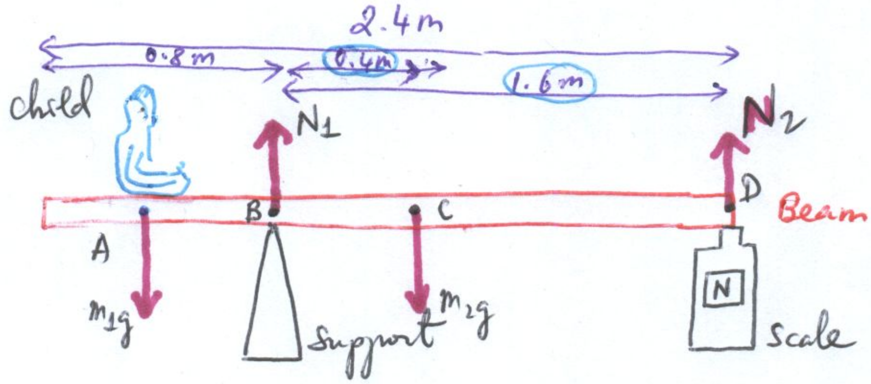
Ch12 Static Equilibrium

Physics: (i) No linear motion $\leftrightarrow$ $\sum_i \vec{F}_i = 0$ (Net force on system is 0)

(ii) No rotational motion $\leftrightarrow$ $\sum_i \vec{\tau}_i = 0$ (Net torque on system is 0)

12.21

Step 1:



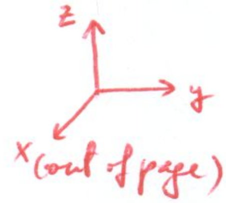
Info (given)

Length of beam $L = 2.4\text{m}$

Mass of child $m_1 = 40\text{kg}$

Mass of beam $m_2 = 60\text{kg}$

Left end to pivot (B) 0.8m



Question: location of child when $N = 100\text{N}$ or 300N?

4 components
- Child (don't know its location and the scale & support are not acting directly on child)
- Beam → Yes! all other 3 components act on beam!
- Support
- Scale

Focus on beam: Forces on beam
- @A: $m_1 g$ (down)
- @B: N_1 (up) (by support on beam)
- @C: $m_2 g$ (down)
- @D: N_2 (up) (by scale on beam)

Step 2: relevant equations (focus on beam)

1) $\sum_i \vec{F}_i = 0 \rightarrow$ on beam: $N_1 + N_2 - m_1 g - m_2 g = 0$

2) $\sum_i \vec{\tau}_i = 0 \rightarrow$ on beam:

↳ Torques: we need to select a center of rotation or pivot. In general the choice of pivot affects the analysis but not the final result!

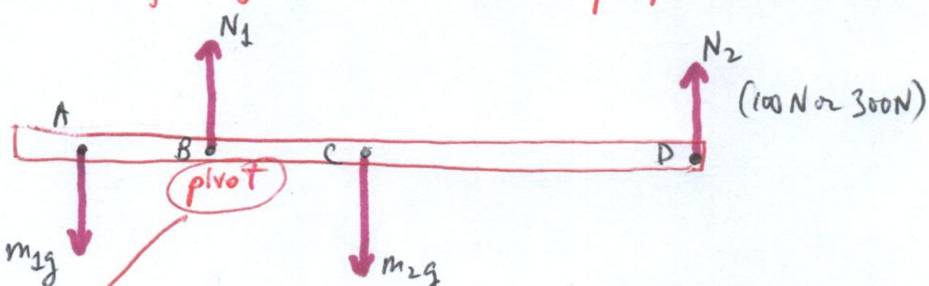
Part of writing torque balance equation is to choose the most convenient center of rotation or pivot:

pivot:
- A : problem: torque by $m_1 g$ is 0 → position of child is not in equation!
- B : no problem, also no need to calculate N_1 → <u>Best choice!</u>
- C : no problem, just need to calculate N_1
- D : problem: torque by N_2 is 0 → can't specify 100N or 300N!

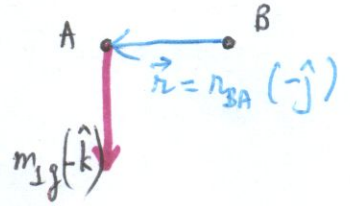
$$\vec{\tau} = \vec{r} \times \vec{F}$$

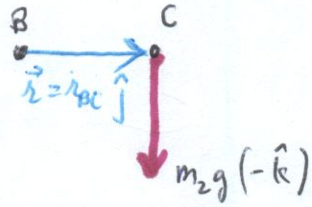
$\vec{r}$: position vector of force application point

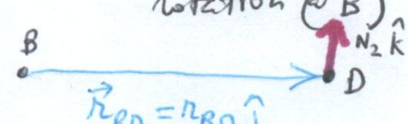
$\vec{F}$: force applied.



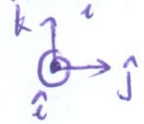
→ To write $\sum_i \vec{\tau}_i = 0 = \vec{\tau}_{m_1 g} + \vec{\tau}_{m_2 g} + \vec{\tau}_{N_2}$ ($\vec{\tau}_{N_1} = 0$ for this center of rotation @ B)





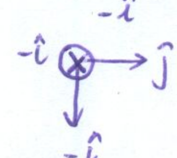


$$\vec{\tau}_{m_1 g} = \vec{r}_{BA} \times m_1 g(-\hat{k}) = r_{BA} m_1 g \underbrace{(\hat{j} \times \hat{k})}_{\hat{i}}$$



$$= r_{BA} m_1 g\, \hat{i}$$

$$\vec{\tau}_{m_2 g} = \vec{r}_{BC} \times m_2 g(-\hat{k}) = r_{BC} m_2 g \underbrace{\hat{j} \times (-\hat{k})}_{-\hat{i}}$$



$$= -r_{BC} m_2 g\, \hat{i}$$

$$\vec{\tau}_{N_2} = \vec{r}_{BD} \times N_2 \hat{k} = r_{BD} N_2 \underbrace{(\hat{j} \times \hat{k})}_{\hat{i}} = r_{BD} N_2 \hat{i}$$

$$\sum_i \vec{\tau}_i = 0 \implies (r_{BA} m_1 g - r_{BC} m_2 g + r_{BD} N_2)\hat{i} = 0$$

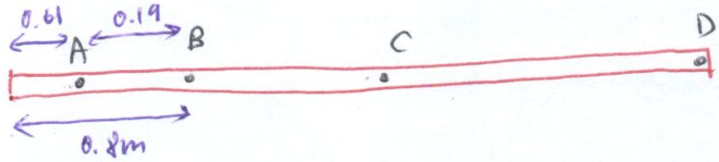
<u>Step 3:</u> Solve for position of child:

$$r_{BA} = \frac{r_{BC} m_2 g - r_{BD} N_2}{m_1 g} = \begin{cases} = \dfrac{0.4 \cdot 60 \cdot 9.81 - 1.6 \cdot \boxed{100}}{40 \cdot 9.81} = +0.19\,m \\ = \dfrac{0.4 \cdot 60 \cdot 9.81 - 1.6 \cdot \boxed{300}}{40 \cdot 9.81} = -0.62\,m \end{cases}$$

wrt left end of beam

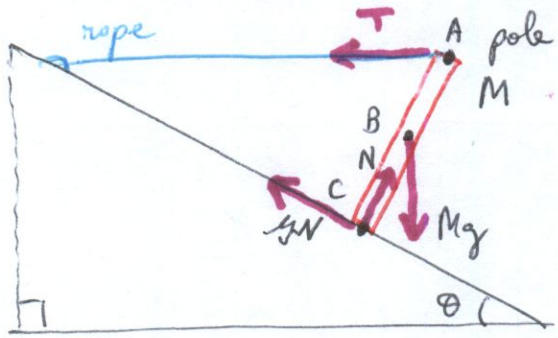
$$r_{BA} = \begin{cases} N_2 = 100N \rightarrow r_{BA} = 0.19m \text{ (A left of B)} \rightarrow \boxed{0.61m} \\ N_2 = 300N \rightarrow r_{BA} = -0.62m \text{ (A right of B)} \rightarrow 0.8m + 0.62m = \boxed{1.42m} \end{cases}$$

position of child



$\boxed{12.55}$

Step 1:

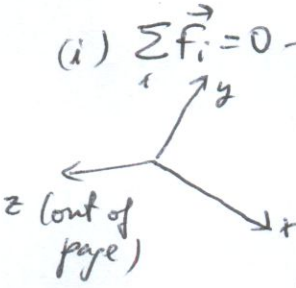


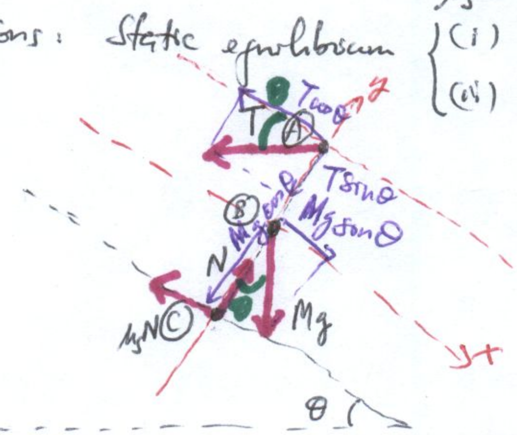
Min coeff. friction b/w pole & slope for pole to be in static equilibrium.
↓
Clear focus on pole

Focus on pole → Forces on pole
- (A): tension T by rope (top of pole)
- (B): its CM → weight of pole or Mg
- (C):
  - Normal force by slope on pole N
  - Friction force: pole tends to rotate CCW wrt its CM (B), at (C) pole tends to slip downhill → friction force will point uphill! → $\mu_s N$

Step 2: Relevant equations: Static equilibrium
$$\begin{cases} (i) \ \sum_i \vec{F}_i = 0 \\ (ii) \ \sum_i \vec{\tau}_i = 0 \end{cases}$$

(i) $\sum_i \vec{F}_i = 0$





$$\begin{cases} x: Mg\sin\theta - T\cos\theta - \mu_s N = 0 & (1) \\ y: N - Mg\cos\theta - T\sin\theta = 0 & (2) \end{cases}$$

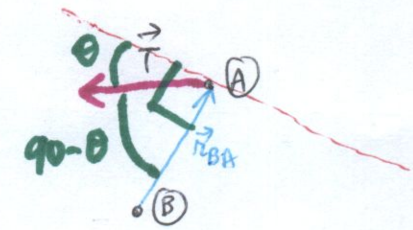
(ii) $\sum_i \vec{\tau}_i = 0 \rightarrow$ select most convenient center of rotation: $\begin{cases} A \\ B \rightarrow \text{our choice} \\ C: \text{problem: torque by friction is zero} \rightarrow \mu_s \text{ is not in the equation} \end{cases}$

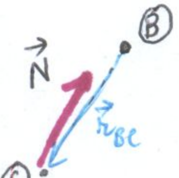
Selecting (B) as our center of rotation allows us to ignore M and L (length of pole)

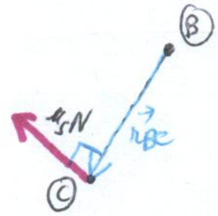
B = pivot:

$$\downarrow$$

$$\sum_i \vec{\tau}_i = \vec{\tau}_N + \vec{\tau}_{\mu_s N} + \vec{\tau}_T$$







$\vec{\tau}_N = \vec{r}_{BC} \times \vec{N} = 0$

($\sin 180° = 0$)

cross-product of any two parallel vectors is 0

$$\vec{\tau}_{\mu_s N} = \vec{r}_{BC} \times \mu_s N(-\hat{i})$$
$$= r_{BC}\mu_s N \underbrace{(-\hat{j})\times(-\hat{i})}_{-\hat{k}\ (RHR)}$$

$$\vec{\tau}_T = \vec{r}_{BA} \times \vec{T}$$
$$= r_{BA} T \underbrace{\sin(90-\theta)}_{\cos\theta}\ \hat{k} \quad (RHR)$$

$$\sum_i \vec{\tau}_i = 0 \Rightarrow -r_{BC}\mu_s N + r_{BA} T\cos\theta = 0$$

(B) CM of pole: $r_{BC} = r_{BA} \equiv \frac{L}{2}$

$$\rightarrow -\mu_s N + T\cos\theta = 0 \rightarrow \boxed{T = \frac{\mu_s N}{\cos\theta}} \quad (3)$$

(From $\sum_i \vec{\tau}_i = 0$ with (B) as pivot.)

Step 3: solve for μ_s:

$$2)\ N = Mg\cos\theta + T\sin\theta \stackrel{3)}{=} Mg\cos\theta + \underbrace{\frac{\mu_s N}{\cos\theta}\sin\theta}_{= \tan\theta}$$

$$N(1-\mu_s\tan\theta) = Mg\cos\theta \rightarrow \boxed{N = Mg\frac{\cos\theta}{1-\mu_s\tan\theta}} \quad (4)$$

$$1)\ Mg\sin\theta - 2\mu_s N = 0 \stackrel{4)}{=} \cancel{Mg}\sin\theta - \frac{2\cancel{Mg}\mu_s\cos\theta}{1-\mu_s\tan\theta} = 0$$

$$(1-\mu_s\tan\theta)\times\left[\sin\theta - \frac{2\mu_s\cos\theta}{1-\mu_s\tan\theta} = 0\right] \rightarrow (1-\mu_s\tan\theta)\sin\theta - 2\mu_s\cos\theta = 0$$

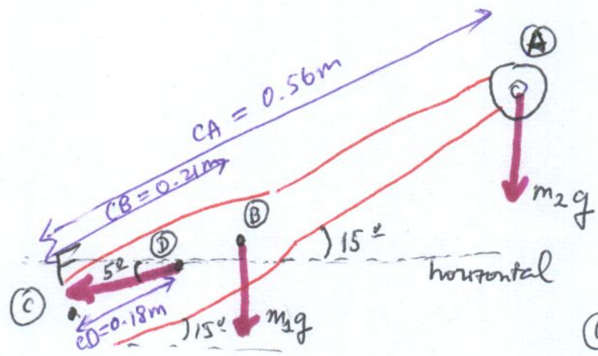
$$\sin\theta = \mu_s(\tan\theta\sin\theta + 2\cos\theta) \Rightarrow \boxed{\mu_s = \frac{\sin\theta}{\tan\theta\sin\theta + 2\cos\theta}}$$

$$\left[\mu_S = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\tan\theta\sin\theta + 2\cos\theta}{\cos\theta}} = \boxed{\frac{\tan\theta}{\tan^2\theta + 2}}\right.$$

min coeff. static friction to keep pole from slipping downhill.
(neither M or length of pole L matter!)

12.27)

Step 1:



$m_2 = 6\,kg$
$m_1 = 4.2\,kg$
$r_{CA} = 0.56\,m$
$r_{CB} = 0.21\,m$
$r_{CD} = 0.18\,m$

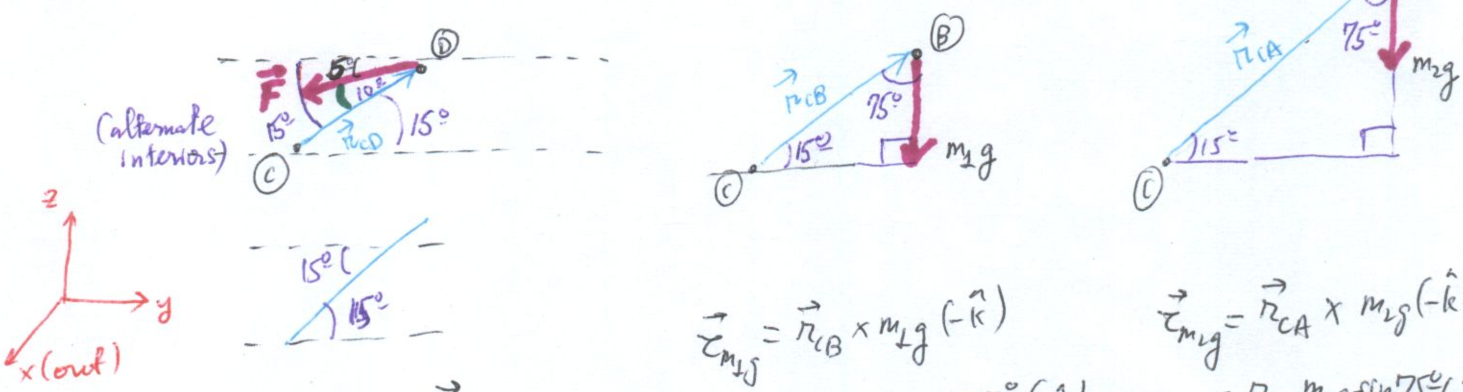
Ⓒ: shoulder joint = center of rotation (specified in problem)

Focus on arm → forces on arm
- Ⓐ: hand → weigh of mass $m_2 g$
- Ⓑ CM of arm: weigh of arm $m_1 g$
- Ⓓ Force F by deltoid muscle is applied (5° below horizontal)

Step 2:

$$\begin{cases} \sum_i \vec{F}_i = 0 \\ \sum_i \vec{\tau}_i = 0 \iff \text{center of rotation is Ⓒ shoulder joint!} \end{cases}$$

$$\hookrightarrow \vec{\tau}_F + \vec{\tau}_{m_1 g} + \vec{\tau}_{m_2 g} = 0$$



$$\vec{\tau}_F = \vec{r}_{CD} \times \vec{F} = r_{CD} F \sin 10^\circ\ \hat{i} \quad \text{(RHR)}$$

$$\vec{\tau}_{m_1 g} = \vec{r}_{CB} \times m_1 g(-\hat{k}) = r_{CB} m_1 g \sin 75^\circ (-\hat{i}) \quad \text{RHR}$$

$$\vec{\tau}_{m_2 g} = \vec{r}_{CA} \times m_2 g(-\hat{k}) = r_{CA} m_2 g \sin 75^\circ (-\hat{i}) \quad \text{RHR}$$

$$\Rightarrow \sum_i \vec{\tau}_i = 0 \Rightarrow \boxed{r_{CD} F \sin 10^\circ - r_{CB} m_1 g \sin 75^\circ - r_{CA} m_2 g \sin 75^\circ = 0}$$

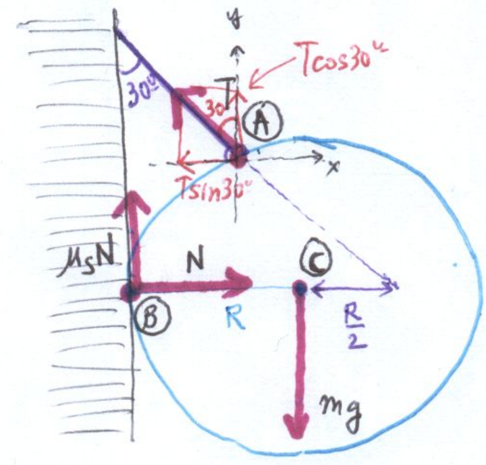
<u>Step 3:</u> solve for F : $F = \dfrac{(r_{CB}\, m_1 + r_{CA}\, m_2)\, g \sin 75^\circ}{r_{CD} \sin 10^\circ}$

$$= \frac{(0.21 \cdot 4.2 + 0.56 \cdot 6)\, 9.81 \cdot \sin 75^\circ}{0.18 \cdot \sin 10^\circ} = \frac{40.2}{0.18 \cdot \sin 10^\circ} = 1280\,N$$

$$= 1.28\,kN$$

12.28

Step 1:

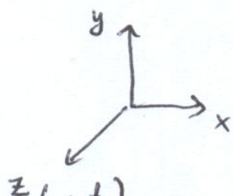


Uniform sphere of radius R held by a rope attached to a vertical wall forming an angle of 30°. Sphere is in contact with wall.

$\mu_{s,min}$ (b/w sphere & wall) so sphere is in static equilibrium

→ Special points where forces apply on sphere:
- (A) Tension T by rope
- (B) Normal force N & friction $\mu_s N$
- (C) Weigh of sphere on its CM = mg

→ Directions of forces using unit vectors:



(A) $\vec{T} = -T\sin 30\,\hat{i} + T\cos 30\,\hat{j}$

(B) $\vec{N} = N\hat{i}$

$\vec{F}_s = \mu_s N\hat{j}$ (sphere tends to rotate CCW wrt its center)

(C) $\vec{W} = mg\,(-\hat{j})$

Step 2: Relevant equations: conditions for static equilibrium:

(i) $\sum_i \vec{F}_i = 0$ (ii) $\sum_i \vec{\tau}_i = 0$ (on sphere)

$$(i)\quad \vec{F}_{net} = 0 \begin{cases} x: & N - T\sin 30^\circ = 0 \quad (1) \\ y: & T\cos 30^\circ + \mu_s N - mg = 0 \quad (2) \end{cases}$$

(ii) $\vec{\tau}_{net} = 0$: select most convenient center of rotation:

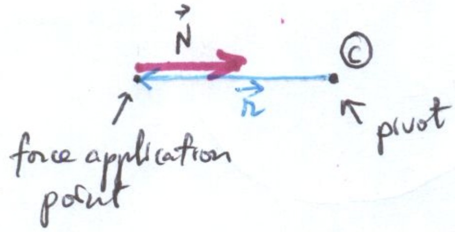
center of rotation:
- A : problem: we know direction of tension $\vec{T}$
- B : problem: $\vec{\tau}_{\mu_s N} = 0$ friction is not in the equation
- C no problem: we would not need to know m!

C = center of sphere is selected as the center of rotation (pivot)

$$\rightarrow \vec{\tau}_{mg} = 0 \quad (\vec{r} = 0)$$

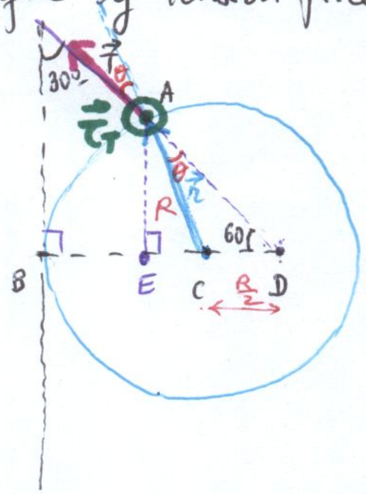
$$\vec{\tau}_{net} = \vec{\tau}_T + \vec{\tau}_{\mu_s N} + \vec{\tau}_N = 0$$

a) $\vec{\tau}_N$: torque by normal force $\vec{N}$ wrt C



$$\vec{\tau}_N = \vec{r} \times \vec{N} = rN \underbrace{\sin 180}_{0} \overset{(-\hat{i} \times \hat{i})}{} = 0$$

b) $\vec{\tau}_T$: torque by tension force T wrt C



$$\vec{\tau}_T = \vec{r} \times \vec{T} = R \cdot T \cdot \sin\boxed{\theta}\ \hat{k} \quad \text{(RHR)}$$

Focus on triangle $\overset{\triangle}{ACD}$: apply sine theorem:

$$\frac{\sin\theta}{CD} = \frac{\sin 60^\circ}{CA}$$

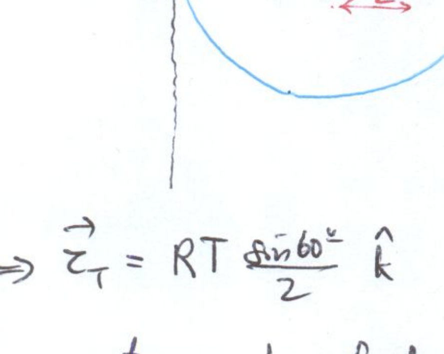
or

$$\frac{\sin\theta}{\frac{R}{2}} = \frac{\sin 60^\circ}{R}$$

$$\boxed{\sin\theta = \frac{\sin 60^\circ}{2}}$$

$$\Rightarrow \vec{\tau}_T = RT \frac{\sin 60^\circ}{2} \hat{k}$$

c) $\vec{\tau}_{\mu_s N}$: torque by friction force $\mu_s N$ wrt C



$$\vec{\tau}_{\mu_s N} = \vec{r} \times \mu_s N \hat{j} = R \mu_s N \underbrace{(-\hat{i} \times \hat{j})}_{-\hat{k}} = \mu_s RN(-\hat{k})$$

$$\rightarrow \sum_i \vec{\tau}_i = \vec{\tau}_T + \vec{\tau}_{\mu_s N} = \left(RT \frac{\sin 60^\circ}{2} - \mu_s RN\right)\hat{k} = 0$$

Step 3: Solve for μ_s : $\cancel{R}T \frac{\sin 60^\circ}{2} - \mu_s \cancel{R}N = 0 \Rightarrow \boxed{\mu_s = \frac{T \sin 60^\circ}{2N}}$ (3)

1) $N - T\sin 30^\circ = 0 \rightarrow \frac{T}{N} = \frac{1}{\sin 30^\circ}$

2) $T\cos 30 + \mu_s N - mg = 0$

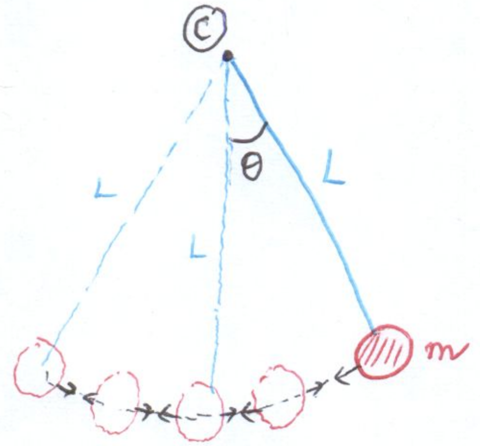
3) $\mu_s = \frac{\sin 60^\circ}{2} \frac{T}{N}$

$$\boxed{\mu_s = \frac{\sin 60^\circ}{2} \frac{1}{\sin 30^\circ} = \sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866}$$

Ch13: Oscillatory Motion

Motion
- → linear ✓
- → rotational ✓
- → oscillatory motion ✓
- → wave motion
- → fluid motion

1) **Pendulum**: bob & string (of negligible mass) with one end fixed.



θ: angle of string (bob) wrt vertical direction

$-\theta_M \leq \theta \leq \theta_M$ (θ_M when pendulum turns back)

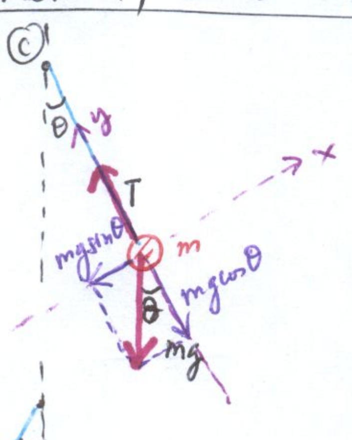
C: center of rotation

L: separation of bob to C, constant,

or bob has tangential but not radial motion (not toward C nor away from C)

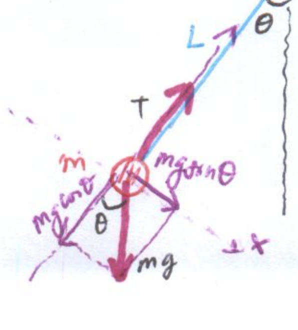
Equation of motion for pendulum:

1st method, linear motion of bob using Newton's 2nd Law: $\vec{F}_{net} = m\cdot\vec{a}$



Coord. system: x: parallel to tangential direction or direction of motion of bob; y: radial direction or direction of tension

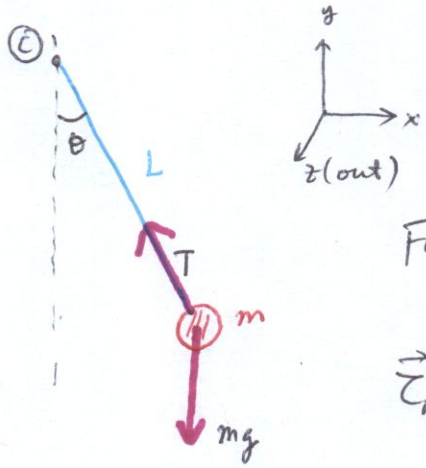
$$\begin{cases} F_{net,x} = -mg\sin\theta = m\cdot a \Rightarrow a = -g\sin\theta \\ F_{net,y} = T - mg\cos\theta = m\cdot 0 \end{cases}$$



$$\begin{cases} F_{net,x} = mg\sin\theta = m\cdot a \Rightarrow a = +g\sin\theta \\ F_{net,y} = T - mg\cos\theta = 0 \end{cases}$$

2nd Method, rotational motion of bob using analog's of Newton's 2nd Law

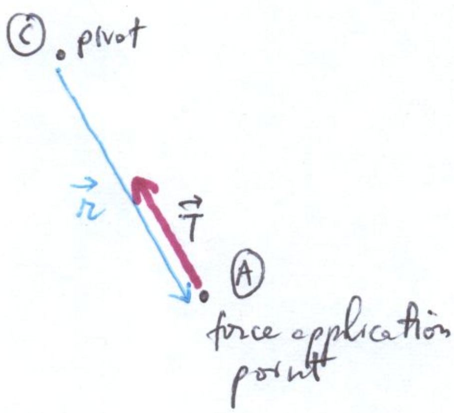
$$\vec{\tau}_{net} = I \cdot \vec{\alpha}$$



Focus on bob: $\begin{cases} mg\,(-\hat{j}) \\ \vec{T} \end{cases}$

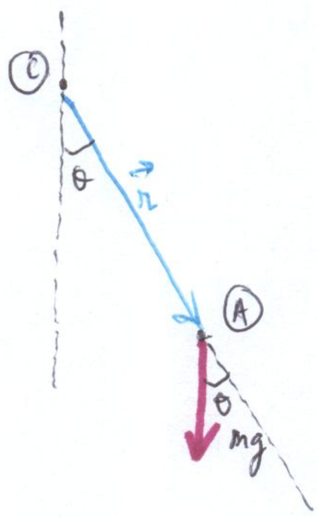
$\vec{\tau}_{net} = \vec{\tau}_T + \vec{\tau}_{mg}$ → Center of rotation is C

$\vec{\tau}_T$:



$$\vec{\tau}_T = \vec{r} \times \vec{T} = L \cdot T \sin 180^\circ = 0$$

$\vec{\tau}_{mg}$:



$$\vec{\tau}_{mg} = \vec{r} \times mg(\hat{-j}) = Lmg\,(-\hat{k})\sin\theta = Lmg\sin\theta\,(-\hat{k})$$

$$\Rightarrow \vec{\tau}_{net} = Lmg\sin\theta\,(-\hat{k}) = I \cdot \alpha\,(\hat{k})$$

$$\Rightarrow -\cancel{L}\cancel{m}g\sin\theta = \cancel{m}L^{\cancel{2}} \cdot \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

Same exact eq. of motion for a pendulum

Small angle approx: $\sin\theta \simeq \theta$ // $\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta}$ → SHM = $\boxed{\theta(t) = \theta_M \cos\omega t}$ osc. motion

Important application of pendulum:

First derive equation for angular frequency ω:

$\theta(t) = \theta_M \cos\omega t \longrightarrow$ into 2nd order linear differential eq:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\frac{d}{dt}\left(-\theta_M \omega \sin\omega t\right) = -\frac{g}{L}\theta_M \cos\omega t$$

$$-\cancel{\theta_M}\omega^2 \cancel{\cos\omega t} = -\frac{g}{L}\cancel{\theta_M}\cancel{\cos\omega t}$$

$$\omega^2 = \frac{g}{L} \quad \text{or} \quad \boxed{\omega = \sqrt{\frac{g}{L}}}$$

Consequences: 1) Longer string or larger L → smaller ω less osc. per second or it takes longer for a pendulum to complete one cycle or larger T (period)

2) Larger $g\left(= G\frac{M_E}{(R_E+h)^2}\right)$, @ R_E if there is a change of density underground

(Earth ~ uniform sphere or same material / density everywhere)

→ ω changes: → underground water pocket

ω = angular freq (# osc. per second) $\left(\frac{rad}{s} \text{ or } s^{-1}\right)$

T : period (# seconds per osc.) $= \frac{2\pi}{\omega}$ (s)

f : linear freq (# linear osc. per second) $= \frac{\omega}{2\pi}$ (Hertz or Hz)

2) Torsional pendulum :

Torsional Law : $\tau = -\kappa \cdot \Delta\theta$

κ : kappa : torsional constant (size & material)

$\Delta\theta$ = change of angle

τ = recovery torque

$-$: it opposes the twisting

bar + disk → can twist bar by turning disk

→ rotates back & forth wrt its axis of rotation (center axis)

$$\tau = -\kappa \cdot \theta = I \cdot \alpha = I \cdot \frac{d^2\theta}{dt^2} \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta}$$

2nd order linear diff eq.

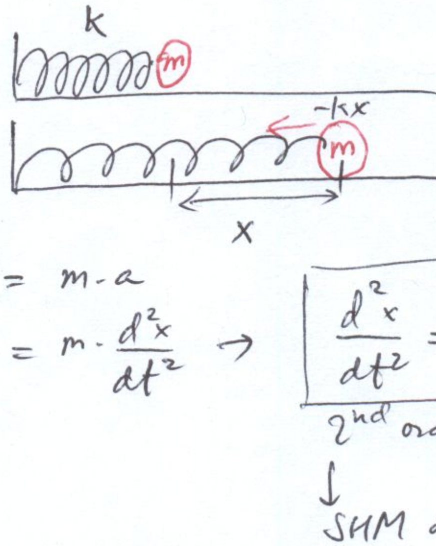
↓

SHM / osc. motion :

$\theta(t) = \theta_M \cos(\omega t)$

$\boxed{\omega = \sqrt{\frac{\kappa}{I}}}$ (s^{-1})

3) Spring & Bob :



Eq. motion :

$F = m \cdot a$

$$-kx = m \cdot \frac{d^2x}{dt^2} \rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

2nd order linear differential eq.

↓

SHM or osc. motion

$x(t) = X_m \cos \omega t$

$\boxed{\omega = \sqrt{\frac{k}{m}} \ (s^{-1})}$

a: tangential accel of bob :
α: angular accel. of bob : $\alpha = \frac{a}{R} = \frac{a}{L}$

$$\left.\begin{cases} a = \alpha L = \frac{d^2\theta}{dt^2} L \\ a = -g\sin\theta \end{cases}\right\} \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g\sin\theta}{L}}$$

- Exact equation for a pendulum
- 2nd order <u>non-linear</u> differential equation

  $\sin\theta$

- With "small angle approximation"

  θ small $\Rightarrow$ $\sin\theta \simeq \theta$

  $$\rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta}$$

  → Eq for a pendulum with small angle approximation

  → Solutions are SHM (simple harmonic motion)

  $$\theta(t) = \theta_M \cos(\omega t)$$

  θ_M = oscillation amplitude

  ω = angular frequency of the oscillation (# osc. per second)

Simple Harmonic Motion: (SHM)

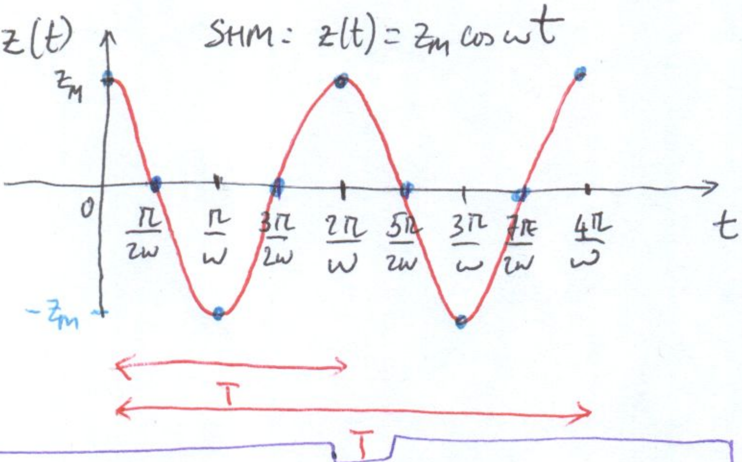
eq. of motion $\dfrac{d^2 z}{dt^2} = -\dfrac{a}{b} z$

solution: $z(t) = z_M \cos \omega t$

$\omega \equiv \sqrt{\dfrac{a}{b}}$

	z	a	b
pendulum	θ	g	l
torsional pendulum	θ	κ	I
spring & bob	x	k	m

SHM: $z(t) = z_M \cos \omega t$

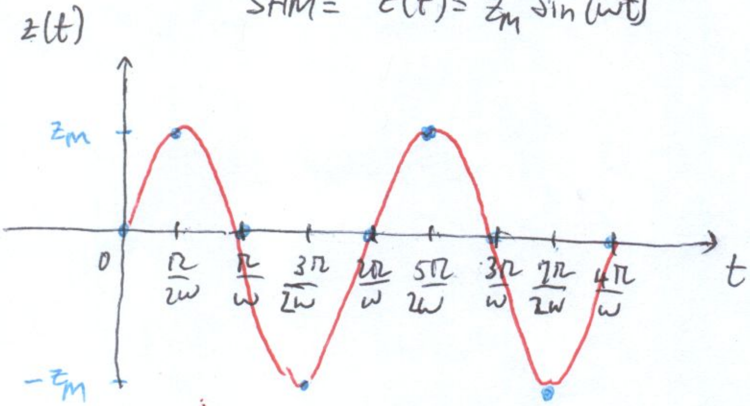


Max
- $\cos 0 = 1 = \cos 2\pi = \cos 4\pi \ldots$
- $\omega t = 2\pi \rightarrow t = \dfrac{2\pi}{\omega} = T$
- $\omega t = 4\pi \rightarrow t = \dfrac{4\pi}{\omega} = 2T$

Min
- $\cos \pi = -1 = \cos 3\pi = \cos 5\pi = \ldots$
- $\omega t = \pi \rightarrow t = \dfrac{\pi}{\omega} = \dfrac{T}{2}$
- $\omega t = 3\pi \rightarrow t = \dfrac{3\pi}{\omega} = \dfrac{3}{2}T$

Zeroes
- $\cos \dfrac{\pi}{2} = 0 = \cos \dfrac{3\pi}{2} = \cos \dfrac{5\pi}{2} = \ldots$
- $\omega t = \dfrac{\pi}{2} \rightarrow t = \dfrac{\pi}{2\omega} = \dfrac{T}{4}$
- $\omega t = \dfrac{3\pi}{2} \rightarrow t = \dfrac{3\pi}{2\omega} = \dfrac{3}{4}T$

SHM = $z(t) = z_M \sin(\omega t)$



Max
- $\sin \dfrac{\pi}{2} = 1 = \sin \dfrac{5\pi}{2} = \sin \dfrac{9\pi}{2} = \ldots$
- $\omega t = \dfrac{\pi}{2} \rightarrow t = \dfrac{\pi}{2\omega} =$
- $\omega t = \dfrac{5\pi}{2} \rightarrow t = \dfrac{5\pi}{2\omega}$

Min
- $\sin \dfrac{3\pi}{2} = -1 = \sin \dfrac{7\pi}{2} = \sin \dfrac{11\pi}{2} \ldots$
- $\omega t = \dfrac{3\pi}{2} \rightarrow t = \dfrac{3\pi}{2\omega}$

Note: if we shift $\sin \omega t$ by $\frac{\pi}{2}$ or $90°$ to the left we get $\cos \omega t$!

Damped-SHM:

equation of motion: $\dfrac{d^2 z}{dt^2} = -\dfrac{a}{b} z \underbrace{- \dfrac{c}{d}\dfrac{dz}{dt}}_{\text{damping term}}$

ω: angular freq; $T = \dfrac{2\pi}{\omega}$.

solution: $z(t) = z_M \underbrace{e^{-\frac{c}{2d}t}}_{\text{exponential decay}} \cos(\omega t + \phi)$

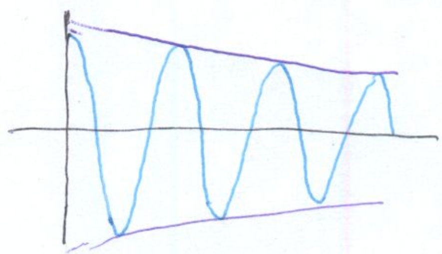
ϕ: "phi" phase shift

Damped SHM: $z(t) = Z_M e^{-\frac{c}{2d}t} \cdot \cos(\omega t + \phi)$

↳ Two time-scales
- Time constant (decaying time): $t_d = \frac{2d}{c}$
  ↳ when $t = t_d \Rightarrow$ osc. is decreased by a factor of e $\left(e^{-1} = \frac{1}{e}\right)$
- Period $T = \frac{2\pi}{\omega}$ or time to complete one full oscillation.

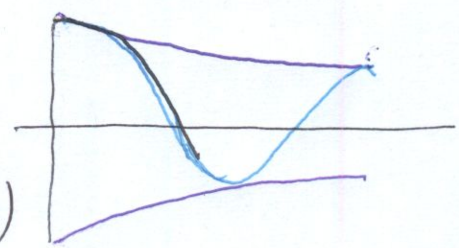
↳ (i) $T \ll t_d$

(many osc. before amplitude is decayed by factor of e)



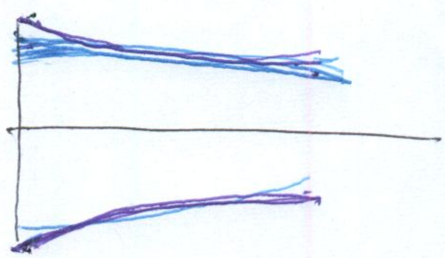
(ii) $T \sim t_d$

(about one osc. and amplitude is decayed by factor of e)

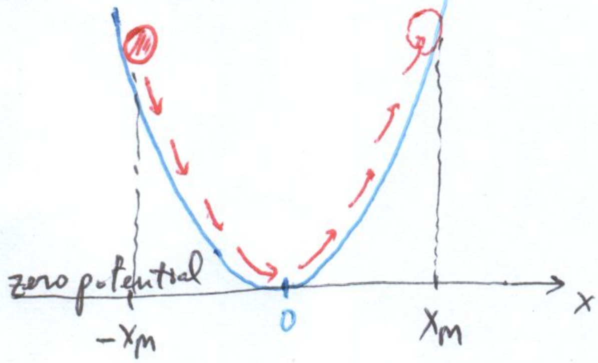


(iii) $T \gg t_d$

can't see the osc. as it is masked by the decay



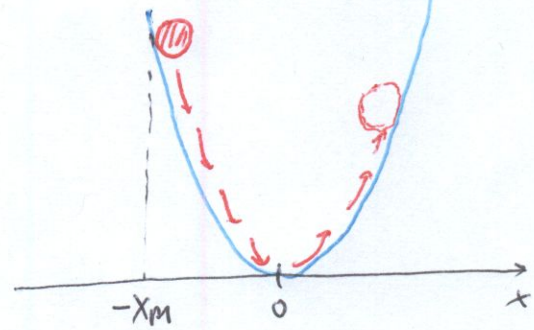
4) Particle trapped in a potential well



No friction

Position along x-axis: SHM:

$$x(t) = x_M \cos(\omega t + \pi)$$



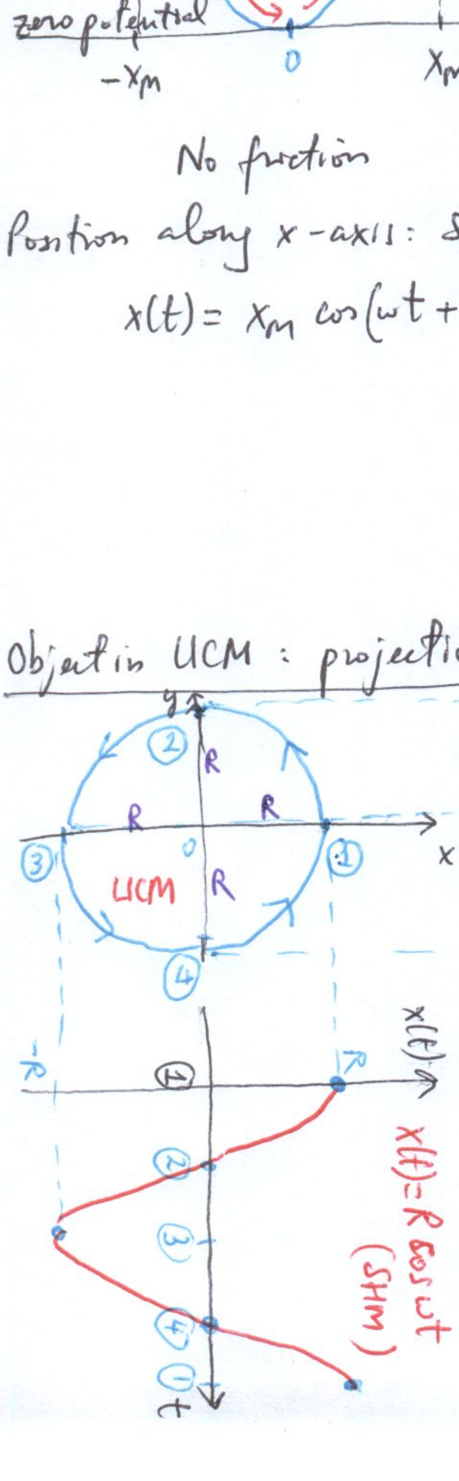
with friction

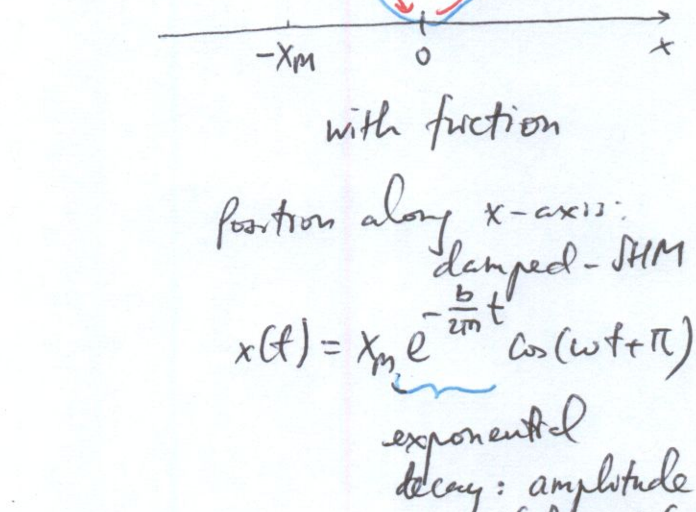
Position along x-axis: damped-SHM

$$x(t) = \underbrace{x_M e^{-\frac{b}{2m}t}}_{} \cos(\omega t + \pi)$$

exponential decay: amplitude of oscillation of ball tends to 0 @ $t \to \infty$

5) Object in UCM: projections of its position onto x & y-axes are SHM's:





object in UCM $\begin{cases} x(t) = R\cos\omega t \\ y(t) = R\sin\omega t \end{cases}$

Both are SHM's shifted by $\frac{\pi}{2}$ or 90° or $\frac{T}{4}$

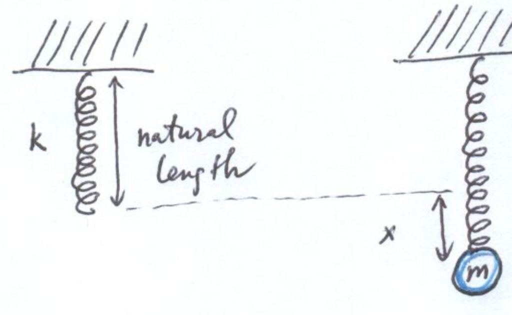
(140)

13.67 Unstreched spring ($m=0$, $k = 74\frac{N}{m}$), a bob is added ($m = 0.49kg$)
hang vertically

→ spring is stretched, then recovers due to Hooke's Law

→ bob under SHM. → a) Amplitude? b) Period T?

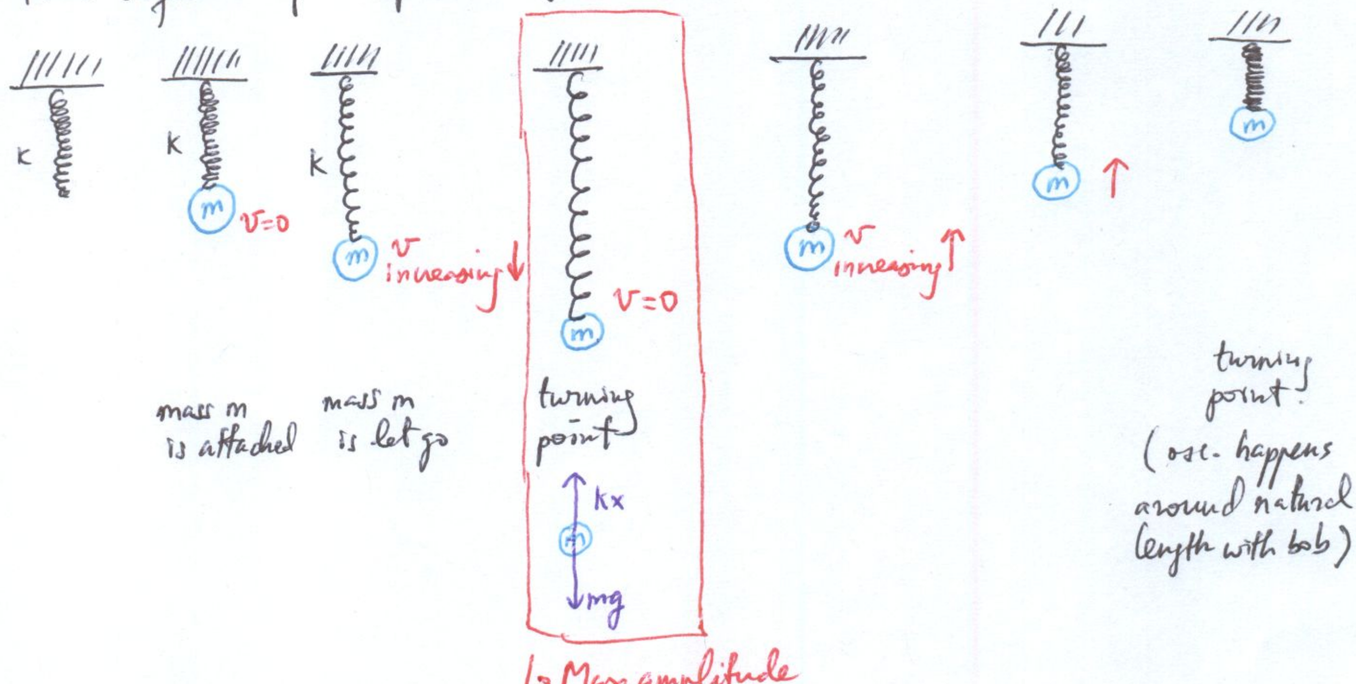
Step 1:



x = displacement from natural length

$k = 74\frac{N}{m}$; $m = 0.49\,kg$

Time sequence of snapshots after bob is added:



Step 2: relevant equations Bob → SHM: $x(t) = x_M \cos(\omega t)$

$$v(t) = \frac{dx}{dt} = -x_M \omega \sin(\omega t)$$

$$a(t) = \frac{d^2x}{dt^2} = -x_M \omega^2 \cos(\omega t)$$

turning point: $\begin{cases} x(t) = x_M \text{ or } \cos\omega t = 1 \rightarrow \sin\omega t = 0 \rightarrow v = 0 \text{ (as expected)} \\ a(t) = -x_M\omega^2 \quad (\cos\omega t = 1) \end{cases}$

2nd Newton's Law @ turning point: $F_{net} = ma$

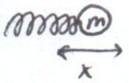
$$kx_M - mg = m(-x_M\omega^2)$$

Step 3: solve for x_M: $x_M(k + m\omega^2) = mg \Rightarrow x_M = \frac{mg}{k + m\omega^2}$

Ch 14 Wave Motion

Oscillatory Motion

Time repeating (periodic) variation of a linear or angular position



→ periodic perturbation : local

→ in time : $z(t) = z_m \cos \omega t$

Osc. Motion { - local
- periodic variation in time

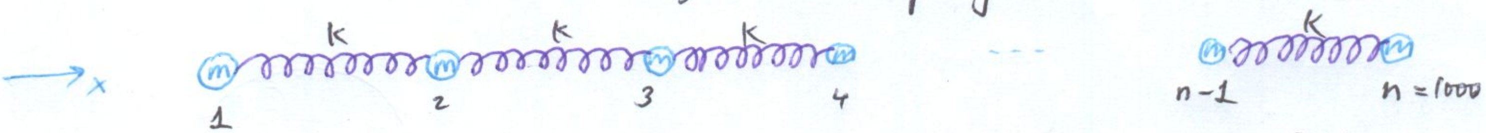
Wave Motion

A step further : the periodic in time perturbation / variation / oscillation is propagated in space

Wave Motion { - propagation
- variation in both time & space

Wave Motion:

1) Propagation: an example of a longitudinal wave : system of identical bobs connected by identical springs:



If I perturb bob #2 giving it a displacement in the horizontal direction:

i) Bob #2 will undergo a time-repeating or periodic variation of position or oscillation (SHM). What happens to bob # 900 at that time? → It is still at rest : perturbation given to #2 is local.

ii) Perturbation on #2 propagates to #3, 4, 5, -- or propagation happens @ finite speed, which depends on the medium: spring constant k

→ a) Perturbation is in x-direction, propagation is also in x-direction : longitudinal wave.

→ b) Bob #2 stays around its position : its perturbation (SHM) is propagated but bob is not !

Wave motion: there is a propagation of the perturbation, not of matter or material

This is different than linear or rotational motion, here the object is not moving, only its perturbation is moving!

↓

No translation of CM

Wave motion: objects involved stay local while their perturbation reaches as far as possible

- Sound waves:
  - matter: air molecules
  - perturbation: change of air density or pressure
- light waves
  - matter: no
  - perturbation: osc. of electric & magnetic fields.

Ch 14 Wave Motion

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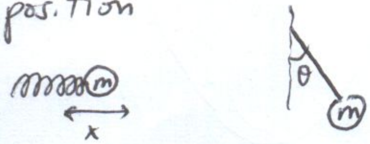

periodic perturbation : local

in time : $z(t) = z_m \cos \omega t$

Osc. Motion
- local
- periodic variation in time

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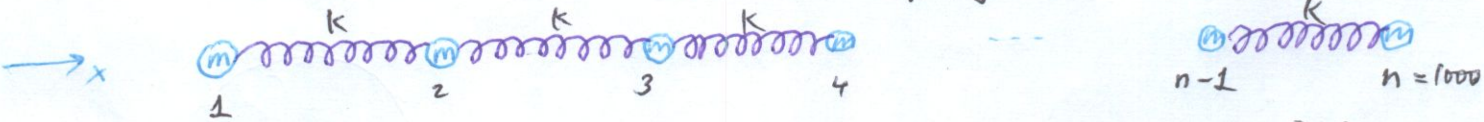

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  - matter: no
  - perturbation: osc. of electric & magnetic fields.

2) Types of waves:

(i) Longitudinal: perturbation & propagation are both in the same direction:

Examples: spring & bob system, seismic waves, ..

(ii) Transverse: perturbation & propagation are perdicular to each other

Examples: wave on a guitar string, EM waves, water ripples, etc.

3) Math description of a transverse wave:

Perturbation in y
Propagation in +x

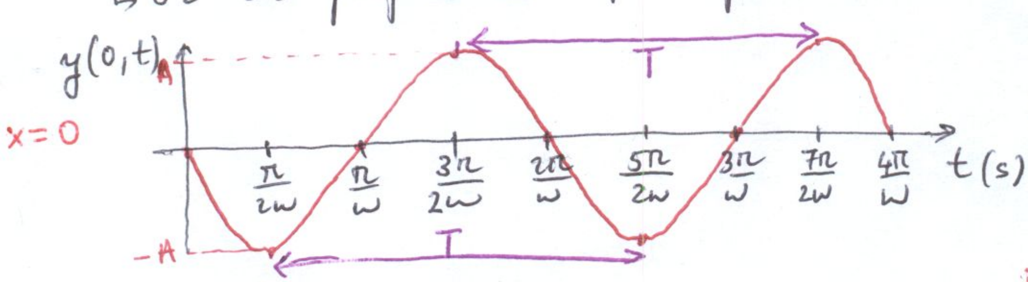
$$\boxed{y(x,t) = A \sin(kx - \omega t)}$$

- k: wave number; number of wavelengths λ in 2π: $k = \frac{2\pi}{\lambda}$ (SI unit is m^{-1})
- λ = wavelength, space separation b/w two consecutive peaks (SI unit: m)
- ω = angular frequency; number of periods T in 2π: $\omega = \frac{2\pi}{T}$ (SI unit is s^{-1})
- T: period, time separation b/w two consecutive peaks (SI unit: s)

4) Graphical description of a transverse wave:

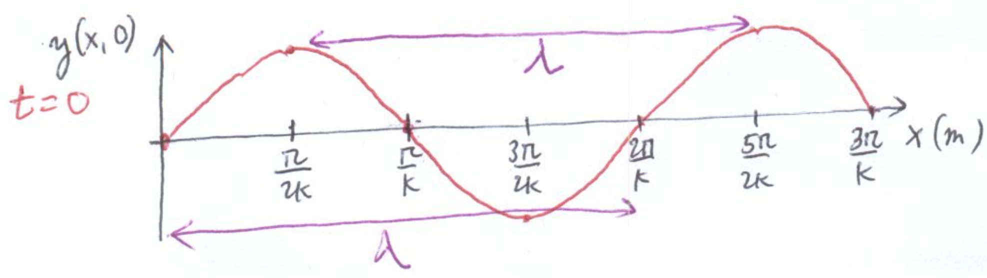
↳ Space & time variation → 3D graphics

↳ Or 2D profile @ a fixed position or @ a fixed time:



This describes how the perturbation at position $x=0$ varies over time.

$y(0,t) = A\sin(-\omega t)$
$= -A\sin\omega t$



This describes the perturbation at $t=0$ over different positions

$y(x,0) = A\sin kx$

14.56 wave on a ~~wire~~ or string given by $y(x,t) = 1.5 \sin(0.1x - 560t)$

Tension $T = 28N$ $\left\{ \begin{array}{l} x, y \text{ are in cm} \\ t \text{ is in s} \end{array} \right.$

– This is transverse wave: ($y(x,t) = A \sin(kx - \omega t)$) where perturbation is in y & the propagation is in x.

(a) Wave amplitude $A = 1.5\,cm$

(b) Wavelength λ

$$\left.\begin{array}{l} k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} \\ k = 0.1\,cm^{-1} \end{array}\right\} \lambda = \frac{2\pi}{0.1}\,cm = 20\pi\,cm \quad \lambda = 62.8\,cm$$

(c) Period T

$$\left.\begin{array}{l} \omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega} \\ \omega = 560\,s^{-1} \end{array}\right\} T = \frac{2\pi}{560}\,s = 11.2 \cdot 10^{-3}\,s \quad T = 11.2\,ms$$

(d) Wave speed : $\boxed{v = \frac{\lambda}{T}}$ (it takes a period to travel a wavelength)

$$v = \frac{62.8 \cdot 10^{-2}\,m}{11.2 \cdot 10^{-3}\,s} = 56\,\frac{m}{s}$$

speed of a transverse wave in a wire

Average speed of a car in highways:

$$65\,\frac{mi}{h} \cdot \frac{1609\,m}{1\,mi} \cdot \frac{1h}{3600\,s} = 18.1\,\frac{m}{s}$$

Note: linear frequency $f = \frac{\omega}{2\pi}$ (Hz), how many periods in one second

$$\left[f = \frac{\frac{2\pi}{T}}{2\pi} = \frac{1}{T} \right] \rightarrow \boxed{v = \frac{\lambda}{T} = \lambda \cdot f}$$

(e) Power carried by the wave: $\boxed{\overline{P} = \frac{1}{2}\mu\omega^2 A^2 v}$

μ: linear density of wire (thick wires carry more power than thin wires)
ω: angular freq.
A: wave amplitude
v: wave speed

$v = \sqrt{\frac{T}{\mu}}$ (can be derived apply Newton's 2nd Law on an element of the wire)

T: tension in wire

$$v = \sqrt{\frac{T}{\mu}} \rightarrow v^2 = \frac{T}{\mu} \rightarrow \mu = \frac{T}{v^2}$$

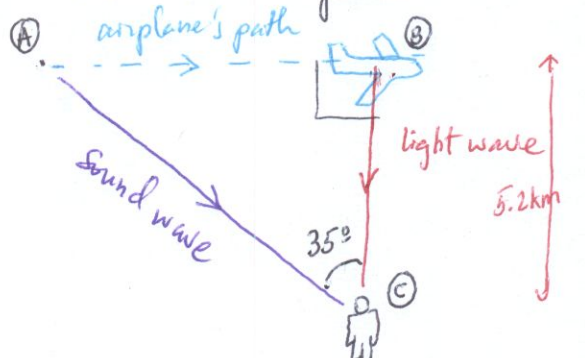
$$\boxed{\bar{P} = \frac{1}{2}\mu\omega^2 A^2 \cancel{v} = \frac{1}{2}\frac{T}{\cancel{v^2}}\omega^2 A^2 \cancel{v} = \frac{1}{2}\frac{\overset{\text{Tension}}{T}}{v}\omega^2 A^2}$$

$$\bar{P} = \frac{1}{2}\,\frac{28 \cdot 560^2 \cdot 0.015^2}{56} = 17.4\,W$$

$$\cancel{\mu = \sqrt{\frac{T}{v}} = \sqrt{\frac{28}{56}}} \quad \mu = \frac{T}{v^2} = \frac{28}{56^2}\left(\frac{kg}{m}\right)$$

14.63

Step 1: See airplane overhead, hear sound not from overhead but 35° back on it's path:



observer @ C, light comes from B, sound comes from A

→ When observer sees plane @ B, he hears sound that the plane made when it passed A (not yet the sound it makes @ B). Reason: $C = 3 \cdot 10^8 \frac{m}{s}$ $v_s = 330\,\frac{m}{s}$

↳ Consequence: $t_s = t_p$ $\begin{cases} t_s: \text{ time for sound (jet noise) to travel AC} \\ t_p: \text{ time for airplane to travel AB} \end{cases}$

Step 2: $$v_{plane} = \frac{d_{AB}}{t_p} = \frac{d_{AB}}{t_s} = \frac{d_{AB}}{\frac{d_{AC}}{v_s}}$$

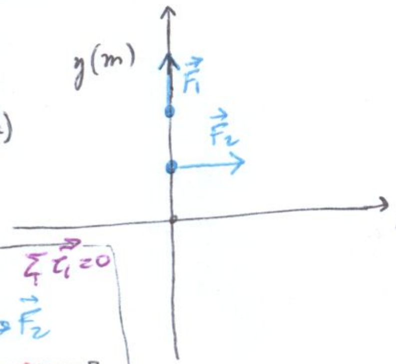
$$= \frac{d_{AB}}{d_{AC}} v_s = \frac{\text{opposite of } 35°}{\text{hypothenuse}} v_s = (\sin 35°)\, v_s$$

Step 3: $$v_{plane} = 330 \cdot \sin 35° = 189\,\frac{m}{s}$$

$$189\,\frac{m}{s} \cdot 3.6 = 680\,\frac{km}{h}$$

12.16

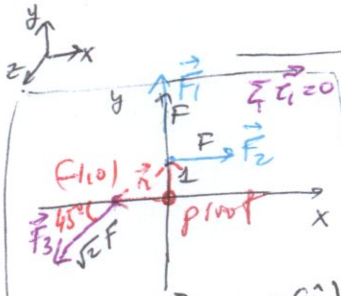
(a)



is there a $\vec{F_3}$ $\begin{cases} (i) \; \vec{F_1}+\vec{F_2}+\vec{F_3}=0 \\ (ii) \; \vec{\tau}_{F_1}+\vec{\tau}_{F_2}+\vec{\tau}_{F_3}=0 \end{cases}$

(i) Yes : $\vec{F_3} = -\vec{F_1} - \vec{F_2} = -\left(\vec{F_1}+\vec{F_2}\right)$

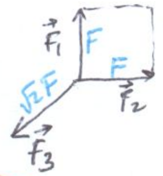
(ii) Can this $\vec{F_3}$ satisfy $\sum_i \vec{\tau}_i = 0$?

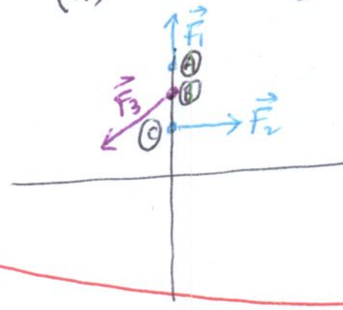


$\vec{\tau}_{F_1} = 0$; $\vec{\tau}_{F_2} = 1 \cdot F \cdot (-\hat{k})$

$\vec{\tau}_{F_3} = 1 \cdot \sqrt{2} F \sin 45^\circ (\hat{k})$
$= \sqrt{2} F \frac{\sqrt{2}}{2} \hat{k} = F\hat{k}$

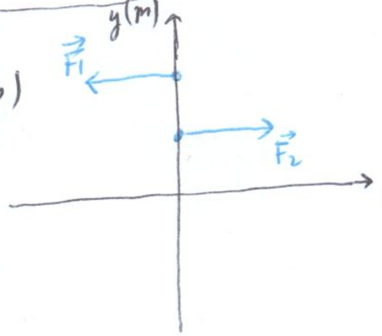
$\vec{F_3}$ =





In this example wether center of rotation is @ (A) or (B) or (C) the torques do not add up to 0 → No for this center of rotation but Yes for center of rotation @ origin

(b)



(i) No : $\vec{F_1}+\vec{F_2}=0$ if we add a 3rd force
$\Rightarrow \vec{F_1}+\vec{F_2}+\vec{F_3} = \vec{F_3} \neq 0$

13.46

Pendulum in a rocket : $\begin{cases} \text{(a) at rest on launchpad} \\ \text{(b) up @ } a = \frac{g}{2} \\ \text{(c) down @ } a = \frac{g}{2} \\ \text{(d) down @ } a = g \text{ (Free fall)} \end{cases}$

↳ SHM : $\omega = \sqrt{\frac{g}{L}}$

↳ Find its period $T = \frac{2\pi}{\omega}$

(a) $T_{(a)} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}} = 2\pi\sqrt{\frac{L}{g}}$

$a=0$

(b) $T_{(b)} = 2\pi\sqrt{\frac{2L}{g}}$

$\uparrow a = g/2 \Rightarrow g' = g - a = g - \frac{g}{2} = \frac{g}{2}$

(c) $T_{(c)} = 2\pi\sqrt{\frac{2L}{3g}}$

$\downarrow a = \frac{g}{2} \Rightarrow g' = g + a = g + \frac{g}{2} = \frac{3g}{2}$

(d) $T_{(d)} = 2\pi\sqrt{\frac{L}{g'}} = \infty$

$\downarrow a = g \rightarrow g' = 0$

takes ∞ time to complete one osc. → doesn't oscillate

Wave properties

Wave superposition:

↳ 3 very important phenomena

(i) Beats: tuning of string instruments, tuning of airplane engines (bimotors)

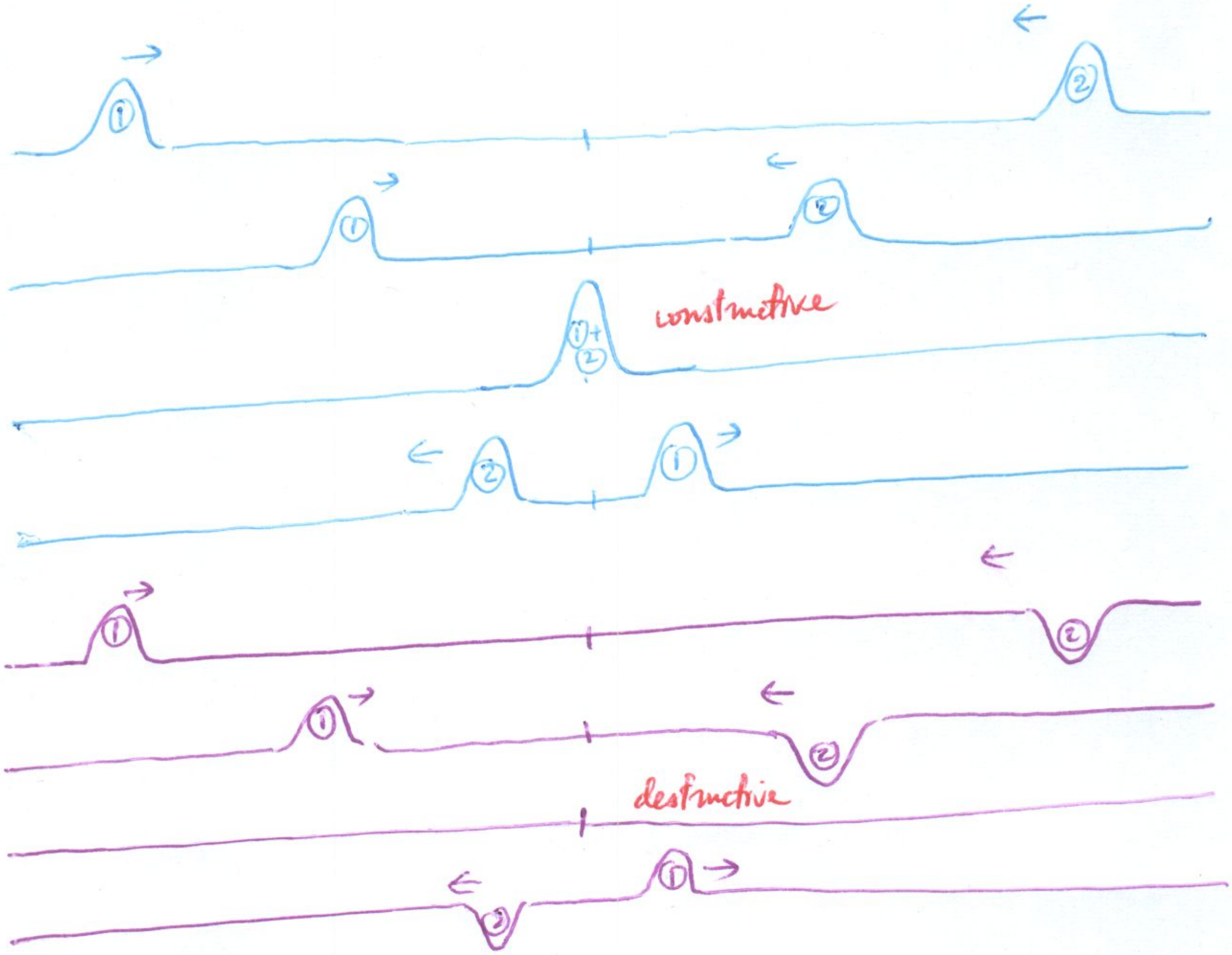
(ii) Standing waves: wind instruments (pipes, flutes, ...)

(iii) Wave interference: constructive / destructive

1 + 1 = 0

Doppler effect: when wave source is also moving → LIDAR = (speed traps)

Wave superposition:



Beat phenomena: math description:

- Two transverse waves traveling in the same direction

$\hookrightarrow \begin{cases} \text{same amplitudes } A \\ \text{different frequencies } w_1, w_2 \end{cases}$ (and different wave numbers k_1, k_2)

$$y_1 = A \sin(k_1 x - w_1 t)$$

$$y_2 = A \sin(k_2 x - w_2 t)$$

- Wave superposition: they combine:

At $x=0$ $\Rightarrow$ $y(0,t) = y_1(0,t) + y_2(0,t)$

$$= -A \sin w_1 t - A \sin w_2 t$$

$$= -A(\sin w_1 t + \sin w_2 t)$$

- Trigonometry: $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)$

$$y(0,t) = -2A \sin\left[\frac{(w_1+w_2)}{2} t\right] \cdot \cos\left[\frac{(w_1-w_2)}{2} t\right]$$

average of w_1 & w_2 — difference of w_1 & w_2

- If $w_1 \sim w_2$ $\begin{cases} \frac{w_1+w_2}{2} \sim w_1 \\ \frac{w_1-w_2}{2} \text{ very small compared } w_1 \text{ or } w_2 \end{cases}$

$\hookrightarrow$ When we can't hear w_1 or w_2 (two fast for ear drums) we can hear $w_1 - w_2$ → **beats**.