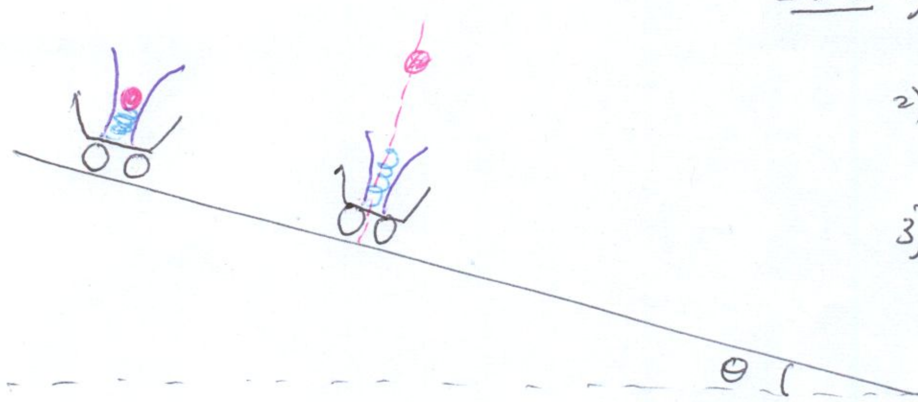
Variation of ball & spring on cart experiment:

Use a ramp instead of a horizontal track



Same: 1) No friction between cart wheels & track
2) Negligible air resistance
3) Motions along perpendicular directions are independent.

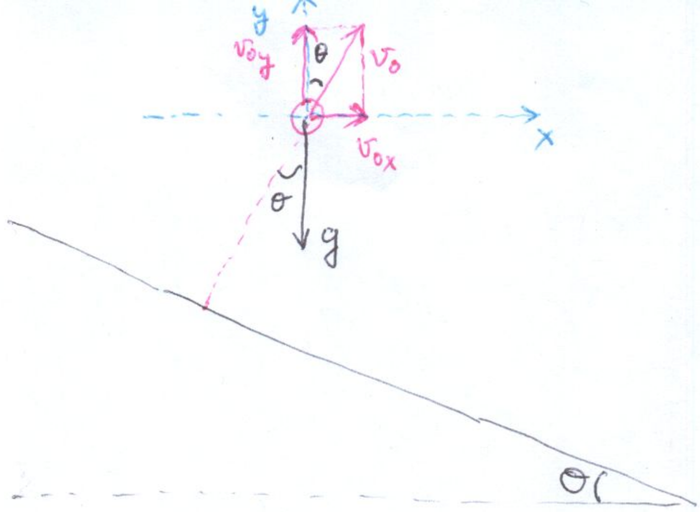
Changes: 1) Cart now has an acceleration in the direction of motion due to a component of gravity force that is parallel to ramp.

2) Ball behind cart | in front | into funnel
8 | 9 | 2

3) Ball, before it gets launched by spring, has a same constant acceleration motion as the cart.

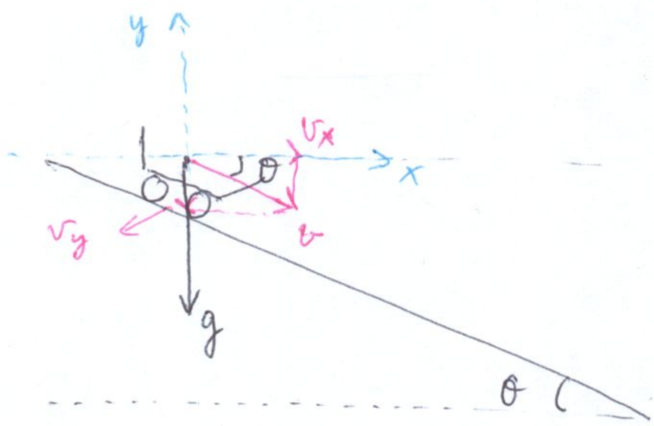
Conclusions: with two 3's (same & change), the additional motion ball acquires due to the spring launch is perpendicular to its other motion down the ramp (same as cart, same acceleration)

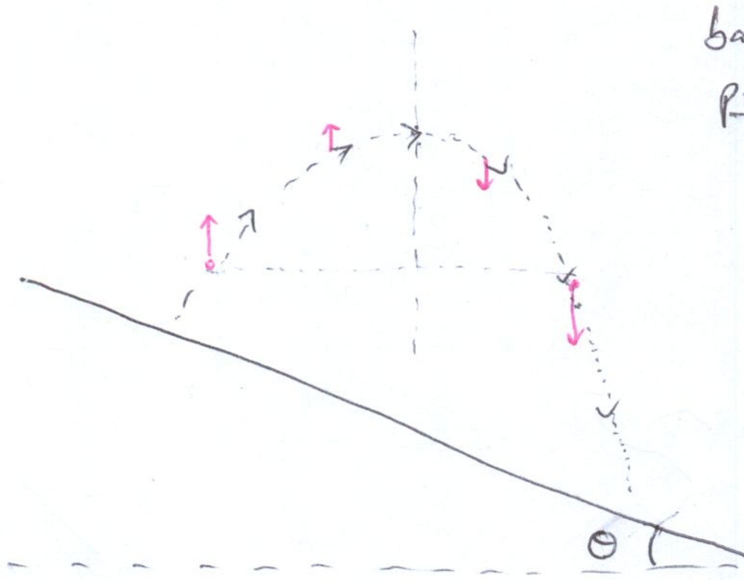
Ball



gravity doesn't affect our previous conclusion.

Cart





ball motion ↓

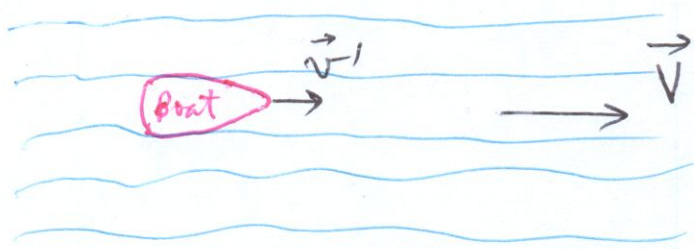
projectile motion

↳ { horizontal uniform motion
simultaneously
with
vertical constant acceleration motion
(due to gravity)

Relative Motion $\boxed{\vec{v} = \vec{v}' + \vec{V}}$

1) Boat going down stream in a river

view from above



$\vec{V}$: velocity of water (down stream is to the right)

$\vec{v}'$: velocity of boat w.r.t. water

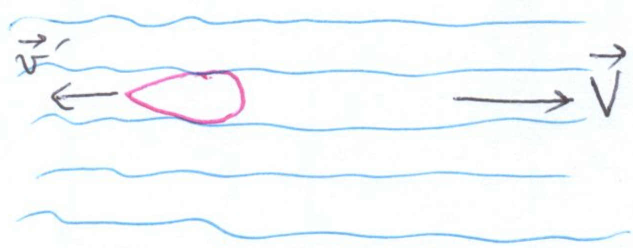
$\vec{v}$: velocity of boat w.r.t. ground : $\boxed{\vec{v} = \vec{v}' + \vec{V}}$

Unit vectors to specify directions $y\,(\hat{j})$, $x\,(\hat{\imath})$

$$\begin{cases} \vec{V} = V\hat{\imath} \\ \vec{v}' = v'\hat{\imath} \\ \vec{v} = v'\hat{\imath} + V\hat{\imath} \end{cases}$$

$$= (v' + V)\,\hat{\imath}$$

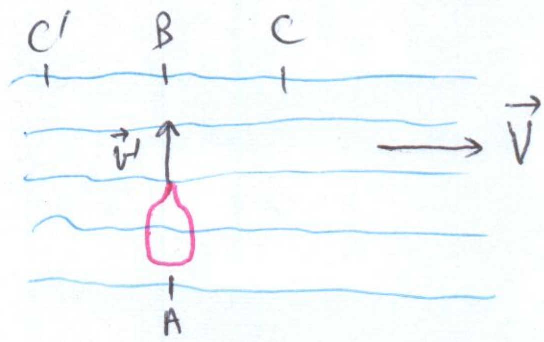
2) Boat going up stream



$$\vec{v} = (-v' + V)\,\hat{\imath}$$

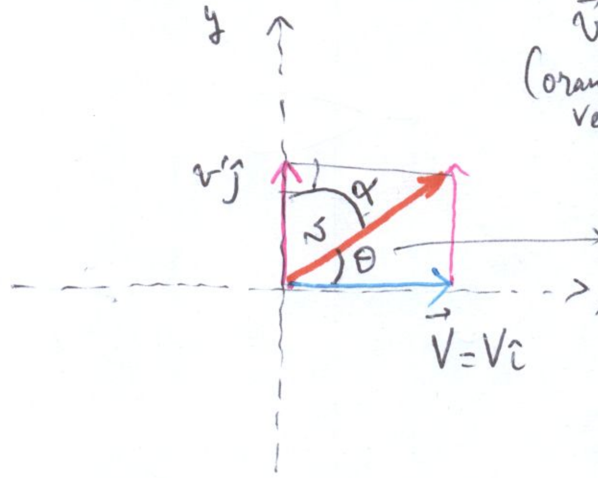
3) Boat crossing a river :

$y\,(\hat{j})$, $x\,(\hat{\imath})$



Unit vectors $\begin{cases} \text{Velocity of water } \vec{V} = V\hat{\imath} \\ \text{Velocity of boat wrt water } \vec{v}\,' = v'\hat{\jmath} \\ \text{velocity of boat wrt ground } \boxed{\vec{v} = \vec{v}\,' + \vec{V}} = \end{cases}$ 

$\vec{v} = v'\hat{\jmath} + V\hat{\imath}$ (orange vector) ($\vec{v}$ points @ an angle !)

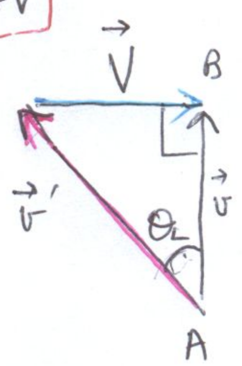


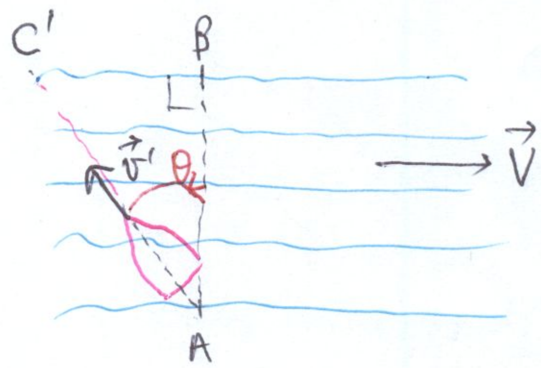
$$\begin{cases} \theta = \tan^{-1}\left(\dfrac{v'}{V}\right) \rightarrow \alpha = \begin{cases} 90-\theta \\ \text{or} \\ \tan^{-1}\left(\dfrac{V}{v'}\right) \end{cases} \\ v = \sqrt{v'^2 + V^2} \end{cases}$$

Consequence: 1) If you row straight across the river you won't get straight across river (in previous diagram, you'll end up @ C)

2) To go from A to B I have to aim @ C' @ angle θ_L left of AB

$\boxed{\vec{v} = \vec{v}\,' + \vec{V}}$





$$\begin{cases} \theta_L = \sin^{-1}\left(\dfrac{V}{v'}\right) \\ v = \sqrt{v'^2 - V^2} \end{cases}$$

Kinematic Eqs for constant acceleration in 1D & 2D:

1D (x, v, a) | 2D $(\vec{r}, \vec{v}, \vec{a})$

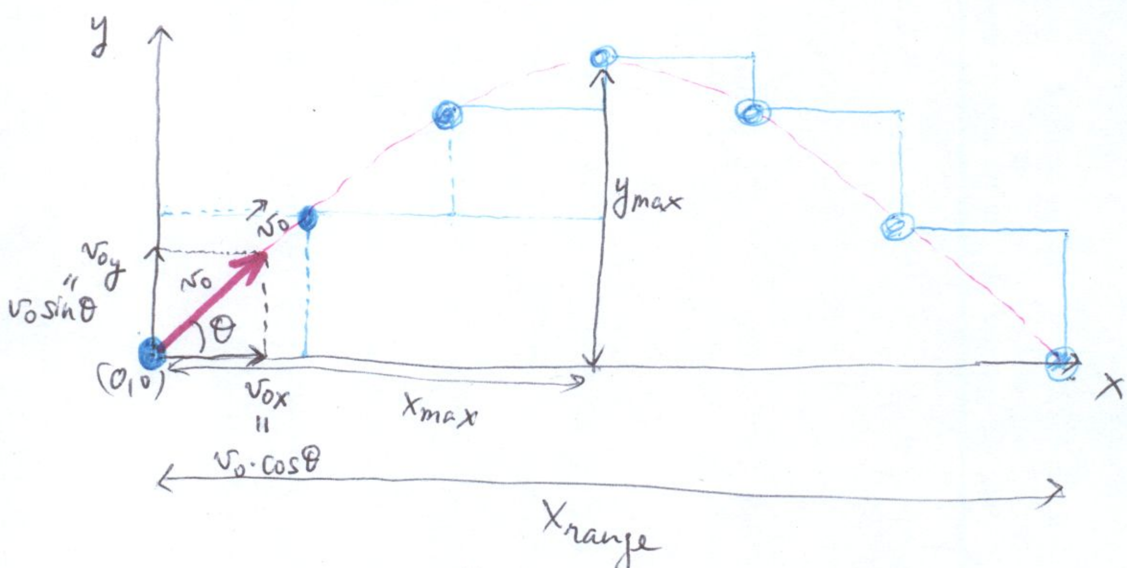
1) $v = v_0 + a \cdot t$
2) $x = x_0 + v_0 \cdot t + \frac{1}{2} a \cdot t^2$

1) $\vec{v} = \vec{v}_0 + \vec{a} \cdot t$
2) $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \vec{a} \cdot t^2$

Projectile Motion → simultaneous { uniform horizontal motion & constant acceleration vertical motion

1) (i) $v_x = v_{0x} + a_x \cdot t = v_{0x}$ $(a_x = 0)$
(ii) $v_y = v_{0y} + a_y \cdot t = v_{0y} \mp g \cdot t$ (−: upward motion, +: downward motion)

2) (i) $x = x_0 + v_{0x} \cdot t + \frac{1}{2} a_x t^2 = x_0 + v_{0x} \cdot t$ $(a_x = 0)$
(ii) $y = y_0 + v_{0y} \cdot t \mp \frac{1}{2} g t^2$ (− up ; + down)



1) (i) $v_x = v_0 \cos\theta$
(ii) $v_y = v_0 \sin\theta \mp g \cdot t$ (− up, + down)

2) (i) $x = x_0 + v_0 \cos\theta \cdot t$
(ii) $y = y_0 + v_0 \sin\theta \cdot t \mp \frac{1}{2} g t^2$ (−: up, +: down)

Trajectory eq:
$$\boxed{y = x\cdot\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2}$$

Max. altitude point: $(x_{max}, y_{max}) = \left(\frac{v_0^2\sin(2\theta)}{2g}, \frac{v_0^2\sin^2\theta}{2g}\right)$

Range: $x_{Range} = 2\cdot x_{max}$

3.45 Particle's position: $\vec{r}(t) = (\underbrace{ct^2 - 2dt^3}_{x(t)})\hat{i} + (\underbrace{2ct^2 - dt^3}_{y(t)})\hat{j}$

(c & d are >0)

a) when will particle move in x-direction?

$(\vec{r}, \vec{v}, \vec{a}) \Rightarrow$

$$v_y = 0 = \frac{dy}{dt} = \frac{d}{dt}(2ct^2 - dt^3)$$

$$0 = 4ct - 3dt^2$$

(t=0 is a trivial solution) ← $0 = 4c - 3dt \Rightarrow \boxed{t = \frac{4c}{3d}}$

b) when will it move in y-direction?

$$v_x = 0 = \frac{dx}{dt} = \frac{d}{dt}(ct^2 - 2dt^3)$$

$$0 = 2ct - 6dt^2$$

$$0 = 2c - 6dt \Rightarrow \boxed{t = \frac{c}{3d}}$$

Projectile Motion - $\begin{cases} \rightarrow \text{Uniform motion in horizontal direction} \\ \text{Simultaneous} \\ \rightarrow \text{Constant acceleration motion in vertical direction} \end{cases}$

Trajectory equation: Rearranging Kinematic Eq 2) :

↳ Kinematic equations $\begin{cases} 1) \begin{cases} v_x = v_0 \cos\theta \\ v_y = v_0 \sin\theta - gt \end{cases} \\ 2) \begin{cases} x = v_0 \cos\theta \cdot t \\ y = v_0 \sin\theta \cdot t - \frac{1}{2} g t^2 \end{cases} \end{cases}$

$(x_0, y_0) = (0,0)$

For a projectile motion starting from origin of coordinates.

2) $x = v_0 \cos\theta \cdot t \rightarrow t = \dfrac{x}{v_0 \cos\theta}$

$$y = \cancel{v_0} \sin\theta \cdot \frac{x}{\cancel{v_0} \cos\theta} - \frac{1}{2} g \cdot \frac{x^2}{v_0^2 \cdot \cos^2\theta}$$

$$\boxed{y = x \cdot \tan\theta - \frac{g x^2}{2 v_0^2 \cdot \cos^2\theta}}$$

"Trajectory equation" (t was eliminated from kin-eq. 2))

θ: direction of initial velocity

v_0: magnitude of initial velocity.

Conclusion: $\begin{cases} 1) \ \theta \ \& \ v_0 \text{ determines the trajectory of the projectile motion} \\ 2) \ y(x) \text{ is a quadratic polynomial} \Rightarrow \text{parabolic trajectory} \end{cases}$

Max. Altitude Point: $(x_{max}, y_{max}) = \left(\dfrac{v_0^2 \sin(2\theta)}{2g}, \dfrac{v_0^2 \sin^2\theta}{2g} \right)$

↳ Kin. eq. 1): $v_y = v_0 \sin\theta - g \cdot t$

@ $(x_{max}, y_{max}) \rightarrow v_y = 0 \rightarrow 0 = v_0 \sin\theta - g t_{max} \rightarrow \boxed{t_{max} = \dfrac{v_0 \sin\theta}{g}}$

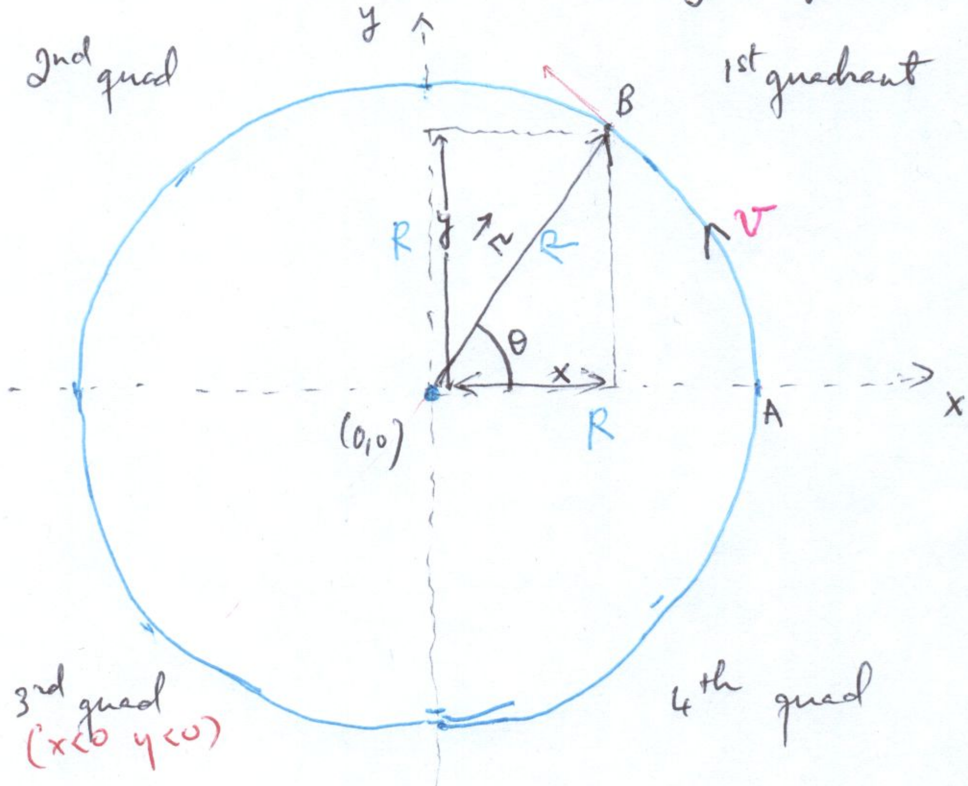
$\rightarrow$ kin eq 2) : $\begin{cases} x_{max} = v_0 \cos\theta \cdot \dfrac{v_0 \sin\theta}{g} = \dfrac{v_0^2 \sin(2\theta)}{2g} \\ \uparrow \text{ Trig: } 2\cos\theta \sin\theta = \sin(2\theta) \\ y_{max} = v_0 \sin\theta \cdot \dfrac{v_0 \sin\theta}{g} - \dfrac{1}{2\cancel{g}} \dfrac{v_0^2 \sin^2\theta}{g^{\cancel{2}}} = \dfrac{v_0^2 \sin^2\theta}{2g} \end{cases}$

Due to symmetry of parabolic trajectory of:

Range point: $\left(2x_{max}, 0\right) = \left(\dfrac{v_0^2 \sin(2\theta)}{g}, 0\right)$

$\boxed{\text{Uniform}}$ Circular Motion: a particle with this type of motion has $\boxed{\text{constant speed } v}$ along its circular trajectory

(UCM)



Let's say particle covers arc AB in time t, sweeping angle θ in same time: from geometry: $\boxed{\theta = \dfrac{\text{arc}}{R} = \dfrac{v \cdot t}{R}}$

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$= R\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + R\sin\left(\frac{v \cdot t}{R}\right)\hat{j}$$

$$= R\Big[\underbrace{\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}}_{\text{unit vector}}\Big]$$

→ In UCM, position vector $\vec{r}$ has a fixed length R but varying orientation.

Velocity in a UCM:

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = R\left[-\frac{v}{R}\cdot \sin\left(\frac{v\cdot t}{R}\right)\hat{\imath} + \frac{v}{R}\cos\left(\frac{v\cdot t}{R}\right)\hat{\jmath}\right]$$

$$= v\underbrace{\left[-\sin\left(\frac{v\cdot t}{R}\right)\hat{\imath} + \cos\left(\frac{v\cdot t}{R}\right)\hat{\jmath}\right]}_{\text{unit vector}}$$

→ In UCM, velocity vector $\vec{v}$ has a fixed length v (constant speed of UCM) but varying orientation

Acceleration in UCM:

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = v\left[-\frac{v}{R}\cdot \cos\left(\frac{v\cdot t}{R}\right)\hat{\imath} - \frac{v}{R}\sin\left(\frac{v\cdot t}{R}\right)\hat{\jmath}\right]$$

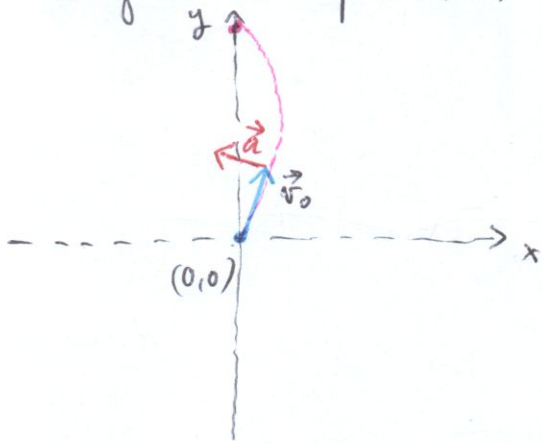
$$= -\frac{v^2}{R}\underbrace{\left[\cos\left(\frac{v\cdot t}{R}\right)\hat{\imath} + \sin\left(\frac{v\cdot t}{R}\right)\hat{\jmath}\right]}_{\text{Unit vector}}$$

→ In UCM, acceleration vector $\vec{a}$ has a fixed length $\boxed{\frac{v^2}{R}}$ (direction radially inward ⇔ minus sign) but a varying orientation

or toward center of curvature

3.54

Step 1: Diagram & information



$\vec{v}_0 = 11\hat{i} + 14\hat{j}$ $\frac{m}{s}$

2D, 1st quad.

$\vec{a} = -1.2\hat{i} + 0.26\hat{j}$ $\frac{m}{s^2}$

constant, 2D, 2nd quad.

⇓

pulling particle towards y-axis

⇓

a) When does particle cross y-axis t?

Step 2 Relevant equations:

Kinematic equations for constant acceleration in 2D:

1) $\vec{v} = \vec{v}_0 + \vec{a}\cdot t$ $\begin{cases} v_x = v_{0x} + a_x \cdot t \\ v_y = v_{0y} + a_y \cdot t \end{cases}$

2) $\vec{r} = \vec{r}_0 + \vec{v}_0\cdot t + \frac{1}{2}\vec{a}\cdot t^2$ $\begin{cases} \boxed{x = v_{0x}\cdot t + \frac{1}{2}a_x\cdot t^2} \\ y = v_{0y}\cdot t + \frac{1}{2}a_y\cdot t^2 \end{cases}$

$(x_0, y_0) = (0,0)$

Step 3: When particle crosses y-axis $\Rightarrow$ $x = 0 = \cancel{v_{0x}}\ 11\cdot t + \frac{1}{2}(-1.2)\cdot t^2$

$0 = 11t - 0.6t^2 \Rightarrow 0 = 11 - 0.6t \Rightarrow \boxed{t = \frac{11}{0.6}s = 18.3\,s}$

b) What is $y(t = 18.3s)$? $y = v_{0y}\cdot t + \frac{1}{2}a_y\cdot t^2$

$= 14\cdot 18.3 + \frac{1}{2}0.26\cdot 18.3^2 = 300\ m$

c) How fast & at what direction does it go at that time?

2nd quad

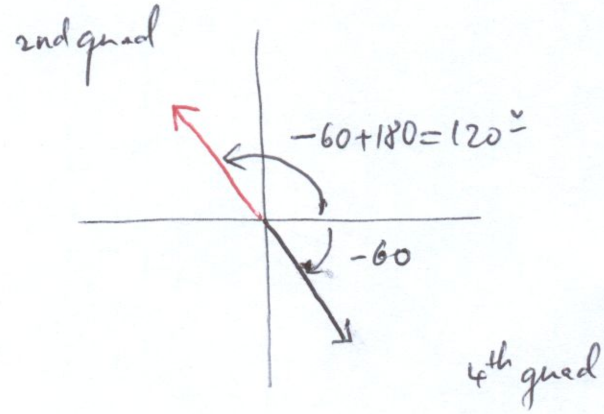
$$\begin{cases} v_x = 11 - 1.2 \cdot 18.3 = \boxed{-10.96\ \frac{m}{s}} \\ v_y = 14 + 0.26 \cdot 18.3 = \boxed{18.8\ \frac{m}{s}} \end{cases}$$

magnitude $v = \sqrt{10.96^2 + 18.8^2} = 21.7 \frac{m}{s}$

angle $\theta_v = \tan^{-1}\left(\frac{18.8}{-10.96}\right) = -60^\circ$

Cartesian components of $\vec{v}$

Polar components of $\vec{v}$



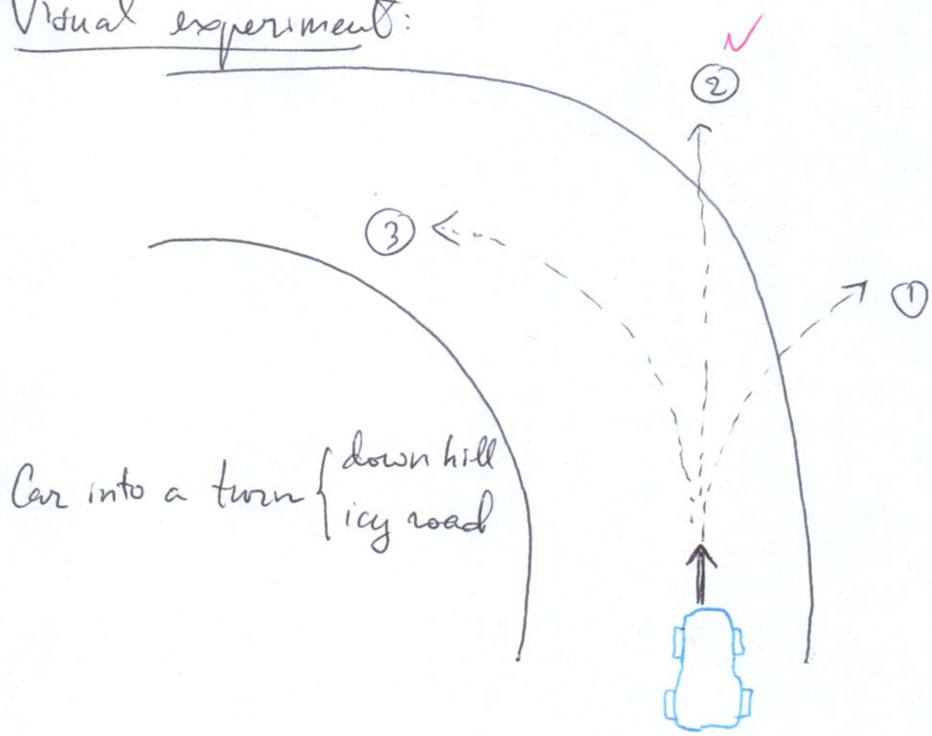
$$\Rightarrow \vec{v} \begin{cases} \text{magnitude} = 21.7 \frac{m}{s} \\ \text{angle } \theta_v = 120^\circ \end{cases}$$

Ch 4 Motion & Forces : (How forces change motion)

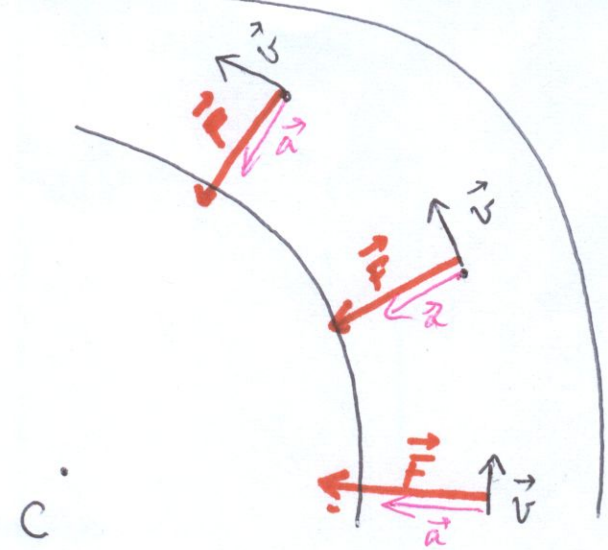
$\vec{r}, \vec{v}, \vec{a}, t$ $\qquad$ $\vec{F}$

Statement: Force $\vec{F}$ is the agent that causes the acceleration $\vec{a}$, changing the motion

Visual experiment:



- We rely on force of friction to drive and turn. In absence of this force or agent to change direction of motion, motion of vehicle stays straight 2
- If friction force was present it would have caused $a = \frac{v^2}{R}$ accel. toward center of curvature that keeps an object (car) in UCM
- A force is needed to change a motion.



Conclusion:
1) $\vec{F}$ is a vector, pointing towards center of curvature
2) $\vec{F}$ is the agent to change direction of $\vec{v}$

Since car tends to go off the curve, there needs to be a friction force that changes the direction of velocity vector or causing the acceleration towards center of curvature $\left(a = \frac{v^2}{R}\right)$

Newton's Laws (3)

1st Law or Law of Inertia:

A body at rest will continue at rest, a body in uniform motion will continue in uniform motion unless there is a net force acting on the body (then 2nd Law)

2nd Law:

$$\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$$

$\vec{p}$: linear momentum $\vec{p} = m \cdot \vec{v}$ $\left(kg \cdot \frac{m}{s} \right)$

$\vec{F}_{net}$: superposion of all forces acting on the body

$$\vec{F}_{net} = \frac{d}{dt}(m\vec{v}) = \underbrace{\frac{dm}{dt} \cdot \vec{v}}_{\substack{\text{normally} \\ \text{negligible} \\ \text{since } \frac{dm}{dt} \simeq 0}} + m \underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}$$

($\frac{dm}{dt}$ is important e.g. in space shuttle which burns lot of fuel)

$$\text{If } \frac{dm}{dt} = 0 \Rightarrow \boxed{\vec{F}_{net} = m\vec{a}}$$

$$[F] = M \frac{L}{T^2} \xrightarrow{SI} kg \frac{m}{s^2} \equiv N \text{ (for Newton)}$$

3rd Law: or Law of Action & Reaction:

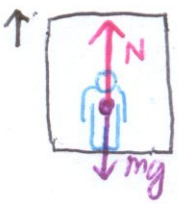
If A exerts a force on B, then B exerts an equal and opposite force on A

Law of Action & Reaction:

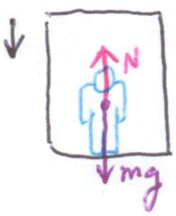
1) Feeling our weight in an elevator

by pressure on
our feet.

→ When we stand, our weight applies a force on the floor ($m \cdot g$), by "action & reaction" floor exerts an equal and opposite force ("normal force") on us through our feet

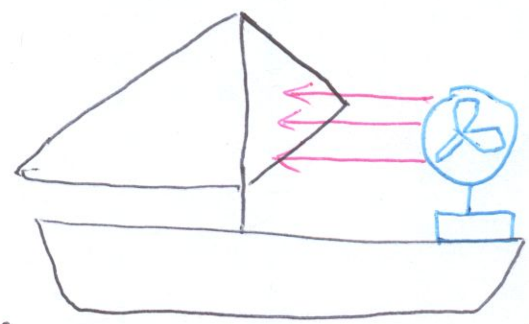


→ When an elevator accelerates upward, this "normal force" is larger than our weight mg making us feeling heavier.



→ When it accelerates downward, the "normal force" is less than our weight mg, making us feel lighter

2) Sailing without wind:



a) We fixed a fan on our boat, then blow air on sail

b) Fan blows air : microscopically blades hit air molecules in forward direction

c) Air molecules then hit sail applying a force on sail

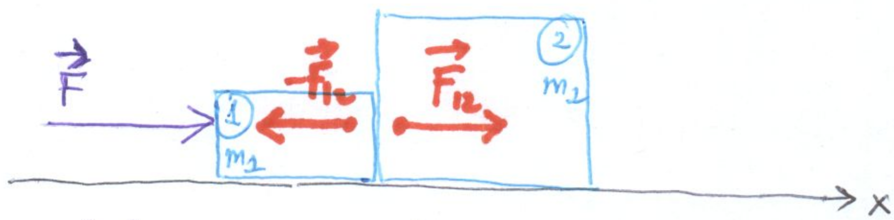
d) But this doesn't work because air molecules apply an equal and opposite force on the blades (and fan is fixed onto boat)

Newton's Laws:

1) Two boxes ~~next to~~ in contact with each other on a horizontal surface:

(i) No friction

(ii) A force $\vec{F}$ is applied on box #1 causing system of the two boxes to accelerate in +x-direction: $\vec{F} = (m_1 + m_2)\vec{a}$



a) If $\vec{F}$ is applied on m_1, system moves forward with acceleration $\vec{a}$

Force that makes m_2 moves is $\vec{F}_{12}$ (force by m_1 on m_2)

↳ Clarifications a1) If system moves together with acceleration $\vec{a}$: $\vec{F} = (m_1 + m_2)\vec{a}$

a2) Then $\vec{F}_{12}$ can't be $\vec{F}$ since $\vec{F}_{12} = m_2 \cdot \vec{a}$

b) What ~~force~~ is the net force on m_1?

b1) It can't be $\vec{F}$, since by 2nd Newton's Law net force on m_1 is $m_1\vec{a}$

b2) By 3rd Newton's Law, there is a reaction force by m_2 on m_1 of equal value as F_{12}, pointing in opposite direction

b3) Then 2nd Newton's Law for m_1 is $\vec{F} - \vec{F}_{12} = m_1\vec{a}$

Summary:

Total force on system (or net)	Net force on m_1	Net force on m_2
$\vec{F}$	$\vec{F} - \vec{F}_{12}$	$\vec{F}_{12}$

i) Net force on m_1 + Net force on m_2

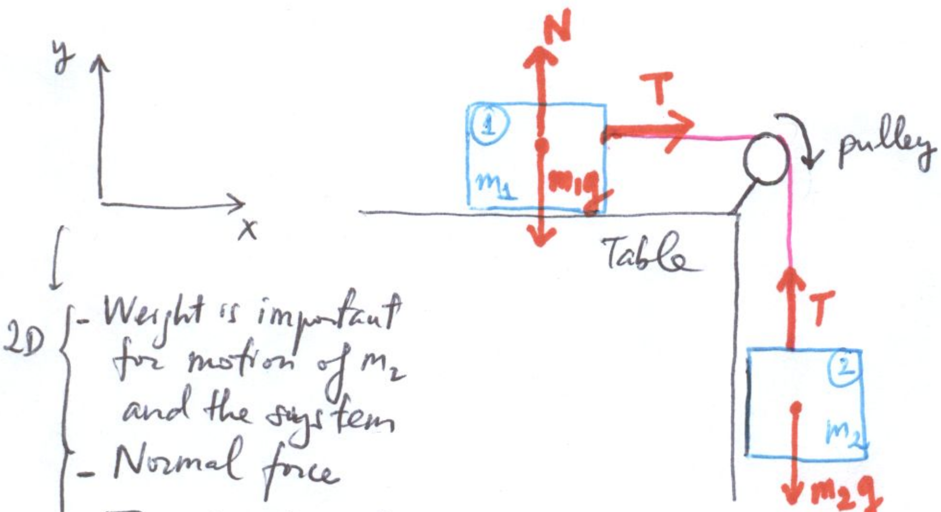
= net force on system

ii) $\vec{F}_{12}$ & $\vec{F}_{21} = -\vec{F}_{12}$ are internal forces that cancel each other out by pairs

2) Two boxes connected by a massless rope & a pulley:

(i) No friction

(ii) Rope : sufficiently small mass compared to m_1 & m_2



Since friction is negligible system will move CW @ pulley as shown

2D { - Weight is important for motion of m_2 and the system
- Normal force
- Tension force by the rope plays important role in motion of system

Box #1

Forces on m_1 $\begin{cases} \text{weight} : -m_1 g \hat{j} \\ \text{normal} = N\hat{j} \quad (N = m_1 g \text{ by } 3^{rd} \text{ Law}) \\ \text{tension} : T\hat{i} \end{cases}$

2^{nd} Law : $\vec{F}_{net_1} = m_1 \vec{a} \begin{cases} F_{net_{1,x}} = \boxed{T = m_1 \cdot a} \; (1) \\ F_{net_{1,y}} = N - m_1 g = 0 \end{cases}$

{ i) Action & Reaction
{ ii) $a_y = 0$

Box #2

Forces on m_2 $\begin{cases} \text{weight} : -m_2 g \hat{j} \\ \text{tension} : T\hat{j} \end{cases}$

2^{nd} Newton's Law :

$\vec{F}_{net_2} = m_2 \vec{a} \begin{cases} F_{net_{2,x}} = 0 \\ F_{net_{2,y}} = \boxed{T - m_2 g = -m_2 a} \; (2) \end{cases}$

* This $\vec{a}$ has same magnitude as that for m_1 (rope) but different direction (pulley)

downward acceleration for m_2

Example: given m_1 & m_2, find a & T

All equations we can write for this system are

$$T = m_1 \cdot a \quad (1)$$

$$T - m_2 g = -m_2 a \quad (2)$$

→ Solve for a: 1) into 2) $m_1 a - m_2 g = -m_2 a$

$$(m_1 + m_2) a = m_2 g \Rightarrow \boxed{a = \frac{m_2}{m_1 + m_2} g}$$

→ Solve for T: back in 1) $\boxed{T = \frac{m_1 m_2}{m_1 + m_2} g}$

Observations: a) If we double the masses what happens to a?

same since $a' = \frac{\cancel{2} m_2}{\cancel{2} m_1 + \cancel{2} m_2} g = a$

b) If I double m_2, will a be doubled?

$$a' = \frac{2 m_2}{m_1 + 2 m_2} g \neq 2a$$ (larger but not doubled)

Spring Forces (Hooke's Law)

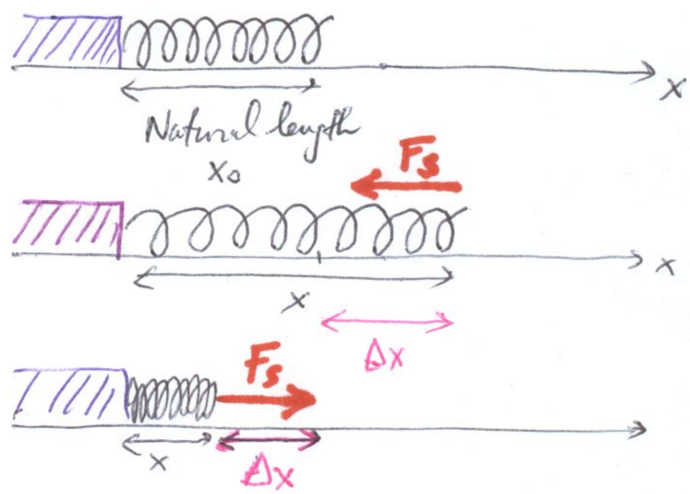
If a spring is stretched or compressed it reacts with a force in the opposite direction. $F_s = -k \cdot \Delta x$

− : resistance to change of length

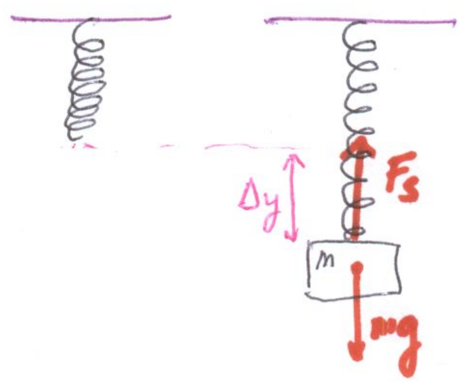
k : spring constant (material & size of spring) $\frac{N}{m}$

Δx : change of length from its natural length

Horizontal spring



Vertical spring



When m arrives at equilibrium:

$$F_{net} = m \cdot a = 0$$

$$k\Delta y - mg = 0 \Rightarrow \Delta y = \frac{mg}{k}$$

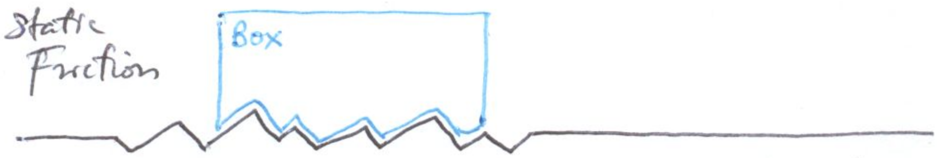
Friction forces: are present whenever an object is in contact with a surface

→ Types

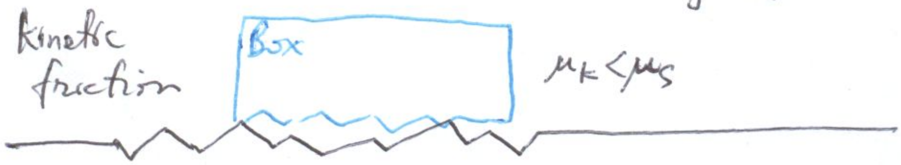
- **Static friction:** $F_s = \mu_s N$
  - → when object is in contact with surface but not in relative motion
  - → we notice when we need to move a box that has been sitting on a surface
- **Kinetic friction:** $F_k = \mu_k N$
  - → when object is in contact & relative motion with surface

- μ_s; μ_k coefficients of static/kinetic, respectively, depend on the material & roughness of surface
- N: normal force by surface on body (by reaction)
- For a same surface & object $\boxed{\mu_s > \mu_k}$

microscopic explanation



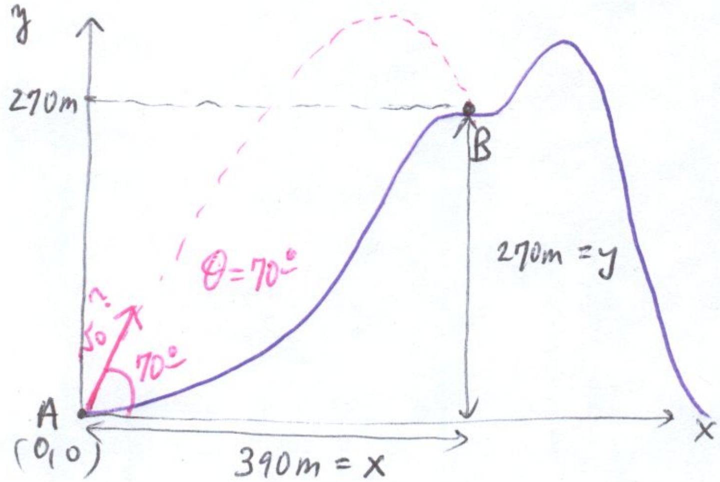
Any surface has some degree of roughness



- When we push heavy boxes, after we overcome the static friction, the box acquires an acceleration $F_s - F_k = m\boxed{a}$

3.70

Step 1:



What is the initial speed v_0?

Step 2: Relevant equation: projectile motion $\begin{cases} \text{- Uniform horizontal motion simultaneously with} \\ \text{- Constant acceleration vertical motion.} \end{cases}$

kinematic eqs in 2D: v_x, v_y, x, y $\xrightarrow{\text{eliminate } t}$ Trajectory equation

$$\boxed{y = x \tan\theta - \frac{g}{2} \frac{x^2}{v_0^2 \cos^2\theta}}$$

Step 3: Solve for v_0:

$$\frac{g}{2v_0^2 \cos^2\theta} x^2 = x \tan\theta - y$$

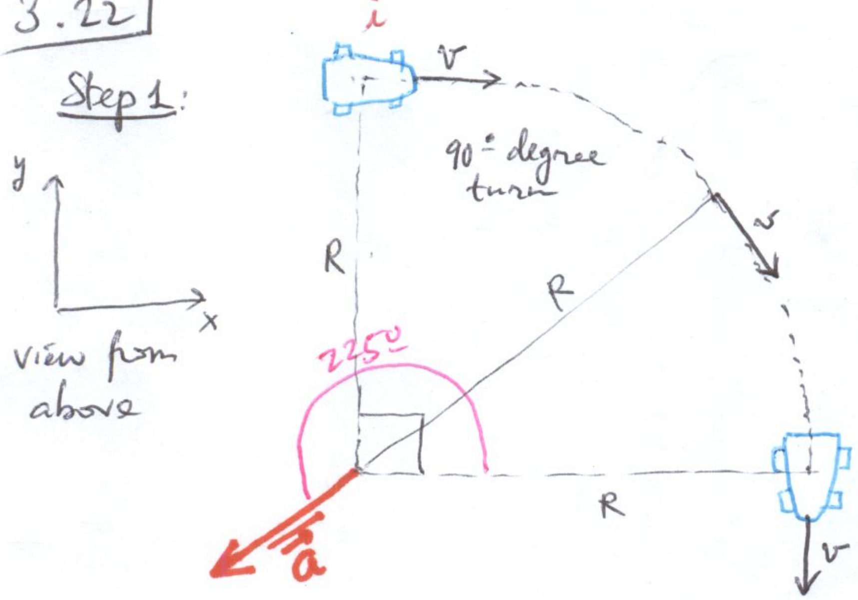
$$v_0^2 = \frac{g x^2}{2\cos^2\theta\,(x \tan\theta - y)}$$

$$v_0 = \sqrt{\frac{9.81 \cdot 390^2}{2\cos^2 70°\,(390 \cdot \tan 70° - 270)}} = 89.2 \frac{m}{s}$$

Although value for v_0 is unique, this is not the only way to solve for it.

3.22

Step 1:



* speedometer remains constant through the turn → car follows UCM

* Find direction of car's average acceleration.

Step 2: Equation for the average acceleration:

$$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t}$$, once we get its Cartesian components → $\theta_{\bar{a}} = \tan^{-1}\left(\frac{\bar{a}_y}{\bar{a}_x}\right)$

Step 3: $$\vec{\bar{a}} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{-v\hat{j} - v\hat{i}}{\Delta t} = \underbrace{\left(-\frac{v}{\Delta t}\right)}_{\bar{a}_x}\hat{i} + \underbrace{\left(\frac{-v}{\Delta t}\right)}_{\bar{a}_y}\hat{j}$$

3rd quad.

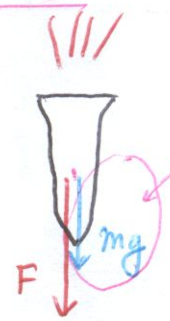
$$\theta_{\bar{a}} = \tan^{-1}\left(\frac{-\frac{\cancel{v}}{\cancel{\Delta t}}}{-\frac{\cancel{v}}{\cancel{\Delta t}}}\right) = 45^{\circ} \longrightarrow + 180^{\circ} = 225^{\circ}$$

calculator

4.55 * Thrust force F to accelerate a rocket of mass m

a) Downward $a = 1.4g$ near Earth

Step 1:



Step 2: Relevant eq. = 2nd Newton's Law:

$$F_{net} = m \cdot a$$

$$F + mg = m \cdot a$$

Step 3: $F = m(a - g) = m \cdot 0.4g$

$$\boxed{F = 0.4\,mg}$$

b) Upward $a = 1.4g$ near Earth

Step 1:

Step 2: $F_{net} = m \cdot a$

$$F - mg = ma$$

Step 3: $\boxed{F = m(a+g) = 2.4\,mg}$

c) In interstellar space (all directions behave the same since there are no gravitational attractions.)

$a = 1.4g$

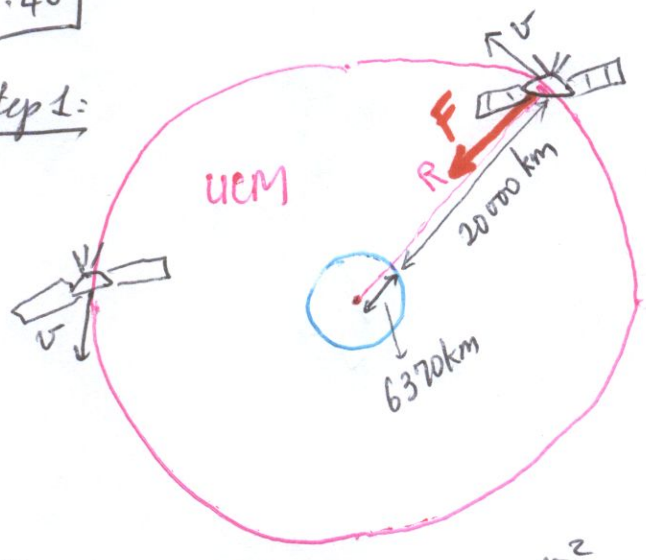
Step 1:

Step 2: $F_{net} = m \cdot a$

Step 3: $\boxed{F = m \cdot a = 1.4\,mg}$

3.40

Step 1:



* GPS satellites circle Earth
↔ circular orbits ↔ $\boxed{\text{UCM}}$
* Orbit radius $R = 26{,}370$ km
* In this orbit $g \longrightarrow \boxed{\frac{5.8}{100} g}$
* Find orbital period: T or time for satellite to complete one circumference $2\pi R$

Step 2: UCM $\longleftrightarrow$ $a = \frac{v^2}{R}$ is required for an object to conform to a circular trajectory $\longleftrightarrow$ a force or agent to create this acceleration. In this case it's $F = m \cdot \frac{5.8}{100} g$

Relevant equations:

1) $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{5.8 g R}{100}}} = 20\pi \sqrt{\frac{R}{5.8 g}}$

2) $F_{net} = m \cdot a$

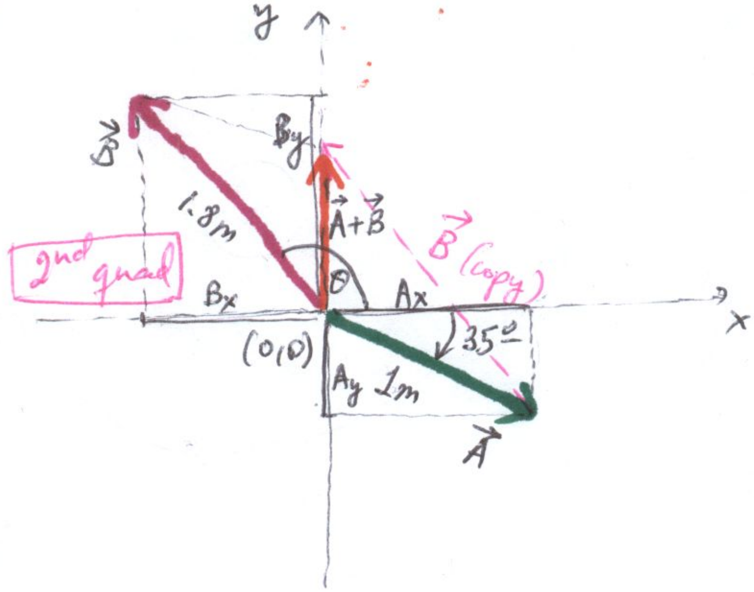
$\cancel{m} \frac{5.8}{100} g = \cancel{m} \cdot \frac{v^2}{R} \Rightarrow v = \sqrt{\frac{5.8 g R}{100}}$

Step 3: $T = 20\pi \sqrt{\frac{26.370 \cdot 10^6 \, \cancel{m}}{5.8 \cdot 9.81 \frac{\cancel{m}}{s^2}}} = 42774 \, \cancel{s} \cdot \frac{1 \text{ hr}}{3600 \cancel{s}} = 11.88 \text{ hrs}$

$\simeq 12$ hrs.

3.42

Step 1:



* Vector addition in 2D

$\vec{A} = (1m, 35°$ cw from x axis) or below x-axis

$\vec{B} = (1.8m, \theta?)$ → so that $\vec{A}+\vec{B}$ is in y-direction

(x-component of $\vec{A}+\vec{B}$ is 0 !)

* $\vec{A}+\vec{B}$ is the diagonal of a parallelogram formed by $\vec{A}$ & $\vec{B}$. It should point in +y-direction.

Step 2: Relevant equation:

Adding vectors mathematically using unit vectors

$$\vec{A}+\vec{B} = A_x\hat{i} + A_y\hat{j} + B_x\hat{i} + B_y\hat{j} = \boxed{\underbrace{(A_x+B_x)}_{=0}}\hat{i} + (A_y+B_y)\hat{j}$$

Step 3: $A_x = 1\cdot\cos 35°$; $B_x = 1.8\cos\theta$ (θ is in 2nd quad or $90° < \theta < 180°$)

$A_y = -1\cdot\sin 35°$; $B_y = 1.8\sin\theta$ (θ 2nd quad.)

$\sin(-35°) = -\sin 35°$

$$A_x + B_x = 0$$

$$\cos 35° + 1.8\cos\theta = 0 \rightarrow \cos\theta = -\frac{\cos 35°}{1.8} \rightarrow \theta = \cos^{-1}\left[-\frac{\cos 35°}{1.8}\right]$$

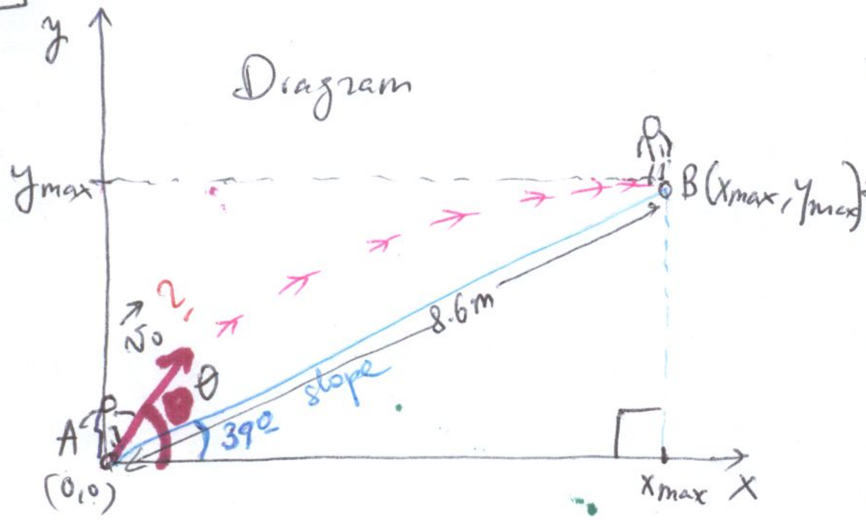
$$90° < \boxed{\theta = 117°} < 180°$$

Second solution if we assume $\vec{A}+\vec{B}$ is along −y-direction.

$\vec{B}$ is the 3rd quad; since $\cos\theta$ is an even function $\theta = -117°$ or $243°$

3.62

Step 1: Diagram



* Protein bar in a projectile motion
* Companion @ max. altitude point (bar should reach him going horizontally) $v_y = 0$
* Initial velocity $\vec{v_0}$? for protein bar

$$B = (x_{max}, y_{max}) = (8.6 \cos 39^\circ,\ 8.6 \cdot \sin 39^\circ) = (6.68\text{m},\ 5.41\text{m})$$

Step 2: Relevant equations : at least two for $\begin{cases}(v_{0x}, v_{0y}) \\ \text{or} \\ (v_0, \theta)\end{cases}$

↓

kinematic equations for constant acceleration ($a=g$) in 2D.

$$\begin{cases} 1)\ v = v_0 + a \cdot t \\ 2)\ x = x_0 + v_0 \cdot t + \frac{1}{2} a t^2 \\ 3)\ \frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a \quad (\text{no time}) \end{cases}$$

$$\begin{cases} 3a)\ \frac{v_x^2 - v_{0x}^2}{x - x_0} = 2 \cdot a_x = 0 \quad (\text{trivial}) \\ 3b)\ \boxed{\frac{v_y^2 - v_{0y}^2}{y - y_0} = 2 \cdot a_y = -2g} \quad (\text{upward : 1st half of parabola!}) \end{cases}$$

↓ Solve for v_{0y} $\begin{cases} v_y = 0 \\ y - y_0 = y_{max} = 5.41\text{m} \\ g = 9.81 \frac{\text{m}}{\text{s}^2} \end{cases}$

Step 3: i) solve for v_{0y}: $0 - v_{0y}^2 = -2g \cdot y_{max} \rightarrow v_{0y} = \sqrt{2g \cdot y_{max}}$

$$\boxed{v_{0y}} = \sqrt{2 \cdot 9.81 \cdot 5.41} = \boxed{10.3 \frac{\text{m}}{\text{s}}}$$

ii) solve for v_{0x}: $\begin{cases} 1)\ v_y = 0 = v_{0y} - g \cdot t \quad \boxed{t = \frac{v_{0y}}{g}} \\ 2)\ \boxed{v_{0x} = \frac{x_{max}}{t}} = \frac{6.68}{(10.3/9.81)} = \boxed{6.36 \frac{\text{m}}{\text{s}}} \end{cases}$

$\vec{v}_0$

Cartesian ← → Polar

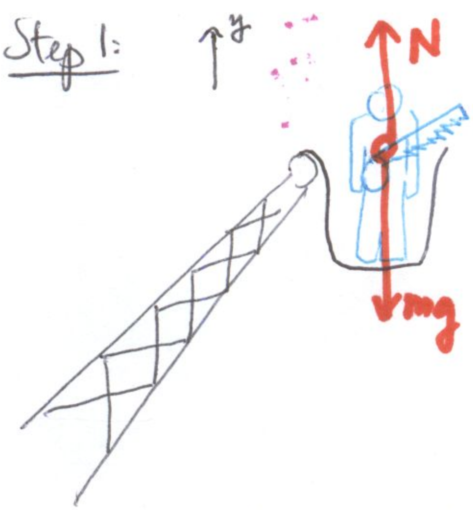
49

$$\vec{v}_0 = \left(6.36\,\frac{m}{s},\ 10.3\,\frac{m}{s}\right) = \Big(\underbrace{v_0 = \sqrt{6.36^2 + 10.3^2}}_{12.1\,\frac{m}{s}},\ \underbrace{\theta = \tan^{-1}\frac{10.3}{6.36}}_{58.3^\circ}\Big)$$

$> 39^\circ$!

4.40

Step 1: $\uparrow y$



$m = 74\,kg$

* Normal force N exerted by lift on tree surgeon
  a) upward constant speed
  b) downward @ constant speed
  c) upward @ $a = 1.7\,\frac{m}{s^2}$
  d) downward @ $a = 1.7\,\frac{m}{s^2}$

Step 2: Relevant equation: 2nd Newton's Law: $F_{net} = m \cdot a$

Step 3: a) & b) $a = 0$ → $N - mg = 0 \Rightarrow N = mg$

$= 74 \cdot 9.81$

$= 725\,N$

c) $a = +1.7\,\frac{m}{s^2}$ ⟶ $N - mg = m \cdot a$

$N = m(g + a)$

$= 74\,(9.81 + 1.7)$

$= 851\,N$ (feels heavier)

d) $a = -1.7\,\frac{m}{s^2}$ ⟶ $N = m(g + a)$

$= 74\,(9.81 - 1.7)$

$= 599\,N$ (feels lighter)