 0.63 \cdot 10^{-3} kg$

Step 3: $$N = \frac{(1 - 0.18)\frac{4}{3}\pi \cdot (14 \cdot 10^{-2})^3 - 1.6 \cdot 10^{-3}}{0.63 \cdot 10^{-3}}$$

$= 12.42$

$N = 12$ clips before balloon loses buoyancy

To hang more clips: increase $V_{balloon}$ (more He)

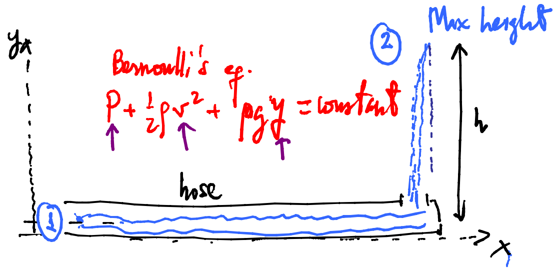
15.55 | Water in garden hose @ 140 kPa $\boxed{\text{gauge pressure}}$ moving @ negligible speed ($v_1 = 0$), hose terminates in a sprinkler w/ many small holes. Max. height of water?

$\rightarrow$ Fluid @ different vertical position: $\begin{cases} \text{hose @ ground level} \\ \text{water to max. } h. \end{cases}$ $\rightarrow$ Conservation of energy $P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$

$1 Atm = 1.013 \cdot 10^5\, Pa$

Gauge pressure is pressure of fluid w/o atmospheric pressure:

$\hookrightarrow P = P_{Gauge} + P_{Atm} = 140\, kPa + 101.3\, kPa$



Bernoulli's eq.

$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$

$P_2 = P_{Atm}$

$v_2 = 0$ (max. height)

$y_2 = h$?

$$\begin{cases} P_1 = P_{Gauge} + P_{Atm} \\ v_1 = 0 \\ \text{(problem said negligible speed)} \\ y_1 = 0 \end{cases}$$

Conservation of energy

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_{gauge} + P_{Atm} + \frac{1}{2}\rho \cdot 0^2 + \rho g \cdot 0 = P_{Atm} + \frac{1}{2}\rho \cdot 0^2 + \rho g h$$

$$P_{Gauge} = \rho g h \rightarrow h_{max} = \frac{P_{Gauge}}{\rho_W g} = \frac{140 \cdot 10^3 \, Pa}{10^3 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2}} = 14\, m$$

Ch10&11: Rotational motion & C.A.M.

1) Cross-product of two vectors is vector

- Torque vector : $\vec{\tau} = \vec{r} \times \vec{F}$ { i) Pivot or center of rotation; ii) Identify force application point; iii) position vector $\vec{r}$ from pivot to force app. point }
- Angular momentum vector : $\vec{L} = \vec{r} \times \vec{p}$ { same as above, with ii) replaced by object's position. } $= I \cdot \vec{\omega}$ (rotational)

2) Know right-hand rule (RHR) to find directions for $\vec{\tau}$ & $\vec{L}$

3) Analog of 2nd Newton's Law :

Linear: $\vec{F}_{net} = m \cdot \vec{a}$ Rotational: $\vec{\tau}_{net} = I \cdot \vec{\alpha}$; $I = cMR^2$ (Mom. of inertia I)

more generally: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

CLM → $\vec{P}_i = \vec{P}_f$ CAM → $\vec{L}_i = \vec{L}_f$

Shape	c
disk	$\frac{1}{2}$
ring	1
sphere	$\frac{2}{5}$
rod	$\frac{1}{12}$

Ch12: Static equilibrium

4 questions { 1) What is the object; 2) What is the pivot or center of rotation; 3) What are the forces on object; 4) What are the force application points } determine $\vec{r}_i$ for torque calculation: $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{F}_{net} = \sum_i \vec{F}_i = 0$

$\vec{\tau}_{net} = \sum_i \vec{\tau}_i = 0$ → $\vec{\tau}_i = \vec{r}_i \times \vec{F}_i = r_i F_i \sin\theta \, \hat{\tau}$

($\vec{r}_i$: pivot to force app point)

{ θ is angle b/w $\vec{r}_i$ & $\vec{F}_i$; $\hat{\tau}$ is direction as given by RHR: right-hand fingers along $\vec{r}_i$, closing toward $\vec{F}_i$, thumb shows direction of torque }

Ch13: oscillatory motion

SHM: $\frac{d^2z}{dt^2} = -\frac{a}{b} z$ → solution $z(t) = A\cos\omega t$; $\omega = \sqrt{\frac{a}{b}}$

	a	b	ω
pendulum	g	L	$\sqrt{\frac{g}{L}}$
torsional pendulum	K	I	$\sqrt{\frac{K}{I}}$
spring & box	k	m	$\sqrt{\frac{k}{m}}$

Energy in SHM: stays constant.

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

Ch 14: waves (periodic variations in both space & time)

wave superpositions

- Beats: 2 waves, same direction, same A's, different k's & ω's

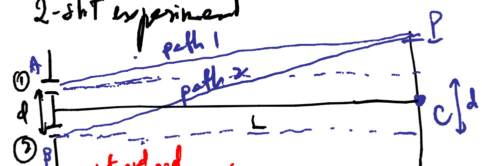
$y_T(0,t) = -2A\cos\left(\frac{\omega_1 - \omega_2}{2} t\right)\sin\left(\frac{\omega_1 + \omega_2}{2} t\right)$

Low freq. modulations → Beats

- Standing waves: 2 waves, incident & reflected (amplitudes +A, −A, respectively) in opposite directions, same k's & ω's

$y_T(x,t) = 2A\sin kx \sin\omega t$ { Fixed end @ $x=L$: $\sin kL = 0$ → $kL = n\pi$ ($n=1,2,3,\ldots$); Open end @ $x=L$: $\sin kL = 1$, $kL = (2n+1)\frac{\pi}{2}$ ($n=0,1,2,3\ldots$) }

- Interference: 2-slit experiment



$\Delta path = AP - BP = \sqrt{L^2 + \left(y + \frac{d}{2}\right)^2} - \sqrt{L^2 + \left(y - \frac{d}{2}\right)^2}$

$\Delta path = n\lambda$ ($n = 1,2,3,\ldots$) → constructive interference

$= (2n+1)\frac{\lambda}{2}$ ($n = 0,1,2,3\ldots$) → destructive (1st, 2nd, 3rd min ...)

Doppler's effect: moving source $f' = \frac{f}{1 \mp \frac{u}{v}}$ { −: approaching source; +: receding source. }

$$\lambda' = \lambda\left(1 \mp \frac{u}{v}\right)$$

ch 15
fluid motion

1) Hydrostatic equilibrium $\frac{dP}{dh} = g\rho \rightarrow F_B = g\rho V$

2) Conservation of mass : no leaks $\rightarrow$ $v \cdot A = \text{constant}$

3) Conservation of energy or Bernoulli's eq : $\frac{1}{2}\rho v^2 + \rho g y + P = \text{constant}$