Ch7 Conservation of Energy

Motion → Forces → Work & Energy

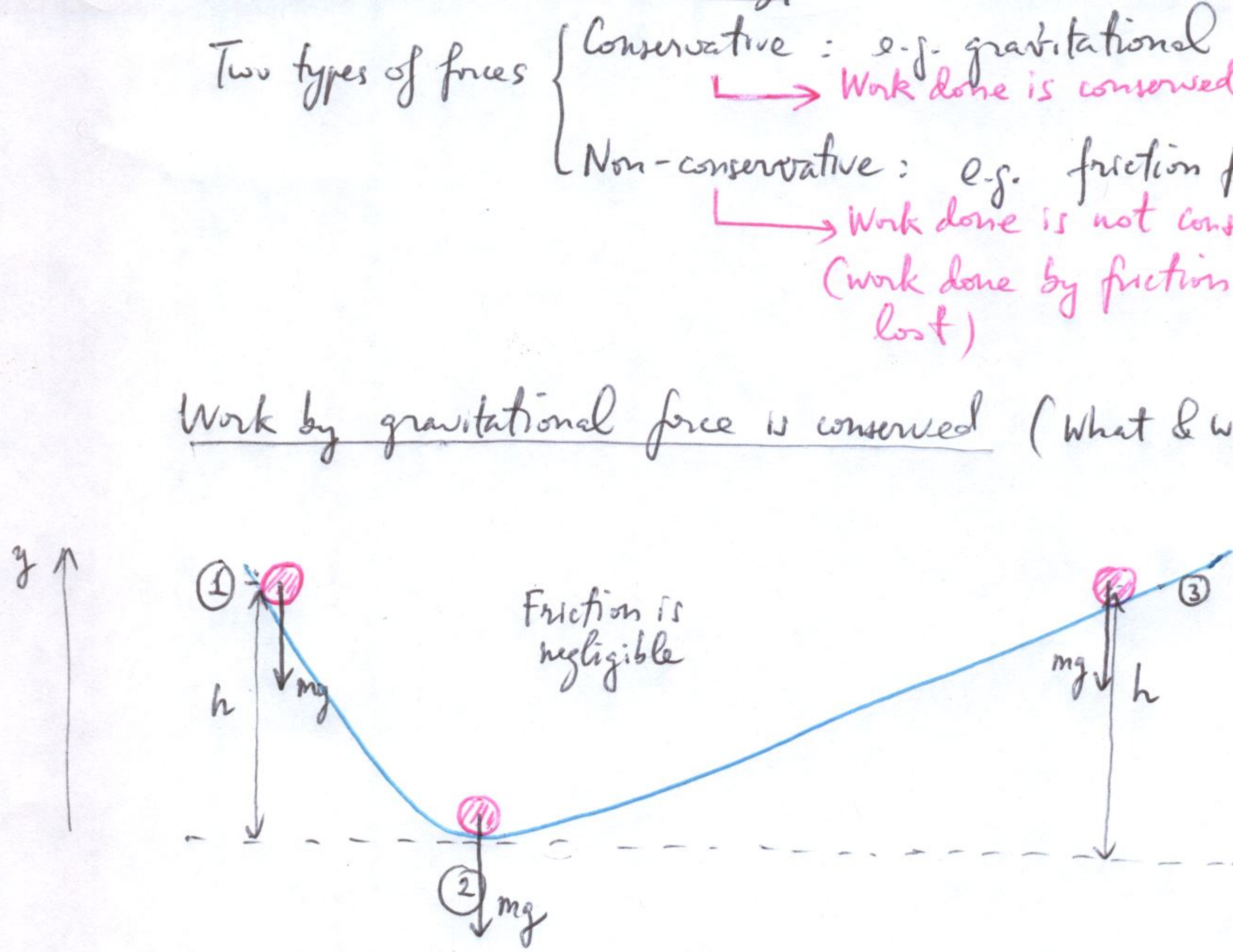
(Kinematic equations for constant acceleration) (Newton's equations)
- Definition
- Conservation of energy

} Alternatives to describe motions

Forces & Conservation of Energy

Two types of forces {
- Conservative : e.g. gravitational force
  → Work done is conserved
- Non-conservative : e.g. friction force
  → Work done is not conserved (work done by friction is always lost)

Work by gravitational force is conserved (What & why?)



Work definition : $W = \int \vec{F} \cdot d\vec{r}$.

$$W = -mg\hat{j} \cdot \underbrace{\int d\vec{r}}_{\Delta\vec{r}} = -mg\hat{j} \cdot \Delta\vec{r}$$

mg$\hat{j}$ does not do any work in horizontal direction ⇒ ignore horizontal displacement

① → ②

$\Delta\vec{r} = (0-h)\hat{j} = -h\hat{j}$

$W_{12} = -mg\hat{j} \cdot (-h\hat{j}) = mgh\,\hat{j}\cdot\hat{j} = mgh$

$\vec{A}\cdot\vec{B} \equiv AB\cos(\theta) \Rightarrow \hat{j}\cdot\hat{j} = 1 \cdot 1 \cos(0) = 1$

② → ③

$\Delta\vec{r} = (h-0)\hat{j} = h\hat{j}$

$W_{23} = -mg\hat{j} \cdot (h\hat{j}) = - mgh$

Gravity received back the work it did b/w ① & ②.

Conclusions:

1) Gravity did work on ball from ① to ②, in the amount of mgh (ball's $\underbrace{\text{grav. potential energy}}_{GPE}$ decreased by mgh from ① to ②)

2) Gravity received back work from ② to ③, in the same amount (sign negative to indicate gravity received work) mgh (ball's $\underbrace{\text{gravitational potential energy}}_{GPE}$ increased by mgh from ② to ③)

3) Total work done by gravity after ball comes back to same height is conserved: $W_{12} + W_{23} = mgh - mgh = 0$
(Work done by grav. force, which is a conservative force, is conserved)

4) As ball rolls down track from ① its GPE decreases while its kinetic energy ($KE = \frac{1}{2}mv^2$) increases. KE is max @ ② where GPE is min. Between ② & ③ KE decreases when GPE increases, KE is back to 0 @ ③ when GPE is back to its initial value.

5) Sum of GPE & KE is ME (mechanical energy) is conserved (if all forces involved are conservative!)

6) Most general ME also includes an elastic potential energy (EPE)

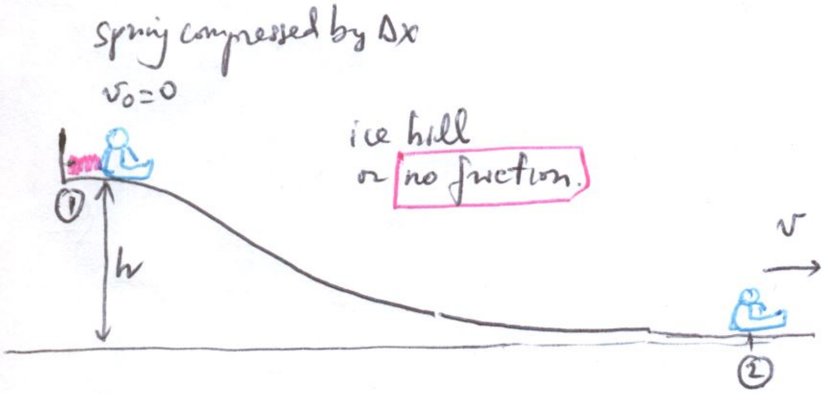
$$ME = GPE + KE + EPE$$

→ Work done by spring force $F = -k \cdot x$
$$W = \int \vec{F} \cdot d\vec{x} = -k \int_0^x x \cdot dx = -\frac{1}{2}kx^2$$
→ object attached to spring $EPE = \frac{1}{2}kx^2$

→ Work done by 2nd Newton's Law of motion $F = m\frac{dv}{dt}$
$$W = \int \vec{F} \cdot d\vec{x} = m \int^v \frac{dv}{dt} \cdot dx = \frac{1}{2}mv^2$$

→ Work done by gravity force $F = -mg\hat{j}$
$$W = \int \vec{F} \cdot d\vec{x} = -mg\hat{j} \int_h^0 dy\,\hat{j} = -mg(-h) = mgh$$

Example where all 3 forms of mechanical energy are involved:

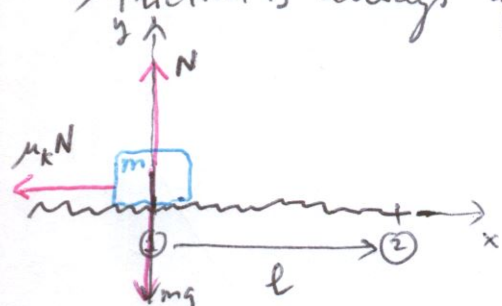


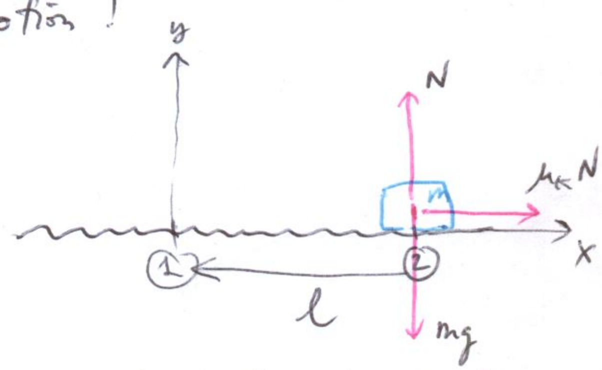
1

$$\underbrace{mgh + \cancel{\tfrac{1}{2} m \cdot 0^2} + \tfrac{1}{2} k \Delta x^2}_{ME_1} = \underbrace{\cancel{mg \cdot 0} + \tfrac{1}{2} m v^2 + \cancel{0}}_{ME_2}$$

Work by friction (non-conservative force) is $\boxed{\text{not}}$ conserved: Why?

1) Pushing a box of mass m   1 → 2 → 1
2) Friction is always against motion!





Work done by friction 1 → 2

$$W_{12} = \int \vec{F} \cdot d\vec{r} = -\mu_k N \hat{i} \cdot \int_1^2 d\vec{r}$$

$$= -\mu_k N \hat{i} \cdot (\ell - 0)\hat{i}$$

$$= -\mu_k N \ell$$

Work done by friction 2 → 1

$$W_{21} = \int \vec{F} \cdot d\vec{r} = +\mu_k N \hat{i} \cdot \cancel{(0-\ell)\hat{i}} \cdot \int_2^1 d\vec{r}$$

$$= \mu_k N \hat{i} \cdot (0 - \ell)\hat{i}$$

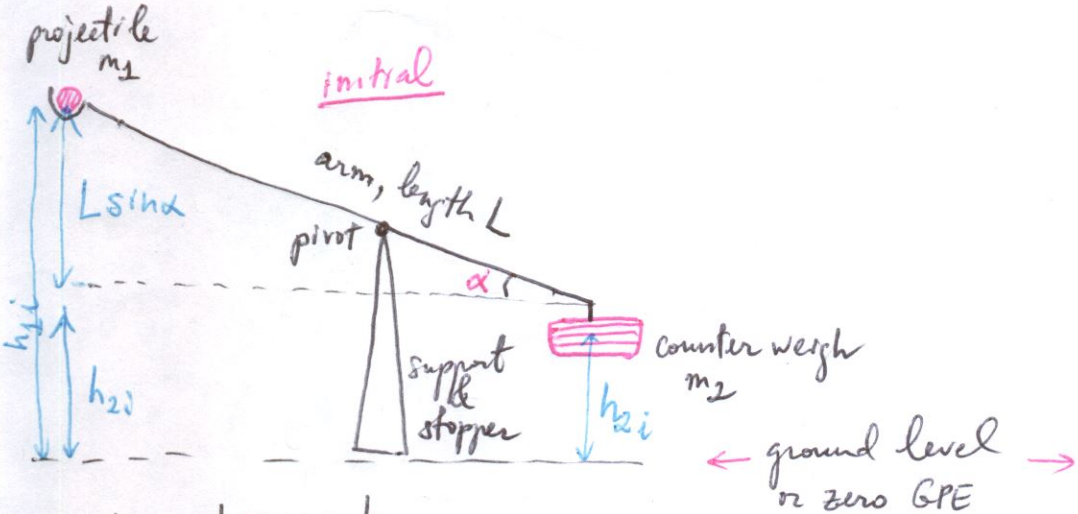
$$= -\mu_k N \ell$$

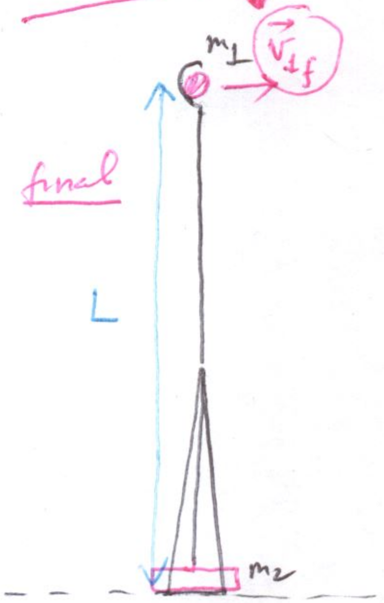
Conclusions: 1) "Work done" by friction is always negative! Friction never does work, it only receives work

2) Total work done by frictions when box goes back to same position is $W_{12} + W_{21} = -2\mu_k N \ell \neq 0$. Work done by friction is NOT conserved, or friction is a non-conservative force

PP set 3 7.1 | Physics of a catapult or trebuchet: they use a counterweight to launch a projectile

- Projectile motion (second)
- Conservation of mechanical energy (first)





$h_{1i} = L\sin\alpha + h_{2i}$

The support also stops the arm from swinging past vertical for a max. range for the projectile

Analysis goal:
- (i) Calculate $\vec{v}_{1f}$ using conservation of mechanical energy
- (ii) Calculate range of projectile once we have $\vec{v}_{1f}$, using projectile motion
  - uniform motion in horizontal direction
  - constant acceleration motion in vertical direction)

(i) Calculate $\vec{v}_{1f}$ (final velocity of projectile): system of projectile m_1 & counter weight m_2 (ignoring mass of arm):

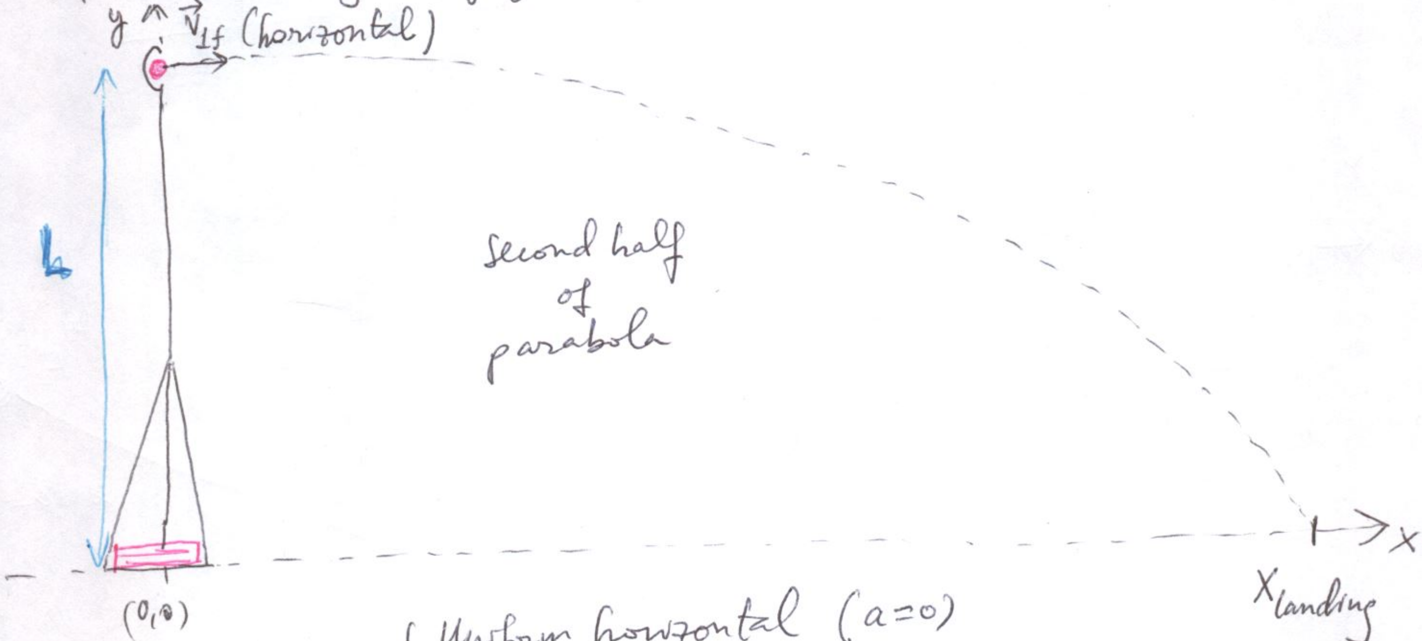
a) Initial position: arm @ angle α, both m_1 & m_2 are held in place or no initial KE, only GPE's

b) Final position: arm is vertical,
- $m_1 \Rightarrow$ KE + GPE
- $m_2 \Rightarrow$ neither

$$ME_i = ME_f$$

$$m_1 g h_{1i} + m_2 g h_{2i} = \frac{1}{2} m_1 v_{1f}^2 + m_1 g L \Rightarrow v_{1f} = \sqrt{\frac{2}{m_1}(m_1 g h_{1i} + m_2 g h_{2i} - m_1 g L)}$$

(ii) Calculate range of projectile



Projectile motion $\begin{cases} \text{Uniform horizontal } (a=0) \\ \text{Constant acceleration vertical } (a=+g) \end{cases}$

$t \equiv$ Time for projectile to go from $x=0$ to $x_{Landing}$ = $\underbrace{\text{time for it to go from } y=L \text{ to } y=0}$

2nd kin. eq. for constant acceleration motion

$$L = \cancel{v_{0y}}^{\,0} \cdot t + \frac{1}{2} g t^2$$

$$L = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2L}{g}}$$

$$\Rightarrow x_{Landing} = v_{0x} \cdot t = v_{1f} \cdot \sqrt{\frac{2L}{g}} = \sqrt{\frac{4L}{m_1 g}\left(m_1 g h_{1i} + m_2 g h_{2i} - m_1 g L\right)}$$

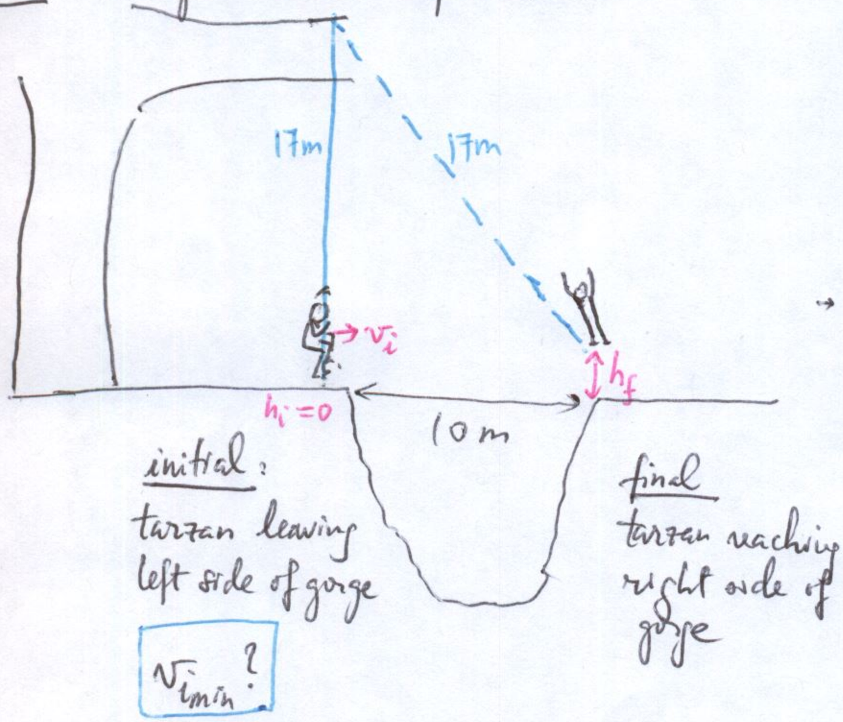
$$= \sqrt{\frac{4L}{m_1 g}\left[m_1 g (L \sin\alpha + h_{2i}) + m_2 g h_{2i} - m_1 g L \right]}$$

$$x_{Landing} = \sqrt{\frac{4L}{m_1 g}\left[\underbrace{m_1 g L(\sin\alpha - 1)}_{\text{negative}} + \underbrace{(m_1 + m_2) g h_{2i}}_{\text{positive}} \right]}$$

$$h_{2i} = \frac{\dfrac{x_{Landing}^2 \, m_1 g}{4L} + m_1 g L (1 - \sin\alpha)}{(m_1 + m_2) g}$$

7.62

Step 1: Diagram with information



Tarzan runs toward vine (length 17m) with initial velocity v_i. He swings to the other side of a 10-m-wide gorge.

→ Focus on motion of tarzan:

(i) If he ran towards vine @ $v_{i_{min}}$, that would be just enough to elevate to h_f without any speed when he lands on the other side of gorge ($v_f = 0$)

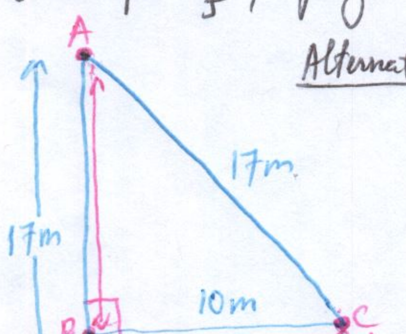
(ii) If he ran towards vine @ $v_i > v_{i_{min}}$ he will land with some speed on the other side ($v_f > 0$)

Step 2: Relevant equation: Conservation of ME:

$$ME_i = ME_f$$

$$\frac{1}{2}\cancel{m} v_{i_{min}}^2 = \cancel{m} g h_f \Rightarrow v_{i_{min}} = \sqrt{2gh_f}$$

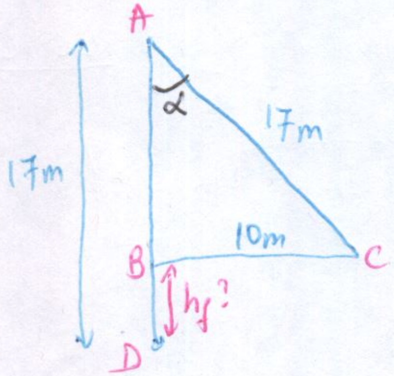
Step 3: Solve for h_f, plug in to calculate $v_{i_{min}}$



Alternative #1: $\triangle ABC$: $AB^2 + 10^2 = 17^2 \rightarrow AB = \sqrt{17^2 - 10^2}$

$$h_f = AD - AB = 17 - \sqrt{17^2 - 10^2} = 3.25\,m$$

Alternative #2: using trig.



$AB = 17\cos\alpha$ ✓

$\sin\alpha = \frac{10}{17} \rightarrow \alpha = \sin^{-1}\frac{10}{17} = 36^\circ$

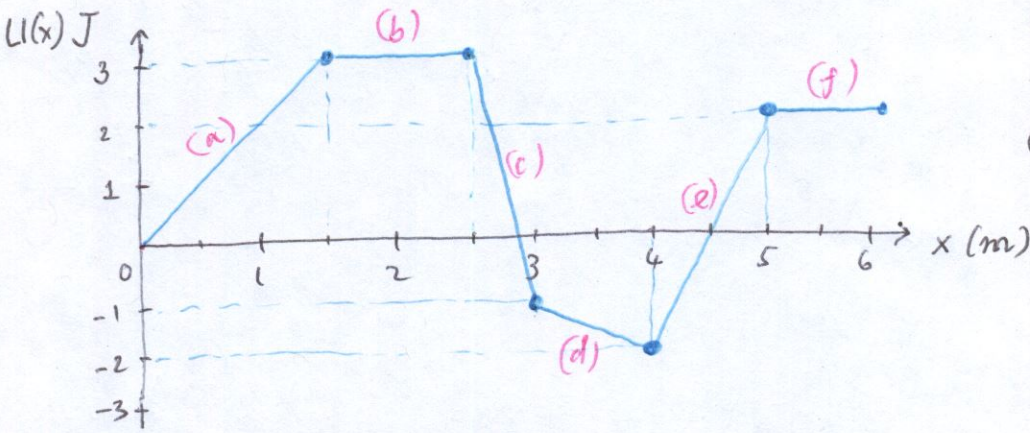
$\rightarrow AB = 17\cos 36^\circ =$

$h_f = AD - AB = 17 - 17\cos 36^\circ = 17(1-\cos 36^\circ) = 3.25m$

$$\rightarrow v_{i_{min}} = \sqrt{2 \cdot 9.81 \cdot 3.25} = 7.98\ \frac{m}{s}$$

7.26

Given potential energy curve calculate force applied.



Step 1: Copy Diagram

Step 2: relevant equation: potential $\underline{\text{energy}}^{U}$ & $\underline{\text{force}}$

$$\boxed{\begin{array}{c} U = -W \\ \text{An object must receive work to increase its potential energy} \end{array}}$$

work W

$\hookrightarrow W = \int F \cdot dx$ (1D)

"dot" product is arithmetic product

$$\boxed{F = -\frac{dU}{dx}}$$

$$\boxed{F = \frac{dW}{dx}}$$

Geometrically the derivative of a function represents its slope

Our task is to find the $\boxed{\text{slope}}$ of U(x) in each of the six segments (a) to (f)

$\hookrightarrow \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Step 3:

(a) slope $= \frac{3-0}{1.5-0} = 2 \rightarrow F_a = -2N$

(b) slope $= \frac{0}{1} = 0 \rightarrow F_b = 0N$

(c) slope $= \frac{-1-3}{0.5} = -8 \rightarrow F_c = +8N$

(d) slope: $\frac{-2-(-1)}{4-3} = -1 \Rightarrow F_d = +1N$

(e) slope: $\frac{2-(-2)}{5-4} = \frac{4}{1} \rightarrow F_e = -4N$

(f) slope: $\frac{0}{1} = 0 \rightarrow F_f = 0N$

Ch 8 Gravitation

So far: $F = mg$, but this only applies around Earth's surface

Surface	General
$F = mg$	$\boxed{F = G\frac{M_E m_2}{r^2}}$
GPE $U = mgh$	$U = -\frac{GM_E m}{r}$

Universal Law of Gravitation: (Universal: applies Earth, moon, planets, glaxies, universe)

- Essential for space exploration, cellular & GPS satellites, etc..

$$F = G\frac{m_1 \cdot m_2}{r^2}$$

- F: force of gravitational attraction b/w two masses m_1 & m_2
- G: universal grav. constant $= 6.67\times10^{-11}\frac{Nm^2}{kg^2}$
- m_1 & m_2: masses
- r: center-to-center separation b/w m_1 & m_2

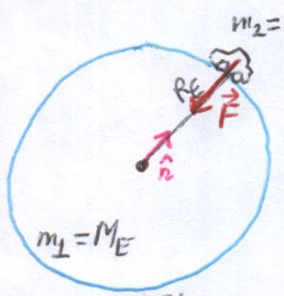
$\boxed{\frac{1}{r^2} \rightarrow \text{"inverse square law"}}$

$$G = 6.67\times10^{-11}\frac{Nm^2}{kg^2}$$

$\vec{F}$ is a vector $\Rightarrow \boxed{\vec{F} = -G\frac{m_1 \cdot m_2}{r^2}\hat{r}}$

$\hat{r}$ is the radial unit vector

Example: calulate grav. attraction of Earth on an object at surface



- Direction of grav. attraction by Earth on car is radial & towards Earth's center ($-\hat{r}$)
- Magnitude is $F = G\frac{M_E \cdot m}{r^2} = \boxed{6.67\cdot10^{-11}\cdot\frac{5.97\cdot10^{24}}{[6.37\cdot10^6]^2}}\,m$

Units $\frac{Nm^2}{kg^2}\cdot\frac{kg}{m^2} = \frac{kg\frac{m}{s^2}}{kg}$ ⟷ $9.81\frac{m}{s^2}$ or $9.81344065\frac{m}{s^2}$

$M_E = 5.97\times10^{24}kg$

$R_E = 6.37\times10^{6}m \rightarrow r \simeq R_E$ (height of car's center of mass is about 1m, negligible)

Conclusions: (i) $F = mg$ $\left(g = 9.81\,\frac{m}{s^2}\right)$ is a particular case of the Universal Law of Gravitation when $\boxed{r \cong R_E}$. $(h \ll R_E)$

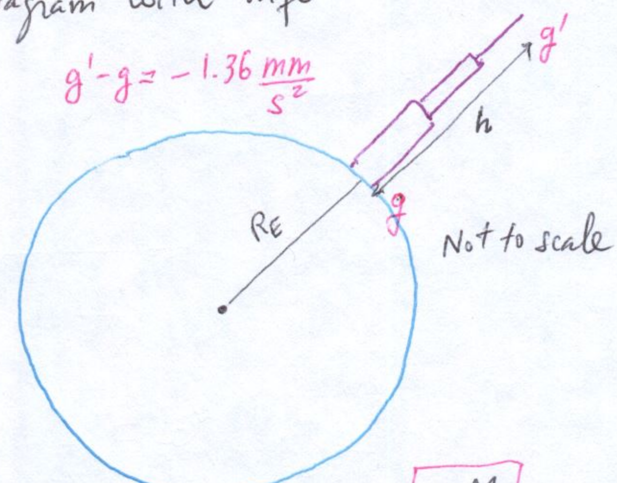
(ii) For objects way above surface we need to use

$$\boxed{r = R_E + h} \rightarrow \text{different (lower) values for } g$$

$$g' < g$$

8.17 Application of Univ. Law of Grav. to calculate a building height. Chicago Willis Tower (Sears Tower)

Step 1: Diagram with info



surface, top of Sears Tower

or $\Delta g = g - g' = 1.36\,\frac{mm}{s^2}$

Guess:

$h =$ 10m, 100m ✓, 1000m, 10000m

Step 2: Relevant equation: $F = \boxed{G\frac{M_E}{r^2}}\, m$

$$\left\{\begin{array}{l} g' = \dfrac{GM_E}{(R_E+h)^2} \\ g = \dfrac{GM_E}{R_E^2} \end{array}\right\} \quad \Delta g = g - g' = 1.36\,\frac{mm}{s^2}$$

Then solve for h

Step 3: Solve for h:

$$\Delta g = g - g' = GM_E\left[\frac{1}{R_E^2} - \frac{1}{(R_E+h)^2}\right] = GM_E \cdot \frac{(R_E+h)^2 - R_E^2}{R_E^2 (R_E+h)^2}$$

$$= GM_E \frac{2R_E h + h^2}{R_E^2 (R_E+h)^2} = \underbrace{\boxed{\frac{GM_E}{R_E^2}}}_{g} \boxed{\frac{2R_E h + h^2}{(R_E+h)^2}}$$

$\hookrightarrow$ approximation $\left\{\begin{array}{l} R_E + h \simeq R_E \\ \underbrace{(2R_E + h)}_{2R_E} h \simeq 2R_E h \end{array}\right.$

$$\Delta g = g\frac{2\cancel{R_E} h}{R_E^{\cancel{2}}} = g\frac{2h}{R_E}$$

$$\boxed{h = \frac{\Delta g}{g}\frac{R_E}{2}} = \frac{1.36 \cdot 10^{-3}}{9.81}\,\frac{6.37 \cdot 10^{6}}{2} = 442\,m$$

Step 3: Alternative #2 to calulation of h:

$$g' = \frac{GM_E}{(R_E+h)^2} \quad \& \quad g' = g - \Delta g = \boxed{\frac{GM_E}{R_E^2}} - \Delta g = \frac{6.67\cdot10^{-11}\cdot 5.97\cdot10^{24}}{(6.37\cdot10^{6})^2} - \Delta g$$

$$= 9.81344065 - 0.00136 = 9.81208065 \frac{m}{s^2}$$

(Using $9.81\frac{m}{s}$ here would be equivalent to neglecting Δg!)

$$\Rightarrow R_E + h = \sqrt{\frac{GM_E}{g'}} \quad \text{or} \quad h = \sqrt{\frac{GM_E}{g'}} - R_E =$$

$$h = \sqrt{\frac{6.67\cdot10^{-11}\cdot 5.97\cdot10^{24}}{9.81208065}} - 6.37\cdot10^{6} = 6370444.441 - 6370000$$

$$h = 441.441\ m$$

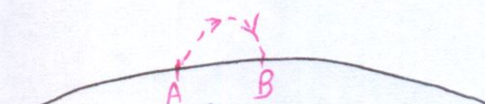
Projectile Motion:

Chapter 3: $\hookrightarrow$ { Uniform horizontal
Constant acc. in vertical direction $a = \mp g$

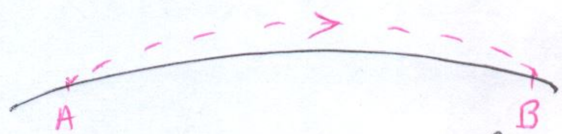
Projectile that goes up high above surface & long range requires correction due to universal law of gravitation

(i) grav. attraction is radial not always vertical (specially long-range projectile: intercontinental missile, etc.)

(ii) value of g changes with altitude.



parabola OK for short-range



If range is sufficiently long, projectile trajectory is more elliptical than a parabola

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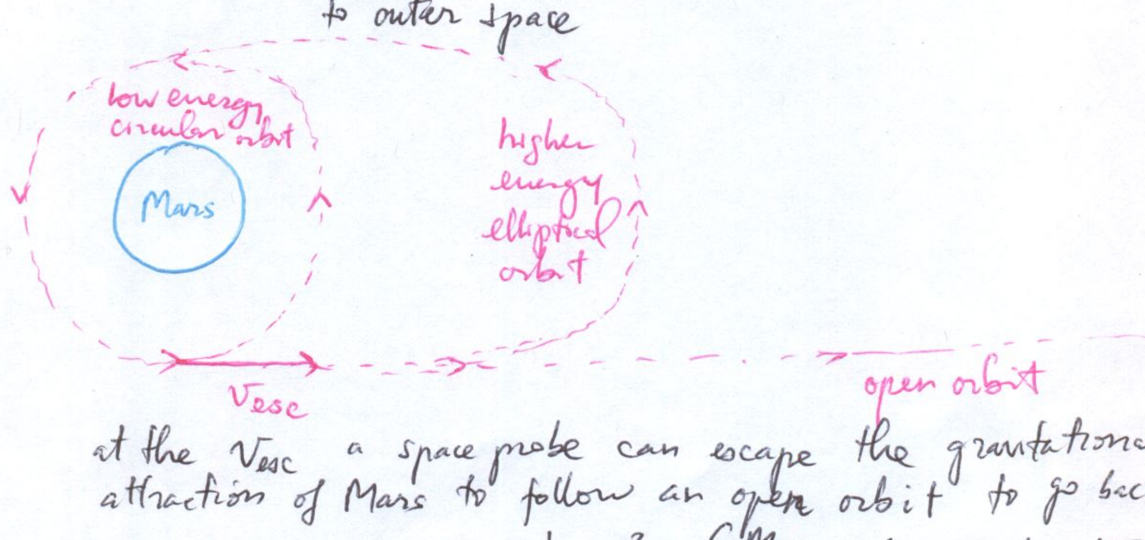
Escape Velocity:

min velocity to escape gravitational attraction:
→ space shuttle must reach this velocity to go to outer space



at the v_{esc} a space probe can escape the gravitational attraction of Mars to follow an open orbit to go back home.

$$ME = KE + GPE = \frac{1}{2}mv^2 - \frac{GMm}{r} < 0$$

when $ME = 0$
$\rightarrow v = v_{escape}$

$$\frac{1}{2}\cancel{m} v_{esc}^2 - \frac{GM\cancel{m}}{r} = 0$$

$$\boxed{v_{esc} = \sqrt{\frac{2GM}{r}}}$$

(i) v_{esc} is same regardless of the object mass m

(ii) v_{esc} is lower further way from a planet

At Earth's surface $r = R_E$, for any object:

$$v_{esc} = \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 5.97 \cdot 10^{24}}{6.37 \cdot 10^6}} = 11.2 \frac{km}{s} = 40320 \frac{km}{h}$$

Gravitational Potential Energy: $U = -\frac{GMm}{r}$ as extension of mgh

We derived mgh : $W = \int F \cdot dy = -mg \int_0^h dy = -mgh$

$U = -W = mgh$ $F = -mg$

Now use $F = -\frac{GMm}{r^2}$

$$\Delta W_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = -GMm \int_A^B \frac{dr}{r^2} = GMm \left[\frac{1}{r}\right]_A^B$$

Direction of $\vec{F}$ is radial $\rightarrow$ 1D

$$= GMm\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\Delta U_{AB} = -\Delta W_{AB} = GMm\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

Def: $\boxed{U(r) \equiv -\frac{GMm}{r}}$ $\longleftrightarrow$ $\Delta U_{AB} = U_B - U_A$

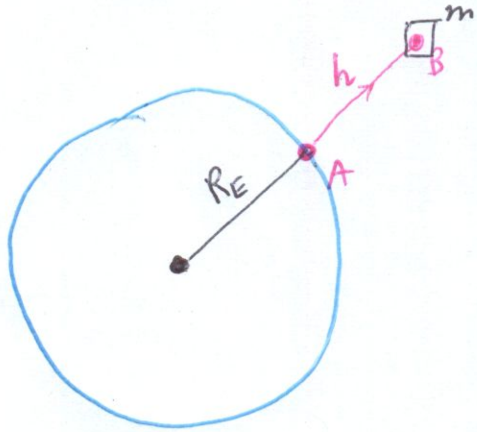
Reference for zero potential energy: $r \to \infty$ (object very far away has zero potential energy!) $U_\infty = 0$

$$\Delta U_{\infty B} = U_B - U_\infty = U_B = -\frac{GMm}{r_B}$$

Apply this result to surface of Earth: $r = R_E \rightarrow U = -\frac{GM_E}{R_E} m \frac{R_E}{R_E}$

$$= -\underbrace{\frac{GM_E}{R_E^2}}_{g} m R_E$$

→ Gravitational Potential Energy $\boxed{U = -\dfrac{GMm}{r}}$:



Raising object of mass m from surface (point A) to height h (point B): determine ΔU_{AB}:

$$\Delta U_{AB} = U_B - U_A = -GM_E m\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$= -GM_E m\left(\frac{1}{R_E+h} - \frac{1}{R_E}\right)$$

$$= -GM_E m \, \frac{R_E - (R_E+h)}{R_E(R_E+h)} = +GM_E m \frac{h}{R_E(R_E+h)}$$

(exact result)

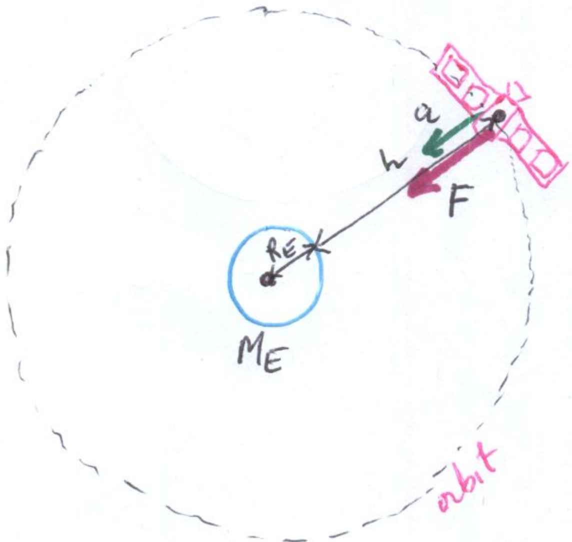
For $h \ll R_E$, up to 10m, very good approximation is:

$$\boxed{R_E + h \approx R_E} \Rightarrow \boxed{\Delta U_{AB} = \underbrace{\left(\frac{GM_E}{R_E^2}\right)}_{g} mh = mgh}$$

Circular Orbital Motion

↳ Motion of natural or artificial satellites

$\begin{cases} \text{UCM} \leftrightarrow \text{accel. towards center of curvature } a = \dfrac{v^2}{R} \\ \text{Force of gravitational attraction} \end{cases}$



Center-to-center separation Earth–Satellite is $R = R_E + h$, is the orbital radius

Orbit's length: $2\pi(R_E+h)$

Orbital period = T : time for satellite to complete one orbit

UCM → $T = \dfrac{2\pi(R_E+h)}{v}$

Final expression for T: write v in terms of the gravitational attraction of the Earth of mass M_E.

Only force on satellite is $\boxed{F = G\dfrac{M_E \cancel{m}}{(R_E+h)^{\cancel{2}}}} = \boxed{F_{net} = m \cdot a} = \cancel{m} \cdot \dfrac{v^2}{\cancel{(R_E+h)}}$

Univ. Law of Grav.   2nd Newton's Law

$$\Rightarrow v = \sqrt{\frac{GM_E}{R_E+h}} = \frac{(GM_E)^{\frac{1}{2}}}{(R_E+h)^{\frac{1}{2}}}$$

$$\Rightarrow \boxed{T = \frac{2\pi(R_E+h)}{(GM_E)^{\frac{1}{2}}}(R_E+h)^{1/2} = \frac{2\pi}{(GM_E)^{\frac{1}{2}}}(R_E+h)^{3/2}}$$

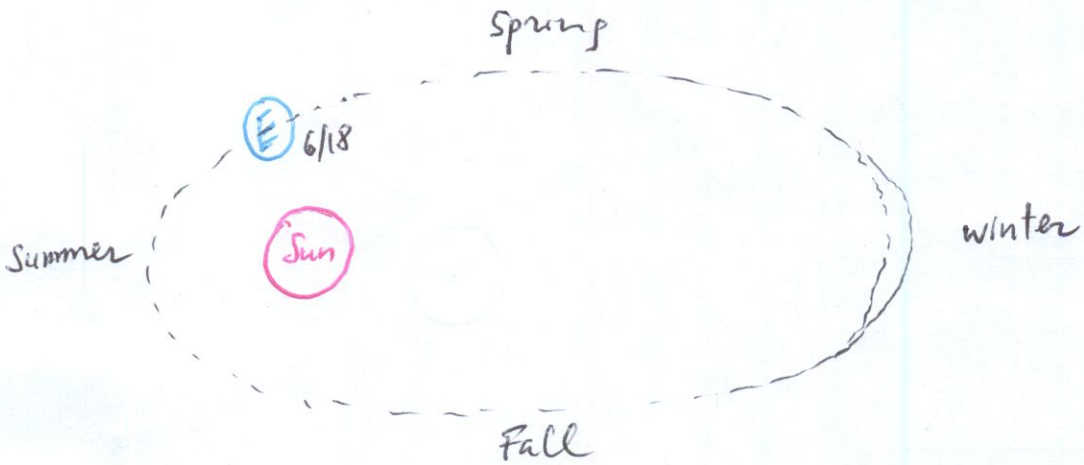
↓

$$\boxed{T^2 = \frac{4\pi^2}{GM_E}(R_E+h)^3}$$

Planetary orbital motion: $R_E + h = R$

$$T^2 = \frac{4\pi^2}{GM_E} R^3 \quad \text{or} \quad \boxed{T^2 \propto R^3}$$

Generalized → Kepler's Third Law: period squared is directly proportional (Elliptical orbits) to the semimajor axis cubed.



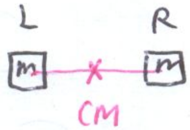
Cell phone satellites: $h = 250\text{km}$ (above surface)

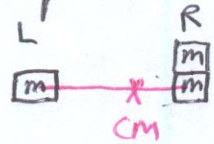
$$T = \frac{2\pi}{(6.67\cdot10^{-11}\cdot 5.97\cdot10^{24})^{1/2}} (6.37\cdot10^{6} + 0.25\cdot10^{6})^{3/2} = 5400\,\text{s}\ \frac{1\text{h}}{3600\,\text{s}} = 1.5\,\text{hrs.}$$

Ch9 System of Particles

When we describe two or more particles it is important to define the center of mass (CM) so to relate to the previous equations we introduced that applied to a single particle.

CM: average position of all components of a system, weighted by the masses of the components





The position of the right component carries a double weight compared to that of the left component in computing the average position.

→ CM position vector $\vec{R}$:

Discrete systems:

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{M}$$

- m_i : mass of component i
- $\vec{r}_i$: position vector of component i
- $M = \sum_i m_i$, total mass of system

"average position weighted by m_i"

Continuous systems:

$$\vec{R} = \frac{\int \vec{r}\, dm}{M}$$

- dm : mass of an infinitesimal component
- $\vec{r}$: position vector of the infinitesimal component
- $M = \int dm$, total mass of system

"average position weighted by dm"

Relate CM to Newton's 2nd Law for a system of particles:

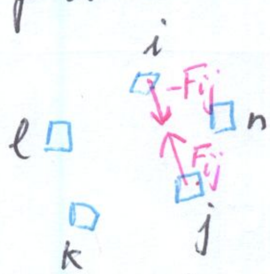
$$\boxed{\vec{F}_{net} = M \frac{d^2\vec{R}}{dt^2}}$$

- $\vec{F}_{net}$: net force on system of particles
- M : total mass system
- $\vec{R}$: position vector of CM of system.

Important clarifications:

(i) $\vec{F}_{net}$ & force of interaction b/w particles:

System with 5 particles:



→ interaction forces: $\begin{cases} i \text{ attracts } j \text{ by } F_{ij} \\ j \text{ attracts } i \text{ by } \boxed{-F_{ij}} \end{cases}$ Newton's 3rd Law: ~~Re~~Action & Reaction Law

→ in pairs: equal & opposite by action & reaction

→ Internal interaction forces b/w particles are not part of $\vec{F}_{net}$

→ they are not relevant to the motions of their CM.

→ Motion of CM of a system depends only on the net external force on system.

(ii) Linear momentum of a system of particles: $\vec{P}$

$$\vec{P} \equiv M\cdot\vec{V} = M\cdot\frac{d\vec{R}}{dt} = \cancel{M}\cdot\frac{d}{dt}\frac{\sum_i m_i\vec{r}_i}{\cancel{M}} = \sum_i m_i \underbrace{\frac{d\vec{r}_i}{dt}}_{\vec{v}_i} = \sum_i \underbrace{m_i\vec{v}_i}_{\vec{p}_i}$$

M: total mass

$\vec{V}$: velocity of CM $= \dfrac{d\vec{R}}{dt}$

$\vec{R}$: position vector of CM

$\vec{v}_i$: velocity vector of component i

$\vec{p}_i$: linear momentum vector of component i

$$\vec{P} = \sum_i \vec{p}_i$$

2nd Newton's Law for a system of particles:

$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}}$$

$\begin{cases} \vec{F}_{net}: \text{ net } \underline{\text{external}} \text{ force on system} \\ \vec{P}: \text{ linear momentum of system.} \end{cases}$

→ if $\vec{F}_{net} = 0 \longrightarrow \boxed{\vec{P} = \text{constant}}$ Conservation of total linear momentum of a system of particle (Collisions)

→ if $M = \text{constant}$ $\vec{F}_{net} = M\dfrac{d\vec{V}}{dt} = M\dfrac{d^2\vec{R}}{dt^2}$

Collisions b/w two particles: two colliding billiard balls, marbles, cars, etc..

↳ Ball 1 hits ball 2 on a 2D surface: at collision, net external force on the system is 0 : $\vec{F}_{net} = 0 \rightarrow \vec{P} = \text{constant}$ before & after collision

Collision $\leftrightarrow$ $\vec{P}$ = constant before & after

$$\underbrace{\vec{P}_{1i} + \vec{P}_{2i}}_{\vec{P}_i} = \underbrace{\vec{P}_{1f} + \vec{P}_{2f}}_{\vec{P}_f}$$

Conservation of total linear momentum

Collision

Elastic: in addition to conservation of total linear momentum, the total kinetic energy is also conserved.

$$\vec{P}_i = \vec{P}_f \quad \text{or} \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$KE_i = KE_f \quad \text{or} \quad \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

Inelastic: total KE is not conserved: the two colliding particles stick together after the collision. Part of KE is lost into deformation, damage, breaking, heat, etc.

$$\vec{P}_i = \vec{P}_f \quad \text{or} \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

if they stick together: $\vec{v}_{1f} = \vec{v}_{2f} \equiv \vec{v}_f$

$$\Rightarrow m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

1D elastic collision:

↳ 2 equations

$$\left\{ \begin{array}{l} m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_1 v_{2f}^2 \end{array} \right\}$$

$\boxed{\text{can solve for 2 unknowns}}$

↳ Given $m_1, m_2, v_{1i}, v_{2i} \Rightarrow$ solve for v_{1f} & v_{2f}

1) $$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

2) $$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

3) $$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

2D elastic collision:

↳ 3 equations

$$\left\{ \begin{array}{l} P_{ix} = P_{fx} \\ P_{iy} = P_{fy} \\ KE_i = KE_f \end{array} \right\}$$

$\boxed{\text{Can solve for 3 unknowns}}$

(more restrictive)

↳ we can't calculate all of $\vec{v}_{1f}$ & $\vec{v}_{2f}$ $(v_{1fx}, v_{1fy}, v_{2fx}, v_{2fy})$

↳ We need at least one piece of information for the final velocities!

↳ Can be derived

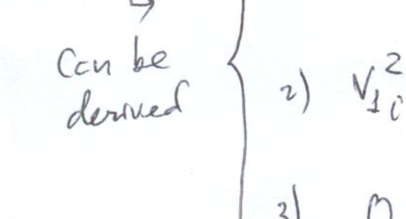
1) $$v_{1i}^2 = v_{1f}^2 + \frac{m_2^2}{m_1^2} v_{2f}^2 + \frac{2m_2}{m_1} v_{1f} v_{2f} \cos(\theta_2 - \theta_1)$$

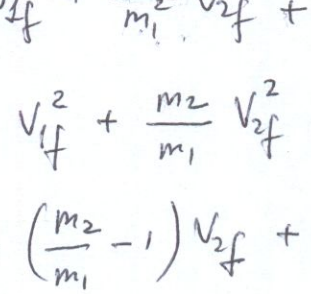
2) $$v_{1i}^2 = v_{1f}^2 + \frac{m_2}{m_1} v_{2f}^2$$

3) $$0 = \left(\frac{m_2}{m_1} - 1\right) v_{2f} + 2 v_{1f} \cos(\theta_1 - \theta_2)$$

if $m_1 = m_2 \Rightarrow$ 3) $$0 = 2 v_{1f} \underbrace{\cos(\theta_1 - \theta_2)}_{0} \Rightarrow \boxed{\theta_1 - \theta_2 = 90^\circ}$$

Final vel. are perpendicular to each other.



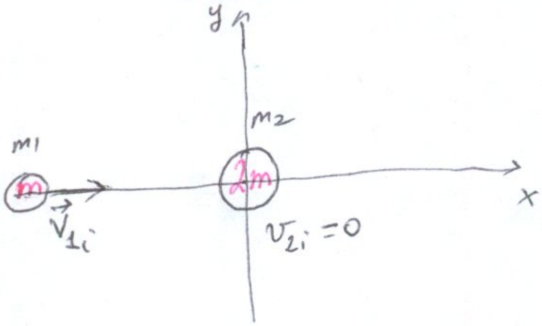


89

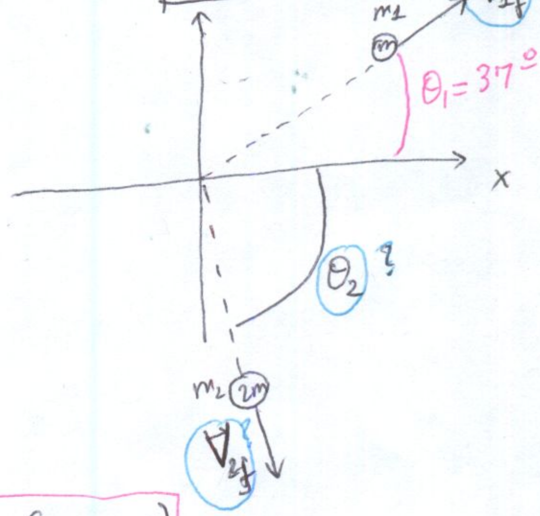
9.71 | **2D elastic collision** proton ($m_1 = 1u$) against deuteron ($m_2 = 2u$) initially @ rest

Step 1

initial



final (after collision)



Step 2:

$$\text{3 eqs.}\left\{\boxed{\begin{aligned} V_{1i}^2 &= V_{1f}^2 + 4V_{2f}^2 + 4V_{1f}V_{2f}\cos(\theta_2 - 37°) \\ V_{1i}^2 &= V_{1f}^2 + 2V_{2f}^2 \\ 0 &= V_{2f} + 2V_{1f}\cos(\theta_2 - 37°) \end{aligned}}\right.$$

Conservation of total linear momentum:

$$P_{ix} = P_{fx}$$

$$m_1 V_{1i} = m_1 V_{1f}\cos 37° + m_2 V_{2f}\cos\theta_2$$

$$\cos 37° \times \left[V_{1i} = V_{1f}\cos 37° + 2V_{2f}\cos\theta_2 \right] \rightarrow V_{1i}\cos 37° = V_{1f}\cos^2 37° + 2V_{2f}\cos\theta_2\cos 37° \quad (i)$$

$$P_{iy} = P_{fy}$$

$$\sin 37° \times \left[0 = V_{1f}\sin 37° - 2V_{2f}\sin\theta_2 \right] \rightarrow 0 = V_{1f}\sin^2 37° - 2V_{2f}\sin\theta_2\sin 37° \quad (ii)$$

$$(i) + (ii) \rightarrow \quad V_{1i}\cos 37° = V_{1f} + 2V_{2f}\underbrace{(\cos\theta_2\cos 37° - \sin\theta_2\sin 37°)}_{\cos(\theta_2 - 37°)}$$

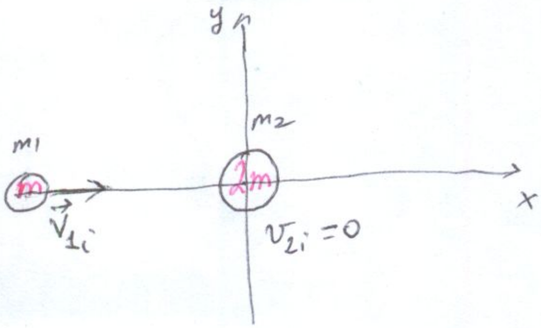
$$\rightarrow \boxed{V_{1i}\cos 37° = V_{1f} + 2V_{2f}\cos(\theta_2 - 37°)}$$

Step 3: solve for V_{1f}, V_{2f}, θ_2

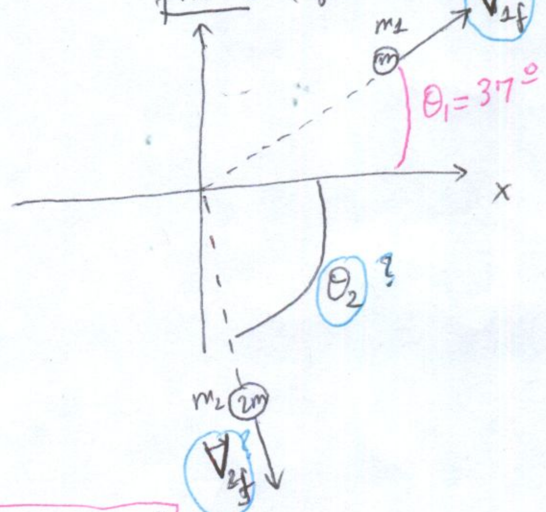
9.71 | **2D elastic collision** proton ($m_1 = 1u$) against deuteron ($m_2 = 2u$) initially @ rest

Step 1

initial



final (after collision)



Step 2:

3 eqs.

$$\boxed{\begin{aligned} v_{1i}^2 &= v_{1f}^2 + 4v_{2f}^2 + 4v_{1f}v_{2f}\cos(\theta_2 - 37°) \quad (1)\\ v_{1i}^2 &= v_{1f}^2 + 2v_{2f}^2 \quad (2)\\ 0 &= v_{2f} + 2v_{1f}\cos(\theta_2 - 37°) \quad (3)\end{aligned}}$$

Conservation of total linear momentum:

$$P_{ix} = P_{fx}$$

$$m_1 v_{1i} = m_1 v_{1f}\cos 37° + m_2 v_{2f}\cos\theta_2$$

$$\cos 37° \times \left[v_{1i} = v_{1f}\cos 37° + 2v_{2f}\cos\theta_2\right] \rightarrow v_{1i}\cos 37° = v_{1f}\cos^2 37° + 2v_{2f}\cos\theta_2\cos 37° \quad (i)$$

$$P_{iy} = P_{fy}$$

$$\sin 37° \times \left[0 = v_{1f}\sin 37° - 2v_{2f}\sin\theta_2\right] \rightarrow 0 = v_{1f}\sin^2 37° - 2v_{2f}\sin\theta_2\sin 37° \quad (ii)$$

$$(i) + (ii) \rightarrow \quad v_{1i}\cos 37° = v_{1f} + 2v_{2f}\underbrace{(\cos\theta_2\cos 37° - \sin\theta_2\sin 37°)}_{\cos(\theta_2 - 37°)}$$

$$\boxed{v_{1i}\cos 37° = v_{1f} + 2v_{2f}\cos(\theta_2 - 37°)} \quad (4)$$

Step 3: solve for v_{1f}, v_{2f}, θ_2

$$v_{2f} \times (3) \Rightarrow 0 = v_{2f}^2 + \underbrace{2v_{1f}v_{2f}\cos(\theta_2 - 37^\circ)}_{v_{1f}\left[\underbrace{2v_{2f}\cos(\theta_2-37^\circ)}_{(4)\to\; v_{1i}\cos 37^\circ - v_{1f}}\right]}$$

$$0 = v_{2f}^2 + v_{1f}v_{1i}\cos 37^\circ - v_{1f}^2 \quad \text{(quadratic eq. in } v_{1f}) \quad (5)$$

Eliminating v_{2f}:

$$(2) \Rightarrow v_{2f}^2 = \frac{1}{2}\left(v_{1i}^2 - v_{1f}^2\right)$$

Plugging this in (5):

$$2\times \left[0 = \frac{1}{2}\left(v_{1i}^2 - v_{1f}^2\right) + v_{1f}v_{1i}\cos 37^\circ - v_{1f}^2\right]$$

$$0 = v_{1i}^2 - v_{1f}^2 + 2v_{1f}v_{1i}\cos 37^\circ - 2v_{1f}^2$$

$$0 = -3v_{1f}^2 + (2v_{1i}\cos 37^\circ)v_{1f} + v_{1i}^2$$

$$3v_{1f}^2 - (2v_{1i}\cos 37^\circ)v_{1f} - v_{1i}^2 = 0 \quad \text{(quadratic eq. in } v_{1f})$$

$$\hookrightarrow v_{1f} = \frac{2v_{1i}\cos 37^\circ \pm \sqrt{4v_{1i}^2\cos^2 37^\circ + 12v_{1i}^2}}{6} \quad \text{(quadratic formula)}$$

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v_{1f} = \frac{2v_{1i}\cos 37^\circ \pm 2v_{1i}\sqrt{\cos^2 37^\circ + 3}}{6}$$

$$\boxed{v_{1f} = v_{1i}\underbrace{\left[\frac{\cos 37^\circ}{3} + \frac{\sqrt{\cos^2 37 + 3}}{3}\right]}_{} = 0.902\, v_{1i}}$$

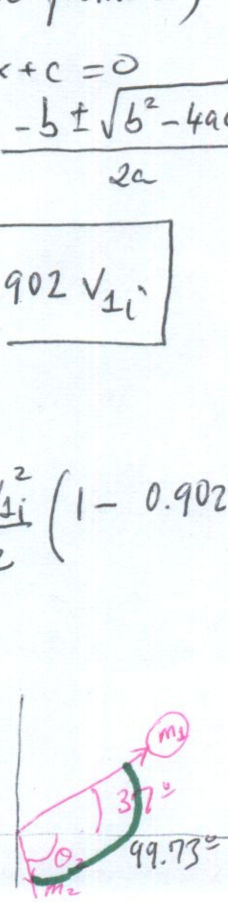
magnitudes of velocity are always positive

$$(2) \Rightarrow v_{2f}^2 = \frac{1}{2}\left(v_{1i}^2 - v_{1f}^2\right) = \frac{v_{1i}^2}{2}\left(1 - \frac{v_{1f}^2}{v_{1i}^2}\right) = \frac{v_{1i}^2}{2}\left(1 - 0.902^2\right)$$

$$v_{2f}^2 = 0.93\, v_{1i}^2 \Rightarrow \boxed{v_{2f} = 0.305\, v_{1i}}$$

$$(3) \Rightarrow \cos(\theta_2 - 37^\circ) = -\frac{v_{2f}}{2v_{1f}} = -\frac{0.305\, \cancel{v_{1i}}}{2 \cdot 0.902\, \cancel{v_{1i}}}$$

$$\boxed{\theta_2 - 37^\circ} = \cos^{-1}\left[-\frac{0.305}{1.804}\right] = 99.73^\circ \Rightarrow \boxed{\theta_2 = 62.73^\circ}$$



Fraction of KE of proton transferred to deuteron?

initial $\qquad\qquad$ final (after collision)

$$KE_{p,i} + \cancel{KE_{d,i}}_{0} = KE_{p,f} + KE_{d,f}$$

$$KE_{1,i} \quad \text{(deuteron was at rest)} \quad = KE_{1,f} + KE_{2,f}$$

deuteron acquired some KE after collision with proton:

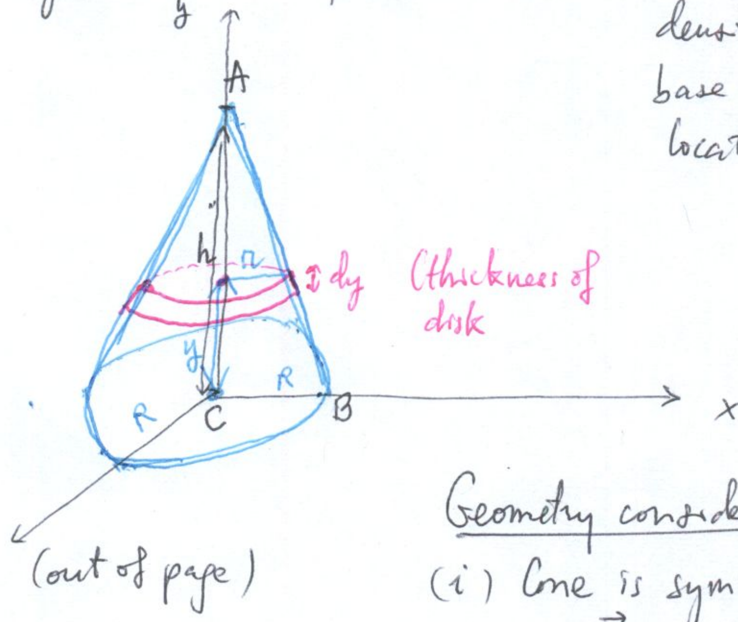
$$\frac{KE_{d,f}}{KE_{p,i}} = \frac{KE_{2,f}}{KE_{1,i}} = \frac{KE_{1i} - KE_{1f}}{KE_{1,i}} = 1 - \frac{KE_{1f}}{KE_{1i}} = 1 - \frac{\cancel{\frac{1}{2}m_1} v_{1f}^2}{\cancel{\frac{1}{2}m_1} v_{1i}^2}$$

$$= 1 - \frac{v_{1f}^2}{v_{1i}^2} = 1 - \frac{0.902^2 \cancel{v_{1i}^2}}{\cancel{v_{1i}^2}} = 1 - 0.902^2 = 0.186$$

or 18.6 %
(fraction of KE transferred from proton to deuteron)

9.41)

Step 1: Diagram with information: solid cone, uniform mass density ρ; height h, base radius R → find location of CM.



Geometry considerations:

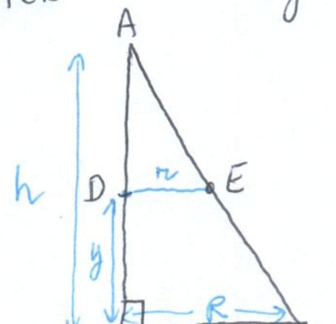
(i) Cone is symmetric wrt y-axis

$$\Rightarrow \vec{R} = y_{cm}\,\hat{j} \Rightarrow y_{cm} = \frac{\int y\,dm}{M}$$

$$\vec{R} \equiv \frac{\int \vec{r}\,dm}{M}$$

(3D problem but 1D calculation!)

(ii) $\overset{\triangle}{ACB}$ is a right triangle:



Similar triangles: $\overset{\triangle}{ACB}$ & $\overset{\triangle}{ADE}$

$$\hookrightarrow \frac{r}{h-y} = \frac{R}{h}$$

$$r = \frac{R}{h}(h-y)$$

$$\boxed{r = R\left(1 - \frac{y}{h}\right)}$$

Step 2: Relevant equations:

1) Def of CM: $y_{cm} = \dfrac{\int y\,dm}{M}$

2) Density $\rho = \dfrac{dm}{dvol} \Rightarrow dm = \rho \underbrace{dvol}$

infinitesimal volume of disk of radius r & thickness dy

3) Volume of a disk or cylinder: base area × height (thickness)

$\hookrightarrow dvol = \pi r^2 dy$

$\Rightarrow dm = \rho \pi r^2 dy = \rho \pi R^2 \left(1 - \frac{y}{h}\right)^2 dy$

$\boxed{\text{4) volume of a cone of height } h\text{, base radius } R \text{ is } \frac{\pi R^2 h}{3}}$

Step 3: solve for y_{cm}:

$$y_{cm} = \frac{1}{M}\int_0^h \rho \pi R^2\, y\left(1 - \frac{y}{h}\right)^2 dy = \frac{\rho \pi R^2}{M}\int_0^h \left(y - \frac{2y^2}{h} + \frac{y^3}{h^2}\right) dy$$

$$= \frac{\rho \pi R^2}{M}\left[\frac{y^2}{2} - \frac{2}{3h}y^3 + \frac{1}{4h^2}y^4\right]_0^h = \frac{3}{h}\left[\frac{h^2}{2} - \frac{2}{3}h^2 + \frac{h^2}{4}\right]$$

$$= \frac{3}{2}h - 2h + \frac{3}{4}h$$

$$= \frac{6h - 8h + 3h}{4} = \boxed{\frac{h}{4}}$$

$$\rho = \frac{M}{V_{cone}} = \frac{3M}{\pi R^2 h}$$

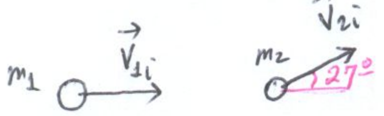
$$\frac{\rho \pi R^2}{M} = \frac{3\cancel{M}}{\cancel{\pi R^2} h}\,\frac{\cancel{\pi R^2}}{\cancel{M}} = \frac{3}{h}$$

$$\Rightarrow \boxed{y_{cm} = \frac{h}{4}}$$

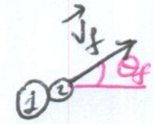
9.74

Step 1: Diagram with info:

initial



final (after collision)



inelastic 2D collision $\longleftrightarrow$ $\boxed{\text{they stick together}}$: $m = 48u$

$\vec{v}_f = v_f \cos\theta\,\hat{i} + v_f \sin\theta\,\hat{j}$

$\vec{v}_{1i} = 580\,\hat{i}$ $\frac{m}{s}$

$m_1 = 32u$

$\vec{v}_{2i} = 870\cos 27°\,\hat{i} + 870 \sin 27°\,\hat{j}$ $\frac{m}{s}$

$m_2 = 16u$

Step 2: $\vec{F}_{net} = 0 \leftrightarrow$ Conservation of total linear momentum:

$$\vec{P}_i = \vec{P}_f$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1+m_2)\vec{v}_f$$

in x : $m_1 v_{1i} + m_2 v_{2i}\cos 27° = (m_1+m_2) v_f \cos\theta_f$

in y : $0 + m_2 v_{2i} \sin 27° = (m_1+m_2) v_f \sin\theta_f$

Step 3:

$30962.8 = 32\cdot 580 + 16\cdot 870 \cdot \cos 27° = 48\, v_f \cos\theta_f$ or $645 = v_f\cos\theta_f$

$6319.5 = 16\cdot 870 \cdot \sin 27° = 48\, v_f \sin\theta_f$ or $131.66 = v_f \sin\theta_f$

$$\frac{131.66}{645} = \tan\theta_f \rightarrow \theta_f = \tan^{-1}\frac{131.66}{645} = 11.53°$$

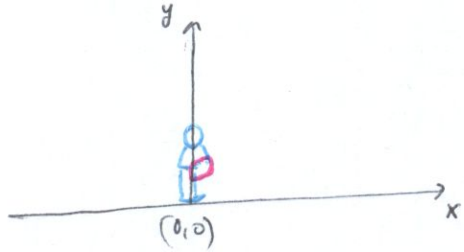
$$v_f = \frac{645}{\cos 11.53°} = 658.29\ \frac{m}{s}$$

9.57

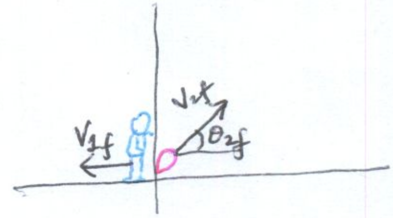
Step 1: Diagram with info:

(frictionless ice $\rightarrow \vec{P}_{net} = 0$)

initial



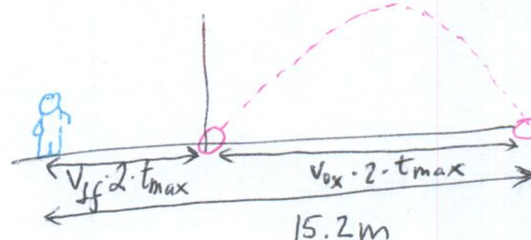
final (after "collision")



person: $m_1 = 65\,kg$
rock: $m_2 = 4.5\,kg$
$\vec{v}_{1i} = \vec{v}_{2i} = 0$
(person was holding rock before he tossed it)

$v_{2f} = 12\,\frac{m}{s}$
θ_{2f} ?

Additional info:



t_{max}: time for rock to gets to its max. altitude point:

1st kin. eq: $v_y = v_{0y} - g\,t$

$0 = 12 \cdot \sin\theta_{2f} - g\,t_{max}$

$$\boxed{t_{max} = \frac{12 \sin\theta_{2f}}{g}}$$

range for rock: uniform horizontal motion:

$$v_{0x} \cdot 2 \cdot t_{max} = 12(\cos\theta_{2f}) \cdot 2 \cdot t_{max}$$

$$= \frac{12^2 \cdot 2 \cdot \sin\theta_{2f} \cdot \cos\theta_{2f}}{g}$$

Step 2: Conservation linear momentum:

in x direction: $m_1 v_{1f} + m_2 v_{2f} \cos\theta_{2f} = 0 \rightarrow v_{1f} = -\frac{m_2}{m_1} v_{2f} \cos\theta_{2f}$

$$\boxed{v_{1f} = -0.83 \cos\theta_{2f}}$$

$$15.2m = 2 v_{0x} t_{max} - v_{1f} \cdot 2 \cdot t_{max} = 2 \cdot t_{max}(v_{0x} - v_{1f}) = 2 \cdot \frac{12 \cdot \sin\theta_{2f}}{g}(12 \cos\theta_{2f} + 0.83 \cos\theta_{2f})$$

$$15.2 = \frac{12}{g}\, 12.83\, \underbrace{2\sin\theta_{2f}\cos\theta_{2f}}_{\sin(2\theta_{2f})}$$

$$15.2 = \frac{12\cdot 12.83}{9.81}\sin(2\theta_{2f}) \rightarrow 2\theta_{2f} = \sin^{-1}\frac{15.2\cdot 9.81}{12\cdot 12.83}$$

$$\boxed{\theta_{2f} = \frac{75.58°}{2} = 37.8°}$$

8.32

$v_{esc} = 30\,\frac{km}{s}$ For Earth $\left\{\begin{array}{l} M_E = 5.97\cdot 10^{24}\,kg \\ R_E = 6.37\cdot 10^{6}\,m \end{array}\right\}$ $v_{esc} = 11.2\,\frac{km}{s}$

$\downarrow$

what is R_E'?

To escape grav. attraction $\rightarrow$ $ME = \frac{1}{2}mv_{esc}^2 - \frac{GM_E m}{R_E} = 0$

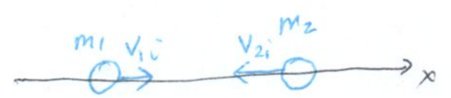
$$\Rightarrow R_E' = \frac{2GM_E \cancel{m}}{\cancel{m}\, v_{esc}^2} = \frac{2\cdot 6.67\cdot 10^{-11}\cdot 5.97\cdot 10^{24}}{(30\cdot 10^3)^2}\,m$$

$$= 8.85\cdot\frac{10^{13}}{10^{8}} = 0.885\cdot 10^{6}\,m$$

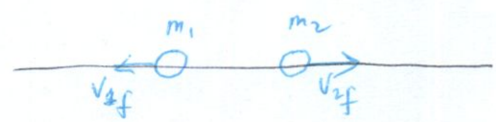
9.33

Step 1:

initial



final (after collision)



$m_1 = 1u$
$m_2 = 1u$ $\}$ $m_1 = m_2 = m$

$v_{1i} = 6.9\cdot 10^6\,\frac{m}{s}$

$v_{2i} = -11\cdot 10^6\,\frac{m}{s}$

v_{1f}?
v_{2f}?

1D elastic collision

$\left\{\begin{array}{l} P_i = P_f \\ KE_i = KE_f \end{array}\right\}$ $\rightarrow$ can solve for 2 unknowns.

Step 2: Relevant equations: 1D elastic collision:

1) $v_{1f} = \frac{m_1 - m_2}{m_1+m_2}v_{1i} + \frac{2m_2}{m_1+m_2}v_{2i}$ $\overset{m_1=m_2}{=}$ v_{2i}

2) $v_{2f} = \frac{2m_1}{m_1+m_2}v_{1i} + \frac{m_2-m_1}{m_1+m_2}v_{2i}$ $=$ v_{1i}

when masses are equal the two colliding particles exchange their velocities

Step 3: $v_{1f} = v_{2i} = -11\cdot 10^6\,\frac{m}{s}$; $v_{2f} = v_{1i} = 6.9\cdot 10^6\,\frac{m}{s}$

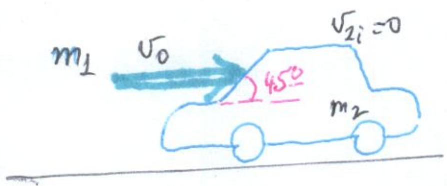
9.45

Step 1: Diagram with info :

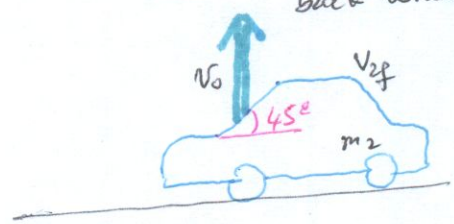
→ car initially at rest, received a push by a jet of water hitting its back window horizontally (leaving vertically), car acquired an acceleration a?

→ no friction $F_{net}=0$ (no net external force on system of car & water)

initial



final (after water collided with back window)



m_1 : water

$\vec{v}_{1i} = v_0 \hat{i}$

m_2 = car

$v_{2i} = 0$ (at rest)

$$\vec{F}_{net}=0 \;\Downarrow\; \vec{P}_i = \vec{P}_f$$

$\vec{v}_{1f} = v_0 \hat{j}$

$\vec{v}_{2f}$ car after collision.

Step 2:

$$\vec{P}_i = \vec{P}_f$$

$$m_1 \vec{v}_{1i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Step 3: Solve for $\vec{v}_{2f} = \dfrac{m_1 \vec{v}_{1i} - m_1 \vec{v}_{1f}}{m_2} = \left(\dfrac{1}{m_2}\vec{v}_{1i} - \dfrac{1}{m_2}\vec{v}_{1f}\right) m_1$

$$\vec{v}_{2f} = \frac{v_0}{m_2}\left(\hat{i} - \hat{j}\right) m_1$$

Acceleration $\vec{a}$ acquired by car: $\vec{a} = \dfrac{d\vec{v}_{2f}}{dt} = \dfrac{v_0}{m_2}(\hat{i}-\hat{j})\dfrac{dm_1}{dt}$

⇓ $v_0, m_2, \hat{i}, \hat{j}$ are time independent!

→ a) Forward acceleration of car: $\boxed{a_x = \dfrac{v_0}{m_2}\dfrac{dm_1}{dt}}$

(downward acceleration is felt by car's suspension : $a_y = -\dfrac{v_0}{m_2}\dfrac{dm_1}{dt}$)

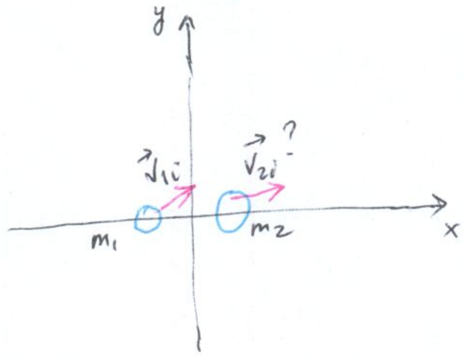
b) Max speed car can reach? ⇒ speed of water v_0

Once car reaches speed of v_0 in x-direction water can no longer provide a push for any further acceleration

9.28

Step 1:

Initial



$m_1 = 1u$

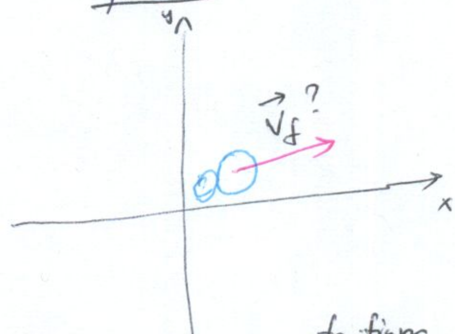
$m_2 = 2u$

$\vec{v}_{1i} = 28\hat{i} + 17\hat{j}\ (\cdot 10^6 \frac{m}{s})$

$\vec{v}_{2i} = ?$

$\vec{F}_{net} = 0$

final



they combine as tritium

$m_1 + m_2 = 3u$

$\vec{v}_f = 12\hat{i} + 20\hat{j}\ (10^6 \frac{m}{s})$

Step 2:

$$\vec{P}_i = \vec{P}_f$$

$$(28\hat{i} + 17\hat{j}) + 2(v_{2ix}\hat{i} + v_{2iy}\hat{j}) = 3(12\hat{i} + 20\hat{j})$$

Step 3:

$$\begin{cases} \text{in } x: & 28 + 2v_{2ix} = 36 \rightarrow v_{2ix} = \frac{36-28}{2} = 4 \cdot 10^6 \frac{m}{s} \\ \text{in } y & 17 + 2v_{2iy} = 60 \rightarrow v_{2iy} = \frac{60-17}{2} = 21.5 \cdot 10^6 \frac{m}{s} \end{cases}$$