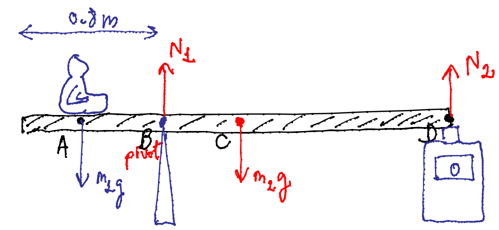
Ch 12 Static Equilibrium

Physics $\begin{cases} \text{i)} \ \sum_i \vec{F}_i = 0 & (\vec{F}_{net} = 0) \to \text{no linear motion} \\ \text{ii)} \ \sum_i \vec{\tau}_i = 0 & (\vec{\tau}_{net} = 0) \to \text{no rotational motion} \end{cases}$

12.18

Step: 1



Board: $m_2 = 60\,kg$, $L = 2.4\,m$

Child: $m_1 = 40\,kg$

location so scale reads 0

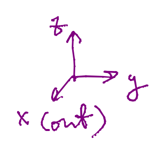
Static equilibrium : focus on board $\begin{cases} \vec{F}_{net} = 0 \\ \vec{\tau}_{net} = 0 \end{cases}$

$\downarrow$

$(\vec{r} \times \vec{F})$

To arrive at an equation in step 2: we need to identify all forces acting on board along with their application points. Also a pivot point is needed to write the torque equation $\to$ B

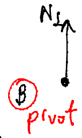
Step 2: Cartesian coord. system:



on the board $\begin{cases} 1) \ \vec{F}_{net} = 0 \ \to \ \sum_i F_i = N_1 + N_2 - m_1 g - m_2 g = 0 \\ 2) \ \vec{\tau}_{net} = 0 \ \to \ \sum_i \vec{\tau}_i = 0 + 0 + \underbrace{r_{BA} m_1 g \, \hat{i} - r_{BC} m_2 g \, \hat{i}} = 0 \end{cases}$

Note: Some torque needs to be negative for possible static equilibrium!

Choosing (B) as center of rotation of pivot.



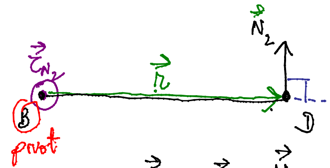
$\vec{\tau}_{N_1} = \vec{r} \times \vec{N}_1 = 0$

$\vec{r}$: position vector of force application point wrt center of rotation or pivot

For N_1 : $\vec{r} = 0$

Note: often a pivot point is choosen at a force application point where the force is unknown

→ In this problem (B) is the right choice for pivot since N_1 is the only unknown ($N_2 = 0$)



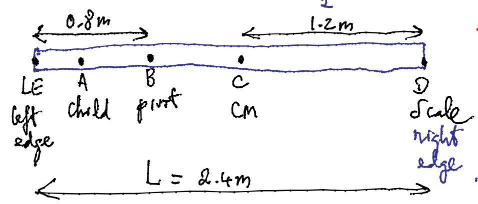
$\vec{\tau}_{N_2} = \vec{r} \times \vec{N}_2 = 0$

$\vec{r} \neq 0$

$N_2 = 0$

Note: if $\vec{N}_2 \neq 0$, by RHR: thumb points out of page which is the direction of $\vec{\tau}_{N_2}$

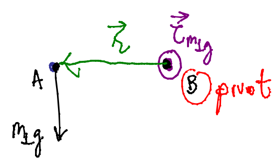
$$\boxed{\vec{\tau}_{N_2} = r_{BD} \cdot N_2 \underbrace{\sin 90}_{1} \hat{i} = 1.6 N_2 \hat{i}}$$



$r_{BD} = 1.6m$

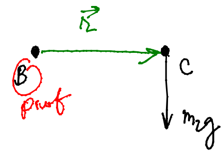
r_{AB} ?

$r_{BC} = 0.4m$



$$\vec{\tau}_{m_1g} = \vec{r} \times (-m_1g \hat{k}) = r m_1g \underbrace{(\hat{j} \times (-\hat{k}))}_{\text{RHR: out of page } \hat{i}}$$

$$= r_{BA} m_1 g \hat{i}$$



$$\vec{\tau}_{m_2g} = \vec{r} \times (-m_2g \hat{k})$$

$$= r_{BC} m_2 g \,(\hat{j} \times (-\hat{k}))$$

RHR: into page $(-\hat{i})$

$$= -r_{BC} m_2 g \hat{i}$$

Step 3: Solve for position of the child or r_{BA} $(= r_{AB})$ so N_2 reads 0:

$$\vec{\tau}_{net} = 0 = r_{BA} m_1 g \hat{i} - r_{BC} m_2 g \hat{i} \Rightarrow r_{BA} m_1 - r_{BC} m_2 = 0$$

$$\boxed{r_{BA} = \frac{m_2}{m_1} r_{BC}}$$

$r_{BA} = \frac{60}{40} 0.4m = 0.6m$ → { child sits 0.6m left of pivot or 0.2m right of left edge

12.19 | Same as 12.18 but { a) $N_2 = 100$ N, b) $N_2 = 300$ N

Step 1: Same as above

Step 2: Same as above { 1) $\vec{F}_{net} = 0$ → $N_1 + N_2 - m_1g - m_2g = 0$

2) $\vec{\tau}_{net} = 0$ → $0 + \underbrace{r_{BD}}_{1.6m} N_2 \hat{i} + \underbrace{r_{BA}}_{?} m_1 g \hat{i} - \underbrace{r_{BC}}_{0.4m} m_2 g \hat{i} = 0$

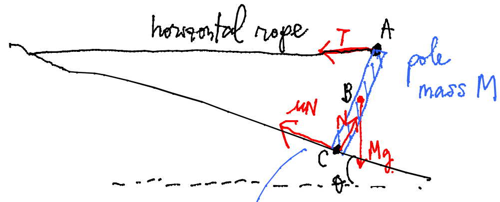
Step 3: solve for r_{BA} : $r_{BA} = \dfrac{r_{BC} m_2 g - r_{BD} N_2}{m_1 g}$

a) $N_2 = 100$N → $r_{BA} = \dfrac{0.4 \cdot 60 \cdot 9.81 - 1.6 \cdot 100}{40 \cdot 9.81} = 0.19$ m { Child sits @ 0.19m left of pivot or $0.8 - 0.19 = 0.61$ m right of left edge

b) $N_2 = 300$N → $r_{BA} = -0.62$ m ⟶ { Child sits @ 0.62 Right of pivot or $0.8m + 0.62m = 1.42m$ from left edge

12.53 Static equilibrium for a pole on an incline: what is μ_{min} to keep pole from slipping?
b/w bottom pole & incline

Step 1:



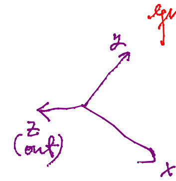
1) Object: pole of mass M
2) Forces on object: tension T, Mg, N, μN
3) Force application points: A, B, C, C
4) Pivot point or center of rotation
Ⓑ

opposes slipping motion of pole down the incline
Since T is applied @ A, pole tends to rotate CCW about its CM @ Ⓑ

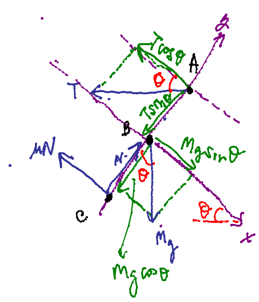
Pivot point selection: 1) choice of pivot for analysis using torques will not affect the physics of the problem
2) Any of the force application points can be selected as pivot: A, B, C.
Strategy is to eliminate the torque by an unknown force out of the equation

pivots →
A: will lead to different equation $\vec{\tau}_{net}=0$, but same answer for μ!
B: natural ✓
C: can't be pivot b/c we need μ in our equation

Step 2: Static equilibrium
i) $\vec{F}_{net}=0$
ii) $\vec{\tau}_{net}=0$

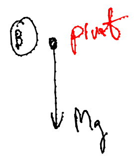


FBD:



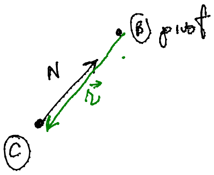
1) $\vec{F}_{net}=0$
$$F_{net,x}=0 = Mg\sin\theta - T\cos\theta - \mu N = 0 \quad (1)$$
$$F_{net,y}=0 = N - Mg\cos\theta - T\sin\theta = 0 \quad (2)$$

2) $\vec{\tau}_{net}=0 = \vec{\tau}_{Mg} + \vec{\tau}_{N} + \vec{\tau}_{\mu N} + \vec{\tau}_{T}$
$= 0 + 0 - r_{BC}\,\mu N\,\hat{k} + r_{BA}\,T\cos\theta\,\hat{k}$ $\Rightarrow$ $\boxed{r_{BA}\,T\cos\theta = r_{BC}\,\mu N}$ (3)

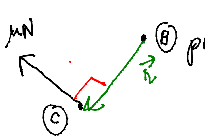


$\vec{r}=0$ Force app. point & pivot are same!

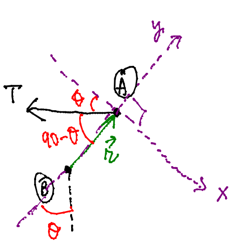
$\vec{\tau}_{Mg} = \vec{r}\times\vec{F}_{Mg} = 0$



angle b/w $\vec{r}$ & $\vec{N}$ is 180°
$\sin 180° = 0$
$\vec{\tau}_N = 0$



angle b/w $\vec{r}$ & $\vec{F}_{\mu N}$ is 90°
$\sin 90° = 1$
$\vec{\tau}_{\mu N} = \vec{r}\times\vec{F}_{\mu N} = r_{BC}\,\mu N\,\underbrace{(-\hat{k})}_{RHR}$ (CW rotation of pole w.r.t. pivot Ⓑ)



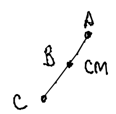
angle b/w $\vec{r}$ & $\vec{T}$ is $90-\theta$
(θ is angle of the incline)
$\vec{\tau}_T = \vec{r}\times\vec{T} = r_{BA}\,T\,\underbrace{\sin(90-\theta)}_{\cos\theta}\,\underbrace{(\hat{k})}_{RHR} = r_{BA}\,T\cos\theta\,\hat{k}$

Step 3: Equations

$$\left\{\begin{array}{l} 1)\ Mg\sin\theta - T\cos\theta - \mu N = 0 \\ 2)\ N - Mg\cos\theta - T\sin\theta = 0 \\ 3)\ r_{BA} T\cos\theta = r_{BC}\mu N \end{array}\right\}$$ static equilibrium of the pole

Solve for μ: unknowns $\left\{\begin{array}{l} N \\ T \\ M \end{array}\right.$

$$3)\quad T = \underbrace{\frac{r_{BC}}{r_{BA}}}_{1}\frac{\mu N}{\cos\theta} = \frac{\mu N}{\cos\theta} \rightarrow 2)\quad N - Mg\cos\theta - \mu N\underbrace{\frac{\sin\theta}{\cos\theta}}_{\tan\theta} = 0$$



$$N(1-\mu\tan\theta) = Mg\cos\theta \rightarrow N = \frac{Mg\cos\theta}{1-\mu\tan\theta} \quad 4)$$

$$1)\quad Mg\sin\theta - 2\mu N = 0 \xrightarrow{4)} \cancel{Mg}\sin\theta - 2\mu\frac{\cancel{Mg}\cos\theta}{1-\mu\tan\theta} = 0$$

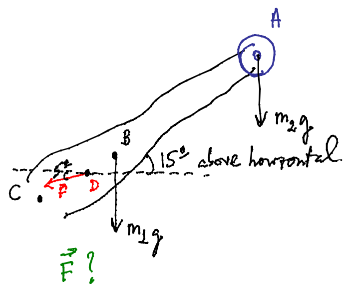
$$\sin\theta = \frac{2\mu\cos\theta}{1-\mu\tan\theta} \rightarrow \tan\theta = \frac{2\mu}{1-\mu\tan\theta}$$

$$\rightarrow \tan\theta - \mu\tan^2\theta = 2\mu \rightarrow \tan\theta = \mu(2+\tan^2\theta) \rightarrow \boxed{\mu = \frac{\tan\theta}{2+\tan^2\theta}}$$

Min. coefficient of static friction to keep pole from slipping (neither M or L determine μ!, only θ!)

12.25

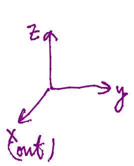
Step 1:

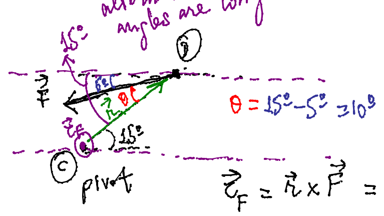


mass @ A : $m_2 = 6$ kg
mass of arm @ B (CM) : $m_1 = 4.2$ kg
center of rotation @ shoulder : C
Length of arm : $r_{CA} = 0.56$ m
location of CM $r_{CB} = 0.21$ m
Biceps apply force $\vec{F}$ @ D, 5° below horizontal (deltoid muscles)

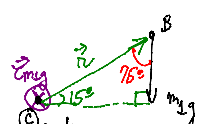
Answer:
1) Object: focus on arm
2) Forces on object: on arm: $\vec{F}$, $m_1 g$, $m_2 g$
3) Force app. points: → D, B, A
4) Choice of pivot or center of rotation: a point beyond D, B, A → C : natural center of rotation → 3 torques:

Step 2:
$$\left\{\begin{array}{l} 1)\ \vec{F}_{net} = 0 \\ 2)\ \vec{\tau}_{net} = 0 = \vec{\tau}_F + \vec{\tau}_{m_1 g} + \vec{\tau}_{m_2 g} \end{array}\right.$$
$$= r_{CD}F\sin 10^\circ\,\hat{i} - r_{CB}m_1 g\sin 75^\circ\,\hat{i} - r_{CA}m_2 g\sin 75^\circ\,\hat{i}$$

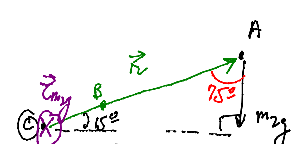




$$\vec{\tau}_F = \vec{r}\times\vec{F} = r_{CD}F\sin\theta\ \underbrace{\hat{i}}_{RHR}$$



$$\vec{\tau}_{m_1 g} = \vec{r}\times\vec{F}_{m_1 g} = r_{CB}m_1 g\sin 75^\circ\,(-\hat{i})$$



$$\vec{\tau}_{m_2 g} = \vec{r}\times\vec{F}_{m_2 g} = r_{CA}m_2 g\sin 75^\circ\,(-\hat{i})$$

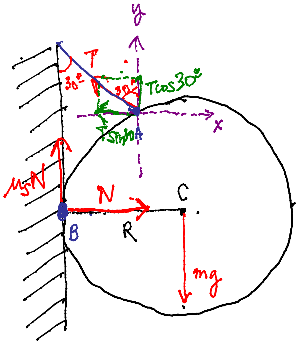
<u>Step 3:</u> solve for F $\rightarrow$ $r_{CD} F \sin 10° - r_{CB} m_1 g \sin 75° - r_{CA} m_2 g \sin 75° = 0$

$$F = \frac{r_{CB} m_1 g + r_{CA} m_2 g}{r_{CD}} \cdot \frac{\sin 75°}{\sin 10°} = \frac{0.21 \cdot 4.2 + 0.56 \cdot 6}{0.18} \cdot \frac{\sin 75°}{\sin 10°} \cdot 9.81$$

$$F = 1280\,N = 1.28\,kN$$

12.26

Step 1:



Static equilibrium: $\begin{cases} \vec{F}_{net} = 0 \\ \vec{\tau}_{net} = 0 \end{cases}$

1) What is the object? sphere

2) Forces on object: $mg, N, \mu_s N, T$

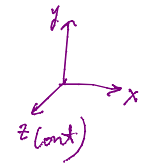
3) Force app. points: C, B, B, A

4) What is pivot point or center of rotation: C

↳ A: not natural pivot; we want τ_T in the equation to use the 30° data
B: not natural; we want $\tau_{\mu N}$ in the equation to calculate $\mu_{s,min}$
C: natural pivot; we are also OK w/o τ_{mg} in the equations.

Uniform sphere, radius R held by rope attached to a vertical wall forming an angle of 30°. Sphere in contact with wall $\mu_{s,min}$ (b/w sphere & wall) so sphere in static equilibrium

Friction force: b/c rope tension sphere tends to rotate ccw or down @ B. Friction always opposes motion.



Step 2: Conditions for static equilibrium $\begin{cases} 1) \ \vec{F}_{net} = \sum_i \vec{F}_i = 0 \\ 2) \ \vec{\tau}_{net} = \sum_i \vec{\tau}_i = 0 \end{cases}$

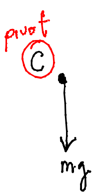
1) $\vec{F}_{net} = 0 \begin{cases} F_{net,x} = 0 : \quad N - T\sin 30° = 0 \quad (i) \\ F_{net,y} = 0 : \quad T\cos 30° + \mu_s N - mg = 0 \quad (ii) \end{cases}$

2) $\vec{\tau}_{net} = 0 = \underbrace{\vec{\tau}_{mg}}_{0} + \vec{\tau}_T + \vec{\tau}_N + \vec{\tau}_{\mu N}$
$= 0 + RT\sin\theta\,\hat{k} + 0 - R\mu_s N\,\hat{k}$

$\tau_{net}' = 0$ or $\cancel{R}T\sin\theta - \cancel{R}\mu_s N = 0$

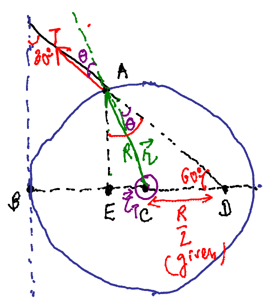
$\boxed{T\sin\theta = \mu_s N}$ (iii)

pivot



$\vec{\tau}_{mg} = \underset{0}{\vec{r}} \times \vec{F}_{mg}$

Force app. point = pivot.



RHR (CCW rotation)

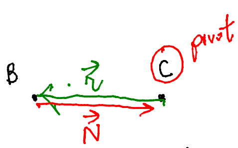
$\vec{\tau}_T = \vec{r} \times \vec{T} = r_{CA} T \sin\theta\,\hat{k}$ → needs θ or $\sin\theta$

△AED → ∠A = 30°; ∠E = 90° ⇒ ∠D = 60°

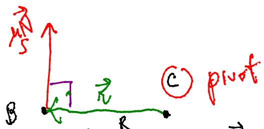
△ACD: I know ∠D = 60° & its opposite side $r_{CA} = R$; I don't know ∠A = θ but I know its opposite side $CD = \frac{R}{2}$ →

Sine theorem: $\frac{\sin\theta}{\cancel{R}/2} = \frac{\sin 60}{\cancel{R}}$

$\boxed{\sin\theta = \frac{\sin 60°}{2}}$



$\vec{\tau}_N = \vec{r} \times \vec{N} = rN\underbrace{\sin 180}_{0} = 0$



$\vec{\tau}_{\mu_s N} = \vec{r} \times \vec{F}_{\mu N} = R\mu_s N \underbrace{\sin 90}_{1} \underbrace{(-\hat{k})}_{RHR}$

Step 3: solve for $\mu_{s,min}$ → $\mu_s = \frac{T\sin\theta}{N} = \frac{T\sin 60°}{2N}$ → Needs $\frac{T}{N}$

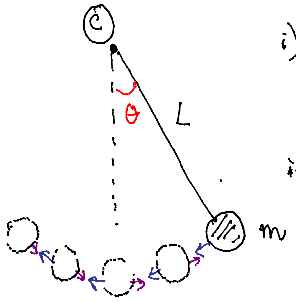
$N - T\sin 30° = 0$ (i) → $\frac{T}{N} = \frac{1}{\sin 30°}$ → ↓ $\mu_{s,min} = \frac{\sin 60°}{2\sin 30°} = \sin 60° = \frac{\sqrt{3}}{2} = 0.866$

$T\cos 30° + \mu_s N - mg = 0$ (ii)

Ch 13 Oscillatory Motion:

Describe motion
- linear — Ch 3-5
- rotational — Ch 9-10
- oscillatory motion — Ch 13
- wave motion — Ch 14
- fluid motion — Ch 15

1) **Pendulum**: bob & string (negligible mass), one end is fixed



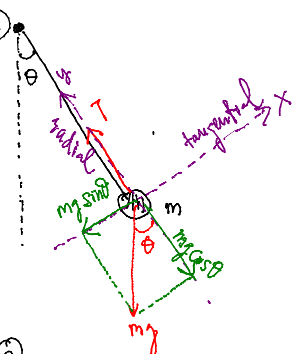
i) bob oscillates back & forth :→ description of position via angle θ wrt vertical direction $\underbrace{-\theta_M}_{\text{left of vertical}} \le \theta \le \theta_M$; $\theta = \pm\theta_M$ when pendulum turns

ii) bob should be sufficiently heavy to keep L constant @ all time
→ bob follows tangential motion, not radial (it will not move closer or further away from center of rotation C)

→ Equation of motion for a pendulum → equation for oscillatory motion!

Method #1: using 2nd Newton's Law $\underbrace{\vec{F}_{net}}_{\text{weight of bob}} = m\vec{a}$ (object = bob with mass m)

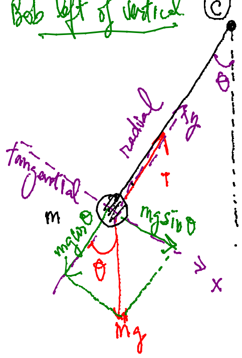
Bob right of vertical



$F_{net,x} = -mg\sin\theta = ma \rightarrow a = -g\sin\theta$

$F_{net,y} = T - mg\cos\theta = 0$

Bob left of vertical



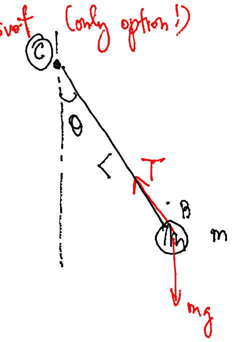
$F_{net,x} = mg\sin\theta = m\cdot a \rightarrow a = +g\sin\theta$

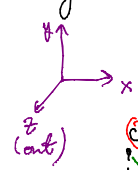
$F_{net,y} = T - mg\cos\theta = 0$

acceleration a:
i) a is not constant
ii) a = 0 @ θ = 0 (vertical) → it changes sign when bob crosses vertical
built-in property of oscillatory motion

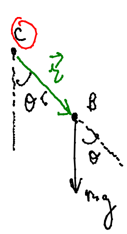
Method #2: Rotational motion of bob using the analog of 2nd Newton's Law: $\vec{\tau}_{net} = I\cdot\alpha$

pivot (only option!)



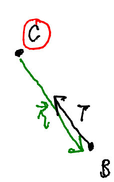


$\vec{\tau}_{net} = \vec{\tau}_{mg} + \vec{\tau}_T$



$\vec{\tau}_{mg} = \vec{r} \times \vec{F}_{mg}$
$= Lmg\sin\theta\,\underbrace{(-\hat{k})}_{RHR}$

$\vec{\tau}_{net} = -Lmg\sin\theta\,\hat{k} = I\cdot\alpha\,\hat{k}$



$\vec{\tau}_T = \vec{r} \times \vec{T}$
$= L\cdot T\sin 180^\circ = 0$

$$-\cancel{L}mg\sin\theta = \cancel{m}L^{\cancel{2}}\frac{d^2\theta}{dt^2} \rightarrow -g\sin\theta = L\frac{d^2\theta}{dt^2}$$

$$\rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

$\boxed{\text{exact}}$ equation for a pendulum

Non-linear differential eq. of 2nd order

$\sin\theta$ is non-linear in θ

Oscillatory motion: simplified version: small angle approximation (pendulum not far away from vertical):

θ small $\Rightarrow \sin\theta \simeq \theta$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta} \rightarrow \text{Simple Harmonic Motion (SHM)} \rightarrow \theta(t) = \theta_M \cos\omega t$$

$\boxed{\omega = \sqrt{\frac{g}{L}}}$ omega angular frequency

Connection to rotational motion

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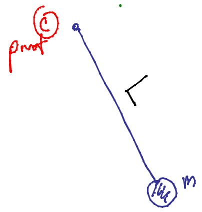
Summary: 1) Pendulum

- → 2^{nd} Newton's Law (Method #1): $a = -g\sin\theta \rightarrow \boxed{a = \alpha \cdot L} = \frac{d^2\theta}{dt^2}\cdot L$

  $L\frac{d^2\theta}{dt^2} = -g\sin\theta \rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$

- → Rotational analog of 2^{nd} Newton's Law: $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$

Proof ①



2) After small angle approximation: $\sin\theta \simeq \theta$: SHM → $\begin{cases} \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \\ \theta(t) = \theta_M \cos\omega t \end{cases}$

ω: angular frequency

$\boxed{\omega = \sqrt{\frac{g}{L}}}$: Proof: Substitute solution into SHM's equation of motion:

angular frequency: $\frac{d\theta}{dt} = -\theta_M\omega\sin\omega t$; $\begin{cases} \text{LHS } \frac{d^2\theta}{dt^2} = -\theta_M\omega^2\cos\omega t \\ \text{RHS } -\frac{g}{L}\theta = -\frac{g}{L}\theta_M\cos\omega t \end{cases}$ $\xrightarrow{\text{SHM}} \omega^2 = \frac{g}{L}$. $\rightarrow \omega = \sqrt{\frac{g}{L}}$

How fast a pendulum oscillates:

i) g changes based on altitude (center-to-center separation to center of Earth) also on underground material density → underground water pocket detection.

$\frac{GM_E}{(R_E+h)^2}$

ii) ω decreases with increasing length L

3) ✓ Angular frequency ω ("omega"): # oscillations per second, SI unit $\frac{rad}{s}$ or s^{-1}

more intuitive $\begin{cases} \text{Linear frequency } f & : \text{ \# linear osc. per second } f = \frac{\omega}{2\pi} \text{ (Hz or Hertz)} \\ \text{Period } T = \frac{1}{f} & : \text{ \# seconds per oscillation (s)} \end{cases}$

Simple Harmonic Motion (SHM)

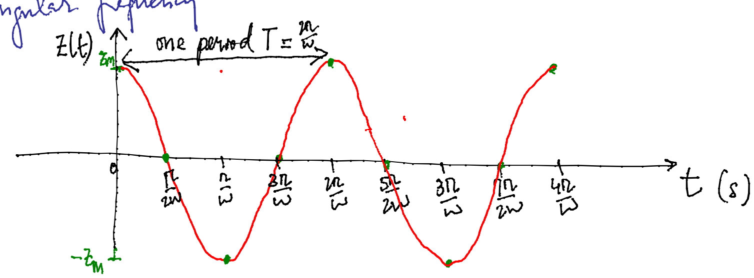
↓

$$\frac{d^2z}{dt^2} = -\frac{a}{b}z$$

	z	a	b
pendulum	θ	g	L
torsional pendulum	θ	κ	I
Spring & bob	x	k	m

Solution: $z(t) = z_M\cos\omega t$; $\omega = \sqrt{\frac{a}{b}}$

$\begin{cases} z_M: \text{ amplitude of SHM} \\ \omega: \text{ angular frequency} \end{cases}$

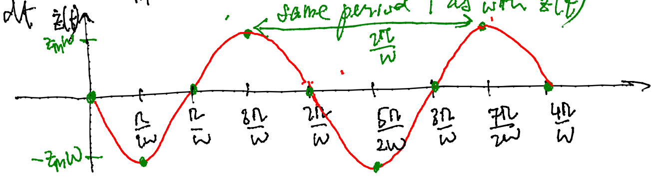


t	ωt	cos ωt	
0	0	1	Max
$\frac{\pi}{2\omega}$	$\frac{\pi}{2}$	0	zero
$\frac{\pi}{\omega}$	π	-1	Min
$\frac{3\pi}{2\omega}$	$\frac{3\pi}{2}$	0	zero
$\frac{2\pi}{\omega}$	2π	1	Max
$\frac{5\pi}{2\omega}$	$\frac{5\pi}{2}$	0	zero
$\frac{3\pi}{\omega}$	3π	-1	Min
$\frac{7\pi}{2\omega}$	$\frac{7\pi}{2}$	0	zero
$\frac{4\pi}{\omega}$	4π	1	Max
	...	...	

$\omega t = \frac{\pi}{2} \rightarrow t = \frac{\pi}{2\omega}$

period T: time separation b/w two consecutive peaks or number of seconds per oscillation

Velocity of SHM: $\dot{z} = \frac{dz}{dt} = -z_m \omega \sin \omega t$



t	ωt	$\cos \omega t$	$\sin \omega t$
0	0	1	0
$\frac{\pi}{2\omega}$	$\frac{\pi}{2}$	0	1
$\frac{\pi}{\omega}$	π	-1	0
$\frac{3\pi}{\omega}$	$\frac{3\pi}{2}$	0	-1
$\frac{2\pi}{\omega}$	2π	1	0
$\frac{5\pi}{2\omega}$	$\frac{5\pi}{2}$	0	1
$\frac{3\pi}{\omega}$	3π	-1	0
$\frac{7\pi}{2\omega}$	$\frac{7\pi}{2}$	0	-1
$\frac{4\pi}{\omega}$	4π	1	0
	...	...	

Note: $\sin \omega t$ & $\cos \omega t$ are similar but shifted by $\frac{\pi}{2}$

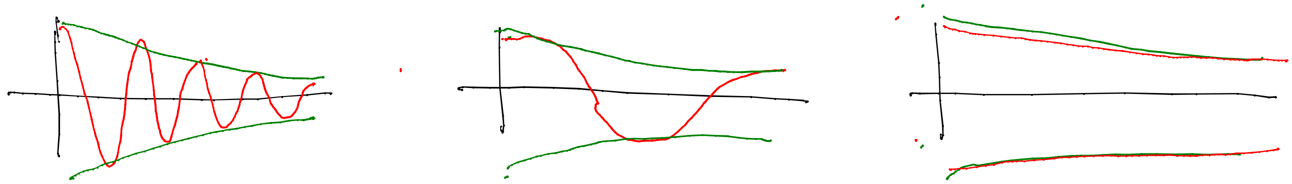
Damped - SHM: SHM where amplitude decays over time:

$$\frac{d^2 z}{dt^2} = -\frac{a}{b} z - \underbrace{\frac{c}{d}\frac{dz}{dt}}_{\text{damping term}}$$

Solution $z(t) = z_m e^{-\frac{c}{2d}t} \cos(\omega t + \phi)$

ϕ: phase shift "phi"

Two time-scales
1) Time constant: decaying time ; $t_d = \frac{2d}{c}$
   when $t = t_d$ → oscillation amplitude is decayed by a factor e
2) Period of oscillation $T = \frac{2\pi}{\omega}$ or time per oscillation

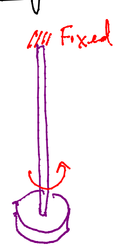


i) $T \ll t_d$
many oscillations before the amplitude decays by factor of e
"underdamped SHM"

ii) $T \sim t_d$
about one oscillation when amplitude is decayed by factor of e
"critical or resonant"

iii) $T \gg t_d$
can't observe oscillations as they are masked by the amplitude decay
"overdamped SHM"

2) Torsional pendulum



if bar is twisted, pendulum rotates back & forth about its vertical center axis

pendulum (gravity)

$\tau = -mgl \sin\theta$

torsional pendulum (torsion)

$\tau = -K \cdot \Delta\theta$ (torsional law)

K (kappa): torsional constant (size & material of the bar)

$\Delta\theta$: change of angle

τ: recovery torque by pendulum

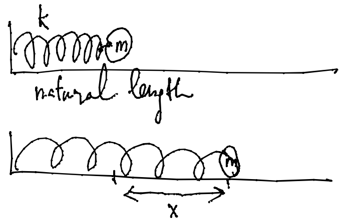
-: it opposes any twisting

Eq. of motion: $\tau_{net} = I \cdot \alpha$

$-K \cdot \theta = I \cdot \frac{d^2\theta}{dt^2} \rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{K}{I}\theta} \rightarrow \theta(t) = \theta_m \cos \omega t$

SHM: $\omega = \sqrt{\frac{K}{I}}$ (s^{-1})

3) Spring & bob:



Recovery force: $F = -k\cdot x$

k: spring constant ($\frac{N}{m}$)

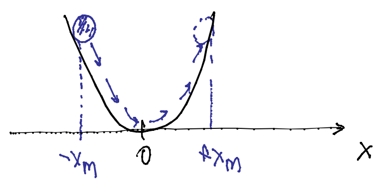
Linear eq of motion

$F_{net} = m\cdot a$

$-k\cdot x = m\cdot \frac{d^2x}{dt^2} \rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x} \rightarrow x(t) = x_m \cos \omega t$

SHM $\omega = \sqrt{\frac{k}{m}}$ (s^{-1})

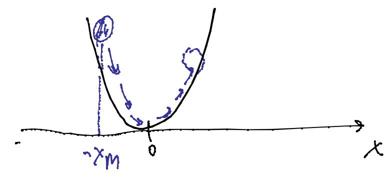
4) Particle trapped in potential well:



If there is no friction: SHM

Position along x-axis: $x(t) = x_m \cos(\omega t + \pi)$

($t=0 \rightarrow x(0) = -x_m$)



If there is friction → damped-SHM

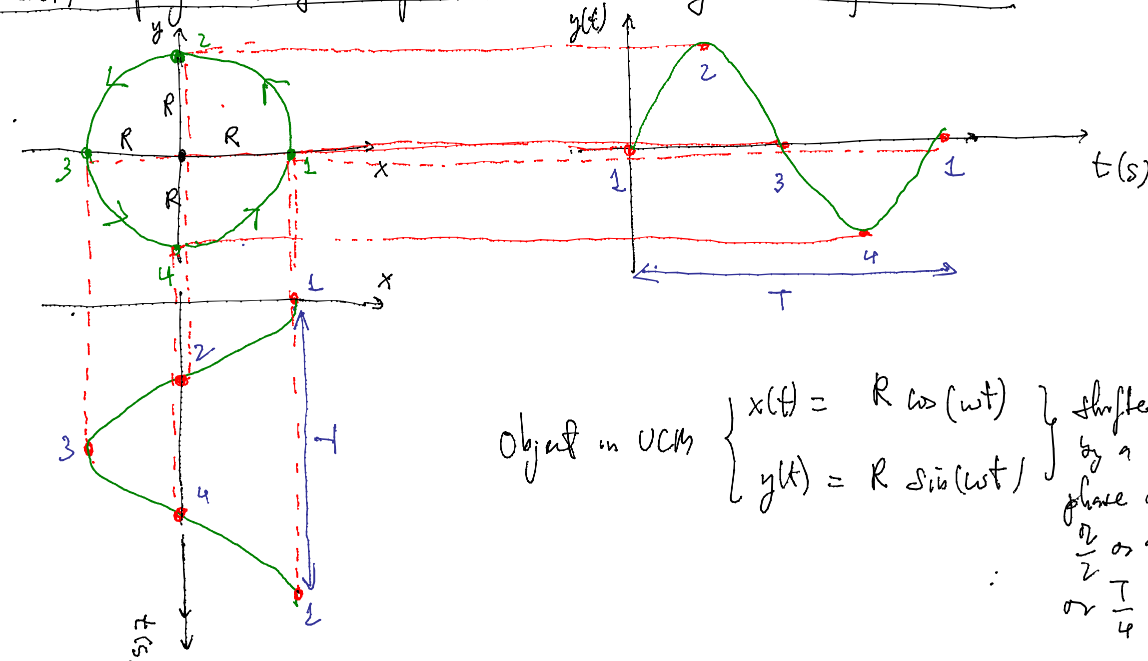
Position along x-axis:

$x(t) = \underbrace{x_m e^{-\frac{b}{2m}t}}$ $\cos(\omega t + \pi)$

amplitude of oscillation decays to 0 when $t \rightarrow \infty$

5) Object in UCM: projections of its position onto x- & y-axes follow SHM's

UCM



Object in UCM $\begin{cases} x(t) = R\cos(\omega t) \\ y(t) = R\sin(\omega t) \end{cases}$ shifted by a phase of $\frac{\pi}{2}$ or 90° or $\frac{T}{4}$

Ch 14 Wave Motion

Oscillation

Time repetition of linear or angular position, overall $\boxed{\text{local}}$. Object oscillates back & forth about certain point

1) local
2) Periodic variation in time

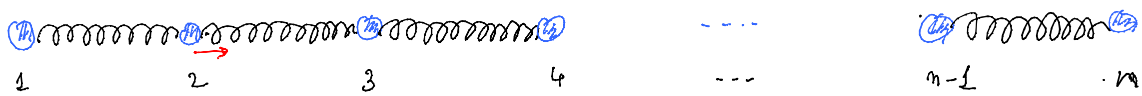
Wave

Oscillation also propagates in space
→ $\boxed{\text{non-local}}$

1) Non-local
2) Periodic variation in both space & time

Wave motion visual experiment:

Longitudinal wave: system of horizontal identical bobs connected by identical springs



1) I perturb bob#2 pulling it from equilibrium, along horizontal direction. It will follow a SHM. If n = 1000, what happends to bob #999 at that time? Still @ rest
   Oscillation is a local phenomenon!
2) Since bobs & springs are connected → oscillation propagats to bob #3, then #4, etc...
   Propagation takes time: it has finite speed (depends of medium, materials)
3) Here oscillation & propagation are in same direction (horizontal)
   → longitudinal wave
4) Motion of bob # 2 : is about its equilibrium position → { oscillation & local
   → What is propagating? energy or the wave

Wave motion: - propagation of the perturbation (energy), not of matter or material
- in wave motion, an object is not moving, only its perturbation is!

↓

Sound waves: { matter or object : air molecules
perturbation : change of air density → pressure

Light waves: { matter : none
perturbation : oscillation of electric & magnetic fields. (create each other)

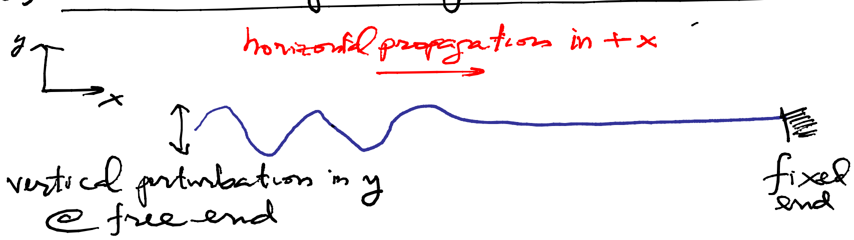
Linear motion	Rotational	Wave
Translation of CM	Rotational about CM	Object not moving neither rotating only its perturbation (energy) is moving

2) Types of waves:

(i) Longitudinal: perturbation & propagation are both in same direction.
Example: Springs & slinky, earthquake waves, etc.

(ii) Transverse: perturbation & propagation are perpendicular to each other
Examples: wave on a guitar string, water ripples, electromagnetic waves (light, radio, cell phone signals, etc --

3) Math description of a transverse wave:



$$\boxed{y(x,t) = A\sin(kx - \omega t)}$$

A : wave amplitude

k : wave number
number of wavelengths λ in 2π $\rightarrow k = \frac{2\pi}{\lambda}$ (SI unit: m^{-1})

λ : wavelength
wave is periodic in both time & space, space separation b/w two consecutive peaks is a wavelength λ (SI unit: m)

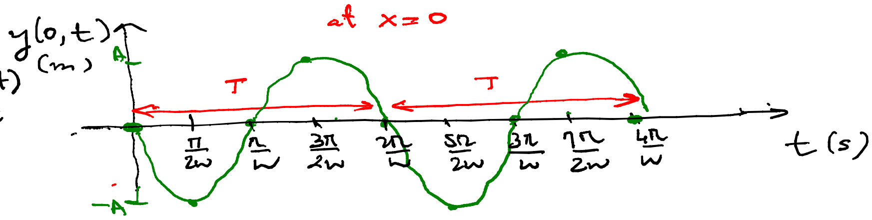
ω : angular frequency
number of periods T in 2π $\rightarrow \omega = \frac{2\pi}{T}$ (SI unit: s^{-1})

T : period
wave is periodic in both time & space, time separation b/w two consecutive peaks is a period T (SI unit: s)

4) Graphical description of a transverse wave :

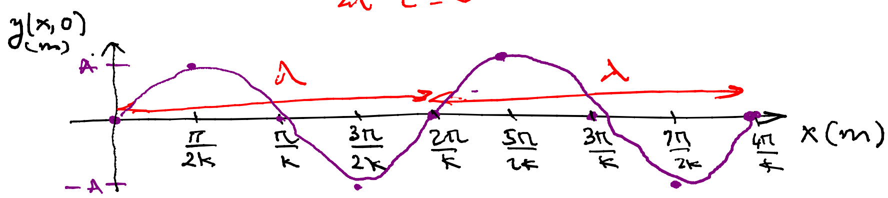
3D graphics → 2D profiles $\begin{cases}\text{fixed position} \\ \text{or} \\ \text{fixed time}\end{cases}$

$y(0,t) = A\sin(-\omega t)$
$= -A\sin\omega t$



This graph describes how the perturbation at point x=0 varies over time
It shows 2 periods

$y(x,0) = A\sin kx$



This graph describes a snapshot of wave profile at t=0
It shows 2 wavelengths

14.54] Wire, T = 32.8 N, carries wave: $y(x,t) = 1.75\sin(0.211x - 466t)$
x, y in cm, t in s.

Transverse wave: $y(x,t) = A\sin(kx - \omega t)$
↳ { perturbation in y
  { propagation in x

a) Wave amplitude : $A = 1.75$ cm

b) Wavelength : $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.211} = 29.78$ cm

c) Period : $T = \frac{2\pi}{\omega} = \frac{2\pi}{466} = 13.5\cdot10^{-3}$ s $= 13.5$ ms (mili seconds)

d) Wave speed : $v = \frac{\lambda}{T} = \frac{0.2978 \text{ m}}{13.5\cdot10^{-3}\text{ s}} = 22.09 \frac{\text{m}}{\text{s}}$
↓
it takes a period to travel a wavelength

speed of the transverse wave in this wire

For comparison : $65 \frac{\text{mi}}{\cancel{\text{h}}}\cdot\frac{1\cancel{\text{h}}}{3600\text{ s}}\cdot\frac{1609\text{ m}}{1\text{ mi}} = 18.1 \frac{\text{m}}{\text{s}}$

Alternative equation for wave speed : $v = \lambda\cdot f$

$f = \frac{1}{T} \longrightarrow v = \frac{\lambda}{T} = \lambda\cdot f$
$= \frac{\omega}{2\pi} = \frac{\frac{2\pi}{T}}{2\pi}$

e) Power carried by a wave : $\overline{P} = \frac{1}{2}\mu\omega^2A^2v$

μ: linear density of the wire
ω: wave's angular frequency
A: wave's amplitude
v = wave's speed

Speed of a transverse wave in a wire $v = \sqrt{\frac{T}{\mu}}$ (can be derived using 2nd Newton's Law)

T: tension in wire

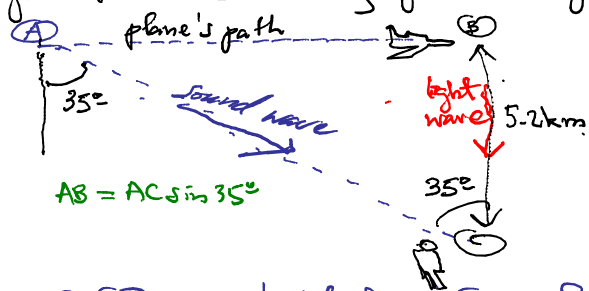
$$\bar{P} = \frac{1}{2}\mu\omega^2A^2 v = \frac{1}{2}\frac{T}{v^{\cancel{2}}}\omega^2A^2\cancel{v} = \frac{T\omega^2A^2}{2v} = \frac{32.8\cdot 466^2\cdot(1.75\cdot10^{-2})^2}{2\cdot 22.01} = 49.6\ \mathrm{W}$$

Tension! (99)

$$v^2 = \frac{T}{\mu} \to \mu = \frac{T}{v^2}$$

14.61) See airplane 5.2km straight overhead, sound seems to come from a point back along plane's path @ 35° to vertical

Step 1:



speed of airplane? v_{plane} ?

observer @ C sees light from B & hears sound from A

When observer sees plane @ B he hears the sound that the plane made when it was @ A (the sound it makes at B didn't reach him yet!)

Reason: $c = 3\cdot10^8\ \frac{m}{s}$; $v_s = 330\ \frac{m}{s}$ (~a million times slower)

↳ $t_{s_{AC}} = t_{\ell_{BC}}$ { $t_{s_{AC}}$: time for sound wave to travel AC; $t_{\ell_{BC}}$: time for light wave to travel BC }

Step 2: Solve $v_{plane} = \frac{d_{AB}}{t_{s_{AC}}} = \frac{d_{AB}}{t_{\ell_{BC}}}$

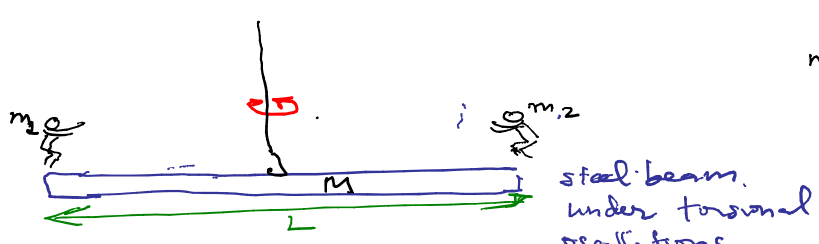
Step 3: $v_{plane} = \frac{d_{AB}}{\frac{d_{AC}}{v_s}} = \frac{\cancel{d_{AC}}\cdot\sin 35°\ v_s}{\cancel{d_{AC}}} = 330\cdot\sin 35° = 189\ \frac{m}{s}$

For comparison: $189\ \frac{\cancel{m}}{\cancel{s}}\cdot\frac{1\,mi}{1609\,\cancel{m}}\cdot\frac{3600\,\cancel{s}}{1\,h} = 423\ \frac{mi}{h}$

13.51)

$m_1 = m_2 = 82.4\ kg$

Step 1:



ω_0 = w/o steel workers
ω = w/ steel workers
} $\omega = 0.79\,\omega_0$ (diminished by 21%)

M?

Step 2: Torsional pendulum → SHM $\omega = \sqrt{\frac{\kappa}{I}}$ κ torsional constant

$I_0 = I_M = \frac{1}{12}ML^2$ (L: length of beam)

$I = \frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2\times 2$

$$\frac{w}{w_0} = 0.79 = \frac{\sqrt{\frac{K}{I}}}{\sqrt{\frac{K}{I_0}}} = \sqrt{\frac{I_0}{I}} \rightarrow 0.79^2 = \frac{I_0}{I} = \frac{\frac{1}{12}M\cancel{L^2}}{\frac{1}{12}M\cancel{L^2} + \frac{1}{2}m\cancel{L^2}}$$

$$0.79^2 = \frac{M}{M + 6m} \rightarrow M(1 - 0.79^2) = 0.79^2 \cdot 6 \cdot m$$

$$M = \frac{0.79^2 \cdot 6 \cdot m}{1 - 0.79^2}$$

Step 3:

$$M = \underbrace{\frac{0.79^2 \cdot 6}{1 - 0.79^2}}_{9.96} 82.4\,kg = 820.84\,kg$$

(100)