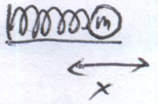
# Ch 14 Wave Motion

## Oscillatory motion

Time-repeating variation of a position or angle

↓



→ periodic variation

→ perturbation (periodic)

## Wave motion

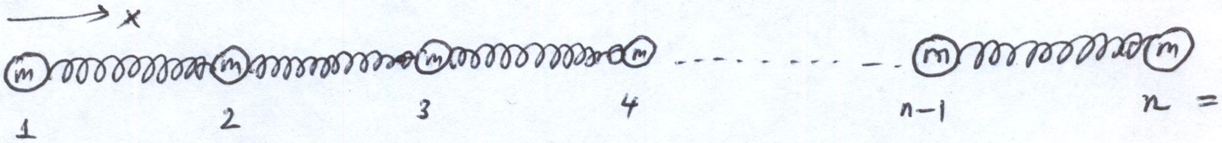
A step beyond: the oscillation/ time (periodic) variation/perturbation is propagated in space

→ variation in both time & space

→ there is propagation

1) Propagation: → an example of a longitudinal wave

→ on a system of identical bobs connected by identical springs



If I give bob #2 a displacement in the horizontal direction:

a) Bob #2: will undergo a time-repeating variation of position or oscillation or SHM. At this time what happens to bob #900? It is still at rest: perturbation given to a bob is local

b) Then the perturbation will propagate to #3, #4, #5, etc..

Propagation happens at finite speed, it is not instant! The speed depends on the medium (spring types, masses)

b1) { Perturbation on #2 is in x-direction.
Propagation of this perturbation is in x-direction
→ longitudinal wave

b2) What happens to #2 as perturbation is propagated?
#2 stays around its original position →

b2) **Conf**: propagation is of the perturbation or oscillation, not of matter or material. Clearly wave motion is different than previously studied motions: linear or rotational motion of matter

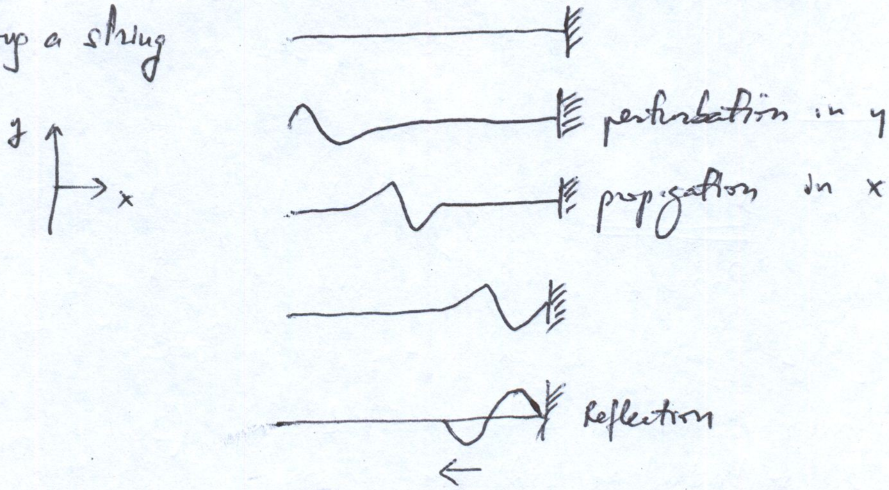
**Propagation**: of a perturbation is such that the objects involved (e.g.: springs & bobs) stay local while the perturbation reaches as far as possible:

→ **Waves**
- **Sound**:
  - matter: air molecules
  - perturbation: air pressure change is propagated as far as possible
- **Light**:
  - matter: no
  - perturbation: oscillation of electric & magnetic fields.

2) **Waves**: there are both time & space variations:

**Transverse wave**: perturbation & propagation are perpendicular to each other { For example: propagation along x-directi while the perturbation is along y-direction:

Wave along a string



**Mathematically**: space & time oscillation

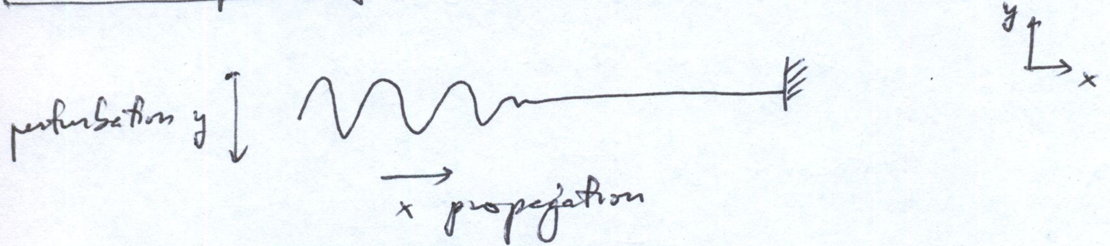
$$y(x,t) = A\sin(kx - \omega t)$$

wave { perturbed in y direction
propagated in x direction

- A: wave amplitude
- k: wave number $k = \frac{2\pi}{\lambda}$
- ω: angular freq.

3) Types of waves:
- → Longitudinal: perturbation & propagation are in same direction (springs & Lobs-, seismic waves, etc.)
- → Transverse: perturbation & propagation are perpendicular (wave in a guitar string; EM waves, etc)

4) Math description of transverse waves:



$$\boxed{y(x,t) = A \sin(kx - \omega t)}$$

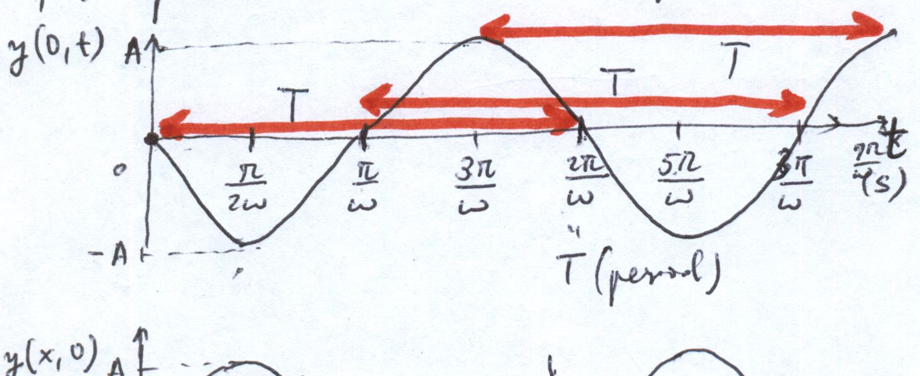
→ $k$: wave number: number of wavelengths in $2\pi$ $= \frac{2\pi}{\lambda}$ $(m^{-1})$

→ $\lambda$: 'lambda': wavelength (m): space separation b/w two consecutive peaks

→ $\omega$: angular frequency $=$ $\omega = \frac{2\pi}{T}$ $(s^{-1})$ $\longrightarrow$ $T = \frac{2\pi}{\omega}$

→ $T$: period (s): time separation b/w two consecutive peaks

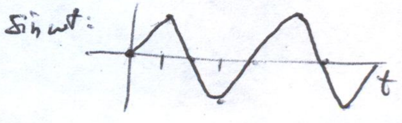
x=0



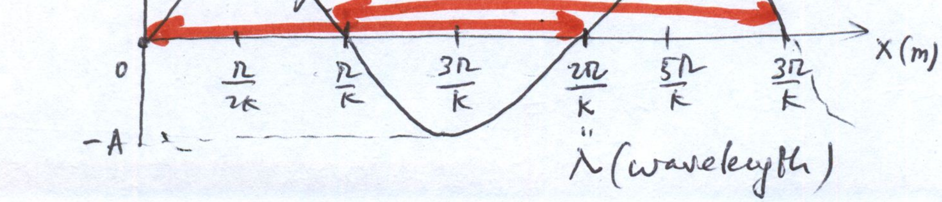
How the perturbation @ position x=0 varies over time:

$y(0,t) = A \sin(-\omega t)$

$= -A \sin(\omega t)$



t=0



How the perturbation @ t=0 varies over position

$y(x,0) = A \sin(kx)$

(14.56) Wave in a wire : $y(x,t) = 1.5 \sin(0.1x - 560t)$ $\begin{cases} x, y \text{ are in cm} \\ t \text{ is in s} \end{cases}$

$\underline{T = 28N}$

↳ wave:

1) perturbation in y, propagation in x
→ transverse wave

2) wave amplitude $\boxed{A = 1.5\,\text{cm}}$ → a)

3) wave number: $k = 0.1\ \text{cm}^{-1}$

$\downarrow$

$k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.1} = 20\pi\ \text{cm}$

$\boxed{\lambda = 62.8\,\text{cm}}$ b)

4) angular freq: $\omega = 560\ s^{-1}$

$\downarrow$

$\omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\cancel{560}} = \frac{2\pi}{560} = 11.2 \times 10^{-3} s$

$\boxed{T = 11.2\,\text{ms}}$ c)

$y(x,t) = A \sin(kx - \omega t)$
transverse

d) <u>Wave speed</u> : $\boxed{v = \frac{\lambda}{T}}$ (it takes a t = T to propagate a distance ~~ep~~of λ)

$$= \frac{62.8 \times 10^{-2}\ m}{11.2 \times 10^{-3}\ s} = 56\ \frac{m}{s}$$ (transverse wave in wire)

compared to ~~cars~~ average speed in highways:

$$65\ \frac{mi}{h} \sim 100\ \frac{km}{h} \cdot \frac{1000m}{1km}\ \frac{\cancel{1h}}{3600s} = \frac{100}{3.6}\ \frac{m}{s} = 27.8\ \frac{m}{s}$$

<u>other equations related to wave speed</u> : linear frequency f : how many cycles or periods fit in 1s : $f = \frac{1s}{T}$

$\rightarrow \boxed{v = \lambda \cdot f}$

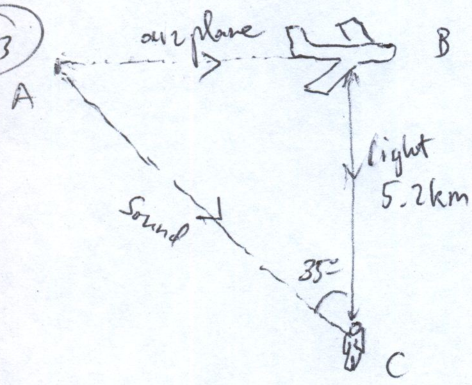
e) <u>Power carried by this wave</u> : (average power) $\overline{P} = \frac{1}{2}\mu\omega^2 A^2 v$

μ : linear density of wire (thin wire carries less power than a thick one)
ω : angular freq ; A : wave amplitude ; v wave speed.
→ Need to find μ : tension of wire T = 28N : $v = \sqrt{\frac{T}{\mu}} \rightarrow \mu = \frac{T}{v^2}$

$\mu = \frac{28}{56^2}\ \frac{kg}{m}$

$$\overline{P} = \frac{1}{2} \times \frac{28}{56^2} \times 560^2 \times 0.015^2 \times 56 \underset{\underset{\text{Watts}}{\downarrow}}{W} = 17.4\,W$$

(14.63)



→ Airplane straight overhead @ B

→ Observer @ C

→ Sound coming from A (AC forms 35° with AB)

⟹ Plane speed v? assuming $v_s = 330\,\frac{m}{s}$

Statement: $\begin{cases} t_s = \text{time for sound (jet noise) to travel from A to C} \\ t_p = \text{time for plane to travel AB} \end{cases}$

Facts: observer @ C see plane @ B but hears its noise that was made when plane passed A → $\boxed{t_s = t_p}$

Note: speed of light c = 300,000 $\frac{km}{s}$ → time for light to travel BC (5.2km) is negligible → instantaneous!

$$v_{plane} = \frac{d_{AB}}{t_p} = \frac{d_{AB}}{t_s} = \frac{d_{AB}}{\frac{d_{AC}}{v_s}} = v_s \underset{\underset{\frac{\text{opposite side to }35^\circ}{\text{hypothenuse}} = \sin 35^\circ}{\downarrow}}{\frac{d_{AB}}{d_{AC}}} = v_s \sin 35^\circ$$

$$= 330 \cdot \sin 35^\circ = 189\,\frac{m}{s}$$

$$v_{plane} = 189 \times 3.6\,\frac{km}{h} = 680\,\frac{km}{h}$$

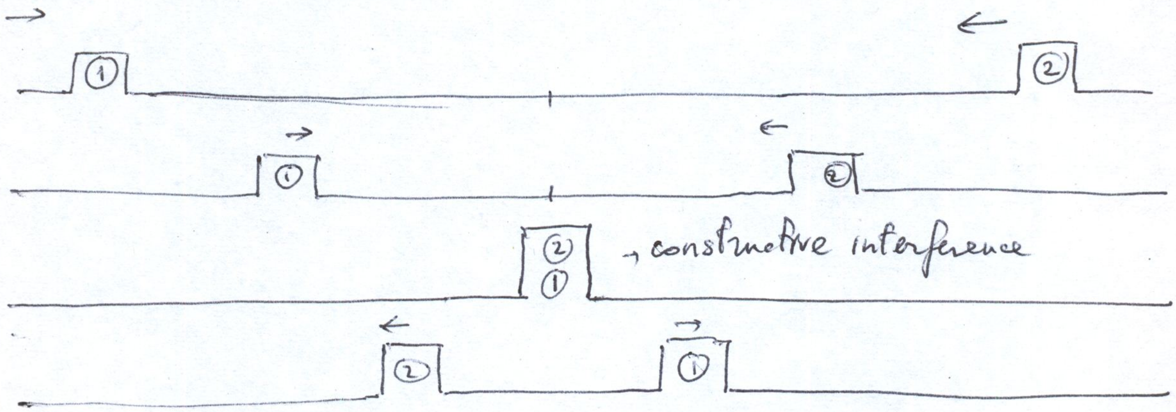
## ch14 (cont.) Wave Superposition:

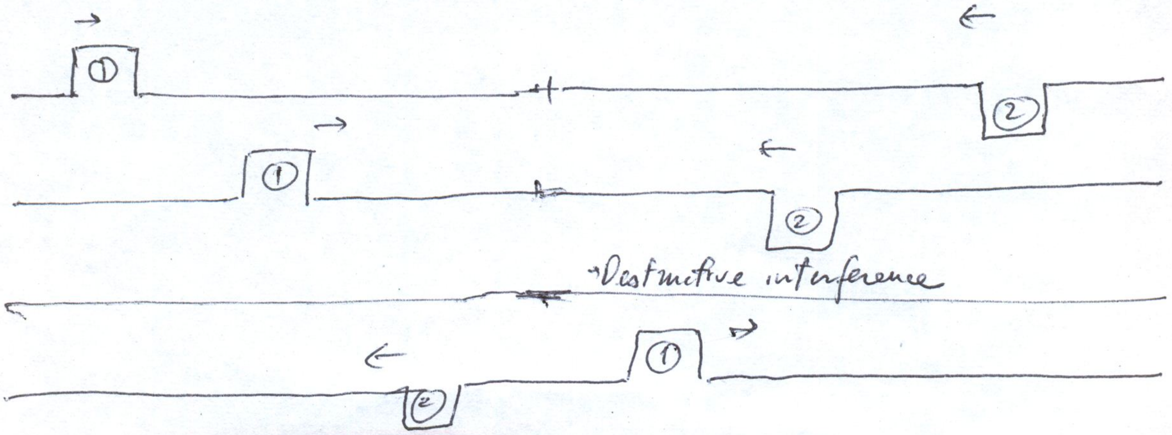
- → **Beat** phenomena: tuning of string instruments, tuning of engines etc.
- → Standing waves: waves in pipes, flutes, etc.
- → Wave interference
  - constructive
  - destructive: ↓ 1+1=0

unique properties of waves → 1+1=0

→ Doppler effect: when source of wave is moving (LIDAR: speed trap)

Wave superposition:





## Quantitative description of wave superposition: → Beat phenomenon

Two transverse waves: going in same direction
- same amplitudes A
- different frequencies: $\omega_1$, $\omega_2$ (different wave numbers $k_1$, $k_2$)

↳ $$\begin{cases} y_1(x,t) = A\sin(k_1 x - \omega_1 t) \\ y_2(x,t) = A\sin(k_2 x - \omega_2 t) \end{cases}$$

Superposition of these two waves @ $x = 0$ $$\begin{cases} y_1(0,t) = A\sin(-\omega_1 t) \\ y_2(0,t) = A\sin(-\omega_2 t) \end{cases}$$

↳ $$y(0,t) = y_1(0,t) + y_2(0,t) = -A\left[\sin\omega_1 t + \sin\omega_2 t\right]$$

$$\boxed{\text{Trigonometry:}\quad \sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)}$$

→ $$y(0,t) = -2A\,\sin\left(\boxed{\frac{\omega_1+\omega_2}{2}}\cdot t\right)\cdot\cos\left(\boxed{\frac{\omega_1-\omega_2}{2}}\cdot t\right)$$

$$= \underbrace{-2A\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)}_{\text{Modulated amplitude}}\cdot\ \sin\left(\frac{\omega_1+\omega_2}{2}\cdot t\right)$$

↓

If $\omega_1 \sim \omega_2$ → Beat phenomenon (tuning string instruments etc...)

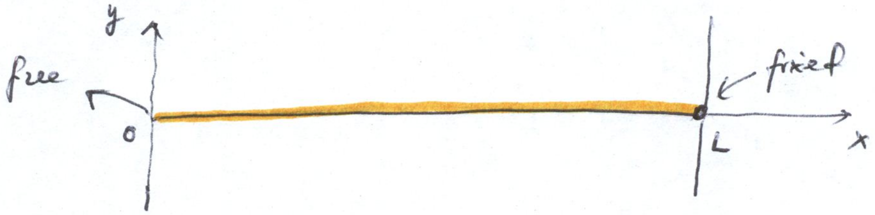
↓

Oscillating at much lower frequencies

one is reflection of the other
(same $A, \omega, k$)

## Wave Superposition: two waves going in opposite directions → Standing waves

- String of length $L$ attached to a fixed point:



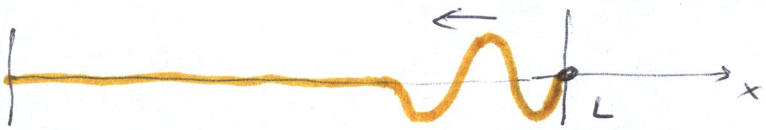
- Perturb free end by moving it up & down (in $y$ direction)

This perturbation generate a wave that propagate in $+x$ direction

$$y_1(x,t) = A\cos(kx - \omega t) \qquad \text{(incoming wave)}$$

+x propagation

- When it reaches the fixed end → it gets reflected: same wave (same $A$, same $\omega$, same $k$!) that will travel in $-x$ direction



$$y_2(x,t) = A\cos(-kx - \omega t) = A\cos(kx + \omega t)$$

$\downarrow$ (reflected wave) $\qquad \Downarrow \qquad$ $-x$ propagation

- If I keep sending incoming waves from left, they will ~~superp~~ superimpose with reflected waves (same $A, \omega, k$)

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\cos(kx - \omega t) - A\cos(kx + \omega t)$$

↑ Reflection: adds 180° phase to the incoming wave

Trigonometry: $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$$\boxed{y(x,t) = -2A\left[\sin(kx)\cdot\sin(-\omega t)\right] = 2A\sin(kx)\cdot\sin(\omega t)}$$

↓ Total wave = incoming + reflection

Why "standing" waves?

@ fixed end: $x = L$ : amplitude of total wave is 0

$$y(L,t) = \boxed{0 = 2A\cdot\sin(kL)\cdot\sin(\omega t)}$$

$2A\cdot\sin kL\cdot\sin\omega t$ has to be 0 @ any time. $\Rightarrow \sin(kL) = 0$

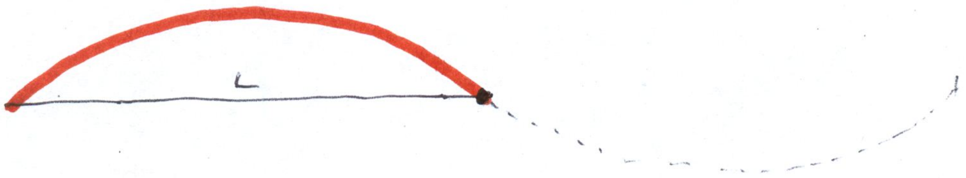
$\boxed{kL = n\pi}$ $(n = 1, 2, 3, \text{etc}\ldots)$

$$\frac{2\pi}{\lambda}L = n\pi$$

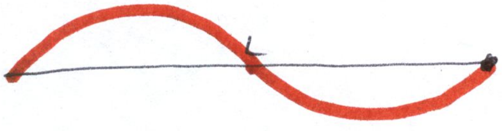
$$\lambda = \frac{2\cancel{\pi}L}{n\cancel{\pi}} = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots)$$

→ wave will stand in this string @ wavelengths: $\lambda = 2L, L, \frac{2L}{3}, \frac{L}{2}$, etc...

$\lambda = 2L$

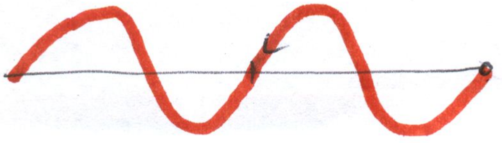


$\lambda = L$



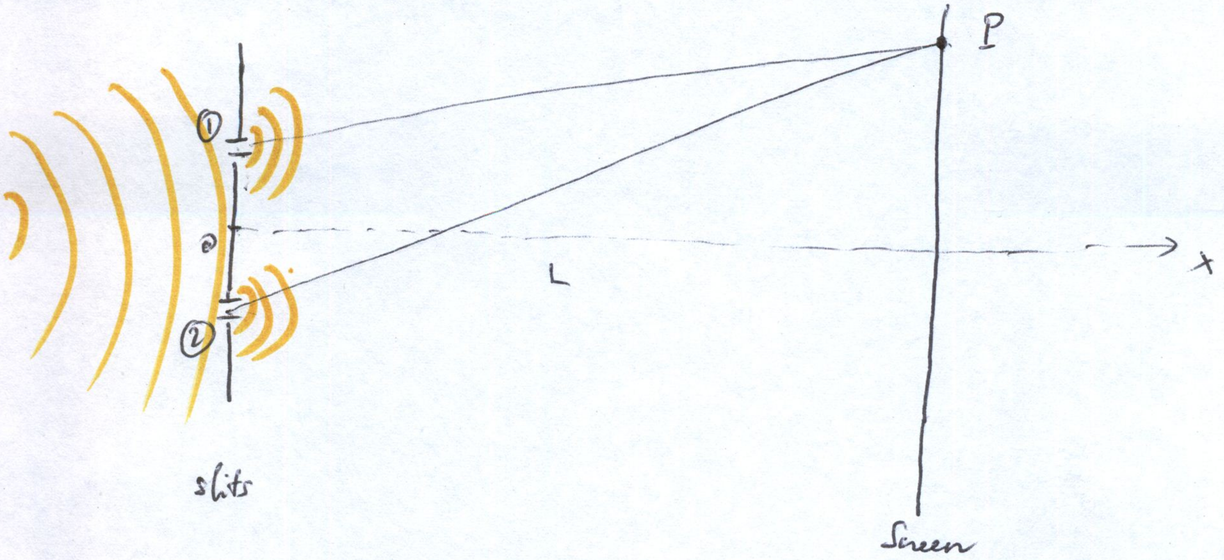
$\lambda = \frac{2L}{3}$

$\lambda = \frac{L}{2}$

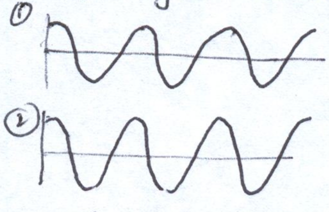


→ Superposition of two identical waves at a same point after traveling different paths: constructive & destructive interference

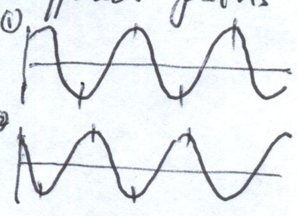
2 slits w/a "screen" @ certain distance from slits



Waves 1 & 2 are identical @ slits after traveling different paths they arrive @ P @ different phase

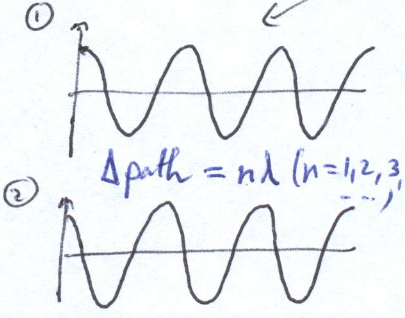


in phase @ slits



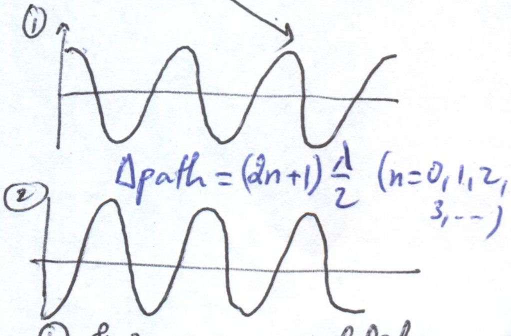
out of phase @ P
still same A, ω, k

if P slides along screen



$\Delta path = n\lambda$ ($n = 1, 2, 3, \ldots$)

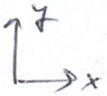
1 & 2 are in phase
↓
constructive interference
(bright spots, loud)

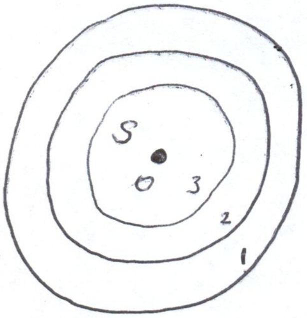


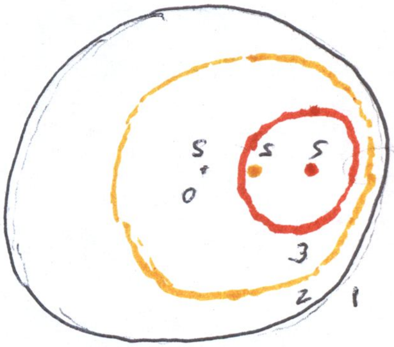
$\Delta path = (2n+1)\frac{\lambda}{2}$ ($n = 0, 1, 2, 3, \ldots$)

① & 2 are completely out of phase
↓
destructive interference
(dark spot, silence)

**Doppler Effect:** not wave superposition; moving source (of wave)







- source is @ rest

① came out first
then ②, then ③
① oldest, ③ youngest

- Source is moving in +x.

① came out then source moved ② came out, source continues to move

Consequence: waves closer in front ($\overset{\text{shorter } \lambda}{\text{higher frequency}}$), further apart in back ($\overset{\text{longer } \lambda}{\text{lower frequency}}$)

approaching source −

$$\underset{\substack{\downarrow \\ \text{effective} \\ \text{wavelength}}}{\lambda'} = \underset{\substack{\downarrow \\ \text{original} \\ \text{wavelength}}}{\lambda} \boxed{-} \underset{\substack{\text{source} \\ \text{speed}}}{u} T \longrightarrow \text{period of sound wave}$$

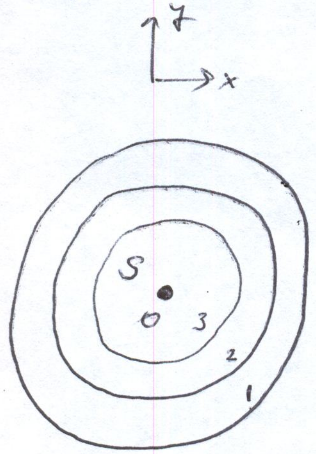
$$f' = \frac{f}{1 \boxed{-} \frac{\boxed{u} \rightarrow \text{source speed}}{\boxed{v} \rightarrow \text{wave speed}}}$$

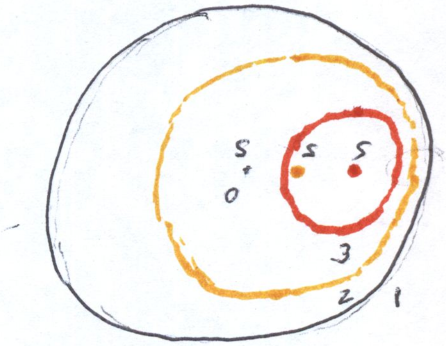
receding source +

$$\lambda' = \lambda \boxed{+} uT$$

$$f' = \frac{f}{1 \boxed{+} \frac{u}{v}}$$

**Doppler Effect:** not wave superposition; moving source (of wave)





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approaching source −

$$\underbrace{\lambda'}_{\text{effective wavelength}} = \underbrace{\lambda}_{\text{original wavelength}} - \underbrace{u}_{\text{source speed}}\overbrace{T}^{\text{period of sound wave}}$$

$$f' = \frac{f}{1 - \frac{u \;\rightarrow\; \text{source speed}}{v \;\rightarrow\; \text{wave speed}}}$$

receding source +

$$\lambda' = \lambda + uT$$

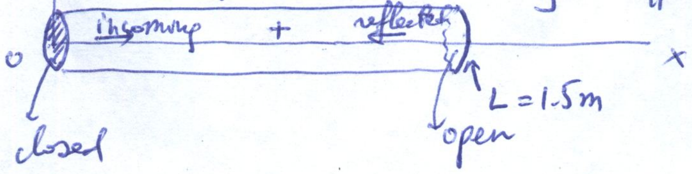
$$f' = \frac{f}{1 + \frac{u}{v}}$$

(14.76)

Pipe one end open : wave superposition → standing waves ↓ frequencies for different modes n

waves of same A, k, ω traveling in ~~different~~ opposite direction

$$\begin{cases} f_n = 225\,\text{Hz} \\ f_{n+1} = 375\,\text{Hz} \\ f_0\ ? \end{cases}$$



→ Same equation for standing wave as with waves in a string with a fixed end but different condition @ x = L

$$y(x,t) = 2A \cdot \sin kx \cdot \sin \omega t \quad \begin{cases} @\ x=0 \rightarrow y(0,t) = 0 \quad \text{trivial} \\ @\ x=L \rightarrow y(L,t) = 2A \underbrace{\sin kL}_{} \sin \omega t = \underline{\underline{\max}} \end{cases}$$

$$\Rightarrow \boxed{\sin kL = \pm 1}$$

$$kL = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \text{etc.}$$

$$\boxed{kL = (2n+1)\frac{\pi}{2}} \quad (n = 0, 1, 2, 3, ..)$$

→ Wavelengths for standing waves in this pipe:

$$\frac{2\cancel{\pi}}{\lambda} L = (2n+1)\frac{\cancel{\pi}}{2} \rightarrow \boxed{\lambda_n = \frac{4L}{2n+1}} \quad (n = 0, 1, 2, 3 ..)$$

→ Frequencies for these standing waves:

$$\underset{\substack{\uparrow \\ \text{wave speed}}}{v} = \frac{\lambda}{T} = \lambda \cdot f \Rightarrow \boxed{f = \frac{v}{\lambda}}$$

$\frac{1}{T}$ is how many periods or cycles fit in one second : cycles per second or f

$$\boxed{f_n = \frac{v}{\lambda_n} = \frac{v}{\left(\frac{4L}{2n+1}\right)} = \frac{2n+1}{4L} v} \quad (n = 0, 1, 2, 3, ...)$$

a) Standing wave frequencies on this pipe: $f_n = \frac{2n+1}{4L} v \quad (n=0,1,2,3...)$

$$\frac{f_{n+1}}{f_n} = \frac{375}{225} = \boxed{\frac{5}{3} = \frac{\frac{2(n+1)+1}{\cancel{4L}}\cancel{v}}{\frac{2n+1}{\cancel{4L}}\cancel{v}} = \frac{2n+3}{2n+1}}$$

$$\frac{2n+3}{2n+1} = \frac{5}{3} \quad \rightarrow \quad n=1 \text{ works!} \qquad \begin{cases} f_1 = 225\,Hz \\ f_2 = 375\,Hz \end{cases}$$

$f_0$ ?

one method: $f_0 = \frac{1}{4L} v$

$$f_1 = \frac{3}{4L} v \Rightarrow f_0 = \frac{f_1}{3} = \frac{225}{3} = 75\,Hz$$

second method:

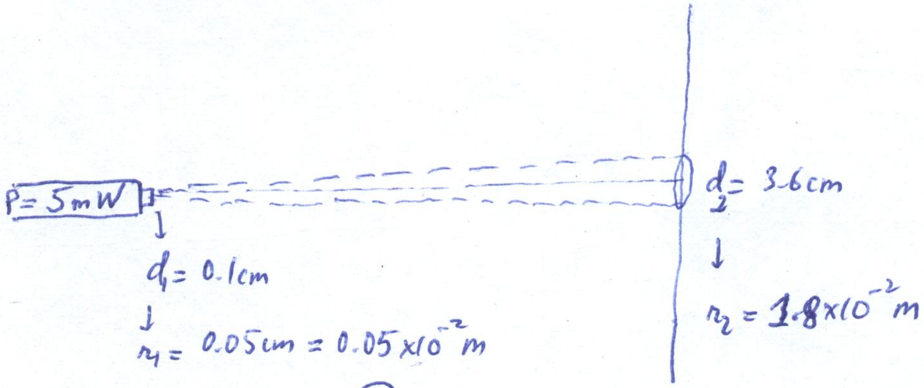
$f_2 - f_1 = 375 - 225 = 150\,Hz$

$f_1 - f_0 = 150\,Hz \rightarrow f_0 = f_1 - 150\,Hz = 75\,Hz.$

third method: find $\left[ v = \frac{4L}{3} f_1 = \frac{4\times 1.5}{3} \times 225 = 450 \frac{m}{s} \right]$ b)

$$f_0 = \frac{v}{4L} = \frac{450}{4\times 1.5} = 75\,Hz.$$

14.54



Beam intensity $I = \frac{P}{A}$ (power per unit area) $\left\{ \begin{array}{l} I_{d_1} ? \\ I_{d_2} ? \end{array} \right.$

A : circular cross-sectional area of beam

$$I_1 = \frac{5\times10^{-3}}{\pi(0.5\times10^{-3})^2}$$

$$I_2 = \frac{5\times10^{-3}}{\pi(18\times10^{-3})^2}$$

$$I_1 = 6.37 \frac{kW}{m^2}$$

$$I_2 = 4.91\times10^{-3} \frac{kW}{m^2}$$

14.43 Vocal tract ≃ pipe with one end closed

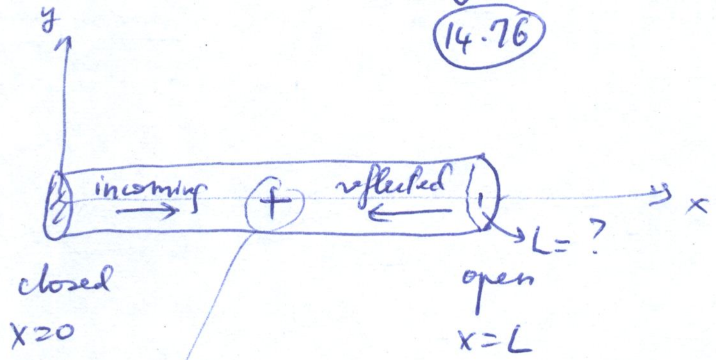
⇓

14.76

$f_0 = 620Hz$

$v = 354 m/s$

sound speed in human body



$$y(x,t) = A\cos(kx-\omega t) - A\cos(kx+\omega t) = 2A\sin kx \sin\omega t$$

$$\hookrightarrow \text{open } @ \ x=L \Rightarrow y(L,t) = \max = 2A\underbrace{\sin kL}_{\pm1}\sin\omega t$$

$$\sin kL = \pm1 \Rightarrow \boxed{kL = (2n+1)\frac{\pi}{2}} (n=0,1,2,3...)$$

$$\lambda_n = \frac{4L}{2n+1} \quad (n=0,1,2,3...)$$

$$f_n = \frac{2n+1}{4L}v \quad (n=0,1,2,3...)$$

$$f_0 = \frac{1}{4L}v$$

$$L = \frac{1}{4f_0}v = \frac{354}{4\times620}m =$$

# Ch15 Fluid Motion :

↳ { Gas : $\rho$ can be variable (gas is compressible)
Liquid: $\rho$ is constant (liquid is incompressible)

1) density = $\rho = \frac{\text{mass}}{\text{vol.}}$ or $\frac{dm}{dV}$ (SI: $\frac{kg}{m^3}$)

2) pressure: $P = \frac{\text{normal force}}{\text{area}} = \frac{F}{A}$ or $\frac{dF}{dA}$

SI unit = $\frac{N}{m^2} \equiv Pa$ for Pascal

→ Alternative unit: Atm (Atmosphere)

1Atm = $1.013 \times 10^5$ Pa

Equations for fluids

1) Hydrostatic equilibrium.

$$\boxed{\frac{dP}{dh} = g\rho}$$

↳ for constant g & $\rho$ ⇒ $P = g\rho h$

↳ Bouyancy : $F_{\text{bouyancy}} = P.A = g\rho \underbrace{hA}_{\text{vol.}}$

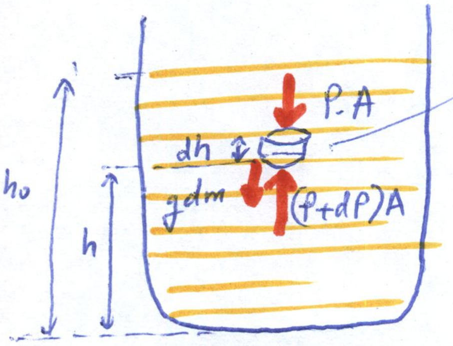
2) Conservation of mass:

$$\boxed{vA = \text{constant}}$$

3) Conservation of energy: $\frac{1}{2}\rho v^2 + \rho g y + P = \text{constant}$

Bernoulli's equation.

1) Hydrostatic equilibrium : on planet



vase filled w/ water

infinitesimal element of water (cylinder)

→ mass dm, cross-sectional area A

→ Pressure @ top of cylinder P

→ Pressure @ bottom of cylinder is higher (more fluid above it) P + dP

$dV = Adh$

↑

cylinder

→ Net force on this element of fluid → $F_{net} = 0$ (2nd Newton's Law)

$$F_{net} = (P+dP)A - P.A - g\,dm = 0$$

$$A\,dP - g\,dm = 0 \rightarrow A\,dP = g\,dm$$

$$\rho = \frac{dm}{\underset{vol}{dV}} \longrightarrow = g\rho\,dV$$

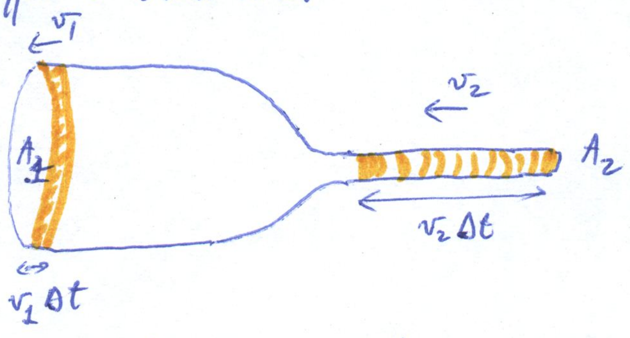
$$= g\rho A\,dh$$

$dV = Adh$

$$\cancel{A}\,dP = g\rho\cancel{A}\,dh \rightarrow \boxed{\frac{dP}{dh} = g\rho}$$

2) Conservation of mass : (no leaking or loss of fluid molecules)

Pipe with different cross-sectional areas :



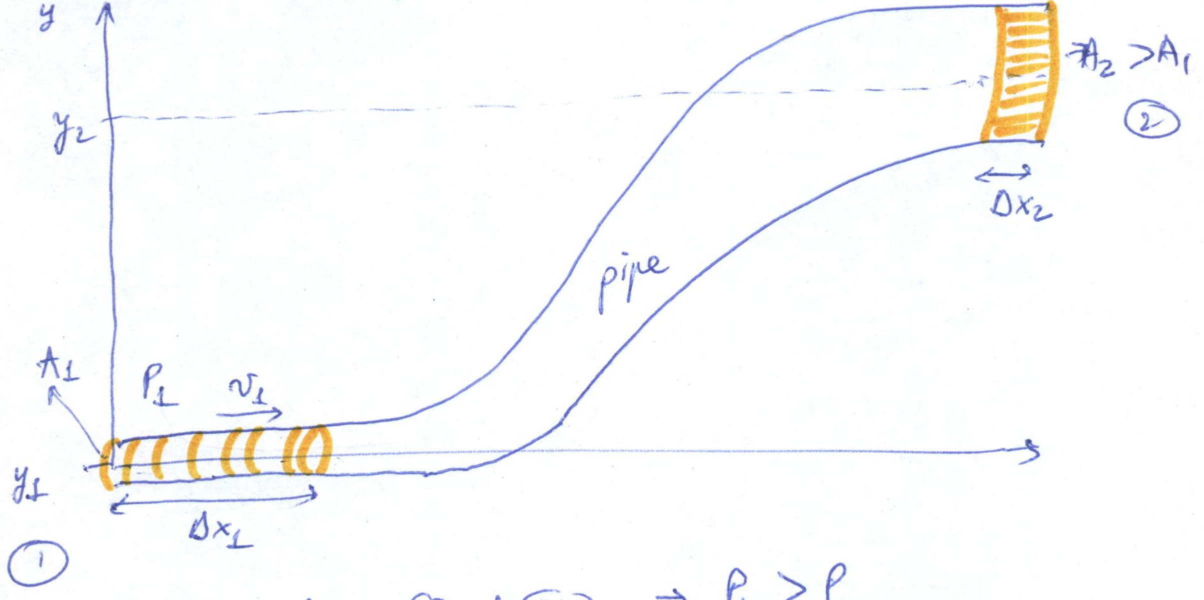
Smaller A, larger v

→ $m_1 = m_2$

$$m_1 = \rho V_1 = \rho\, v_1 \Delta t A_1 = m_2 = \rho V_2 = \rho\, v_2 \Delta t A_2$$

$$\cancel{\rho}\, v_1 \cancel{\Delta t} A_1 = \cancel{\rho}\, v_2 \cancel{\Delta t} A_2 \Rightarrow v_1 A_1 = v_2 A_2 \text{ or } \boxed{vA = \text{constant}}$$

## 3) Conservation of energy:



Water to go from ① to ② ⇒ $P_1 > P_2$

Work done by pressure to push water from ① to ②:

$$\Delta W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

conservation of energy → goes into fluid $\begin{cases} \Delta KE \\ \Delta GPE \end{cases}$

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

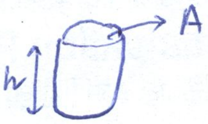
$$\frac{1}{2} m v^2 + mgy + PA\Delta x = \text{constant}$$

Dividing by Vol $V = A\Delta x$

$$\frac{1}{2}\frac{m}{V} v^2 + \frac{m}{V} gy + \frac{P\cancel{A\Delta x}}{\cancel{A\Delta x}} = \text{const.}$$

$$\boxed{\frac{1}{2}\rho v^2 + \rho g y + P = \text{constant}}$$ Bernoulli's eq.

(15.48) Glass beaker

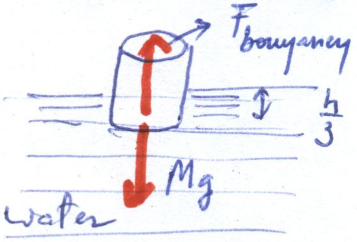


Cylinder

$d = 0.04\,m$

$h = 0.1\,m$

Empty: $\frac{h}{3}$ submerged in water



Floats: $F_{net} = 0$

$F_{bouyancy} - Mg = 0$

$P \cdot A$ ↓ by water ↘ beaker

$g\rho(h/3) \cdot A$ ↓ in

→ $F_{bouyancy}$ is proportional to volume of fluid displaced

$$\boxed{Mg = g\rho\frac{h}{3}A}$$

Sinks: h submerged on water

more vol. water displaced → larger $F_{bouyancy}$ → can add weights (rocks) to beaker!

Sinks:

$$g\rho hA - Mg - n \times 15 \times 10^{-3} g = 0$$

~~$g\rho h = 3Mg$~~

~~$2Mg = n \times 15 \times 10^{-3} g$~~

$n = 2x$

$$\frac{2}{3}\cancel{g}\rho hA = n \times 15 \times 10^{-3}\cancel{g}$$

$$n = \frac{\frac{2}{3}\rho hA}{15 \times 10^{-3}} = \frac{\frac{2}{3}1000 \times 0.1 \times \pi \times 0.02^2}{15 \times 10^{-3}}$$

$= 5.59$

$$\boxed{\rho_{water} = 1000\,\frac{kg}{m^3}}$$

→ about to sink: $n = 5$ rocks

## Exam 3 Ch 10 – 15

## Ch10: Rotational motion

1) New quantities

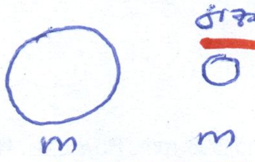
$\vec{\tau}$ : torque $= \vec{r} \times \vec{F}$   ($\vec{r}$ : pivot to force application point)

$\vec{L}$ : angular momentum $= \vec{r} \times \vec{p}$ (general)   ($\vec{r}$ : pivot to location of object)

$= I \cdot \vec{\omega}$ (rotations)

↑

size does matter



same linear motion but very different rotational motion

2) Analog of 2nd Newton's Law: $\vec{\tau} = I \cdot \vec{\alpha}$

I : moment of inertia wrt pivot

$\vec{\alpha}$ : angular acceleration

$$\vec{\tau} = \begin{cases} \frac{d\vec{L}}{dt} & \text{general} \\ I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} & \text{rotations} \end{cases}$$

## Ch11: Angular momentum conservation:

$$\vec{L} = \begin{cases} \vec{r} \times \vec{p} \text{ (general)} & (\vec{p} = m\vec{v}) \\ I\vec{\omega} \text{ (rotations)} \end{cases}$$

$\vec{L} = \vec{r} \times \vec{p}$   ($\vec{r}$ : from pivot to location of object)

↓

cross product

- $\vec{L} \perp \vec{r}$
- $\vec{L} \perp \vec{p}$
- Direction by RHR

Magnitude is $rp\sin\theta$ ($\theta$ is smaller angle b/w $\vec{r}$ & $\vec{p}$

$\vec{\tau}_{net} = 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L}$ is constant : $L_i = L_f$ (clearly define initial & final situations)

## Ch12: Static equilibrium

$\sum_i \vec{F}_i = 0$ (no linear motion)

$\sum_i \vec{\tau}_i = 0$ (no rotational motion)

↳ define pivot : select among different force application points to eliminate unknown forces

↳ requires angles b/w $\vec{r}$ & $\vec{F}$ ; use geometry to find these angles.

**Ch13:** Oscillatory Motion:

↓

Repeating → SHM

small θ ↓

Pendulum: $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \Rightarrow \theta(t) = \theta_M \cos\omega t$

$\omega = \sqrt{\frac{g}{L}}$

Torsional pendulum: $\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta \to \theta(t) = \theta_M \cos\omega t$

$\omega = \sqrt{\frac{\kappa}{I}}$

Spring & bob: $\frac{d^2x}{dt^2} = -\frac{k}{m}x \Rightarrow x(t) = x_M \cos\omega t$

$\omega = \sqrt{\frac{k}{m}}$

↳ Total energy stays constant:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \underbrace{\frac{1}{2}k x_M^2}_{\text{time independent}}$$

**Ch14:** Wave Motion:

$\boxed{\text{transverse}}$: $\underbrace{\text{oscillation/perturbation}}$ ⊥ propagation

longitudinal: ∥ propagation

wave in a string

$$y(x,t) = A\sin(kx \overset{\text{prop in }+x}{-} \underset{\text{prop in }-x}{+} \omega t)$$

↳ tension T → $\underset{\text{wave speed}}{v} = \sqrt{\frac{T}{\underset{\uparrow \text{ linear mass density of string}}{\mu}}}$ ∥ $v = \frac{\lambda}{T} = \lambda \cdot f$

λ → wavelength

T → wave period

**Wave superposition:** 2 waves

Beat phenomenon: same direction, same A, differen k & ω

$$y(x,t) = A\sin(k_1x - \omega_1 t) + A\sin(k_2x - \omega_2 t)$$

$$@\ x=0 \to y(0,t) = \underbrace{-2A\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)}_{\text{modulation}} \sin\left(\frac{\omega_1+\omega_2}{2}\cdot t\right)$$

beats when $\omega_1 - \omega_2$ is small!

Standing waves: opposite directions (incoming + reflected)

same A, k, ω

$$y(x,t) = A\cos(kx - \omega t) - A\cos(kx + \omega t) = 2A\sin kx \sin\omega t$$

↳ @ x=L : fixed or closed: $\sin kL = 0$

$kL = n\pi \quad (n = 1, 2, 3 \ldots)$

$\frac{2\pi}{\lambda}L = n\pi \to \boxed{\lambda_n = \frac{2L}{n}}$

@ x=L open or free: $\sin kL = \max = \pm 1$

$kL = (2n+1)\frac{\pi}{2}$

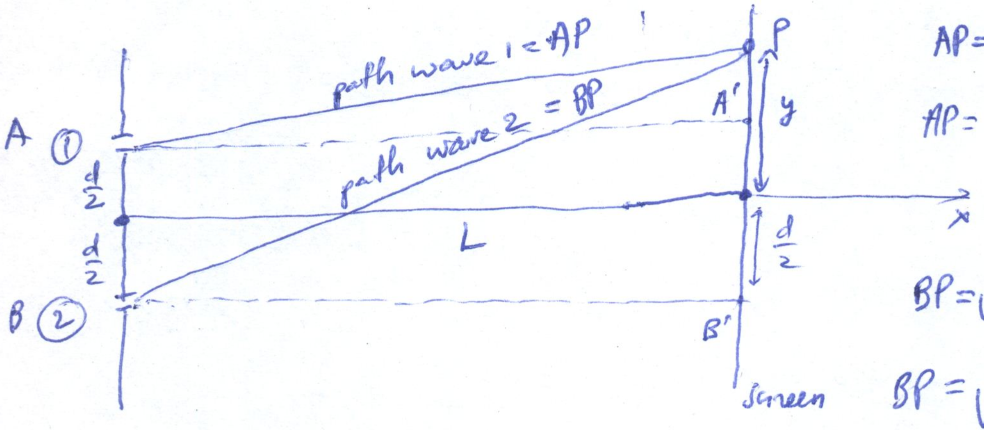
$\to \lambda_n = \frac{4L}{2n+1} \quad (n = 0, 1, 2, 3 \ldots)$

Two identical waves (same $A, K, \omega$) traveling different paths to P

- Constructive interference @ P = BP − AP = $n\lambda$ $(n=0,1,2,3\ldots)$

$$\sqrt{L^2+(y+\tfrac{d}{2})^2}-\sqrt{L^2+(y-\tfrac{d}{2})^2}=n\lambda$$

- destructive interference @ P : BP − AP = $(2n+1)\frac{\lambda}{2}$ $(n=0,1,2,3\ldots)$



$$AP=\sqrt{AA'^2+A'P^2}$$

$$AP=\sqrt{L^2+(y-\tfrac{d}{2})^2}$$

$$BP=\sqrt{BB'^2+(y+\tfrac{d}{2})^2}$$

$$BP=\sqrt{L^2+(y+\tfrac{d}{2})^2}$$

## Ch 15: Fluid motion:

3 equations

→ Hydrostatic equilibrium: $\frac{dP}{dh}=g\rho$

↳ $F_{\text{bouyancy}} = g\underbrace{\rho}_{\text{fluid density}}\underbrace{h\cdot A}_{\text{vol. of fluid displaced}}$

→ conservation of mass: $\underbrace{v}_{\text{speed of fluid}}\underbrace{A}_{\text{cross-sectional area}} = \text{constant}$

- Conservation of energy: $\frac{1}{2}\rho v^2 + \rho g h + P = \text{constant}$