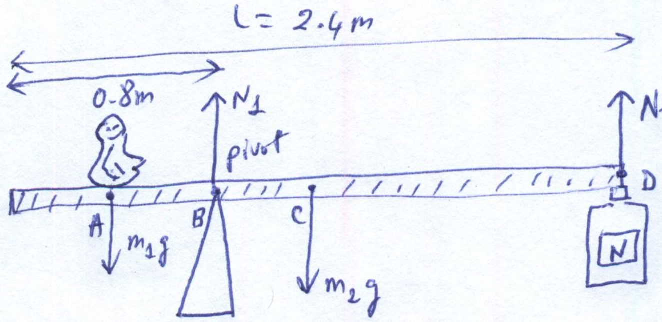
Ch12 Static Equilibrium

→ No linear motion: 1) $\sum_i \vec{F}_i = 0$ (Net force on systems is 0)

→ No rotational motion 2) $\sum_i \vec{\tau}_i = 0$ (Net torque on system is also 0)

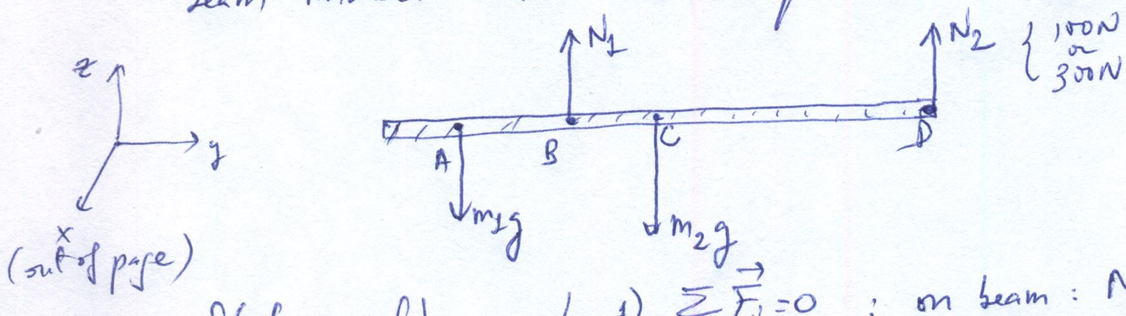
12.21



length of beam $L = 2.4m$
mass of beam: $m_2 = 60kg$
mass of child $m_1 = 40kg$

Names:
- N_1: normal force on beam by support @ pivot point B
- N_2 = " " " " by scale @ point D
- Point A: child's location
- Point C: CM of beam

Object to satisfy the two equations for static equilibrium:
We four objects: child, beam, support, scale, but the beam interacts with all components involved!



Static equilibrium
1) $\sum_i \vec{F}_i = 0$: on beam: $N_1 + N_2 - m_1 g - m_2 g = 0$
2) $\sum_i \vec{\tau}_i = 0$:

1) Torques $\vec{\tau} = \vec{r} \times \vec{F}$

↳ Need to select the center of rotation

$\vec{r}$ = position vector: position of the force application point with respect to the center of rotation or pivot

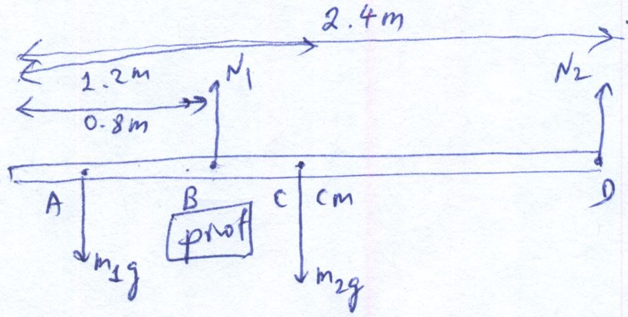
2) "Select": Center of rotation or pivot can be chosen to simplify the calculation of torques. The choice does not affect final result!

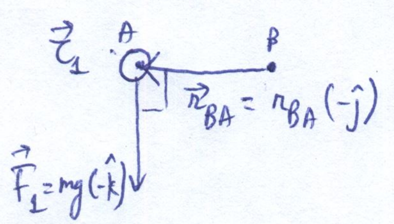
3) How would the selection of a pivot point simplify torque calculation?

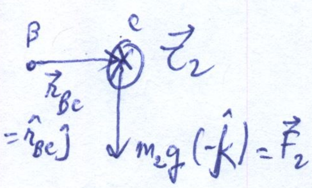
Beam: choose as pivot
- A
- B ✓ → since N_1 is not given (although can be calculated from $\sum_i \vec{F}_i = 0$)
- C
- D

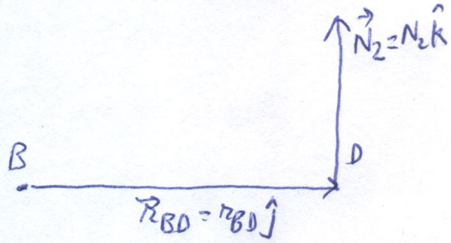
$$\vec{\tau}_{N_1} = \underbrace{\vec{r}_{BB}}_{0} \times \vec{N}_1 = 0$$

⇒ Need to calculate 3 torques: $\vec{\tau}_1$ (by child), $\vec{\tau}_2$ (by beam), $\vec{\tau}_{N_2}$ (by scale)









$$\sum_{i=1}^{3} \vec{\tau}_i = r_{BA}\, m_1 g\,(\hat{i}) + r_{BC}\, m_2 g\,(-\hat{i}) + r_{BD}\, N_2\, \hat{i} = 0$$

(r_{BA} ? ; r_{BC} = 0.4m ✓ ; r_{BD} = 1.6m ✓)

a) $N_2 = 100\,N$ → $r_{BA} = \dfrac{r_{BC}\, m_2 g - r_{BD}\, N_2}{m_1 g} = \dfrac{0.4 \times 60 \times 9.81 - 1.6 \times 100}{40 \cdot 9.81}$

= 0.19m (positive! or A is left of B)

→ Position of child from left edge of the beam is: $0.8 - 0.19 = \boxed{0.61m}$

b) $N_2 = 300\,N$ $\quad r_{BA} = \dfrac{0.4 \cdot 60 \cdot 9.81 - 1.6 \times 300}{40 \cdot 9.81} = -0.62\,m$ (negative! or A is right of B!)

→ Position of child from left edge of beam: $\boxed{1.42m}$ (right of beam's CM)

child exerts more pressure on scale by sitting closer to it, makes sense!

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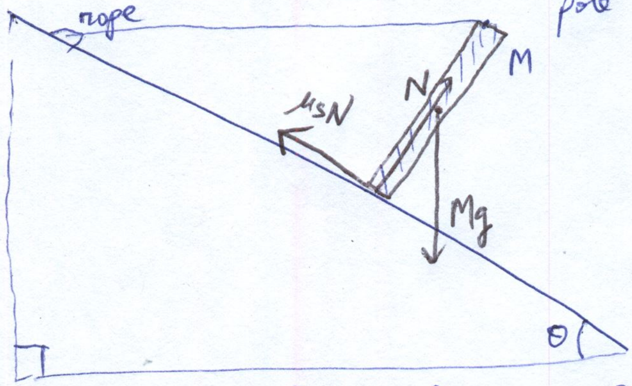
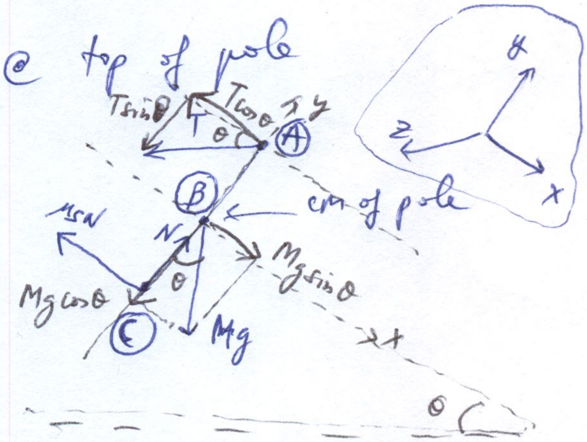
→ Pole of mass M
→ Horizontal rope
→ Friction b/w pole & incline μ?

(μ_{min} in term of θ?)
→ θ: angle of incline



μ_{min} so pole does not slip?

. Forces on pole
- N: normal to incline on pole @ contact point b/w pole & incline
- Mg: weight of pole @ its CM
- $\mu_s N$: friction @ contact b/w pole & incline. B/w of tension by rope, pole tends to rotate CCW wrt its CM → lower part tends to slip down the ramp → friction points up the ramp!
- T: tension by rope @ top of pole



→ Static equilibrium

i) $\sum_i \vec{F}_i = 0$

$$\sum_{i=1}^{4} \vec{F}_{x_i} = Mg\sin\theta - \mu_s N - T\cos\theta = 0 \quad (1)$$

$$\sum_i \vec{F}_{y_i} = N - Mg\cos\theta - T\sin\theta = 0 \quad (2)$$

ii) $\sum_i \vec{\tau}_i = 0$ → Select pivot
- CM or (B) ✓
- Top of pole or A
- Bottom of pole or C

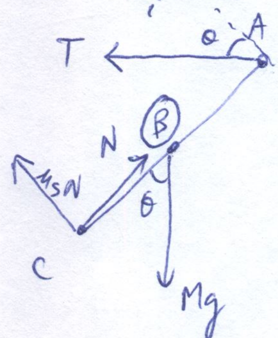
(B) is selected as our pivot → position vector $\vec{r}$ for all forces will refer to this pivot
→ $\vec{\tau}_{Mg} = 0$

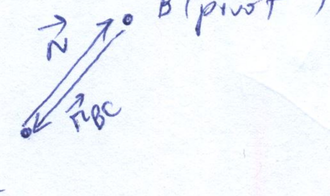
(C) is not a good choice as we need μ in our equations

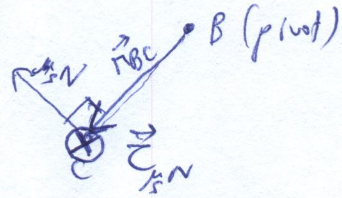
$\vec{\tau} = \vec{r} \times \vec{F}$

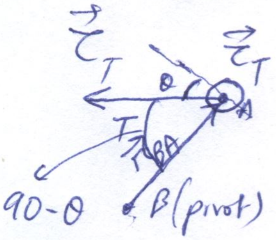
Pivot = B

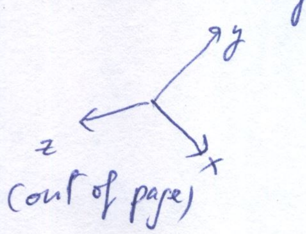
$$\sum_i \vec{\tau}_i = \vec{\tau}_N + \vec{\tau}_{\mu_s N} + \vec{\tau}_T$$











$$\vec{r}_{BC} \times \vec{N} \overset{0}{} + r_{BC}\,\mu_s N(-\hat{k}) + r_{BA}\,T \overbrace{\sin(90-\theta)}^{\cos\theta}(\hat{k})$$

(When $\vec{r} \parallel \vec{F}$ → no torque since $\sin 180° = \sin 0° = 0$)

$-\hat{k}$: into page by RHR; $\hat{k}$: out of page

$$0 = -r_{BC}\,\mu_s N + r_{BA}\,T\cos\theta$$

Note: B middle of pole (CM) : $r_{BC} = r_{BA} = \frac{L}{2}$ (L: length of pole)

$$0 = -\mu_s N + T\cos\theta \rightarrow \boxed{T = \frac{\mu_s N}{\cos\theta}}$$ (3) From torque balance equation.

Force balance equations:

2) ⇒ $N = Mg\cos\theta + T\sin\theta = Mg\cos\theta + \mu_s N \frac{\sin\theta}{\cos\theta}$ (tan θ) ⇒ $\boxed{N = \frac{Mg\cos\theta}{1-\mu_s\tan\theta}}$

1) ⇒ $Mg\sin\theta - \mu_s N - \frac{\mu_s N}{\cancel{\cos\theta}}\cancel{\cos\theta} = Mg\sin\theta - 2\mu_s N = 0$

$$\left[\cancel{Mg}\sin\theta - 2\mu_s \frac{\cancel{Mg}\cos\theta}{1-\mu_s\tan\theta} = 0\right] \times (1-\mu_s\tan\theta)$$

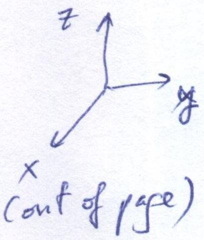
$$\left[\sin\theta - \mu_s\sin\theta\tan\theta - 2\mu_s\cos\theta = 0\right] \times \frac{1}{\cos\theta}$$

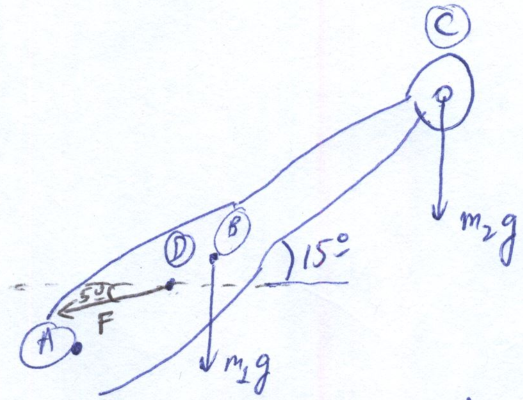
$$\tan\theta \underbrace{-\mu_s\tan^2\theta - 2\mu_s}_{-\mu_s(2+\tan^2\theta)} = 0$$

$$\boxed{\mu_s = \frac{\tan\theta}{2+\tan^2\theta}}$$ at this value for μ_s → $\left\{ \begin{array}{l} \sum \vec{F}_i = 0 \\ \sum \vec{\tau}_i = 0 \end{array} \right.$

→ This is actually $\mu_{s\,min}$!

12-27





arm raised @ 15° angle above horizontal

→ Mass of arm $m_1 = 4.2$ kg @ CM (B) (AB = 0.21 m)

→ Mass of weight $m_2 = 6.0$ kg @ (C) (AC = 0.56 m)

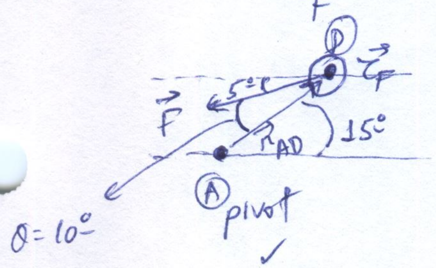
→ Shoulder is point (A)

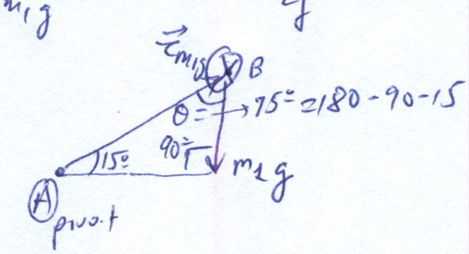
→ Deltoid muscle force [F]? applied @ D (AD = 0.18 m)
F is pointing 5° below horizontal

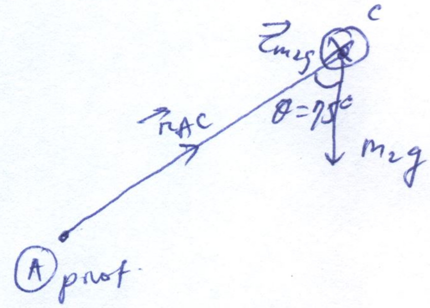
Statements

1) Select (A) as pivot → Three torques ($\vec{\tau} = \vec{r} \times \vec{F}$)

$$\vec{\tau}_F + \vec{\tau}_{m_1g} + \vec{\tau}_{m_2g} = 0$$





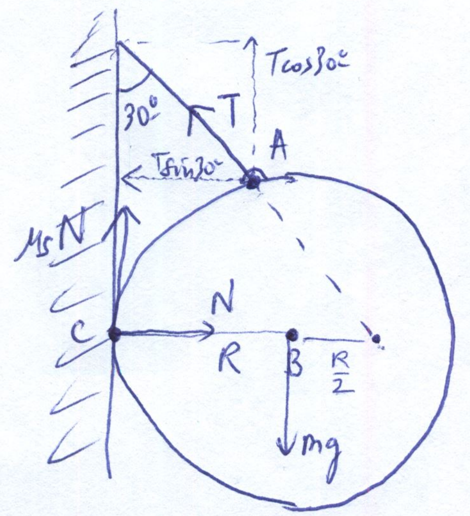


$$\underbrace{r_{AD}}_{0.18\text{m}} \cdot F \sin 10° \,\hat{i} + \underbrace{r_{AB}}_{0.21\text{m}} m_1 g \sin 75° (-\hat{i}) + \underbrace{r_{AC}}_{0.56\text{m}} m_2 g \sin 75° (-\hat{i}) = 0$$

$$F = \frac{0.21 \times 4.2 \times 9.81 \times \sin 75° + 0.56 \times 6 \times 9.81 \times \sin 75°}{0.18 \times \sin 10°}$$

$$= \frac{40.2}{0.18 \cdot \sin 10°} = 1280\text{N} = 1.28\text{kN}$$

12.28



→ Uniform sphere of radius R supported by a rope attached to a vertical wall forming an angle of 30° with it.

→ Question: min μ_s b/w sphere & wall

<u>special points</u>: where a force is applied:

A (Tension by rope is applied on sphere)

B (weight of sphere is applied)

C (Normal force by wall & friction by wall is applied)

<u>forces</u>: $T, \underbrace{mg, \; N, \; \mu_s N}$ ⇒ y, x, z (out of page)

$\hookrightarrow \vec{T} = -T\sin 30°\,\hat{i} + T\cos 30°\,\hat{j}$

$W = mg\,(-\hat{j})$

$\vec{N} = N\,\hat{i}$

$\vec{F_s} = \mu_s N\,\hat{j}$ (sphere tends to rotate CCW due to $T_x = -T\sin 30°$)

<u>Static equilibrium</u>: $\begin{cases} 1) \; \sum_i \vec{F_i} = 0 = \vec{F}_{net} & \text{(on sphere)} \\ 2) \; \sum_i \vec{\tau_i} = \vec{\tau}_{net} = 0 & \text{(on sphere)} \end{cases}$

$\vec{F}_{net} = 0 \rightarrow \begin{cases} F_{net_x} = 0 : & -T\sin 30° + N = 0 & (1a) \\ F_{net_y} = 0 ; & T\cos 30° + \mu_s N - mg = 0 & (1b) \end{cases}$

<u>Pivot or center of rotation</u>: A $\quad \overset{\checkmark}{B} \quad \cancel{C}$ → position vectors of A, B, C (force application points) will be referred to B!

↑ we need μ_s

pivot is B

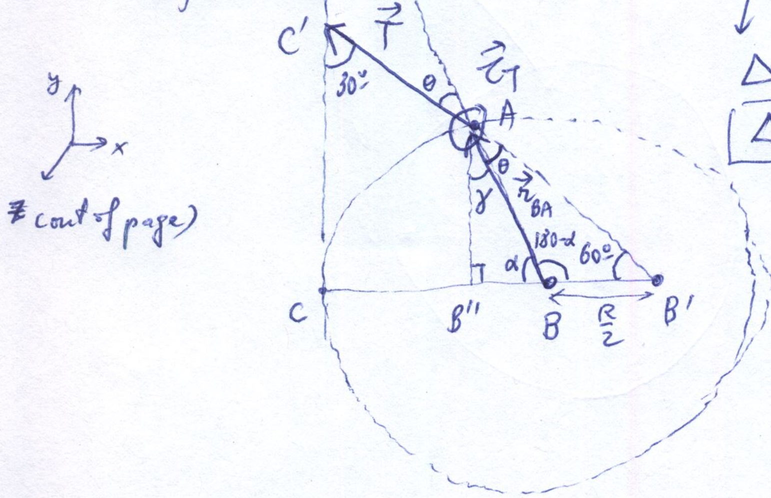
$\vec{\tau}_{net} = 0 = \vec{\tau}_T + \vec{\tau}_N + \vec{\tau}_{\mu_s N}$

$\vec{r}_{BB} = 0 \Rightarrow \vec{\tau}_{mg} = 0$

$\vec{\tau} = \vec{r} \times \vec{F}$

Torque by rope tension:

$$\vec{\tau}_T = \overset{R}{\overset{\uparrow}{r_{BA}}}\, T\, \boxed{\sin\theta}\, \hat{k}$$



$\Delta\ C'CB' \Rightarrow \angle B' = 60°$

$\boxed{\Delta\ ABB'} \Rightarrow \theta + 180° - \alpha + 60° = 180°$

$\theta + 60° = \alpha$

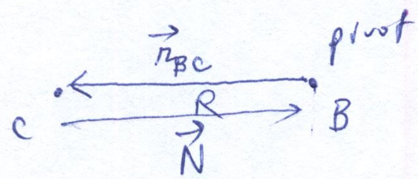
sine theorem: $\dfrac{\sin\theta}{BB'} = \dfrac{\sin 60°}{AB}$

$$\sin\theta = \frac{BB'}{AB}\sin 60° = \frac{\frac{R}{2}}{R}\sin 60°$$

$$\boxed{\sin\theta = \frac{\sin 60}{2}}$$

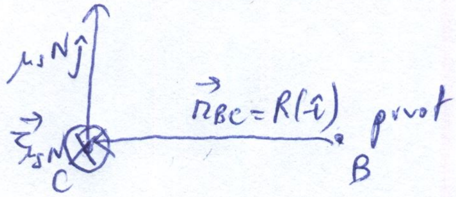
$$\vec{\tau}_T = RT\,\frac{\sin 60°}{2}\,\hat{k}$$

Torque by normal force $\vec{N}$:



$$\left.\begin{cases} \vec{r}_{BC} = R(-\hat{i}) \\ \vec{N} = N\hat{i} \end{cases}\right\} \vec{\tau}_N = 0$$

Torque by friction force $\mu_s N \hat{j}$



$$\left.\begin{cases} \vec{r}_{BC} = -R\hat{i} \\ \vec{F}_s = \mu_s N\hat{j} \end{cases}\right\} \vec{\tau}_{\mu_s N} = R\mu_s N \underbrace{(-\hat{i})\times\hat{j}}_{-\hat{k}}$$

$$\vec{\tau}_{net} = \vec{\tau}_T + \vec{\tau}_{\mu_s N} = \left(RT\frac{\sin 60}{2} - R\mu_s N\right)\hat{k} = 0 \qquad (2)$$

1a) $N - T\sin 30^\circ = 0$

1b) $T\cos 30^\circ + \mu_s N - mg = 0$

2) $T\frac{\sin 60^\circ}{2} - \mu_s N = 0$

} 3 eqs & 3 unknowns: N, T, μ_s

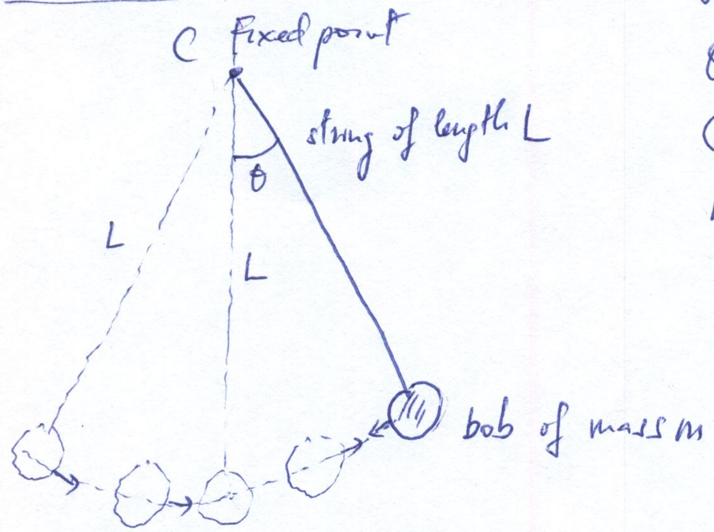
1a) $N = T\sin 30^\circ \Rightarrow \frac{T}{N} = \frac{1}{\sin 30^\circ}$

2) $\mu_s = \frac{T}{N}\frac{\sin 60^\circ}{2} = \frac{1}{2}\frac{\sin 60^\circ}{\sin 30^\circ} = 0.866 \rightarrow \mu_{s\,min}$

Ch 13 Oscillatory Motion

Neither linear nor exactly rotational

1) **Pendulum**: bob and string w/ negligible mass and one end fixed



θ: angle of string wrt. vertical

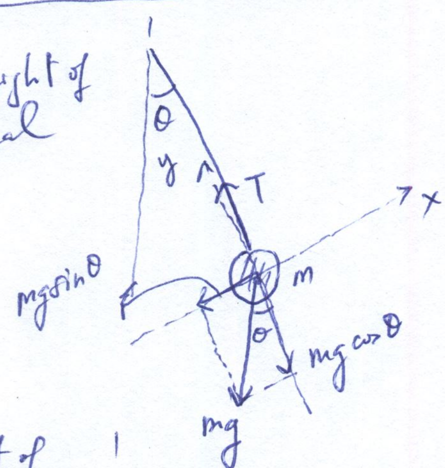
C: "center of rotation"

L: bob is always at separation L from pivot C → bob has tangential but not radial motion

Derive equation of motion for pendulum:

1st Method, Newton's 2nd Law on bob: $\vec{F}_{net} = m\vec{a}$

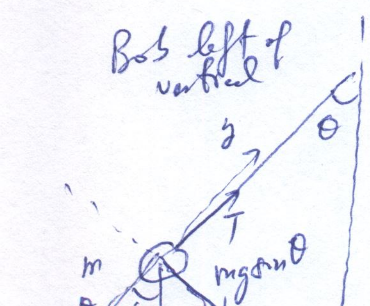
Bob right of vertical



In this coordinate system, @ this time, motion is along x-direction

$$\begin{cases} F_{net_x} = -mg\sin\theta = m\cdot a \rightarrow \boxed{a = -g\sin\theta} \\ F_{net_y} = T - mg\cos\theta = 0 \end{cases}$$

Bob left of vertical



$F_{net_x} = mg\sin\theta = m\cdot a \rightarrow \boxed{a = +g\sin\theta}$

→ acceleration keeps switching signs → "oscillation!"

Eq. of motion → equations in θ:

$$\alpha = \frac{a}{L} \rightarrow \left[a = \alpha L = \frac{d^2\theta}{dt^2} L = -g\sin\theta \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}\right.$$

2nd order non-linear differential equation

Common approximation: "small angle approximation": $\boxed{\theta \text{ is small}}$

$\sin\theta \simeq \theta$

→ simple solution:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

↓

solution: $\theta(t) = \theta_M \cos(\omega t)$

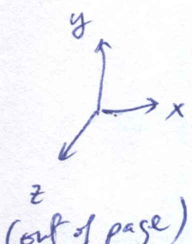
↓ periodic function in time

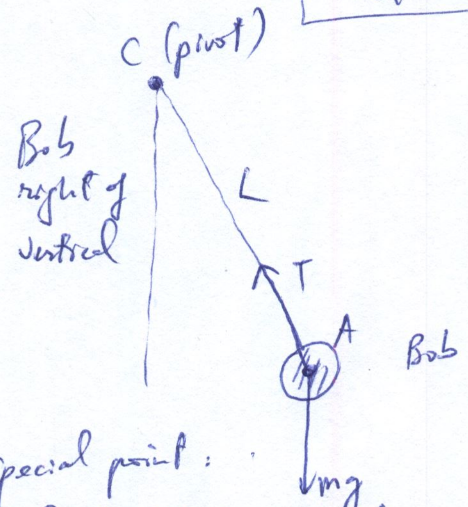
θ_M : amplitude of oscillation

ω : angular frequency of oscillation (# osc. per second)

2nd Method, Analog of Newton's 2nd Law for rotational motion on bob

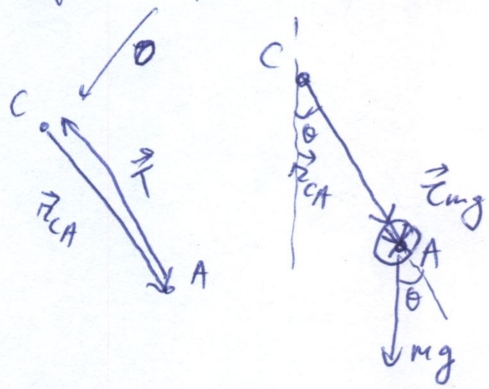
$$\boxed{\vec{\tau}_{net} = I \cdot \vec{\alpha}}$$





$\vec{\tau}_{net} = \vec{\tau}_T + \vec{\tau}_{mg} = Lmg\sin\theta(-\hat{k})$

(with $\vec{\tau}_T$ crossed out, ↙ 0)



→ Special point:
A (center of bob) (Two forces: T, mg)

→ pivot has to be C

$\vec{\tau}_{net} = mgL\sin\theta\,(-\hat{k})$

$I = mL^2$

$-mgL\sin\theta = mL^2\alpha$

$-g\sin\theta = \alpha L \; (=a)$

$\alpha = -\frac{g}{L}\sin\theta$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

small angle θ → $\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\theta$

→ $\theta(t) = \theta_M\cos(\omega t)$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \quad \text{(with small angle approx.)}$$

↓

$$\theta(t) = \theta_M \cos(\omega t) \quad \text{(position of bob in term of its angle)}$$

$$\frac{d\theta}{dt} = -\theta_M \omega \sin(\omega t) \quad ; \quad \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(-\theta_M \omega \sin(\omega t)\right) = -\theta_M \omega^2 \cos(\omega t)$$

$$\hookrightarrow -\omega^2 \theta_M \cos(\omega t) = -\frac{g}{L}\theta_M \cos(\omega t) \Rightarrow \omega^2 = \frac{g}{L} \Rightarrow \boxed{\omega = \sqrt{\frac{g}{L}}}$$

Observations: 1) If string is longer, ω is smaller: less oscillations per second

2) g depends on distance from surface;

$\hookrightarrow G\frac{Mm}{r^2} \Rightarrow \frac{GM}{R^2}\cdot m$ (with $\frac{GM}{R^2}$ circled and labelled g)

- dependence on M → used uniform sphere
- → oil, water pockets with different densities

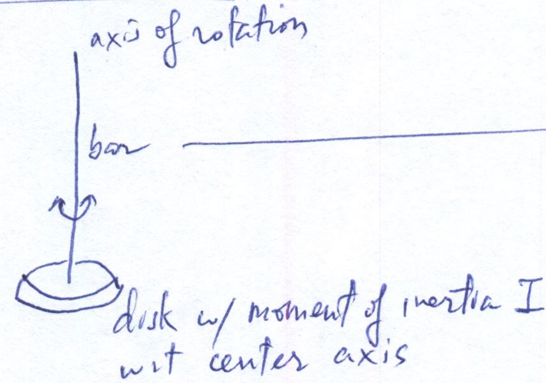
Find possible water pocket: $\boxed{\omega = \sqrt{\frac{g}{L}}}$

ω: angular frequency (# osc. per second) (s^{-1})

T: period (# seconds per oscillation) $= \frac{2\pi}{\omega}$ (s)

f: linear frequency (# linear osc. per second) $= \frac{\omega}{2\pi}$ (Hz "Hertz")

2) Torsional Pendulum:



$\text{ᘮᘮᘮ} \rightarrow$ Spring's law: $F = -K\Delta x$

Torsional Law: $\tau = -\kappa \cdot \Delta\theta$

- κ "kappa": torsional constant (dimension & material)
- $\Delta\theta$: change of angle

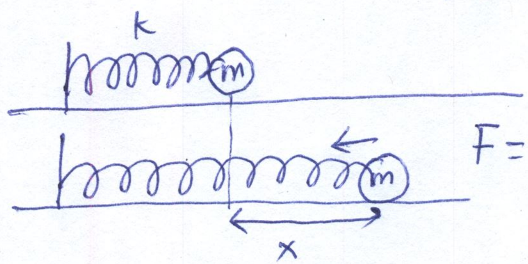
Eq. of motion: $\tau = I \cdot \alpha$

$$-\kappa\theta = I \cdot \frac{d^2\theta}{dt^2} \rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta}$$

$$\theta(t) = \theta_M \cos(\omega t)$$

$$\omega = \sqrt{\frac{\kappa}{I}} \quad (s^{-1})$$

3) Spring & Bob:



$F = -kx$

Equation of motion: $F = m \cdot a$

$$-kx = m \cdot \frac{d^2x}{dt^2} \rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

$$x(t) = x_M \cdot \cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Simple Harmonic Motion (SHM) $\boxed{\frac{d^2z}{dt^2} = -\frac{a}{b}z} \Rightarrow z(t) = z_M \cos(\omega t) \rightarrow \omega = \sqrt{\frac{a}{b}}$

	z	a	b
pendulum	θ	g	L
torsional pendulum	θ	κ	I
spring & bob	x	k	m

Damped - SHM:

$$\frac{d^2z}{dt^2} = -\frac{a}{b}z \underbrace{-\frac{c}{d}\frac{dz}{dt}}_{\text{damping term}}$$

$\downarrow$

$$z(t) = Z_M e^{-\frac{c}{2d}t} \cos(\omega t + \phi)$$

ϕ ↓ "phi" for phase

Damped - SHM:

$$\frac{d^2 z}{dt^2} = -\frac{a}{b} z - \underbrace{\frac{c}{d}\frac{dz}{dt}}_{\text{damping term}}$$

↓

$$z(t) = z_M \underbrace{e^{-\frac{c}{2d}t}}_{\text{exponential decay}} \cos(\omega t + \phi)$$

ϕ ↓ "phi" for phase

, ω: angular frequency

$T = \frac{2\pi}{\omega}$

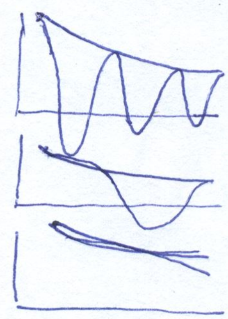
Two time scales
- → Time constant: $t_d = \frac{2d}{c}$
  - ↳ $t = t_d$ → oscillation is decayed by $\frac{1}{e}$
- → Period $T = \frac{2\pi}{\omega}$
  - ↳ time to complete one full oscillation

↓

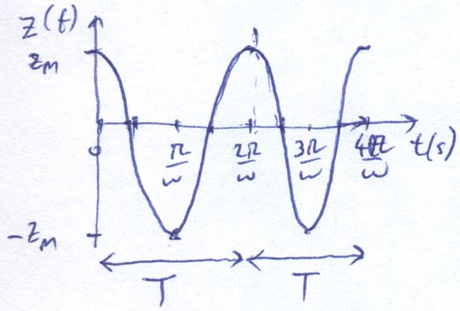
(i) $T \ll t_d$:

(ii) $T \sim t_d$:

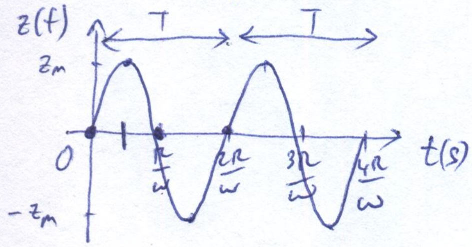
(iii) $T >> t_d$:



SHM: $z(t) = z_M \cos(\omega t)$

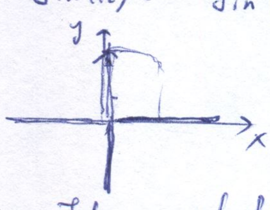


SHM $z(t) = z_M \sin(\omega t)$



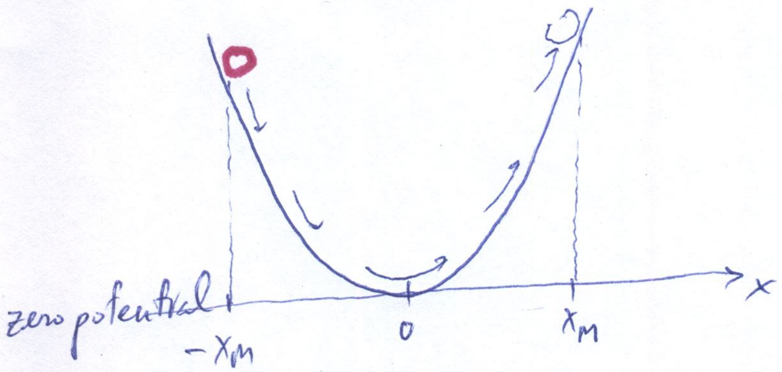
$\sin(0) = 0$ $\sin\left(\frac{\pi}{2}\right) = 1$

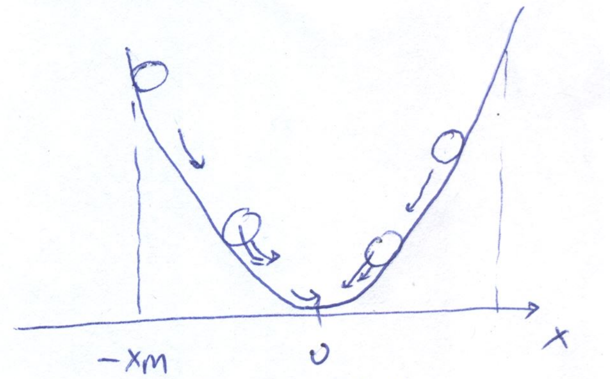
$\sin(\pi) = 0$ $\sin\left(\frac{3\pi}{2}\right) = -1$



If you shift $\sin(\omega t)$ by $\frac{\pi}{2}$ or $90°$ to the left → you get $\cos(\omega t)$

4) Particle trapped in a potential well:





No friction

Position along x-axis $x(t) = x_M \cos(\omega t + \pi)$

↳ SHM

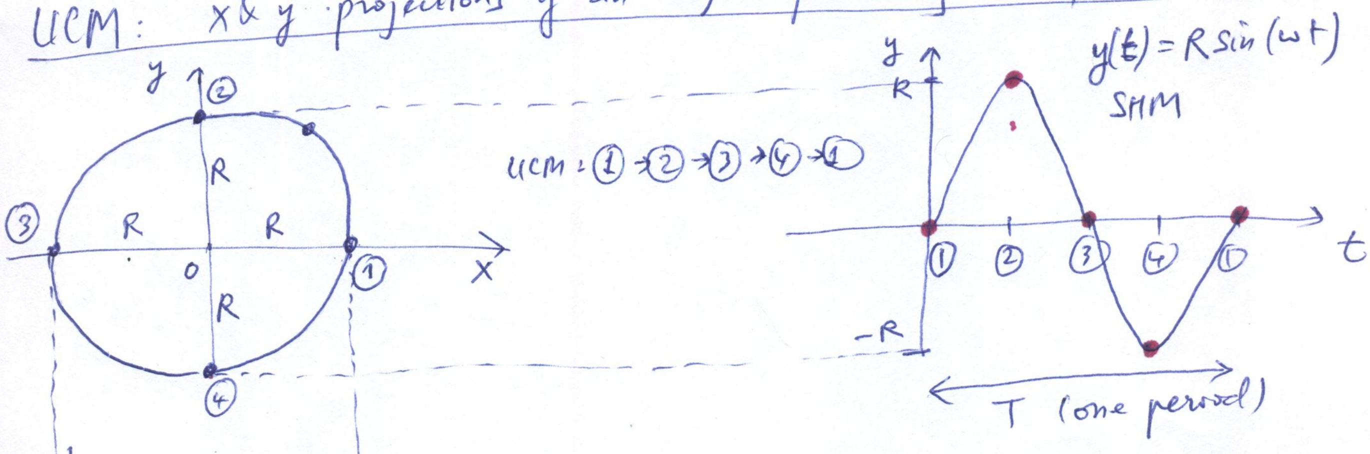
with friction

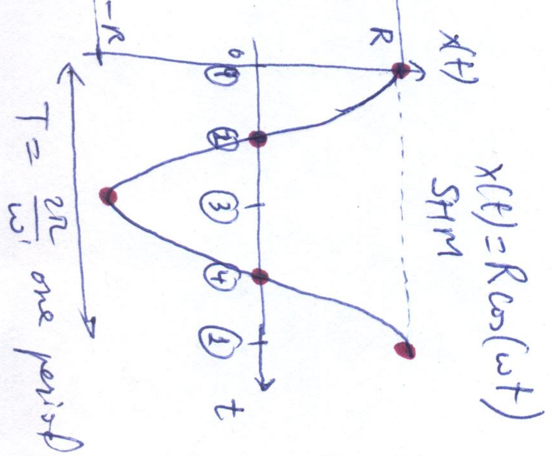
Position along x-axis $x(t) = x_M e^{-\frac{b}{2m}t} \cos(\omega t + \pi)$

↳ Damped-SHM

↓ ($e^{-\frac{b}{2m}t}$) exponential decay will bring amplitude of oscillations to zero

5) UCM: x & y projections of an object following UCM are SHM's





Object in UCM:

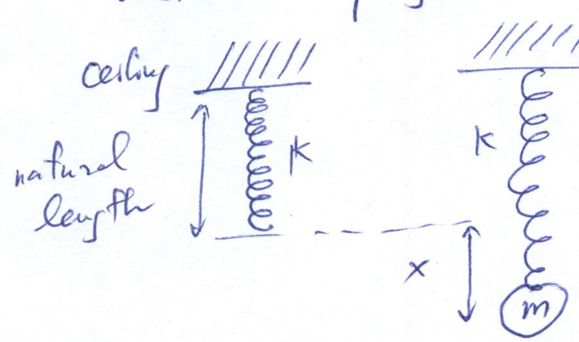
$x(t) = R\cos(\omega t)$

$y(t) = R\sin(\omega t)$

Both are SHM's shifted by $\frac{\pi}{2}$ or 90°

$\frac{1}{4}$ of a period.

13.67 Unstretched spring then add a bob and released



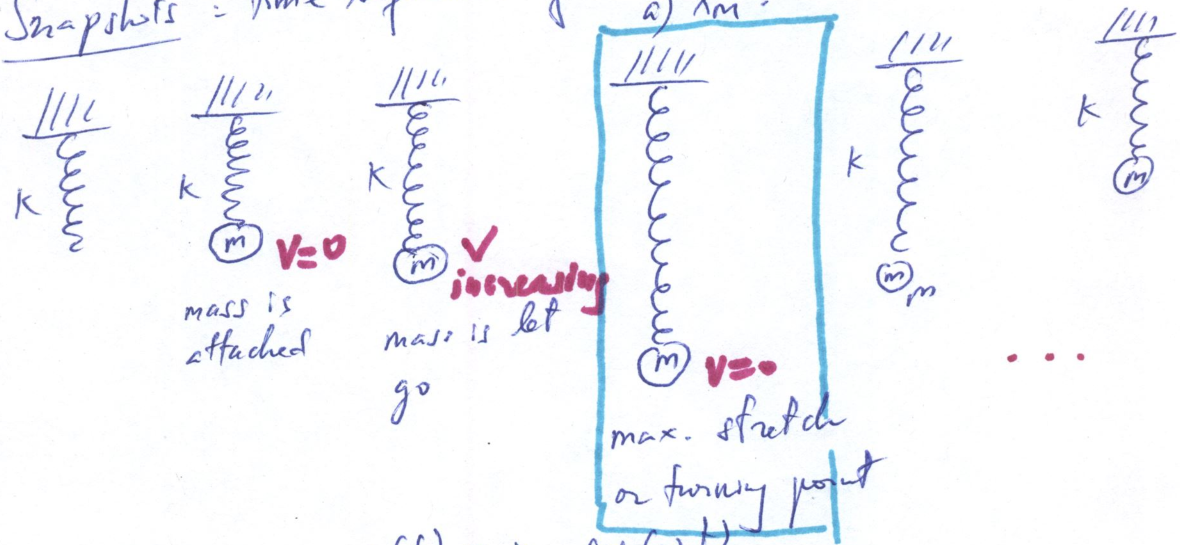
x: displacement from natural length

$K = 74 \frac{N}{m}$

$m = 0.49 kg$

a) x_M, amplitude of oscillation? b) T: period of SHM?

Snapshots: time sequence of events:



Bob → SHM:
$$x(t) = x_M \cos(\omega t)$$
$$v(t) = \frac{dx}{dt} = -x_M \omega \sin(\omega t)$$
$$a(t) = \frac{dv}{dt} = -x_M \omega^2 \cos(\omega t)$$

a) x_M → we will focus on the turning point:

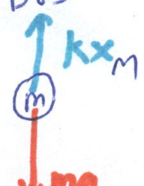
$x(t) = x_M \Rightarrow \boxed{\cos(\omega t) = 1} \Rightarrow \sin(\omega t) = 0 \Rightarrow v(t) = 0$

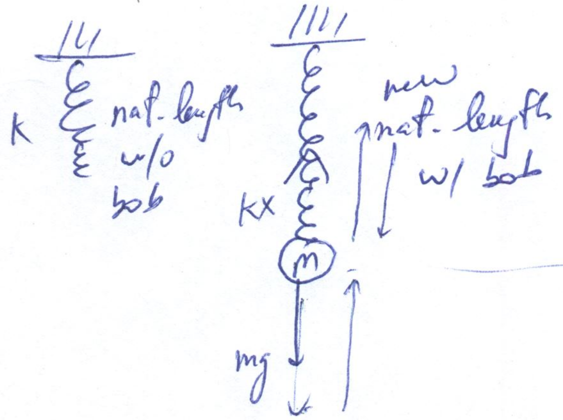
$\Rightarrow a(t) = -x_M \omega^2$

Newton's 2nd Law on Bob: $F_{net} = m \cdot a$

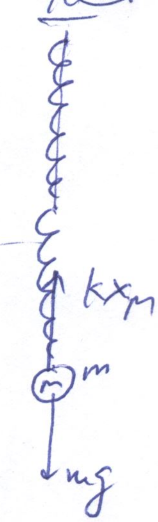
$K x_M - mg = m(-x_M \omega^2) \Rightarrow x_M (K + m\omega^2) = mg$

$$x_M = \frac{mg}{K + m\omega^2}$$





osc. occurs about the new natural length!



@ New Natural length: $F_{net} = 0 \Rightarrow kx - mg = 0$

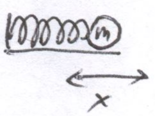
$kx_M - mg = m(-x_M\omega^2)$

Ch 14 Wave Motion

Oscillatory motion

Time-repeating variation of a position or angle

↓



→ periodic variation

→ perturbation (periodic)

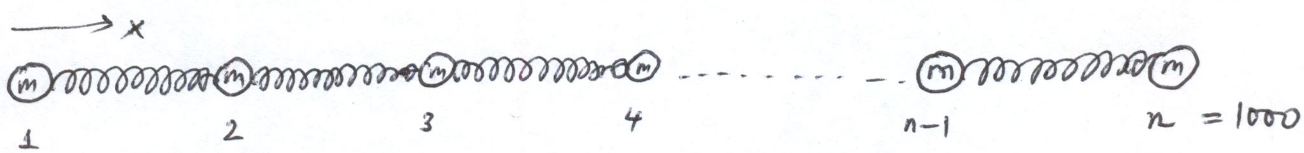
Wave motion

A step beyond: the oscillation/time (periodic) variation/perturbation is propagated in space

→ variation in both time & space

→ there is propagation

1) Propagation: → an example of a longitudinal wave
→ on a system of identical bobs connected by identical springs



If I give bob #2 a displacement in the horizontal direction:

a) Bob #2: will undergo a time-repeating variation of position or oscillation or SHM. At this time what happens to bob #900? It is still at rest: perturbation given to a bob is local

b) Then the perturbation will propagate to #3, #4, #5, etc..
Propagation happens at finite speed, it is not instant!
The speed depends on the medium (spring types, masses)

b1) { Perturbation on #2 is in x-direction.
{ Propagation of this perturbation is in x-direction
→ longitudinal wave

b2) What happens to #2 as perturbation is propagated?
#2 stays around its original position →

b2) Conf: propagation is of the perturbation or oscillation, not of matter or material. Clearly wave motion is different than previously studied motions: linear or rotational motion of matter

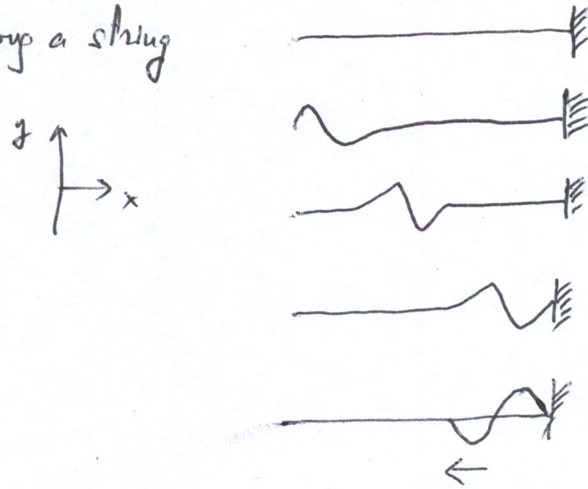
Propagation: of a perturbation is such that the objects involved (e.g.: springs & bobs) stay local while the perturbation reaches as far as possible:

→ Waves
- Sound:
  - matter: air molecules
  - perturbation: air pressure change is propagated as far as possible
- Light:
  - matter: no
  - perturbation: oscillation of electric & magnetic fields.

2) Waves: there are both time & space variations:

Transverse wave: perturbation & propagation are perpendicular to each other { For example: propagation along x-direction while the perturbation is along y-direction:

Wave along a string



Mathematically: $y(x,t) = A \sin(kx - \omega t)$
- A: wave amplitude
- k: wave number, $k = \frac{2\pi}{\lambda}$
- ω: angular freq.

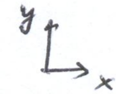
space & time oscillation

wave { perturbed in y direction; propagated in x direction

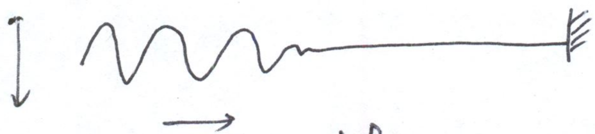
3) Types of waves:

- → Longitudinal: perturbation & propagation are in same direction (springs & Lobs, seismic waves, etc.)
- → Transverse: perturbation & propagation are perpendicular (wave in a guitar string; EM waves, etc)

4) Math description of transverse waves:



perturbation y



x propagation

$$y(x,t) = A\sin(kx - \omega t)$$

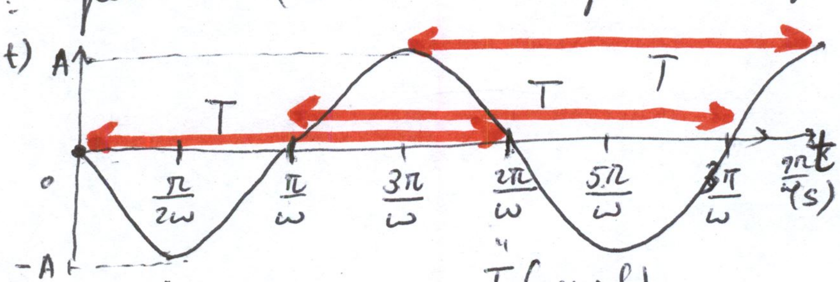
→ k: wave number: number of wavelengths in $2\pi = \frac{2\pi}{\lambda}$ (m^{-1})

→ λ: 'lambda': wavelength (m): space separation b/w two consecutive peaks

→ ω: angular frequency = $\omega = \frac{2\pi}{T}$ (s^{-1}) ⟶ $T = \frac{2\pi}{\omega}$

→ T: period (s): time separation b/w two consecutive peaks

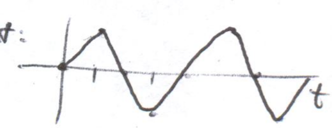
x=0



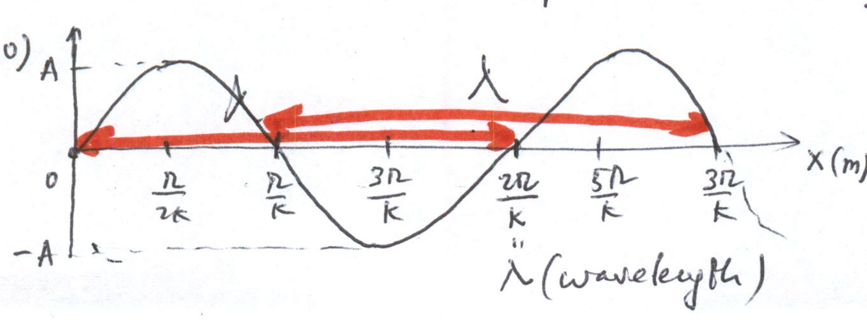
How the perturbation @ position x=0 varies over time:

$y(0,t) = A\sin(-\omega t)$

$= -A\sin(\omega t)$



t=0



How the perturbation @ t=0 varies over position

$y(x,0) = A\sin(kx)$

(14.56) Wave in a wire : $y(x,t) = 1.5 \sin(0.1x - 560t)$ { x, y are in cm; t is in s }

$T = 28N$

↳ wave:
1) perturbation in y, propagation in x → transverse wave
2) wave amplitude $\boxed{A = 1.5\,cm}$ → a)
3) wave number: $k = 0.1\ cm^{-1}$
↓
$k = \frac{2\pi}{\lambda}$ → $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.1} = 20\pi\,cm$

$\boxed{\lambda = 62.8\,cm}$ b)

4) angular freq: $\omega = 560\ s^{-1}$
↓
$\omega = \frac{2\pi}{T}$ → $T = \frac{2\pi}{\omega} = \frac{2\pi}{560} = 11.2\times10^{-3}s$

$\boxed{T = 11.2\,ms}$ c)

↙

$y(x,t) = A \sin(kx - \omega t)$
transverse

d) Wave speed : $\boxed{v = \frac{\lambda}{T}}$ (it takes a $t = T$ to propagate a distance of λ)

$= \frac{62.8\times10^{-2}\,m}{11.2\times10^{-3}\,s} = 56\ \frac{m}{s}$ (transverse wave in wire)

compared to cars average speed in highways:

$65\ \frac{mi}{h} \sim 100\ \frac{km}{h} \cdot \frac{1000m}{1km} \cdot \frac{1h}{3600s} = \frac{100}{3.6}\ \frac{m}{s} = 27.8\ \frac{m}{s}$

other equations related to wave speed : linear frequency f: how many cycles or periods fit in 1s : $f = \frac{1s}{T}$

→ $\boxed{v = \lambda \cdot f}$

e) Power carried by this wave :(average power) $\bar{P} = \frac{1}{2}\mu\omega^2 A^2 v$

μ: linear density of wire (thin wire carries less power than a thick one)
ω: angular freq ; A : wave amplitude ; v wave speed.

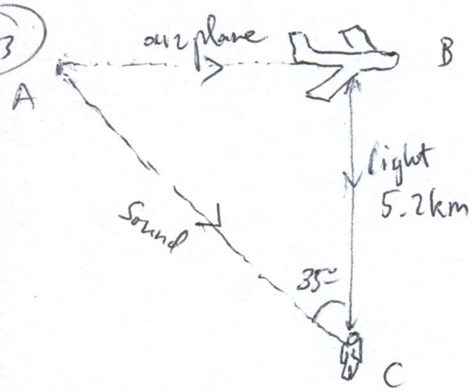
→ Need to find μ: tension of wire $T = 28N$: $v = \sqrt{\frac{T}{\mu}}$ → $\mu = \frac{T}{v^2}$

$\mu = \frac{28}{56^2}\ \frac{kg}{m}$

$\bar{P} = \frac{1}{2} \times \frac{28}{56^2} \times 560^2 \times 0.015^2 \times 56 \quad W = 17.4\,W$

↓ Watts

(14.63)



→ Airplane straight overhead @ B

→ Observer @ C

→ Sound coming from A (AC forms 35° with AB)

→ Plane speed v? assuming $v_s = 330\,\frac{m}{s}$

Statement: $\begin{cases} t_s = \text{time for sound (jet noise) to travel from A to C} \\ t_p = \text{time for plane to travel AB} \end{cases}$

Facts: observer @ C see plane @ B but hears its noise that was made when plane passed A → $\boxed{t_s = t_p}$

Note: speed of light $c = 300{,}000\,\frac{km}{s}$ → time for light to travel BC (5.2km) is negligible → instantaneous!

$$v_{plane} = \frac{d_{AB}}{t_p} = \frac{d_{AB}}{t_s} = \frac{d_{AB}}{\frac{d_{AC}}{v_s}} = v_s \frac{d_{AB}}{d_{AC}} = v_s \sin 35°$$

$\frac{d_{AB}}{d_{AC}}$ ↓ $\frac{\text{opposite side to } 35°}{\text{hypothenuse}} = \sin 35°$

$$= 330 \cdot \sin 35° = 189\,\frac{m}{s}$$

$$v_{plane} = 189 \times 3.6\,\frac{km}{h} = 680\,\frac{km}{h}$$

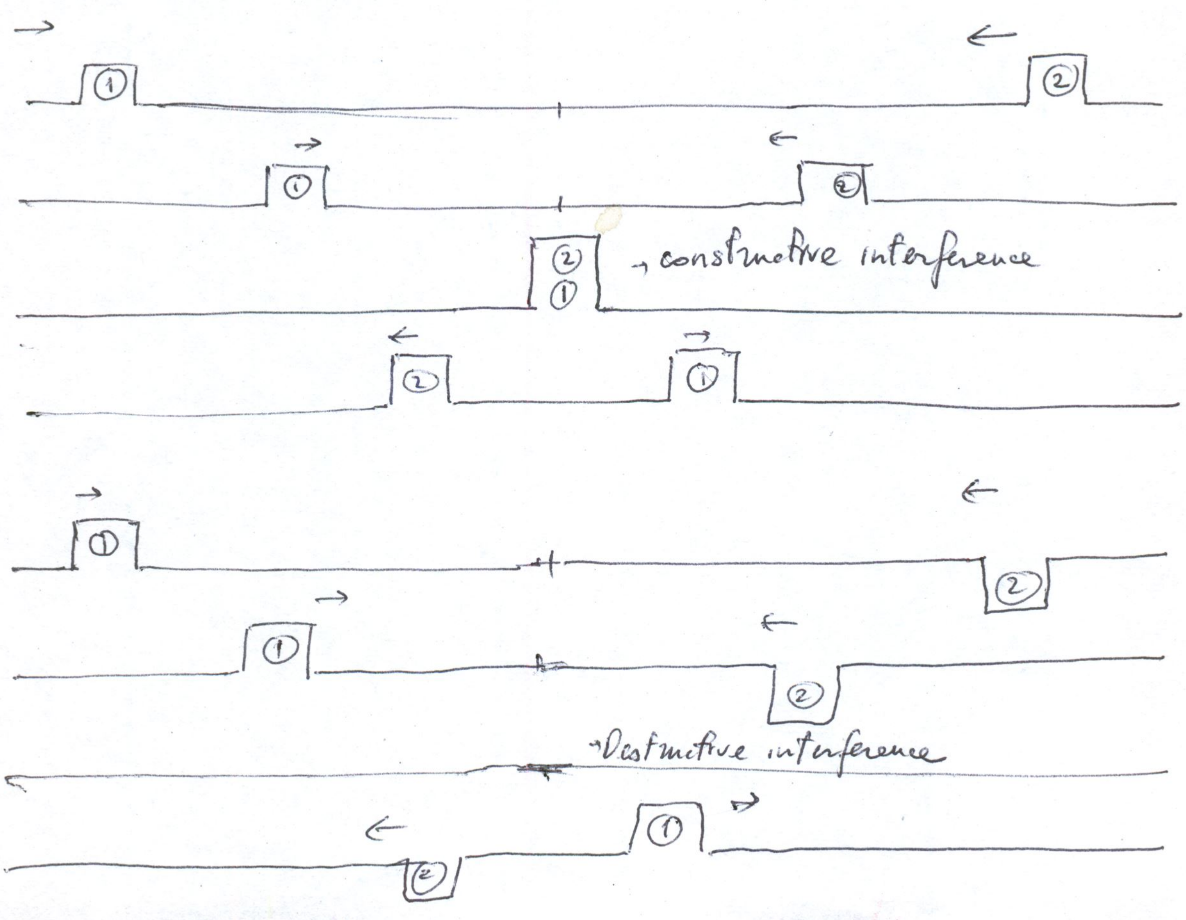
Ch 14 (cont.) Wave Superposition:

- → **Beat phenomena**: tuning of string instruments; tuning of engines etc.
- → **Standing waves**: waves in pipes, flutes, etc.
- → **Wave interference**
  - constructive
  - destructive: 1 + 1 = 0

unique properties of waves → 1 + 1 = 0

→ **Doppler effect**: when source of wave is moving (LIDAR: speed trap)

Wave superposition:



Quantitative description of wave superposition → Beat phenomenon

Two transverse waves: going in same direction
- same amplitudes A
- different frequencies: ω_1, ω_2 (different wave numbers k_1, k_2)

↳
$$\begin{cases} y_1(x,t) = A\sin(k_1 x - \omega_1 t) \\ y_2(x,t) = A\sin(k_2 x - \omega_2 t) \end{cases}$$

Superposition of these two waves @ $x=0$
$$\begin{cases} y_1(0,t) = A\sin(-\omega_1 t) \\ y_2(0,t) = A\sin(-\omega_2 t) \end{cases}$$

↳ $y(0,t) = y_1(0,t) + y_2(0,t) = -A[\sin\omega_1 t + \sin\omega_2 t]$

Trigonometry: $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)$

→ $y(0,t) = -2A\sin\left(\frac{\omega_1+\omega_2}{2}\cdot t\right)\cdot\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)$

$= -2A\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)\cdot\sin\left(\frac{\omega_1+\omega_2}{2}\cdot t\right)$

Modulated amplitude (the $-2A\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)$ term)

↓

If $\omega_1 \sim \omega_2$ → Beat phenomenon (tuning string instruments etc...)

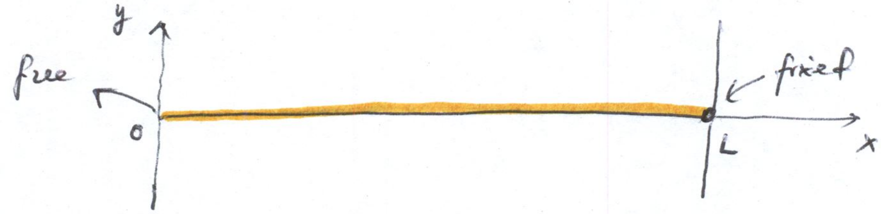
↓

Oscillating at much lower frequencies

one is reflection of the other
(same A, ω, k)

<u>Wave Superposition: two waves going in opposite directions</u> → <u>Standing waves</u>

- String of length L attached to a fixed point:



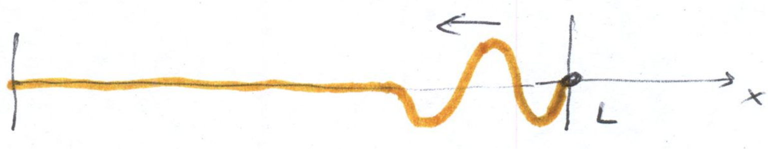
- Perturb free end by moving it up & down (in y direction)

This perturbation generate a wave that propagate in $+x$ direction

$$y_1(x,t) = A\cos(kx - \omega t) \quad \text{(incoming wave)}$$

($-$: $+x$ propagation)

- When it reaches the fixed end → it gets reflected: same wave (same A, same ω, same k!) that will travel in $-x$ direction



$$y_2(x,t) = A\cos(-kx - \omega t) = A\cos(kx + \omega t)$$

↓ (reflected wave) ($+$: $-x$ propagation)

- If I keep sending incoming waves from left, they will ~~[illegible]~~ superimpose with reflected waves (same A, ω, k)

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\cos(kx - \omega t) - A\cos(kx + \omega t)$$

↑ Reflection: adds 180° phase to the incoming wave