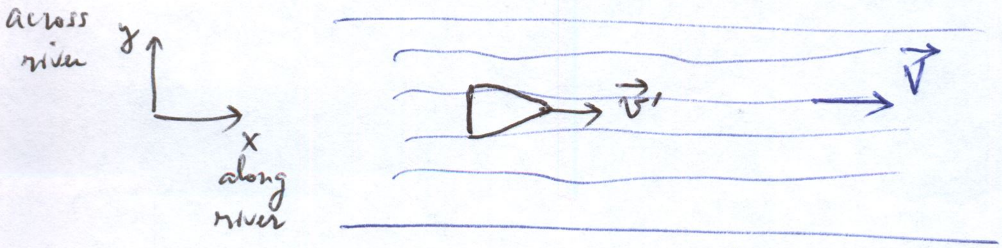
Relative Motion

1) Boat going down stream

Top view

(down stream: right)



Velocity of water: $\vec{V} = V\hat{i}$ (upper case V)

Velocity of boat wrt water $\vec{v}' = v'\hat{i}$

Velocity of boat wrt ground or river banks $\vec{v} = v\hat{i}$

$$\boxed{\vec{v} = \vec{v}' + \vec{V}}$$

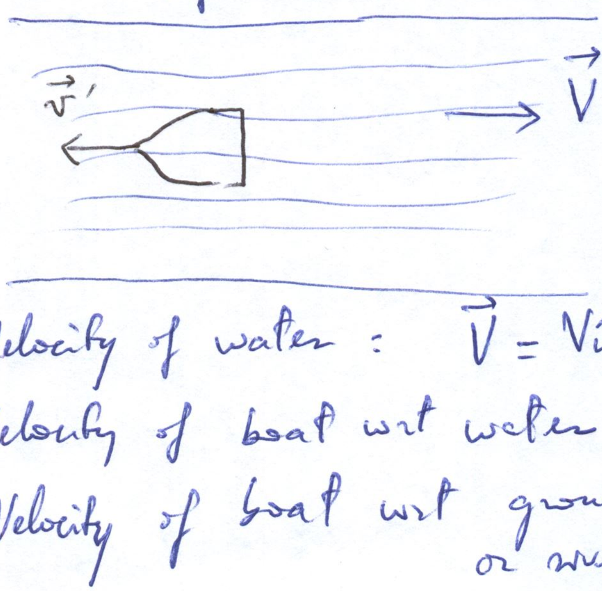
Relative motion equation

$\vec{v} = v'\hat{i} + V\hat{i} = (v' + V)\hat{i}$

$\Rightarrow \boxed{v = v' + V}$

2) Boat going up stream

Top view



Velocity of water: $\vec{V} = V\hat{i}$

Velocity of boat wrt water: $\vec{v}' = -v'\hat{i}$

Velocity of boat wrt ground or river banks: $\vec{v} = v\hat{i}$

$$\boxed{\vec{v} = \vec{v}' + \vec{V}}$$

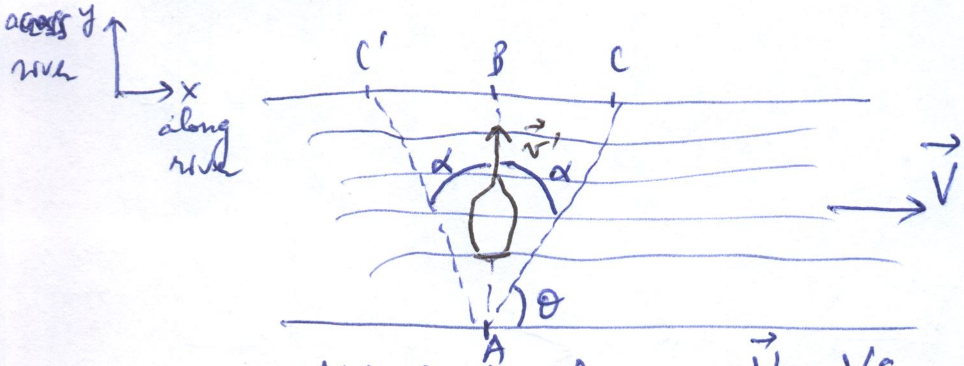
Relative Motion equation

$\vec{v} = -v'\hat{i} + V\hat{i}$

$= (-v' + V)\hat{i}$

$\rightarrow \boxed{v = -v' + V}$

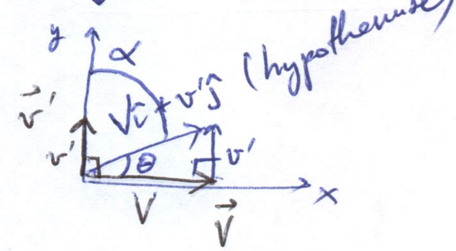
3) Boat going across river:



- Velocity of water: $\vec{V} = V\hat{i}$
- Velocity of the boat wrt water: $\vec{v}' = v'\hat{j}$
- Velocity of boat wrt ground or river banks: $\boxed{\vec{v} = \vec{v}' + \vec{V}}$

Relative Motion Equation

$$\vec{v} = v'\hat{j} + V\hat{i}$$
$$= V\hat{i} + v'\hat{j}$$



Magnitude of the velocity of boat wrt. ground $\longrightarrow$ $v = \sqrt{V^2 + v'^2}$

Direction of that velocity wrt x-axis $\longrightarrow$ $\theta = \tan^{-1}\dfrac{v'}{V}$

wrt. x-axis

$\alpha = 90 - \theta$ (wrt y-axis)

- Consequence of the relative motion equations: boat started out @ A will arrive @ C
- To arrive @ B, aim @ C' (symmetrically opposite to C wrt. AB or y-axis or straight across river direction) to compensate for the velocity of water.

Kinematic Equations for Constant Acceleration in 1D & 2D

1D	2D
x, v, a	$\vec{r}, \vec{v}, \vec{a}$

$v = v_0 + a \cdot t \quad (1) \longrightarrow \vec{v} = \vec{v}_0 + \vec{a} \cdot t \ (1) \begin{cases} v_x = v_{0x} + a_x \cdot t \\ v_y = v_{0y} + a_y \cdot t \end{cases}$

$x = x_0 + v_0 \cdot t + \frac{1}{2} a t^2 \ (2) \longrightarrow \vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \vec{a} \cdot t^2 \ (2)$

$\begin{cases} x = x_0 + v_x \cdot t + \frac{1}{2} a_x t^2 \\ y = y_0 + v_y \cdot t + \frac{1}{2} a_y t^2 \end{cases}$

2D: $\vec{r} = x\hat{i} + y\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dt} = \underbrace{\frac{dx}{dt}}_{v_x} \hat{i} + \underbrace{\frac{dy}{dt}}_{v_y} \hat{j}$ $\left(\frac{d\hat{i}}{dt} = \frac{d\hat{j}}{dt} = 0 :\right.$ fixed reference frames $\left.\right)$

$= v_x \hat{i} + v_y \hat{j}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \underbrace{\frac{dv_x}{dt}}_{a_x} \hat{i} + \underbrace{\frac{dv_y}{dt}}_{a_y} \hat{j} = a_x \hat{i} + a_y \hat{j}$$

Eq(1): $\vec{v} = \vec{v}_0 + \vec{a} \cdot t$

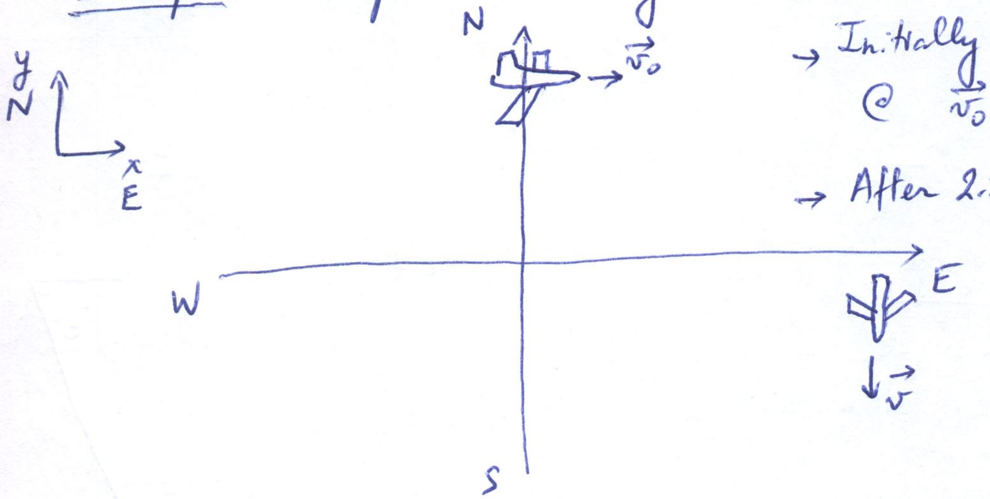
$$\boxed{v_x \hat{i}} + v_y \hat{j} = v_{0x} \hat{i} + v_{0y} \hat{j} + (a_x \hat{i} + a_y \hat{j}) \cdot t$$

$$= \boxed{(v_{0x} + a_x \cdot t)\hat{i}} + (v_{0y} + a_y \cdot t)\hat{j}$$

$$\rightarrow \begin{cases} v_x = v_{0x} + a_x \cdot t \\ v_y = v_{0y} + a_y \cdot t \end{cases}$$

Eq (2): do it yourself.

Example: airplane taking a turn (2D motion)



→ Initially flying eastward @ $\vec{v}_0 = 2100 \frac{km}{h} \hat{i}$

→ After 2.5 min it turns southward with a final velocity $\vec{v} = -1800 \frac{km}{h} \hat{j}$

Average acceleration vector? (SI : $\frac{m}{s^2}$)

$$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{\Delta t} = \frac{-500\hat{j} - 583.3\hat{i}}{150 s} \left(\frac{m}{s}\right) = \boxed{-3.9\hat{i} - 3.3\hat{j}} \frac{m}{s^2}$$

3rd quadrant.

$$\begin{cases} \vec{v}_0 = 2100 \frac{\cancel{km}}{\cancel{h}} \cdot \frac{1\cancel{h}}{3600 s} \cdot \frac{1000 m}{1 \cancel{km}} \hat{i} = \frac{2100}{3.6} \frac{m}{s} \hat{i} = 583.3 \frac{m}{s} \hat{i} \\ \vec{v} = -1800 \frac{km}{h} \hat{j} = -\frac{1800}{3.6} \frac{m}{s} \hat{j} = -500 \frac{m}{s} \hat{j} \end{cases}$$

$$\vec{\bar{a}} = \begin{cases} \text{Cartesian} \begin{cases} a_x = -3.9 \ m/s^2 \\ a_y = -3.3 \ m/s^2 \end{cases} \\ \text{Polar} \begin{cases} \bar{a} = \sqrt{3.9^2 + 3.3^2} = 5.1 \frac{m}{s^2} \\ \theta_{\bar{a}} = \tan^{-1} \frac{a_y}{a_x} = \end{cases} \end{cases}$$

(Magnitude of average acceleration vector)

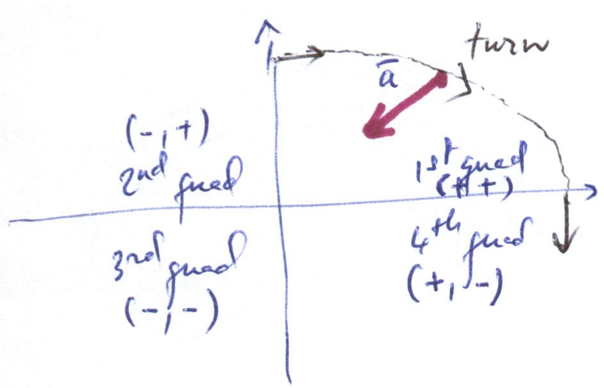
$$= \tan^{-1} \frac{-3.3}{-3.9} = 40.24^\circ$$

1st quad.

$\boxed{+180^\circ}$ Needed after calculator.

$\boxed{\theta_{\bar{a}} = 220.24^\circ}$

3rd quad!



Projectile Motion:

New physics? New equations? No, just a particular type of 2D motion of constant acceleration $\begin{cases} a_x = 0 \\ a_y = \pm g \end{cases}$

- Examples: baseball, basketball, soccer ball, bullets, short-range missiles (ground is flat); water from a sprinkler, etc...

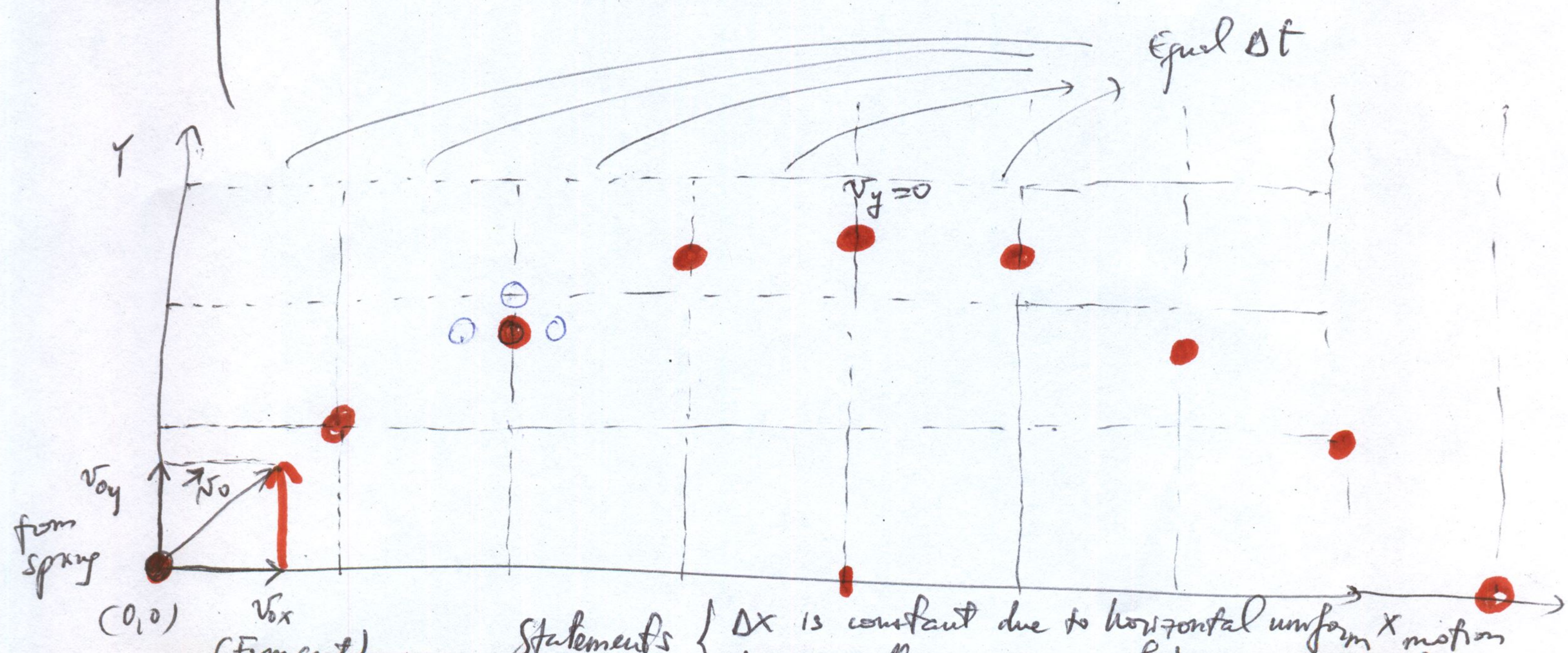
  → basically any object with an initial velocity & let go under effect of gravity.

- Example we will focus on: ball ejected upward from rolling cart (visual experiment #1) →

Statements:

parabolic trajectory for ball : $\vec{r}$ $\begin{cases} x : \text{uniform motion (constant velocity)} \\ y : \text{constant acceleration } a_y = \begin{cases} -g & \text{upward} \\ +g & \text{downward} \end{cases} \end{cases}$

Middle point = max altitude



Statements $\begin{cases} \Delta x \text{ is constant due to horizontal uniform } x \text{ motion} \\ \Delta y \text{ is smaller on way up \& bigger on way down.} \end{cases}$

<u>Equations for projectile motion:</u> (no new equations! only 2D kinematic equations for constant acceleration $\vec{a} \begin{cases} a_x = 0 \\ a_y = \pm g \end{cases}$

$y \uparrow \rightarrow x$ $\left(a_y = \begin{cases} -g & \text{up} \\ +g & \text{down} \end{cases} \right)$

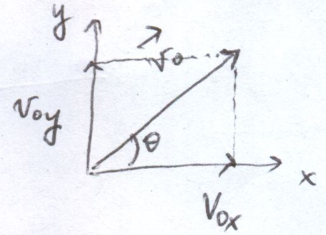
1) $\vec{v} = \vec{v}_0 + \vec{a} \cdot t$ $\begin{cases} v_x = v_{0x} \\ v_y = v_{0y} \mp g \cdot t \end{cases}$ $\begin{cases} - : \text{upward part} \\ + : \text{downward part} \end{cases}$

2) $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2} \vec{a} \cdot t^2$ $\begin{cases} x = v_{0x} \cdot t \\ y = v_{0y} \cdot t \mp \frac{1}{2} g t^2 \end{cases}$ $\begin{cases} - : \text{upward part} \\ + : \text{downward part} \end{cases}$

($\vec{r}_0 = (0, 0)$ @ origin)

Introduce: $\boxed{\text{aim angle } \theta}$ or the angle of the initial velocity $\vec{v}_0$

Easy & useful measurement in practical applications of projectile



$$\vec{v}_0 = \begin{cases} v_{0x} = v_0 \cos\theta \\ v_{0y} = v_0 \cdot \sin\theta \end{cases}$$

2) $\begin{cases} x = v_{0x} \cdot t = v_0 \cos\theta \cdot t \\ y = v_0 \sin\theta \cdot t \mp \frac{1}{2} g t^2 \end{cases}$ $\longrightarrow t = \frac{x}{v_0 \cos\theta}$

$$y = \cancel{v_0} \sin\theta \cdot \frac{x}{\cancel{v_0} \cos\theta} \mp \frac{1}{2} g \cdot \frac{x^2}{v_0^2 \cos^2\theta}$$

$$\boxed{y = x \tan\theta \mp \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta}}$$

<u>Trajectory equation</u> for a projectile motion

If θ & v_0 are known, this equation determines pairs of (x, y) which give the trajectory.

Maximum altitude point: $(x_{max}, y_{max}) = \left(\frac{v_0^2 \sin 2\theta}{2g}, \frac{v_0^2 \sin^2\theta}{2g} \right)$

Proof: From kinematic eqs in 2D for constant acceleration:

y_{max}

Eq 1) $v_y = \underbrace{v_0 \sin\theta}_{v_{0y}} - gt$ (upward part)

@ max altitude: $v_y = 0 = v_0 \sin\theta - gt \rightarrow \boxed{t_{max} = \frac{v_0 \sin\theta}{g}}$

Eq 2) $y_{max} = \underbrace{v_0 \sin\theta}_{v_{0y}} \cdot \underbrace{\frac{v_0 \sin\theta}{g}}_{t_{max}} - \frac{1}{2}\cancel{g} \underbrace{\frac{v_0^2 \sin^2\theta}{g^{\cancel{2}}}}_{t_{max}^2} = \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g}$

$x_{max} = v_{0x} \cdot t_{max} = v_0 \cos\theta \cdot \frac{v_0 \sin\theta}{g} = \frac{v_0^2 \cos\theta \sin\theta}{g} = \frac{v_0^2 \sin 2\theta}{2g}$

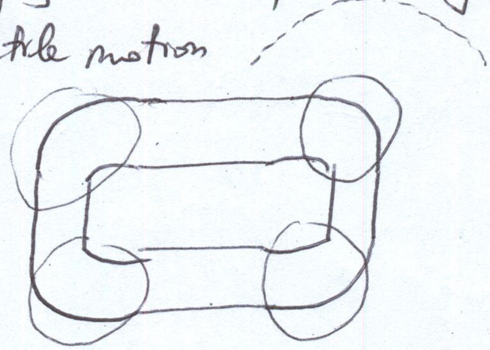
Trigonometry: $\cos\theta \sin\theta = \frac{1}{2} \sin 2\theta$

Uniform Circular Motion : UCM : Circular motion with constant speed! (not constant velocity)

velocity: includes direction $\vec{v} = (v, \theta_v)$ → an object can have
$\downarrow$ speed

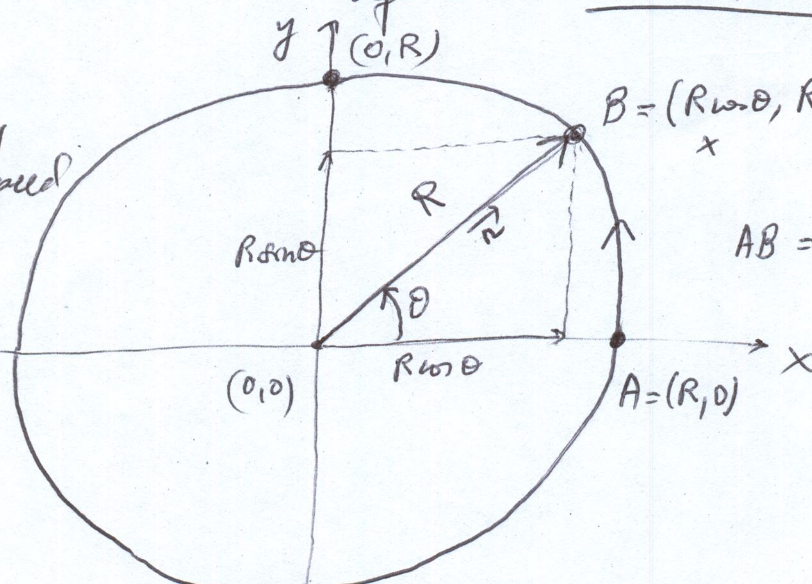
constant speed but changing angle → changing velocity: This is what happens in UCM: the speed is uniform along circular trajectory however as direction is changing → velocity is changing!

2D Motions → Projectile motion, → UCM



Changing velocity: ⟶ in direction, not in magnitude, but still requires an acceleration

UCM
Object traveling @ uniform speed v



$$AB = \text{arc} \rightarrow \left[\theta = \frac{\text{arc}}{R} = \frac{v.t}{R}\right]$$

@ time t, object is @ B defined by position vector $\vec{r} = x\hat{i} + y\hat{j}$

$$\vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j} = R\cos\left(\frac{v.t}{R}\right)\hat{i} + R\sin\left(\frac{v.t}{R}\right)\hat{j}$$

UCM $= @t$ $\quad \vec{r} = R \cdot \cos\left(\frac{v \cdot t}{R}\right)\hat{i} + R\sin\left(\frac{v \cdot t}{R}\right)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \cancel{R}\left[-\frac{v}{\cancel{R}}\sin\left(\frac{v \cdot t}{R}\right)\hat{i} + \frac{v}{\cancel{R}}\cos\left(\frac{v \cdot t}{R}\right)\hat{j}\right]$$

$\boxed{\vec{v} \text{ is changing over time}}$

$$v = |\vec{v}| = \left| v\left(-\sin\left(\frac{v \cdot t}{R}\right)\hat{i} + \cos\left(\frac{v \cdot t}{R}\right)\hat{j}\right)\right|$$

$$= v \underbrace{\sqrt{\left(-\sin\frac{v \cdot t}{R}\right)^2 + \left(\cos\frac{v \cdot t}{R}\right)^2}}_{\sin^2\alpha + \cos^2\alpha = 1} = \text{constant}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -v\frac{d}{dt}\left[\sin\left(\frac{v \cdot t}{R}\right)\hat{i} - \cos\left(\frac{v \cdot t}{R}\right)\hat{j}\right]$$

$$= -v\left[\frac{v}{R}\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \frac{v}{R}\sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]$$

$$= -\frac{v^2}{R}\underbrace{\left[\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]}_{\text{Magnitude} = 1}$$

$$\boxed{|\vec{a}| = \frac{v^2}{R}}_{\text{UCM}}$$

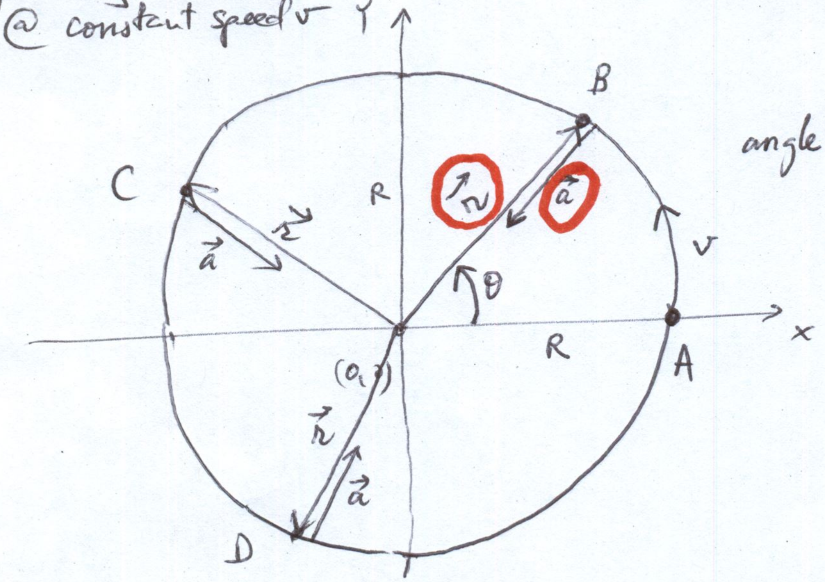
Acceleration connected with the change of direction of velocity.

Summary on UCM results:

object following circular motion @ constant speed v

Position vector: $\vec{r} = R\left[\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]$

Acceleration vector $\vec{a} = -\frac{v^2}{R}\left[\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]$



angle $\theta = \frac{\text{arc}}{R} = \frac{v \cdot t}{R}$

Statements:
- $\vec{r}$ & $\vec{a}$ have identical square brackets which define their directions
- $\vec{a}$ is in radial direction & toward center of curvature (opposite to $\vec{r}$)

[→ whenever an object takes a turn there has to be an acceleration toward center of curvature.]

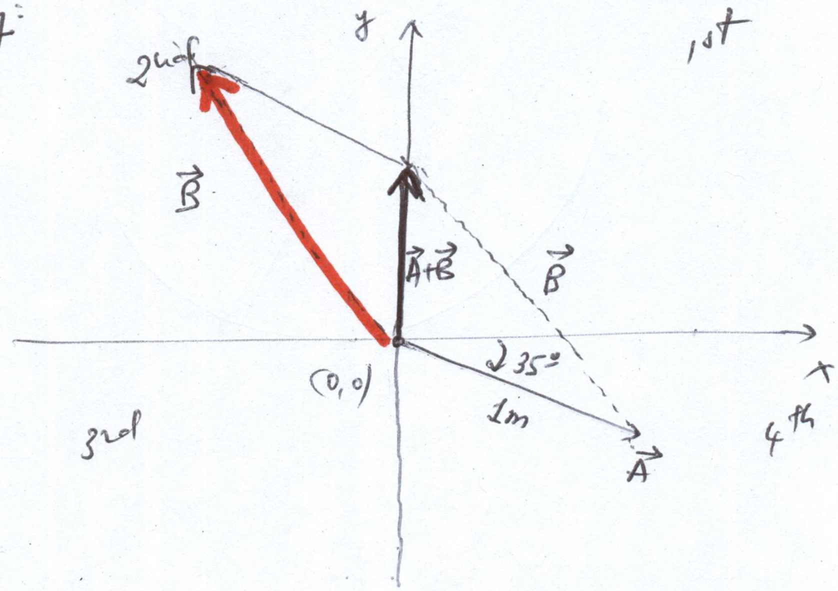
3.42 **Vector addition in 2D**

Polar

$$\begin{cases} \vec{A} = (1m,\ 35^\circ \text{ CW from } x\text{-axis}) \\ \vec{B} = (1.8m,\ \theta) \end{cases}$$

$\uparrow$ so that $\vec{A}+\vec{B}$ is in y-direction

Graphically:



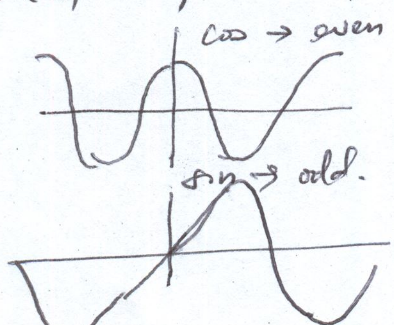
Statement: $\vec{B}$ will be in 2^{nd} quad.
so diagonal of quadrilateral of sides $\vec{A}$ & $\vec{B}$ will point along $+y$-axis

Mathematically: → Cartesian coords are best suited for addition & subtraction
→ Polar coords " " " " multiplication & division.

$\vec{A} = (1, -35^\circ) = (1\cos(-35^\circ),\ 1\sin(-35^\circ)) = (\cos 35^\circ, -\sin 35^\circ)$

$\vec{A} = \cos 35^\circ\,\hat{i} - \sin 35^\circ\,\hat{j}$

Review:



$\vec{B} = (1.8, \theta) = 1.8\cos\theta\,\hat{i} + 1.8\sin\theta\,\hat{j}$

$\vec{A}+\vec{B} = (\cos 35^\circ + 1.8\cos\theta)\hat{i} + (-\sin 35^\circ + 1.8\sin\theta)\hat{j}$

For $\vec{A}+\vec{B}$ to be along y-direction: $\Rightarrow$ x-component should be zero: $\cos 35^\circ + 1.8\cos\theta = 0$

$$\underbrace{\cos\theta = -\frac{\cos 35^\circ}{1.8}} \rightarrow \theta = \cos^{-1}\left[-\frac{\cos 35^\circ}{1.8}\right]$$

$$= 117^\circ \quad (2^{nd}\text{ quad}).$$

Now if $\vec{A}+\vec{B}$ points in $\ominus$y-direction $\theta = -117^\circ$ (3^{rd} quad.).

b/c: cos is an even function

$(\cos 117^\circ = \cos(-117^\circ))$

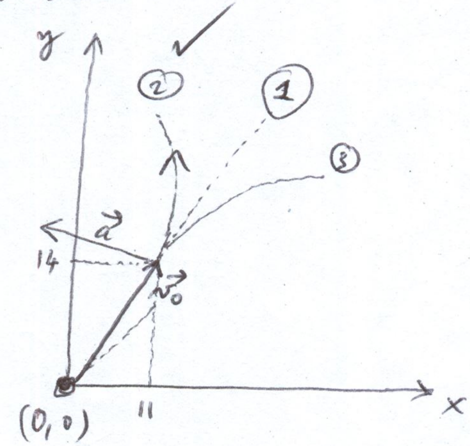
(3.54) Statement: particle undergoing constant acceleration 2D.

Info: $\vec{v}_0 = 11\hat{i} + 14\hat{j}\ \frac{m}{s}$ @ $\vec{r} = (0,0)$ origin

$\vec{a} = -1.2\hat{i} + 0.26\hat{j}\ \frac{m}{s^2}$

a) When does particle cross y-axis?

Does this question make sense? Answer will help understand this problem.



→ $\vec{v}_0$ is more vertical than horizontal, will stay along direction 1 unless there is a change of velocity (including direction)

→ $\vec{a}$ points to 2^{nd} quad. → particle will bend to the left.

→ Yes, it will cross y-axis @ some point

Statement: t can be calculated using kinematic eqs for constant acceleration in 2D eqs 1 and/or 2

Eq 1: $\vec{v} = \vec{v_0} + \vec{a} \cdot t$

Eq 2: $\vec{r} = \vec{r_0} + \vec{v_0} \cdot t + \frac{1}{2}\vec{a} \cdot t^2$ ⟸ since we have some info on final position: $\boxed{x=0}$ when it crosses y-axis.

$(x_0, y_0) = (0,0)$

$$\begin{cases} x = 0 = 0 + v_{0x} \cdot t + \frac{1}{2} a_x t^2 \rightarrow 11 \cdot \cancel{t} + \frac{1}{2}(-1.2) t^{\cancel{2}} = 0 \rightarrow t = \frac{22}{1.2} s \\ y = 0 + v_{0y} \cdot t + \frac{1}{2} a_y t^2 \end{cases}$$

$\boxed{t = 18.3 s}$

@ this time particle crosses y-axis.

b) Particle y position @ t = 18.3s

$$y = 14 \times 18.3 + \frac{1}{2} \times 0.26 \times 18.3^2 = 300 \text{ m}$$

c) Final $\vec{v}$ @ t = 18.3s

$$\vec{v} = 11\hat{i} + 14\hat{j} + (-1.2\hat{i} + 0.26\hat{j}) \times 18.3$$

$$= (11 - 1.2 \times 18.3)\hat{i} + (14 + 0.26 \times 18.3)\hat{j} \quad m/s$$

$$= -10.96\,\hat{i} + 18.8\,\hat{j} \quad m/s$$ (2nd quad.)

Magnitude & direction of $\vec{v} = (v, \theta)$

$$= \left(\underbrace{\sqrt{(-10.96)^2 + 18.8^2}}_{21.7}, \underbrace{\tan^{-1}\left(\frac{18.8}{-10.96}\right)}_{-60^\circ + 180^\circ}\right)$$

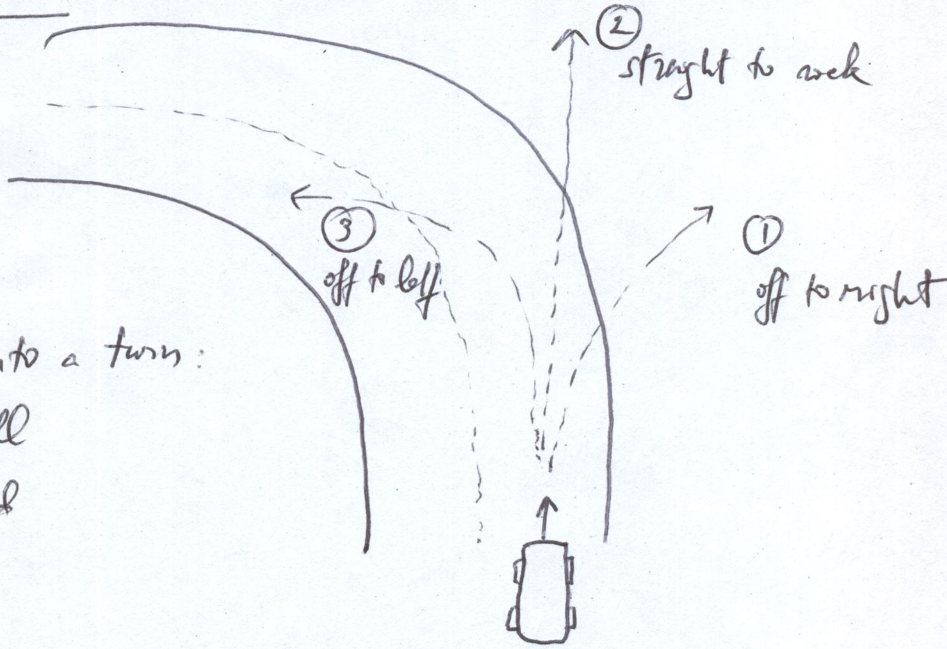
$$\vec{v} = (21.7 \tfrac{m}{s}, 120^\circ)$$

Ch 4 Motion & Forces

description: $\vec{r}, \vec{v}, \vec{a}, t$ $\leftarrow$ $\vec{F}$

Statement: Force is the agent that causes the accel. $\vec{a}$ or a change of motion

Visual experiment: to introduce $\vec{F}$ & its connection with $\vec{a}$

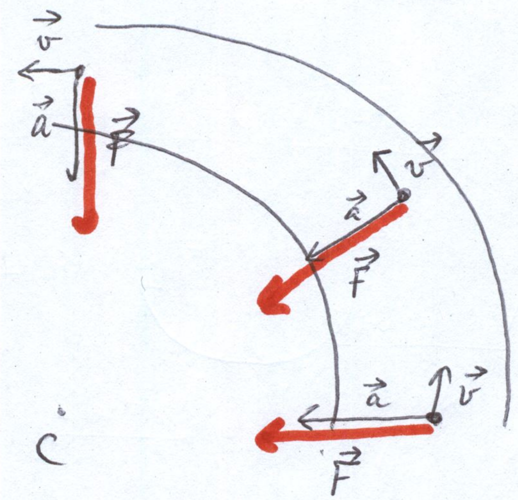


Driving a car into a turn:
- Downhill
- Icy road

Vehicle will follow path ②. Why? Lack of acceleration toward center of curvature b/c lack of agent or force to provide that acceleration which is the friction b/w tires & road.

Conclusion: vehicle entering a curve in forward direction will continue to do so if there is no force or agent that changes its direction.

A force is needed to change a motion!



Conclusions: 1) Force is a vector (it changes direction)

2) Force is agent to change direction of $\vec{v}$

Newton's Laws:

$\boxed{1^{st}}$: a body at rest will continue at rest, a body in uniform motion will continue in uniform motion unless there is a net force acting on the body.

Law of inertia

$\boxed{2^{nd}}$: $$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$\vec{p}$: linear momentum $\vec{p} = m\vec{v}$ $\left(kg\,\frac{m}{s}\right)$

$\vec{F}_{net}$ = superposition of all forces involved.

$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt} = \underbrace{\frac{dm}{dt}}_{\text{important when mass is changing over time}}\vec{v} + m\underbrace{\boxed{\frac{d\vec{v}}{dt}}}_{\vec{a}}$$

If m is constant:

$$\boxed{\vec{F}_{net} = m.\vec{a}}$$

$$[F] = \frac{ML}{T^2} \xrightarrow{SI} kg\,\frac{m}{s^2} \equiv N$$

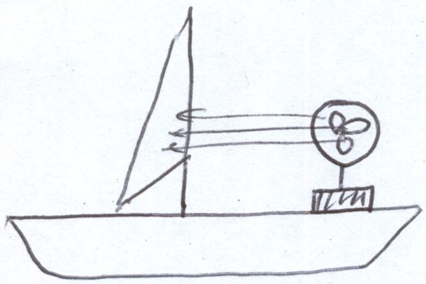
(Newton)

$\boxed{3^{rd}}$ Law of action & reaction:

If A exerts a force on B, B exerts an equal and opposite force on A.

Sailing without wind: Law of action & reaction

Info: → fan is fixed on boat
→ blows air on sail



Will boat move forward?

Yes: ? → Fan blows air molecules which in turn push sail (pushes)

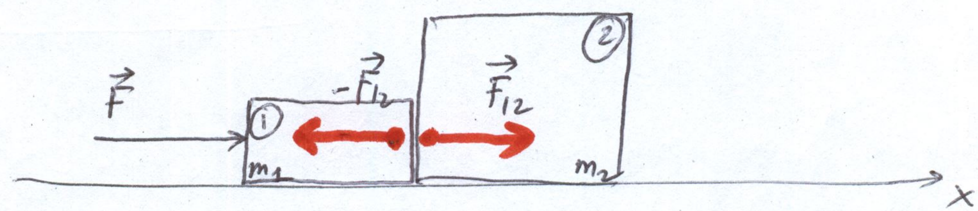
→ Law of action & reaction: air molecules push back on fan same force in opposite direction.

Since fan is attached to boat → air pushes back on boat → $\vec{F}_{net} = 0$

No.

1) Two boxes next to each other on a horizontal surface (no friction) force $\vec{F}$ is applied on box ① causing system of ① & ② to accelerate in x-direction : $\vec{F} = (m_1 + m_2)\vec{a}$

$\vec{a}$



a) What force is applied on box ②? (Net force)

Can it be $\vec{F}$? $\boxed{\text{No}}$ $\vec{a}_1 = \vec{a}_2 = \vec{a}$

if $\vec{F} = (m_1 + m_2)\vec{a}$ it can't be also $\vec{F} = m_2\vec{a}$

→ What force makes m_2 move? $\vec{F}_{12}$: force applied by box ① on box ②. Without friction this is also the net force on box ② ⇒ $\boxed{F_{12} = m_2 a}$

b) What is the net force on box ①?

$$F_{net①} = \boxed{F - F_{12} = m_1 a}$$

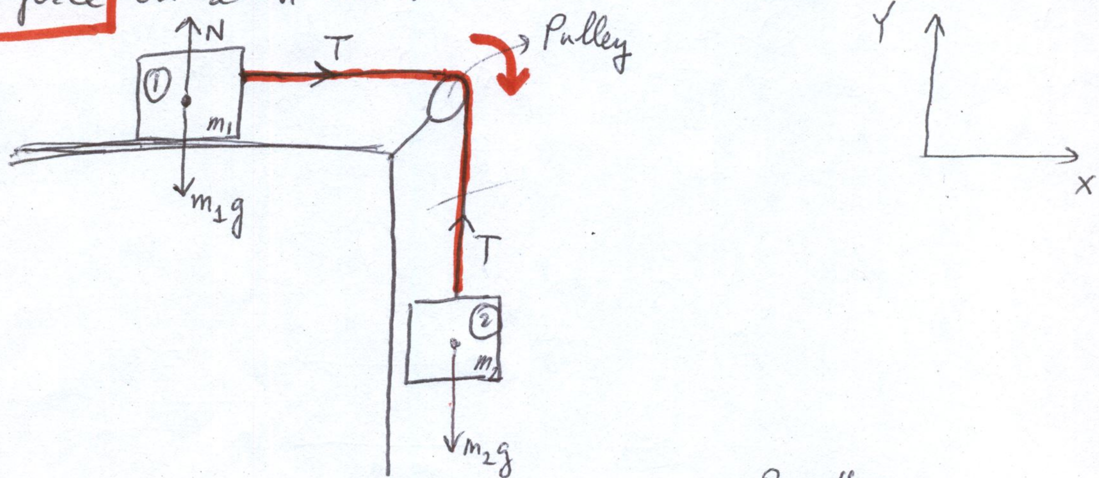
c) Summary:

Net/Total force on system	Net force on ①	Net force on ②
F	$F - F_{12}$	F_{12}

Note: Net force on ① + Net force on ② = $F - F_{12} + F_{12} = F$ = Net force on system (F_{12} & $-F_{12}$ are internal forces for this system)

2) Two boxes connected by a $\boxed{\text{massless string/rope}}$ (sufficiently small compared to m_1 & m_2) (no friction)

$\boxed{\text{Net force}}$ on each box.



Box #1

Forces acting on this box:

- weight $m_1 g$
- tension T
- normal N (by table)

Net force: F_{net_1}

on #1:
- $\rightarrow$ in x: T
- $\rightarrow$ in y: $N - m_1 g$

2nd Newton's Law:

$$\vec{F}_{net_1} = m_1 \boxed{\vec{a}} \begin{cases} T = m_1 a & (1) \\ N - m_1 g = 0 \end{cases}$$

Box # 2

- weight $m_2 g$
- tension T (same throughout due to massless rope)

Net force: F_{net_2}

on #2:
- in x: 0
- in y: $T - m_2 g$

2nd Newton's Law:

$$\vec{F}_{net_2} = m_2 \boxed{\vec{a}} \begin{cases} x = \text{no motion} \\ T - m_2 g = -m_2 a & (2) \end{cases}$$

not in the same directions but same magnitude since boxes are connected by the rope (box #1 goes right & box #2 goes $\boxed{\text{down}}$)

These equations allow us to solve ~~for~~ any situation:

Example: given m_1 & m_2 find a & T:

→ <u>To calculate a:</u> plug eq (1) into eq (2)

$$m_1 a - m_2 g = -m_2 a \quad \Rightarrow \quad (m_1 + m_2) a - m_2 g = 0$$

$$\Rightarrow \boxed{a = \frac{m_2}{m_1 + m_2} g}$$

Check: acceleration for m_2 is $\underbrace{\frac{m_2}{m_1+m_2}}_{<1} g < g$ slower than free fall why?

→ <u>To calculate T:</u> eq (1) : $\boxed{T = m_1 a = \frac{m_1 m_2}{m_1 + m_2} g}$

Check: $a = \frac{m_2}{m_1 + m_2} g$

1) If we double the masses $\left.\begin{array}{l} m_1 \to 2m_1 \\ m_2 \to 2m_2 \end{array}\right\} \Rightarrow$ same acceleration

2) If we double m_2 only : $a' = \frac{2m_2}{m_1 + 2m_2} g > a$

$$a = \frac{2m_2}{2m_1 + 2m_2} g$$

Spring forces: Hooke's Law:

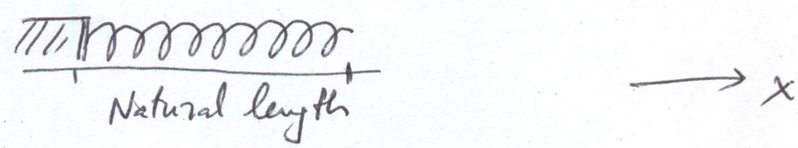
$$F_s = -k\,\underbrace{\Delta x}_{}$$

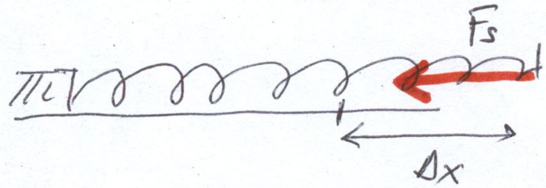
change of length from the natural length } or stretch or compression

resistance to stretch/compression

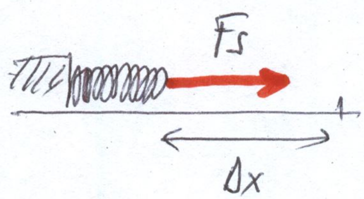
k: spring constant ($\frac{N}{m}$ in S.I.)

Horizontal:

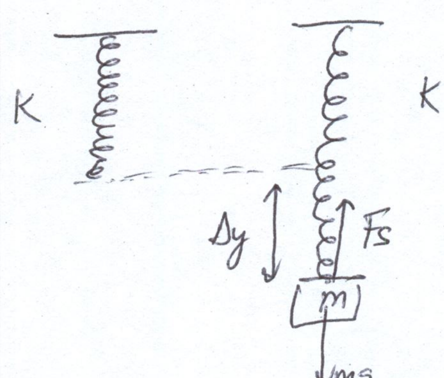




$F_s = -k\Delta x$



Vertical:



If m is static:

$$F_s - mg = m \cdot 0$$

$$F_s = mg$$

$$+k\Delta y = mg$$

$$\boxed{\Delta y = \frac{mg}{k}}$$

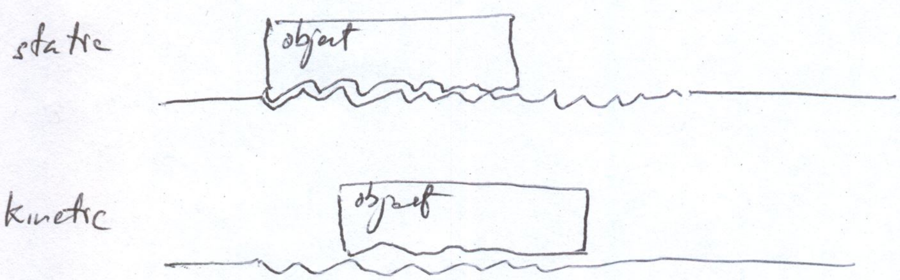
F_s in $+y$

<u>Friction forces</u> : are present whenever an object is in contact with a surface

→ 2 types:
- static friction : in contact but not in relative motion
  $F_S = \mu_S N$
- kinetic friction : in contact & in relative motion
  $F_K = \mu_K N$

For a same object & surface:
- μ_S : coeff. of static friction (a number w/o units)
- N : normal force by surface on object
- μ_K : coeff of kinetic friction

<u>Microscopically : b/w bottom of object & the surface :</u>



if we look close enough: roughness on any surface

$\mu_S > \mu_K$

When we ~~move~~ push heavy boxes, after we overcome the static friction : the box acquires an acceleration $F_S - F_K = m \cdot a$

3.40 → Orbital period of GPS satellite @ 20,000 km above surface

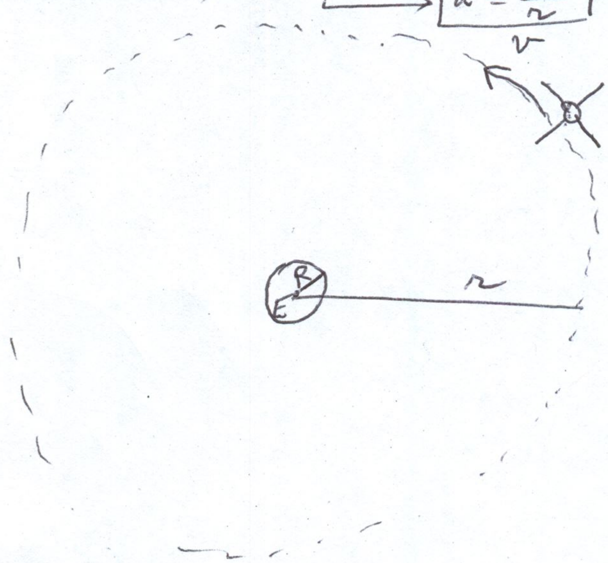
$g' = 0.058g$

Statement: 1) UCM constant speed v → $\boxed{a = \frac{v^2}{r}}$

2) Separation to center of circular trajectory:

$r = 20{,}000\,km + 6{,}370\,km$

$= 26{,}370\,km$



$R_E = 6370\,km$

Orbital period: time to complete one orbit or one turn:

$$T = \frac{2\pi r}{v}$$

$$g' = \frac{v^2}{r}$$ (g' allows satellite to follow circular orbit)

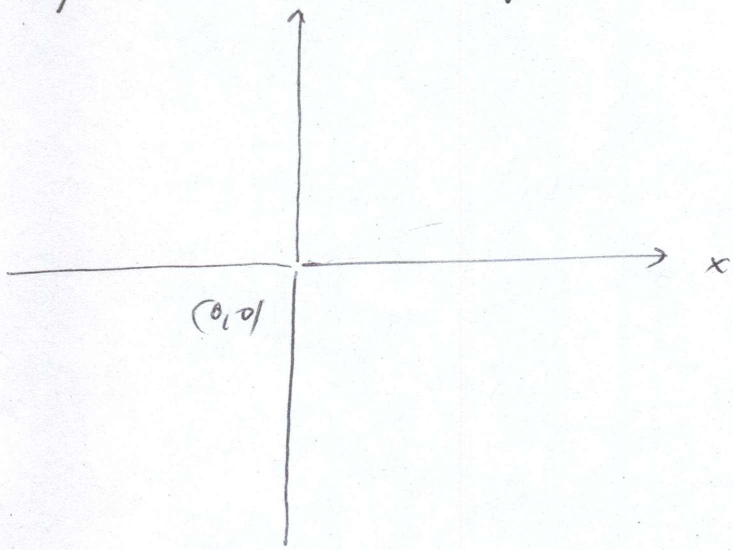
$$v = \sqrt{g'r}$$

$$T = \frac{2\pi r}{\sqrt{g'r}} = 2\pi\sqrt{\frac{r}{g'}} = 2\pi\sqrt{\frac{2.637\times10^7}{0.058\times9.81}} = 42774\,s$$

$$T = \frac{42774}{3600}\,hr = 11.88\,hrs \simeq 12\,hrs.$$

3.45 $\vec{r} = (ct^2 - 2dt^3)\hat{i} + (2ct^2 - dt^3)\hat{j}$ $\quad c, d > 0$

a) Find $t > 0$ when particle will be moving in x-direction



$y = 0$

$2ct^2 - dt^3 = 0$

or $2c - dt = 0$

$\boxed{t = 2\frac{c}{d}}$

@ this time it will be crossing the x-axis

$$\vec{v} = \frac{d\vec{r}}{dt} = (2ct - 6dt^2)\hat{i} + (\underbrace{4ct - 3dt^2}_{v_y = 0})\hat{j}$$

$4ct - 3dt^2 = 0$

$4c - 3dt = 0$

$\boxed{t = \frac{4}{3}\frac{c}{d}}$ ✓

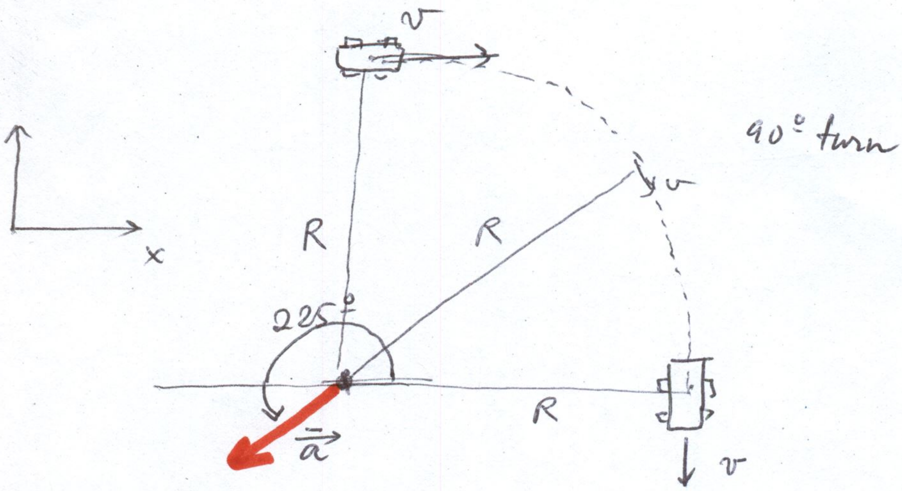
@ this time it will be moving in the x-direction.

b) It will be moving in y-direction:

$v_x = 2ct - 6dt^2 = 0$

$2c - 6dt = 0$ or $\boxed{t = \frac{1}{3}\frac{c}{d}}$

3.22



speedometer reading constant :→ UCM

↓

$$a = \frac{v^2}{R}$$

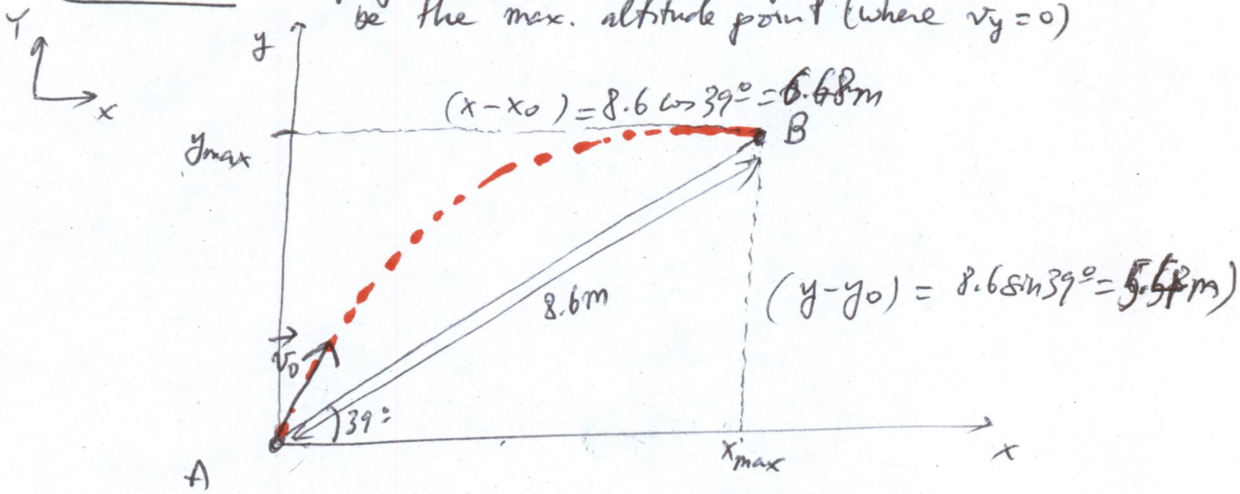
Direction of car average acceleration vector?

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{-v\hat{j} - v\hat{i}}{\Delta t} = \frac{1}{\Delta t}(-v\hat{i} - v\hat{j}) \quad \boxed{3^{rd}\text{ quad.}}$$

$$\theta_{\vec{a}} = \tan^{-1}\left(\frac{-\frac{v}{\Delta t}}{-\frac{v}{\Delta t}}\right) = \tan^{-1}\left(\frac{-1}{-1}\right) = 45° + 180°$$

$$= 225°$$

3.62 **Statement:** projectile motion for protein bar; B needs to be the max. altitude point (where $v_y = 0$)



We need to find $\vec{v}_0$ for bar as it leaves A. θ_{v_0} has to be larger than 39°!

Alternative #1:

$$\begin{cases} x_{max} = \dfrac{v_0^2 \sin 2\theta_{v_0}}{2g} = 8.6 \cos 39° = 6.68\,m \\ y_{max} = \dfrac{v_0^2 \sin^2\theta_{v_0}}{2g} = 8.6 \sin 39° = 5.4\,m \end{cases}$$

Two eqs with 2 unknowns v_0 & θ_{v_0} → can solve.

Alternative #2: remember eqs for x_{max} & y_{max} were derived from kinematic eqs for constant acceleration in 2D!

Eq 3:

$$\begin{cases} 3a) \quad \dfrac{v_x^2 - v_{0x}^2}{(x-x_0)} = 2 \cdot a_x & (a_x = 0) \\ 3b) \quad \dfrac{v_y^2 - v_{0y}^2}{(y-y_0)} = 2 \cdot a_y & (a_y = -g \text{, 1st half of parabola: upward motion}) \end{cases}$$

$v_y = 0$ (@ B)

3b) $$\frac{0 - v_{0y}^2}{5.4} = -2 \times 9.81 \Rightarrow v_{0y} = \sqrt{2 \times 9.81 \times 5.4} = 10.3 \frac{m}{s}$$

Now find v_{0x} (initial vel. in x) $= v_x$ (uniform motion in x!)

Note: bar needs to go 6.68m in x-direction in same time

Statement it needs to go 5.4m in y-direction!

↓

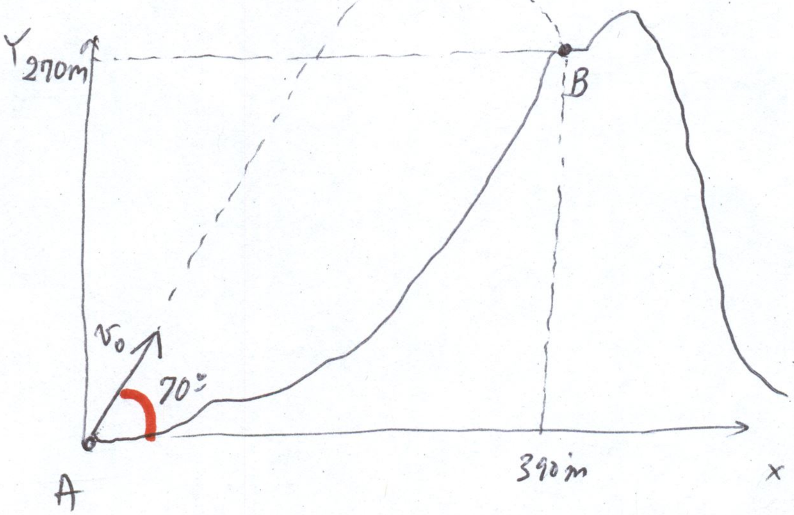
$$v_y = 0 = v_{0y} - g \cdot t \Rightarrow t = \frac{v_{0y}}{g} = \frac{10.3}{9.81}$$

$=$

$$\Rightarrow v_{0x} = \frac{6.68}{\frac{10.3}{9.81}} = 6.36\ m/s$$

$$\Rightarrow \vec{v}_0 = 6.36\,\hat{i} + 10.3\,\hat{j}\ \frac{m}{s} \xrightarrow{\text{Polar}} \begin{cases} v_0 = \sqrt{6.36^2 + 10.3^2} = 12.1\ m/s \\ \theta_{v_0} = \tan^{-1}\dfrac{10.3}{6.36} = 58.3° > 39° \checkmark \end{cases}$$

3.70



Statement: projectile motion for medical packet, B is a point on parabola (being A the initial point)

Trajectory eq:

$$y_B = x_B \tan\theta_{v_0} - \frac{g}{2}\frac{x_B^2}{v_0^2 \cos^2\theta_{v_0}}$$

Solve for v_0:

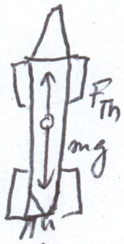
$$v_0^2 = \frac{g}{2}\frac{x_B^2}{(x_B \tan\theta_{v_0} - y_B)\cos^2\theta_{v_0}}$$

$$v_0 = \sqrt{\frac{9.81}{2}\frac{390^2}{(390 \tan 70° - 270)\cos^2 70}} = 89.2 \frac{m}{s}$$

4.55 Statement: Application of Newton's Law.

a)

y ↑



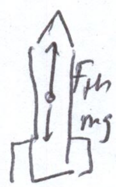
↓ $a = 1.4g$

$$F_{net} = F_{th} - mg = -m \times 1.4g$$

$$F_{th} = (-1.4 + 1)mg$$

$$= -0.4mg.$$

b)



↑ $a = 1.4g$.

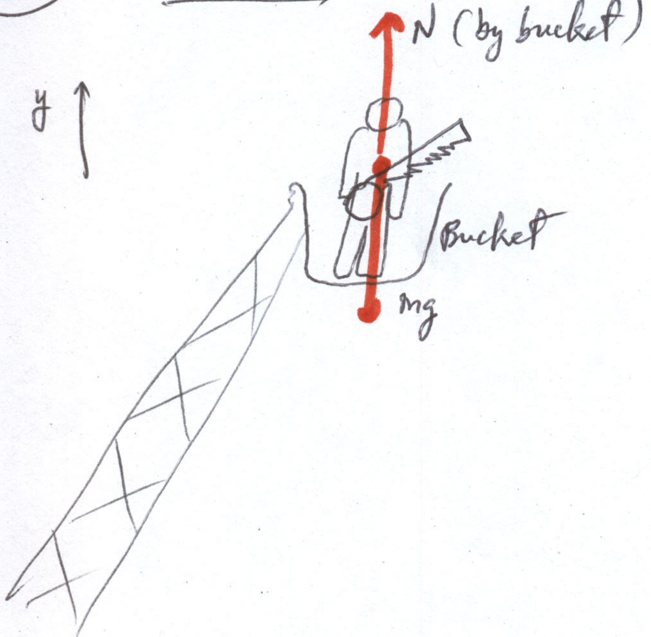
$$F_{th} - mg = +1.4mg.$$

$$F_{th} = 2.4mg$$

c) interstellar space far from any planet (no weight)

$$F_{th} = 1.4m.g$$

4.40 Statement: Application of second Newton's Law: $F_{net} = m \cdot a$



a) Bucket @ rest: $v = 0 = a$

$$F_{net} = N - mg = 0 \Rightarrow N = mg$$
$$= 74 \times 9.81$$
$$\boxed{N = 725N}$$

b) Bucket moving up @ steady $v = 2.4 m/s$
$$\Downarrow$$
$$a = 0$$

$$F_{net} = 0 \Rightarrow \boxed{N = 725N}$$

c) Bucket moving down @ steady $v = 2.4 m/s$
$$\Downarrow$$
$$a = 0$$

$$F_{net} = 0 \Rightarrow \boxed{N = 725N}$$

d) Bucket accelerating up @ $1.7 m/s^2 = a$

$$F_{net} = m \cdot a$$
$$N - mg = m \cdot a$$
$$N = m(g + a)$$
$$= 74(9.81 + 1.7)$$
$$\boxed{N = 851N}$$
(Feels heavier)

e) Bucket accelerating down @ $1.7 m/s^2$

$$F_{net} = -m \cdot a$$
$$N - mg = -ma \Rightarrow N = m(g - a) = 74(9.81 - 1.7)$$
$$\boxed{N = 599N}$$
(Feels lighter)