Wave properties

Wave Superposition:

→ 3 very important phenomena

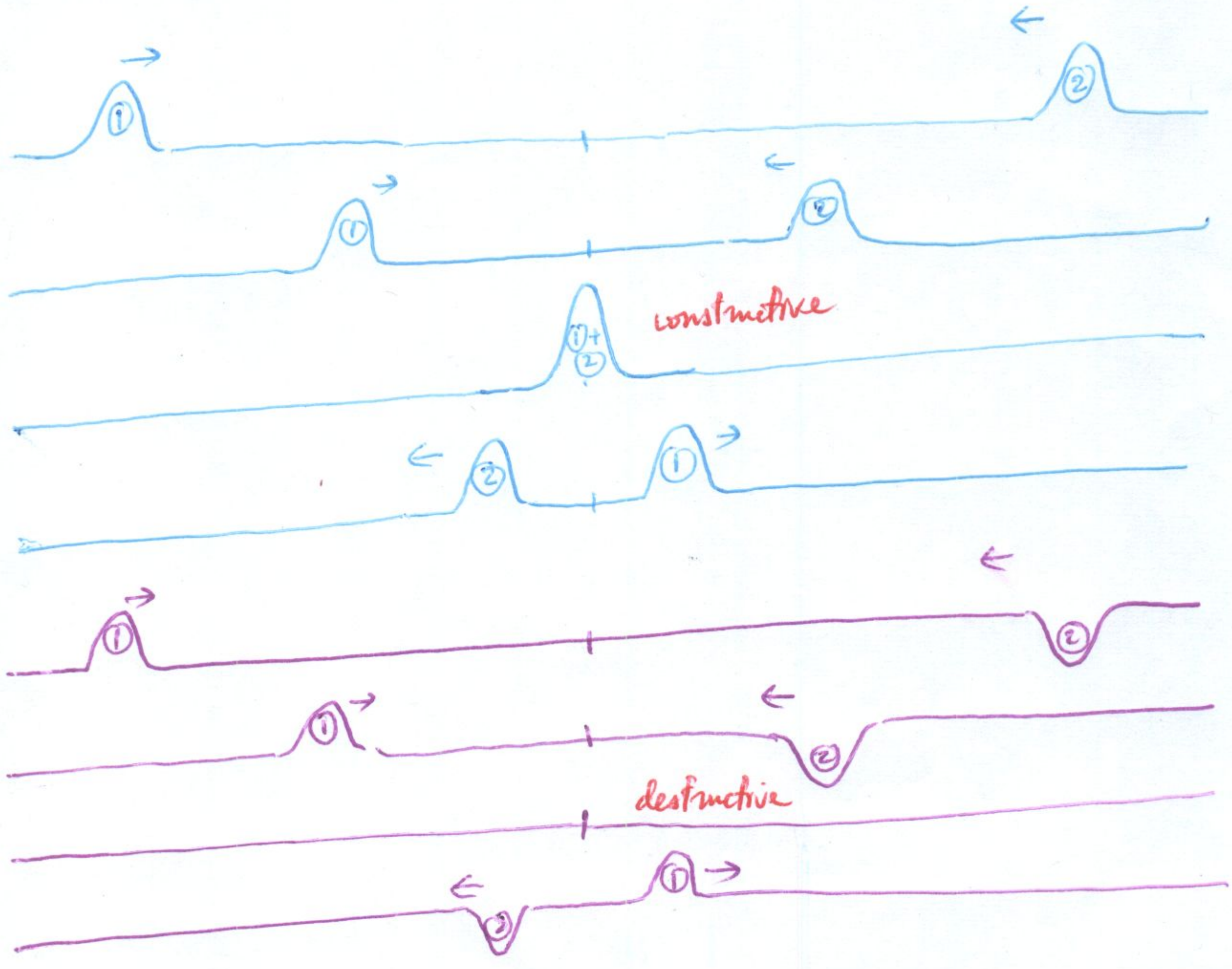
(i) Beats: tuning of string instruments, tuning of airplane engines (bimotors)

(ii) Standing waves: wind instruments (pipes, flutes, ...)

(iii) Wave interference
- constructive
- destructive
  $1+1=0$

Doppler effect: when wave source is also moving → LiDAR: (speed traps)

Wave superposition:



1) Beat phenomena : math description :

. Two transverse waves traveling in the same direction

↳ { same amplitudes A
  different frequencies w_1 , w_2 (and different wave numbers k_1, k_2)

$$y_1 = A \sin(k_1 x - w_1 t)$$

$$y_2 = A \sin(k_2 x - w_2 t)$$

. Wave superposition : they combine :

At $x=0$: → $y(0,t) = y_1(0,t) + y_2(0,t)$

$$= -A \sin w_1 t - A \sin w_2 t$$

$$= -A(\sin w_1 t + \sin w_2 t)$$

. Trigonometry : $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)$

$$y(0,t) = -2A \sin\left[\underbrace{\frac{w_1+w_2}{2}}_{\text{average of } w_1 \& w_2} t\right]\cdot\cos\left[\underbrace{\frac{w_1-w_2}{2}}_{\text{difference of } w_1 \& w_2} t\right]$$

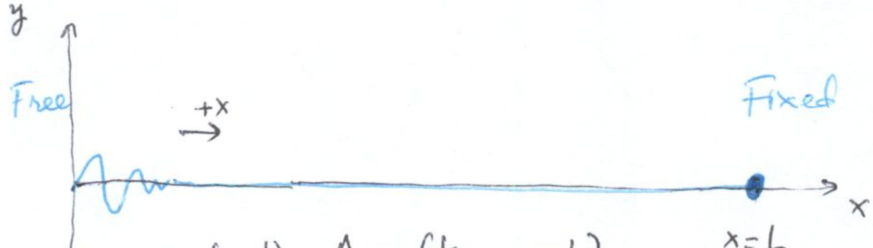
. If $w_1 \sim w_2$ $\begin{cases} \frac{w_1+w_2}{2} \sim w_1 \\ \frac{w_1-w_2}{2} \text{ very small compared } w_1 \text{ or } w_2 \end{cases}$

↳ when we can't hear w_1 or w_2 (two fast for ear drums) we can hear $w_1 - w_2$ → beats.

Beats can be easily heard as they oscillate at low frequency ($w_1 - w_2$)

2) Wave superposition : Standing waves :

a wave propagating in $+x$
& its reflection propagating in $-x$ } → standing wave (pipes, flutes, strings



$y_1(x,t) = A\cos(kx - \omega t)$
incident wave
(will reflect at $x=L$, fixed end)

$y_2(x,t) = -A\cos(kx + \omega t)$ (reflected)
reflected wave, inverted amplitude
(propagates in the same string, opposite direction)
↳ wave superposition

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\cos(\underbrace{kx-\omega t}_{\alpha}) - A\cos(\underbrace{kx+\omega t}_{\beta})$$

Trigonometry : $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

↳ $\boxed{y(x,t) = -2A\sin kx \sin(-\omega t) = 2A\sin kx \sin\omega t}$

$\underbrace{\text{incident + reflected waves}}$

Fixed point @ $x=L \Rightarrow y(L,t) = 0$

$\Leftrightarrow 2A\,\underbrace{\boxed{\sin kL}}_{=0}\,\underbrace{\sin\omega t}_{\neq 0} = 0 \quad \forall t$

$\boxed{\sin kL = 0 \Longleftrightarrow kL = n\pi \ (n = 1, 2, 3, \text{etc...})}$

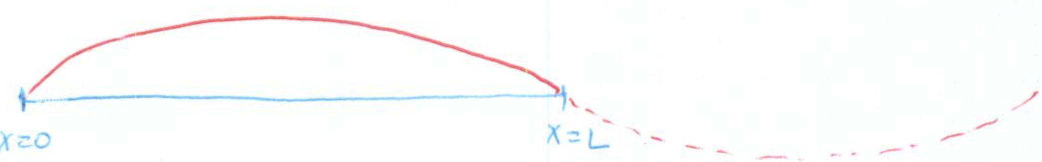
requirement for reflection: fixed point @ $x=L$
↓
$\boxed{\text{standing wave} \Leftrightarrow \text{fixed point}}$

Standing waves $\leftrightarrow$ fixed point $\Leftrightarrow$ $kL = n\pi$ $(n = \cancel{0}, 1, 2, 3, \ldots)$

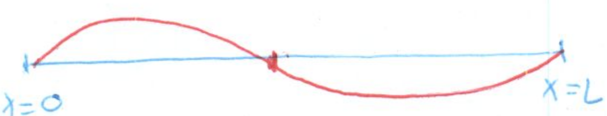
$$k = \frac{2\pi}{\lambda} \rightarrow \frac{2\cancel{\pi}}{\lambda} \cdot L = n\cancel{\pi} \quad (n = \cancel{0}, 1, 2, 3, \ldots)$$

$$\text{or } \lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots) = \left\{ \underset{\substack{\downarrow \\ \lambda_1 \\ \text{longest}}}{2L}, \underset{\substack{\downarrow \\ \lambda_2}}{L}, \underset{\substack{\downarrow \\ \lambda_3}}{\frac{2L}{3}}, \underset{\substack{\downarrow \\ \lambda_4}}{\frac{2L}{4}}, \ldots \right\}$$

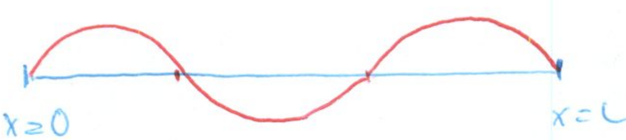
$\lambda_1 = 2L$



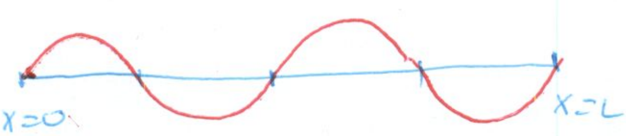
$\lambda_2 = L$



$\lambda_3 = \frac{2L}{3}$



$\lambda_4 = \frac{L}{2}$

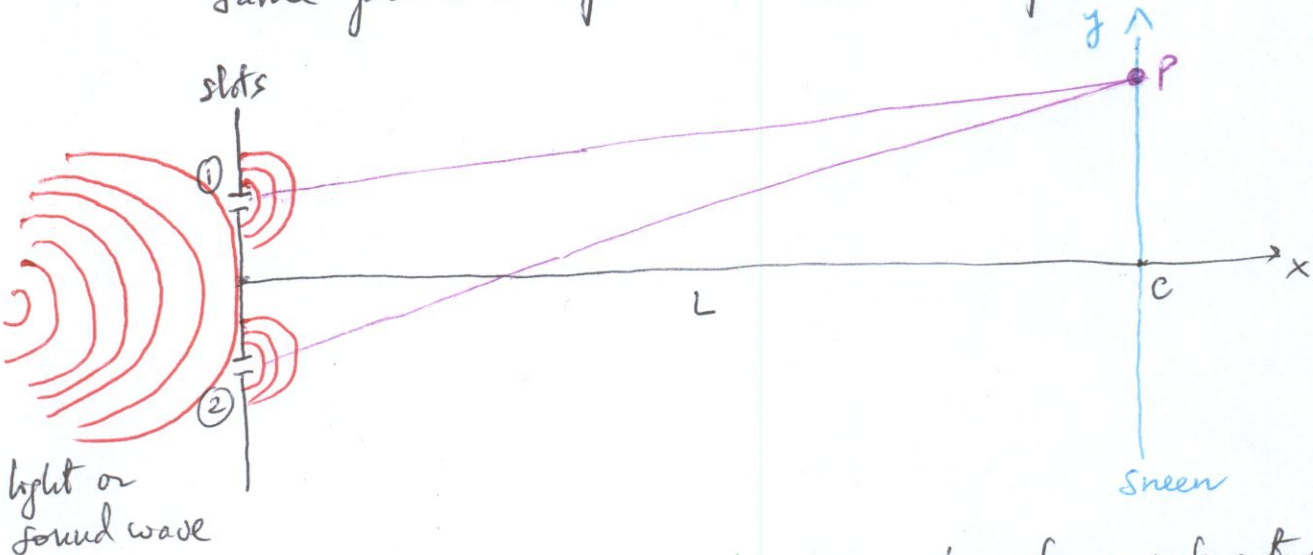


As λ decrease, # oscillations in the standing wave increases (shorter λ, high f)

$$v = \frac{\lambda}{T} = \lambda \cdot f$$

3) Wave superposition: interference

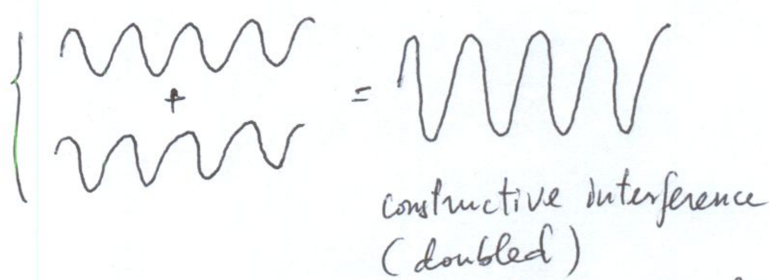
Two identical waves traveling different paths arriving at a same point in space → wave interference



As wave hits slits ① & ②, by Huyghens' principle, its wavefront creates two identical baby waves

Waves ① & ② are identical traveling different paths to P (on sreen), they arrive @ P with different phases (they will arrive with same phase at C)

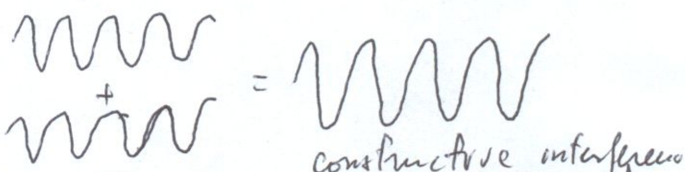
@ C: ① & ② same paths



@ P: ① & ② different paths

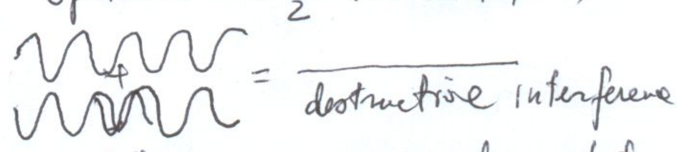
(i) Phase difference is a multiple of wavelength

$$\Delta path = n\lambda \quad (n = 1, 2, 3, \dots)$$



(ii) Phase difference is an odd multiple of $\frac{\lambda}{2}$

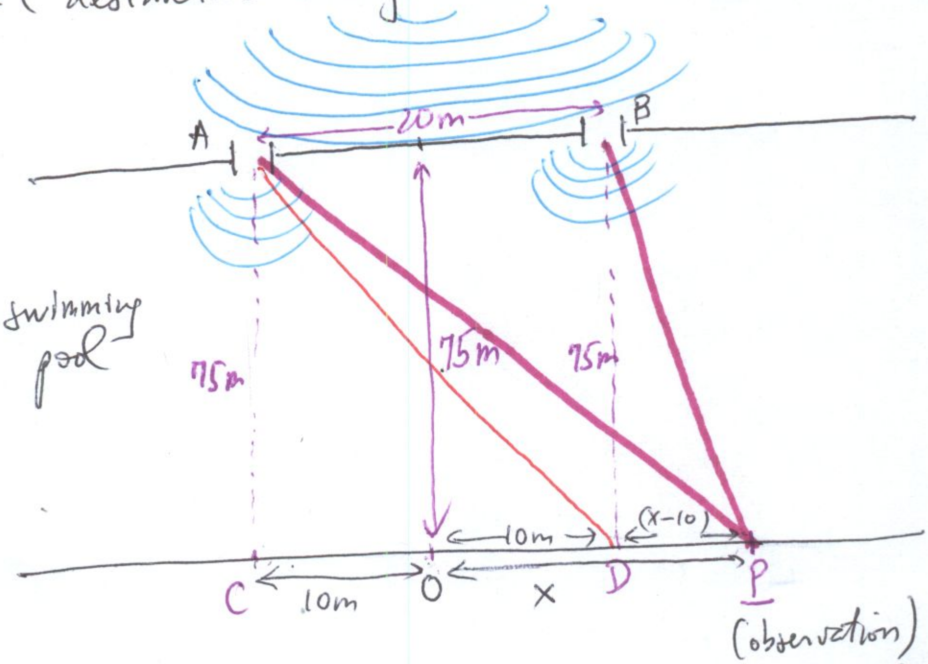
$$\Delta path = (2n+1)\frac{\lambda}{2} \quad (n = 0, 1, 2, 3, \dots)$$



(iii) Phase difference is some value in between

Water Wave Interference:

Swimming pool with 2 breakwater openings on a long side, separated by distance $a = 20m$. We observe water wave interference the other side of the pool a distance $d = 75m$. Find locations for maxima (constructive interference) & minima (destructive interference) $\lambda_{water} = 16m$



Two identical water waves A & B traveling different paths AP, BP, respectively,:

at P {

maxima if $AP - BP = n\lambda \quad (n = 1, 2, 3, \ldots)$

$$\sqrt{75^2 + (10+x)^2} - \sqrt{75^2 + (10-x)^2} = n\lambda$$

1^{st} max: $n=1 \Rightarrow \sqrt{75^2+(10+x_1)^2} - \sqrt{75^2+(10-x_1)^2} = \lambda$

solve for x_1

2^{nd} max: $n=2 \Rightarrow \sqrt{75^2+(10+x_2)^2} - \sqrt{75^2+(10-x_2)^2} = 2\lambda$

...

minima if $AP - BP = (2n+1)\dfrac{\lambda}{2} \quad (n = 0, 1, 2, 3, \text{etc}\ldots)$

1^{st} min: $n=0 \rightarrow \sqrt{75^2+(10+x)^2} - \sqrt{75^2+((10-x)^2} = \dfrac{\lambda}{2}$

2^{nd} min: $n=1 \quad \sqrt{75^2+(10+x)^2} - \sqrt{75^2+(10-x)^2} = \dfrac{3\lambda}{2}$

...

Example: finding location of 1st max:

$$\sqrt{75^2+(10+x_1)^2} - \sqrt{75^2+(10-x_1)^2} = 16$$

$$75^2+(10+x_1)^2 + 75^2+(10-x_1)^2 \underbrace{-2\sqrt{75^2+(10+x_1)^2}\sqrt{75^2+(10-x_1)^2}} = 256$$

$$\sqrt{75^2+(10+x_1)^2}\sqrt{75^2+(10-x_1)^2} = 5597 + x^2$$

$$\underbrace{\left[75^2+(10+x_1)^2\right]\left[75^2+(10-x_1)^2\right]} = x^4 + 11194x + 5597^2$$

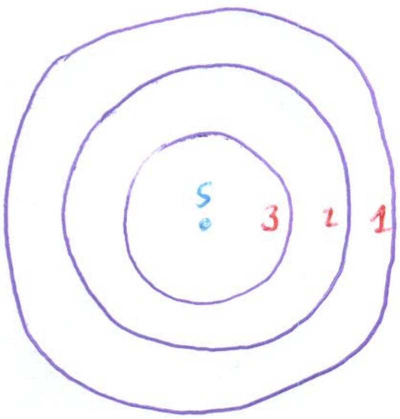
$$5625^2 + 5625\left[\underbrace{(10-x_1)^2+(10+x_1)^2}_{2x^2+100}\right] + \underbrace{\left[(10-x_1)(10+x_1)\right]^2}_{(x^2-100)^2} =$$

$$11050x^2 - 11194x + 1449216 = 0$$

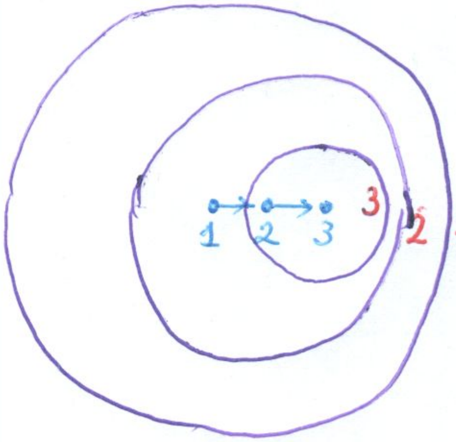
$$x = \frac{11194 \pm \sqrt{11194^2 - 4\cdot 11050\cdot 1449216}}{2\cdot 11050}$$

$$\boxed{x = \pm 33m}$$

4) Doppler Effect: source of wave is moving



Source at rest



Source moving to the right (+x)

Source @ 1 when it produces wave 1
2 — 2
3 — 3

Wave 1 is centered @ 1
2 — 2
3 — 3

→ wave is
- compressed @ front side → shorter λ
- spaced out @ back side → longer λ

$\boxed{\text{change of } \lambda \text{ due to moving source} \Leftrightarrow \text{Doppler's effect.}}$

Source approaching
$$\lambda' = \lambda - uT$$
λ': new wavelength
λ: original wavelength (source @ rest)
u: source speed
T: wave period

$$f' = \frac{f}{1 - \frac{u}{v}}$$
f': new freq.
f: original freq (source @ rest)
u: source speed
v: wave speed

Source receding
$$\lambda = \lambda + uT$$
$$f' = \frac{f}{1 + \frac{u}{v}}$$

Step 3: (i) solve for

$$\left.\begin{aligned} f_0 &= \frac{v}{\lambda_0} = \frac{v}{\frac{4L}{1}} = \frac{v}{4L} \\ f_1 &= \frac{v}{\lambda_1} = \frac{v}{\frac{4L}{3}} = 3 \cdot \frac{v}{4L} = 3 \cdot f_0 \end{aligned}\right\} \quad f_0 = \frac{f_1}{3} = \frac{225\,Hz}{3} = 75\,Hz$$

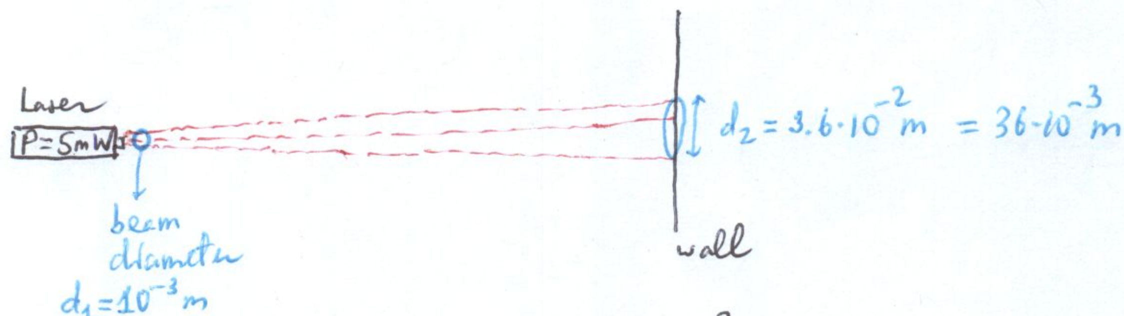
(ii) $v = 4L \cdot f_0 = 4 \cdot 1.5 \cdot 75 = 450\ \frac{m}{s}$

14.54) Laser beam intensity → $\frac{\text{Power}}{\text{Area}}$ a) at laser b) at the wall

5mW

a) 1mm diameter beam

b) 3.6 cm diameter spot



I_1?

I_2?

$$I_1 = \frac{P}{A_1} = \frac{P}{\pi R_1^2} = \frac{4P}{\pi d_1^2}$$

$$= \frac{4 \cdot 5 \cdot 10^{-3}}{\pi \cdot 10^{-6}} = \frac{20}{\pi} \cdot 10^{3} \frac{W}{m^2}$$

$$= 6.37 \cdot \frac{kW}{m^2}$$

$$I_2 = \frac{4P}{\pi d_2^2}$$

$$= \frac{4 \cdot 5 \cdot 10^{-3}}{\pi \cdot (36 \cdot 10^{-3})^2}$$

$$= \frac{I_1}{36^2} = 4.91 \cdot 10^{-3} \frac{kW}{m^2}$$

Ch 15 Fluid Motion :

- Gas : density ρ (rho) is variable (compressible)
  or
- Liquid : density ρ is constant (non-compressible)

Fluid description
- **Density ρ** : mass per unit volume : $\rho = \frac{dM}{dV}$ or $\frac{\text{Mass}}{\text{Vol.}}$ $\left(\frac{kg}{m^3}\right)$
  $\rho_{air} = 1\,kg/m^3$; $\rho_{H_2O} = 1000\,kg/m^3$ $\rightarrow$ $\rho_{liquid} > \rho_{gas}$
- **Pressure P** normal force per unit area $= P = \frac{F}{A} \rightarrow \frac{dF}{dA}$ $\left(\frac{N}{m^2}\right)$

Units
- SI: $\frac{N}{m^2} \equiv Pa$ (Pascal)
- Atm (Atmosphere) : $1\,Atm = 1.013 \cdot 10^5\,Pa$

Fluid motion equation

1) Hydrostatic equilibrium : $\boxed{\frac{dP}{dh} = g\rho}$

   - P = pressure
   - h : height or vertical distance
   - $g = 9.81\,m/s^2$
   - ρ = fluid density

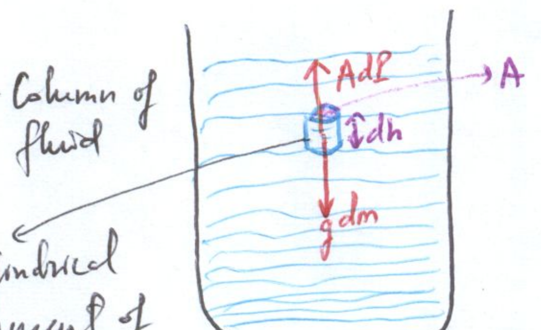
   meaning: if ρ = constant & g constant
   $\Rightarrow \frac{dP}{dh}$ = constant or Pressure ~~was~~ increases linearly with depth h

2) Conservation of mass : $v \cdot A$ = constant
   - v = fluid speed
   - A : cross-sectional area

3) Conservation of energy : Bernoulli's equation
   $$\frac{1}{2}\rho v^2 + \rho g y + P = \text{constant}.$$

1) Hydrostatic equilibrium: $\frac{dP}{dh} = g\rho$ Why?

From 2nd Newton's Law:



fluid { height dh, cross-sectional area = A, mass dm

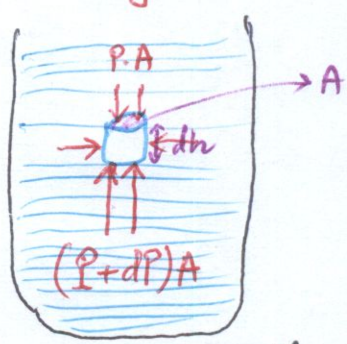
$\rho = \frac{dm}{A\,dh}$ or $dm = \rho A\,dh$

Forces on this element of fluid:

(i) its weight $g\,dm$ (down)

(ii) bouyant force $A\,dP$: (up)

↳ Why?



Pressure up on bottom is $P+dP$

Pressure down on top is P

(lower in fluid: more molecules or higher density so higher pressure)

→ Net force by fluid on element of fluid is upward or bouyant force

$$\underbrace{(P+dP)A}_{up} - \underbrace{P\cdot A}_{down} = \underbrace{A\,dP}_{up}$$

Origin of bouyant force: higher pressure at bottom and lower pressure at top of a fluid

(iii) 2nd Newton's Law: hydrostatic equilibrium:

$$\vec{F}_{net} = m\cdot\vec{a} = 0$$

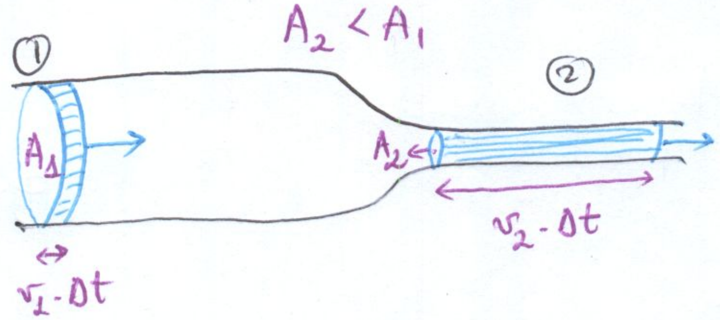
$$A\,dP - g\,dm = 0 \Rightarrow \cancel{A}\,dP = g\,dm = g\rho\cancel{A}\,dh$$

$$\Rightarrow \boxed{\frac{dP}{dh} = g\rho}$$

2) Conservation of Mass : no leaking fluid

(if there is no leak, mass in = mass out)

Fluid moving in a pipe with different cross-sectional areas



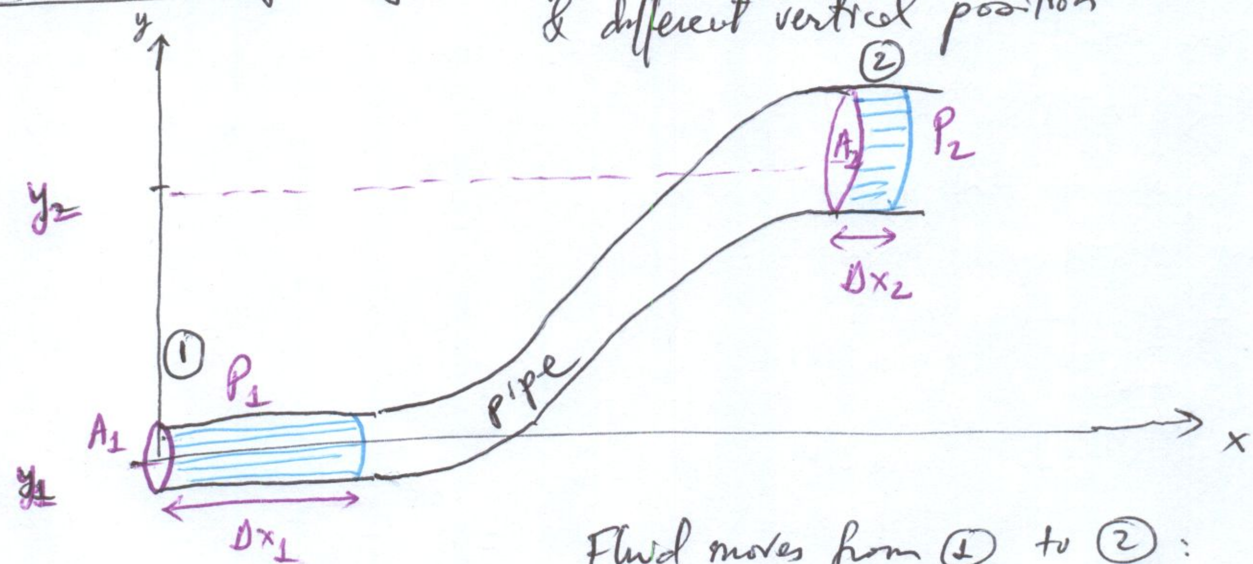
$$m_1 = \rho V_1 = \rho A_1 \cdot v_1 \cdot \Delta t \quad = \quad m_2 = \rho V_2 = \rho A_2 v_2 \cdot \Delta t$$

$$\cancel{\rho} A_1 v_1 \cancel{\Delta t} = \cancel{\rho} A_2 v_2 \cancel{\Delta t} \Rightarrow v_1 A_1 = v_2 A_2$$

or $\boxed{v \cdot A = \text{constant}}$ in fluid motion

(if A is smaller $\Rightarrow$ v higher)

3) Conservation of energy ? pipe with different cross-sectional areas & different vertical position



Fluid moves from ① to ② :

↳ Pressure $P_1 > P_2$ (pushes fluid from ① to ②) conservation of energy

Fluid motion ① → ② $\Delta W \overset{\uparrow}{=} \Delta(KE + PE) = \Delta KE + \Delta PE$

$\downarrow$

$F_1 \cdot \Delta x_1 - F_2 \cdot \Delta x_2$

$\downarrow$

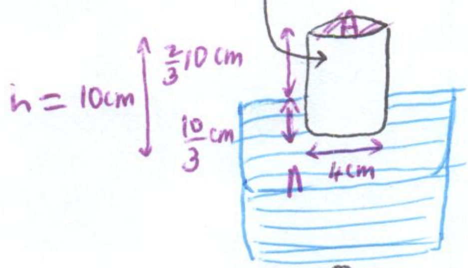
$$P_1 \cdot A_1 \Delta x_1 - P_2 \cdot A_2 \cdot \Delta x_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

$$\frac{1}{2} m v_1^2 + mgy_1 + P_1 A_1 \Delta x_1 = \frac{1}{2} m v_2^2 + mgy_2 + P_2 A_2 \Delta x_2$$

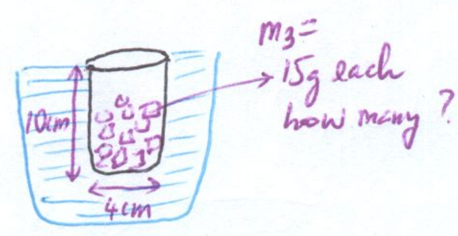
or $\left[\frac{1}{2} m v^2 + mgy + PA\Delta x = \text{constant} \right] \times \frac{1}{Vol} = \frac{1}{A \cdot \Delta x}$

$$\boxed{\frac{1}{2}\rho v^2 + \rho g y + P = \text{constant}}$$ Conservation of energy for a fluid or Bernoulli's eq.

15.48 Glass beaker 10cm high & 4 cm diameter

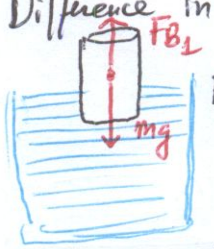


①



②

Difference in bouyant force b/w ② & ① = weigh of rocks.



$F_{B_1} = mg$

$$F_{B_2} - F_{B_1} = g\rho_W \underbrace{\frac{2}{3}(h \cdot A)}_{\Delta V_W}$$

m = mass of beaker

Bouyant force: $\frac{dP}{dh} = g\rho \rightarrow dP = g\rho\, dh \rightarrow P = g\rho h$

$$F_B = P.A = g\rho \underbrace{h \cdot A}_{\text{Volume } V} = g\rho_W V_W$$

$\hookrightarrow F_B = g\rho_W V_W$ { V_W: volume of water displaced by beaker } more volume of water displaced, more bouyant force

A = cross-sectional area of beaker

$\begin{cases} h: \text{height of beaker} = 10\text{ cm} = 10^{-1}\text{ m} \\ N = \text{number of rocks} \\ m_3 = 15\cdot 10^{-3}\text{ kg} \quad \text{mass of each rock}) \\ \rho_W = \text{density of water} = 1000\text{ kg/m}^3 \end{cases}$

$$F_{B_2} - F_{B_1} = N m_3 \cdot g$$

$$\cancel{g}\rho_W \frac{2}{3} h \cdot A = N m_3 \cdot \cancel{g}$$

$$N = \frac{\rho_W \frac{2}{3} h \cdot A}{m_3} = \frac{1000 \cdot \frac{2}{3} \cdot 10^{-1} \cdot \pi \cdot (2\cdot 10^{-2})^2}{15 \cdot 10^{-3}} = 5.6 \rightarrow 5 \text{ rocks before it sinks}$$

15.58] Helium balloon in air, how many paper clips before it loses buoyancy

Fluid motion

$\downarrow$

$$\frac{dP}{dh} = g\rho$$

$\downarrow$

$$P = g\rho h$$

$$F_B = P \cdot A = g\rho V$$

15.48

Glass beaker

in liquid

added rocks

buoyancy by water or liquid

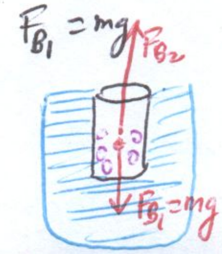
$\downarrow$

$$F_B = g\rho_W V_W$$

V_W: vol of water displaced by beaker = volume of beaker sumerged

$$F_{B_2} - F_{B_1} = N m_3 \cdot g$$

$$g\rho_W \frac{2}{3} V_{Beaker} = N m_3 \cdot g$$



15.58

Helium balloon

in air

added paper clips

buoyancy by air or gas

$\downarrow$

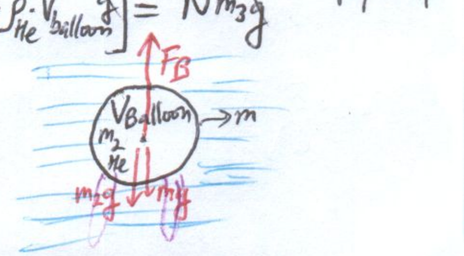
$$F_B = g\rho_{air} V_{air}$$

V_{air} = vol of air displaced by He balloon = volume of entire He balloon

$$F_B - mg - m_2 g = N m_3 g$$

$$\left[g\rho_{air} V_{Balloon} - mg - \rho_{He} V_{balloon} g \right] = N m_3 g$$

$\begin{cases} m: \text{mass of balloon} \\ m_2 = \text{mass of He inside balloon} \\ m_3 = \text{mass per paper clip} \end{cases}$



Solve for N (# paper clips):

$$N = \frac{\rho_{air} V_{Balloon} - m - \rho_{He} V_{Balloon}}{m_3}$$

$$= \frac{(\rho_{air} - \rho_{He}) V_{Balloon} - m}{m_3}$$

$$= \frac{(1 - 0.18)\frac{4}{3}\pi(0.15)^3 - 0.85 \cdot 10^{-3}}{10^{-3}}$$

$$N = 0.82 \cdot \frac{4}{3}\pi \cdot (1.5)^3 - 0.85 = 10.74 \text{ clips}$$

$\rightarrow N = 10$ clips before balloon & clips lose buoyancy.

$$\begin{cases} \rho_{air} = 1 kg/m^3 \\ V_{Balloon} = \frac{4}{3}\pi R^3 \\ \quad = \frac{4}{3}\pi \cdot (15 \cdot 10^{-2})^3 \\ m = 0.85 \cdot 10^{-3} kg \\ \rho_{He} = 0.18 kg/m^3 \\ m_3 = 10^{-3} kg \end{cases}$$

Ch 10 & 11

Rotational Motion & Conservation of angular momentum

1) New vectors given as cross-product of two vectors
   - Torque vector $\vec{\tau} = \vec{r} \times \vec{F}$
   - Angular momentum vector $\vec{L} = \vec{r} \times \vec{p}$ $= I \cdot \vec{\omega}$ (for rotations)

   need to define a center of rotation or pivot

2) Know right hand rule (RHR) to find directions of $\vec{\tau}$ & $\vec{L}$

3) Analog of Newton's 2nd Law
$$\begin{cases} \vec{F}_{net} = m \cdot \vec{a} \\ \vec{\tau}_{net} = I \cdot \vec{\alpha} \end{cases}$$

($I = c\, mR^2$ $\begin{cases} c = \frac{1}{2} & \text{disk} \\ c = 1 & \text{ring} \\ c = \frac{2}{5} & \text{sphere} \\ c = \frac{1}{12} & \text{rod} \end{cases}$)

More generally:
$$\begin{cases} \vec{F}_{net} = \frac{d\vec{p}}{dt} = 0 \rightarrow \vec{p}_i = \vec{p}_f \\ \vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0 \rightarrow \vec{L}_i = \vec{L}_f \end{cases}$$

Ch: 12

Static Equilibrium

$$\sum_i \vec{F}_i = 0$$

$$\sum_i \vec{\tau}_i = 0$$ $\leftrightarrow$ select a convenient center of rotation or pivot among the force application points.

$$\vec{\tau} = \vec{r} \times \vec{F} = r \cdot F \sin\theta \underbrace{\hat{\tau}}_{RHR}$$

- $\vec{r}$: position vector of the force application point from the pivot point
- θ: angle b/w $\vec{r}$ & $\vec{F}$
- $\hat{\tau}$: direction of torque given RHR

Ch 13:

Oscillatory motion or SHM

Equation: $\frac{d^2 z}{dt^2} = -\frac{a}{b} z$; solution: $z(t) = A \cos \omega t$; $\omega = \sqrt{\frac{a}{b}}$

	a	b	ω
pendulum	g	L	$\sqrt{\frac{g}{L}}$
torsional pend.	K	I	$\sqrt{\frac{K}{I}}$
spring & bob:	k	m	$\sqrt{\frac{k}{m}}$

Total energy in SHM stays constant:
$$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$
(spring & bob)

ch14 Wave Motion:

Wave superposition

Beats: 2 waves, same direction of propagation, same A, different k's & ω's

$$y_T(0,t) = -\boxed{2A\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)}\sin\left(\frac{\omega_1+\omega_2}{2}\cdot t\right)$$

low freq. varying amplitude ⟺ Beats.

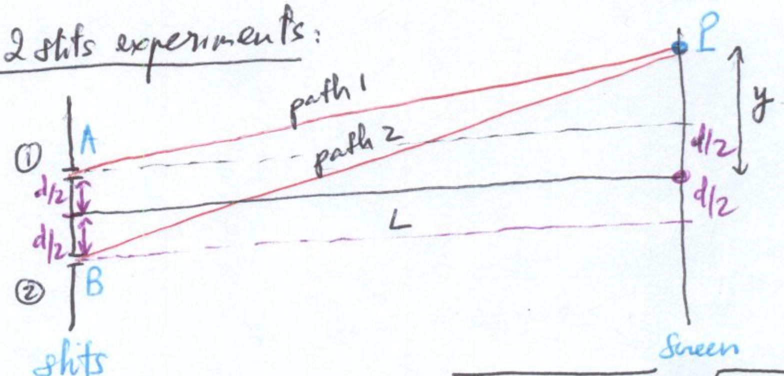
Standing waves: 2 waves, incident & reflected propagating in opposite directions. Amplitudes are A, & −A, same k's & ω's

$$y_T(x,t) = 2A\sin kx \sin \omega t$$

- Fixed end @ $x=L$: $\sin kL = 0 \rightarrow kL = n\pi$ $(n=1,2,3,\ldots)$
- Open end @ $x=L$: $\sin kL = 1 \rightarrow kL = (2n+1)\frac{\pi}{2}$ $(n=0,1,2,3\ldots)$

wave interference
- constructive: $\Delta path = n\lambda$ $(n=0,1,2,3\ldots)$
- destructive: $\Delta path = (2n+1)\frac{\lambda}{2}$ $(n=0,1,2,3,\ldots)$

Δpath in 2 slits experiments:



$$\Delta path = path\,2 - path\,1 = BP - AP = \sqrt{L^2+\left(y+\frac{d}{2}\right)^2} - \sqrt{L^2+\left(y-\frac{d}{2}\right)^2}$$

$\Delta path = \begin{cases} 0 & \text{(center spot)} \\ \lambda & \text{(1st max on screen)} \\ 2\lambda & \text{(2nd max)} \\ \ldots \end{cases}$

$\Delta path = \begin{cases} \frac{\lambda}{2} & \text{(1st min)} \\ \frac{3\lambda}{2} & \text{(2nd min)} \\ \frac{5\lambda}{2} & \text{(3rd min).} \\ \ldots \end{cases}$

Doppler's effect: moving source $f' = \dfrac{f}{1 \mp \frac{u}{v}}$ or $\dfrac{1}{\lambda'} = \dfrac{\frac{1}{\lambda}}{1 \mp \frac{u}{v}} \rightarrow \lambda' = \lambda\left(1 \mp \frac{u}{v}\right)$ $(v = \lambda\cdot f)$

−: approaching source
+: receding source

Ch 15:

Fluid Motion

1) Hydrostatic equilibrium: $\leftrightarrow$ bouyancy

$$\frac{dP}{dh} = g\rho \rightarrow F_B = g\rho V = g\rho \underbrace{hA}$$

vol of fluid displaced

ρ: density of fluid displaced.

2) Conservation of mass of fluid: $v \cdot A = \text{constant}$

v: speed of fluid

A: cross-sectional area of fluid

3) Conservation of energy for a fluid or Bernoulli's eq:

$$\frac{1}{2}\rho v_i^2 + \rho g h_i + P_i = \frac{1}{2}\rho v_f^2 + \rho g h_f + P_f$$