Ch 5 Applications of Newton's Laws

1) Static equilibrium ($\vec{F}_{net} = 0$)
2) Multiple objects ($\vec{F}_{net,i} = m_i \vec{a}_i$: "i" object i)
3) Frictional forces ($\vec{F}_s = \mu_s \vec{N}$; $\vec{F}_k = \mu_k \vec{N}$)
4) Circular motion

Solution strategies: 3-step process

Step 1) Understand the problem & information given

Step 2) a) Convenient coordinate system / b) Free-body diagram (net force) / c) Write Newton's 2^{nd} Law for each object

2a) Convenient = simplifies the analysis
- i) Coord. system where most force point along its axes (no need to project components $F_x = F\cos\theta$; $F_y = F\sin\theta$)
- ii) Motion of interest should be along an axis

2b) FBD: when we only need $\vec{F}_{net}$ on a body, we replace it with a dot (free-body) then draw all forces applied on the dot using arrows.

- (i) Once we have FBD, can quickly add force to find $\vec{F}_{net}$
- (ii) Can easily see which force is off axes (inclined) so to project its components onto the axes.

2c) Write Newton's 2^{nd} Law for each object along each direction in the chosen coord. system

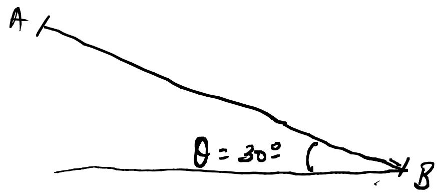
$$2D: \quad \vec{F}_{net} = m\vec{a} \quad \begin{cases} F_{net,x} = m a_x \\ F_{net,y} = m a_y \end{cases}$$

(m is a scalar, no components)

Step 3) Solve for unknown(s) by plugging given numeric values in correct units. Then check if your answers make sense.

PP 5.1 A scooter starts from rest at top of 30° iced slope, 10 m long, ignore friction. How long does it take to reach bottom?

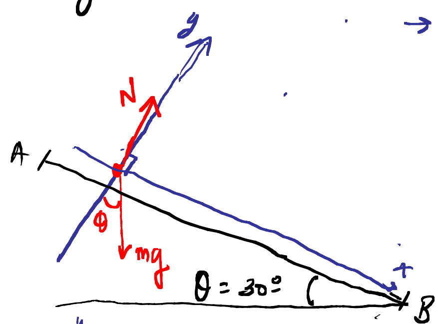
Step 1)



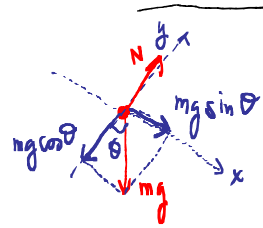
$AB = 10\,m$, $v_A = 0$
No friction
t_{AB} ?

Scooter will slide down the slope due to gravity, with some acceleration, calculate t_{AB} to go 10 m.

Step 2a) Convenient coordinate system: 2D : since motion of interest is along slope → x axis will be ∥ slope & y axis ⊥ slope as shown.



Step 2b) FBD



(i) Project mg into its cartesian components

(ii) $F_{net,x} = mg\sin\theta$

$F_{net,y} = N - mg\cos\theta = 0$

Step 2c) 2nd Newton's Law: $\vec{F}_{net} = m\vec{a}$

$$\begin{cases} F_{net,x} = \cancel{m}g\sin\theta = \cancel{m}a \\ F_{net,y} = N - mg\cos\theta = 0 \end{cases}$$

Step 3) Solve for a : $a = g\sin\theta$

→ kinematic eq 2): $x - x_0 = v_{0x}t + \frac{1}{2}at^2$

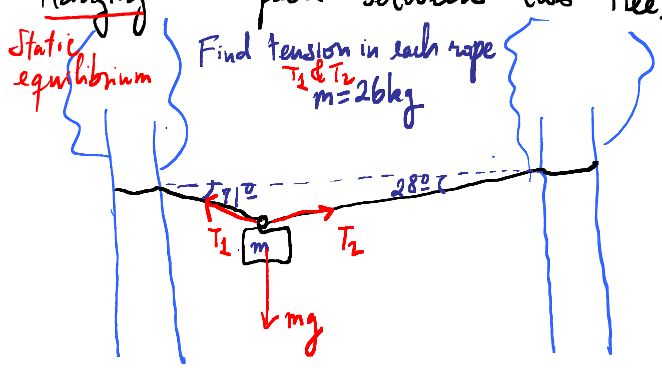
$10\,m = \quad + \frac{1}{2}g\sin\theta\, t^2 \rightarrow t = \sqrt{\frac{2\cdot 10}{9.81\sin 30}}$

$t = \sqrt{\frac{40}{9.81}} \simeq 2\,s$

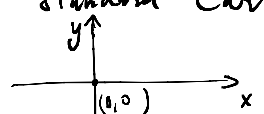
1) Static equilibrium: $\vec{a} = 0 \leftrightarrow$ Newton's 2^{nd} Law: $\vec{F}_{net} = 0 \Rightarrow \begin{cases} F_{net,x} = 0 \\ F_{net,y} = 0 \end{cases}$

5.38 | Hanging a pack between two trees using separate ropes of different lengths

Step 1:



Step 2a: Most convenient coord. system: since angle b/w $\vec{T}_1$ & $\vec{T}_2$ are not 90° our best option is standard Cartesian coord. system where its y-axis is along vertical



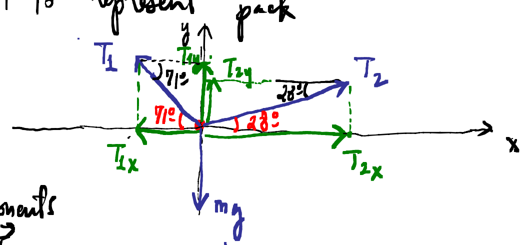
Step 2b: Dot to represent pack

(Parallel lines: alternate interior angles are equal)

FBD

↓

Need components for $\vec{T}_1$, $\vec{T}_2$



$$\begin{cases} \vec{T}_1 = T_{1x}\hat{\imath} + T_{1y}\hat{\jmath} = T_1\cos 71°\,\hat{\imath} + T_1 \sin 71°\,\hat{\jmath} \\ \vec{T}_2 = T_{2x}\hat{\imath} + T_{2y}\hat{\jmath} = T_2\cos 28°\,\hat{\imath} + T_2 \sin 28°\,\hat{\jmath} \end{cases}$$

$\vec{F}_{net}$ on pack

$$\begin{cases} F_{net,x} = T_{2x} - T_{1x} = T_2\cos 28° - T_1 \cos 71° \\ F_{net,y} = T_{1y} + T_{2y} - mg = T_1 \sin 71° + T_2 \sin 28° - mg \end{cases}$$

Step 2c: Write 2^{nd} Newton's Law in each direction, for back pack

$$F_{net,x} = ma_x = m\cdot 0 = 0 \rightarrow T_2\cos 28° - T_1\cos 71° = 0 \quad (1)$$

$$F_{net,y} = ma_y = m\cdot 0 = 0 \rightarrow T_1 \sin 71° + T_2 \sin 28° - mg = 0 \quad (2)$$

Step 3: Solve for T_1 & T_2 ($m = 26$kg): system of 2 linear equations with 2 unknowns

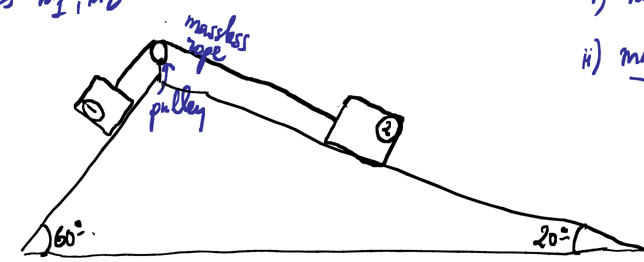
Solve for T_1 from (1): $T_1 = T_2 \dfrac{\cos 28°}{\cos 71°} \longrightarrow$ in (2): $T_2 \dfrac{\cos 28° \cdot \sin 71°}{\cos 71°} + T_2 \sin 28° = 26 \cdot 9.81$

$$\rightarrow T_2 = \frac{26 \cdot 9.81}{\cos 28° \tan 71° + \sin 28°} = 84\text{N} \rightarrow \text{in (1)}\ T_1 = 84 \cdot \frac{\cos 28°}{\cos 71°} = 228\text{ N}$$

$T_1 = 228$N is almost three times $T_2 = 84$N (yes since pack is much closer to tree #1 & rope angle is much larger there)

2) Multiple objects: $\vec{F}_{net,i} = m\vec{a}_i$ (Apply 2^{nd} Newton's Law on each object!)

Step 1: Two boxes of masses m_1, m_2 connected by a massless rope slope angles are 60° & 20°



i) no friction b/w boxes and slopes

ii) massless rope: its mass is negligible compared to m_1 & m_2

→ tension is same throughout rope

Describe motion of these two boxes

(i) Rope going CW @ pulley (m_1 up & m_2 down)

(ii) Rope going CCW @ pulley (m_1 down & m_2 up)

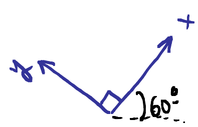
(iii) Static equilibrium, $a=0$

We will start analysis assuming (i), sign & value for a will tell us what is the actual outcome

- $a>0$ → confirms (i)
- $a<0$ → (ii)
- $a=0$ → (iii)

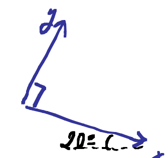
Step 2a: Select most convenient coord. system: on an incline direction of motion should be along an axis

Box #1



Motion along incline 60°

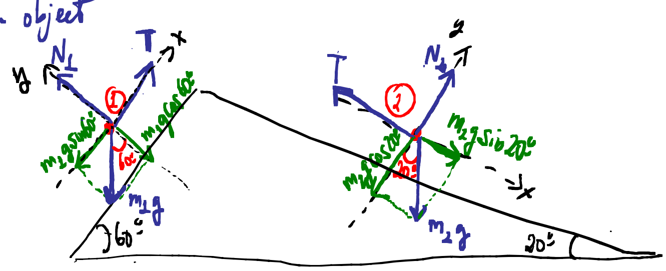
Box #2



Motion along incline 20°

Step 2b: FBD for each object

Components for $m_1 g$ & $m_2 g$



$$\vec{F}_{net,1} = \begin{cases} F_{net,1,x} = T - m_1 g \sin 60^\circ \\ F_{net,1,y} = N_1 - m_1 g \cos 60^\circ \end{cases}$$

$$\vec{F}_{net,2} = \begin{cases} F_{net,2,x} = m_2 g \sin 20^\circ - T \\ F_{net,2,y} = N_2 - m_2 g \cos 20^\circ \end{cases}$$

Step 2c: complete 2^{nd} Newton's Law for each box: $\vec{F}_{net,i} = m_i \vec{a}$ ($\vec{a}_i = \vec{a}$ since two boxes are connected by a rope)

Box #1			Box #2		
	$T - m_1 g \sin 60^\circ = m_1 a$	(1)		$m_2 g \sin 20^\circ - T = m_2 a$	(3)
	$N_1 - m_1 g \cos 60^\circ = 0$	(2)		$N_2 - m_2 g \cos 20^\circ = 0$	(4)

Step 3: solve for unknowns

For example masses are given, find acceleration of the system

↳ unknowns $\left\{ \begin{array}{c} a \\ T \\ N_1 \\ N_2 \end{array} \right\}$ with 4 equations ✓

↳ Solution: i) let's eliminate T from (1): $T = m_1 a + m_1 g \sin 60°$

ii) plug this into (3): $m_2 g \sin 20° - m_1 a - m_1 g \sin 60° = m_2 a$

$$\boxed{a = \frac{m_2 g \sin 20° - m_1 g \sin 60°}{m_1 + m_2}}$$

Note: the "−" sign here is very important, if it was a "+" sign it would be incorrect since a would never be negative, which is perfectly possible if m_1 is sufficiently heavier than m_2 when system turns left @ pulley and a<0

Options:

(i) $m_2 g \sin 20° > m_1 g \sin 60°$ or $\frac{m_2}{m_1} > \frac{\sin 60°}{\sin 20°} = 2.53 \rightarrow a > 0 \rightarrow$ system turns CW @ pulley

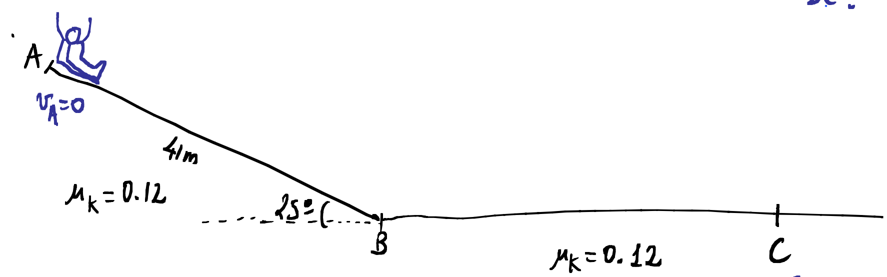
(ii) $m_2 g \sin 20° < m_1 g \sin 60°$ or $\frac{m_2}{m_1} < 2.53 \rightarrow a < 0 \rightarrow$ system turns CCW @ pulley

(iii) $m_2 g \sin 20° = m_1 g \sin 60°$ or $\frac{m_2}{m_1} = 2.53 \rightarrow a = 0 \rightarrow$ static equilibrium

3) Frictional forces :

Child sledding down a hill of 41m length & 25° angle, then on a flat bottom till it stops. $\mu_k = 0.12$ distance traveled on flat bottom? BC?

Step 1:



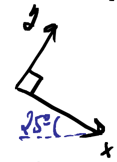
Understand the problem
- (i) A to B : constant acceleration motion, gravity contribution is larger than frictional force.
- (ii) B to C : constant deceleration motion, gravity is cancelled by normal force, only frictional force in opposite direction as motion
- (iii) at C : $v_C = 0$ makes sense!

↓

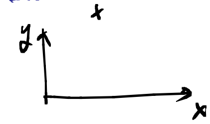
what physics will explain this problem (we will use in step 3)
1) kinematic eq. constant acceleration
2) 2^{nd} Newton's Law

Step 2a: Convenient coord. system :

A to B :



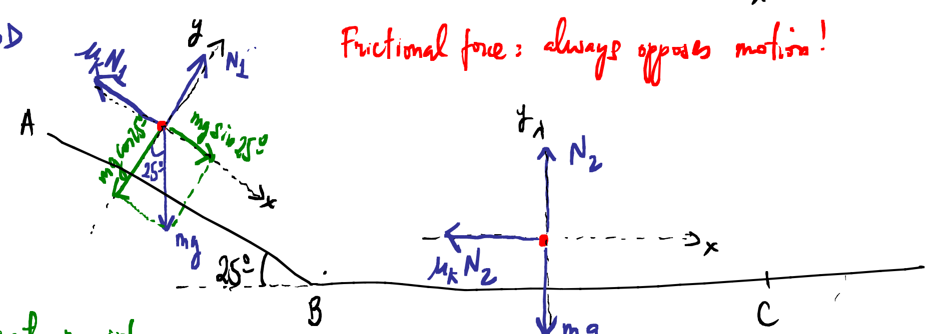
B to C :



Direction of motion along x-axis for each part of trajectory.

Step 2b: FBD

Frictional force : always opposes motion!



Project mg into x & y components

No need for components for any force here!

A to B
$$\begin{cases} F_{net,x} = mg\sin 25° - \mu_k N_1 \\ F_{net,y} = N_1 - mg\cos 25° \end{cases}$$

B to C
$$\begin{cases} F_{net,x} = -\mu_k N_2 \\ F_{net,y} = N_2 - mg \end{cases}$$

Step 2c: Complete 2^{nd} Newton's Law : $\vec{F}_{net} = m\vec{a}$

A to B
$$\begin{cases} mg\sin 25° - \mu_k N_1 = m a_1 & (1) \\ N_1 - mg\cos 25° = 0 & (2) \end{cases}$$

B to C
$$\begin{cases} -\mu_k N_2 = m a_2 & (3) \\ N_2 - mg = 0 & (4) \end{cases}$$

Step 3 : Need BC : kinematic equation # 3: $\frac{v^2 - v_0^2}{x - x_0} = 2\cdot a \rightarrow \frac{\overset{0}{v_C^2} - v_B^2}{(x-x_0)_{BC}} = 2\cdot a_2$

↳ Need
- $a_2 \rightarrow$ From 2^{nd} Newton's Law
- v_B using another kin. eq. #3 b/w A&B : $\frac{v_B^2 - \overset{0}{v_A^2}}{41\,m} = 2\cdot a_1$

<u>Step 3</u>: Need BC : Kinematic equation # 3: $\frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a \rightarrow \frac{\overset{0}{\uparrow}{v_C}^2 - v_B^2}{(x-x_0)_{BC}} = 2 \cdot a_2$

↳ Need $\begin{cases} a_2 \rightarrow \text{From } 2^{nd} \text{ Newton's Law} \\ v_B \text{ using another kin. eq. \#3 b/w A \& B}: \frac{v_B^2 - \overset{0}{\uparrow}{v_A}^2}{41\,m} = 2 \cdot a_1 \end{cases}$

To calculate v_B we need a_1 → from Newton's 2nd Law equations 1) & 2):

$$A \text{ to } B \begin{cases} mg\sin 25° - \mu_k N_1 = m a_1 \quad (1) \rightarrow g\sin 25° - \mu_k g\cos 25° = a_1 \\ N_1 - mg\cos 25° = 0 \quad (2) \rightarrow N_1 = mg\cos 25° \end{cases}$$

$$\Rightarrow v_B = \sqrt{2 \cdot (x-x_0)_{AB} \cdot a_1} = \sqrt{2 \cdot (x-x_0)_{AB} \cdot g\,(\sin 25° - \mu_k \cos 25°)} = \sqrt{2 \cdot 41 \cdot 9.81\,(\sin 25° - 0.12 \cdot \cos 25°)}$$

$v_B = 15.9 \frac{m}{s}$

To calculate a_2 use Newton's 2nd Law equations 3) & 4)

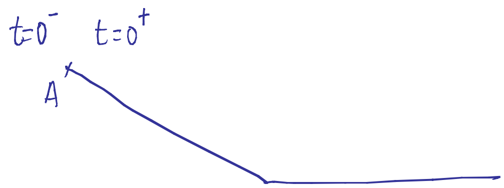
$$B \text{ to } C \begin{cases} -\mu_k N_2 = m a_2 \quad (3) \rightarrow -\mu_k \cancel{m} g = \cancel{m} a_2 \rightarrow a_2 = -\mu_k g \\ N_2 - mg = 0 \quad (4) \rightarrow N_2 = mg \end{cases}$$

To calculate $(x-x_0)_{BC}$ → Kinematic eq #3: $\frac{\overset{0}{\uparrow}{v_C}^2 - v_B^2}{(x-x_0)_{BC}} = 2 \cdot a_2$

$$\Rightarrow (x-x_0)_{BC} = \frac{0 - v_B^2}{2 a_2} = -\frac{15.9^2}{2 \cdot (-0.12 \cdot 9.81)} = 107\ m$$

This is the distance child & sled will travel on flat bottom till they stop.

<u>Question</u>: since system started @ A w/ $v_A = 0$ do we need to look at static friction?



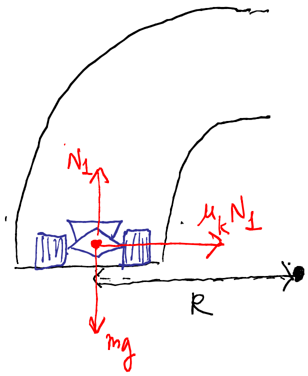
i) $t = 0^+$ only kinetic friction, system already in motion

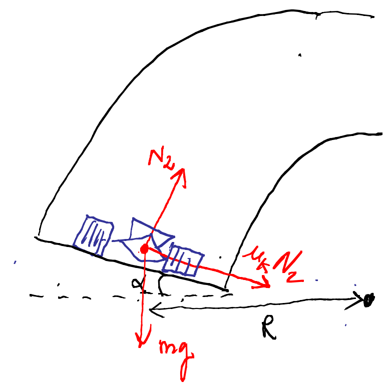
ii) child & sled can be assumed to sit @ A for short time & pressure would melt ice not much of bonding b/w sled & ice for static friction

4) Circular motion: with friction

Race car tracks:

Step 1: Flat turn — Slanted turn

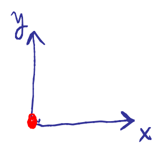




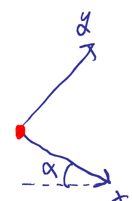
i) Race Cars take a turn in UCM, since they go at technical speed limit. If you exceed technical speed limit car will slide off

ii) In UCM a centripetal acceleration is needed $\left(a = \frac{v^2}{R}\right)$ which is provided by the force of friction: $\mu_k N = m \cdot \frac{v^2}{R} \rightarrow v_{max} = \sqrt{\frac{\mu_k N R}{m}}$

Step 2a: most convenient coord. system for each case:

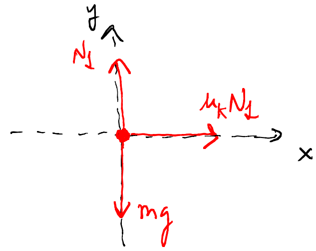


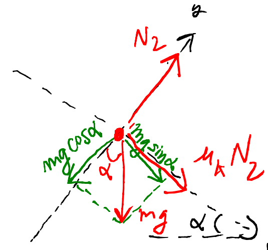
Flat track



Slanted track

Step 2b: FBD





$\vec{F}_{net}$:

$$\begin{cases} F_{net,x} = \mu_k N_1 \\ F_{net,y} = N_1 - mg \end{cases}$$

$$\begin{cases} F_{net,x} = \mu_k N_2 \boxed{+ \; mg\sin\alpha} \\ F_{net,y} = N_2 - mg\boxed{\cos\alpha} \end{cases}$$

Step 2c: Complete Newton's 2nd Law by adding RHS = ma

Flat track

$$\begin{cases} \mu_k N_1 = m \cdot \frac{v_F^2}{R} \\ N_1 - mg = 0 \rightarrow N_1 = mg \end{cases}$$

Slanted track

$$\mu_k N_2 + mg\sin\alpha = m \cdot \frac{v_S^2}{R}$$

$$N_2 - mg\cos\alpha = 0 \rightarrow N_2 = mg\cos\alpha$$

Step 3: solve for v:

$$v_F = \sqrt{\mu_k g R}$$

$$v_S = \sqrt{gR(\mu_k \cos\alpha + \sin\alpha)}$$

Example: $\alpha = 20°$, $\mu_K = 0.2$

$v_F = \sqrt{0.2 \cdot 9.81 \cdot R}$ $\quad$ $v_S = \sqrt{9.81 \cdot R\left(0.2 \cdot \cos 20° + \sin 20°\right)}$

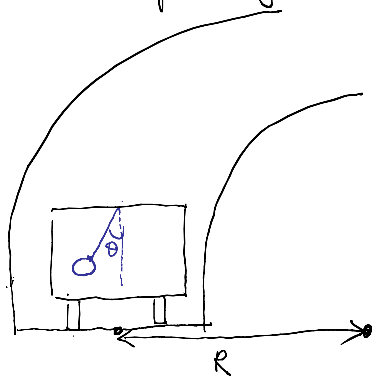
$$\frac{v_S}{v_F} = \sqrt{\frac{0.2\cos 20° + \sin 20°}{0.2}} = 1.63$$ → slanted track allows higher physics speed limit @ turns!

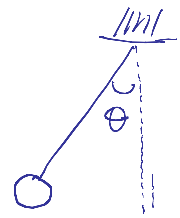
5.27 Train derailed when taking a turn (flat or unbanked) $\left\{ R = 150\,m,\ v_{legal} = 35\,\frac{km}{h} \right.$

Unused strap hanged @ $\theta = 15°$ to vertical → calculate v to compare with v_{legal}

Step 1:

view from back of wagon



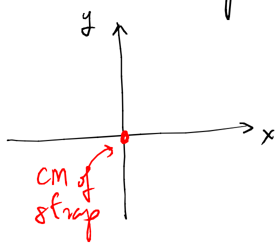


i) Train & strap in UCM → centripetal acceleration $a = \frac{v^2}{R}$

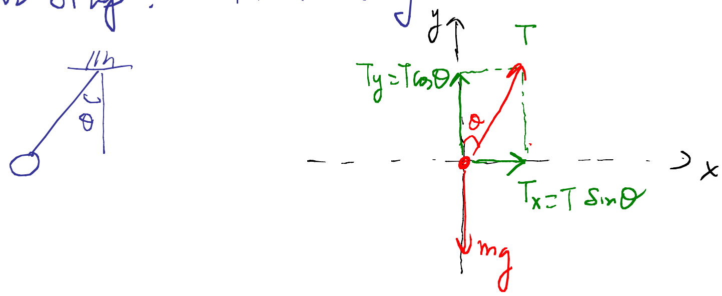
ii) 2nd Newton's Law: $\boxed{F_{net}} = m \cdot \frac{v^2}{R}$

To find v we need F_{net} on strap!

Step 2a: convenient coord. system: direction of motion: centripetal acceleration is toward center of curvature



Step 2b: FBD for strap: CM in ring



$$\vec{F}_{net} = \begin{cases} F_{net,x} = T\sin\theta \\ F_{net,y} = T\cos\theta - mg \end{cases}$$

Step 2c: Complete RHS of 2nd Newton's Law:
$$\begin{cases} F_{net,x} = T\sin\theta = m\frac{v^2}{R} & (1) \\ F_{net,y} = T\cos\theta - mg = 0 & (2) \end{cases}$$

Step 3: solve for v: $\quad$ (2) $T = \frac{mg}{\cos\theta}$ → (1) $\cancel{m}g\tan\theta = \cancel{m}\frac{v^2}{R}$ → $v = \sqrt{gR\tan\theta}$

Speed of strap (& train) at time of derailment was $v = \sqrt{9.81 \cdot 150 \cdot \tan 15°} = 19.86\,\frac{m}{s}$

$v = 19.86\,\frac{\cancel{m}}{\cancel{s}} \cdot \frac{3600\cancel{s}}{1h} \cdot \frac{1km}{1000\cancel{m}} = 19.86 \cdot 3.6\,\frac{km}{h} = 71.48\,\frac{km}{h} > v_{legal} = 35\,\frac{km}{h}$!

Ch 6 Work, Energy, and Power

Analyze motion of an object
- i) Kinematic equations (x, v, a, t) (Ch. 2 & 3)
- ii) Newton's Laws (F_{net}, m, a) (ch 4 & 5)
- iii) Work & energy → conservation of mechanical energy or sum of kinetic & potential energy (Ch6&7)
  - ↳ very useful, for example when analyzing motion down an irregular slope (angle is unknown)

Work & scalar product: when a constant force $\vec{F}$ is applied on an object causing a displacement $\Delta\vec{r}$ (or change in position vector), work $\equiv \vec{F}\cdot\Delta\vec{r}$ ($\vec{F}$: vector, $\Delta\vec{r}$: vector)

Scalar product:
- i) product b/w two vectors → work equals "$\vec{F}$ dot $\Delta\vec{r}$"
- ii) Scalar product b/w $\vec{A}$ & $\vec{B}$ is "$\vec{A}$ dot $\vec{B}$":

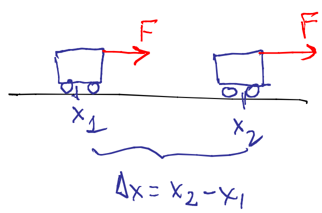
$$\underset{\text{vector}}{\vec{A}}\cdot\underset{\text{vector}}{\vec{B}} = \underbrace{AB\cos\theta}_{\text{scalar or number}}$$

$A \equiv |\vec{A}|$ or magnitude of $\vec{A}$
$B \equiv |\vec{B}|$ or magnitude of $\vec{B}$
θ is angle b/w vector $\vec{A}$ & vector $\vec{B}$

Note: if $\vec{A}\perp\vec{B} \rightarrow \theta = 90°$ $\rightarrow \cos 90° = 0$ $\rightarrow \vec{A}\cdot\vec{B} = 0$

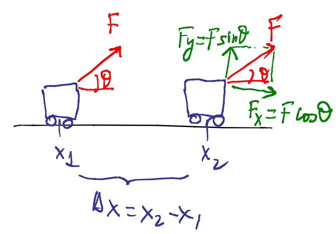
- iii) Scalar product produces a scalar or number
  Work & energy (& power) are scalar

Work:
- i) Unit: $N\cdot m \equiv J$ (Joule)
- ii) Work $= \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta$: work is affected by F, Δr, and θ



F points in same direction as displacement → $\theta = 0°$

Work $= F\Delta x\cos 0 = F\Delta x$



F point @ angle θ from direction of displacement

Work $= F\Delta x\cos\theta$
$= \underbrace{F\cos\theta}\,\Delta x$
$= F_x\cdot\Delta x$

i) Only that component of force applied in direction of displacement produces work

ii) $F_y = F\sin\theta$ doesn't produce work since it is perpendicular to direction of displacement

iii) Technically F_y "produces" indirectly some work by reducing N so reducing $\mu_k N$ making work done by F_x more efficient

Work and Power:

Car #1 $\quad \begin{cases} m_1 = m_2 = m \\ v_{0_1} = v_{0_2} = 0 \\ v_1 = v_2 = 40\ \frac{mi}{h} \end{cases} \quad$ Car #2

$W_1 \quad = \quad W_2$

$t_1 \qquad\qquad t_2 = \frac{t_1}{2}$

$\Rightarrow P_1 = \frac{P_2}{2}$

Car #2 produces same amount of work in half time → its power is double that of car #1

Average power : $\bar{P} = \frac{\Delta W}{\Delta t}$ (work per unit time)

Instantaneous power $P = \frac{dW}{dt}$ (time derivative of work)

Power & Velocity:

$$P = \frac{d}{dt}(\underset{\text{constant}}{\vec{F}} \cdot \Delta\vec{r}) = \vec{F} \cdot \underbrace{\frac{d\Delta\vec{r}}{dt}}_{\vec{v}} = \vec{F} \cdot \vec{v}$$

SI unit : $N \cdot \frac{m}{s} = \frac{J}{s} = W$ (Watt) $\quad \begin{cases} 1\ H.P. = 746W \\ 1\ \frac{Btu}{h} = 0.293W \end{cases}$

Cars $\begin{cases} \text{sedan } P \cong 150\ H.P. \\ \text{SUV } P \cong 250\ H.P. \end{cases}$

Non-constant Force:

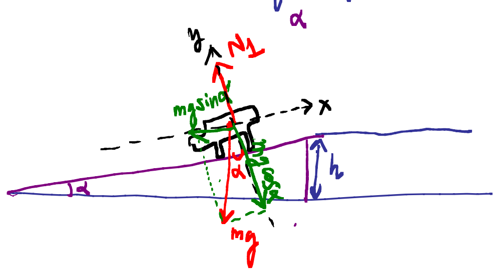
Work = $\begin{cases} \vec{F} \cdot \Delta\vec{r} & (\vec{F} \text{ constant along displacement}) \\ \int \vec{F} \cdot d\vec{r} & (\vec{F} \text{ varying along displacement}) \end{cases}$

example: spring force : $F_s = -k\Delta x$

↳ work by spring $= -k \int_1^2 x\, dx = -\frac{1}{2}k(x_2^2 - x_1^2)$

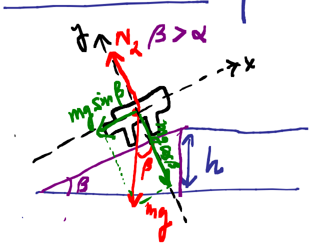
Piano mover:

#1 long ramp α



$F_{p_1} = mg\sin\alpha$

#2 short ramp



$F_{p_2} = mg\sin\beta > F_{p_1}$!

Steeper ramp requires stronger push

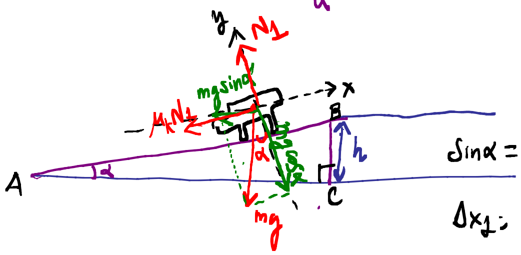
Piano mover:

#1 long ramp

α

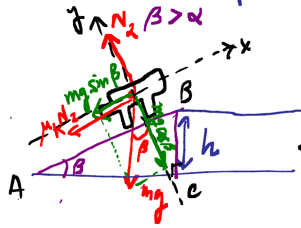
Coefficient of kinetic friction: μ_K

#2 short ramp



$\sin\alpha = \dfrac{h}{\Delta x_1}$ (ΔABC)

Δx_1: long ramp length



$\sin\beta = \dfrac{h}{\Delta x_2}$ (ΔABC)

Δx_2 = short ramp length

$F_{P_1} = mg\sin\alpha + \mu_K N_1$
$\quad = mg(\sin\alpha + \mu_K\cos\alpha)$

$F_{P_2} = mg\sin\beta + \mu_K N_2 = mg(\sin\beta + \mu_K\cos\beta)$

Steeper ramp requires stronger push

i) We assume F_{P_1} & F_{P_2} are constant along displacement $\begin{cases} \#1: \Delta x_1 = \dfrac{h}{\sin\alpha} \\ \#2: \Delta x_2 = \dfrac{h}{\sin\beta} \end{cases}$

ii) $F_{P_1} \parallel \Delta x_1$ & $F_{P_2} \parallel \Delta x_2$: both forces are in the direction of displacement $\rightarrow \theta = 0° \rightarrow \begin{cases} \#1: \vec{F}_{P_1} \cdot \Delta\vec{r}_1 = F_{P_1}\Delta x_1 \\ \#2: \vec{F}_{P_2} \cdot \Delta\vec{r}_2 = F_{P_2}\Delta x_2 \end{cases}$

#1

$W_1 = F_{P_1}\Delta x_1 = mg(\sin\alpha + \mu_K\cos\alpha)\cdot\dfrac{h}{\sin\alpha}$

$W_1 = mgh\left(1 + \dfrac{\mu_K}{\tan\alpha}\right)$

#2

$W_2 = F_{P_2}\cdot\Delta x_2 = mg(\sin\beta + \mu_K\cos\beta)\dfrac{h}{\sin\beta}$

$W_2 = mgh\left(1 + \dfrac{\mu_K}{\tan\beta}\right)$

i) If friction is negligible $\mu_K \simeq 0 \rightarrow W_1 = W_2$

ii) If friction is not negligible: $\dfrac{W_1}{W_2} = \dfrac{1 + \dfrac{\mu_K}{\tan\alpha}}{1 + \dfrac{\mu_K}{\tan\beta}}$

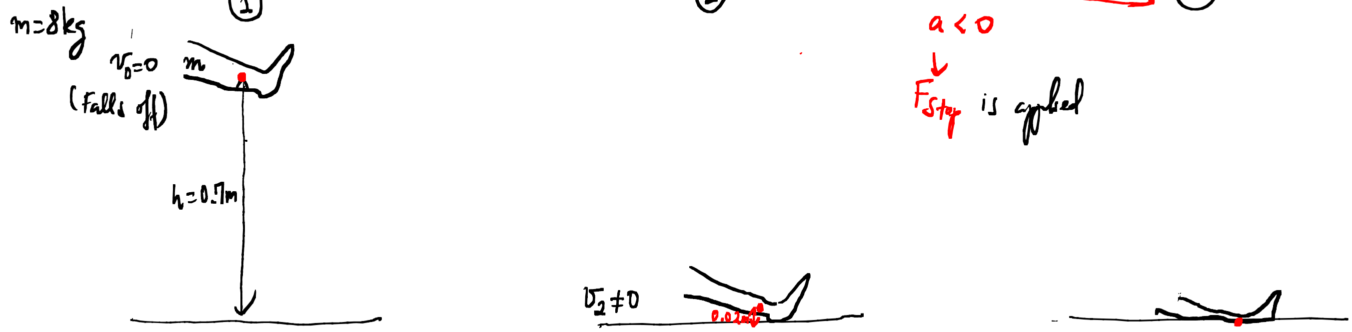
$\left\{\begin{array}{l} \text{Example: } \mu_K = 0.62 \\ \alpha = 15° \\ \beta = 30° \\ \dfrac{W_1}{W_2} = \dfrac{28.5}{17.8} = 1.6 \end{array}\right.$

Long ramp	Short ramp
less force	more force
more work	less work

6.89

Step 1: Falling leg
$\begin{cases} 1^{st}) & \text{Free fall during first } 0.7m \text{ until heel is about to touch the floor} \\ 2^{nd}) & \text{Crashing against floor till a complete stop, during } 0.02m \end{cases}$

Leg about to fall off stretcher ① — Heel touches floor ② → Leg came to complete stop ③



Free fall : a=g
constant acceleration

- Not yet at complete stop
- CM still above floor
- Max speed v_2 before the crash

- Leg came to complete stop
- CM @ floor level
- Zero speed $v_3=0$

Step 2: physics :

Alternative #1 : Work & Energy :

①

$KE_1 = \frac{1}{2}mv_0^2 = 0$

$PE_1 = mgh_1$

gravitational potential energy :

$ME_1 = KE_1 + PE_1 = mgh_1$

②

$KE_2 = \frac{1}{2}mv_2^2$

$PE_2 = mgh_2 \quad (h_2 = 0.02m)$

$\boxed{ME_1 = ME_2} = \underbrace{\frac{1}{2}mv_2^2}_{\text{big part}} + \underbrace{mgh_2}_{\text{very small}}$

③

$KE_3 = \frac{1}{2}mv_3^2 = 0$

$PE_3 = 0 \quad (h_3 = 0 \text{ @ floor})$

$ME_3 = 0$

(where did total mechanical energy go ? → damage leg (bruises, broken bones) & heat up floor)

Leg looses all its energy (mgh_1) during a distance of $\Delta x = 0.02m$ to come to complete stop, agent to cause this change of motion is F_{stop}

$$mgh_1 = F_{stop} \cdot \Delta x \rightarrow F_{stop} = \frac{mgh_1}{\Delta x} = \frac{8 \cdot 9.81 \cdot 0.7}{0.02} = 2744\,N$$

much larger than 80N !

Alternative #2 : physics $\begin{cases} \text{Kinematic eq. for constant acceleration} \\ 2^{nd} \text{ Newton's Law} \end{cases}$

① → ② Free fall a=g , no time information → kin. eq. #3 $\frac{v_2^2 - 0}{h} = 2 \cdot g \rightarrow v_2 = \sqrt{2gh}$

$= \sqrt{2 \cdot 9.81 \cdot 0.7}$

$= 3.7 \frac{m}{s}$

② → ③ Constant deceleration, no time info → kin. eq #3 $\frac{0 - v_2^2}{\Delta x} = 2a \rightarrow a = -\frac{v_2^2}{2\Delta x}$

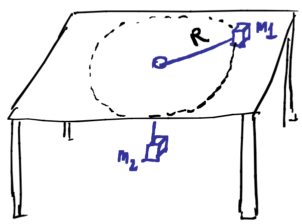
$= \frac{-3.7^2}{2 \cdot 0.02}$

$a = -343.35 \frac{m}{s^2}$

2^{nd} Newton's Law: $F_{Stop} = m \cdot a = 8 \cdot (-343.35) = -2774\,N$

5.39

Step 1:

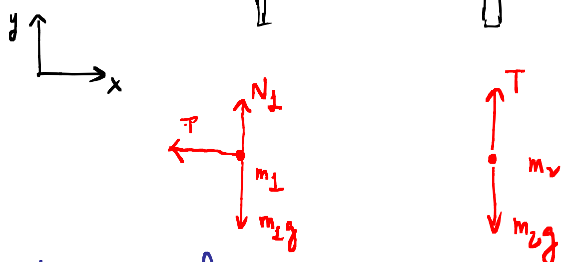


UCM m_1 & stationary m_2 connected by a massless string → same Tension T

a) T?   b) period of circular motion for m_1

Step 2a)

Step 2b)



Step 2c) 2^{nd} Newton's Law for each object

① $\begin{cases} T = m_1 \frac{v_1^2}{R} \\ N_1 - m_1 g = 0 \end{cases}$ ② $\{ T - m_2 g = m \cdot 0 = 0$

since m_2 stationary

T provides centripetal acceleration for m_1 to stay in UCM

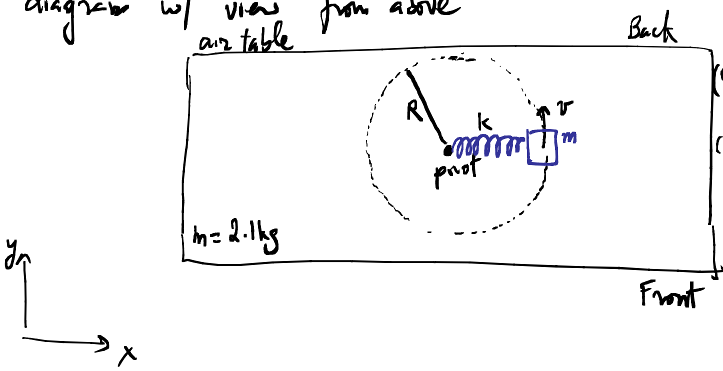
Step 3) a) Solve for tension T: $T = m_2 g$

b) Period for UCM of m_1 $= \frac{2\pi R}{v_1}$ ⟶ period $= \frac{2\pi\sqrt{R}}{\sqrt{\frac{m_2}{m_1} g}}$

$m_1 \frac{v_1^2}{R} = m_2 g$ (same tension) → $v_1 = \sqrt{\frac{m_2}{m_1} g R}$

5.67 Mass m attached to a spring $k = 150\frac{N}{m}$, they are on air cushion → no friction, m in UCM $v = 1.4\frac{m}{s}$ → R?

Step 1: diagram w/ view from above

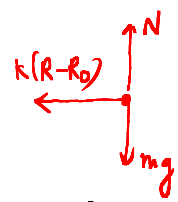


(i) spring w/ unstretched length 0.18 m ⇒ $R_0 = 0.18\,m$

(ii) m in UCM, spring is stretched to R will pull $-k(R - R_0)$ that provides centripetal acceleration for m $(a = \frac{v^2}{R})$

Step 2a:

Step 2b: FBD : view from front side of table :



→ $\begin{cases} F_{net,x} = -k(R - R_0) \overset{2c)}{=} m \cdot \frac{-v^2}{R} \\ F_{net,y} = N - mg \overset{2c)}{=} 0 \end{cases}$

Step 3: Solve for R: $\begin{cases} Rk(R - R_0) = m v^2 \Rightarrow kR^2 - kR_0 R - mv^2 = 0 \\ \text{Quadratic eq: } aR^2 + bR + c = 0 \rightarrow R = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{cases}$

$$\rightarrow R = \frac{kR_0 \pm \sqrt{k^2 R_0^2 + 4kmv^2}}{2k} = \frac{150 \cdot 0.18 \pm \sqrt{150^2 \cdot 0.18^2 + 4 \cdot 150 \cdot 2.1 \cdot 1.4^2}}{2 \cdot 150} = \begin{cases} \boxed{R = 0.279\ \text{m}} \\ \text{negative} \end{cases}$$