Ch5 Application of Newton's Equations

1) Static Equilibrium ✓
2) Multiple Objects ✓
3) Frictional Forces ✓
4) Circular Motion

Solution strategy: five-step process

1) Understand the problem: statements & sketch
2) Select a $\boxed{\text{convenient}}$ coordinate system
   - convenient → to simplify the analysis
     - → Most forces pointing along axes of our selected coordinate system
     - → Motion of interest pointing along an axis of the coordinate system
3) Free-body diagrams (one per object)
   - → To facilitate the calculation of net force on each object
   - → Draw components (x, y) for those forces not already aligned along the axes
4) Write Newton's 2nd Law for each object in each direction $\vec{F}_{net} = m\vec{a} \rightarrow \begin{cases} F_{net,x} = m \cdot a_x \\ F_{net,y} = m \cdot a_y \end{cases}$

   In each direction (x or y) we can combine forces arithmetically (addition or subtraction).

5) Solve for the unknowns using these equations. Write numeric answers with units and check if these numbers make sense.

1) <u>Static Equilibrium</u> : $\vec{a}=0 \leftrightarrow \vec{F}_{net}=0 \begin{cases} F_{net,x}=0 \\ F_{net,y}=0 \end{cases}$

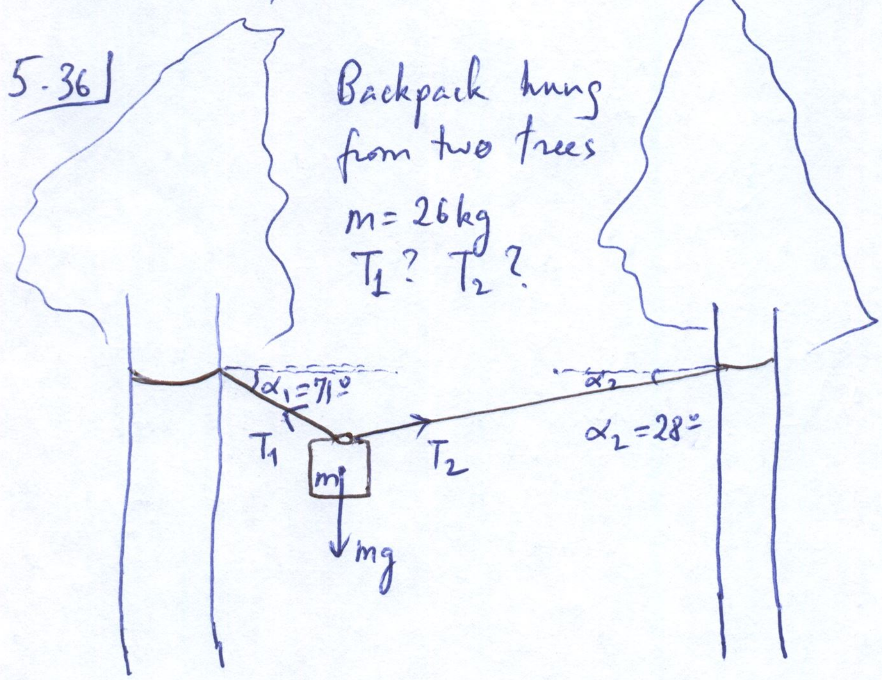
Net force on each object in each direction is zero

<u>5-36</u>

Backpack hung from two trees

$m = 26\,kg$

T_1? T_2?



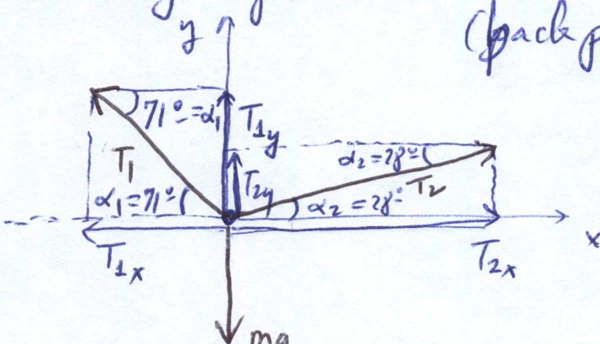
<u>Step 1</u>: <u>Statement</u>: Backpack is in static equilibrium $\vec{F}_{net}=0$

<u>Step 2</u>: Select <u>most convenient</u> coord. system (y ↑, → x)

Our 3 forces on the backpack point along different directions

→ Use standard (y ↑, → x) (mg points along $-y$)

<u>Step 3</u>: Free-body diagram : a dot will represent each object (backpack)



$$\vec{T}_1 = \underbrace{T_1 \cos 71°}_{T_{1x}}\,\hat{i} + \underbrace{T_1 \sin 71°}_{T_{1y}}\,\hat{j}$$

$$\vec{T}_2 = \underbrace{T_2 \cos 28°}_{T_{2x}}\,\hat{i} + \underbrace{T_2 \sin 28°}_{T_{2y}}\,\hat{j}$$

$$F_{net_x} = T_{2x} - T_{1x} = T_2 \cos 28° - T_1 \cos 71°$$

$$F_{net_y} = T_{1y} + T_{2y} - mg = T_1 \sin 71° + T_2 \sin 28° - mg$$

Step 4: Write Newton's 2nd Law in each direction:

$$F_{net,x} = m \cdot a_x = 0 \qquad \Big| \qquad F_{net,y} = m \cdot a_y = 0$$

a) $T_2 \cos 28° - T_1 \cos 71° = 0$

b) $T_1 \sin 71° + T_2 \sin 28° - mg = 0$

Step 5: Solve for unknowns T_1 & T_2 : system of two equations w/ two unknowns

a) $T_1 = T_2 \dfrac{\cos 28°}{\cos 71°}$

b) $T_2 \dfrac{\cos 28°}{\cos 71°} \sin 71° + T_2 \sin 28° - mg = 0$

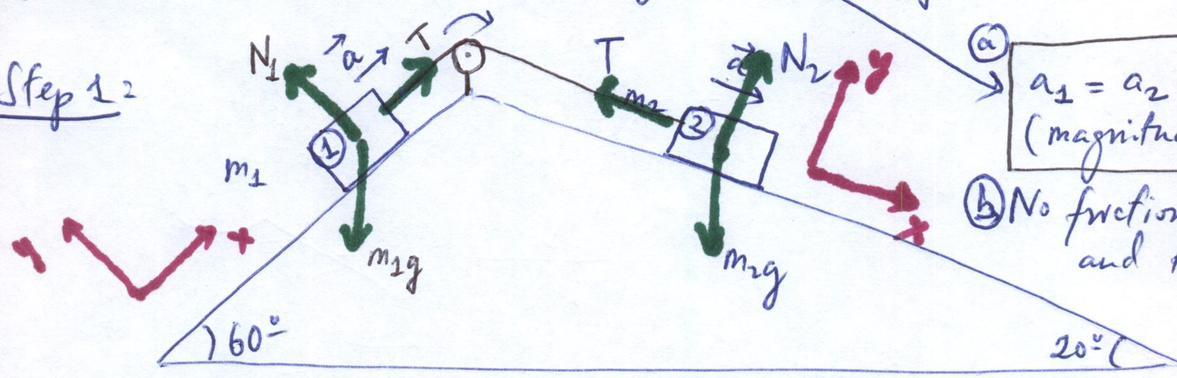
$$\Rightarrow T_2 = \frac{mg}{\cos 28° \cdot \tan 71° + \sin 28°} = 84\,N$$

$\rightarrow$ a) $T_1 = 84 \dfrac{\cos 28°}{\cos 71°} = 228\,N > T_2$! (makes sense)

2) Multiple Objects: Five-step solution

Two boxes m_1 & m_2 connected by a massless rope (tension is the same throughout the rope)

Step 1:



ⓐ $\boxed{a_1 = a_2 = a \text{ (magnitudes)}}$

ⓑ No friction b/w boxes and the surfaces

ⓒ Directions of motions:

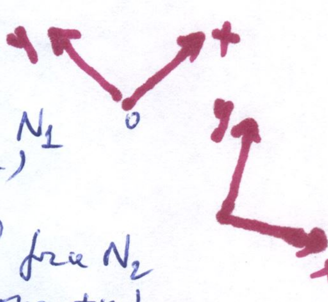
↳ 3 scenarios
- i) Box ① moves up Box ② moves down
- ii) Box ① moves down Box ② moves up
- iii) Static equilibrium $(a=0)$

We will start the analysis assuming either i) or ii)

If a turns out positive → assumption = outcome or actual happening. If a turns out negative → outcome or actual happening is the opposite to assumption.

Step 2: Most convenient coordinate system

↳ forces & direction of motion along axis



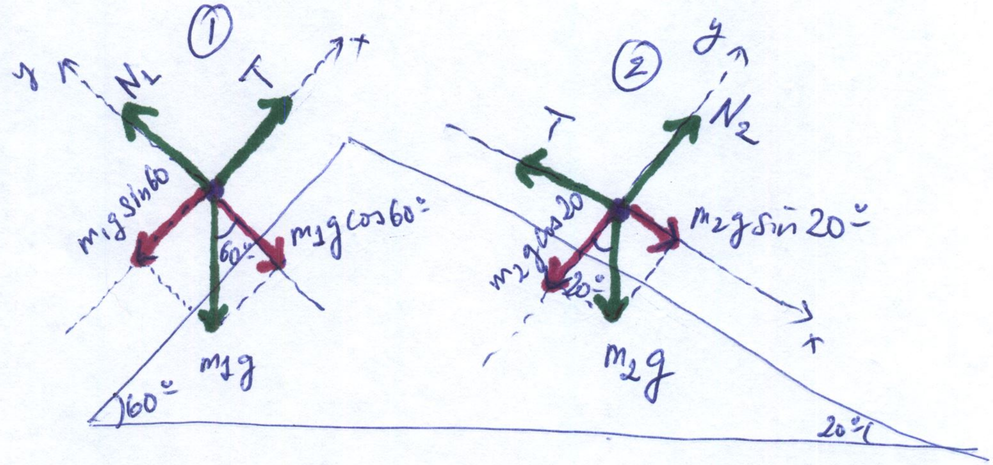
Box 1: . motion along slope 60° (in x)

. vertical weight, tension T (along $+x$), normal force N_1 (along $+y$)

Box 2: . motion along slope 20° (along $+x$)

. vertical weight, tension T (along $-x$), normal force N_2 (along $+y$)

Step 3: FBD's (two, one per object)



→ Angle: two angles whose sides are perpendicular to each other, the angles are congruent.

→ Weights are off axes → find components along axes

Net forces

Box 1
$$F_{net_x} = T - m_1 g \sin 60°$$
$$F_{net_y} = N_1 - m_1 g \cos 60°$$

Box 2
$$F_{net_x} = m_2 g \sin 20° - T$$
$$F_{net_y} = N_2 - m_2 g \cos 20°$$

Step 4: Write Newton's 2nd Law for each object in each direction

Newton's 2nd Law $\vec{F}_{net} = m \cdot \vec{a}$

Box 1
1) $T - m_1 g \sin 60° = m_1 \cdot a$ (accel. in +x was a)
2) $N_1 - m_1 g \cos 60° = 0$

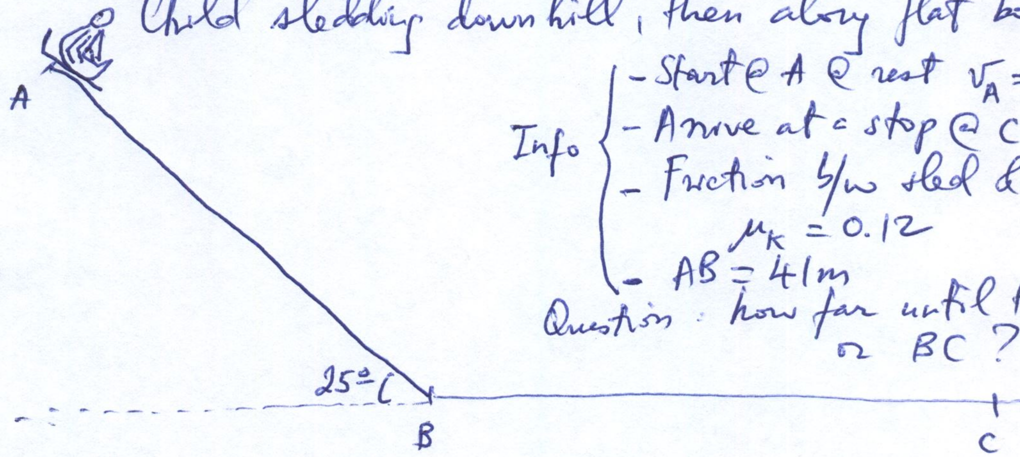
Box 2
3) $m_2 g \sin 20° - T = m_2 \cdot a$
4) $N_2 - m_2 g \cos 20° = 0$

Boxes are not jumping on slopes

(boxes are connected by a rope)

3) Frictional Forces: Five-step solution

5.50 Child sledding downhill, then along flat bottom $A \to B \to C$



Info:
- Start @ A @ rest $v_A = 0$
- Arrive at a stop @ C $\to v_C = 0$
- Friction b/w sled & slope: $\mu_K = 0.12$
- $AB = 41m$

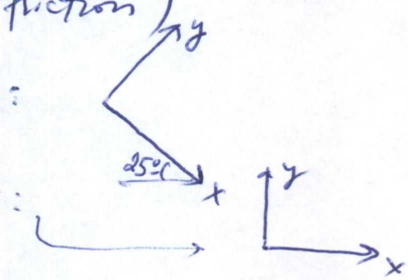
Question: how far until they stop or BC?

Step 1:

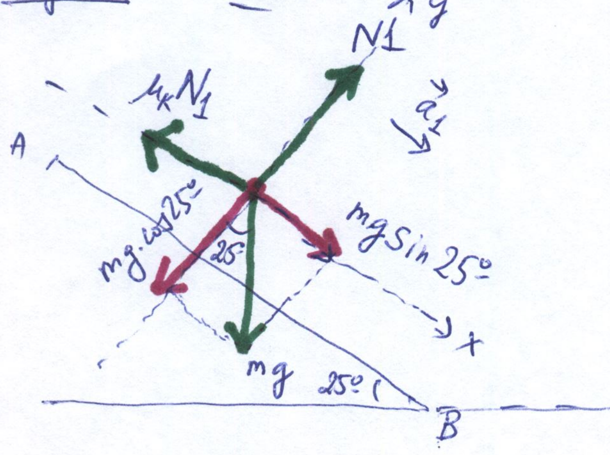
Statements:
a) Why do they stop @ C? Due to friction
b) From A to B: constant acceleration (assuming they can overcome static friction)
c) From B to C: constant deceleration (b/c friction)

Step 2 Convenient coord. systems $\to$ direction of motion along x-axis (1D motion)

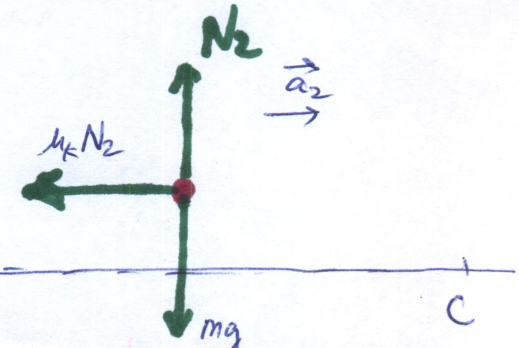
A to B:
B to C:



Step 3: FBD



Note: Frictional forces always point in the opposite direction as the motion



Step 5: Solve for unknowns:

For example: assume we know the masses m_1 & m_2
→ find the acceleration of the system a

Check: given m_1 & m_2 → 4 unknowns T, a, N_1, N_2,
also 4 equations → Yes!

Eliminate T, then solve for a

1) $T = m_1 a + m_1 g \sin 60°$

→ 3) $m_2 g \sin 20° - m_1 g \sin 60° - m_1 a = m_2 a$

$$a = \frac{m_2 g \sin 20° - m_1 g \sin 60°}{m_1 + m_2}$$

This sign has to be ⊖ → 3 options for a

a) $m_2 \sin 20 = m_1 \sin 60°$

or $\boxed{\frac{m_2}{m_1} = \frac{\sin 60°}{\sin 20°}}$

a=0 or Static Equilibrium

b) $m_2 \sin 20 > m_1 \sin 60°$

or $\boxed{\frac{m_2}{m_1} > \frac{\sin 60°}{\sin 20°}}$

a>0 System goes CW @ pulley (Box 1 up, Box 2 down)

c) $m_2 \sin 20 < m_1 \sin 60°$

or $\boxed{\frac{m_2}{m_1} < \frac{\sin 60°}{\sin 20°}}$

a<0 System goes CCW at pulley (Box 1 down, Box 2 goes up)

Note: if that sign was ⊕ ~~these three~~ only ~~options~~ option 2 is possible, which is not true for any values for m_1 & m_2.
So a ⊖ there is correct, a ⊕ there would be an error.

~~Step~~ Net forces

A to B:
$$\begin{cases} F_{net\,x} = mg\sin 25^\circ - \mu_K N_1 \\ F_{net\,y} = N_1 - mg\cos 25^\circ \end{cases}$$

B to C:
$$\begin{cases} F_{net\,x} = -\mu_K N_2 \\ F_{net\,y} = N_2 - mg \end{cases}$$

Step 4: Write Newton's 2nd Law:

Newton's 2nd Law $\vec{F}_{net} = m \cdot \vec{a}$

A to B:
$$\begin{cases} 1)\ mg\sin 25^\circ - \mu_K N_1 = m a_1 \\ 2)\ N_1 - mg\cos 25^\circ = 0 \end{cases}$$

B to C:
$$\begin{cases} 3)\ -\mu_K N_2 = m \cdot a_2 \\ 4)\ N_2 - mg = 0 \end{cases}$$

Step 5: Find distance BC → need solve for $a_2 = \frac{-\mu_K N_2}{m}$

$a_2 \overset{4)}{=} -\mu_K \frac{\cancel{m}g}{\cancel{m}}$

$\boxed{a_2 = -\mu_K g}$

$x - x_0$; $v_C = 0$; v_B

Kinematic eq 3)
$$\boxed{\frac{\cancel{v_C^2}^{\,0} - v_B^2}{(x-x_0)_{BC}} = 2 \cdot a_2}$$

Need v_B

Motion b/w A & B: need a_1 → 1) & 2) $\cancel{m}g\sin 25^\circ - \mu_K \cancel{m}g\cos 25^\circ = \cancel{m}a_1$

$$a_1 = g(\sin 25^\circ - \mu_K \cos 25^\circ)$$

v_B

$v_A = 0$

$AB = 41m$

Kinematic eq 3)
$$\frac{v_B^2 - \cancel{v_A^2}^{\,0}}{\underbrace{(x-x_0)_{AB}}_{41m}} = 2 \cdot a_1 \rightarrow v_B = \sqrt{2 \cdot g(\sin 25^\circ - \underset{0.12}{\mu_K}\cos 25^\circ) \cdot 41}$$

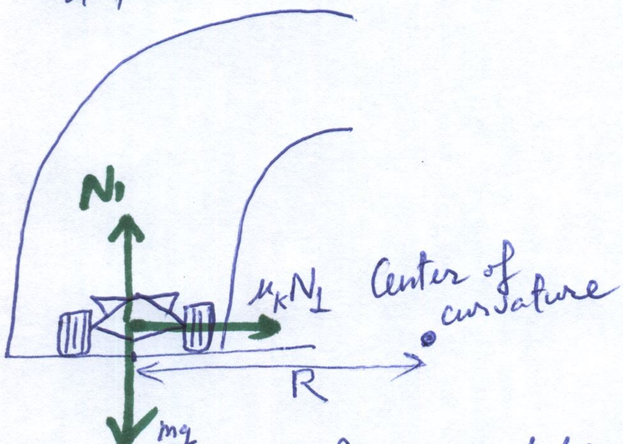
$\boxed{v_B = 15.9\ \frac{m}{s}}$

$$(x-x_0)_{BC} = \frac{-v_B^2}{2 \cdot (-\mu_K g)} = \frac{15.9^2}{2 \cdot 0.12 \cdot 9.81} = \boxed{107m}$$

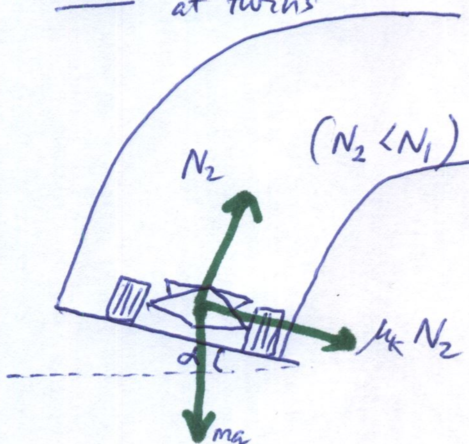
4) Circular Motion: Race car tracks 5-step solution

Step 1:

Flat race car track at turns



Slanted race car track at turns

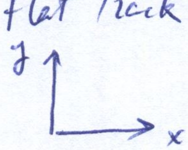


Statements: a) Racecars taking turns in UCM (constant speed but not constant velocity $a = \frac{v^2}{R}$)

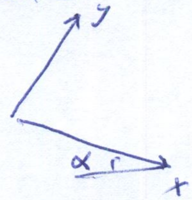
b) What keep cars on track? Friction is the agent to cause the change of direction and provide the acceleration toward center of curvature $a = \frac{v^2}{R}$ → Wide tries: more friction → faster turn & sharper.

Step 2:

Flat track:

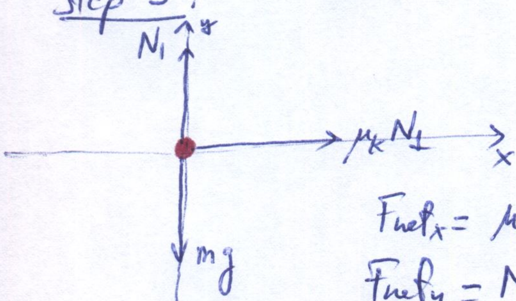


Slanted track



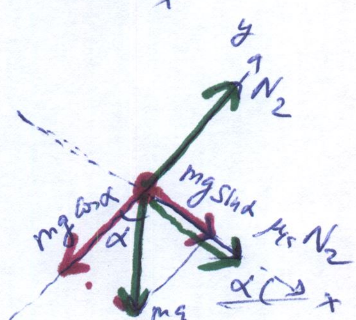
Step 3:

FBD's



$F_{net_x} = \mu_k N_1$

$F_{net_y} = N_1 - mg$



$F_{net_x} = \mu_k N_2 + mg \sin\alpha$

$F_{net_y} = N_2 - mg \cos\alpha$

Step 4: Newton's 2nd Law

Newton's 2nd Law
$\vec{F}_{net} = m \cdot \vec{a}$

Flat track: $\begin{cases} \mu_K N_1 = m \dfrac{v^2}{R} \\ N_1 - mg = 0 \end{cases}$ $\mu_K \cancel{m} g = \cancel{m} \dfrac{v^2}{R}$

$$\boxed{v_{flat} = \sqrt{\mu_K g R}}$$

Slanted track $\begin{cases} \mu_K N_2 + mg \sin\alpha = m \cdot \dfrac{v^2}{R} \\ N_2 - mg\cos\alpha = 0 \end{cases}$

$$\cancel{m} g(\mu_K \cos\alpha + \sin\alpha) = \cancel{m} \frac{v^2}{R}$$

$$\boxed{v_{slanted} = \sqrt{gR(\mu_K \cos\alpha + \sin\alpha)}}$$

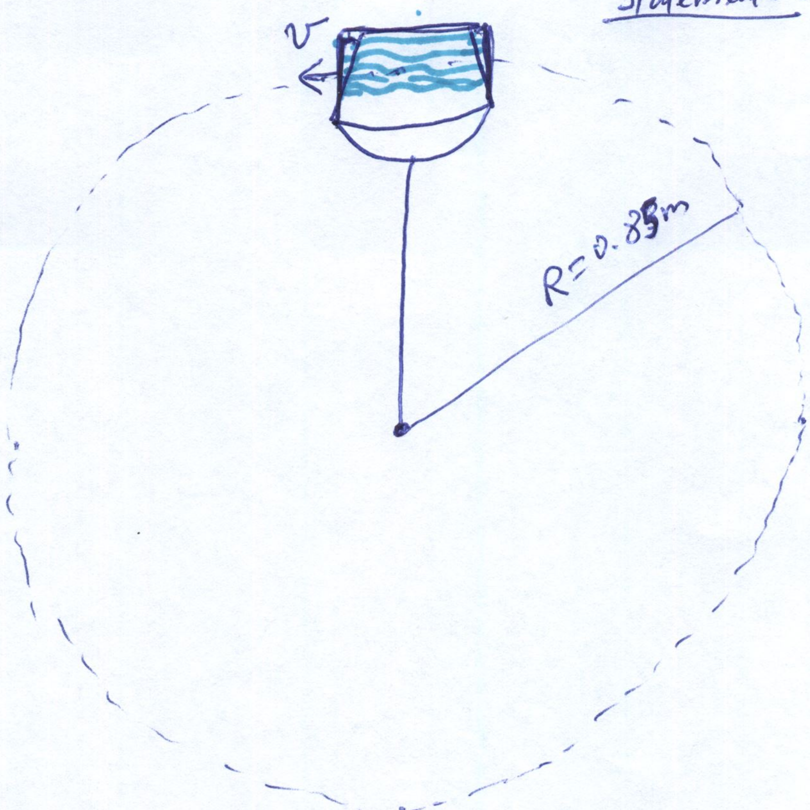
Let's compare: $\dfrac{v_{slanted}}{v_{flat}} = \sqrt{\dfrac{\mu_K \cos\alpha + \sin\alpha}{\mu_K}} = 1.63$

$\left.\begin{matrix} \alpha = 20° \\ \mu_K = 0.2 \end{matrix}\right\}$

Slanted track allows faster speed @ turns!

5-42 Bucket of water in vertical circle ($R = 0.85m$)

Step 1: Hardest point to keep water in bucket is when bucket is inverted!



Statements: 1) Bucket & water: UCM (@ constant v. Velocity $\vec{v}$ is changing → $|\vec{a}| = \frac{v^2}{R}$

2) Find v_{min}

$\boxed{v < v_{min}}$: water will fall out

$\boxed{v > v_{min}}$: water will stick to bottom of bucket (will stay)

$\boxed{v = v_{min}}$: water barely touches bottom of bucket

Step 2: convenient coord. system for our object: water inside bucket

Step 3: FBD (water)

at v_{min} $N = 0$
(when $v > v_{min}$ $N \neq 0$ pointing also downward)

$F_{net} = mg$

Step 4: Newton's 2nd Law for water:

$F_{net} = m \cdot a$

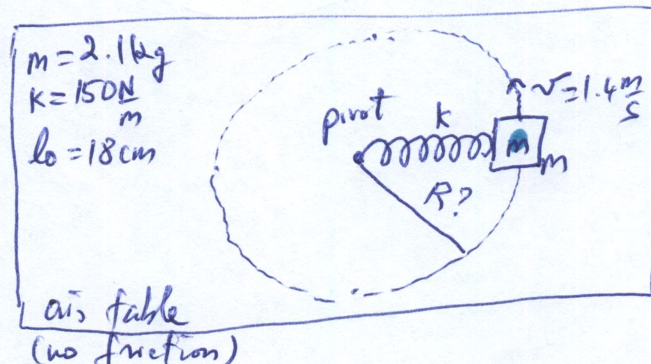
$\cancel{m} g = \cancel{m} \cdot \frac{v_{min}^2}{R} \Rightarrow v_{min} = \sqrt{g \cdot R}$

Step 5: $v_{min} = \sqrt{9.81 \cdot 0.85} = 2.89 \frac{m}{s}$

5.65 Five-step solution

Step 1: mass m attached to a spring, both in UCM ($v = 1.4 \frac{m}{s}$) on an air table (no friction), about a pivot. Find R radius of circular trajectory

View from above (2D):

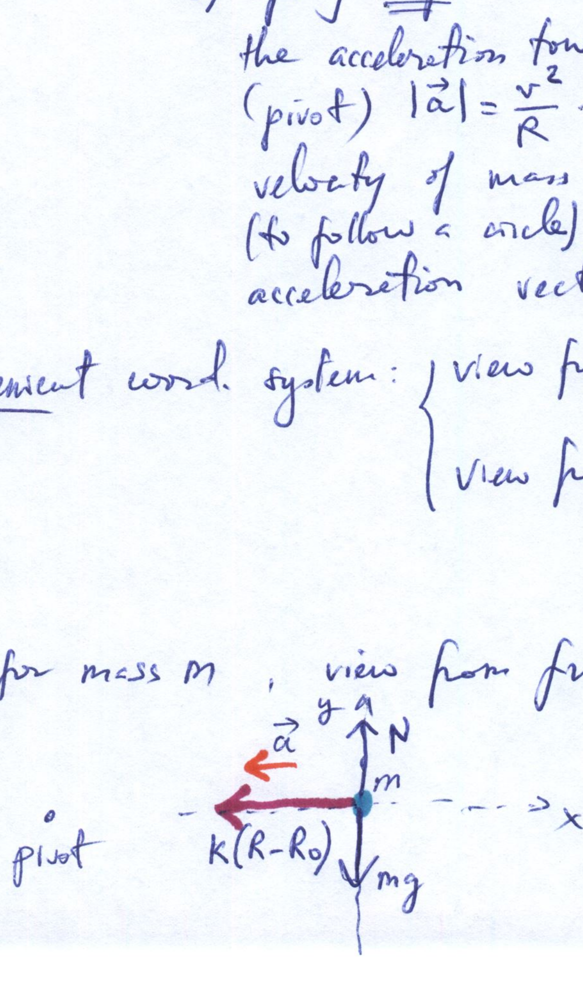


Statements: a) Mass m in UCM @ constant speed $v = 1.4 \frac{m}{s}$
Its velocity $\vec{v}$ is not constant $\rightarrow |\vec{a}| = \frac{v^2}{R}$

b) Spring keeps m in UCM: it is providing ($k\Delta x$) the acceleration toward center of curvature (pivot) $|\vec{a}| = \frac{v^2}{R}$. The spring force keeps velocity of mass m changing direction (to follow a circle) and provides the changing acceleration vector $\vec{a}$

Step 2: convenient coord. system: { view from front side: y up, x to the right
{ view from above: $y \odot \rightarrow x$
(y: going out of screen toward us)

Step 3: FBD for mass m, view from front side:



$F_{net_x} = -K(R - R_0)$

$F_{net_y} = N - mg$

Step 4: Newton's 2nd Law for mass m : $\vec{F}_{net} = m\cdot\vec{a}$

$$\hookrightarrow \begin{cases} \text{in } x : & \boxed{\underbrace{-k(R-R_0)} = m\cdot\underbrace{\left(-\frac{v^2}{R}\right)}} \\ \text{in } y : & N - mg = m\cdot 0 \end{cases}$$

Step 5: Solve for R :

$$kR(R-R_0) = mv^2 \rightarrow \boxed{\overset{a}{k}R^2 \overset{b}{-kR_0}R \overset{c}{-mv^2} = 0}$$

$(R_0 = l_0 = 18\,cm = 0.18\,m)$

$\hookrightarrow$ Quadratic equation in R

$$\boxed{aR^2 + bR + c = 0}$$

$$R = \frac{-b \pm \sqrt{b^2 - 4a\cdot c}}{2a}$$

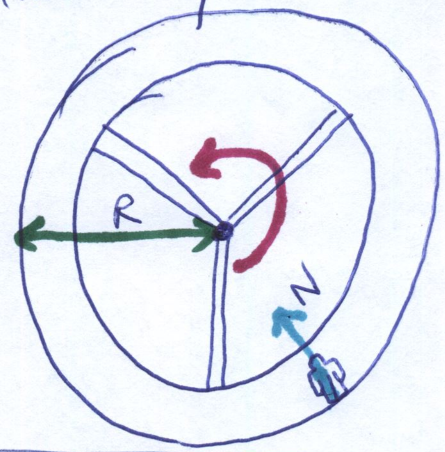
$$\rightarrow R = \frac{kR_0 \pm \sqrt{k^2R_0^2 + 4kmv^2}}{2k}$$

$$= \frac{150\cdot 0.18 \pm \sqrt{(150\cdot 0.18)^2 + 4\cdot 150\cdot 2.1\cdot 1.4^2}}{2\cdot 150} \begin{cases} 0.279\,m \\ \cancel{\text{negative}} \end{cases}$$

5.58 Five-step solution

Step 1:
- Hollow ring space station $R = \frac{D}{2} = 225\,m$
- Ring rotates CCW to simulate effect of gravity

RPM (how many rev's or turns per minute?)



Statements:

a) In outer space
$\rightarrow$ no gravitational attraction

b) Astronaut standing on outer edge of ring
$\rightarrow$ since ring in UCM
$\rightarrow$ astronaut also in UCM
$\vec{v}$ not constant $\rightarrow |\vec{a}| = \frac{v^2}{R}$
(toward center of curvature or center of ring)

c) What force is causing this Astronaut's acceleration?
Normal force N by outer edge that he stands on

Step 2: 1D coord. system $\quad y \uparrow$

Step 3: FBD for astronaut

$\uparrow N$

$F_{net_y} = N$

Step 4: Newton's 2nd Law: $\quad F_{net} = m \cdot a$

$$N = \underbrace{m}_{\text{mass of astronaut}} \cdot \frac{v^2}{R}$$

To simulate effect of gravity: $N = mg$ (when he stands on Earth)

Step 5: Solve for v: (speed of rotation of space station and of the astronaut)

$$\cancel{m}g = \cancel{m}\frac{v^2}{R}$$

$$v = \sqrt{gR} = \sqrt{9.81 \cdot 225}$$

$$\boxed{v = 46.9\ \frac{m}{s}}$$

$$46.9\ \frac{\cancel{m}}{\cancel{s}} \cdot \frac{60\ \cancel{s}}{min} \cdot \frac{1\ rev}{2\pi \cdot 225\ \cancel{m}} = 1.99\ RPM \text{ or } 1.99\ \frac{rev}{min}$$

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Ch6 Work, Energy, Power

Ch2,3 : described motion

Ch4,5 : included agent – force – that causes change of motion

Ch6 : introducing work → energy : alternative solution to Newton's Equation & Kinematic eqs.

Work & force : pushing a piano up a ramp

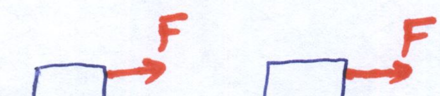
Stronger guy : more force, but what about work performed or energy spent

- more
- less ✓ Why?
- same

Work := $\vec{F} \cdot \Delta\vec{r}$ (Force applied – a vector – "dot" displacement vector $\Delta\vec{r}$)

"Dot": scalar product of 2 vectors that produces a number

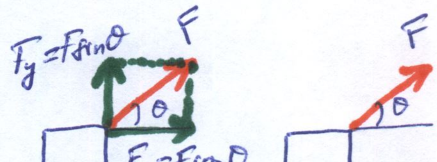
$\vec{A} \cdot \vec{B} = AB\cos(\theta)$ (θ angle b/w vector $\vec{A}$ & vector $\vec{B}$)



Pulling a suitcase from x_1 to x_2 ($\Delta x = x_2 - x_1$) by applying a constant force F : Work $= \vec{F} \cdot \Delta\vec{r}$

$= F \cdot \Delta x \cdot \cos 0$

$= F \cdot \Delta x$



F is applied @ angle θ

Work $= \vec{F} \cdot \Delta\vec{r} = F \cdot \Delta x \cdot \cos\theta$

$= \underbrace{F\cos\theta}_{F_x} \cdot \Delta x$

(work done by F_x parallel to $\Delta\vec{r} = \Delta\vec{x}$)

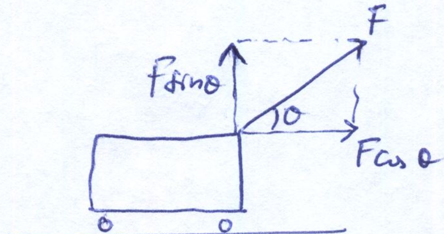
in y : $\vec{F} \cdot \Delta\vec{r} = F_y \cdot 0 \cos 90 = 0$

About work: 1) only that component of the force applied that is parallel to the displacement performs work. The component of the force applied that is perpendicular to the displacement doesn't perform work

2) Unit = N.m = J (Joule)

Energy & Work

Difference?



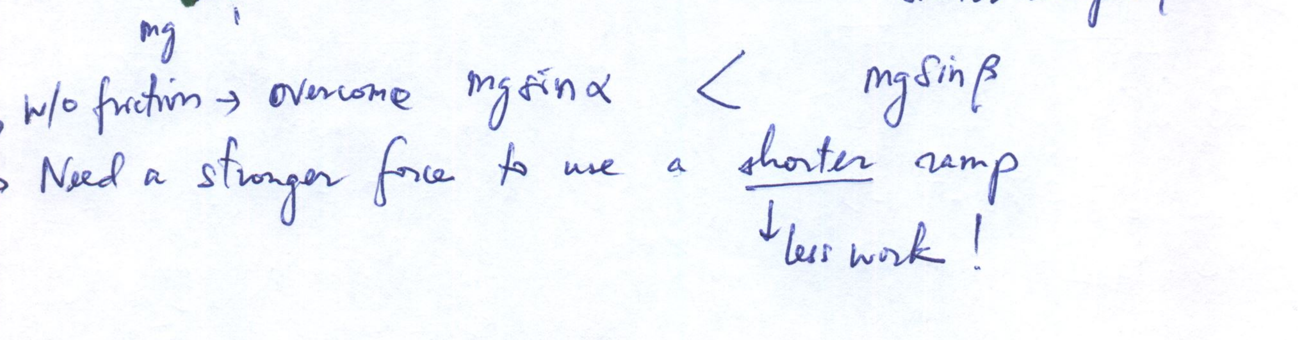
Here { only $F\cos\theta$ performs work

but energy is spent on holding suitcase up with $F\sin\theta$

→ Not efficient to pull the suitcase @ angle θ

Pushing against a wall: no work yet energy is spent

Pushing a piano up a ramp:



→ w/o friction → overcome $mg\sin\alpha$ < $mg\sin\beta$

→ Need a stronger force to use a shorter ramp

↓ less work!

Work & Power:

→ Two different cars going from rest to $40\,\frac{mi}{h}$,

same masses $m_1 = m_2 = m$

$$\text{Work} = \frac{1}{2} m v^2 \quad \text{(can be proved)} \quad \Big\{ \; W_1 = W_2 = W$$

→ Power { car 1: sedan (HP=150)
car 2: Porsche (HP: 300) → takes less time (half) since it has double power.

$\frac{\text{Work}}{\text{time}}$ → Average Power $\bar{P} = \frac{\Delta W}{\Delta t}$

→ Instantaneous Power $P = \frac{dW}{dt}$

→ Power & velocity $P = \frac{dW}{dt} = \frac{d(\vec{F} \cdot \Delta \vec{r})}{dt}$

$$= \vec{F} \cdot \underbrace{\frac{d(\Delta \vec{r})}{dt}}_{\vec{v}}$$

$\vec{F}$ constant

$$\boxed{P = \vec{F} \cdot \vec{v}}$$

$$\underbrace{N \cdot \frac{m}{s}}_{} = kg\,\frac{m^2}{s^2}$$

Units: $W = \frac{J}{s}$

Watts = Joules per second

6.36 $m = 75\,kg$ long jumper $v_0 = 0 \xrightarrow[t=3.1s]{} v = 10\frac{m}{s}$

Power output?

$$\overline{P} = \frac{Work}{\Delta t} = \frac{\Delta KE}{\Delta t} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2}{\Delta t} = \frac{\frac{1}{2}75\cdot 10^2}{3.1} = 1210\,W$$

$$KE = \frac{1}{2}mv^2$$

$$Work = \vec{F}\cdot\Delta\vec{r} \quad (\vec{F} \text{ constant})$$

$$= \int \vec{F}\cdot d\vec{r} \quad (\vec{F} \text{ not constant})$$

$$= m\int \frac{d\vec{v}}{dt}\cdot d\vec{r} = m\int dv\cdot v = \frac{1}{2}mv^2$$

Newton's 2nd Law: $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt}$

6.67 Pushing a crate @ constant speed $v = 0.62\frac{m}{s}$

$m = 95\,kg$

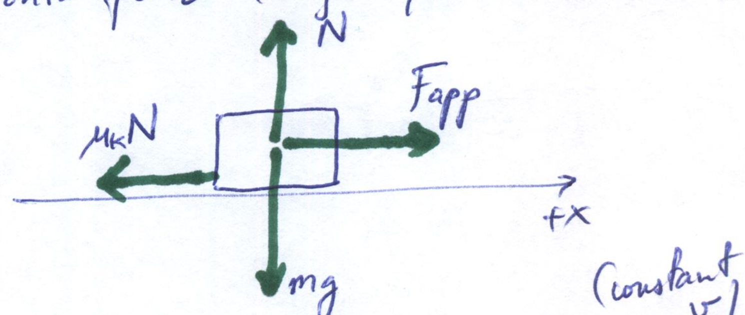
$\mu_K = 0.78$

along horizontal floor (angle b/w $\vec{F}$ & $\vec{v}$ is 0)

$\Bigg\}$ P?

a) $P = F\cdot v$

$\downarrow$ Force applied? $\downarrow$ given



(constant v)

$$Fnet_x = F_{app} - \mu_K N = m\cdot a^{\,0}$$

$$F_{app} = \mu_K N$$

$$\boxed{F_{app} = \mu_K mg}$$

$$P = \mu_K mgv = 0.78\cdot 95\cdot 9.81\cdot 0.62 = 450\,W$$

b) Pushing it 11m $\rightarrow$ $Work = F_{app}\Delta x = \mu_K mg\cdot\Delta x = 0.78\cdot 95\cdot 9.81\cdot 11 = 8000\,J = 8\,kJ$

6.72 How many lifts are needed to burn 230 kCal

↳ each lift = 45 kg up 0.5 m

Conversion: $230 \text{ kCal} = 230 \times 10^3 \cancel{\text{Cal}} \dfrac{4.186 \text{ J}}{1 \cancel{\text{Cal}}} = 963 \times 10^3 \text{ J}$

Work needed per lift: $mg \cdot \Delta h = 45 \cdot 9.81 \cdot 0.5 = 220 \text{ J}$

$\# \text{ lifts} = \dfrac{963 \times 10^3}{\cancel{\frac{1100}{5}}\ 220} = 873 \text{ lifts} \times 5 = 4359 \text{ lifts}.$

6.81 Dropped leg (dropped egg in PP#1)

1) Free fall until about to touch the ground
2) Crashing against the ground.

Statements: Leg falling off a stretcher

$m = 8\,kg$

$v_0 = 0$

1) Free fall: constant acceleration $a = g$ (doesn't hurt)

2) Crashing: there is a time interval from when it touches the ground until it completely stops

$(v \xrightarrow[\Delta t]{} 0)$

This could be a huge deceleration!

① Falling off stretcher

② About to touch the ground (CM still above ground)

③ Completely stopped (CM at ground level)

→ Free fall

→ $a = g$

constant deceleration

stopping distance $= 0.02m$

$$KE = 0J \xrightarrow[\Delta t_{12}]{} KE = \frac{1}{2} m v^2 \xrightarrow[\Delta t_{23}]{} KE = 0$$

(damaging when Δt_{23} is very small!)

Stopping force:

Method #1: Work & energy (Ch 6)

→ Energy acquired ① → ② : $mg\Delta h = 8 \cdot 9.81 \cdot 0.7\,J$

→ Energy absorbed ② → ③ → same

$\rightarrow F_{stopping} \cdot \Delta y = mg\Delta h \rightarrow F_{stopping} = \frac{8 \cdot 9.81 \cdot 0.7}{0.02}$

$= 2744 N$

Method #2 : Kinematic eqs (Ch 2,3) & Newton's 2nd Law (Ch 4,5)

Speed v (before touching ground) :

Kinematic eq 3 : $\frac{v^2 - 0^2}{\Delta h} = 2 \cdot g \rightarrow v = \sqrt{2 \cdot g \cdot \Delta h}$

$v = \sqrt{2 \cdot 9.81 \cdot 0.7} = 3.7 \frac{m}{s}$

Kinematic eq 3 : $\frac{0 - 3.7^2}{0.02} = 2 \cdot a \rightarrow a = \frac{-3.7^2}{0.04} = -343.35 \frac{m}{s^2}$

(very large deceleration!)

Newton's 2nd Law : $F_{stop} = m \cdot a = -8 \cdot 343.35 = -2774 N$

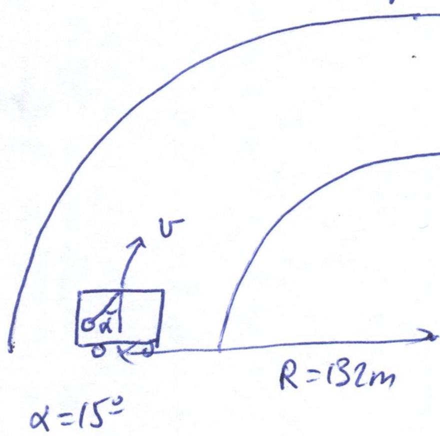
(Much more than $mg = 8 \cdot 9.81 \simeq 80 N$!)

5-25

Train in UCM $\begin{cases} R = 132m \\ v\,? \end{cases}$

Strap hanging @ 15° to vertical

Step 1:

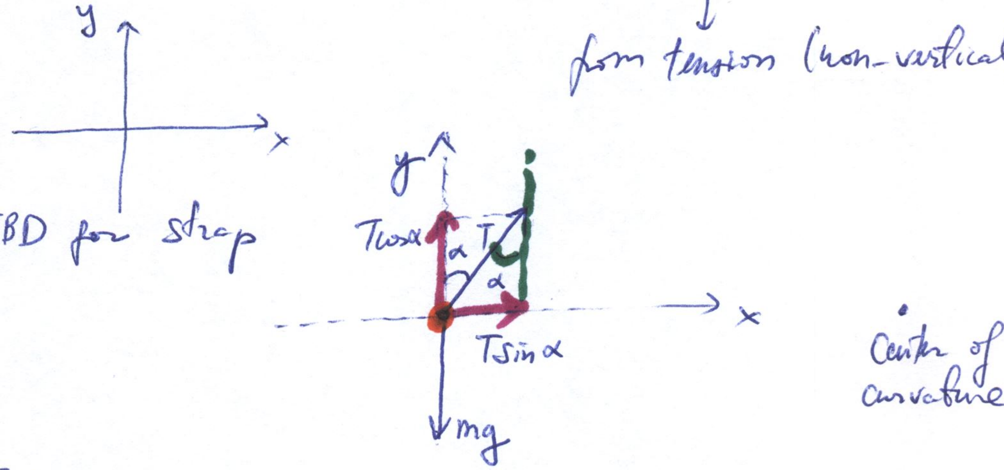


Strap is attached to the train
→ Focus on this strap

Statements:
a) Unbanked track
Strap angle is related to its speed → Why?
b) Strap in UCM → it needs an acceleration toward center of curvature $a = \frac{v^2}{R}$
from tension (non-vertical)

Step 2:

Step 3: FBD for strap



$F_{net_x} = T\sin\alpha$

$F_{net_y} = T\cos\alpha - mg$

Step 4 $\vec{F}_{net} = m\vec{a}$ $\begin{cases} F_{net_x} = T\sin\alpha = m\cdot\frac{v^2}{R} \\ F_{net_y} = T\cos\alpha - mg = m\cdot 0 \end{cases}$

Step 5: Solve for v $(\alpha = 15^\circ)$ $T = \frac{mg}{\cos\alpha} \rightarrow \frac{\cancel{m}g\sin\alpha}{\cos\alpha} = \cancel{m}\frac{v^2}{R}$

$v = \sqrt{g\tan\alpha\, R} = \sqrt{9.81\cdot\tan 15^\circ\cdot 132} = 18.6\ \frac{m}{s}$ above speed limit!

Speed limit $45\ \frac{km}{h}\ \frac{1000m}{1km}\ \frac{1h}{3600s} = \frac{45}{3.6}\ \frac{m}{s} = 12.5\ \frac{m}{s}$