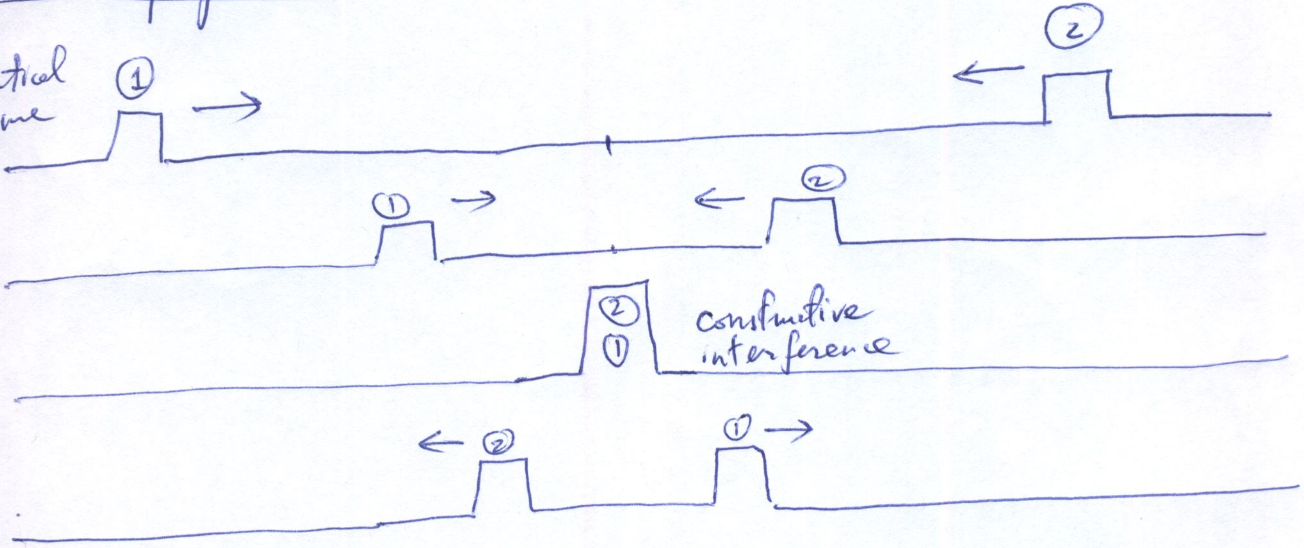
Wave Superposition →
1) Beats : tuning of string instruments; engines etc.
2) Standing waves : pipes, flutes, etc.
3) Wave interference : { constructive
destructive

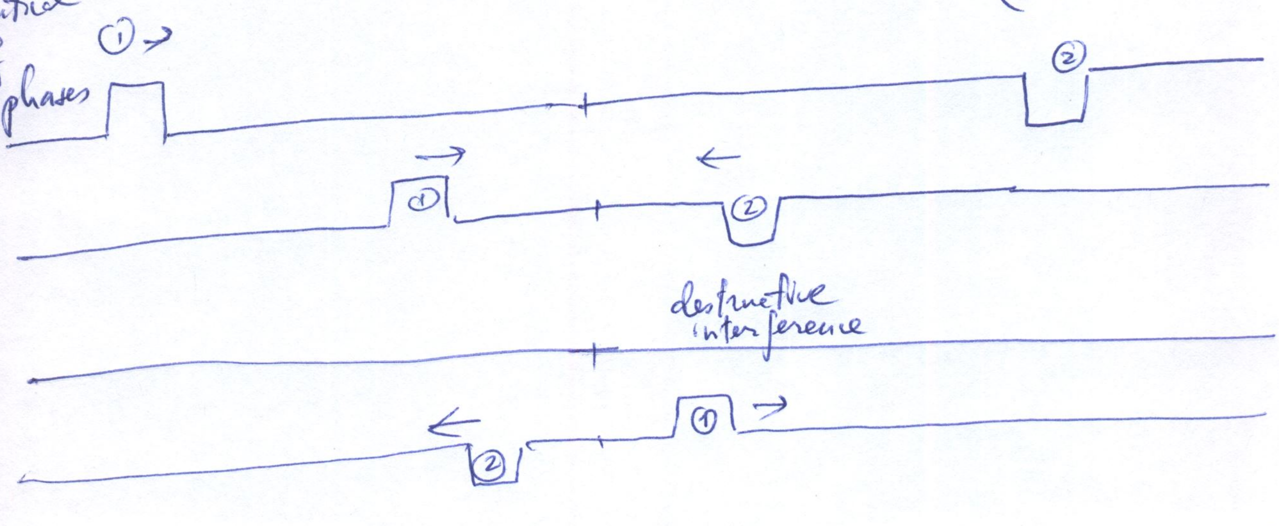
Doppler effect : speed measurement

Wave superposition:

→ Two identical waves same phase



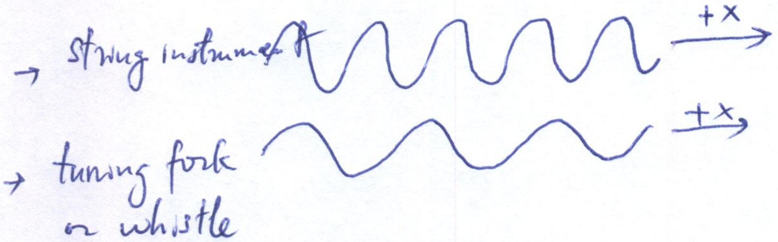
→ Two identical waves of opposite phases



* Energy is conserved,
where does it go ~~when~~ at the instant waves ① & ② cancel each other?
(To the medium)

1) Math description of wave superposition: → Beat Phenomenon

Two transverse waves with the same amplitude but different frequencies ω_1 & ω_2 traveling ~~toward each other~~ in the same direction:



$$y_1(x,t) = A\sin(k_1 x - \omega_1 t)$$
$$y_2(x,t) = A\sin(k_2 x - \omega_2 t)$$

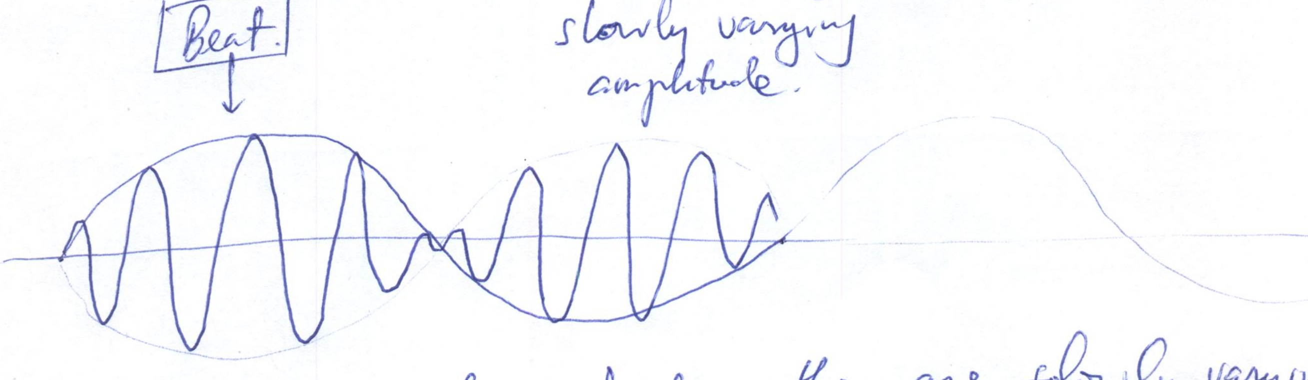
Superposition of these two waves @ a fixed location $x=0$ (our ear):

$$y(0,t) = y_1(0,t) + y_2(0,t)$$
$$= -A[\sin(\omega_1 t) + \sin(\omega_2 t)]$$

$$\boxed{\text{Trigonometry:} \quad \sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$$= -2A\left[\underbrace{\sin\left(\frac{\omega_1-\omega_2}{2}\,t\right)}_{\text{Modulated amplitude}} \underset{\substack{\downarrow \\ \text{average of the} \\ \text{two frequencies}}}{\sin\left(\frac{\omega_1+\omega_2}{2}\,t\right)}\right]$$

↓
small frequency
(if $\omega_1 \sim \omega_2$)
slowly varying
amplitude.

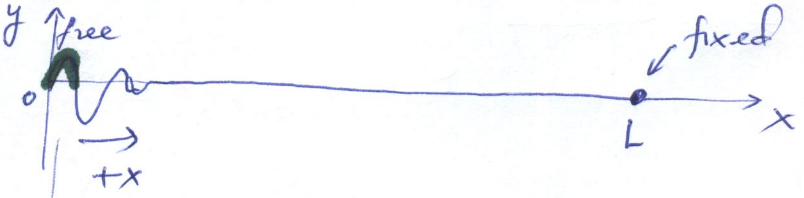


Beats can be easily noticed as they are slowly varying.

2) Wave superposition: a wave in +x & its reflection in −x

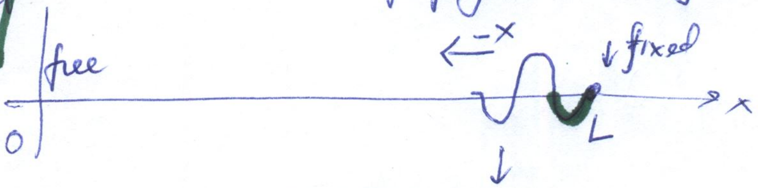
→ Standing Waves

pipes, flutes, string with one fixed end (zero oscillation there)



original wave $\left\{ y_1(x,t) = A\cos(kx \ominus \omega t) \right.$ ↓ propagation along +x

or incident wave



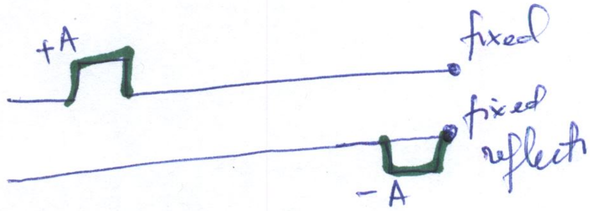
↓

reflected wave
$$\left\{ \begin{aligned} y_2(x,t) &= A\cos(-kx-\omega t) \\ &= A\cos(kx \oplus \omega t) \end{aligned} \right.$$

↓ propagation along −x

When both y_1 & y_2 are present in the string:

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\cos(kx \ominus \omega t) - A\cos(kx \oplus \omega t)$$



reflection = inverted amplitude

$$\boxed{\text{Trigonometry:} \quad \cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$$y(x,t) = -2A\sin(kx)\cdot\sin(-\omega t) = 2A\sin kx\sin\omega t$$

superposition of incident + reflected waves

Fixed end @ $x=L$ ⇒ $y(L,t) = \boxed{0 = 2A\underbrace{\sin kL}_{0}\sin\omega t}$

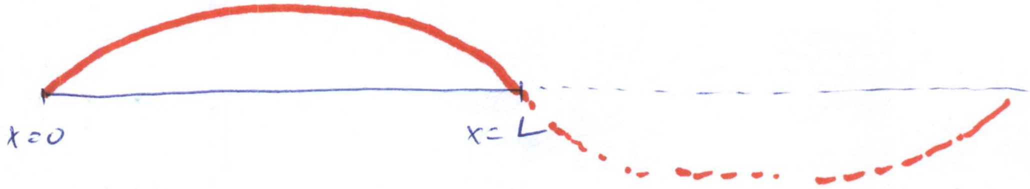
$\sin(kL)=0 \Rightarrow \boxed{kL = n\pi \quad (n=1,2,3,\dots)}$ Standing Waves

$$k\cdot L = n\pi \quad (n=1,2,3,\text{etc.})$$

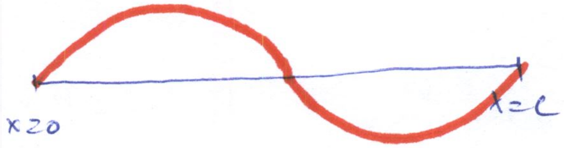
$$\frac{2\cancel{\pi}}{\lambda}\cdot L = n\cancel{\pi} \Rightarrow \lambda = \frac{2L}{n} \Rightarrow \lambda_n = \frac{2L}{n} = \left\{ \overset{\overset{\lambda_1}{\downarrow}}{2L}, \overset{\overset{\lambda_2}{\downarrow}}{L}, \overset{\overset{\lambda_3}{\downarrow}}{\frac{2L}{3}}, \overset{\overset{\lambda_4}{\downarrow}}{\frac{L}{2}}, \text{etc...} \right\}$$

String with fixed end @ $x=L$

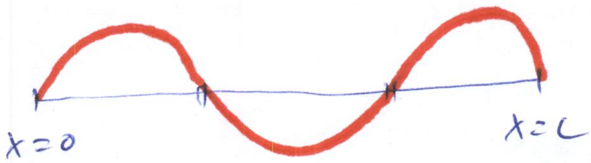
$\lambda_1 = 2L$:



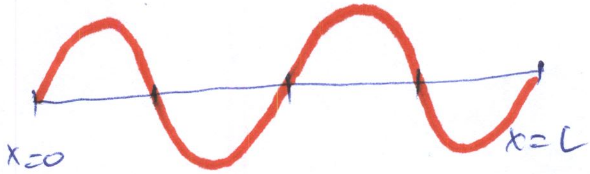
$\lambda_2 = L$



$\lambda_3 = \frac{2L}{3}$

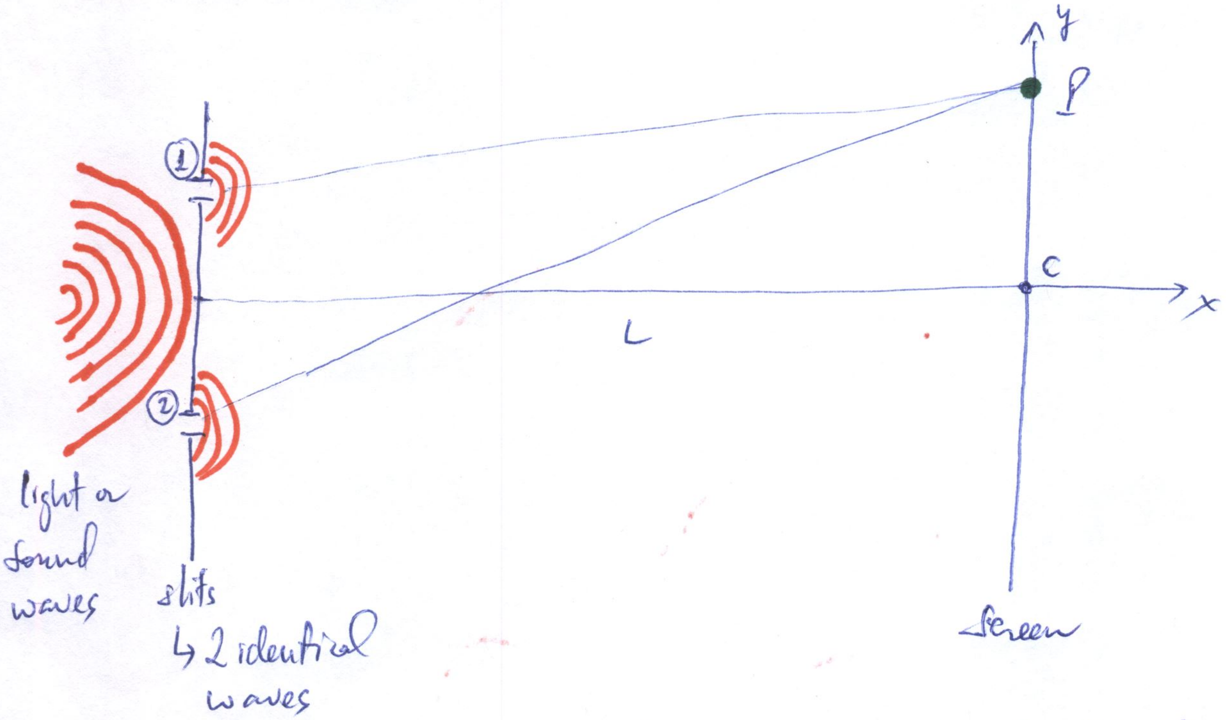


$\lambda_4 = \frac{L}{2}$



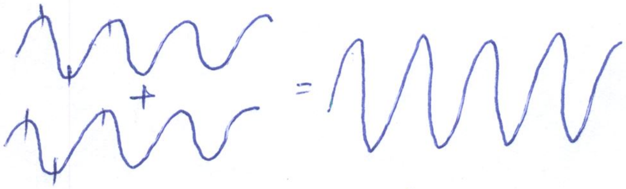
As λ decreases, the # oscillations in the standing waves increases

3) Superposition of two identical waves traveling different paths then arriving at a same point in space → Wave interference



Waves ① & ② are identical at the slits, travel different paths to P
⟹ arrive at P with different phases (they will arrive @ C with same phases!)

→ @ C

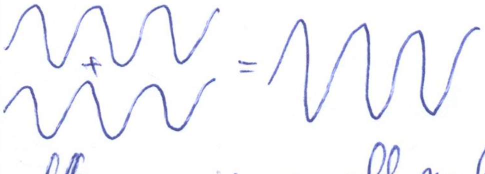


constructive interference

→ @ P

a) Phase difference is a multiple of the wavelength λ:

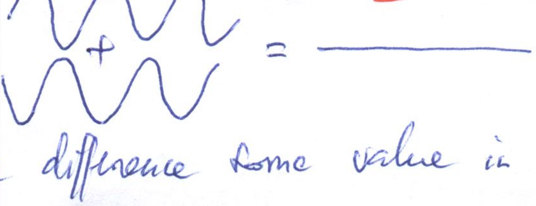
$$\boxed{\Delta path = n\lambda} \quad (n = 1, 2, 3, \text{etc} \ldots)$$



constructive interference

b) Phase difference is an odd multiple of half wavelength $\frac{\lambda}{2}$

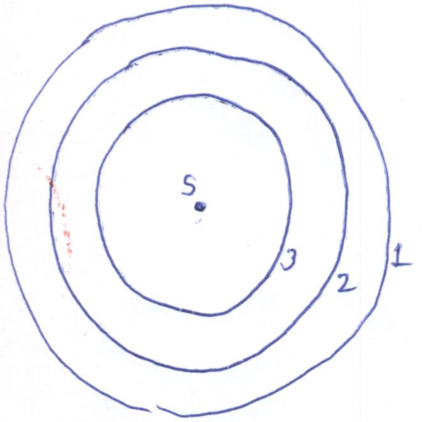
$$\boxed{\Delta path = (2n+1)\frac{\lambda}{2}} \quad (n = 0, 1, 2, 3, \text{etc.})$$

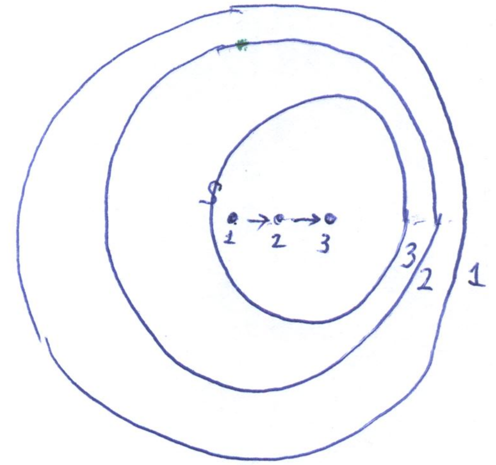


destructive interference

c) Phase difference some value in between.

Doppler Effect: due to a moving source of wave





Source is @ rest
emitting waves of fronts 1, 2, 3

Source is moving to the right
↳ Wavefront ① centered at initial position
↳ Wavefront ② is centered at source position 2
↳ Wavefront ③ centered at source position 3

Doppler effect
↓
Wave is { compressed in front of it (shorter λ)
spaced out behind it (longer λ)

Source is approaching
$$\lambda' = \underset{\substack{\downarrow \\ \text{original wavelength}}}{\lambda} - \underset{\substack{\downarrow \\ \text{source speed}}}{u}\,T \rightarrow \text{wave period}$$

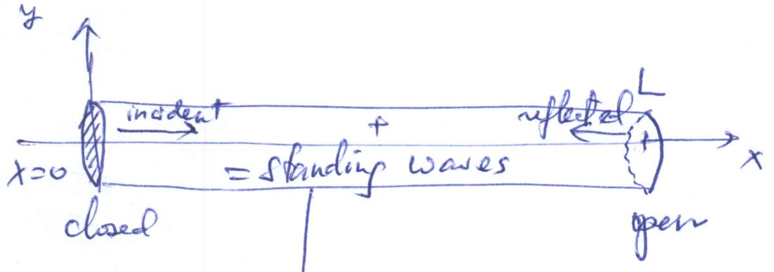
$$f' = \frac{f}{1 - \frac{u}{\boxed{v}}} \rightarrow \text{wave speed } v = \frac{\lambda}{T}$$

Receding source
$$\lambda' = \lambda + uT$$

$$f' = \frac{f}{1 + \frac{u}{v}}$$

14.76

Pipe $L = 1.5\,m$ one end open (oscillation at max amplitude there)



$\hookrightarrow$ $\left.\begin{array}{l} f_n = 225\,Hz \\ f_{n+1} = 375\,Hz \end{array}\right\} \to f_0$ or fundamental frequency

Similar equation as for string with fixed end but condition at $x = L$ is max!

$$y(x,t) = 2A \sin kx \sin \omega t \xrightarrow{x=L} 2A \underbrace{\sin kL}_{1} \sin \omega t = \max \pm 2A$$

$$\Rightarrow \quad \sin kL = \pm 1 \quad \Rightarrow \quad kL = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \text{etc..}$$

$$\boxed{kL = (2n+1)\frac{\pi}{2} \quad (n = 0, 1, 2, 3, \text{etc.})}$$

standing waves in pipe with one open end.

$$\frac{2\cancel{\pi}}{\lambda} L = (2n+1)\frac{\cancel{\pi}}{2} \quad (n = 0, 1, 2, 3, \text{etc.})$$

$$\boxed{\lambda_n = \frac{4L}{2n+1} \quad (n = 0, 1, 2, 3, \text{etc..})}$$

Frequencies: $v = \dfrac{\lambda}{T} = \lambda \cdot f \Rightarrow \boxed{f = \dfrac{v}{\lambda}} \to$ wave speed.

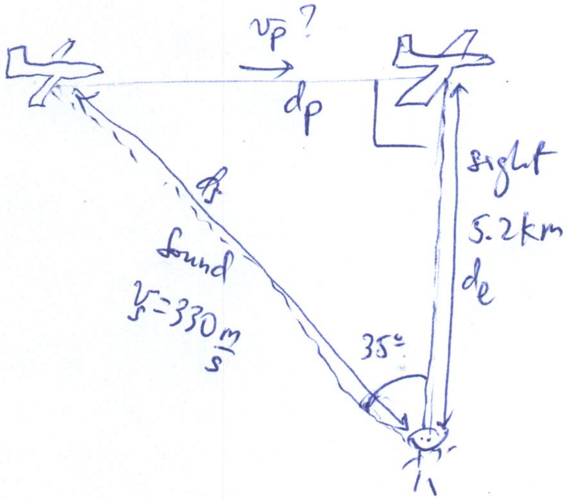
a) $$\frac{f_{n+1}}{f_n} = \frac{\frac{\cancel{v}}{\lambda_{n+1}}}{\frac{\cancel{v}}{\lambda_n}} = \frac{\lambda_n}{\lambda_{n+1}} = \frac{\frac{4L}{2n+1}}{\frac{\cancel{4L}}{2n+3}} = \boxed{\frac{2n+3}{2n+1} = \frac{375}{225} = \frac{15}{9} = \frac{5}{3}}$$

$$\frac{2n+3}{2n+1} = \frac{5}{3} \Rightarrow n = 1 \Rightarrow \left\{\begin{array}{l} f_1 = 225\,Hz \\ f_2 = 375\,Hz \end{array}\right\} f_0 \overset{?}{=}$$

(i) $$\left.\begin{array}{l} f_0 = \dfrac{v}{\lambda_0} = \dfrac{v}{4L} \\ f_1 = \dfrac{v}{\frac{4L}{3}} = 3\,\dfrac{v}{4L} = 3 f_0 \end{array}\right\} \Rightarrow \boxed{f_0 = \frac{f_1}{3} = \frac{225}{3} = 75\,Hz}$$

(ii) find $v = \dfrac{4L}{3} f_1 = \dfrac{4 \times 1.5}{3} \cdot 225 = 450\,\dfrac{m}{s} \to f_0 = \dfrac{v}{4L} = \dfrac{450}{4 \times 1.5} = 75\,Hz$.

14.63]



speed of airplane v_p ?

$$\left(\begin{aligned} \frac{d_s \cos 35^\circ}{\Delta t} &= \frac{d_\ell}{\Delta t} \\ v_s \cos 35^\circ &= v_\ell \end{aligned} \right)$$

$$\frac{d_s \sin 35^\circ}{\Delta t} = \frac{d_p}{\Delta t}$$

$$v_s \sin 35^\circ = v_p$$

$$\boxed{330 \sin 35^\circ = v_p} = 189 \frac{m}{s}$$

$$189 \frac{m}{s} \cdot \frac{1km}{1000} \cdot \frac{3600}{1h} = 189 \cdot 3.6 \frac{km}{h}$$

$$= \frac{189 \cdot 3.6}{1.609} \frac{mi}{h}$$

$$= 424 \frac{mi}{h}$$

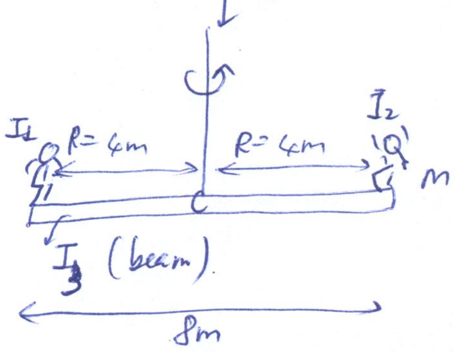
(iii) $f_0 \quad f_1 \quad f_2$ $\begin{cases} f_2 - f_1 = 375 - 225 = 150\,Hz \\ f_1 - f_0 = 150\,Hz \rightarrow f_0 = \cancel{150} f_1 - 150 \end{cases}$

$= \boxed{225 - 150 = 75\,Hz}$

13.59

Torsional pendulum: $\omega = \sqrt{\frac{K}{I}}$

↳ wrt axis of rotation



$\omega_0 = \sqrt{\frac{K}{I_3}}$
(no workers)

$\omega = \sqrt{\frac{K}{I_3 + 2mR^2}} = 0.8\,\omega_0$
(w/ workers)

↓ diminishes by 20 %

$I_1 = I_2 = mR^2$ ⌉

$I_3 = \frac{1}{12} ML^2$

$$\sqrt{\frac{\cancel{K}}{\frac{1}{12}ML^2 + 2mR^2}} = 0.8 \sqrt{\frac{\cancel{K}}{\frac{1}{12}ML^2}} \Rightarrow \frac{\frac{1}{12}ML^2}{\frac{1}{12}ML^2 + 2mR^2} = 0.8^2$$

$$\frac{1}{12}ML^2(1 - 0.8^2) = 2 \times 0.8^2 \cdot 75 \cdot 4^2$$

$$M = \frac{12 \times 2 \times 0.8^2 \times 75 \times 16}{8^2(1 - 0.8^2)}\,kg$$

13.60

2D SHM: $\begin{cases} x(t) = a \sin \omega t \\ y(t) = b \sin(\omega t + \frac{\pi}{2}) = b \cos \omega t \end{cases}$

↑ out of phase by $\frac{\pi}{2}$

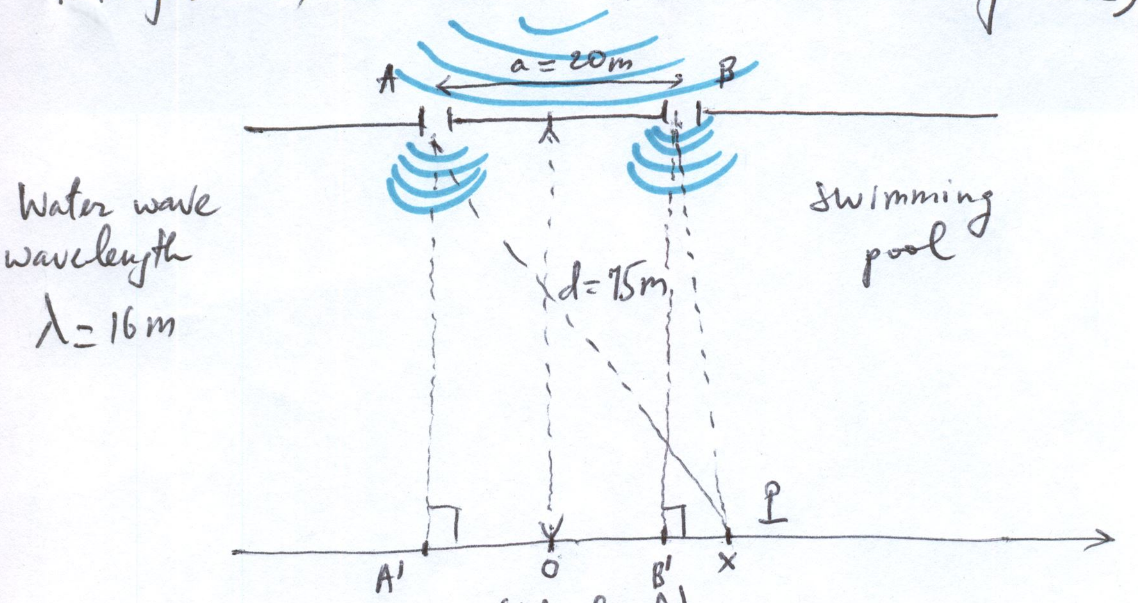
$\vec{r} = x\hat{i} + y\hat{j} \rightarrow r^2 = x^2 + y^2 = \cancel{a^2}$

$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sin^2 \omega t + \cos^2 \omega t = 1}$ equation of ellipse

semi major / minor axes a, b → amplitude of SHM in x, y, respectively

Wave Interference:

Swimming pool with two breakwater openings on one side (water waves), separated a distance $a = 20m$. We observe water wave interference the other side at a distance $d = 75m$. Find locations of maxima (constructive interference) or minima (destructive interference).



# of water waves: 2 (identical)

They combine at each point P along the other side, after traveling different paths AP & BP! If $\Delta path = AP - BP = n\lambda$ ($n = 1, 2, 3$, etc) @P → constructive interference (maxima). If $\Delta path = AP - BP = (2n+1)\frac{\lambda}{2}$ ($n = 0, 1, 2$, etc). @ P → destructive interference (minima)

Maxima: $\triangle AA'P$ is a right triangle: $AP = \sqrt{75^2 + (x+10)^2}$

$A'P = A'O + x = 10 + x$ $BP = \sqrt{75^2 + (x-10)^2}$

$\triangle BB'P$ is also a right triangle

$B'P = x - OB' = x - 10$

$$AP - BP = \sqrt{75^2 + (x+10)^2} - \sqrt{75^2 + (x-10)^2} = n\lambda$$

1st max: $n = 1$: $\sqrt{75^2 + (x+10)^2} - \sqrt{75^2 + (x-10)^2} = \lambda = 16$

2nd max: $n = 2$

etc..

Get rid of $\sqrt{}$: $\left(\sqrt{} - \sqrt{}\right)^2 = 16^2 = 256$

$$256 = 75^2 + (x+10)^2 + 75^2 + (x-10)^2 - 2\sqrt{75^2+(x+10)^2}\sqrt{75^2+(x-10)^2}$$

$$2\sqrt{75^2+(x+10)^2}\sqrt{75^2+(x-10)^2} = 11250 - 256 + 2x^2 + 200 = 11194 + 2x^2$$

$$\sqrt{75^2+(x+10)^2}\sqrt{75^2+(x-10)^2} = 5597 + x^2$$

$$\left[75^2+(x+10)^2\right]\left[75^2+(x-10)^2\right] = x^4 + 11194x + 5597^2$$

$$5625^2 + 5625\left[\underbrace{(x-10)^2 + (x+10)^2}_{2x^2+200}\right] + \underbrace{\left[(x+10)(x-10)\right]^2}_{(x^2-100)^2} = x^4 + 11194x + 5597^2$$

$$5625^2 + 11250x^2 + 11250\times100 + \cancel{x^4} + 10^4 - 200x^2 = \cancel{x^4} + 11194x + 5597^2$$

$$11050x^2 - 11194x + \underbrace{5625^2 - 5597^2 + 11250\times10^2 + 10^4}_{1449216} = 0$$

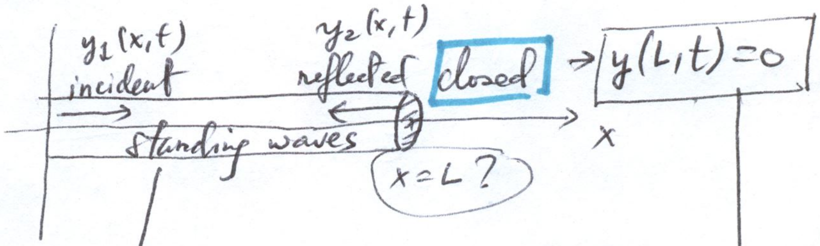
$$x = \frac{11194 \pm \sqrt{11194^2 - 4\times11050\times1449216}}{2\times11050}$$

$$\boxed{x = \pm 33\text{ m}}$$

First minima: $AP - BP = \frac{\lambda}{2}$

2nd minima: $AP - BP = \frac{3\lambda}{2}$

14.43 Vocal tract = pipe with a closed end $\begin{cases} f_0 = 620\,Hz \\ v = 354\,\frac{m}{s} \end{cases}$



$$y(x,t) = \underbrace{A\cos(kx-\omega t)}_{\substack{\text{incident} \\ y_1(x,t)}} - \underbrace{A\cos(kx+\omega t)}_{\substack{\text{reflected} \\ y_2(x,t)}} = 2A\sin kx \sin\omega t$$

$$y(L,t) = 2A \sin kL \sin \omega t = 0$$

$\hookrightarrow \sin(kL) = 0 \rightarrow kL = n\pi$ (n = 1,2,3, etc.)

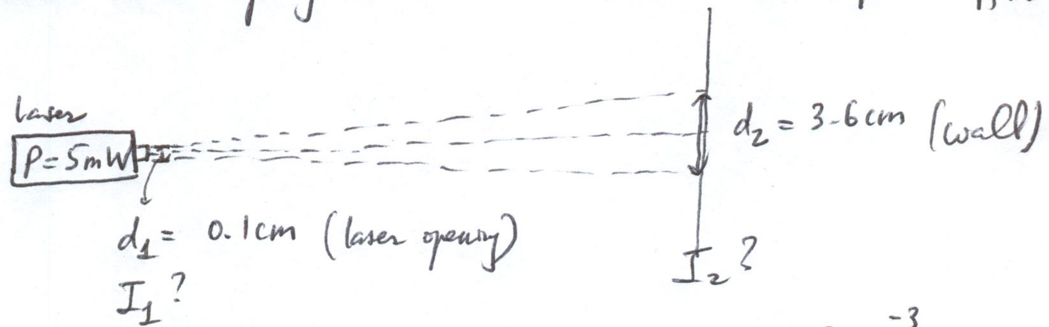
$$\frac{2\pi}{\lambda_n} L = n\pi \rightarrow \boxed{\lambda_n = \frac{2L}{n}} \quad (n = 1,2,3, \text{etc.})$$

$$v = \frac{\lambda}{T} = \lambda f \rightarrow \begin{cases} \text{lowest freq} = 620\,Hz = f \\ \text{longest wave length} = 2L = \lambda \end{cases}$$

$$v = 2L \cdot f.$$

$$354 = 2L \cdot 620 \rightarrow L = \frac{354}{2 \cdot 620}\,m = 0.29\,m$$

14.54 Laser beam projection: beam intensity $I = \frac{P}{Area}$ (power per unit area)



$$I_1 = \frac{5\times10^{-3}}{\pi\times(0.05\times10^{-2})^2} = 6.37\,\frac{kW}{m^2}$$

$$I_2 = \frac{5\times10^{-3}}{\pi\times(1.8\times10^{-2})^2} = 4.91\times10^{-3}\,\frac{kW}{m^2}$$

(more than 1000 times lower)

Ch15 Fluid Motion:

- Gas : density ρ (rho) can be variable (compressibl
- Liquid: density ρ is constant (incompressible)

Density ρ : mass per unit volume $= \frac{M}{Vol} \rightarrow \frac{dM}{dVol} \left(\frac{kg}{m^3}\right)$

$(\rho_{liquid} > \rho_{gas})$ $\rho_{H_2O} = 1000 \frac{kg}{m^3}$ (normal condition)

$\rho_{air} \simeq 1 \frac{kg}{m^3}$

Pressure P = normal force per unit area $= \frac{F}{Area} \rightarrow \frac{dF}{dArea} \left(\frac{N}{m^2}\right)$

$\frac{N}{m^2} = Pa$ (Pascal)

Atm (Atmosphere) → $1 Atm = 1.013 \times 10^5 Pa$

Fluid motion equations

1) Hydrostatic equilibrium: $\boxed{\frac{dP}{dh} = g\rho}$

$g = 9.81 \frac{m}{s^2}$; ρ (rho) density of fluid

P pressure; h = height.

→ Slope of $P(h)$ is a constant or P varies linearly with height → Pressure increases linearly with depth of fluid.

2) Conservation of mass: $v \cdot Area = constant$

(v: speed of fluid; Area: cross sectional area)

3) Conservation of energy: Bernoulli's eq.

$$\frac{1}{2}\rho v^2 + \rho g y + P = constant.$$

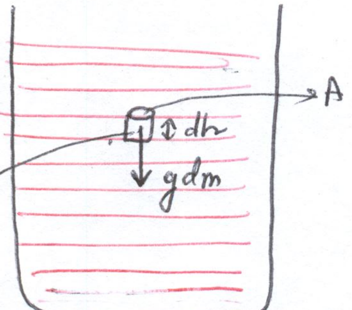
Fluid Motion Equations

1) Hydrostatic equilibrium:

$\frac{dP}{dh} = g\rho$ (P varies linearly with height/depth)

Why? → From $\boxed{2^{nd}\text{ Newton's Law}}$

Column of fluid



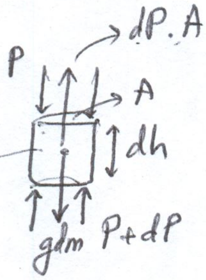
→ Element of fluid: cylinder
- dh
- cross-sectional area A
- dm : infinitesimal mass

→ The infinitesimal element of fluid (dh, A, dm) feels its weight of gdm

→ When this column of fluid is in equilibrium

→ tiny element of fluid is static → what is holding it up? → <u>bouyant force</u>

→ Origin of bouyant force: density of fluid varies with height ↔ high pressure at bottom & low pressure at top → even for our tiny cylinder of fluid!



dm: mass of this element of fluid; dvol is its volume.

$F_{net} = 0$ (static equilibrium)

$$\underbrace{(P+dP)A}_{up} - P.A - dm\,g = 0$$

$$dP.A - dm.g = 0$$

$$dP.A = dm.g$$

$$\rho = \frac{dm}{dvol} = \frac{dm}{A.dh} \rightarrow dm = \rho.A.dh$$

rho (density)

$$dP.\cancel{A} = \rho.\cancel{A}.dh.g$$

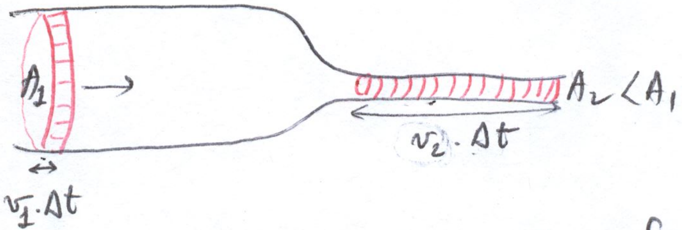
$$\boxed{\frac{dP}{dh} = \rho.g}$$

Pressure increases linearly with h (height or depth) being the slope: $\rho.g$ for gas & liquid!

→ Since $\rho_{liquid} \sim 10^3 \rho_{gas}$ → scuba diving requires prior training to withhold ~~high~~ steep change of pressure

2) Conservation of mass: no leaking of fluid

Fluid moving in a pipe with different cross-sectional areas:



If fluid motion is continuous (~~no~~ traffic bottle neck effect) fluid necessarily moves at higher speed v_2 at smaller cross-sectional area:

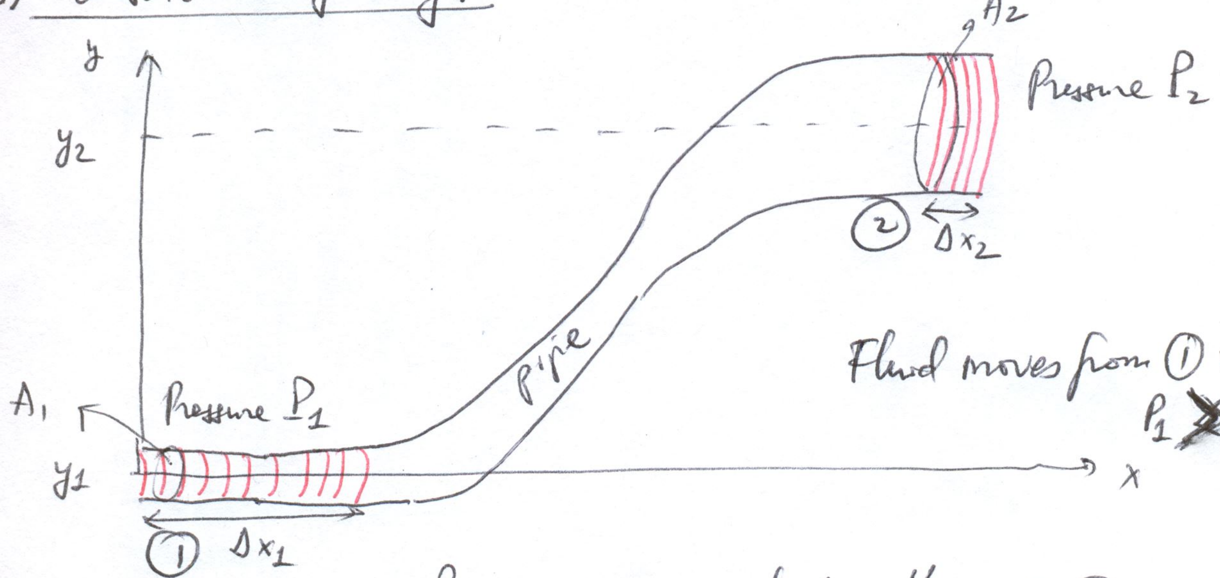
$$m_1 = \rho V_1 = \boxed{\rho \cdot v_1 \Delta t \cdot A_1} = m_2 = \rho V_2 = \boxed{\rho \cdot v_2 \Delta t \cdot A_2}$$

$$\cancel{\rho} v_1 \cancel{\Delta t} A_1 = \cancel{\rho} v_2 \cancel{\Delta t} A_2 \Rightarrow v_1 A_1 = v_2 A_2$$

or $\boxed{v \cdot A = \text{constant}}$

(in traffic v is constant → bottleneck effect!)
even if a lane is blocked

3) Conservation of energy:



Fluid moves from ① to ②
$P_1 > P_2$

Pressure @ ① is higher than @ ② →
Work done, by Pressure, to push fluid from ① → ②.

$$\Delta W = \Delta KE + \Delta P.E$$

$$\Delta W = P_1 A_1 \cdot \Delta x_1 - P_2 A_2 \cdot \Delta x_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + m g y_2 - m g y_1$$

↓

$W = F.d$

↑ Conservation of energy.

$$\frac{1}{2} m v_1^2 + m g y_1 + P_1 A_1 \Delta x_1 = \frac{1}{2} m v_2^2 + m g y_2 + P_2 A_2 \Delta x_2$$

$$\Rightarrow \left[\frac{1}{2} m v^2 + m g y + P A \Delta x = \text{constant} \right] \times \frac{1}{vol} = \frac{1}{A \cdot \Delta x}$$

$$\boxed{\frac{1}{2} \rho v^2 + \rho g y + P = \text{constant}}$$ Bernoulli's eq.

$$\frac{m}{vol} \equiv \rho$$

Ch 10 & Ch 11: Rotational Motion & Conservation of Angular Momentum

1) New quantities involving cross product

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{L} = \vec{r} \times \vec{p} \rightarrow \text{general}$$

$$= I \cdot \vec{\omega} \rightarrow \text{for rotations}$$

Should defined center of rotation or pivot

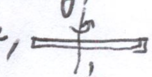
2) RHR to find direction for $\vec{\tau}$ & $\vec{L}$

3) Analog of Newton's 2nd Law

$$\vec{F}_{net} = m \cdot \vec{a}$$

$$\vec{\tau}_{net} = I \cdot \vec{\alpha} \qquad (I = c\, mR^2)$$

$\frac{1}{2}$ disk, 1 ring, $\frac{2}{5}$ sphere, $\frac{1}{12}$



Most general versions:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = 0 \rightarrow \vec{p}_i = \vec{p}_f$$

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0 \rightarrow \vec{L}_i = \vec{L}_f$$

Ch 12: Static Equilibrium:

$$\sum_i \vec{F}_i = 0$$

$$\sum_i \vec{\tau}_i = 0$$

→ (with a selection of pivot among the force application points)

$$\rightarrow \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta\, (\underset{RHR}{\hat{\tau}})$$

$\vec{r}$ from pivot to where $\vec{F}$ is applied

θ angle b/w $\vec{r}$ & $\vec{F}$

$\hat{\tau}$ direction of $\vec{\tau}$, given RHR

ch 13: SHM

$$\rightarrow \frac{d^2 z}{dt^2} = -\frac{a}{b} z \Rightarrow z(t) = A\cos\omega t\ , \quad \omega = \sqrt{\frac{a}{b}}$$

	a	b	ω
pendulum	g	L	$\sqrt{\frac{g}{L}}$
torsional pendulum	K	I	$\sqrt{\frac{K}{I}}$
spring-bob	k	m	$\sqrt{\frac{k}{m}}$

⇔ Total energy stays constant

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

Ch 14: Wave Motion

Wave superposition

- Beats : 2 waves, same direction & A, different k's & ω's

  $$y_T(0,t) = -2A\cos\left(\boxed{\frac{\omega_1-\omega_2}{2}}\cdot t\right)\sin\left(\frac{\omega_1+\omega_2}{2}\cdot t\right)$$

  low freq → Beats

- Standing waves : 2 waves, incident + reflected going in opposite directions (same A, k, ω)

  $$y_T(x,t) = 2A\sin kx\sin\omega t$$

  - Fixed @ $x=L$: $\sin kL = 0 \rightarrow kL = n\pi$ ($n=1,2,3,$ etc)
  - Open @ $x=L$: $\sin kL = 1 \rightarrow kL = (2n+1)\frac{\pi}{2}$ ($n=0,1,2,3,$ etc.)

- Constructive / Destructive Interference

  Constructive: $\Delta path = n\lambda$ ($n=0,1,2,3$ etc.)

  Destructive: $\Delta path = (2n+1)\frac{\lambda}{2}$ ($n=0,1,2,3$ etc.)

Doppler effect : moving source

$$\boxed{f' = \frac{f}{1 \mp \frac{u}{v}}}$$

− → approaching source

+ → receding source

$v = \lambda\cdot f \rightarrow \lambda = \frac{v}{f} \rightarrow f = \frac{v}{\lambda}$

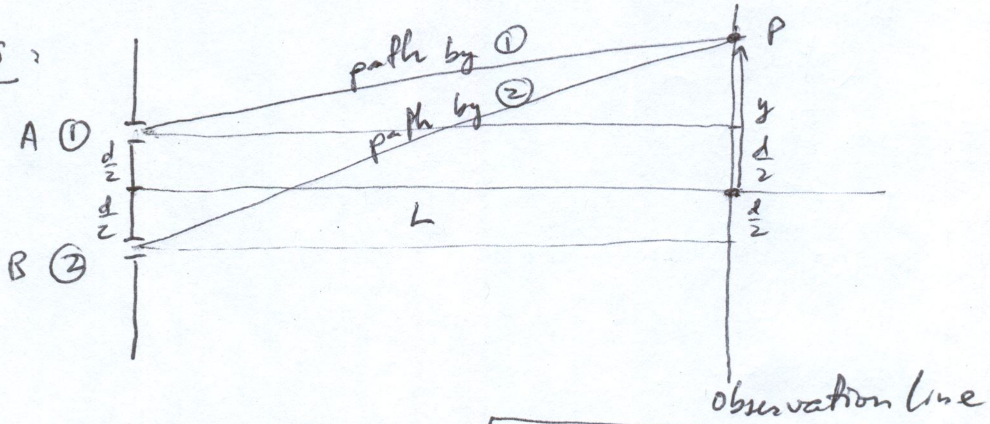
$$\frac{1}{\lambda'} = \frac{\frac{1}{\lambda}}{1 \mp \frac{u}{v}}$$

− → approaching

+ → receding.

$$\rightarrow \boxed{\lambda' = \lambda\left(1 \mp \frac{u}{v}\right)}$$

Two slits :



~~1st~~ Maxima $\Delta path = BP - AP = \sqrt{L^2+\left(y+\frac{d}{2}\right)^2} - \sqrt{L^2+\left(y-\frac{d}{2}\right)^2}$ $= 0$ (center)

$= \lambda$ (1st max

$= 2\lambda$ (2nd max)

…

Minima: $\Delta path = BP - AP = \sqrt{L^2+\left(y+\frac{d}{2}\right)^2} - \sqrt{L^2+\left(y-\frac{d}{2}\right)^2} = \frac{\lambda}{2}$ (1st min.)

$= \frac{3\lambda}{2}$ (2nd min)

$= \frac{5\lambda}{2}$ (3rd min.)

…

Ch 15 Fluid Motion.

3 equations

→ Hydrostatic equilibrium ↔ bouyancy

$$\frac{dP}{dh} = g\rho \quad \text{or} \quad F_b = g\underset{\text{displaced fluid density}}{\rho}\underbrace{h.\text{Area}}_{\text{vol. of fluid displaced}}$$

→ Conservation of mass of fluid → $\underset{\text{speed of fluid}}{v} \cdot \underset{\text{cross-sectional area}}{\text{Area}} = \text{constant}$

→ Conservation of energy or Bernoulli's eq:

$$\frac{1}{2}\rho v^2 + \rho g h + P = \text{constant}$$

$$\frac{1}{2}\rho v_i^2 + \rho g h_i + P_i = \frac{1}{2}\rho v_f^2 + \rho g h_f + P_f$$