Physic 113 Summer 2014 AM

Ch 1 Doing Physics

Dimensional Analysis:

Speed $v = \frac{\Delta s}{\Delta t}$

→ $\boxed{\text{Dimension of speed } [v] = \frac{[\Delta s]}{[\Delta t]} \equiv \frac{L}{T}}$ { "Dimension of speed is length over time"

Δs "change of space" or "increment of space" or distance

Δ "delta"

Δt "increment of time" or time

Acceleration $a = \frac{\Delta v}{\Delta t}$

→ $\boxed{\text{Dimension of acceleration } [a] = \frac{[\Delta v]}{[\Delta t]} = \frac{\frac{L}{T}}{T} = \frac{L}{T^2}}$

Kinetic energy $K.E. = \frac{1}{2}mv^2$

↳ Intuition $\left\{\begin{array}{l} \uparrow v \Rightarrow \uparrow K.E. \\ \uparrow m \Rightarrow \uparrow K.E. \end{array}\right\}$ we will derive in Ch. 6

→ $\boxed{\text{Dimension of K.E. } [K.E.] = \left[\frac{1}{2}mv^2\right]} = \underbrace{\boxed{\left[\frac{1}{2}\right]}}_{=1} \cdot \underbrace{[m]}_{=M} \cdot \underbrace{[v]^2}_{=\frac{L^2}{T^2}} = \boxed{M\frac{L^2}{T^2}}$

numbers have no dimension

→ Use dimensional analysis to check physics formulas:

Which one is a correct formula for speed?

$v_1 = \frac{1}{2}gh^2 \rightarrow [v_1] = [g]\cdot[h]^2 = \frac{L}{T^2}\cdot L^2 = \frac{L^3}{T^2} \neq \frac{L}{T}$

$v_2 = \sqrt{g\cdot h} \rightarrow [v_2] = [g]^{\frac{1}{2}}\cdot[h]^{\frac{1}{2}} = \frac{L^{1/2}}{T}\cdot L^{1/2} = \frac{L}{T}$

g: acceleration of gravity
h: height or vertical position

Conclusion: v_2 is correct formula for speed

limitation ← $\boxed{\text{except}}$ for a multiplicative constant

Units
- S.I. : international system
- British :

Dimension	SI	British	
L	m (meter)	ft (foot)	
T	s (second)	s	
M	kg (kilogram)	lbs (pounds)	1lb = 0.454 kg
[Area] = L^2	m^2	ft^2	
[Volume] = L^3	m^3	ft^3	
[Energy] = $\frac{ML^2}{T^2}$	$\frac{kg\,m^2}{s^2} \equiv J$ (Joule)	$\frac{lb \cdot ft^2}{s^2} \rightarrow$ Btu	

Sub & super units :

nano nm	micron μm	mm	cm	m	km	light-year
$10^{-9}m$	$10^{-6}m$	$10^{-3}m$	$10^{-2}m$	1m	$10^{3}m$	$9.46 \cdot 10^{15}m$

nm^2	μm^2	mm^2	cm^2	m^2	km^2	...
$10^{-18}m^2$	$10^{-12}m^2$	$10^{-6}m^2$	$10^{-4}m^2$	$1m^2$	$10^{6}m^2$	

in	ft	mi
2.54cm	0.3048m	1609m

...	cm^3	m^3	km^3	...
	$10^{-6}m^3$	$1m^3$	$10^{9}m^3$	

femtos	ps	μs	ms	s	min	hr	day	...
$10^{-15}s$	$10^{-12}s$	$10^{-6}s$	$10^{-3}s$	1s	60s	3600s	86400s	

μg	mg	g	kg
$10^{-9}kg$	$10^{-6}kg$	$10^{-3}kg$	1kg

Accuracy & Significant Figures:

. Scientific notation: use powers of 10 & coeffient which is less than 10

$$\Delta s = 6176000\,m = 6.176 \cdot 10^6\,m = \underbrace{6.176}_{\substack{\text{coefficient}\\ <10}}E6\,m$$

$$\Delta t = 3000\,s = 3 \cdot 10^3\,s = 3E3\,s$$

$$\text{speed } v = \frac{\Delta s}{\Delta t} = \frac{6.176 \cdot 10^6}{3 \cdot 10^3} = \frac{6.176}{3} \cdot 10^{6-3} = 2.059 \cdot 10^3\,\frac{m}{s}$$

. Accuracy (addition & subtraction)

$$\pi - 1.14 = 3.1416 - 1.14 = 2.0016$$

(assume $\pi = 3.1416$)

only up to least accuracy of two terms

2.00

. Significant figures s.f.'s (multiplication & division)

s.f's $\underbrace{6\,370\,000}_{3}$ m — end zeroes don't count

$\underbrace{6\,370\,001}_{7}$ m — middle zeroes do count

Circumference of Earth: $2\pi R_E = 2 \cdot 3.1416 \cdot 6.37 \cdot 10^6\,m$

$R_E = 6.37 \cdot 10^6\,m$

$$= 4.002398 \cdot 10^7\,m$$

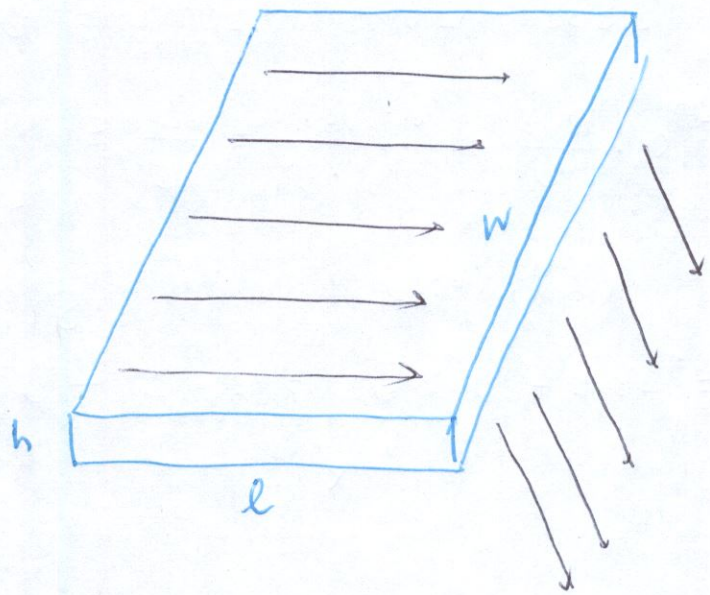
only up to least # of significant figures of the factors involved

$4.00 \cdot 10^7\,m$

1.41 a) Estimate volume of water going over Niagara Falls in 1s

Guess: $6420 \frac{m^3}{s}$

Estimation: water over Falls modeled as a rectangular slab:



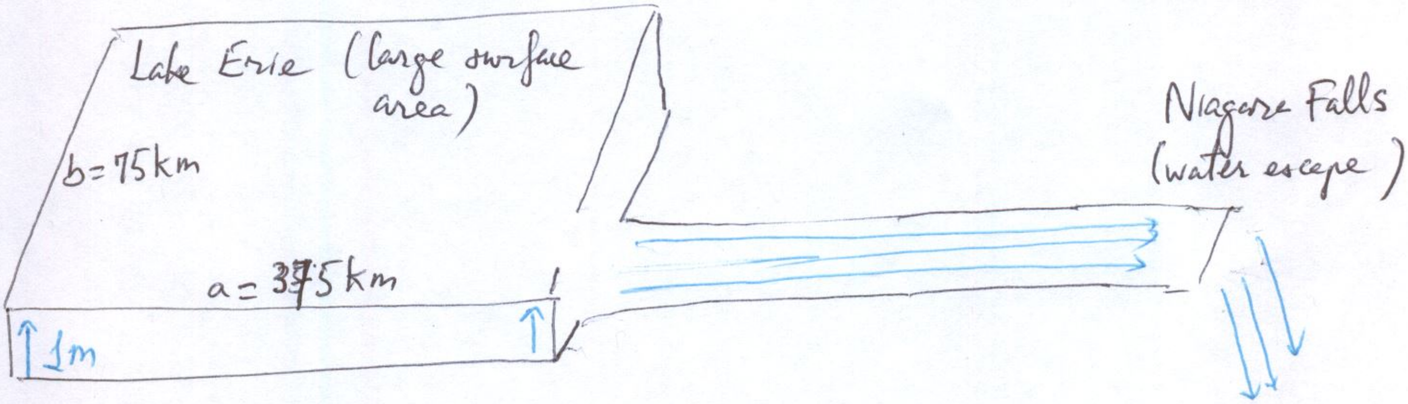
Volume per second or flow rate $= \dfrac{h \cdot l \cdot w}{t}$ $\begin{cases} 1) \dfrac{h}{t} \cdot l \cdot w \\ 2) h \cdot \dfrac{l}{t} \cdot w \checkmark \\ 3) h \cdot l \cdot \dfrac{w}{t} \end{cases}$

Numeric values $\begin{cases} h & : 1m,\ 10m,\ 100m \\ \frac{l}{t} \text{ speed of water} & : 1\frac{m}{s},\ 10\frac{m}{s},\ 100\frac{m}{s} \\ w & : 100m,\ 1000m,\ 10000m \end{cases}$

→ Flow rate: $1m \cdot 1\frac{m}{s} \cdot 1000m = \dfrac{1000\,m^3}{s}$

b) If Niagara Falls are shutoff, water level at Lake Erie will rise (Why?) How long does it take for it to rise 1m?

Niagara Falls serve as an escape for additional water (rains) in Lake Erie. $\Longleftrightarrow$ $\boxed{\text{Flow rate over Falls = water increase rate in Lake Erie}}$ etc.



$\Rightarrow$ How long does it take (t) to accumulate $V = 375\cdot 10^3 \cdot 75 \cdot 10^3 \cdot 1\ m^3$

$$t = \frac{\text{Vol } V}{\text{Flow rate}} = \frac{375\cdot 75\cdot 10^{\cancel{6}\,3}\ \cancel{m^3}}{10^{\cancel{3}}\ \frac{\cancel{m^3}}{s}} = 28125\times 10^3\ \cancel{s}\cdot\frac{1\ \text{day}}{86400\ \cancel{s}} = 326\ \text{days}$$

Flow rate	t
$10^3\ \frac{m^3}{s}$	326 days ~ 1 year
$10^4\ \frac{m^3}{s}$	32.6 days ~ 1 month

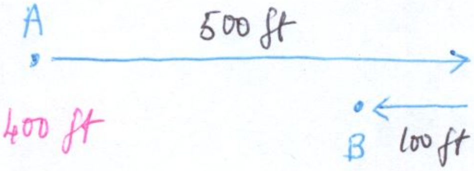
Ch 2 Motion in a Straight Line

either horizontal or vertical

- Average & instantaneous motion
- Speed & velocity

Speed $= \dfrac{\text{distance}}{\text{time}}$ velocity $= \dfrac{\text{displacement}}{\text{time}}$

- distance & displacement:



distance → 600 ft

displacement → 400 ft

- If we travel Boston – NYC – Boston: distance ~ 500 mi; displacent ~ 0 mi

- Between A & B, $t_{AB} = 6\,min$:
  - speed $= 100\ ft/min$
  - velocity $= \dfrac{400\ ft}{6\ min} = 66.67\ \dfrac{ft}{min}$

- Velocity takes consideration of direction (more general)

→ **Average velocity**: $\boxed{\bar{v} = \dfrac{\Delta x}{\Delta t}}$ $\left(\dfrac{m}{s}\right)$

Δx: change of position or displacement

Δt: change of time or time

→ **Instantaneous velocity**: $\boxed{v} = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \boxed{\dfrac{dx}{dt}}$ $\left(\dfrac{m}{s}\right)$ "time derivative of position x"

Calculus

e.g.: $x = a \cdot t^3 \longrightarrow v = \dfrac{dx}{dt} = 3a\,t^2$

$\left(\dfrac{dt^n}{dt} = n\,t^{n-1}\right)$

Acceleration: change of velocity over time

Average acceleration: $\boxed{\bar{a} = \dfrac{\Delta v}{\Delta t}}$ $\left\{ \begin{array}{l} \Delta v = \text{change of velocity} \\ \Delta t = \text{change of time} \end{array} \right.$

$\left(\frac{m}{s^2}\right)$

Instantaneous acceleration: $\boxed{a = \lim_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}}$ "time derivative of velocity v"

$\left(\frac{m}{s^2}\right)$ calculus

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

e.g.: $x = bt^3 \rightarrow v = 3bt^2 \rightarrow a = 6bt$ (acceleration increases linearly with time)

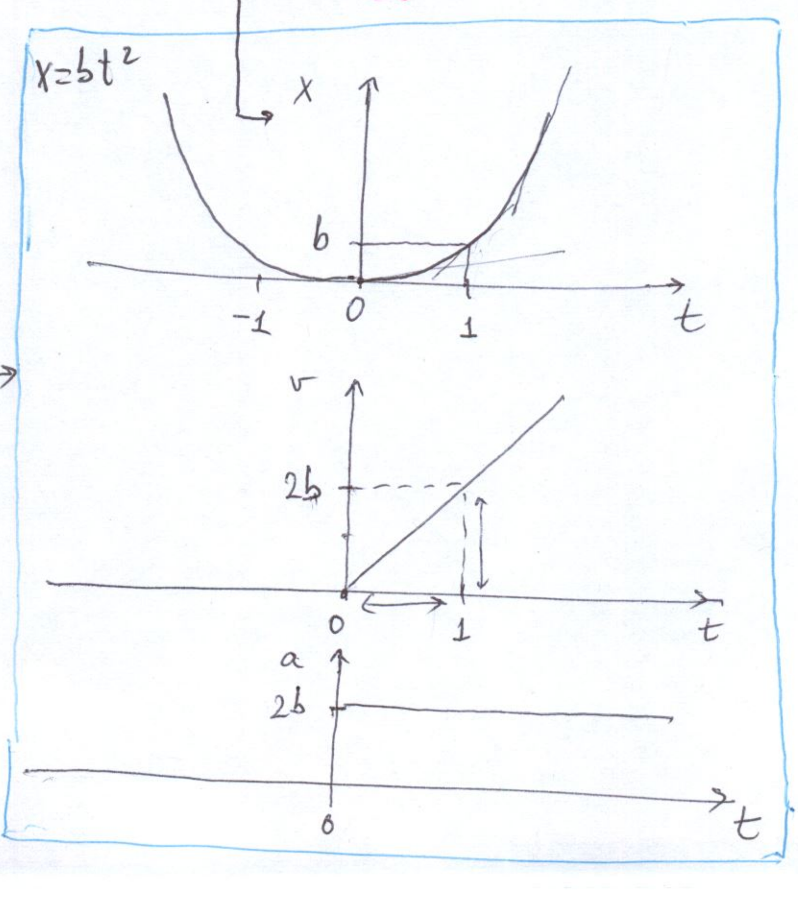
gravity $\rightarrow$ constant acceleration $a = g = 9.81 \frac{m}{s^2}$

Which $x = f(t)$ describes gravity-based motion?

$\hookrightarrow \boxed{x = bt^2} \rightarrow v = 2bt \rightarrow a = 2b$ constant

Gravity-based motion

constant acceleration



From the previous intuitive definitions of $\bar{v}, v, \bar{a}, a$ we will derive kinematic equations for a constant acceleration in 1D (straight line)

Constant acceleration: $\bar{a} = a$

current velocity   initial velocity

$$a = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} \Rightarrow v - v_0 = a \cdot t$$

current time   initial time

$$\rightarrow \boxed{v = v_0 + a \cdot t} \quad (1) \qquad \text{Kinematic eq. \#1}$$

Kinematic eq. #2:

current position   initial position

$$\begin{cases} \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x - x_0}{t - 0} \longrightarrow x = x_0 + \bar{v} \cdot t \quad (A). \\ \bar{v} = \dfrac{\int_0^t dt\, v}{t - 0} \overset{(1)}{=} \dfrac{1}{t} \displaystyle\int_0^t dt\,(v_0 + a \cdot t) \end{cases}$$

mathematical average

$$= \frac{1}{t}\left[v_0 t + \frac{1}{2} a \cdot t^2\right]_0^t = \frac{1}{t}\left[v_0 t + \frac{1}{2} a t^2\right]$$

$$= v_0 + \frac{1}{2} a \cdot t$$

Math review

$$\begin{cases} \int v_0\, dt = v_0 \underbrace{\int dt}_{t} = v_0 \cdot t \\ \int t\, dt = \frac{1}{2} t^2 \\ \int t^n dt = \dfrac{t^{n+1}}{n+1} \end{cases}$$

$$\bar{v} = v_0 + \frac{1}{2} a \cdot t = \frac{1}{2} v_0 + \underbrace{\frac{1}{2} v_0 + \frac{1}{2} a \cdot t}_{\frac{1}{2}(v_0 + a \cdot t)}$$

$$\downarrow$$

$$\bar{v} = \frac{1}{2}(v_0 + v) \quad (B)$$

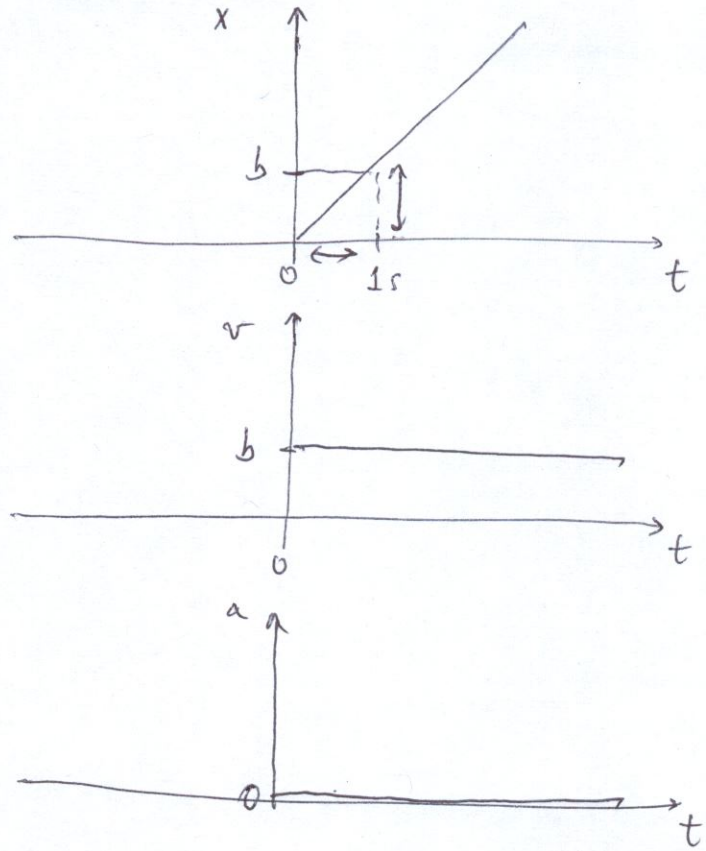
$$(A) = \; x = x_0 + \bar{v} \cdot t = x_0 + \frac{1}{2}(v_0 + v) \cdot t = x_0 + \frac{1}{2}(v_0 + v_0 + a \cdot t) \cdot t$$

$$(B) = \; \bar{v} = \frac{1}{2}(v_0 + v) \nearrow \qquad \qquad \overset{\uparrow}{(1)}$$

If $x = bt \rightarrow \boxed{v = b} \rightarrow a = 0$

↓

<u>Uniform motion</u>



$$\boxed{x = x_0 + v_0 \cdot t + \frac{1}{2} a \cdot t^2} \quad (2) \quad \text{Kinematic eq. \#2}$$

Summary: to describe a $\boxed{\text{constant acceleration}}$ motion in 1D:

1) $v = v_0 + a \cdot t$

2) $x = x_0 + v_0 \cdot t + \frac{1}{2} a \cdot t^2$

3) Eliminating time from 1) & 2): $\frac{v^2 - v_0^2}{x - x_0} = 2a$

- When time is not given or asked for → start with Kinematic Eq. #3. Otherwise use 1) or 2) or both!

x_0 : initial position (m); x : current position (m)
v_0 : " velocity (m/s); v : " velocity
a = constant acceleration $\left(\frac{m}{s^2}\right)$; t: current time (s); $t_0 = 0$ initial time
$\bar{a} = a$

Example: 2.33

1) After reading it carefully, write down information

$v_0 = 50 \frac{mi}{h}$

"Begins slowing down @ constant rate 100 ft short of a stop light"

$\rightarrow \begin{cases} x - x_0 = 100\,ft \\ a = \text{constant } (a?) \end{cases}$

"Car comes to a stop just @ light" $v = 0$

2) Time is not involved ↔ eq#3 $\frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a$

3) Units are in S.I. $\begin{cases} v_0 = 50 \frac{\cancel{mi}}{\cancel{h}} \cdot \frac{1609\,m}{\cancel{mi}} \cdot \frac{\cancel{1h}}{3600\,s} = 22.35 \frac{m}{s} \\ x - x_0 = 100\,\cancel{ft} \cdot \frac{0.3048\,m}{1\,\cancel{ft}} = 30.48\,m \end{cases}$

$$a = \frac{1}{2}\,\frac{0 - 22.35^2}{30.48} = -8.192\ \frac{m}{s^2}$$

Negative acceleration: makes sense since $\begin{cases} v_0 = 22.35\ \frac{m}{s} \\ v = 0 \end{cases}$

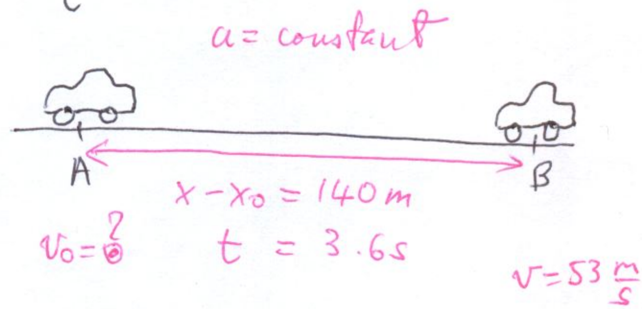
↳ or deceleration

2.59

1) Read carefully, then write down information.

1st sentence $\begin{cases} a = \text{constant} \\ x - x_0 = 140\,m \\ t = 3.6\,s \end{cases}$ 2nd sentence $\begin{cases} v = 53\ \frac{m}{s} \\ v_0\ ? \end{cases}$

Recommend: a diagram



2) Selecting equation to start:

Alternative #1

Eq#2: $x - x_0 = v_0 \cdot t + \frac{1}{2} a \cdot t^2$

Eliminate v_0 using eq#1: $v_0 = v - a \cdot t$

$$x - x_0 = (v - a \cdot t) \cdot t + \frac{1}{2} a \cdot t^2 = v \cdot t - \frac{1}{2} a \cdot t^2$$

↳ Solve for a: $a = [v \cdot t - (x - x_0)] \cdot \frac{2}{t^2}$

3) $$\boxed{a = [53 \cdot 3.6 - 140] \cdot \frac{2}{3.6^2} = 7.83\ \frac{m}{s^2}}$$

Now use eq#1 to find $\boxed{v_0 = v - a \cdot t = 53 - 7.83 \cdot 3.6 = 24.8\ \frac{m}{s}}$

Alternative #2

Start with Eq#1 : $v = v_0 + a\cdot t \rightarrow a = \frac{v-v_0}{t}$

→ Use Eq#2 : $x - x_0 = v_0 t + \frac{1}{2} a\cdot t^2 = v_0\cdot t + \frac{1}{2}\frac{v-v_0}{\cancel{t}}\cdot t^{\cancel{2}}$

$$= v_0\cdot t + \frac{1}{2}(v-v_0)\cdot t$$

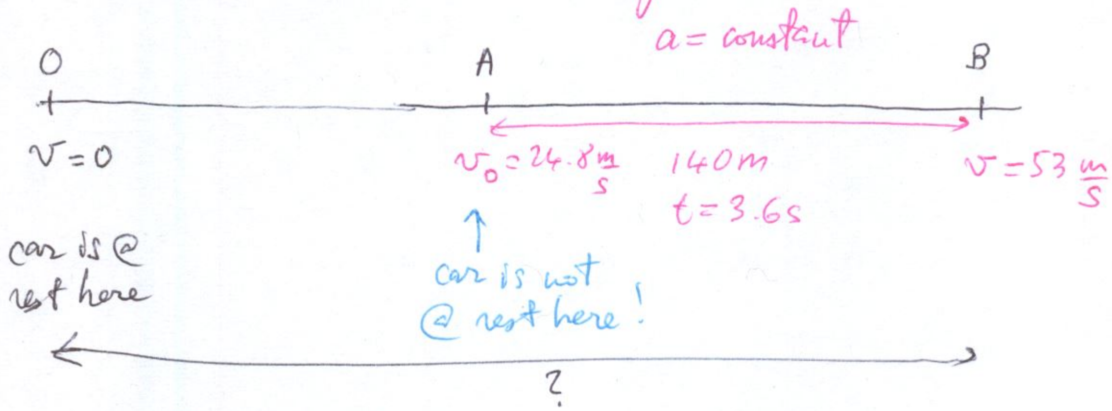
$$= \boxed{\frac{(v+v_0)}{2}\cdot t}$$

$x - x_0 = (v+v_0)\frac{t}{2}$ → Solve for v_0 :

3) $v_0 = (x-x_0)\frac{2}{t} - v = 140\cdot\frac{2}{3.6} - 53 = \boxed{24.8\ \frac{m}{s}}$

b) How far did it travel from rest to end of 140 m distance?

↳ Make sense with a diagram



Continuing with Alternative #1 : OB = OA + 140 m

Find OA , $a = 7.83 \frac{m}{s^2}$, should be same from O to A to B

→ Eq#3 : $\frac{v_A^2 - v_0^2}{(x-x_0)_{OA}} = 2\cdot a \Rightarrow (x-x_0)_{OA} = \frac{v_A^2 - v_0^2}{2\cdot a} = \frac{24.8^2 - 0}{2\cdot 7.83}$

$= 39.4\,m \Rightarrow \boxed{OB = 179.4\,m}$

Continuing with alternative #2:

Calculate $a = \frac{v-v_0}{t} = \frac{53-24.8}{3.6} = 7.83\ \frac{m}{s^2}$

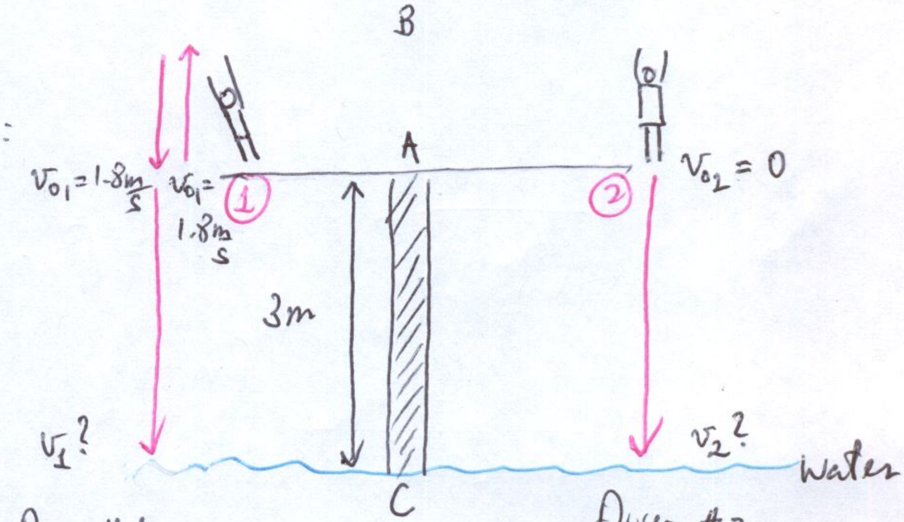
Find OB : Eq#3 :

Find OB: $\frac{v_B^2 - v_0^2}{(x-x_0)_{0B}} = 2 \cdot a \rightarrow (x-x_0)_{0B} = \frac{v_B^2 - v_0^2}{2 \cdot a} = \frac{53^2 - 0}{2 \cdot 7.83}$

$$\boxed{OB = 179.4\,m}$$

2.69

1) Given information:



Diver #1

A → B → A → C

Diver #2

- A → C
- Diver #2 steps off when #1 comes back down by A

questions: (a) v_1 & v_2
(b) which diver enters water first & by how much time?

2) Equation to solve for v_1 & v_2:

→ Predictions or quick answer
i) diver #1 will enter water first since he has an initial downward velocity @ A.
ii) $v = v_0 + a \cdot t = v_0 + g \cdot t$
same $a = g$ for both → $v_1 > v_2$

→ Constant acceleration: no time info or request in a) ⇒ Eq #3

Diver #1

$$\frac{v_1^2 - v_{0_1}^2}{x - x_0} = 2 \cdot g$$

Diver #2

$$\frac{v_2^2 - 0}{x - x_0} = 2 \cdot g$$

3) Numeric solution with correct units: $\begin{cases} x - x_0 = 3m \\ v_{0_1} = 1.8\frac{m}{s} \end{cases}$

$$v_1 = \sqrt{2g(x - x_0) + v_{0_1}^2} \qquad v_2 = \sqrt{2g(x - x_0)}$$

$$v_1 = \sqrt{2 \cdot 9.81 \cdot 3 + 1.8^2} = 7.88\frac{m}{s} > v_2 = \sqrt{2 \cdot 9.81 \cdot 3} = 7.67\frac{m}{s}$$

b) To find time : Eq#1 : $v = v_0 + a \cdot t$

Diver #1

$$t_1 = \frac{v_1 - v_{01}}{g}$$

$$t_1 = \frac{7.88 - 1.8}{9.81} = 0.62s$$

Diver # 2

$$t_2 = \frac{v_2}{g}$$

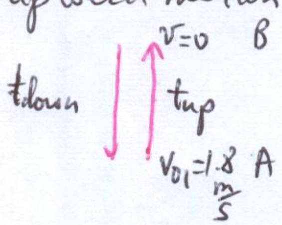
$$t_2 = \frac{7.67}{9.81} = 0.78s$$

$\rightarrow \boxed{t_2 - t_1 = 0.78 - 0.62 = 0.16s}$

Just for fun : $\rightarrow$ If we include time diver #1 takes to go up then down $(A \rightarrow B \rightarrow A)$:

Eq#1 : $v = v_0 + a \cdot t$

upward motion : $\left\{ \begin{array}{l} v = 0 \\ v_{01} = 1.8 \frac{m}{s} \\ a = -g \end{array} \right\}$ $0 = 1.8 - 9.81 \cdot t_{up} \rightarrow t_{up} = \frac{1.8}{9.81} = 0.183s$



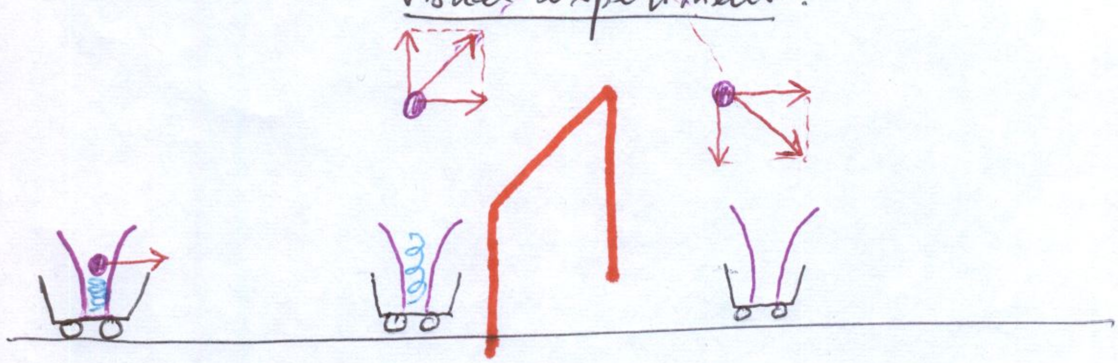
$A \rightarrow B \rightarrow A \rightarrow 2 \cdot t_{up} = 0.366 s$

Ch3 Motion in Two & Three Dimensions

Note: Ch2 : 1D { horizontal : motion of a car
or
vertical : divers }

Ch3 : 2D : horizontal and vertical simultaneous motion

Need a very important assumption
↓
Visual experiment :



1) Negligible friction b/w wheels & track
→ $\boxed{\vec{v}_{cart} = \text{constant \& horizontal}}$

Direction is important in 2D & 3D
→ Use vectors (velocity vector, position vector, acceleration vector)

2) Negligible air resistance (smaller front surface)

3) In cart : a funnel with a compressed spring, a ball sitting on top of it.

4) Gate about middle of long track with a sensor, when cart approaches gate it will release the spring, launching ball $\boxed{\text{vertically upward}}$

5) Before spring is released $\boxed{\text{ball}}$ travels with cart in uniform horizontal motion. After spring is released $\boxed{\text{ball}}$

acquires an additional constant-acceleration vertical motion in addition to the original uniform horizontal motion!

So after being launched upward by the spring, ball has simultaneous horizontal & vertical motion resulting in a parabola.

6) After gate will ball goes $\left\{\begin{array}{l}\text{Behind cart} \\ \boxed{\text{Into cart}} \\ \text{Beyond cart}\end{array}\right.$

7) <u>Big conclusion for this to happen</u>:

The vertical launch (force) by spring didnot affect the ball's horizonal uniform motion. (ball was always above cart traveling @ same uniform horizontal velocity!)

→ Motion along perpendicular directions are independent!

Mathematical descriptions of motion in 2D & 3D

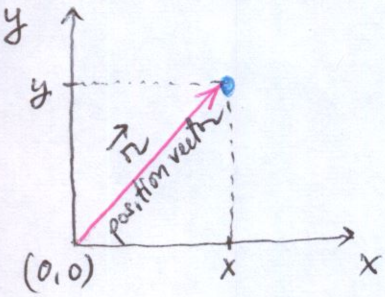
↳ Add & Subtract vectors $\left\{\begin{array}{l}\text{1) Graphically} \\ \text{2) Mathematically using unit vectors}\end{array}\right.$

↓

	<u>1D</u>	<u>2D</u>	<u>3D</u>
			Theta phi
position	x	$\vec{r} = (x, y) = (r, \theta)$	$\vec{r} = (x, y, z) = (r, \theta, \varphi)$
velocity	v	$\vec{v} = (v_x, v_y) = (v, \theta_v)$	$\vec{v} = (v_x, v_y, v_z) = (v, \theta_v, \varphi_v)$
acceleration	a	$\vec{a} = \underbrace{(a_x, a_y)}_{\text{Cartesian coordinates}} = \underbrace{(a, \theta_a)}_{\text{Polar coordinates}}$	$\vec{a} = \underbrace{(a_x, a_y, a_z)}_{\text{Cartesian}} = \underbrace{(a, \theta_a, \varphi_a)}_{\text{Spherical}}$

2D

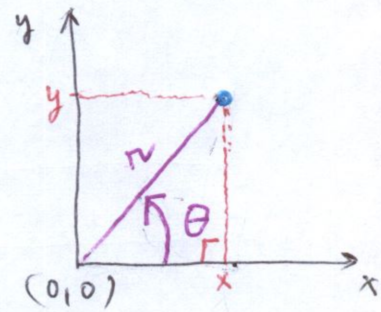
Cartesian



- Position vector $\vec{r}$ ~~has~~ starts @ origin (0,0), ends @ object position
- Projection of $\vec{r}$ onto the x-axis is the x-component; onto the y-axis is y-component

$\vec{r} = (x, y)$

Polar



- We can also locate the same object by knowing its radius r from the origin (0,0) and its angle θ from the x-axis (CCW)

Cartesian (x, y) $\longrightarrow$ Polar (r, θ)

$$\begin{cases} r = \sqrt{x^2 + y^2} & \text{Pythagorean Theorem} \\ \theta = \tan^{-1}\left(\dfrac{y}{x}\right) & \text{Trigonometry} \end{cases}$$

Polar (r, θ) $\longrightarrow$ Cartesian (x, y)

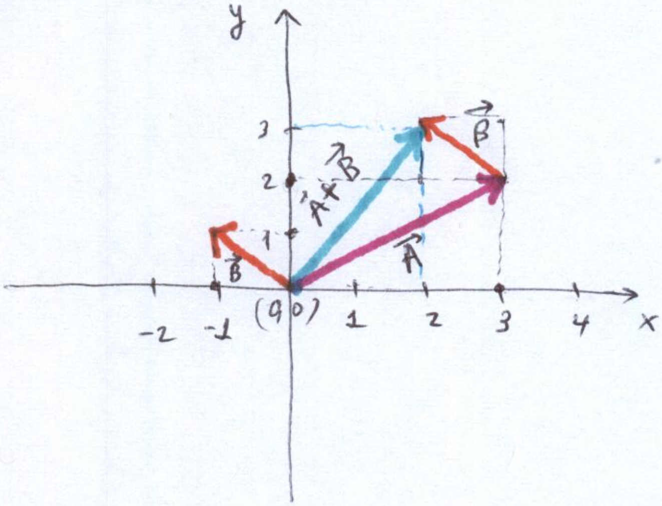
$$\left.\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}\right\} \text{Trigonometry}$$

Add & Subtract vectors

$$\begin{cases} \vec{v} = \vec{v}_0 + \vec{a} \cdot t \\ \vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \vec{a} \cdot t^2 \end{cases}$$

1) Graphical addition & subtraction — two vectors $\begin{cases} \vec{A} = (3,2) \\ \vec{B} = (-1,1) \end{cases}$

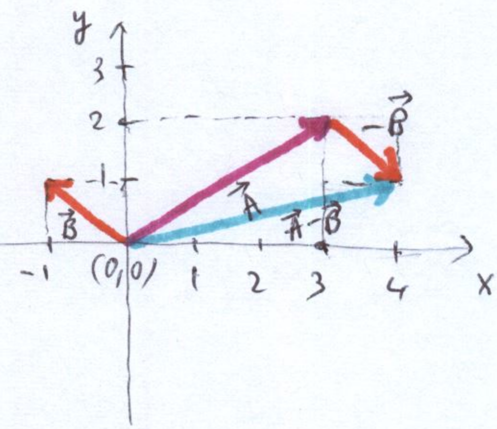
$\vec{A}+\vec{B}$? $\vec{A}-\vec{B}$?



$\vec{A}+\vec{B}$: i) Draw a copy of $\vec{B}$ starting @ tip of $\vec{A}$

ii) $\vec{A}+\vec{B}$ starts from origin of $\vec{A}$ to the tip of the copy of $\vec{B}$

$\vec{A}+\vec{B} = (2,3)$



$\vec{A}-\vec{B}$: i) Draw a copy of $-\vec{B}$ starting @ tip of $\vec{A}$

ii) $\vec{A}-\vec{B}$ from origin of $\vec{A}$ to tip of copy of $-\vec{B}$

$\vec{A}-\vec{B} = (4,1)$

2) Mathematical addition & subtraction using $\boxed{\text{unit vectors}}$ (3D, or other complex situations)

Unit vectors: $\begin{cases} \text{length or magnitude is } 1 \\ \text{direction} : \begin{cases} x \to \hat{i} \quad (\text{"i hat"}) \\ y \to \hat{j} \quad (\text{"j hat"}) \end{cases} \\ \qquad z \to \hat{k} \quad (\text{"k hat"}) \end{cases}$ $\hat{r}$, $\hat{\theta}$, $\hat{\varphi}$

Cartesian unit vectors — Fixed or constant in time

Spherical unit vectors — Not fixed nor constant in time (they can change direction)

Using Cartesian unit vectors:

$\vec{A} = (3,2) = 3\hat{\imath} + 2\hat{\jmath}$ → In general $\vec{A} = (A_x, A_y) = A_x\hat{\imath} + A_y\hat{\jmath}$

$\vec{B} = (-1,1) = -\hat{\imath} + \hat{\jmath}$ → In general $\vec{B} = (B_x, B_y) = B_x\hat{\imath} + B_y\hat{\jmath}$

Addition: $\vec{A} + \vec{B} = 3\hat{\imath} + 2\hat{\jmath} - \hat{\imath} + \hat{\jmath} = 2\hat{\imath} + 3\hat{\jmath} = (2,3)$

Subtraction $\vec{A} - \vec{B} = 3\hat{\imath} + 2\hat{\jmath} - (-\hat{\imath} + \hat{\jmath}) = 4\hat{\imath} + 1\hat{\jmath} = (4,1)$

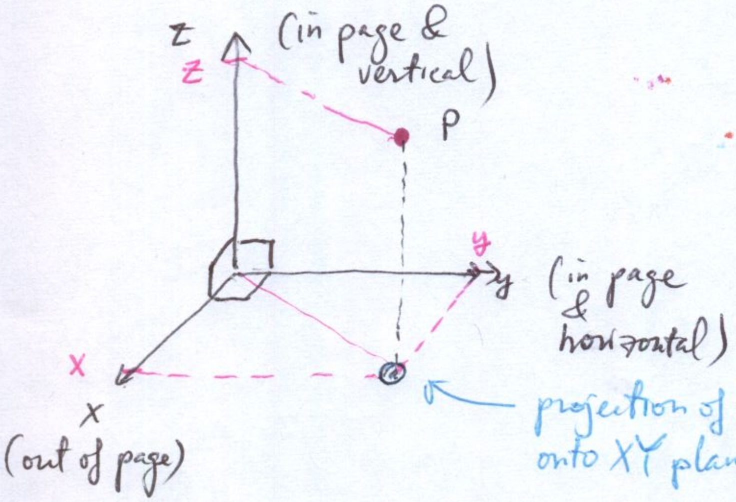
3D

Cartesian

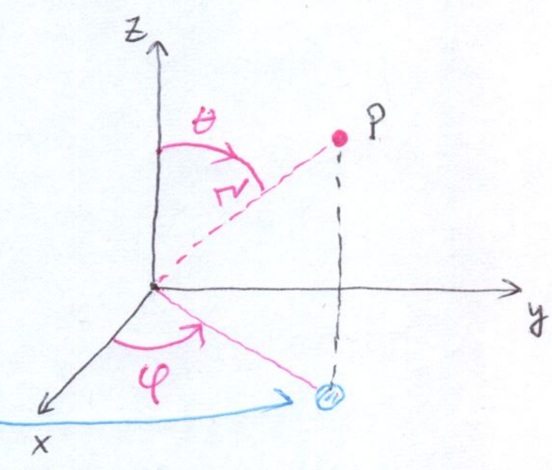
$\vec{r} = (x, y, z)$

Spherical

$\vec{r} = (r, \theta, \varphi)$



projection of P onto XY plane



Note: all 3 axes are perpendicular to each other