Physics 113 Summer 2015 AM

Ch1 Doing Physics

Physics ↔ describes nature ↔ many variables

T { Temperature, Period, Tension }

etc.

Mathematics: manipulate algebraic equations, derivatives, integrals, geometry, trigonometry, etc..

Dimensional Analysis:

i) Speed $v = \dfrac{\Delta s}{\Delta t}$ (distance) / (time)

↓

Dimension of speed: $[v] = \dfrac{[\Delta s]}{[\Delta t]} = \dfrac{L}{T}$ (Length) / (time)

↳ "Dimension of speed is length over time"

Δs "change of space" or "increment of space" or distance

Δt "change of time" or "increment of time" or time

ii) Acceleration $a = \dfrac{\Delta v}{\Delta t}$ (change of speed) / (change of time)

↳ Dimension of acceleration $[a] = \dfrac{[\Delta v]}{[\Delta t]} = \dfrac{\frac{L}{T}}{T} = \dfrac{L}{T^2}$

iii) Kinetic energy $K.E. = \frac{1}{2}mv^2$

↳ Work to stop/lanch an object of mass m from/to speed v

($\uparrow m$, $\uparrow KE$ & $\uparrow v$, $\uparrow KE$)

↳ Dimension of KE : $[KE] = \left[\frac{1}{2}mv^2\right] = \underbrace{\left[\frac{1}{2}\right]}_{\substack{= 1 \\ \text{(number)}}} \cdot [m] \cdot [v]^2$

$= M \cdot \frac{L^2}{T^2}$

Dimensional analysis → 1) Expressing units : SI (international), British
2) Formula check

1)

Dimension	SI	British
L	m (meter)	ft (foot)
T	s (second)	s
M	kg (kilogram)	lb (pound) $1 lb = 0.454 kg$
$[Area] = L^2$	m^2	ft^2
$[Volume] = L^3$	m^3	ft^3
$[Energy] = \frac{ML^2}{T^2}$	$\frac{kg\, m^2}{s^2} = J$ (Joule)	$\frac{lb. ft^2}{s^2} = Btu$

Sub & super units:

nm	μm	mm	cm	m	km
$10^{-9} m$	$10^{-6} m$	$10^{-3} m$	$10^{-2} m$	$1 m$	$10^{3} m$
nm^2	μm^2	mm^2	cm^2	m^2	km^2
$10^{-18} m^2$	$10^{-12} m^2$	$10^{-6} m^2$	$10^{-4} m^2$	$1 m^2$	$10^{6} m^2$

nm^3	μm^3	mm^3	cm^3	m^3	km^3
..	..	..	$10^{-6}\,m^3$	$1\,m^3$	$10^9\,m^3$

time:

femtos	ps	μs	ms	s	min	hr	day	--
$10^{-15}\,s$	$10^{-12}\,s$	$10^{-6}\,s$	$10^{-3}\,s$	1s	60s	3600s	86400s	

mass:

μg	mg	g	kg	--
$10^{-9}\,kg$	$10^{-6}\,kg$	$10^{-3}\,kg$		

2) **Formula check:**

↳ for speed

$$v_1 = \frac{1}{2} g h^2 \rightarrow [v_1] = [g]\cdot[h]^2 = \frac{L}{T^2}\cdot L^2 = \frac{L^3}{T^2} \neq \frac{L}{T} \quad \times$$

$$v_2 = \sqrt{g\cdot h} \rightarrow [v_2] = [g]^{\frac{1}{2}}\cdot[h]^{\frac{1}{2}} = \frac{L^{\frac{1}{2}}}{T}\cdot L^{\frac{1}{2}} = \frac{L}{T} = \frac{L}{T} \quad \checkmark$$

g : acceleration of gravity $\rightarrow [g] = \frac{L}{T^2}$

h : height or vertical distance $\rightarrow [h] = L$

How about $v_3 = 2\sqrt{g\cdot h} \rightarrow [v_3] = \frac{L}{T}$ ✓

Conclusion: dimensional analysis can check a formula in physics up to a constant.

Accuracy & Significant Figures:

1) **Scientific Notation:** coefficient (<10) followed by power of 10 multiplied

$$\Delta s = 6\,185\,000\ m \rightarrow 6.185 \cdot 10^{6}\ m \quad \text{or} \quad 6.185E6\ m$$

$$\Delta t = 5000\ s \rightarrow 5 \cdot 10^{3}\ s \quad \text{or} \quad 5E3\ s$$

$$\text{speed } v = \frac{\Delta s}{\Delta t} = \frac{6.185 \cdot 10^{6}\ m}{5 \cdot 10^{3}\ s} = \frac{6.185}{5} \cdot 10^{6-3}\ \frac{m}{s}$$

$$= 1.237 \cdot 10^{3}\ \frac{m}{s}$$

2) **Accuracy:** (addition & subtraction)

$$\pi - 1.14 = 3.1416 - 1.14 = 2.0016$$

↓ up to lowest accuracy (2 decimal digits)

$$= 2.00$$

3) **Significant figures** (multiplication & division)

$$\underbrace{6\,370\,000}_{3\ s.f.'s}\ m \qquad \underbrace{6\,370\,001}_{7\ s.f.'s}\ m$$

(end zeroes don't count)

Earth circumference: $2\pi R_E = 2 \cdot 3.1416 \cdot 6.37 \cdot 10^{6}\ m$

$R_E = 6.37 \cdot 10^{6}\ m$

$$= 4.002398 \cdot 10^{7}\ m$$

↓ only up to least # of s.f.'s of factors involved in equation

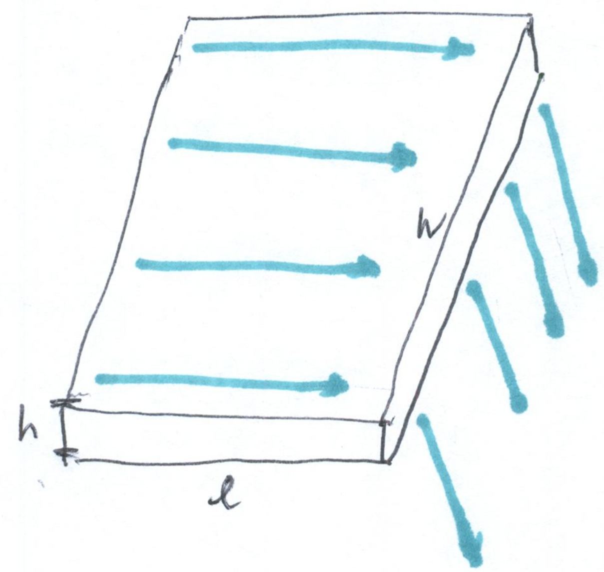
$$= 4.00 \cdot 10^{7}\ m$$

Estimation:

1.45] a) Estimate volume of water (m^3) going over Niagara Falls in 1s

→ **Guess:** 20,000 $\frac{m^3}{s}$

→ **Estimation:** rough model & equation
rectangular slab: h, l, w



Volume of water in this slab in 1s → Flow rate = $\frac{\text{volume}}{\text{time}}$

↓

write Flow rate conveniently for the estimation:

$$\frac{hlw}{t} = \begin{cases} 1)\ \frac{h}{t} \cdot lw \\ 2)\ h \cdot \frac{l}{t} \cdot w \quad ✓ \\ 3)\ h \cdot l \cdot \frac{w}{t} \end{cases}$$

water flow direction can estimate by visual inspection!

Numeric estimates

$$\begin{cases} h : 1m,\ 10m,\ 100m \\ \dfrac{l}{t} : 1\dfrac{m}{s},\ 10\dfrac{m}{s},\ \dfrac{100m}{s} \\ w : 100m,\ 1000m,\ 10\,000m \end{cases}$$

$\downarrow$

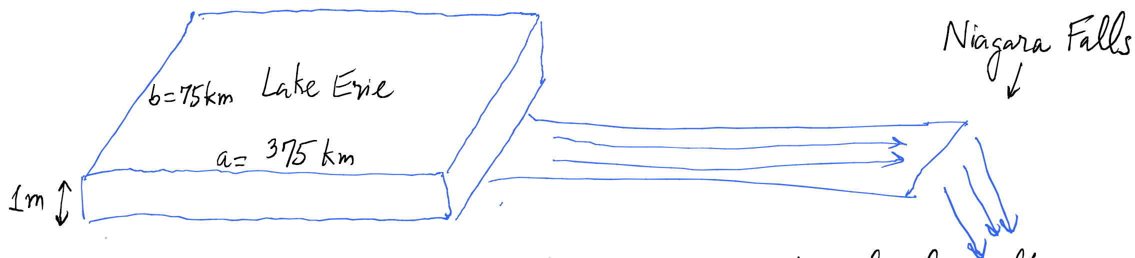
Flow rate estimate $= 1m\ 1\frac{m}{s} \cdot 1000\,m = 1000\frac{m^3}{s} = 10^3\frac{m^3}{s}$

b)

(7)

1.45 b) If Niagara Falls are shut off, water level at lake Erie will rise.
How long does it take for water level there to rise 1m?

Estimation : rough model $\leftrightarrow$ equation
$\downarrow$
Lake Erie $\leftrightarrow$ rectangular slab



If water can't flow down N.Fs $\rightarrow$ water level will rise @ Lake Erie
$\rightarrow$ Water flow rate @ NFs = water rise rate @ L.E. = $\frac{1000\ m^3}{s}$

$\rightarrow$ 1m rise $\rightarrow$ $V_{H_2O} = 1m \cdot 375 \cdot 10^3\ m \cdot 75 \cdot 10^3\ m = 28125 \cdot 10^6\ m^3$

How long? $t = \frac{V_{H_2O}}{\text{Flow rate}} = \frac{28125 \cdot 10^{\cancel{6}\,3}\ \cancel{m^3}}{10^{\cancel{3}}\ \frac{\cancel{m^3}}{s}} = 2.8125 \cdot 10^7\ s$

Convert s to days: $2.8125 \cdot 10^7\ \cancel{s} \cdot \frac{1\ \text{day}}{86400\ \cancel{s}} = 326$ days $\sim$ 1 year

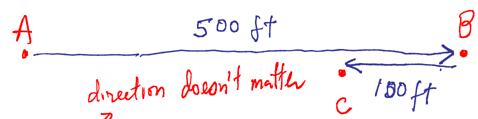
Summary $\left\{ \begin{array}{ll} \text{Flow rate} & \text{time} \\ 1000\ \frac{m^3}{s} & \sim 1\ \text{year} \\ 2\times & \frac{1}{2}\times \end{array} \right.$

Ch 2 Motion in a Straight Line

→ horizontal or vertical

1) Average vs. instantaneous motion
2) Speed vs. velocity

Speed → $= \dfrac{\text{distance}}{\text{time}}$ velocity → $\dfrac{\text{displacement}}{\text{time}}$



$t_{ABC} = 4s$

$$\text{speed} = \frac{600\ ft}{4s} = 150\ \frac{ft}{s}$$

direction doesn't matter

$$\text{velocity} = \frac{400\ ft}{4s} = 100\ \frac{ft}{s}$$

direction matters!

Distance traveled $= 600\ ft$ — direction doesn't matter

Displacement $= 400\ ft$ — direction matters

i) Average velocity $\left(\frac{m}{s}\right)$: $\bar{v} = \dfrac{\Delta x}{\Delta t}$ $\begin{cases} \Delta x: \text{change of position or displacement} \\ \Delta t: \text{change of time or time} \end{cases}$

ii) Instantaneous velocity $\left(\frac{m}{s}\right)$ $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ "time derivative of position x"

Calculus

Example: $x = bt^3 \rightarrow v = \dfrac{dx}{dt} = 3bt^2$

→ $\dfrac{dt^n}{dt} = n\, t^{n-1}$

iii) Average acceleration $\left(\frac{m}{s^2}\right)$: $\bar{a} = \dfrac{\Delta v}{\Delta t}$ $\begin{cases} \Delta v: \text{change of velocity} \\ \Delta t: \text{change of time} \end{cases}$

iv) Instantaneous acceleration: $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$ "time derivative of velocity v"

Calculus

Example: a) $x = bt^3 \rightarrow v = 3bt^2 \rightarrow a = 6bt$ (this acceleration linearly increases with time)

b) Motion due to gravity $\Rightarrow a = g$

– What would x be if it follows motion due to gravity?

$x = \int v\,dt = x_0 + v_0 t + \frac{1}{2} g t^2$ (x_0 is constant)

$\left[\int t^n\, dt = \dfrac{t^{n+1}}{n+1} \right]$

– What would v be if it follows motion due to gravity?

$$v = gt \leftrightarrow a = \frac{dv}{dt} = g$$

More generally $v = v_0 + gt$ (v_0 is constant) $\leftrightarrow a = g$

→ Kinematic equations for $\boxed{\text{Constant acceleration}}$ $(a=g)$ in 1D

↳ Using definitions introduced earlier $\bar{v}, v, \bar{a}, a$

Constant acceleration : $\bar{a} = a$

$$a = \bar{a} = \frac{\Delta v}{\Delta t} \equiv \frac{v - v_0}{t - 0} \Rightarrow v - v_0 = a \cdot t$$

final velocity, initial velocity, final time, initial time

$$\Rightarrow \boxed{v = v_0 + a \cdot t} \quad (1)$$ Kinematic equation #1

Kinematic eq #2: final position, initial position

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \Rightarrow x - x_0 = \bar{v} \cdot t \Rightarrow x = x_0 + \bar{v} \cdot t \quad (i)$$

Definition; final time; initial time

$$\bar{v} = \frac{\int_0^t dt\, v}{t-0} \overset{(1)}{=} \frac{1}{t}\int_0^t dt\,(v_0 + a \cdot t) = \frac{1}{t}\left[v_0 \cdot t + \frac{1}{2}a \cdot t^2\right]_0^t$$

mathematical average

$$= \frac{1}{t}\left[v_0 t + \frac{1}{2}a t^2\right].$$

$$= v_0 + \frac{1}{2}a \cdot t$$

$$\bar{v} = v_0 + \frac{1}{2}a \cdot t$$

$$= \frac{1}{2}v_0 + \underbrace{\frac{1}{2}v_0 + \frac{1}{2}a \cdot t}_{\frac{1}{2}\underbrace{(v_0 + a \cdot t)}_{v \ (\text{K. eq \#1})}}$$

$$\bar{v} = \frac{1}{2}(v_0 + v) \quad (ii)$$

(i) $x = x_0 + \bar{v} \cdot t$

(ii) $\bar{v} = \frac{1}{2}(v_0 + v)$

$$= x_0 + \frac{1}{2}(v_0 + v) \cdot t \underset{(\text{K. eq \#1})}{=} x_0 + \frac{1}{2}(v_0 + v_0 + a \cdot t) \cdot t$$

$$\boxed{x = x_0 + v_0 \cdot t + \frac{1}{2}a t^2} \quad (2)$$

Kinematic equation #2

Summary: to describe a constant acceleration motion in 1D:

1) $v = v_0 + a \cdot t$

2) $x = x_0 + v_0 \cdot t + \frac{1}{2}a t^2$

3) Eliminate time t from eqs 1) & 2) $\quad \frac{v^2 - v_0^2}{x - x_0} = 2a$

Note: when time t is not given or asked for → start solution with kinematic eq. #3 otherwise start with 1) & 2)

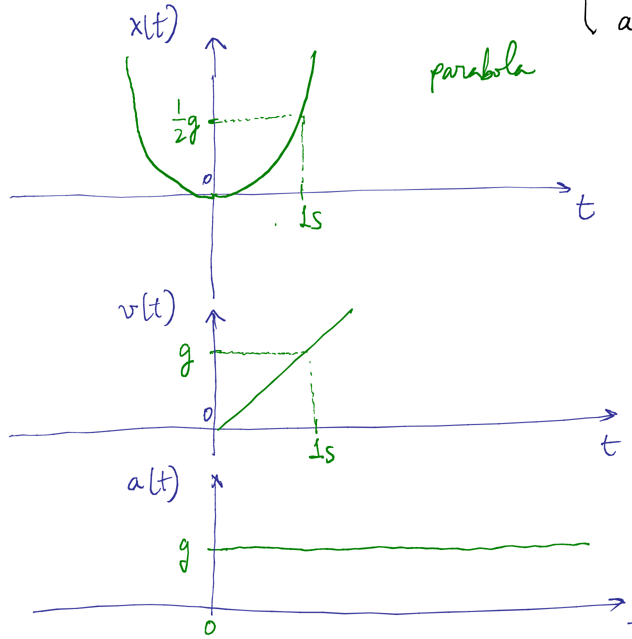
positions (m) $\begin{cases} x_0: \text{initial} \\ x: \text{final} \end{cases}$ velocities $\left(\frac{m}{s}\right)$ $\begin{cases} v_0: \text{initial} \\ v: \text{final} \end{cases}$ acceleration $\left(\frac{m}{s^2}\right)$ $\begin{cases} a = \text{constant} \\ \bar{a} = a \end{cases}$ time (s) $\begin{cases} t: \text{final} \\ 0: \text{initial} \end{cases}$

Graphing: (i) constant acceleration motion

Constant acceleration

$$\begin{cases} x = \frac{1}{2} g t^2 \\ v = \frac{dx}{dt} = gt \\ a = g \end{cases} \qquad (x_0 = 0;\ v_0 = 0)$$

start from rest



2.33 Step 1 Information

$$\begin{cases} \rightarrow \text{Car with } v_0 = 50\ \frac{mi}{h} \\ \rightarrow \text{Constant acceleration (deceleration to stop in } 100\ ft) \end{cases}$$

Equation notation

$$\begin{cases} v_0 = 50\ \frac{mi}{h} \\ x - x_0 = 100\ ft \\ a = ? \\ v = 0 \end{cases}$$

Step 2: select equation to start: Eq#3 $\frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a$

Step 3: make sure units are in correct system before plugging values into equation

$$\left.\begin{aligned} v_0 &= 50\ \frac{mi}{h} \cdot \frac{1609\,m}{1\,mi} \cdot \frac{1\,h}{3600\,s} = 22.35\ \frac{m}{s} \\ x - x_0 &= 100\ ft \cdot \frac{0.3048\,m}{ft} = 30.48\ m \end{aligned}\right\}$$

$$a = \frac{0 - v_0^2}{2(x - x_0)} = -\frac{22.35^2}{2 \cdot (30.48)} = -8.192\ \frac{m}{s^2}$$

deceleration (braking)

$= -8\ \frac{m}{s^2}$

2.61 Given information: 1) Constant acceleration 2) Covers 140 m in 3.6 s $v = 53\frac{m}{s}$ v_0?

$$\begin{cases} x - x_0 = 140m \\ t = 3.6s \\ v = 53\frac{m}{s} \end{cases}$$

a) Alternative #1:

Eq#2: $x - x_0 = v_0 t + \frac{1}{2}at^2$

Eq#1: $v = v_0 + at \rightarrow v_0 = v - at$

$$x - x_0 = (v - at)\cdot t + \frac{1}{2}at^2 = v.t - \frac{1}{2}at^2$$

$$a = [x - x_0 - vt]\cdot\left(-\frac{2}{t^2}\right) = [140 - 53\cdot3.6]\cdot\left(-\frac{2}{3.6^2}\right) = 7.83\frac{m}{s^2}$$

$\Rightarrow$ Eq#1: $v_0 = v - a.t = 53 - 7.83\cdot3.6 = 24.8\frac{m}{s}$

$v_{0_0} = 0\,m/s$ $v_0 = 24.8\frac{m}{s}$ $t = 3.6s$ $v = 53\frac{m}{s}$

O A $x - x_0 = 140m$ B

b) How far from rest (point O; $v_0 = 0\frac{m}{s}$) to end of 140m (point B) → Distance OB ?

Method(i): $OB = OA + 140m \rightarrow$ Find OA

Eq#3: $\frac{v^2 - v_0^2}{x - x_0} = 2a \rightarrow \frac{24.8^2 - 0}{(x - x_0)_{OA}} = 2\cdot7.83$

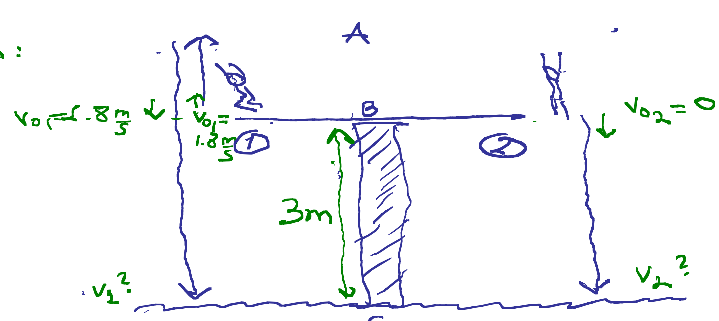
$(x - x_0)_{OA} = \frac{24.8^2}{2\cdot7.83} = 39.4m \Rightarrow OB = 179.4m \rightarrow 180m$

Method(ii): Eq#3 to $(x - x_0)_{OB}$: $\frac{v^2 - \cancel{v_0^2}}{(x - x_0)_{OB}} = 2\cdot a$

$$\frac{53^2 - 0}{(x - x_0)_{OB}} = 2\cdot7.83$$

$$(x - x_0)_{OB} = \frac{53^2}{2\cdot7.83} = 179.4m \rightarrow 180m$$

2.71 Given information:



Diver#1: B→A→C

Diver#2: B→C

Intuition: which diver enters water 1st? $\begin{cases} \#1 : 23 \\ \#2 : 9 \end{cases}$

Physics: equation to calculate v_1 & v_2 → Kinematic Eq #3:

Diver #1

$$\frac{v_1^2 - v_{0_1}^2}{x - x_0} = 2 \cdot g$$

$$\frac{v_1^2 - 1.8^2}{3} = 2 \cdot 9.81$$

$$v_1 = \sqrt{3 \cdot 2 \cdot 9.81 + 1.8^2}$$

$$= 7.88 \ \frac{m}{s}$$

Diver #2

$$\frac{v_2^2 - 0}{x - x_0} = 2 \cdot g$$

$$v_2 = \sqrt{3 \cdot 2 \cdot 9.81}$$

$$= 7.67 \ \frac{m}{s}$$

b) Time for each to reach water: A → C

Eq#1:

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a}$$

Diver #1

$$t_1 = \frac{v_1 - v_{0_1}}{g} = \frac{7.88 - 1.8}{9.81}$$

$$= 0.62s$$

Diver #2

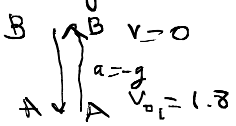
$$t_2 = \frac{v_2 - 0}{g} = \frac{7.67}{9.85} = 0.78s$$

$$t_2 - t_1 = 0.78 - 0.62 = 0.16s$$

For fun: how long for diver #1 to jump up & down ABA?

$$t_{ABA} = 2 \cdot t_{AB}$$

AB: upward motion: constant deceleration $a = -g$



Eq#1: @B

$$v = v_0 + at$$

$$0 = v_0 - g t_{AB} \rightarrow t_{AB} = \frac{v_0}{g} = \frac{1.8}{9.81}$$

$$= 0.183s$$

$$t_{ABA} = 2 \cdot t_{AB} = 0.366s$$

Web page info { user: p113s15
password: kiNemAtic

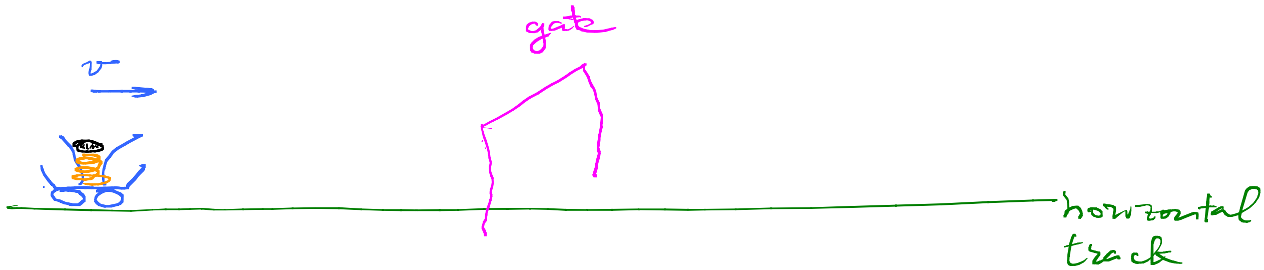
Ch 3: Motion in 2D & 3D

Ch2: motion in 1D { horizontal (car) or vertical (divers)

Ch 3: motion in 2D: simultaneous horizontal & vertical

Need important experimental fact:

visual experiment:



i) when a button is pressed, spring is released, ball is launched vertically upward

ii) negligible friction between car & track → after a push, car will travel @ constant velocity along track

iii) when is in uniform horizontal motion, ball has same motion

iv) gate is placed about middle of track. Before car reaches gate, the button that releases the spring is pressed.

Question: after ball is launched up, when it comes back down will it fall
1) in front of car — 0
2) into car — 18
3) behind car — 8

v) air resistance on ball is negligible

after spring is released ball acquires in addition to its horizontal uniform motion, a vertical motion of constant acceleration → from that point on ball has simultaneous horizontal & vertical motion. Air resistance is important at least on horizontal component

0
18
6

Ball will fall back into car: Reason: additional vertical motion doesn't affect its uniform horizontal motion. In fact ball was still traveling along with car only at temporarily different vertical positions.

Mathematical descriptions of motion in 2D & 3D

↳ Add & Subtract vectors

↳ 1) Graphically
2) Mathematically using unit vectors

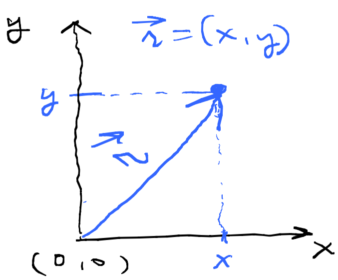
↳ (i) time independent ✓
(ii) time dependent

Notations

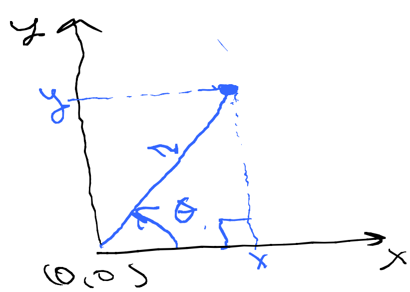
	1D	2D		3D	theta phi
position	x	$\vec{r}=(x,y)$	$=(r,\theta)$	$\vec{r}=(x,y,z)$	$=(r,\theta,\varphi)$
velocity	v	$\vec{v}=(v_x,v_y)$	$=(v,\theta_v)$	$\vec{v}=(v_x,v_y,v_z)$	$=(v,\theta_v,\varphi_v)$
acceleration	a	$\vec{a}=(a_x,a_y)$	$=(a,\theta_a)$	$\vec{a}=(a_x,a_y,a_z)$	$=(a,\theta_a,\varphi_a)$
		Cartesian Coordinates	Polar Coordinates	Cartesian	Spherical

2D

Cartesian (x,y)



Polar (r,θ)



Cartesian ⟶ Polar: $\begin{cases} r=\sqrt{x^2+y^2} \\ \theta = \arctan \frac{y}{x} \text{ or } \tan^{-1}\frac{y}{x} \end{cases}$ Pythagorean theorem

Polar ⟶ Cartesian $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ Trigonometry