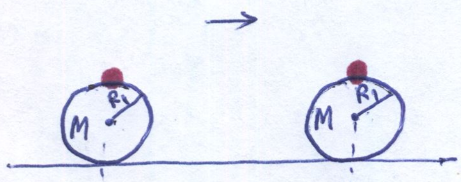
Ch 10 Rotational Motion

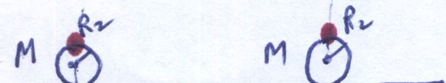
Already described linear motion, UCM (object going around an <u>external</u> center of curvature)

Translation (linear motion)



bowling ball
$\boxed{\text{sliding}}$ on
icy surface

Distance traveled does not depend on size R_1 or R_2

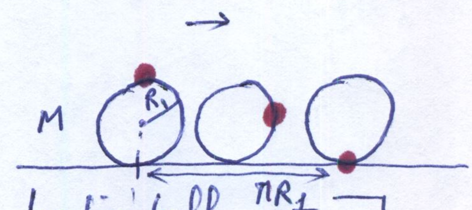


smaller bowling ball sliding on same icy surface

<u>Statements:</u>

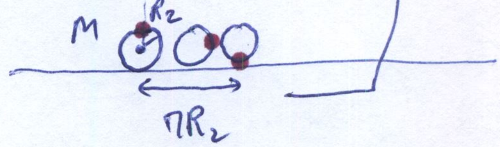
1) Sliding balls of equal masses but different radii ($R_2 < R_1$) have same translational motion. → They can be described as point-like particles of mass M located at their centers of mass. (Size does not matter)
2) Same orientation: top dot stays at top.

Rotation



bowling ball
$\boxed{\text{rolling}}$ on
rough surface

Distance traveled depends on size R_1



<u>Rolling motion</u>

1) In rotational motion, radius does matter!
2) In rolling motion involves both rotation & translation (distance traveled was πR)
3) Is there any pure rotational motion?

Yes, flipped bike wheel on a support rotates but does not translate

1) Car wheels in normal road condition: rolling motion (both translation & rotation)

2) Car wheels stuck in sand: only rotation

3) ABS Braking: anti-locking brake system: allows wheels to slowly roll to a complete stop

4) (i) When brakes are applied on wheels without ABS: only translation (sliding)
(ii) " " " " " " with ABS: rolling motion (both translation & rotation)

$\boxed{\text{shorter stopping distance is shorter}}$

To come to a stop all KE needs to be transferred out:
(i) relying on sliding motion: friction b/w tires & road
(ii) in addition some KE can be transferred to 4 wheels rotation → shorter stopping distance

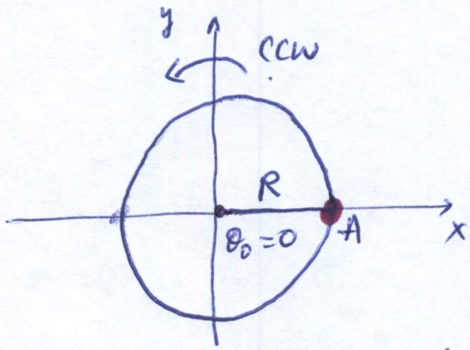
Topics: 1) Rolling motion 2) Angular acceleration α 3) Torque (radius matters)

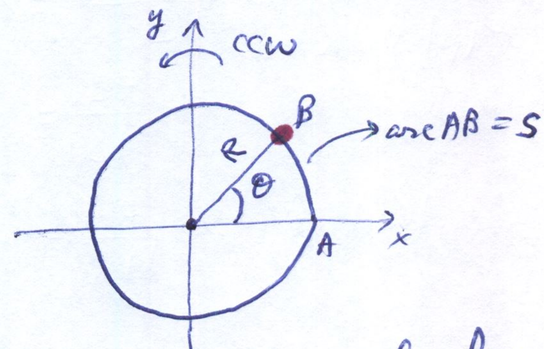
4) Moment of Inertia I (size matters)

5) KE in rotational motion

1) Rolling Motion: quantitative connection b/w translation & rotation (linear vel.) (angular vel)

Focus on a point on bowling ball and how its motion is related to the translation of the CM of the bowling ball





Center of Mass of bowling ball is @ origin of coordinates

a) $\theta = \dfrac{S}{R}$ → translation (since in rolling motion each point of the perimeter of bowling ball will touch the surface one after the other)

rotation

in rolling motion: displacement of CM equals arc S

b) $\dfrac{d}{dt}\left[\theta = \dfrac{S}{R}\right]$

$\omega = \dfrac{1}{R}\dfrac{ds}{dt}$

v (linear vel. of CM)

$\boxed{\omega = \dfrac{v}{R}}$ or $v = \omega R$

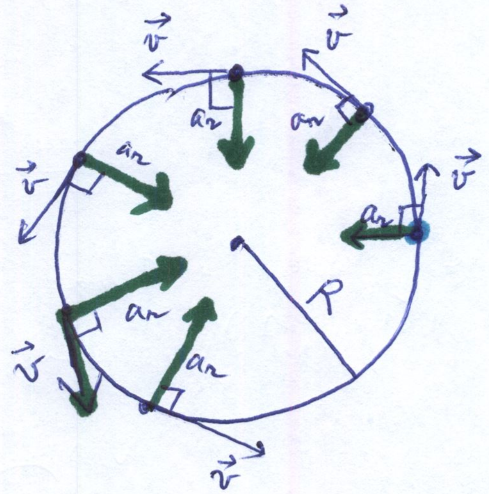
Rolling motion

$\dfrac{rad}{s} = \dfrac{\frac{m}{s}}{m} = s^{-1}$

2) Angular acceleration α

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} \; ; \quad \alpha = \frac{d\omega}{dt} \qquad \left(\frac{rad}{s^2} \text{ or } s^{-2}\right)$$

UCM



$\vec{v}$: same magnitude (Uniform → constant v) but direction is tangential to circular trajectory (direction is always changing) → needs a radial acceleration $a_r = \frac{v^2}{R}$

$$UCM = \begin{cases} a_r = \frac{v^2}{R} \quad \text{(radial acceleration)} \\ a_t = \frac{dv}{dt} = 0 \quad \text{(tangential acceleration)} \end{cases}$$

$$\alpha = \frac{d\omega}{dt} = \frac{1}{R}\frac{dv}{dt} = 0$$

$$\omega = \frac{d\theta}{dt}$$

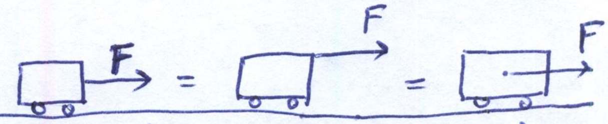
$$\left(\frac{arc}{R} = \theta \rightarrow \frac{1}{R}\underbrace{\frac{d\,arc}{dt}}_{v} = \frac{d\theta}{dt} = \omega \rightarrow \boxed{\begin{array}{c}\omega = \frac{v}{R} \\ \downarrow \\ v = \omega R\end{array}}\right.$$

Non-UCM (non-uniform circular motion): linear speed v around circular trajectory is not constant

$$\begin{cases} a_r = \frac{v^2}{R} \\ a_t = \frac{dv}{dt} \neq 0 \qquad a_t = \frac{d(\omega R)}{dt} = R\underbrace{\frac{d\omega}{dt}}_{\alpha} \text{ or } \boxed{a_t = R\cdot\alpha} \\ \alpha = \frac{1}{R}\frac{dv}{dt} \neq 0 \end{cases}$$

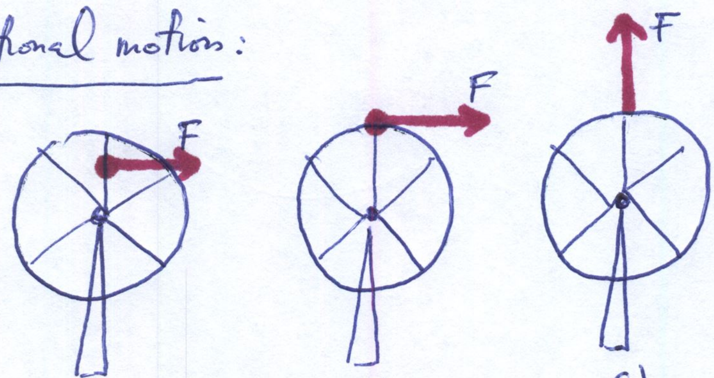
3) Torque $\vec{\tau}$ (tau, vector) (radius matters)

In linear motion:



Force application point does not matter
(in linear motion any object can be described as a single point of mass m located @ its CM)

In rotational motion:



a) b) c)

Best rotation

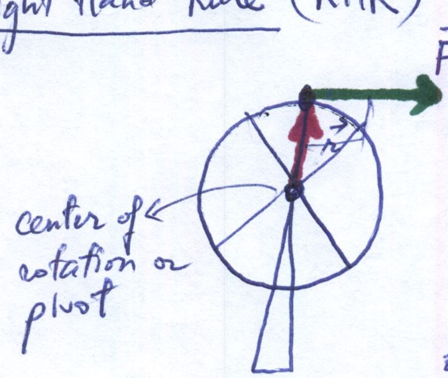
Not c) $\vec{F}$ in radial direction does not help rotation
(direction of $\vec{F}$ is important)

Not a) By applying $\vec{F}$ @ half radius, torque by $\vec{F}$ is only half of that in b). Force application point does matter to get more torque.

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad (\vec{r} \text{ "cross" } \vec{F})$$

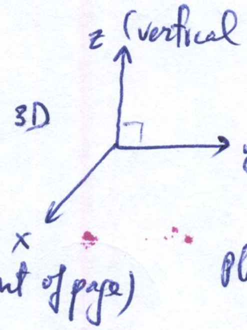
Cross product:
- → product b/w two vectors ($\vec{F}$ & $\vec{r}$) that produces another vector ($\vec{\tau}$) that is perpendicular to both ($\vec{F}$ & $\vec{r}$)
- → $\vec{\tau} = F.r.\sin\theta \; \hat{\tau}$
  - θ angle b/w $\vec{F}$ & $\vec{r}$
  - $\hat{\tau}$: unit vector in direction of $\vec{\tau}$, perpendicular to plane formed by $\vec{F}$ & $\vec{r}$, direction by Right Hand Rule (RHR)
- → Unit: Nm

Right Hand Rule (RHR)



$\vec{F}$ is applied at the highest point as shown

$\vec{r}$: position vector of the force application point from center of rotation

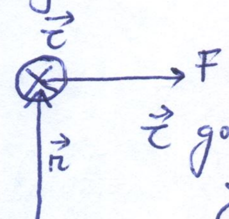


$$\left.\begin{aligned}\vec{F} &= F\hat{j} \\ \vec{r} &= R\hat{k}\end{aligned}\right\} \vec{\tau} = RF\underbrace{\sin 90}_{1}\underbrace{\boxed{\hat{k}\times\hat{j}}}_{(-\hat{\imath})}$$

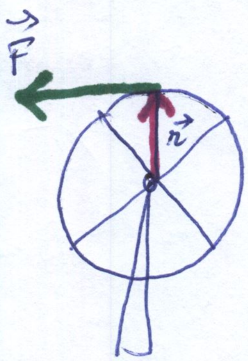
Plane xy = horizontal plane

RHR: $\hat{k}\times\hat{j}$: align RH fingers along 1st vector ($\hat{k}$), close fingers toward second vector ($\hat{j}$), RH thumb indicates direction of $\hat{k}\times\hat{j}$ or direction of $\hat{\tau}$ or $\vec{\tau}$, in this case it's $-\hat{\imath}$ or into the page $\hat{k}\times\hat{j} = -\hat{\imath}$

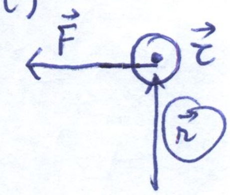
$\rightarrow \vec{\tau} = RF(-\hat{\imath})$



$\vec{\tau}$ goes into page ↓ ⊗

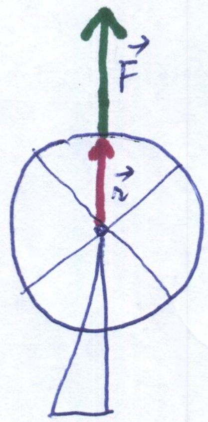


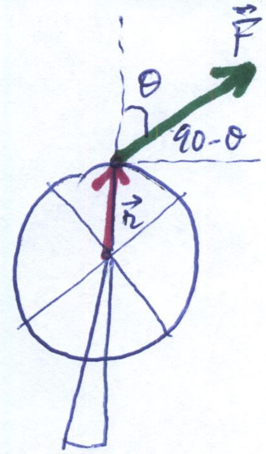
$$\vec{\tau} = RF\sin\theta\,(\underbrace{\hat{k}\times(-\hat{j})}_{-(-\hat{\imath})}) = RF\hat{\imath}$$

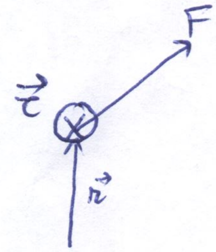


$\vec{\tau}$ goes out of page: ⊙

$\vec{\tau} = \vec{r} \times \vec{F}$

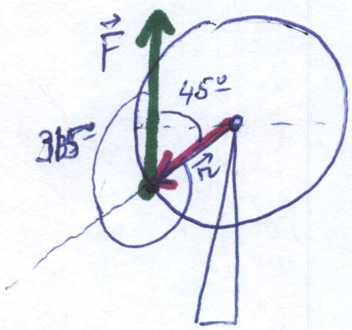






$\vec{\tau} = RF \sin\theta \, (\hat{k} \times \hat{k}) = 0$

$\vec{\tau} = RF \sin\theta \underbrace{(-\hat{i})}_{RHR} = \underbrace{-RF\sin\theta \, \hat{i}}_{<RF}$



$\vec{F}$ is applied @ midpoint of 3rd quadrant

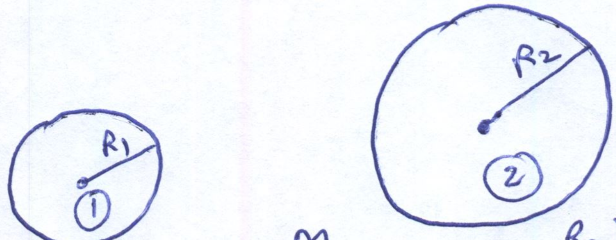
$\vec{\tau} = RF \sin\theta \, (-\hat{i}) \longrightarrow RF \, |\sin\theta| \, (-\hat{i})$ (only rely on RHR for direction of cross product)

$\sin 315° = -\sin 45°$

$\left\{ \begin{array}{l} \text{use smaller angle b/w } \vec{r} \text{ \& } \vec{F} \\ \text{or } |\sin\theta| \end{array} \right.$

4) Moment of Inertia I

- linear motion m → rotational motion I
- size does matter:



M — density ρ_1 (rho)

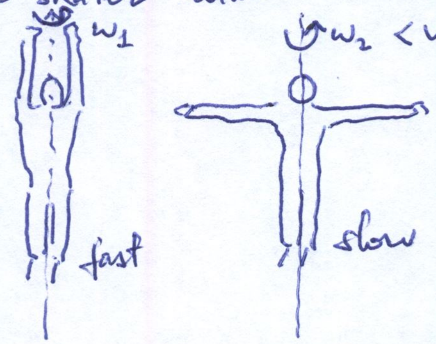
M — $R_2 > R_1$, density $\rho_2 < \rho_1$ (same M is spread over a larger volume)

→ In linear motion these two objects have the same motion (points of same mass M).

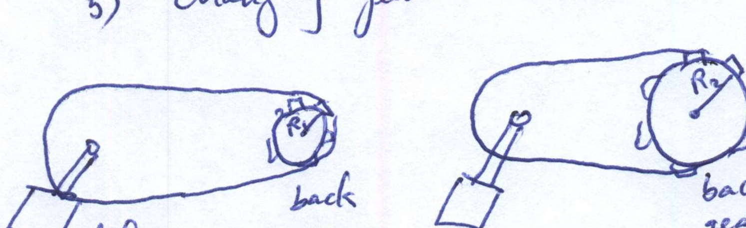
→ In rotational motion the # 2 offers more inertia: Why?

a) Figure skater rotates around their vertical axis

$\omega_2 < \omega_1 \rightarrow I_2 > I_1$

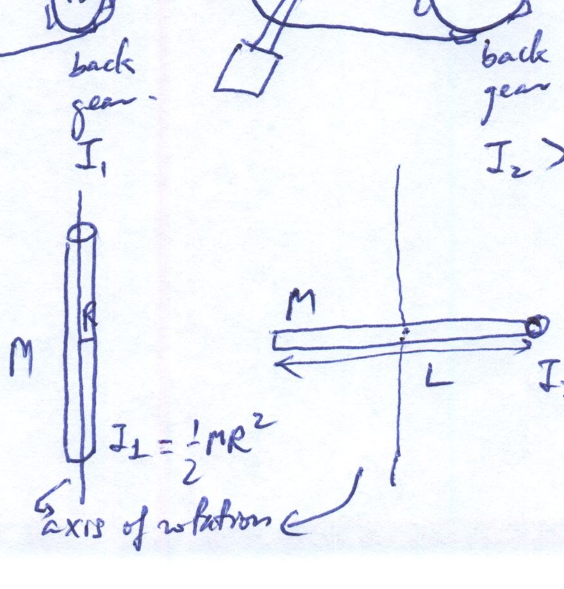


b) Changing gear in bike:



I_1 $\qquad$ $I_2 > I_1$

c)



M $\qquad I_1 = \frac{1}{2}MR^2$ $\qquad I_2 = \frac{1}{12}ML^2$

position wrt axis of rotation

$$I = \begin{cases} \text{Discrete system: } & I = \sum_i m_i r_i^2 \\ \text{Continuous system: } & I = \int r^2\, dm \end{cases}$$

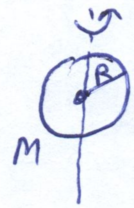
m_i: mass of component i

dm: infinitesimal mass

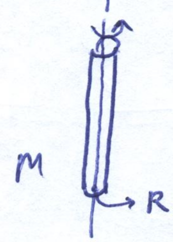
r: position of dm wrt axis of rotation

<u>Simple geometrical shapes</u> : $I = cMR^2$ $\begin{cases} M: \text{total mass} \\ R = \text{radius of mass distribution from axis of rotation} \end{cases}$

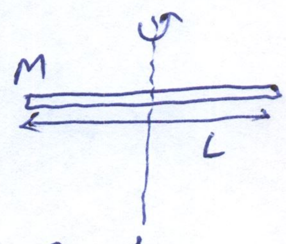
1) Sphere wrt center axis = $c = \frac{2}{5}$



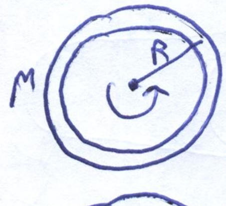
2) Cylinder wrt center axis : $c = \frac{1}{2}$



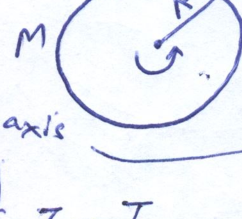
3) Cylinder of length L or rod of length L } wrt middle axis $c = \frac{1}{12}$



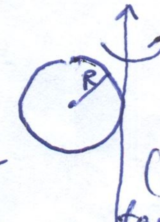
4) Ring wrt center axis $c = 1$



5) Disk wrt center axis $c = \frac{1}{2}$



6) Sphere wrt. tangential axis



$\boxed{\text{Parallel Axis Theorem, applies to any shape}}$: $I = I_{\substack{center\\axis}} + MR^2$

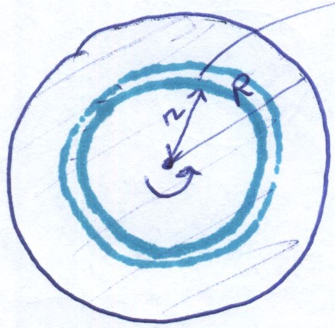
(separation b/w tangential axis & center axis is R)

$I_{\substack{tangential\\axis}} = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

Moment of Inertia for a disk wrt center axis:

$$I = \int r^2\,dm = \int_0^R r^2\,\frac{2M r\,dr}{R^2} = \frac{2M}{R^2}\int_0^R r^3\,dr = \frac{1}{2}\frac{M}{R^2}\left[r^4\right]_0^R = \frac{1}{2}MR^2$$

$\rightarrow$ Tiny ring $\begin{cases} dr \\ \text{mass } dm \rightarrow \text{fraction of } M \end{cases}$

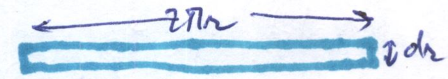


M

$$\boxed{\frac{dm}{M} = \frac{\text{surface area of ring}}{\text{surface area of disk}} = \frac{2\pi r\,dr}{\pi R^2}}$$

Ring

$\rightarrow$

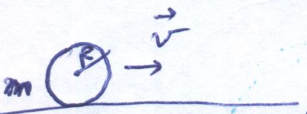


Surface area of ring is $2\pi r\,dr$

$\rightarrow dm = \dfrac{2M r\,dr}{R^2}$

Kinetic Energy in Rotational Motion:

1. linear motion



Disk on frictionless surface

$KE = \frac{1}{2}\boxed{m}v^2$

2. pure rotation

same disk on support pivot at center of disk (no friction at pivot)

No translation $v_{CM} = 0$

Rotation ω

$KE = \frac{1}{2} I \omega^2$

$I = \frac{1}{2} m R^2$

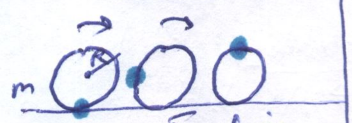
(curiosity: if we call v = speed of a point on outer edge of disk

$KE = \frac{1}{2} \frac{1}{2} m \underbrace{R^2 \omega^2}$

$= \frac{1}{2}\left(\frac{1}{2} m\right) v^2$)

3. rolling motion

$\boxed{v_{CM} = \omega . R}$



Friction sufficient for non-skidding motion

Disk rolling = Translation of CM + rotation wrt center axis of disk

$KE = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I \omega^2$

$= \frac{1}{2} m v_{CM}^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right)\left(\frac{v_{CM}}{R}\right)^2$

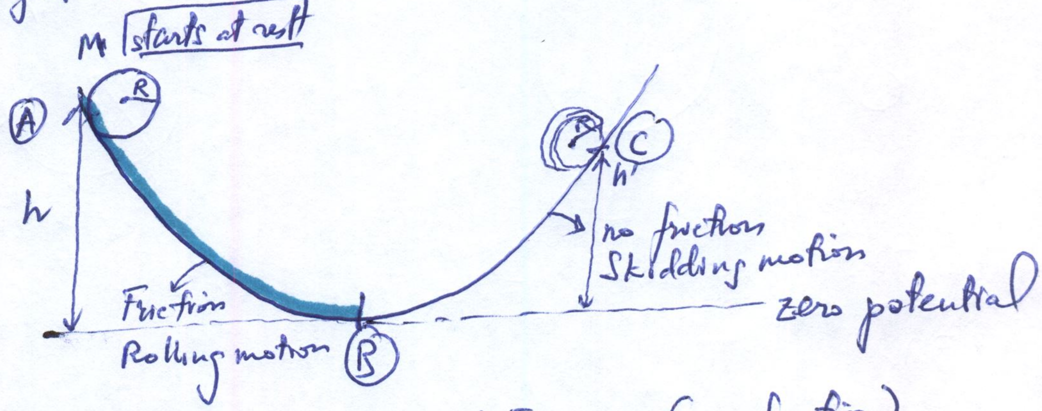
$= \frac{1}{2} m v_{CM}^2 + \frac{1}{4} m v_{CM}^2$

$= \frac{1}{2}\boxed{\left(\frac{3}{2} m\right)} v_{CM}^2$

Rolling increases effective inertia by 50% compared to skidding or sliding

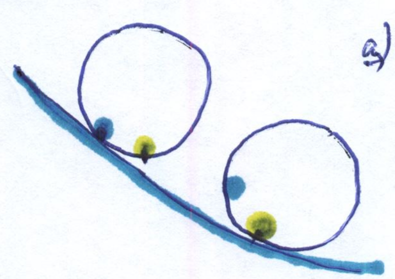
ABS braking system: effective inertia is 1.5m → smaller v_{CM} → shorter stopping distance → traffic safety.

10.64 | Sphere rolls down a parabola on left then slides up on right:



Statements 1) B/w B & C $\quad ME_B = ME_C$ (no friction)

2)



a) There is only one point of contact w/ surface @ any time

b) Sphere does not slide down on rough surface but rolls down on this rough surface → Friction here only allows rotation in rolling motion; it does not oppose motion.

a) $ME_A = ME_B$

3) B is singular or special → B_- (just before B: rolling w/ rotation), B_+ (just after B: only skidding motion)

left side: A (initial) $\quad B_-$ (final)

right side: B_+ (initial) $\quad C$ (final) No further rotation

$$Mgh = \underbrace{\frac{1}{2}Mv_B^2}_{\text{Translation of CM}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{rotation wrt center axis}}$$

$$\frac{1}{2}\cancel{M}v_B^2 = \cancel{M}gh'$$

$$\boxed{h' = \frac{v_B^2}{2g} = \frac{\frac{10}{7}\cancel{g}h}{2\cancel{g}} = \frac{5}{7}h}$$

Sphere wrt center axis → $\boxed{I = \frac{2}{5}MR^2}$ $\quad = \frac{1}{2}Mv_B^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_B}{R}\right)^2$

Rolling Motion → $\boxed{v_B = \omega \cdot R}$ $\quad = \frac{1}{2}Mv_B^2 + \frac{1}{2}\left(\frac{2}{5}M\right)v_B^2$

$$\cancel{M}gh = \frac{1}{2}\left(\frac{7}{5}\cancel{M}\right)v_B^2 \rightarrow v_B^2 = \frac{10}{7}gh$$

Ch 11 Rotational Vectors & Angular Momentum $\vec{L}$

Linear | linear momentum $\vec{p}$ ⟷ conservation ⟷ angular momentum $\vec{L}$ | **rotational**

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$ → **Most general form** or version of Newton's 2nd Law → $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

m is constant → $\vec{F}_{net} = m \cdot \vec{a}$ → Most popular version → $\vec{\tau}_{net} = I \cdot \vec{\alpha}$

(m: linear inertia; I: rotational inertia)

linear momentum $\vec{p} \equiv m\vec{v}$

angular momentum (size matters) $\vec{L} \equiv \vec{r} \times \vec{p}$

of an object wrt the axis of rotation is the cross product b/w its position vector $\vec{r}$ and its linear momentum $\vec{p}$

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1)
$$\left.\begin{aligned} \vec{\tau} &\equiv \vec{r} \times \vec{F} \\ \vec{L} &\equiv \vec{r} \times \vec{p} \end{aligned}\right\} \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

$\begin{cases} \frac{d\vec{p}}{dt} = \vec{F} \\ \vec{r} \text{ constant} \end{cases}$

2)
$$\vec{\tau} = I \cdot \vec{\alpha} \quad \text{(Most popular version)}$$
$$= I \cdot \frac{d\vec{\omega}}{dt}$$
$$\underset{I\ \text{const}}{=} \frac{d}{dt}(I \cdot \vec{\omega})$$

(M, R constant

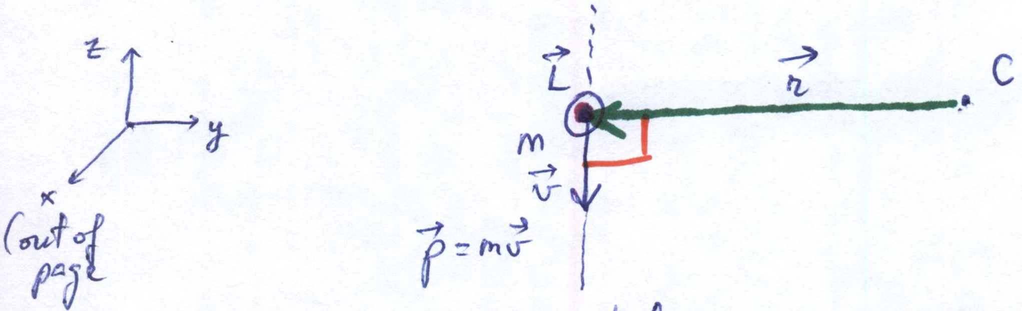
$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \text{(Most general version)}$$

$$\boxed{\vec{L} = I \cdot \vec{\omega}}$$

$$\boxed{\vec{L} = \begin{cases} \text{Most general} = \vec{r} \times \vec{p} \\ \text{Most popular} = I \cdot \vec{\omega} \end{cases}}$$

Angular momentum calculations:

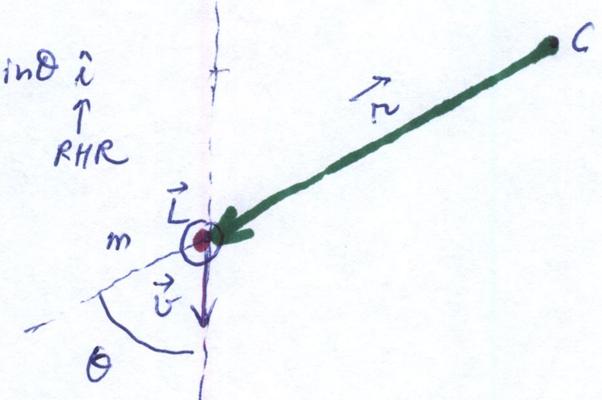
① Object mass m moving along -z axis $(-\hat{k})$, "axis of rotation" is located @ C



Although there is no rotation, once an axis of rotation is selected, we can calculate its angular momentum (wrt axis of rotation) $\vec{L} = \vec{r} \times \vec{p} = rp \underbrace{\sin 90°}_{1} \underbrace{(-\hat{j}) \times (-\hat{k})}_{\hat{i} \text{ (RHR)}} = rp\,\hat{i}$ (out of page)

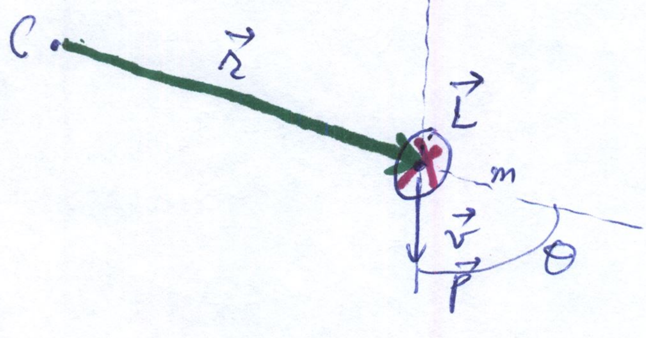
②

$\vec{L} = \vec{r} \times \vec{p} = rp \sin\theta \; \hat{i}$
↑
RHR

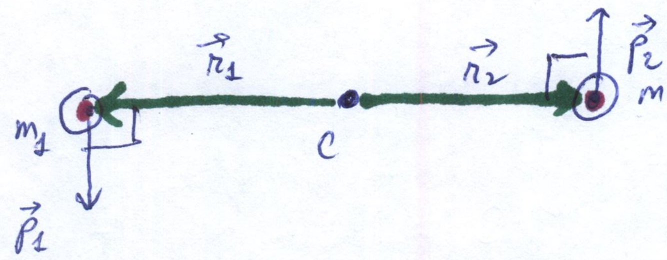


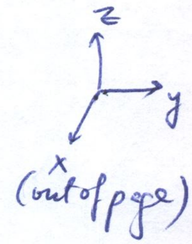
③

$\vec{L} = \vec{r} \times \vec{p} = rp \sin\theta \, (-\hat{i})$
(RHR)



(4)

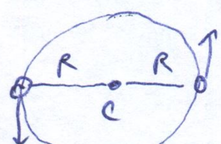




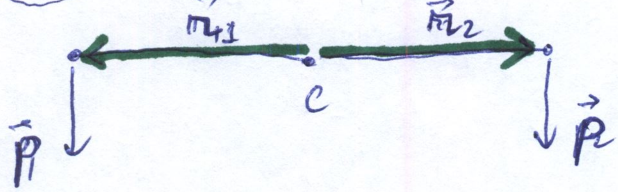
Total angular momentum $\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$

$= r_1 p_1 \hat{i} + r_2 p_2 \hat{i}$

$m_1 = m_2$; $r_1 = r_2 = R$; $p_1 = p_2 \equiv p$ $\rightarrow$ $\vec{L} = 2Rp\,\hat{i}$



(5)



$\vec{L} = \vec{r}_1 \times \vec{p} + \vec{r}_2 \times \vec{p}$

$= Rp\,\hat{i} - Rp\,\hat{i} = 0$

Most general version of analog of Newton's 2nd Law:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

$$\text{If} \quad 0 = \frac{d\vec{L}}{dt} \rightarrow \vec{L} \text{ is constant}$$

Conservation of angular momentum $\boxed{\vec{L}_i = \vec{L}_f}$

Most general $\vec{r}_i \times \vec{p}_i = \vec{r}_f \times \vec{p}_f$

Most popular $I_i \omega_i = I_f \omega_f$

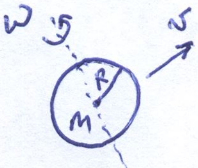
$L_i = L_f$

I flipped the bike wheel (changed direction of rotation)

$0 + L_{wheel} = \underbrace{2L_{wheel}}_{L_{me}} + (-)L_{wheel}$

L_{me} (I started to rotate)

10.36 Baseball: both $\underbrace{\text{translation of CM}}_{v_{cM}}$ & rotation wrt. its $\underbrace{\text{center axis}}_{\omega}$



$M = 0.15\,kg$

$R = 0.037\,m$

$v_{cM} = 33\,\frac{m}{s}$

$\omega = 42\,\frac{rad}{s}$

→ sphere ↓ $I = \frac{2}{5}MR^2$

$$\frac{KE_{rotation}}{KE_{Translation} + KE_{Rotation}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2}$$

$$= \frac{\cancel{\frac{1}{2}}\frac{2}{5}\cancel{M}R^2\omega^2}{\cancel{\frac{1}{2}}\cancel{M}v^2 + \cancel{\frac{1}{2}}\frac{2}{5}\cancel{M}R^2\omega^2} = \frac{\frac{2}{5}R^2\omega^2}{v^2 + \frac{2}{5}R^2\omega^2}$$

$$= \frac{\frac{2}{5}\cdot 0.037^2\cdot 42^2}{33^2 + \frac{2}{5}\cdot 0.037^2\cdot 42^2} = 8.86\times 10^{-4} =$$

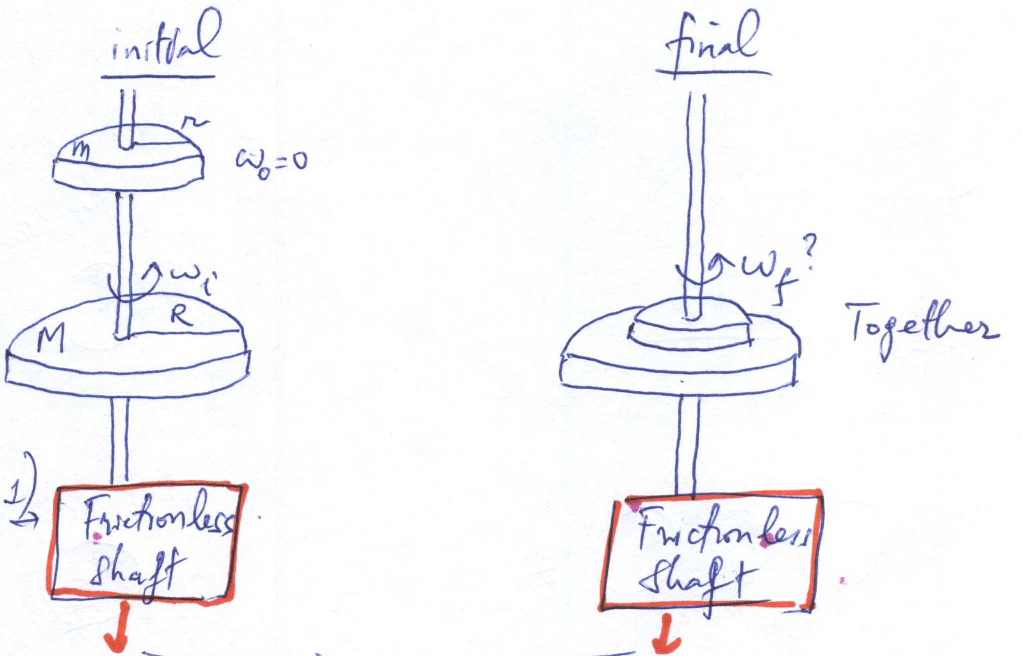
$0.0886\,\%$

$\boxed{\text{Note:}}$

→ Flying baseball $v_{cM} \neq \omega R$ $\left\{ \begin{array}{l} \omega R = 42\cdot 0.037 = 1.554\,\frac{m}{s} = v \\ v_{cM} = 33\,\frac{m}{s} \end{array} \right.$ → This is the linear speed of a point on baseball going around its center in UCM

→ Rolling baseball ~~down a a~~ on a surface $\Rightarrow$ $V_{cM} = \omega R$

11.49



$m = 0.27\,kg$
$r = 0.023\,m$

$M = 0.44\,kg$
$R = 0.035\,m$

$\omega_i = 180\,rpm$

$\vec{\tau}_{net} = 0$ on system of two disks $\Rightarrow \dfrac{d\vec{L}}{dt} = 0 \Rightarrow \boxed{L_i = L_f}$

2) $\boxed{\text{Uniform mass distribution}}$ → CM @ the axis → No torque from weights.

a) $$L_i = L_f$$

$$\underbrace{\cancel{\tfrac{1}{2}}MR^2}_{I_i} \cdot \omega_i = \underbrace{\left(\cancel{\tfrac{1}{2}}MR^2 + \cancel{\tfrac{1}{2}}mr^2\right)}_{I_f} \cdot \omega_f$$

$$\boxed{\frac{I_i}{I_f} = \frac{\omega_f}{\omega_i}} \Leftarrow \omega_f = \boxed{\frac{MR^2}{MR^2 + mr^2}}\,\omega_i = \underbrace{\frac{0.44 \cdot 0.035^2}{0.44 \cdot 0.035^2 + 0.27 \cdot 0.023^2}}_{\text{dimensionless}} 180\,rpm$$

$$\omega_f = 142\ rpm$$

b) Fraction of initial KE lost to friction? $\omega_f < \omega_i$

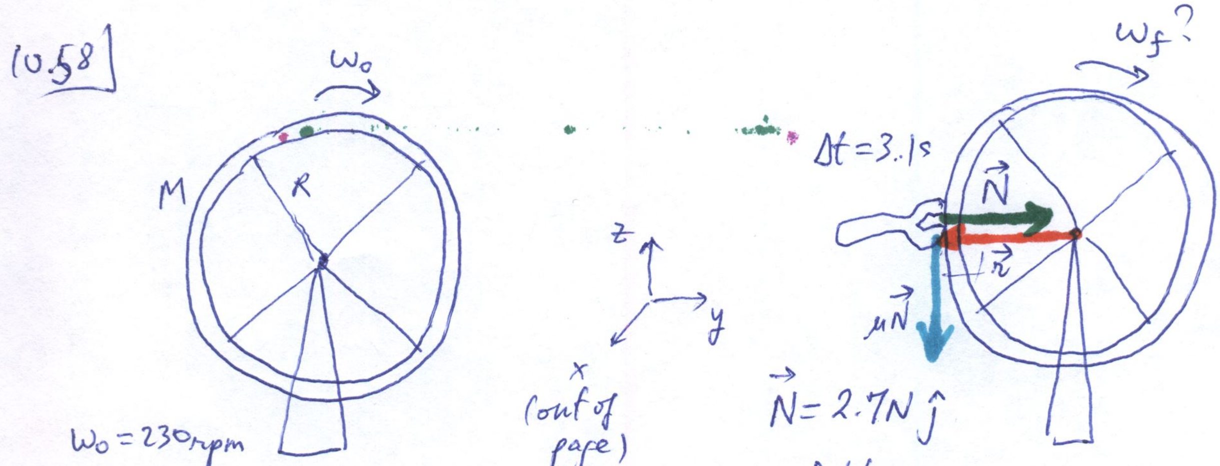
$$\frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{\cancel{\tfrac{1}{2}} I_f\, \omega_f^2}{\cancel{\tfrac{1}{2}} I_i\, \omega_i^2}$$

$$= 1 - \frac{\cancel{\tfrac{1}{2}}MR^2 + \cancel{\tfrac{1}{2}}mr^2}{\cancel{\tfrac{1}{2}}MR^2} \cdot \frac{\omega_f^2}{\omega_i^2} = 1 - \frac{0.44 \cdot 0.035^2 + 0.27 \cdot 0.023^2}{0.44 \cdot 0.035^2} \cdot \left(\frac{142^2}{180^2}\right)$$

$$= 0.265 \text{ or } \boxed{26.5\%}$$

$$\boxed{1 - \frac{KE_f}{KE_i} = 1 - \frac{\cancel{\omega_i}}{\cancel{\omega_f}} \cdot \frac{\omega_f^{\cancel{2}}}{\omega_i^{\cancel{2}}} = 1 - \frac{\omega_f}{\omega_i} = 1 - \frac{142}{180} = 0.265}$$

10.58



$\omega_0 = 230\,rpm$

$\vec{N} = 2.7N\,\hat{j}$

$M = 1.9kg$

$\mu = 0.46$
(wrench - tire)

$R = 0.33m$

→ All M is @ rim @ R = 0.33m ! $I = MR^2$

1) Does wrench apply a torque on wheel? $\vec{\tau}_N = \vec{r} \times \vec{N} = R \cdot N \sin 180 \underbrace{(\hat{j} \times (-\hat{j}))}_{0}$

2) But friction wrench-tire does apply a torque:

$$\vec{\tau}_{\mu N} = \vec{r} \times \vec{F}_{\mu N} = R \cdot \mu N \underbrace{(-\hat{j} \times -\hat{k})}_{\hat{j} \times \hat{k} = \hat{i}} = \mu N R\,\hat{i}$$

This torque is against the CW rotation of wheel.

↳ $\tau_{\mu N} = I \cdot \alpha$

I: wheel

α: angular deceleration

$$-\mu N \cancel{R} = M R^{\cancel{2}} \cdot \frac{\omega_f - \omega_0}{\Delta t} \rightarrow -\frac{\mu N \Delta t}{MR} = \omega_f - \omega_0$$

against rotation

$$\rightarrow \omega_f = \underbrace{\omega_0}_{rpm} - \underbrace{\frac{\mu N \Delta t}{MR}}_{\Delta\omega \,\to\, rpm} = 230\,rpm - \underbrace{\underbrace{\frac{0.46 \times 2.7 \cdot 3.1}{1.9 \cdot 0.33}}_{rad/s} \frac{1\,rev}{2\pi\,rad} \cdot \frac{60\,s}{min}}_{58.6\,rpm}$$

$$\omega_f = 230 - 58.6 = 171\,rpm$$

$\boxed{11\text{-}35}$

$$L = I \cdot \omega = \frac{1}{2} M R^2 \cdot \omega$$

→ Rotor:

$R = 150\,\mu m$, $h = 2\,\mu m$ $\Big\}$ ~ disk $I = \frac{1}{2} M R^2$

$$\omega = 800\,rpm \cdot \frac{2\pi\,rad}{1\,rev} \cdot \frac{1\,min}{60\,s} = \frac{80}{6} \cdot 2\pi \,\frac{rad}{s}$$

→ $M = \rho_{Si} \cdot Vol_{Rotor} = \rho_{Si} \cdot \pi R^2 h = 2330 \,\frac{kg}{m^3} \cdot \pi \left(150\times10^{-6}\right)^2 2\times10^{-6}$

↳ $L = \_\_\_\_\_\_ \; kg\,\frac{m^2}{s}$