Physic 113 Summer 2013 AM

Ch1 Doing Physics

Dimensional Analysis:

Speed: $[v] = \frac{[\Delta s]}{[\Delta t]} = \frac{L}{T}$ { "Dimension of speed is length over time"

"dimension of speed"

Δs "increment of space" or distance : $[\Delta s] = L$ (length)
"delta s"

Δt: "increment of time" or travel time $[\Delta t] = T$ (time)

Acceleration: $[a] = \frac{[\Delta v]}{[\Delta t]} = \frac{\frac{L}{T}}{T} = \frac{L}{T^2}$

Energy → Kinetic energy : $[K.E] = \left[\frac{1}{2}mv^2\right] = \left[\frac{1}{2}\right]\cdot[m]\cdot[v]^2$
$= 1 \cdot M \cdot \frac{L^2}{T^2} = M\frac{L^2}{T^2}$

Application: which of the following two formulas for speed is correct?

$v_1 = \frac{1}{2}gh^2 \rightarrow [v_1] = [g][h]^2 = \frac{L}{T^2}\cdot L^2 = \frac{L^3}{T^2}$ (not a speed!)

$v_2 = \sqrt{gh} \rightarrow [v_2] = [g]^{\frac{1}{2}}\cdot[h]^{\frac{1}{2}} = ([g]\cdot[h])^{\frac{1}{2}} = \left(\frac{L}{T^2}\cdot L\right)^{\frac{1}{2}} = \left(\frac{L^2}{T^2}\right)^{\frac{1}{2}}$
$= \frac{L}{T}$ ✓

g = acceleration of gravity
h = height or vertical position

Limitation: $v_3 = \frac{1}{2}\sqrt{gh}$: $[v_3] = \frac{L}{T}$ → Dimensional analysis can't determine the constant.

Units : { SI : international system
British :

Quantity/Dimension	SI unit
L	m (meter)
T	s (second)
M	kg (kilogram)
Area	m^2
Volume	m^3
Energy	$\frac{kg \cdot m^2}{s^2} = J$ (Joule)

Conversions :

Other	nm	μm "micron"	mm	cm	km	light-year	1 mi	1 ft	in
SI	$10^{-9}m$	$10^{-6}m$	$10^{-3}m$	$10^{-2}m$	$10^{3}m$	$9.46\times10^{15}m$	1609 m	0.3048 m	2.54 cm

$1\,lb = 0.454\,kg$

$1\,min = 60\,s$; $1\,hr = 3600\,s$; $1\,day = 86400\,s$; etc.

$1\,km^2 = (10^3 m)^2 = 10^6\,m^2$

$1\,cm^2 = (10^{-2} m)^2 = 10^{-4}\,m^2$

$1\,mm^2 = 10^{-6}\,m^2$

$1\,km^3 = (10^3 m)^3 = 10^9\,m^3$

$1\,cm^3 = (10^{-2} m)^3 = 10^{-6}\,m^3$

$1\,mm^3 = 10^{-9}\,m^3$

$1\,km^3 = 10^{18}\,mm^3$

$1\,mm^3 = 10^{-18}\,km^3$

Accuracy & Significant Figures:

- **Scientific Notation:** uses powers of 10

$$\Delta s = 6\,176\,000 \text{ m} = 6.176 \times 10^{6} \text{ m} = \underbrace{6.176}_{\substack{\text{Coefficient} \\ (<10)}} \text{E6 m}$$

$$\Delta t = 3000 \text{ s} = 3\text{E}3 \text{ s}$$

$$\text{speed} = \frac{\Delta s}{\Delta t} = \frac{6.176 \times 10^{6}}{3 \times 10^{3}} = \frac{6.176}{3} \times 10^{6-3} = 2.059 \times 10^{3} \frac{\text{m}}{\text{s}}$$

$$= 2.059\text{E}3 \frac{\text{m}}{\text{s}}$$

- **Accuracy:** (addition & subtraction)

$$\pi - 1.14 = 3.1416 - 1.14 \begin{cases} 2.0016 \\ 2.00 \ \checkmark \end{cases}$$

(assume π is 3.1416)

$\boxed{\text{best accuracy is limited by the least accurate quantity in the equation.}}$

- **Significant figures (s.f's)** (multiplication & division).

$\underbrace{6\,370\,000 \text{ m}}_{3 \text{ s.f.'s}}$ (end zeroes don't count)

$\underbrace{6\,370\,001 \text{ m}}_{7 \text{ s.f.'s}}$ (middle zeroes do count)

Circumference of Earth: $2\pi R_E = 2 \times 3.1416 \times 6.37 \times 10^{6}$

$R_E = 6.37\text{E}6$ m

$$= 4.002398 \times 10^{7} \text{ m}$$

$$= 4.00 \times 10^{7} \text{ m}$$

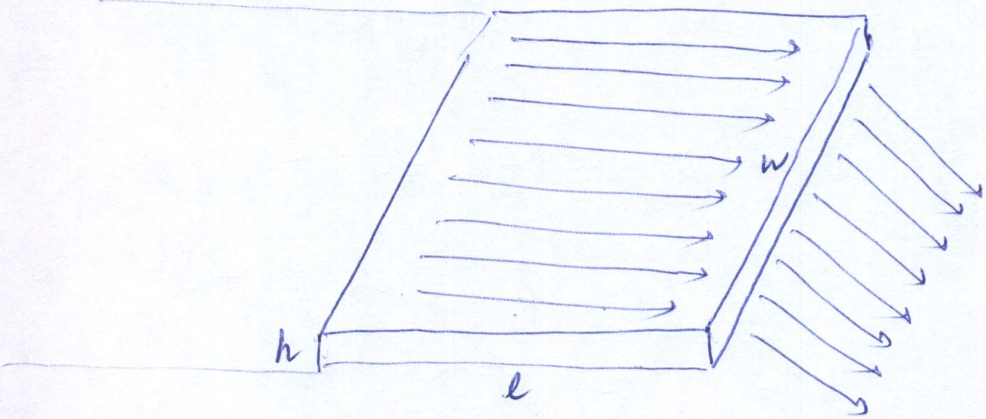
least # of s.f's is 3

1.41

a) Estimate volume (m^3) of water going over Niagara Falls in 1s.

Guesses: $3E6 \frac{m^3}{s}$

Estimation: simple geometrical shape — top level = rectangular slab $h \times l \times w = \text{Volume}$



Volume per second or flow rate : $\frac{\text{volume}}{t} = \frac{hlw}{t}$
1) $\frac{h}{t} lw$
2) $\frac{l}{t} hw$
3) $\frac{w}{t} hl$

Estimation
- $\frac{l}{t}$: speed of water : $1\frac{m}{s}$, $10\frac{m}{s}$ ✓, $100\frac{m}{s}$
- h : slab thickness : $1m$ ✓, $10m$, $100m$
- w : $100m$, $1000m$ ✓, $10000m$

Flow rate: $\frac{l}{t} \cdot h \cdot w = 10\frac{m}{s} \cdot 1m \cdot 1000m = \frac{10{,}000\, m^3}{s}$

b)

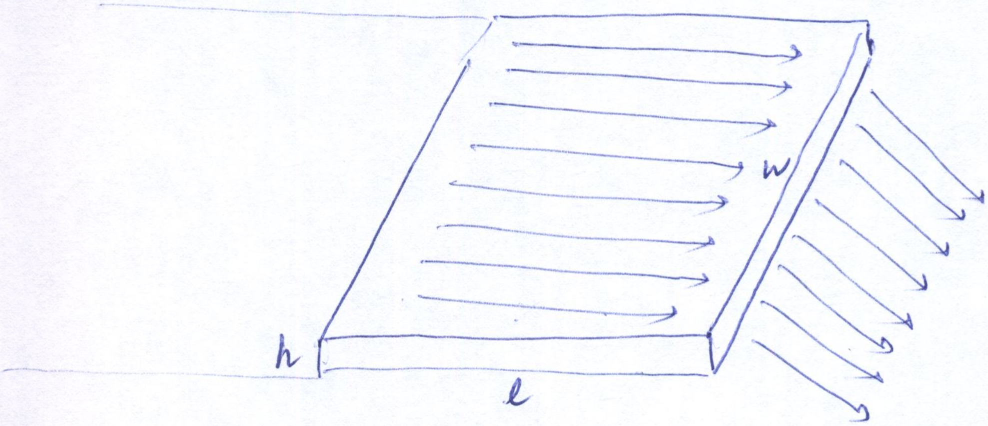
1.41

a) Estimate volume (m^3) of water going over Niagara Falls in 1s.

Guesses: $\left\{ 3E6 \frac{m^3}{s} \right.$

Estimation: simple geometrical shape

top level: rectangular slab $h \times l \times w = $ Volume



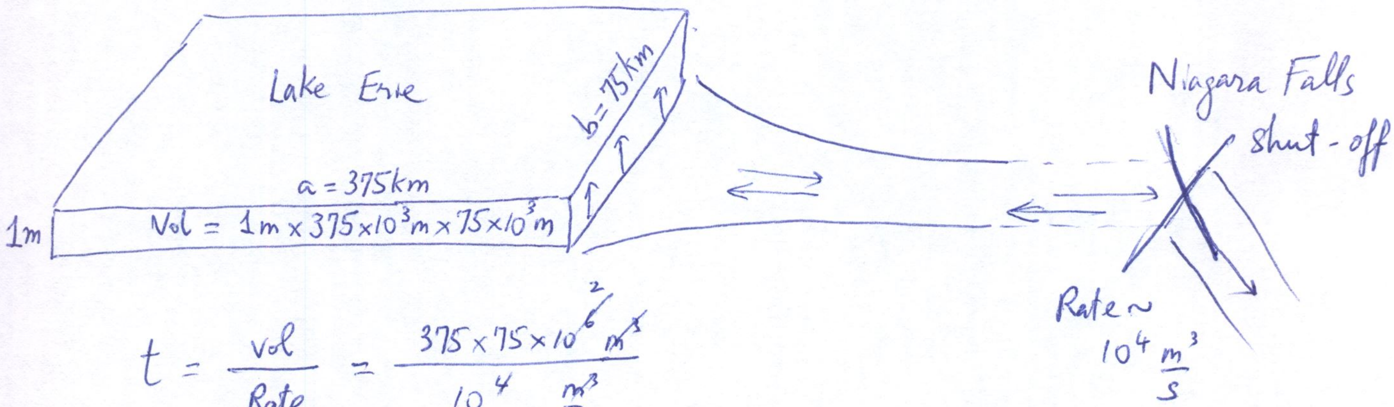
Volume per second or flow rate : $\dfrac{\text{volume}}{t} = \dfrac{hlw}{t}$ $\begin{cases} 1) \frac{h}{t} lw \\ 2) \frac{l}{t} hw \\ 3) \frac{w}{t} hl \end{cases}$

Estimation $\begin{cases} \frac{l}{t} : \text{speed of water} : & 1\frac{m}{s},\ 10\frac{m}{s},\ 100\frac{m}{s} \\ h : \text{slab thickness} & 1m,\ 10m,\ 100m \\ w : & 100m,\ 1000m,\ 10000m \end{cases}$

Flow rate: $\dfrac{l}{t} \cdot h \cdot w = 10\frac{m}{s} \cdot 1m \cdot 1000m = \dfrac{10{,}000\ m^3}{s}$

b)

b) If Niagara Falls are shut-off, water level at Lake Erie will rise, how long does it take to rise 1 m?
$\underbrace{\text{how long}}_{t}$



$$t = \frac{\text{Vol}}{\text{Rate}} = \frac{375 \times 75 \times 10^{\cancel{6}\,2}\ \cancel{m^3}}{10^4\ \frac{\cancel{m^3}}{s}}$$

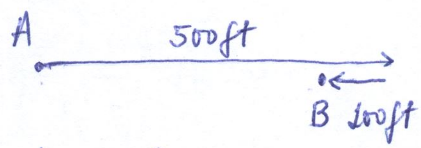
$$= 28125 \times 10^2\ \cancel{s} \cdot \frac{1\ \text{day}}{86400\ \cancel{s}} = 32.6\ \text{days}$$

$$\simeq 1\ \text{month}$$

Ch 2 Motion in a Straight Line (horizontal or vertical)

Average Motion: average velocity & average acceleration

Speed $\left(\frac{m}{s}\right)$	Velocity v $\left(\frac{m}{s}\right)$
$= \frac{\text{distance}}{\text{time}}$	$= \frac{\text{displacement}}{\text{time}}$



$t_{AB} = 6\,min$

$$= \frac{600\,ft}{6\,min} = 100\,\frac{ft}{min}$$

$$= \frac{400\,ft}{6\,min} = 66.67\,\frac{ft}{min}$$

velocity is lower than speed because direction of motion counts

Average velocity: $\bar{v} = \frac{\Delta x}{\Delta t}$ $\left(\frac{m}{s}\right)$

Δx: increment or change of position x or displacement

Δt: increment of time or time

Instantaneous velocity: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ ("time derivative of position")

Example: $x = at^4 \rightarrow v = 4at^3$ (unit: $\frac{m}{s}$)

a: constant

x: (unit: m)

$$\left(\frac{dt^n}{dt} = n\,t^{n-1}\right)$$

Acceleration: change of velocity over time

Average acceleration $\left(\frac{m}{s^2}\right)$ $\quad \bar{a} = \frac{\Delta v}{\Delta t}$

Δv : change of velocity or increment in velocity
Δt : time

Instantaneous acceleration $\left(\frac{m}{s^2}\right)$ $\quad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

(time derivative of velocity)

Example:

$x = ct^4 \, (m) \to v = 4ct^3 \left(\frac{m}{s}\right) \to a = 12ct^2 \left(\frac{m}{s^2}\right)$

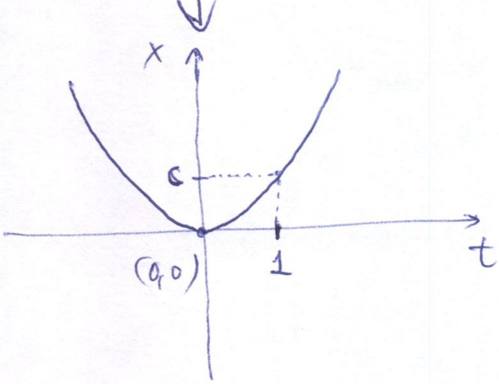
(varies quadratically in time)

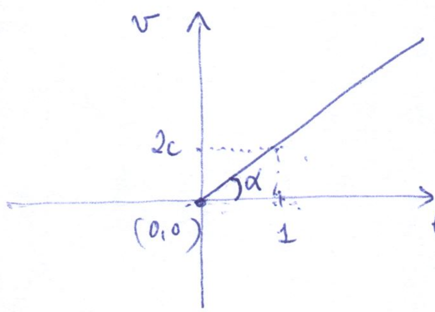
$\boxed{x = ct^2 \to v = 2ct \to a = 2c \left(\frac{m}{s^2}\right)}$

(position varies quadratically w.r.t. time) — constant acceleration

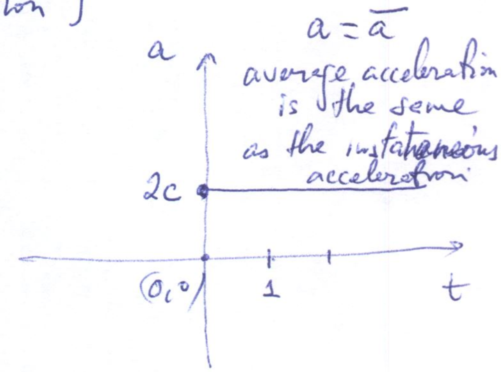
$x = ct \to v = c \to a = 0$

constant velocity (uniform motion)





tan α = slope of the line = 2c



From these basic physics we now derive the kinematic equations to describe a constant acceleration motion in a straight line (1D).

→ **Constant acceleration**:

$$a = \bar{a}$$

current velocity, initial velocity

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} \Rightarrow v - v_0 = at$$

current time, initial time

$$\rightarrow \boxed{v = v_0 + a \cdot t} \quad (1)$$

Kinematic equation #1

Kinematic equation #2:

current position, initial position

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \longrightarrow x = x_0 + \bar{v} \cdot t \quad (A)$$

$$\bar{v} = \frac{\int_0^t v \cdot dt}{t - 0} \overset{(1)}{=} \frac{1}{t}\int_0^t (v_0 + a \cdot t)\, dt$$

Mathematical average

$$= \frac{1}{t}\left[v_0 t + \frac{1}{2} a t^2\right]_0^t = \frac{1}{t}\left[v_0 t + \frac{1}{2} a t^2\right]$$

$$= v_0 + \frac{1}{2} a t$$

$$\int v_0\, dt = v_0 \underbrace{\int dt}_{t} = v_0 t$$

$$\int t\, dt = \frac{t^2}{2}$$

$$\int t^n\, dt = \frac{t^{n+1}}{n+1}$$

$$\boxed{\bar{v} = v_0 + \frac{1}{2} a \cdot t = \frac{1}{2} v_0 + \underbrace{\frac{1}{2} v_0 + \frac{1}{2} a \cdot t}_{\frac{1}{2} v} = \frac{1}{2}(v_0 + v)} \quad (B)$$

$$(A) \Rightarrow x = x_0 + \bar{v}\cdot t = x_0 + \left(\frac{v_0 + v}{2}\right)\cdot t \overset{(1)}{=} x_0 + \frac{1}{2}(v_0 + v_0 + a\cdot t)\cdot t$$

$$(B) \rightarrow \bar{v} = \frac{v_0 + v}{2}$$

$$\boxed{x = x_0 + v_0\cdot t + \frac{1}{2}a\cdot t^2} \quad (2)$$

Kinematic eq. #2

<u>Summary</u>: to describe a constant acceleration motion in 1D:

1) $v = v_0 + a\cdot t$

2) $x = x_0 + v_0\cdot t + \frac{1}{2}a\cdot t^2$

3) $\dfrac{v^2 - v_0^2}{x - x_0} = 2\cdot a$ (can be derived from 1) & 2))

$\rightarrow$ No time variable $\rightarrow$ good equation to start with in those problems where time info is not given

x_0 (m) : initial position ; x (m) : current or final position ;
v_0 $\left(\frac{m}{s}\right)$: initial velocity ; v $\left(\frac{m}{s}\right)$: current velocity
a $\left(\frac{m}{s^2}\right)$: constant acceleration ; t (s) : time ($t_0 = 0$)

$\bar{a} = a$

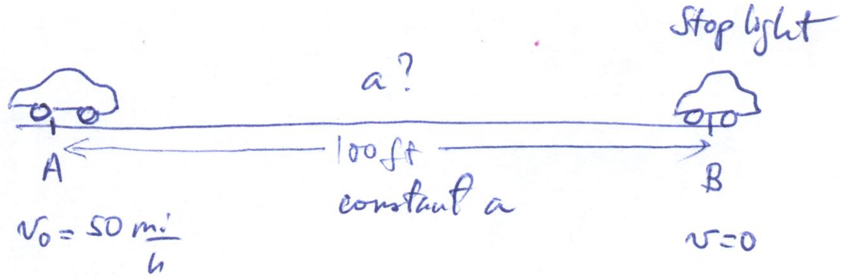
2.33

<u>Info or facts</u>:

- $v_0 = 50 \frac{mi}{h}$
- "begins slowing down at constant rate" = constant acceleration
- "100 ft short of stop light" $\Rightarrow x - x_0 = 100\,ft$
- "car comes to full stop @ light" $\Rightarrow v = 0 \left(\frac{m}{s}\right)$
- a ?

1) $\boxed{\begin{array}{l} v_0 = 50 \frac{mi}{h} \\ \text{constant } a \\ x - x_0 = 100\,ft \\ v = 0 \\ a\,? \end{array}}$

2) Sketch:



3) Write down appropriate equation

Kinematic eq. 3: $\frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a$

<u>Unit conversions</u>: v_0, $x - x_0$ need to be in S.I units

$$v_0 = 50 \frac{\cancel{mi}}{\cancel{h}} \cdot \frac{1609\,m}{\cancel{mi}} \cdot \frac{1\,\cancel{h}}{3600\,s} = 22.35 \frac{m}{s}$$

$$x - x_0 = 100\,\cancel{ft} \cdot \frac{0.3048\,m}{1\,\cancel{ft}} = 30.48\,m$$

$$a = \frac{1}{2} \frac{0 - 22.35^2}{30.48\,m} = -8.192 \frac{m}{s^2}$$

Negative acceleration or deceleration to come to a stop from $v_0 = 50 \frac{mi}{h}$

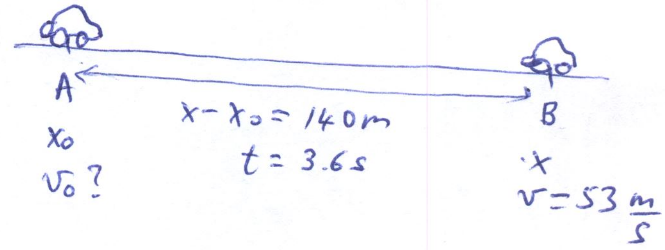
a is constant

2.59

1)

a constant
$x - x_0 = 140\,m$
$t = 3.6\,s$
$v = 53\,\frac{m}{s}$
v_0 ?

2)



3) a) Alternative #1   Eliminating a from eqs 1) & 2)

Eq 1: $v = v_0 + a \cdot t \rightarrow a = \frac{v - v_0}{t}$

Eq 2: $x - x_0 = v_0 \cdot t + \frac{1}{2} a \cdot t^2 = v_0 \cdot t + \frac{1}{2}(v - v_0) \cdot t$

$= \frac{1}{2} v_0 t + \frac{1}{2} v \cdot t$

$\rightarrow 2(x - x_0) = (v_0 + v) \cdot t$

$$v_0 = \frac{2(x - x_0)}{t} - v$$

$$= \frac{2(140)}{3.6} - 53 \rightarrow \boxed{v_0 = 24.8\,\frac{m}{s}}$$

Alternative #2: $\boxed{\text{Find } a}$, then find v_0

Eq. 2: $x - x_0 = v_0 \cdot t + \frac{1}{2} a t^2 \rightarrow$ Need to eliminate v_0

Eq 1: $v = v_0 + a \cdot t \rightarrow v_0 = v - a \cdot t$

$\rightarrow x - x_0 = (v - at) \cdot t + \frac{1}{2} a t^2$

$$= v \cdot t \underbrace{- a t^2 + \frac{1}{2} a t^2}_{-\frac{1}{2} a t^2}$$

$$\frac{1}{2} a t^2 = v \cdot t - (x - x_0)$$

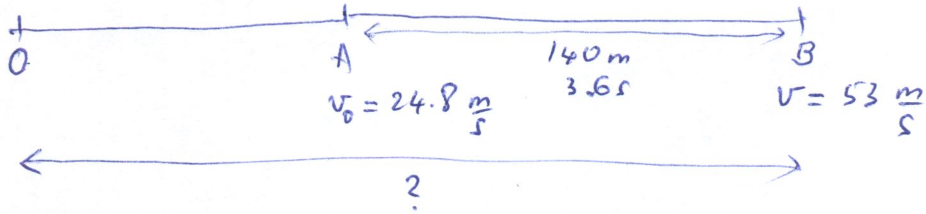
$$a = \frac{2[v \cdot t - (x - x_0)]}{t^2} = \frac{2[53 \cdot 3.6 - 140]}{3.6^2}$$

$$a = 7.83\,\frac{m}{s^2}$$

$$v = v_0 + at \rightarrow \boxed{v_0 = v - at = 53 - 7.83 \cdot 3.6 = 24.8\,\frac{m}{s}}$$

b) How far did it travel from $\underbrace{\text{rest}}_{\text{point O}}$ till $\underbrace{\text{end of 140 m distance}}_{\text{point B}}$

New sketch



Continuing w/ Alternative #1 used in part a)

Find OA $\Rightarrow$ OB = OA + 140 m $= 39.4 + 140 = 179.4\,m$

$$a = \frac{v - v_0}{t} = \frac{53 - 24.8}{3.6} = 7.83 \frac{m}{s^2}$$

$$\text{Eq3} \rightarrow \frac{v^2 - v_0^2}{(x - x_0)_{0A}} = 2 \cdot a \rightarrow (x - x_0)_{0A} = \frac{(v^2 - v_0^2)_{0A}}{2 \cdot a}$$

$$= \frac{24.8^2 - 0}{2 \cdot 7.83} = 39.4\,m$$

Continuing w/ Alternative #2 used in part a)

$a = 7.83 \frac{m}{s^2}$ (same a from 0 to B)

$$\text{Eq 3} \rightarrow \frac{(v^2 - v_0^2)_{0B}}{(x - x_0)_{0B}} = 2 \cdot a \rightarrow (x - x_0)_{0B} = \frac{(v^2 - v_0^2)_{0B}}{2 \cdot a}$$

$$= \frac{(53^2 - 0)}{2 \cdot 7.83}$$

$$= 179.4\,m$$

2.34 e^- $v_0 = 10^8 \frac{m}{s}$ $v = 0$ $\Delta t = 10^{-9} s$

$x - x_0$?

constant acceleration $\rightarrow \begin{cases} 1)\ v = v_0 + a.t \\ 2)\ x - x_0 = v_0.t + \frac{1}{2}at^2 \end{cases}$

1) $a = \frac{v - v_0}{t} = \frac{0 - 10^8}{10^{-9}} = -10^{17} \frac{m}{s^2}$

2) $x - x_0 = 10^8 . 10^{-9} + \frac{1}{2}(-10^{17})\, 10^{-18} = 10^{-1} - \frac{10^{-1}}{2} = 0.05 m$

$= 5 cm$

2.69

1) 3m height

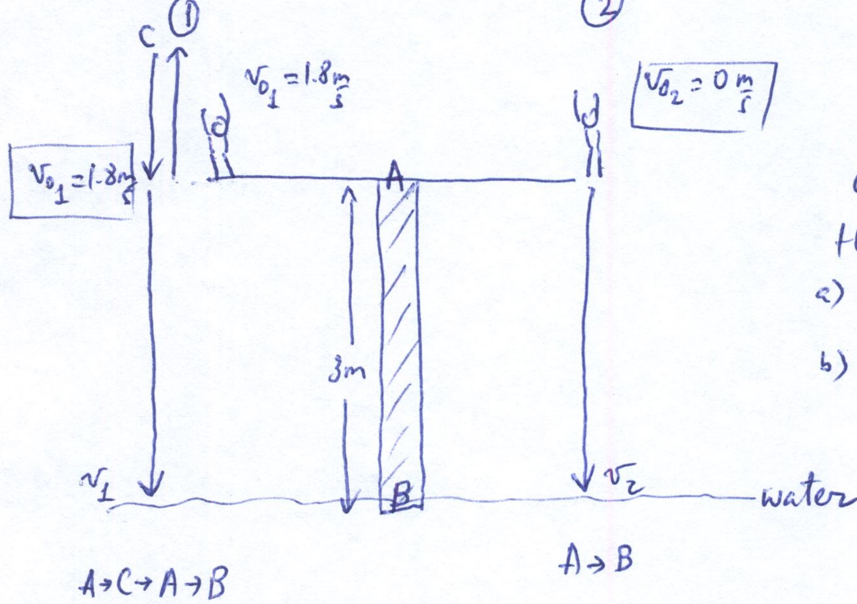
$v_{0_1} = 1.8 \frac{m}{s}$ (up)

$v_{0_2} = 0 \frac{m}{s}$ (down)

constant acceleration: $g = 9.81 \frac{m}{s^2}$

Diver 2 steps off when diver 1 comes back down by the platform.

2)



Compare motion of these two divers:

a) v_1 vs v_2 @ B

b) which diver will enter water first?

3) Use kinematic equations 1) 2) or 3)

Predictions: b) Diver 1 will enter water first since she has the additional $v_0 = 1.8 \frac{m}{s}$

a) Diver 1 will enter water @ higher speed since both are subject to the same g & diver 1 has the addional v_0

Calculations: eq 3 (no time info)
find v_1 & v_2

a)

Diver 1

$$\frac{v_1^2 - v_{0_1}^2}{x - x_0} = 2 \cdot g$$

$$\begin{cases} x - x_0 = AB = 3m \\ g = 9.81 \frac{m}{s^2} \end{cases}$$

$$v_1 = \sqrt{v_{0_1}^2 + 2 \cdot g \cdot (x - x_0)}$$

$$= \sqrt{1.8^2 + 2 \cdot 9.81 \cdot 3}$$

$$= 7.88 \frac{m}{s}$$

Diver 2

$$\frac{v_2^2 - 0}{x - x_0} = 2 \cdot g$$

$$v_2 = \sqrt{2 \cdot g \cdot (x - x_0)}$$

$$= \sqrt{2 \cdot 9.81 \cdot 3}$$

$$= 7.67 \frac{m}{s}$$

b) $v = v_0 + a \cdot t \rightarrow$ eq 1 (have v_1 & v_2 & v_{0_1} & v_{0_2} & g, find t)

$$t_1 = \frac{v_1 - v_{0_1}}{g} = \frac{7.88 - 1.8}{9.81} = 0.62s$$

$$t_2 = \frac{v_2 - v_{0_2}}{g} = \frac{7.67 - 0}{9.81} = 0.78s$$

$\downarrow$

Diver 1 takes less time or she will enter water first.

Sign for g : two alternatives: for placing origin of coordinate

1) Downward is + or

origin @ platform A

$x - x_0 = 3\,m$

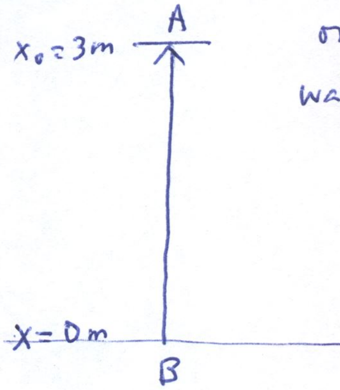
$g = +9.81\ \frac{m}{s^2}$

$v_1 = \sqrt{v_{o_1}^2 + \underbrace{2g(x-x_0)}_{+}}$

$= 7.88\ \frac{m}{s}$

2) Downward is − or

origin @ water surface B



$x - x_0 = -3\,m$

$g = -9.81\ \frac{m}{s^2}$

$v_1 = \sqrt{v_{o_1}^2 + \underbrace{\underset{-}{2g}\,\underbrace{(x-x_0)}_{-}}_{+}}$

$= 7.88\ \frac{m}{s}$

2.49 $x = bt^4$, b constant

$$\boxed{v = \frac{dx}{dt} = 4bt^3}$$

1) Mathematical average of v over time interval $(0, t)$

$$\bar{v} = \frac{1}{t-0}\int_0^t v \cdot dt = \frac{4b}{t}\int_0^t t^3 dt = \frac{b}{t}\left[t^4\right]_0^t$$

$$\boxed{\bar{v} = bt^3}$$

$$\int t^n dt = \frac{t^{n+1}}{n+1}$$

$$\boxed{\bar{v} = \frac{v}{4}}$$

2) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(t) - x(0)}{t-0} = \frac{bt^4 - 0}{t} = bt^3$

2.10

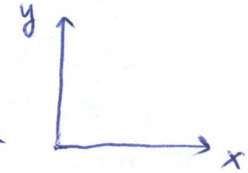
$50\,\frac{km}{h}$ during 1 h & $100\,\frac{km}{h}$ during 1 h

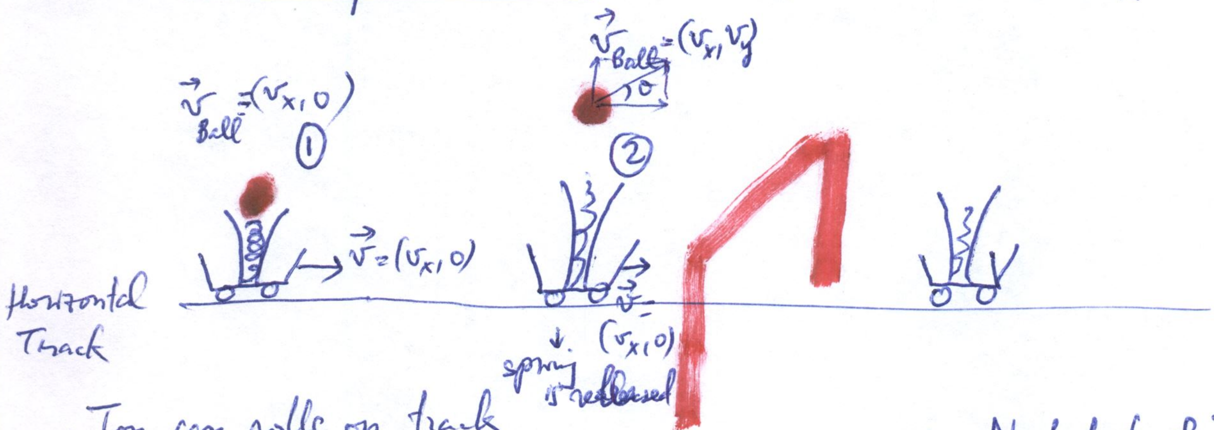
$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{150\text{ km}}{2\text{ h}} = 75\,\frac{km}{h}$$

Ch3 Motion in Two & Three Dimensions

<u>Note:</u> Ch2: horizontal or vertical, now in 2D we will describe motions involving both horizontal & vertical components at the same time.

<u>Visual experiment #1</u> Reference





→ Toy car rolls on track at speed $\vec{v} = (v_x, 0)$ (constant)

→ Spring is released with a switch controlled by the user. It will launch the ball up in the y-direction (spring is vertical)

→ Neglect friction b/w car wheels & track $\vec{v}_{car}$ = constant

→ Neglect any air resistance on the metal ball

→ @ ① ball has 1D motion same as the toy car

@ ② ball has 2D motion (with an angle) toy car still 1D

<u>Big question</u>: will ball make it back to funnel after car passes the gate?

→ Yes given (i) no friction & (ii) no air resistance; also

(iii) Motions in perpendicular directions (in y vertical & in x-horizontal) are independent!
(Ball actually makes it back to funnel!)

Mathematical descriptions

	1D	2D	3D
position	x	$\vec{r} = (x, y) = (r, \theta)$	$\vec{r} = (x, y, z) = (r, \theta, \varphi)$
velocity	v	$\vec{v} = (v_x, v_y) = (v, \theta_v)$	$\vec{v} = (v_x, v_y, v_z) = (v, \theta_v, \varphi_v)$
acceleration	a	$\vec{a} = (a_x, a_y) = (a, \theta_a)$	$\vec{a} = (a_x, a_y, a_z) = (a, \theta_a, \varphi_a)$

$\vec{r}$ = position vector $\begin{cases} (x, y, z) : \text{Cartesian coordinates (in 2D } (x, y)) \\ (r, \theta, \varphi) : \text{Spherical coordinates} \end{cases}$

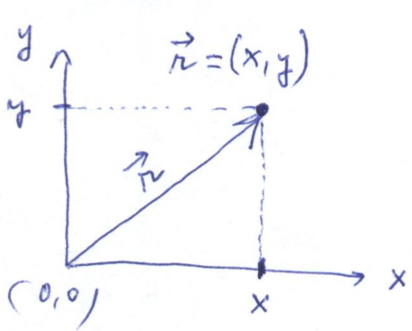
θ ↓ theta, φ ↓ phi

(in 2D (r, θ) : Polar coordinates)

$\vec{v}$ = velocity vector
$\vec{a}$ = acceleration vector

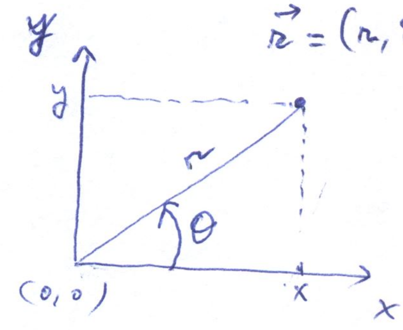
2D

Cartesian



Cartesian coordinates of a position described by position vector $\vec{r}$ are obtained by projecting it onto the x & y axes

Polar



r = length of $\vec{r}$ or its magnitude

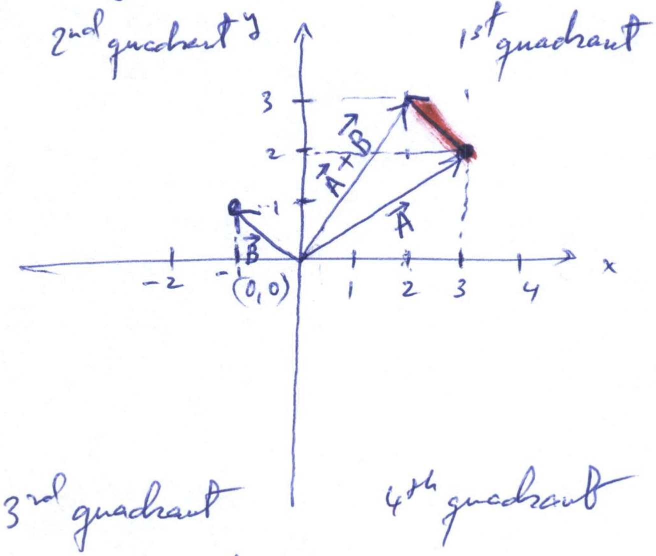
θ = angle formed by $\vec{r}$ from the x-axis (CCW)

Cartesian (x, y) ⟶ Polar $\begin{cases} r = \sqrt{x^2 + y^2} & \text{Pythagorean Theorem} \\ \theta = \tan^{-1} \dfrac{y}{x} & \text{Trigonometry} \end{cases}$

Polar (r, θ) ⟶ Cartesian $\left.\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}\right\}$ Trigonometry

Add & Subtract vectors $(\vec{v}, \vec{a}, \vec{r})$

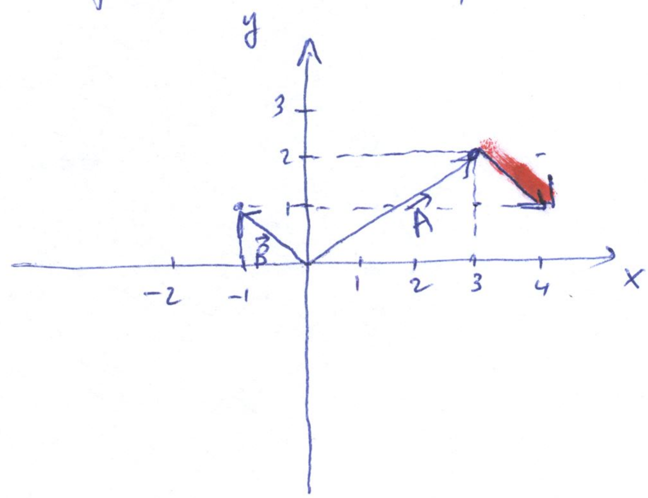
1) Graphically $\quad \vec{A} + \vec{B} \quad ; \quad \vec{A} - \vec{B}$ $\quad \begin{cases} \vec{A} = (3, 2) & 1^{st} \text{ quadrant} \\ \vec{B} = (-1, 1) & 2^{nd} \text{ quadrant} \end{cases}$



$\vec{A} + \vec{B}$: 1) Draw a copy of $\vec{B}$ from tip of $\vec{A}$

2) $\vec{A} + \vec{B}$ starts from origin of $\vec{A}$ to tip of the copy of $\vec{B}$

$\vec{A} + \vec{B} = (2, 3)$



$\vec{A} - \vec{B}$: 1) Draw a copy of $-\vec{B}$ from tip of $\vec{A}$

2) $\vec{A} - \vec{B}$ starts from origin of $\vec{A}$ to tip of copy of $-\vec{B}$

$\vec{A} - \vec{B} = (4, 1)$

2) Mathematically using Unit Vectors:

Unit vectors
- vectors with length or magnitude of 1
- x-direction = $\hat{i}$ (i-hat)
- y-direction = $\hat{j}$ (j-hat)
- z-direction = $\hat{k}$ (k-hat)

$\vec{A} = (3, 2) = (A_x, A_y)$

$\vec{A} = A_x \hat{i} + A_y \hat{j} = \boxed{3\hat{i} + 2\hat{j}}$

$\vec{B} = (-1, 1) = (B_x, B_y)$

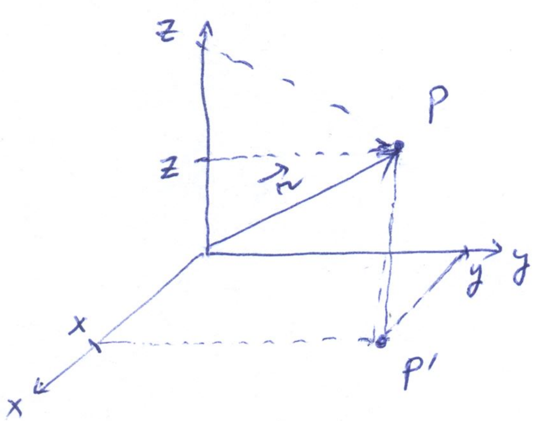
$\vec{B} = B_x \hat{i} + B_y \hat{j} = \boxed{-\hat{i} + \hat{j}}$

Addition: $\vec{A} + \vec{B} = \boxed{3\hat{i} + 2\hat{j}} + \boxed{-\hat{i} + \hat{j}} = 2\hat{i} + 3\hat{j} = (2, 3)$

Subtraction: $\vec{A} - \vec{B} = 3\hat{i} + 2\hat{j} - (-\hat{i} + \hat{j}) = 4\hat{i} + \hat{j} = (4, 1)$

3D

Cartesian: $\vec{r} = (x, y, z)$



Spherical $\vec{r} = (r, \theta, \varphi)$