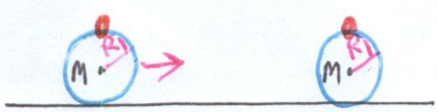
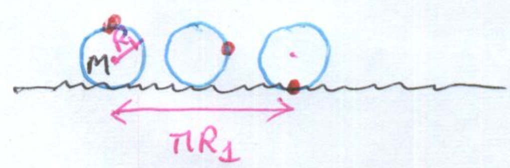
Ch 10: Rotational Motion

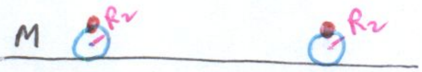
So far: linear motion or translational motion

Translation: sliding bowling ball on frictionless surface



Rotation: rolling bowling ball on rough surface

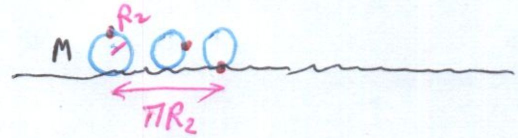




Shrink bowling ball to a smaller radius R_2

(i) Sliding balls of equal masses but different radii ($R_2 < R_1$) will follow same translational motion since they can be described as point-like particles of mass M (size does not matter)

(ii) In translational motion orientation of ball stays the same (red dot stays at top)



(i) In rolling motion size does matter!

(ii) In rolling motion we have:

$\begin{cases} \text{Translation (CM travels } \pi R) \\ \text{Rotation (orientation changes as we follow red dot)} \end{cases}$

(iii) pure rotation?

inverted bike wheel

Cars:

1) Normal road condition (friction): wheels $\leftrightarrow$ rolling motion $\begin{cases} \text{translation} \\ \text{rotation} \end{cases}$

2) Car in sand: wheels $\leftrightarrow$ rotation

3) ABS: anti-blocking braking system: allows wheels to roll slowly till a complete stop

↳ if wheels are blocked: as brakes are applied: car skids forward, wheels have only translational motion

with ABS: car rolls forward to a stop: wheels $\leftrightarrow$ both translation & $\boxed{\text{rotation}}$

↓

shorter stopping distance

↓

saving life

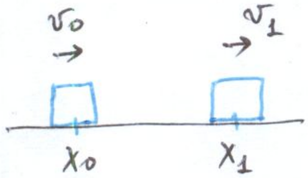
When we want to stop a car (mass m) traveling at speed v:

we need to transfer $\frac{1}{2}mv^2$ out $\rightarrow$ friction: as car skids forward; if in addition there are rotation in 4 wheels $\rightarrow$ shorter stopping distance

Quantitative descriptions

Translational motion

↳ change of position



$$\bar{v} = \frac{v_0 + v_1}{2} \quad \text{(average velocity)}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_1 - x_0}{\Delta t} \quad \text{(average velocity)}$$

$$v = \frac{dx}{dt} \quad \text{(instantaneous velocity)}$$

(Unit SI: $\frac{m}{s}$)

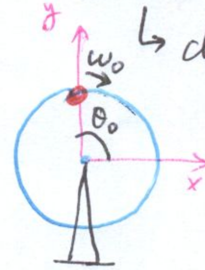
$$a = \frac{dv}{dt} \quad \text{(instantaneous acceleration)}$$

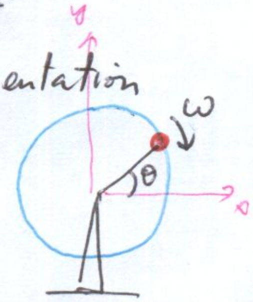
(Unit SI: $\frac{m}{s^2}$)

Definitions

Rotational Motion

↳ change of orientation or angle





θ_0: initial angle
ω_0: initial angular velocity

θ: final angle
ω = final angular velocity.

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \quad \text{(average angular vel.)}$$

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \quad \text{(average ang. vel.)}$$

$$\omega = \frac{d\theta}{dt} \quad \text{(instantaneous ang. vel.)}$$

(Unit SI: $\frac{\text{radian}}{s}$ or s^{-1})

$$\alpha = \frac{d\omega}{dt} \quad \text{(instantaneous ang. acceleration)}$$

(Unit SI: $\frac{\text{radian}}{s^2}$ or s^{-2})

Equations of Motion: (constant acceleration)

1) $v = v_0 + a \cdot t$

2) $x = x_0 + v_0 \cdot t + \frac{1}{2} a t^2$

3) $\frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a$

2nd Newton's Law: $\boxed{F_{net} = m \cdot a}$

1) $\omega = \omega_0 + \alpha \cdot t$

2) $\theta = \theta_0 + \omega_0 \cdot t + \frac{1}{2} \alpha t^2$

3) $\frac{\omega^2 - \omega_0^2}{\theta - \theta_0} = 2 \cdot \alpha$

Analog of 2nd Newton's Law:

$$\boxed{\tau_{net} = I \cdot \alpha}$$

τ_{net}: net torque ; I = moment of inertia
α: angular acceleration

Ch10 Rotational Motion Topics

1) Rolling motion (relate translation of CM to rotation)
2) Angular acceleration α
3) Torque (radius matters)
4) Moment of inertia I (size matters)
5) KE in rotational motion

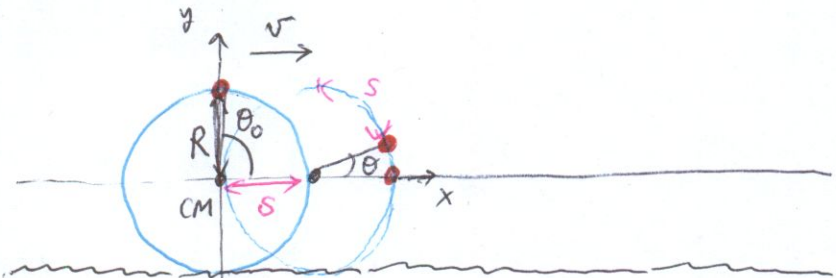
---

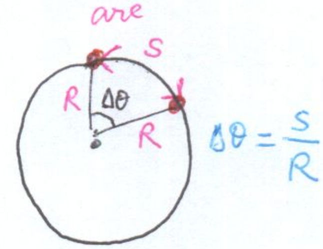
1) Rolling motion: a quantitative connection b/w translation & rotation

translation (linear velocity)

rotation (angular velocity)

Focus on red dot on bowling ball and how its motion is related to the translation of CM of the bowling ball.





(i) CM in linear motion along x-axis at linear velocity v

(ii) @ a later time the red dot is described by angle θ (initially by angle θ_0) $\rightarrow$ angular displacement is $\Delta\theta = \theta - \theta_0$

(iii) $\Delta\theta = \frac{S}{R}$ (geometry)

(iv) Since bowling ball has rolling motion, each point on the perimeter of ball will touch the surface one after the other: length of trace = length of arc s $\Rightarrow$ motion of red dot along arc = motion of CM

(v) $\frac{d}{dt}\left[\Delta\theta = \frac{S}{R}\right]$

$$\omega = \frac{1}{R}\underbrace{\frac{ds}{dt}}_{v \text{ (linear vel. of CM)}} \longrightarrow$$

Rolling motion

$$\boxed{\omega = \frac{v}{R}} \text{ or } v = \omega \cdot R$$

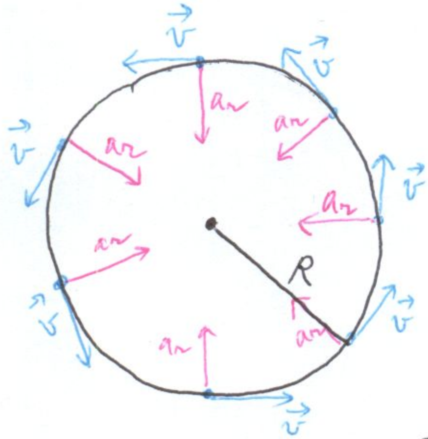
units $s^{-1} = \frac{m}{s} \cdot \frac{1}{m}$

linear vel. of CM equals angular vel. times the radius

2) Angular Acceleration α: $\boxed{\alpha = \frac{a_t}{R}}$

$$\bar{\alpha} \equiv \frac{\Delta\omega}{\Delta t} \quad ; \quad \alpha = \frac{d\omega}{dt} \qquad \left(\frac{rad}{s^2} \text{ or } s^{-2}\right)$$

What is α in UCM?



$\vec{v}$: same magnitude (blue vectors with same length!) v
→ constant linear speed v for uniform circular motion
direction always tangential to circle → there is a need of a linear radial acceleration toward center of curvature $(a_r = \frac{v^2}{R})$ to keep object on circle

UCM
$a_r = \frac{v^2}{R}$ (radial acceleration)
$a_t = \frac{dv}{dt} = 0$ (tangential acceleration or change of speed along circle)
$\hookrightarrow \alpha = \frac{d\omega}{dt} = \frac{d\left(\frac{v}{R}\right)}{dt} = \frac{1}{R}\frac{dv}{dt} = 0$

Why $\omega = \frac{v}{R}$? b/c $\frac{d}{dt}\left[\Delta\theta = \frac{\text{arc } s}{R}\right]$

$$\underbrace{\frac{d\Delta\theta}{dt}}_{\omega} = \frac{1}{R}\underbrace{\frac{ds}{dt}}_{v} \quad \text{or} \quad \omega = \frac{v}{R} \checkmark$$

Non-UCM (non-uniform circular motion) or linear speed v along circle is NOT constant.

$\hookrightarrow$
$a_r = \frac{v^2}{R}$ (now with different magnitudes depending on v)
$a_t = \frac{dv}{dt} \neq 0$
$\hookrightarrow \alpha = \frac{d\omega}{dt} = \frac{1}{R}\frac{dv}{dt} \neq 0$

$$\boxed{a_t = \frac{dv}{dt} = \frac{d(\omega \cdot R)}{dt} = R \cdot \frac{d\omega}{dt} = R \cdot \alpha}$$

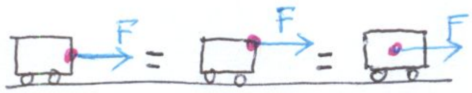
or $\boxed{\alpha = \frac{a_t}{R}}$

3) Torque $\vec{\tau}$ (tau)
vector

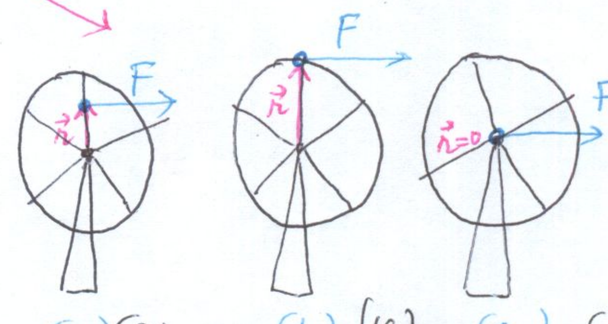
→ "cross product" × between two vectors
(Work = dot product b/w $\vec{F}$ & $\Delta\vec{r}$)

→ Radius matters: Why? Force $\underbrace{\text{application point}}_{\text{where it is applied}}$ matters

Linear motion — Force application point — Rotational motion



Force app. points (in pink) does not matter in linear motion



(a) (3) (b) (19) (c) (3)

which case corresponds to highest speed for wheel?
Force application point does matter
↓
cross-product.

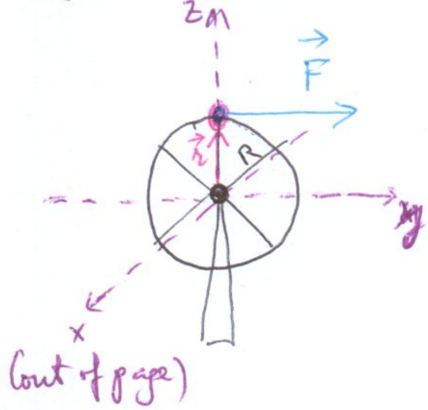
$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$ ($\vec{r}$ "cross" $\vec{F}$) unit: Nm

Essential: (i) pivot point (rotation is with respect to pivot point) → intuitive to find out.
(ii) force application point

→ radial vector $\vec{r}$: from pivot to force application point.
→ force applied $\vec{F}$

Cross-product
- (i) product b/w two vectors ($\vec{r}$ & $\vec{F}$) that produces another vector ($\vec{\tau}$)
- (ii) $\vec{\tau}$ is perpendicular to both $\vec{r}$ & $\vec{F}$, by RHR (right hand rule)
- (iii) $\vec{\tau} = r \cdot F \cdot \sin\theta \ \hat{\tau}$ ($\hat{\tau}$: unit vector in torque direction given by RHR) θ is angle b/w $\vec{r}$ & $\vec{F}$

New item for cross product: direction by RHR:

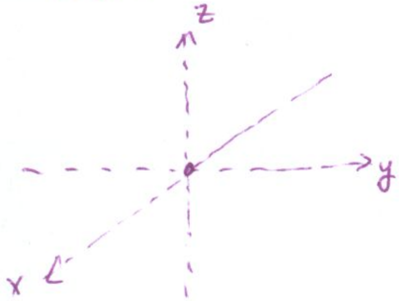


Direction of torque $\vec{\tau}$ when $\vec{F}$ is applied @ top of wheel:

$\begin{cases} \text{pivot} = \text{black dot or center of rotation} \\ \text{force application point: pink dot or top of wheel} \end{cases}$

↳ $\vec{r}$ goes from pivot to force application point.

↳ $\vec{r}$ $\begin{cases} \text{1) radial vector since its origin is the pivot or center of rotation} \\ \text{2) if we place origin of coords. @ pivot we can consider } \vec{r} \text{ as the position vector of the force application point.} \end{cases}$

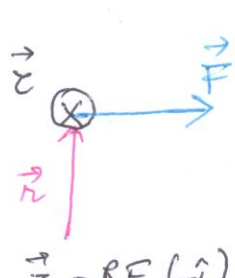


$$\left.\begin{aligned} \vec{F} &= F\,\hat{j} \\ \vec{r} &= R\,\hat{k} \end{aligned}\right\} \quad \vec{\tau} = \vec{r} \times \vec{F} = RF \sin 90 \,(\hat{k} \times \hat{j})$$

$\theta = 90^\circ$

RHR to find direction of $\underset{\text{1st vector}}{\hat{k}} \times \underset{\text{2nd vector}}{\hat{j}}$: 1) align RH fingers along 1st vector $(\hat{k})$, 2) close RH fingers toward 2nd vector $(\hat{j})$ 3) thumb indicates direction of the cross product $\hat{k} \times \hat{j}$ (also direction of torque)

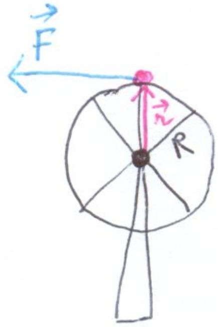
↳ $= -\hat{\imath}$ (torque $\vec{\tau}$ direction is into page)



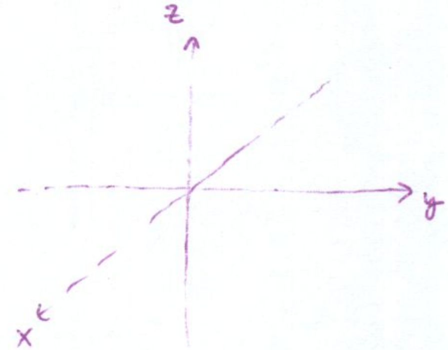
$\otimes$: into page

$\odot$: out of page

$\vec{\tau} = RF(-\hat{\imath}) \Leftrightarrow$ CW rotation



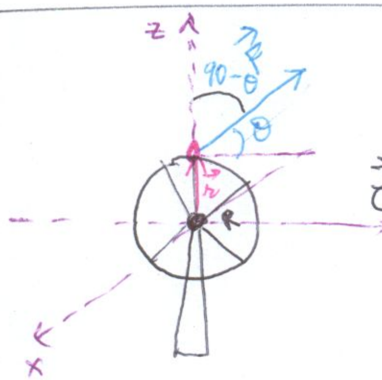
$$\vec{\tau} = RF\,(\underbrace{\hat{k}\times(-\hat{j})}_{\substack{=\hat{i}\\ (RHR)}})$$



$\vec{\tau} = RF\,\hat{i} \Leftrightarrow$ CCW rotation

$\vec{\tau} = RF\underbrace{\sin 0}_{0}\,(\underbrace{\hat{k}\times\hat{k}}_{0}) = 0 \Leftrightarrow$ no rotation

---



$$\vec{\tau} = R\cdot F\sin(90-\theta)\underbrace{(-\hat{i})}_{RHR}$$

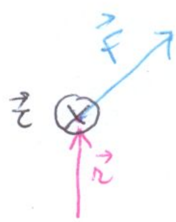
RHR { 1st vector: $\vec{r}$; 2nd vector $\vec{F}$

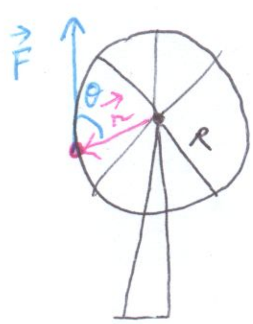
$= -RF\cos\theta\,\hat{i}$

→ CW rotation

→ reduced torque ($\cos\theta < 1$)

→ slower ω





$$\vec{\tau} = R\cdot F\sin\theta\,\underbrace{(-\hat{i})}_{RHR}$$

RHR { 1st vector $\vec{r}$; 2nd vector $\vec{F}$

$= -RF\sin\theta\,\hat{i}$

→ CW rotation

→ reduced torque.

4) Moment of Inertia I

Linear motion

mass m

↳ Two objects with same mass m regardless of sizes, will have same motion (same force applied)

↳ same inertia to linear motion

Rotational motion

moment of inertia I

M smaller vol | M larger volume

$\rho_1 = \frac{M}{V_1}$ | $\rho_2 = \frac{M}{V_2} < \rho_1$

(steel ball) | (tennis ball)

↳ tennis ball offers more inertia to rotational motion

Larger size, same mass ⇒ more rotational inertia:

(i) Figure skater: end of indiv. program: spinning around vertical axis: fast then slow down:

smaller radius

larger radius ⟺ larger rotational inertia

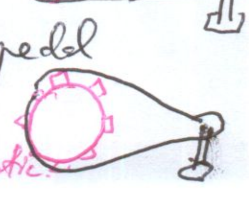
(same mass)

(ii) Bike gear, left handle: 

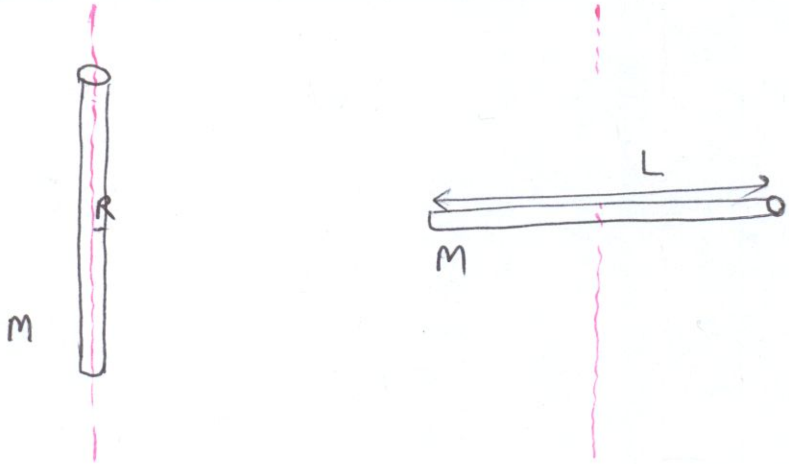
→ flat road, go fast
→ up hill

Flat road { gear 1: smaller disk
gear 3: harder to pedal

larger disk
↓
larger rotational inertia



(iii)



$I_1 = \frac{1}{2} M R^2$ $\qquad$ $I_2 = \frac{1}{12} M \cdot L^2$

For example: $R = 10^{-2}\,m$ $\qquad$ $L = 1\,m$

$$\frac{I_1}{I_2} = \frac{R^2}{L^2} = \frac{10^{-4}}{1} = 10^{-4}$$ → harder to rotate rod in situation ② vs ①

Quantitative formula for I:

- discrete system: $I = \sum_i m_i r_i^2$
  - m_i = mass of component i
  - r_i = position of i wrt axis of rotation
- continuous system:
  $I = \int r^2\, dm$
  - dm: infinitesimal mass
  - r = position of dm wrt axis of rotation

↓ Simple geometrical shapes:

sphere, cylinder, ring, disk:

$I = c M R^2$
- M: total mass of object
- R: radius of mass distribution
- c = coefficient depending on actual shape.

1) Sphere wrt center axis

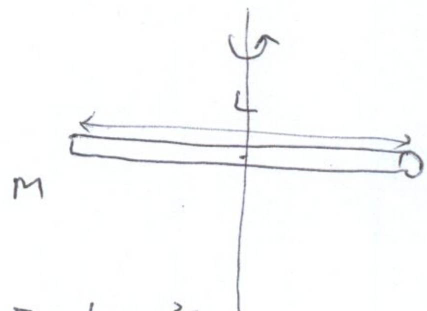
$c = \frac{2}{5}$ $\quad I = \frac{2}{5}MR^2$

2) Cylinder wrt center axis

$c = \frac{1}{2}$

$I = \frac{1}{2}MR^2$

3) Cylinder or rod of length L



$c = \frac{1}{12}$

$I = \frac{1}{12}ML^2$

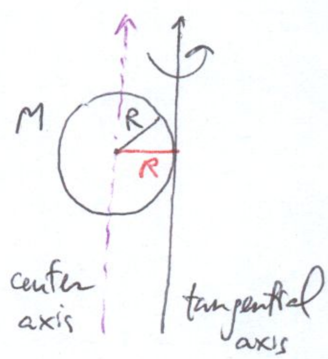
4) Ring wrt center axis

$c = 1$ or $I = MR^2$

5) Disk wrt center axis

$c = \frac{1}{2}$ or $I = \frac{1}{2}MR^2$

6) Sphere wrt tangential axis



$I = \frac{2}{5}MR^2 \quad I = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

Parallel axis Theorem → general, not only for spheres.

$$\boxed{I_{\text{tangential axis}} = I_{\text{center axis}} + \boxed{MR^2}}$$

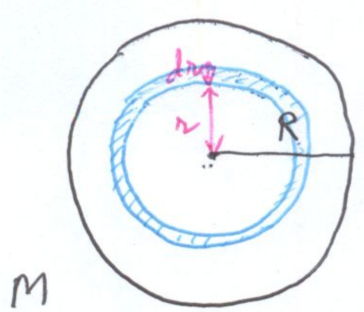
(R = separation b/w center axis & tangential axis

Moment of inertia I for a disk of mass M, radius R, rotating w.r.t. center axis:

$$I = \int r^2 \, dm$$

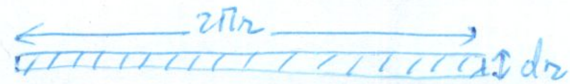
dm: tiny ring on disk of radius r, thickness dr

top view of disk:



in proportion to M (uniform mass distribution for disk)

$$\boxed{\frac{dm}{M} = \frac{2\cancel{\pi} r \, dr}{\cancel{\pi} R^2}}$$



area of a ring of radius r & thickness dr is $2\pi r \, dr$

area of a disk of radius R is πR^2

$$I = \frac{2M}{R^2}\int_0^R r^3 \, dr = \frac{2M}{4R^2}\left[r^4 \right]_0^R = \frac{2M}{2\cancel{4}\cancel{R^2}} R^{\cancel{4}\,2} = \frac{1}{2} M R^2$$

($I = cMR^2$, $c = \frac{1}{2}$ for disk w.r.t. center axis)

5) Kinetic energy in rotational motion:

Linear motion

→ Sliding ~~bowling ball~~ disk on frictionless surface

→ Mass m, radius R

$$KE = \frac{1}{2} m v^2$$

(m: inertia for linear motion)

Pure rotation

→ Rotating disk of mass m, radius R wrt center axis or pivot point (no friction at pivot)

→ No translation or linear motion of CM

$$KE = \frac{1}{2} I \omega^2$$

(I: inertia in rotational motion)

↕

size/radius matters in rotation

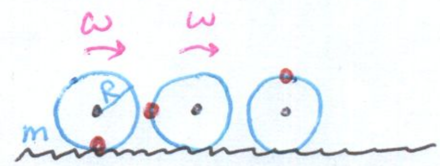
Curiosity: let's call v the speed of a point on the outer edge of disk (red dot)

$\Rightarrow v = R \cdot \omega$

$$KE = \frac{1}{2}\left(\frac{1}{2} m R^2\right)\omega^2$$

$$= \frac{1}{2}\left(\frac{1}{2} m\right) v^2$$

Rolling motion



→ Sufficient friction for rolling motion of disk of mass m, radius R

→ There is rotation wrt center axis ($I = \frac{1}{2} m R^2$) plus translation of CM @ $v_{CM} = \omega \cdot R$

$$KE = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v_{CM}^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right)\omega^2$$

$$= \frac{1}{2} m v_{CM}^2 + \underbrace{\frac{1}{4} m v_{CM}^2}_{\text{half of that of linear motion of CM}}$$

$$= \frac{1}{2}\left(\frac{3}{2} m\right) v_{CM}^2$$

ABS braking: model each wheel as a disk

$$\begin{cases} \text{sliding wheel}: KE = \frac{1}{2} \boxed{m} v_{CM}^2 \\ \text{Rolling wheel}: KE = \frac{1}{2}\left(\boxed{\frac{3}{2} m}\right) v_{CM}^2 \end{cases}$$

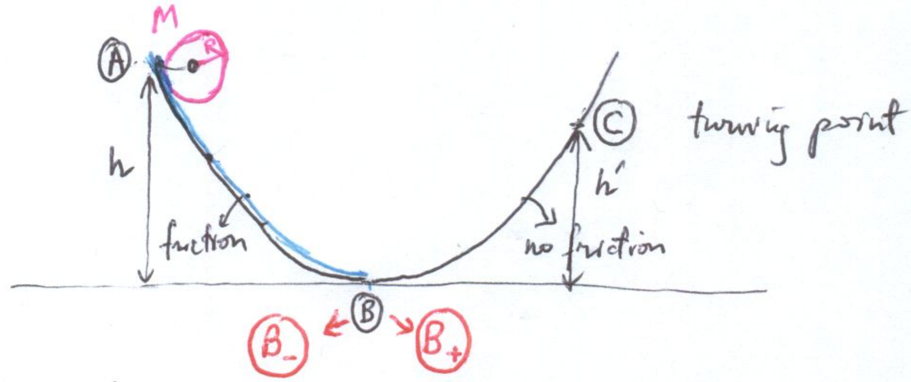
with rolling motion each wheel has an effective mass increase of 50% ! → shorter stopping distance

10.64 Sphere rolls down left side of parabolic potential well from height h, then slides up right side (no friction) to what height h'?

Step 1:

B is a special point.

$\begin{cases} B_- \equiv \text{just before } B \\ B_+ \equiv \text{just after } B \end{cases}$



Focus on motion of sphere:

(i) Friction → $\begin{cases} \text{sliding box} \\ \text{~~sliding~~ rolling sphere} \end{cases}$

: many contact points box & surface → slow down motion

: only one contact point → just to allow rotation wrt center axis in addition to translation of CM

(ii) $ME_{A} = ME_{B_-}$ & $ME_{B_+} = ME_{C}$

$ME_{B_+} = ME_{B_-} - \frac{1}{2}I\omega^2$ (rotational KE part is staying in the left side only)

↳ $h' < h$

Step 2: relevant equations:

1) $\underset{\text{grav. potential}}{\overset{A}{Mgh}} = \overset{B_-}{\underset{\text{linear kinetic}}{\frac{1}{2}Mv_{cm}^2}} + \underset{\text{rotational kinetic}}{\frac{1}{2}I\omega^2}$

2) Sphere rotating wrt center axis
$I = \frac{2}{5}MR^2$

3) Rolling motion: $v_{cm} = \omega \cdot R$
$\omega = \frac{v_{cm}}{R}$

4) $\overset{B_+}{\frac{1}{2}\cancel{M}v_{cm}^2} = \overset{C}{\cancel{M}gh'}$

Step 3: Solve for h'

4) $h' = \dfrac{v_{cm}^2}{2g}$

Find v_{cm}: plug 2) & 3) in 1)

1) $Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\frac{2}{5}M\cancel{R^2}\frac{v_{cm}^2}{\cancel{R^2}} = \frac{1}{2}\left(M + \frac{2}{5}M\right)v_{cm}^2 = \frac{1}{2}\left(\frac{7}{5}M\right)v_{cm}^2$

$\cancel{M}gh = \frac{1}{2}\frac{7}{5}\cancel{M}v_{cm}^2 \Rightarrow \frac{5}{7}h = \dfrac{v_{cm}^2}{2g}$

$\Rightarrow \boxed{h' = \frac{5}{7}h}$

Ch 11 Rotational Vectors & Angular Momentum $\vec{L}$

Linear Motion

2nd Newton's Law $\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$

$\vec{p} \equiv$ linear momentum $\equiv m \cdot \vec{v}$

$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m \cdot \vec{a}$

m constant

most general

Rotational Motion

$\boxed{\vec{\tau}_{net} = \frac{d\vec{L}}{dt}}$ more popular $\vec{\tau}_{net} = I\vec{\alpha}$

$\vec{L} =$ angular momentum vector

$\boxed{\vec{L} \equiv \vec{r} \times \vec{p}}$ more popular $\vec{L} = I\vec{\omega}$

angular momentum of an object wrt axis of rotation is the cross product b/w its position vector $\vec{r}$ & its linear momentum vector $\vec{p}$

---

1) Cross products, 2 so far $\left\{ \begin{array}{l} \vec{\tau} = \vec{r} \times \vec{F} \\ \vec{L} = \vec{r} \times \vec{p} \end{array} \right.$

They are related: $\vec{\tau} = \frac{d\vec{L}}{dt} \underset{\vec{r}\ \text{constant}}{=} \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}$

2) More popular $\vec{\tau}_{net} = I \cdot \vec{\alpha}$

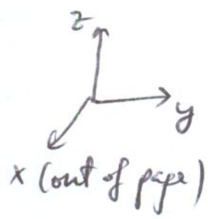
$= I \cdot \frac{d\vec{\omega}}{dt} \underset{\text{if } I \text{ is independent of time}}{=} \frac{d}{dt}(I\vec{\omega}) \Longrightarrow \boxed{\vec{L} = I \cdot \vec{\omega}}$

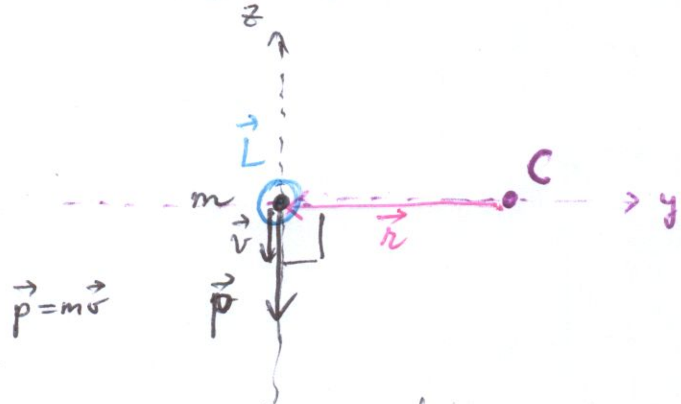
$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

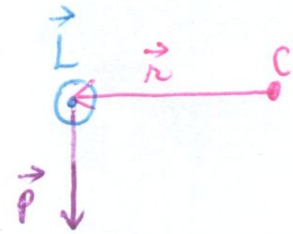
$\left. \begin{array}{l} \vec{\tau}_{net} = I \cdot \vec{\alpha} \\ \vec{L} = I \cdot \vec{\omega} \end{array} \right\}$ are related.

Calculation of angular momentum vector:

1) Object of mass m moving along $-z$ axis, "center of rotation" is @ C





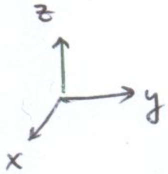


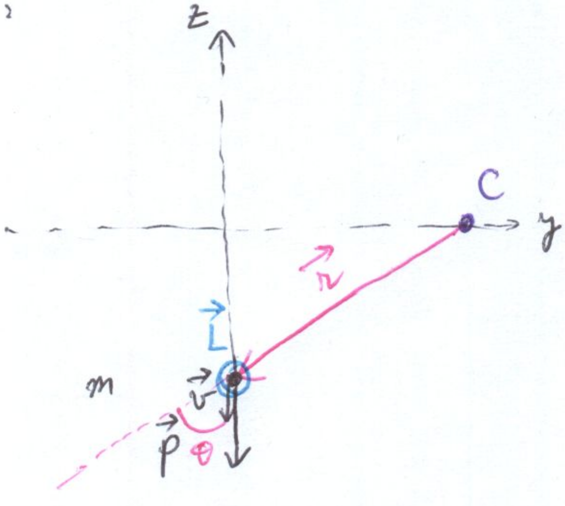
→ There is no obvious rotation, however an angular momentum vector can be calculated : $\vec{L}=\vec{r}\times\vec{p}$ ($\vec{r}$ = position vector of the object wrt. "center of rotation")

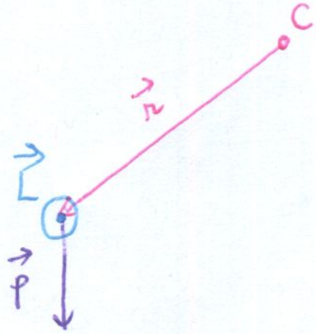
$$\vec{L}=\vec{r}\times\vec{p} = rp\sin 90^\circ \underbrace{(-\hat{j})\times(-\hat{k})}_{RHR\,=\,\hat{i}} = r\cdot p\ \hat{i} \text{ (out of page)}$$

→ Although object travels in straight line it acts like it rotates CCW wrt "center of rotation" C

2) Object mass m moving along $-z$ axis, with "center of rotation" C :



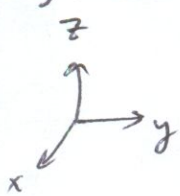


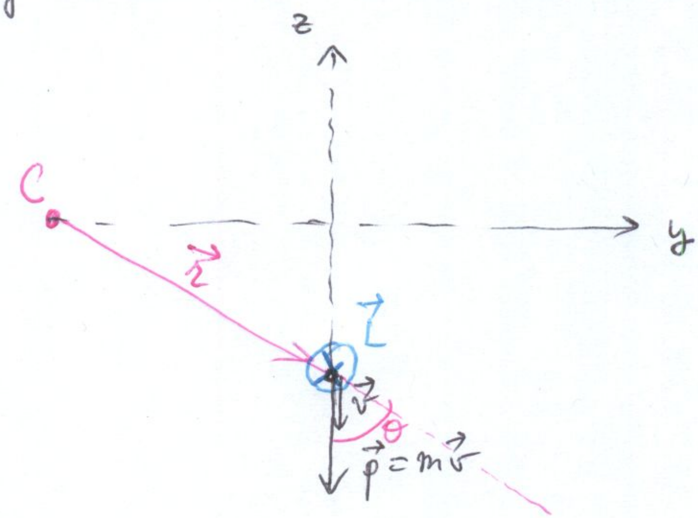


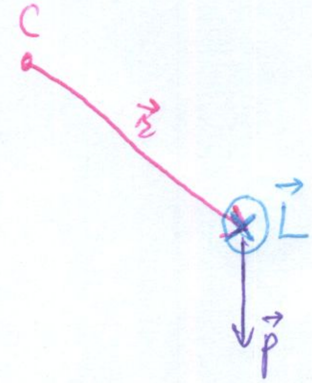
$$\vec{L} = \vec{r} \times \vec{p} = rp\sin\theta \underbrace{\hat{\imath}}_{RHR}$$

<u>Note</u> : $\vec{L}$ had max strength in case 1) Here it is decreased by $\sin\theta < 1$.

3) "Center of rotation" is on the side :



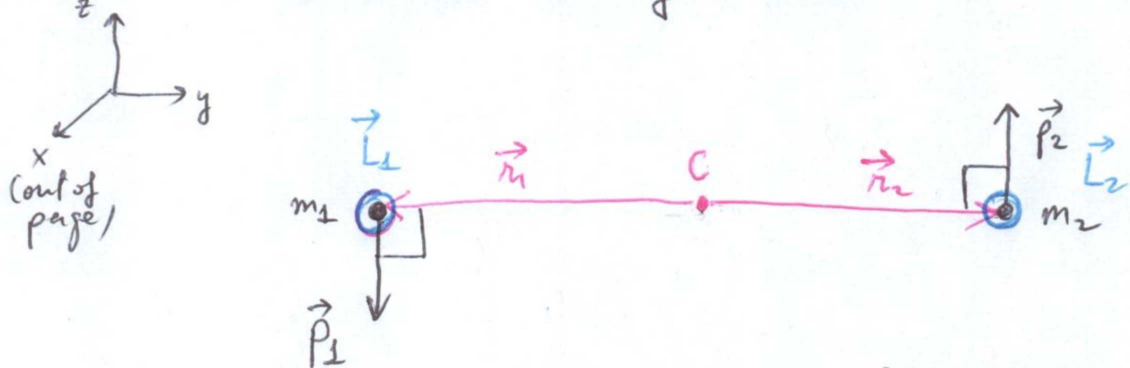




$$\vec{L} = \vec{r} \times \vec{p} = rp\sin\theta \underbrace{(-\hat{\imath})}_{RHR}$$

<u>Note</u>: $\vec{L}$ depends on the selection of the "center of rotation".

4) Two objects of masses m_1 & m_2 moving in opposite directions wrt the middle "center of rotation":



Total angular momentum vector: $\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$

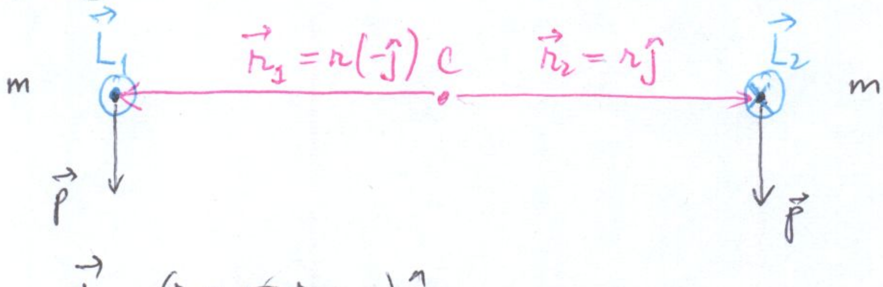
$$= r_1 p_1 \underbrace{\hat{i}}_{RHR} + r_2 p_2 \underbrace{\hat{i}}_{RHR}$$

$$\vec{L} = (r_1 p_1 + r_2 p_2)\,\hat{i}$$

Note: if $m_1 = m_2 \equiv m$ & $r_1 = r_2 \equiv r$ & $v_1 = v_2 \equiv v$ $\rightarrow \vec{L} = 2rp\,\hat{i}$

$\Rightarrow p_1 = p_2 \equiv p$

5) Total $\vec{L}$ is 0:



$$\vec{L} = \underbrace{(r_1 p_1 - r_2 p_2)}_{0 \text{ (if equal } m\text{'s, } r\text{'s, } p\text{'s)}}\,\hat{i}$$

Conservation of momentum

Linear motion

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

if $\vec{F}_{net} = 0 \Rightarrow \vec{P}_i = \vec{P}_f$

conservation of linear momentum

Rotational motion

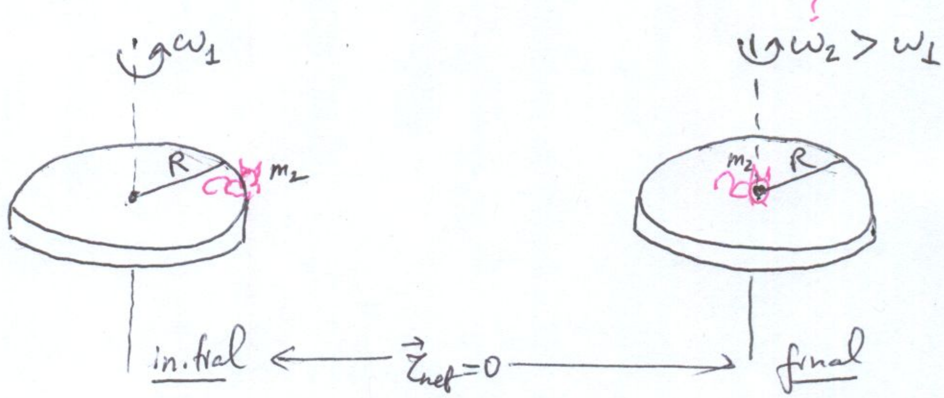
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

if $\vec{\tau}_{net} = 0 \Rightarrow \vec{L}_i = \vec{L}_f$

conservation of angular momentum

11-38

Step 1:



initial $\longleftarrow \vec{\tau}_{net} = 0 \longrightarrow$ final

$R = 0.25 m$

$I_1 = 0.0154 kgm^2$

$\omega_1 = 22 rpm$

$m_2 = 19.5 \cdot 10^{-3} kg$

$$\vec{\tau}_{net} = 0 \;\Downarrow\; \vec{L}_i = \vec{L}_f$$

Turn table (disk, M_1, R) → sub index 1

Mouse (dot, m_2) → sub index 2

Step 2: relevant equations : $\vec{L}_{i_1} + \vec{L}_{i_2} = \vec{L}_{f_1} + \vec{L}_{f_2}$

or 1) $\vec{L}_{1i} + \vec{L}_{2i} = \vec{L}_{1f} + \vec{L}_{2f}$

2) $\vec{L} = \begin{cases} I\vec{\omega} & \text{(continuous)} \\ \vec{r} \times \vec{p} & \text{(discrete)} \end{cases}$

$$1) \& 2) \Rightarrow \quad I_1 \omega_1 + m_2 R^2 \cdot \omega_1 = I_1 \omega_2 + \cancel{0 \cdot \omega_1}^{\,0}$$

Step 3: a) Solve for ω_2

$$\Rightarrow \omega_2 = \frac{(I_1 + m_2 R^2)\,\omega_1}{I_1} = \underbrace{\left(1 + \frac{19.5 \cdot 10^{-3} \cdot 0.25^2}{0.0154}\right)}\, 22 rpm$$

$$\boxed{\omega_2 = 23.74 \text{ rpm}}$$

b) Work done by mouse: $\vec{\tau}_{net} = 0$ or no work done from outside of turntable + mouse system

↳ increased rotational KE: $KE_f - KE_i =$

$$KE_f - KE_i = \frac{1}{2}(I_1 + 0)\,\omega_2^2 - \frac{1}{2}(I_1 + m_2 R^2)\,\omega_1^2$$

$\omega_1 = 22 \frac{rev}{min} \cdot \frac{2\pi rad}{rev} \cdot \frac{1 min}{60 s}$

$= \frac{44\pi}{60} \frac{rad}{s}$

$\omega_2 = \frac{47{,}48\pi}{60} \frac{rad}{s}$

$$= \frac{1}{2} \cdot 0.0154 \cdot \left(\frac{47.48\pi}{60}\right)^2 - \frac{1}{2}\left(0.0154 + 19.5 \cdot 10^{-3} \cdot 0.25^2\right)\left(\frac{44\pi}{60}\right)^2 J$$

$$= 0.0476 J - 0.0441 J = 0.0035 J$$

or 3.5 mJ

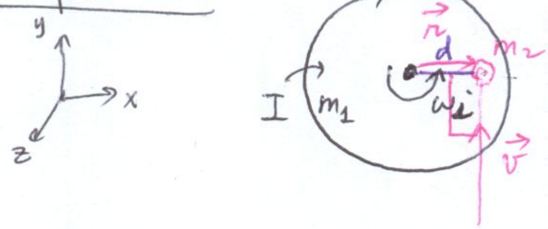
11.45) System of turntable (1) & clay (2) $\rightarrow \vec{\tau}_{net} = 0 \rightarrow \vec{L}_i = \vec{L}_f$

Step 1:

initial: clay about to land on turntable

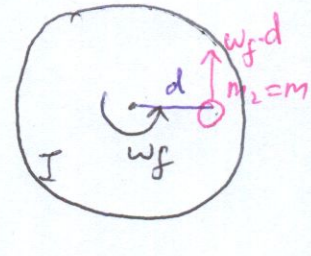
final: clay lands on the turntable

view from above



$\vec{r} = d\hat{i}$

$\vec{p} = mv\hat{j}$



Step 2: relevant equations:

1) $\vec{L}_{1i} + \vec{L}_{2i} = \vec{L}_{1f} + \vec{L}_{2f}$

2) $\vec{L}_i \begin{cases} \vec{L}_{1i} = I \cdot \vec{\omega}_i \\ \vec{L}_{2i} = \vec{r} \times \vec{p} \end{cases}$ $\quad \vec{L}_f = \begin{cases} \vec{L}_{1f} = I\vec{\omega}_f \\ \vec{L}_{2f} = md^2\vec{\omega}_f \end{cases}$

Just Before clay landed (clay goes in linear motion)

After clay landed. (clay goes in rotational motion along with turntable)

1 & 2 $\Rightarrow$ $I\omega_i\hat{k} + dmv \underbrace{\sin 90}_{1} \underbrace{(\hat{i} \times \hat{j})}_{\text{RHR } \hat{k}} = (I + md^2)\omega_f \hat{k}$

$I\omega_i + dmv = (I + md^2)\omega_f \Rightarrow v = \dfrac{(I+md^2)\omega_f - I\omega_i}{m \cdot d}$

Step 3:

a) $v?$ for $\omega_f = \dfrac{\omega_i}{2}$ $\Rightarrow$ $v = \dfrac{\left[\dfrac{(I + md^2)}{2} - I\right]\omega_i}{m \cdot d}$

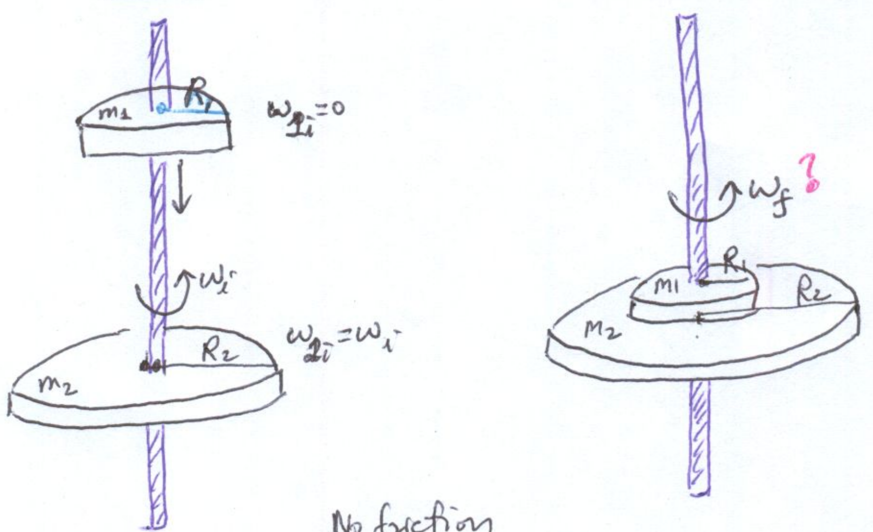
$v = \dfrac{md^2 - I}{2md}\omega_i$

b) $v?$ for $\omega_f = \omega_i$ $\quad v = \dfrac{(I + md^2 - I)\omega_i}{md} = d \cdot \omega_i$

c) $v?$ for $\omega_f = 2\omega_i$ $\quad v = \dfrac{(2I + 2md^2 - I)\omega_i}{md} = \dfrac{I + 2md^2}{md}\omega_i$

11.49 Top disk dropping onto a rotating disk arriving at a final common rotational speed.

Step 1:



$m_1 = 0.27\,kg$

$R_1 = 0.023\,m$

$m_2 = 0.44\,kg$

$R_2 = 0.035\,m$

$\omega_i = 180\,rpm$

No friction

$$\vec{\tau}_{net} = 0 \rightarrow \vec{L}_i = \vec{L}_f$$

Step 2: relevant equations:

1) $\vec{L}_{1i} + \vec{L}_{2i} = \vec{L}_{1f} + \vec{L}_{2f}$

2) $L = I \cdot \omega$ for both rotating disks

3) $I_{Disk} = \frac{1}{2} M R^2$

$$1)\ \&\ 2)\ \&\ 3):\quad 0 + \frac{1}{2} m_2 R_2^2 \cdot \omega_i = \left(\frac{1}{2} m_1 R_1^2 + \frac{1}{2} m_2 R_2^2\right) \cdot \omega_f$$

Step 3: a) solve for ω_f:

$$\omega_f = \frac{m_2 R_2^2}{m_1 R_1^2 + m_2 R_2^2} \omega_i = \frac{0.44 \cdot 0.035^2}{0.27 \cdot 0.023^2 + 0.44 \cdot 0.035^2} \, 180\,rpm$$

$$\omega_f = 142\ rpm$$

b) Fraction of KE_i lost to friction to keep two disks rotating together:

$$\text{lost fraction} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = 1 - \frac{\left(\frac{1}{2} m_1 R_1^2 + \frac{1}{2} m_2 R_2^2\right) \omega_f^2}{\frac{1}{2} m_2 R_2^2 \quad \omega_i^2}$$

$$= 1 - \frac{0.27 \cdot 0.023^2 + 0.44 \cdot 0.035^2}{0.44 \cdot 0.035^2} \left(\frac{142}{180}\right)^2 = 0.211$$

or 21.1%

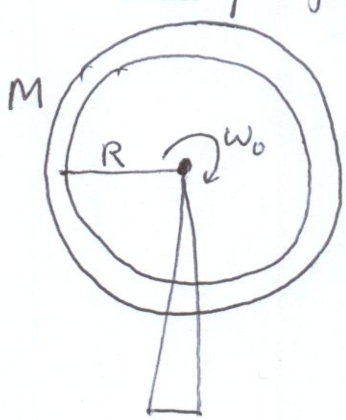
Alternative #2: from a) $\frac{I_i}{I_f} = \frac{\omega_f}{\omega_i} \Rightarrow$ lost fraction $= 1 - \frac{I_f}{I_i} \frac{\omega_f^2}{\omega_i^2} = 1 - \frac{\omega_f}{\omega_i}$

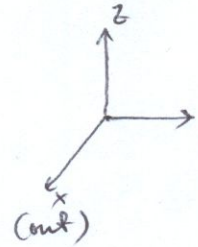
$$= 1 - \frac{142}{180} = 0.211$$

10.58

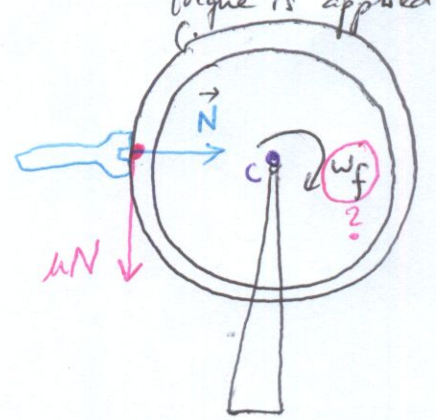
Step 1:

initial
wheel spins freely





final
torque is applied



$w_0 = 230\,rpm$

$M = 1.9\,kg$ (mostly rim)

↳ wheel = ring → $I = MR^2$

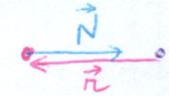
$R = 0.33\,m$

$\vec{N} = 2.7\,\hat{j}$ (N)

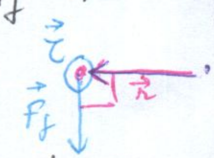
$\mu = 0.46$ (b/w wrench & tire)

↳ friction force $\vec{F}_f = \mu N(-\hat{k})$

Torque $\begin{cases} \vec{\tau}_N = RN \sin 180 = 0 \\ \vec{\tau}_{F_f} = R\mu N \sin 90\,(\hat{i}) \quad \checkmark \end{cases}$



RHR



↳ out of page → CCW rotation → lower w_f !

Step 2: equations: $\tau_{net} = I \cdot \alpha$

$$RN\mu\,\hat{i} = MR^2\left(\frac{w_f - w_0}{\Delta t}\right)\underbrace{(-\hat{i})}_{\text{CW rotation (RHR)}}$$

Step 3: solve for w_f:

$$\underset{rpm}{w_f} = w_0 - \underset{rpm}{\frac{\mu N \cancel{R} \Delta t}{M R^{\cancel{2}}}} = 230\,rpm - \underbrace{\frac{0.46 \cdot 2.7 \cdot 3.1}{1.9 \cdot 0.33}\frac{rad}{s} \cdot \frac{1\,rev}{2\pi\,rad} \cdot \frac{60\,s}{min}}_{58.6\,rpm}$$

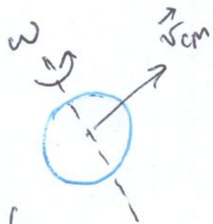
$$\boxed{w_f = 171\,rpm}$$

10.36 Baseball with both linear KE & rotational KE

$m = 0.15\,kg$ $\qquad v_{cm} = 33\,\frac{m}{s}$ $\qquad \omega = 42\,\frac{rad}{s}$

$R = 3.7 \cdot 10^{-2}\,m$

Step 1:



sphere
$\left\{ \begin{array}{l} \text{mass } m \\ \text{radius } R \end{array} \right. \rightarrow I = \frac{2}{5} m R^2$

which fraction of KE is rotational? $\dfrac{KE_{rot}}{KE_{linear} + KE_{rot}}$

Step 2:

$$\text{rot. fraction} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2} = \frac{\frac{2}{5} \cancel{m} R^2 \omega^2}{\cancel{m} v_{cm}^2 + \frac{2}{5} \cancel{m} R^2 \omega^2}$$

$$= \frac{\frac{2}{5} \cdot (3.7 \cdot 10^{-2})^2 \cdot 42^2}{33^2 + \frac{2}{5} (3.7 \cdot 10^{-2})^2 \, 42^2} = 8.86 \cdot 10^{-4} \text{ or } 0.0886\,\%$$

very small

Note: here: flying baseball $\left\{ \begin{array}{l} \rightarrow v_{cm} = 33\,\frac{m}{s} \\ \rightarrow \text{linear speed of a point on outer edge of ball due to rotation } v = \omega \cdot R \end{array} \right.$

$= 42 \cdot 0.0037 = 1.554\,\frac{m}{s}$

↳ very small fraction of rot. KE!

Rolling baseball : $v_{cm} = \omega \cdot R \rightarrow \omega \simeq 20 \cdot 42\,\frac{rad}{s}$
(20 times larger)