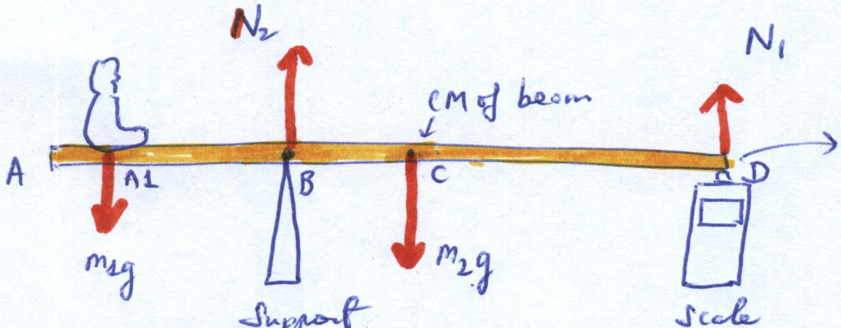
Ch 12 Static Equilibrium

<u>Applications</u> of
- 2nd Newton's Law : $\rightarrow \vec{F}_{net} = m\vec{a} = 0$
- Analog in rotational motion : $\rightarrow \vec{\tau}_{net} = I\vec{\alpha} = 0$
  - → Torque wrt <u>center of rotation</u> (select the most convenient for calculations)
  - → Force application points

12.21



→ Beam (<u>we will focus on the beam</u>)
↓
connects all components in this problem!

Info given:
- m_1 = mass of child = 40 kg
- m_2 = " of beam = 60 kg
- <u>Four forces acting on beam</u> @ these application points
  - A1 : weight of child
  - B : normal by support N_2
  - C : weight of beam
  - D : normal by scale
- AD = 2.4 m → BD = 1.6 m
- AB = 0.8 m, BC = 0.4 m (AC = 1.2 m)

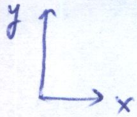
<u>Center of rotation or pivot</u> : Very important before we can talk about torques! $\boxed{\text{is point B.}}$

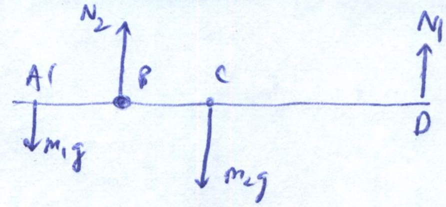
<u>Statement</u>: C could have been selected a the pivot, but B is more convenient! (Re then we don't need to calculate N_2 although we can if C is selected as pivot!)

<u>In principle</u>: 4 forces → 4 torques

Once a pivot is selected among (A1, B, C, D) ⇒ <u>3 torques</u>.

$\vec{\tau} = \vec{r} \times \vec{F}$ ($\vec{r}$: from pivot to force application point)





$N_1 = \begin{cases} 100\,N & a) \\ 300\,N & b) \end{cases}$

Static equilibrium:

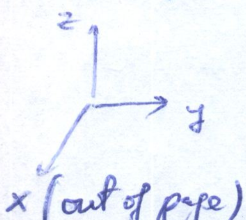
1) $\sum_i \vec{F}_i = \vec{F}_{net} = 0$: $\rightarrow N_2 + N_1 - (m_1 + m_2) g = 0 \rightarrow$ can solve for N_2

2) $\sum_i \vec{\tau}_i = \vec{\tau}_{net} = 0 \rightarrow \boxed{\vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_{N_1} = 0}$

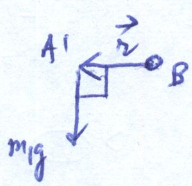
B is pivot

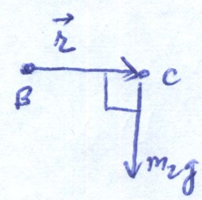
Statements:
a) common simplifying rule in static equilibrium analysis: place pivot @ force application point of the force we don't know
b) If pivot is B then $\vec{N}_2$ applies no torque since $\vec{r}_{N_2} = 0$ $\quad (\vec{\tau}_{N_2} = \vec{r}_{N_2} \times \vec{N}_2)$

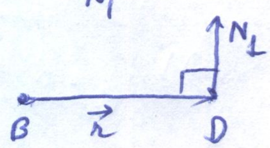
Child position? (since torque involves position of $m_1 g$ → use torque balance equation)



$$\vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_{N_1} = 0$$







$$m_1 g r_{BA'} \underbrace{\hat{i}}_{RHR} + m_2 g r_{BC} \underbrace{-\hat{i}}_{RHR} + N_1 r_{BD} (\hat{i}) = 0$$

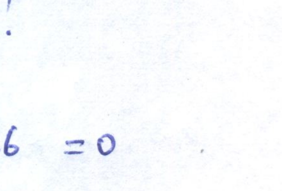
very important!

Torques are in x-direction:

$$m_1 g \underbrace{r_{BA'}}_{?} - m_2 g\, 0.4 + N_1 1.6 = 0$$

$$r_{BA'} = \frac{m_2 g 0.4 - 1.6 N_1}{m_1 g}$$

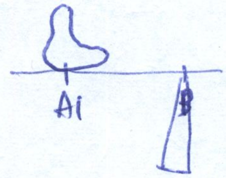
a) $N_1 = 100 N \rightarrow r_{BA'} = \dfrac{60 \times 9.81 \times 0.4 - 1.6 \times 100}{40 \times 9.81} = 0.19\,m$



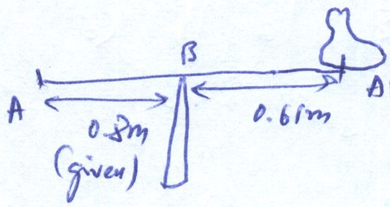
→ Position of child from left edge of beam is 0.61m

b) $N_1 = 300\,N \rightarrow r_{BA1} = \dfrac{60 \times 9.81 \times 0.4 - 1.6 \times 300}{40 \times 9.81} = -0.62\,m$

we assumed child sits left of pivot B

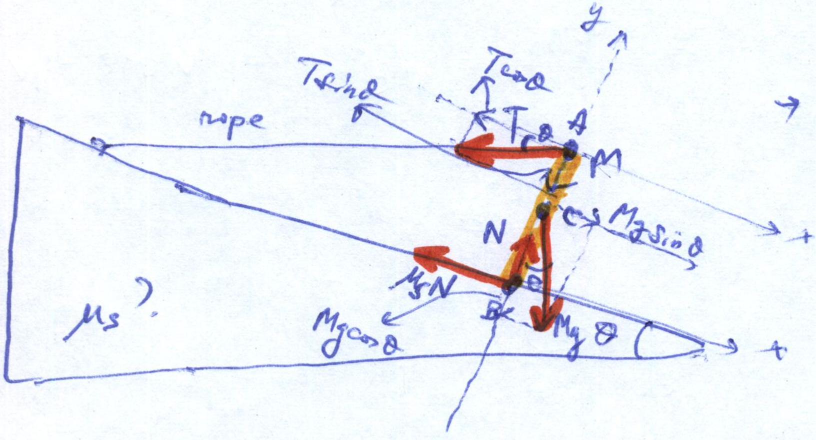


Child has to sit to the right of pivot B for $N_1 = 300\,N$



→ Position of child from left edge of beam is: $0.8 + 0.62 = 1.42\,m$

12.55



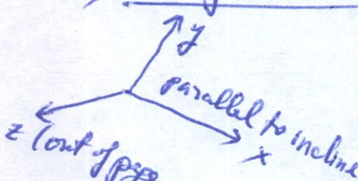
→ We will focus on pole:
Four forces on pole: $T, Mg, N, \mu_s N$
3 force application points: A, B, C

- A: top of pole
- C: CM of pole
- B: contact with incline

→ select convenient pivot or center of rotation: A, ~~B~~, C

- A ↓ eliminates torque of tension (T is unknown)
- B ↓ will eliminate friction torque ↓ need it to find μ_s
- C ↓ will eliminate torque by weight of pole (M is unknown)

→ Let's pick C as center of rotation:



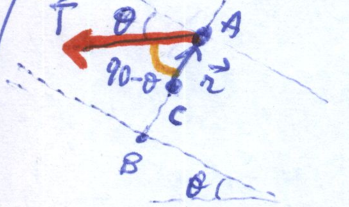
Static equilibrium

1) $\sum_i \vec{F}_i = \vec{F}_{net} = 0$

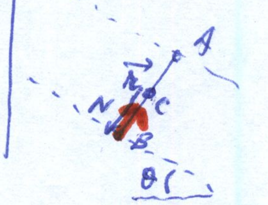
$$\begin{cases} x: Mg\sin\theta - T\cos\theta - \mu_s N = 0 & (1a) \\ y: N - Mg\cos\theta - T\sin\theta = 0 & (1b) \end{cases}$$

2) $\sum_i \vec{\tau}_i = \vec{\tau}_{net} = 0 = \vec{\tau}_T + \vec{\tau}_N + \vec{\tau}_{\mu_s N} = 0 \quad (2)$

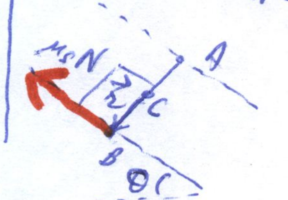
$\vec{\tau}_T = \vec{r}\times\vec{T} = r_{CA} T \underbrace{\sin(90-\theta)}_{\cos\theta} (\hat{k})$ ← RHR



$\vec{\tau}_N = r_{CB} N \underbrace{\sin(180)}_{0}$

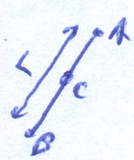


$\vec{\tau}_{\mu_s N} = r_{CB}\mu_s N(-\hat{k})$ ← RHR



→ Torque balance: $r_{CA} T\cos\theta - r_{CB}\mu_s N = 0 \quad (2)$
(z-direction)

since C is CM of pole



$\Rightarrow r_{CA} = r_{CB} = \frac{L}{2}$ (L: pole length)

No unique method from here:

(2) $\frac{L}{2} T\cos\theta - \frac{L}{2}\mu_s N = 0 \Rightarrow T = \frac{\mu_s N}{\cos\theta}$

→ (1b) $N - Mg\cos\theta - T\sin\theta = 0 \Rightarrow$ (1b) $N - Mg\cos\theta - \mu_s N\tan\theta = 0$

(1a) $Mg\sin\theta - \frac{\mu_s N}{\cancel{\cos\theta}}\cancel{\cos\theta} - \mu_s N = 0$

(1a) $Mg \sin\theta = 2\mu_s N$

$$Mg = \frac{2\mu_s N}{\sin\theta}$$

→ (1b) $\cancel{N} - \frac{2\mu_s \cancel{N}}{\sin\theta}\cos\theta - \mu_s \cancel{N}\tan\theta = 0$

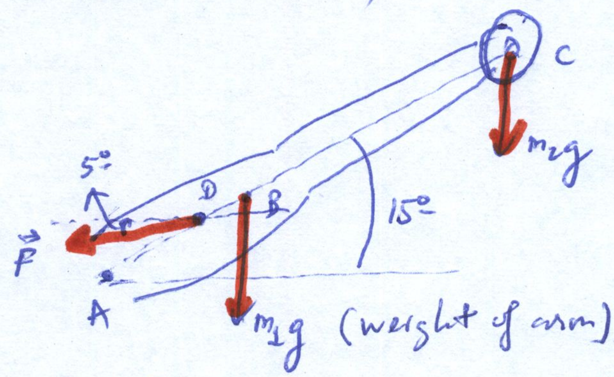
$$1 - \frac{2\mu_s}{\tan\theta} - \mu_s\tan\theta = 0 \rightarrow 1 - \mu_s\left(\frac{2}{\tan\theta} + \tan\theta\right) = 0$$

$$\mu_s = \frac{1}{\tan\theta + \frac{2}{\tan\theta}} = \frac{\tan\theta}{\tan^2\theta + 2}$$

↓ min for pole to stay in static equilibrium

$$\rightarrow \boxed{\mu_s \geq \frac{\tan\theta}{\tan^2\theta + 2}}$$

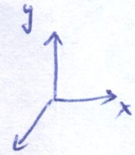
(12.27) Arm holding a weight; arm is 15° above horizontal (unlike beam in 12.21)



$m_2 = 6.0\,kg$ @ C
$m_1 = 4.2\,kg$ @ B
$AC = 0.56\,m$
$AB = 0.24\,m$
$AD = 0.18\,m$

→ Deltoid muscle: $\vec{F}$ applies 5° below horizontal

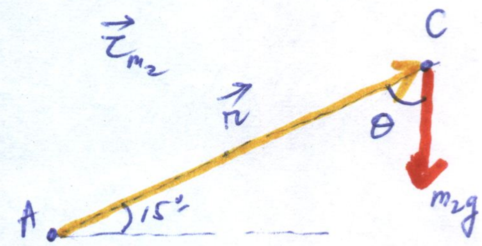
a) Torque about the shoulder (A) due to $\underline{m_1 g}$ & $\underline{m_2 g}$



Pivot or center of rotation: A

$\vec{\tau}_{m_1}$ + $\vec{\tau}_{m_2}$

$\theta = 75°$



$\theta = 75°$

$r_{AB}\, m_1 g \sin 75° (-\hat{k})$ + $r_{AC}\, m_2 g \sin 75 (-\hat{k})$

Total torque by m_1 & m_2 about A is

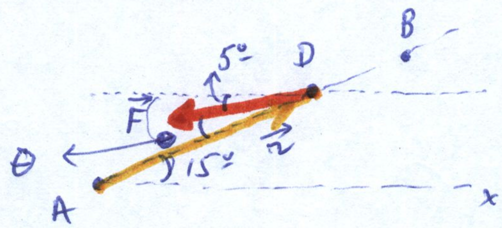
$$\vec{\tau}_{m_1 \& m_2} = (0.4 \times 4.2 \times 9.81 \times \sin 75°)\ (-\hat{k}) + 0.56 \times 6.0 \times 9.81 \times \sin 75°\ (-\hat{k})$$

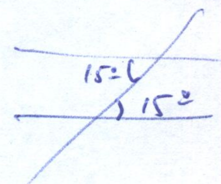
$$= 40.2\ (-\hat{k})\ \text{Nm} \quad \text{(w.r.t. to point A)}$$

b) $\vec{F}$? force applied by deltoid muscle (direction is given: 5° below horizontal ↔ 3rd quadrant in XY plane)

Statement: torque provided by $\vec{F}$ w.r.t. same center of rotation A is needed to cancel $\tau_{m_1 \& m_2} \Rightarrow \vec{\tau}_F = 40.2\ \hat{k}$ Nm

By definition:



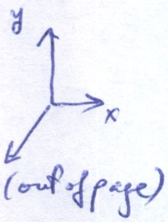


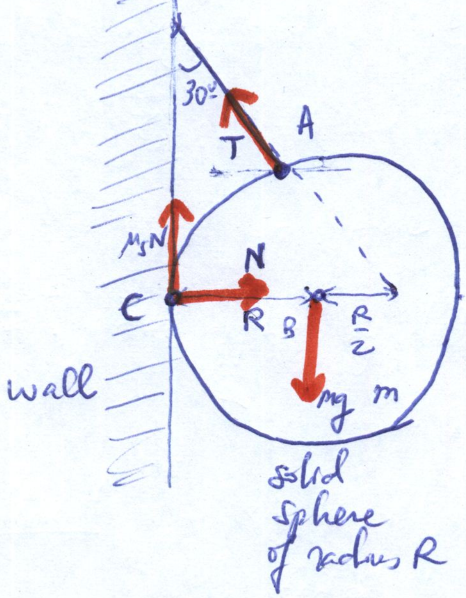
$$\vec{\tau}_F = \vec{r} \times \vec{F} = r_{AD} F \sin\theta\ (\hat{k}) = 40.2\ \hat{k}$$

$\theta = 10°$

$$F = \frac{40.2}{0.18 \times \sin 10°} = 1280\ \text{N} = 1.28\ \text{kN}$$

12.28





Mark all force application points:

→ A (tension T by rope)

$\vec{T} = -T\sin 30°\,\hat{i} + T\cos 30°\,\hat{j}$

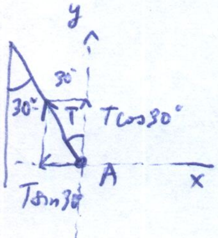
→ C (contact point with wall: Normal $\underline{\underline{N}}$, and friction $\underline{\underline{\mu_s N}}$)

$\vec{N} = N\hat{i}$ $\qquad$ $\vec{F}_f = \mu_s N\hat{j}$

(sphere tends to rotate CCW due to rope $-T\sin 30°\,\hat{i}$)

→ B (weight of sphere @ its center)

$W = -mg\,\hat{j}$



$\vec{T} = -T\sin 30°\,\hat{i} + T\cos 30°\,\hat{j}$

$\underline{\mu_{s\,min} \text{ for equilibrium ?}}$

Focus on sphere (object under analysis)

$$\begin{cases} \sum_i \vec{F}_i = \vec{F}_{net} = 0 \\ \sum_i \vec{\tau}_i = 0 \end{cases}$$

↔ Need to define pivot → center of rotation

→

A $\qquad$ B $\qquad$ C

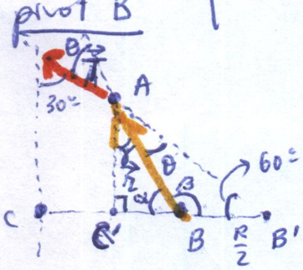
↑ should not be pivot (we look for μ_s!)

$$\vec{F}_{net} = 0 \begin{cases} x: & N - T\sin 30° = 0 & \text{(1a)} \\ y: & T\cos 30° + \mu_s N - mg = 0 & \text{(1b)} \end{cases}$$

(Four forces: $\vec{T}$, $\mu_s N$, N, mg)

$$\vec{\tau}_{net} = 0 \quad \vec{\tau}_T + \vec{\tau}_N + \vec{\tau}_{\mu_s N} + \underbrace{\vec{\tau}_{mg}}_{(\vec{r}_0 = 0)} = 0$$

pivot B



angle b/w $\vec{r}$ & $\vec{T}$ is θ

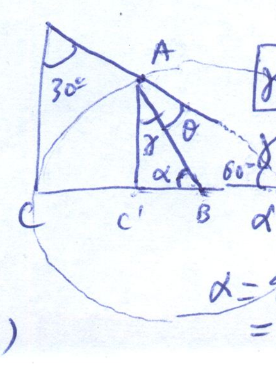
$\theta + \beta + 60° = 180°$

$\alpha + \beta = 180°$

α ?

$\alpha = 60 + \theta \rightarrow 60 + \theta + \beta = 180°$

→ Use sine theorem (next page)



$\boxed{\gamma + \theta = 30°} \rightarrow \gamma = 30° - \theta$

$\gamma + \alpha = 90°$

$\alpha = 90° - \gamma$

$\alpha = 90° - (30° - \theta)$

$= 60° + \theta$

Sine theorem:
ABB'

$$\frac{\sin\theta}{BB'} = \frac{\sin 60}{AB} \Rightarrow \boxed{\sin\theta = \frac{BB'}{AB}\sin 60^\circ = \frac{\sin 60^\circ}{2}}$$

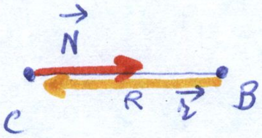
$BB' = \frac{R}{2}$

$AB = R$ (B is center of sphere & A a point on sphere)

$\rightarrow$ $\vec{\tau}_T = r_{BA} T \sin\theta\, \hat{k} = RT \frac{\sin 60^\circ}{2}\hat{k}$

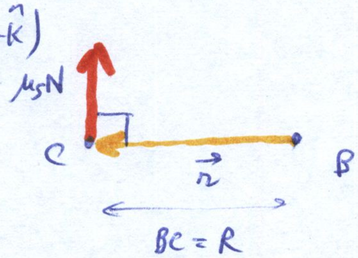
RHR

$\rightarrow$ $\vec{\tau}_N = 0$



$\theta = 180^\circ$

$\rightarrow$ $\vec{\tau}_{\mu_s N} = R\mu_s N(-\hat{k})$



$\theta = 90^\circ$

$$\boxed{\vec{\tau}_{net} = \left(RT\frac{\sin 60^\circ}{2} - R\mu_s N\right)\hat{k}}$$

equilibrium: $\vec{\tau}_{net} = 0$

$$\boxed{\frac{T}{2}\sin 60^\circ - \mu_s N = 0} \quad (2)$$

Summary:

Force balance in x: $N - T\sin 30^\circ = 0$ (1a)

Force balance in y: $T\cos 30^\circ + \mu_s N - mg = 0$ (1b)

Torque balance in z: $\frac{T}{2}\sin 60^\circ - \mu_s N = 0$ (2)

$\boxed{\mu_s = \frac{T}{2N}\sin 60^\circ}$

(1a): $N = T\sin 30^\circ$ $\rightarrow$ $\mu_s = \frac{\cancel{T}}{2\cancel{T}\sin 30^\circ}\sin 60^\circ$

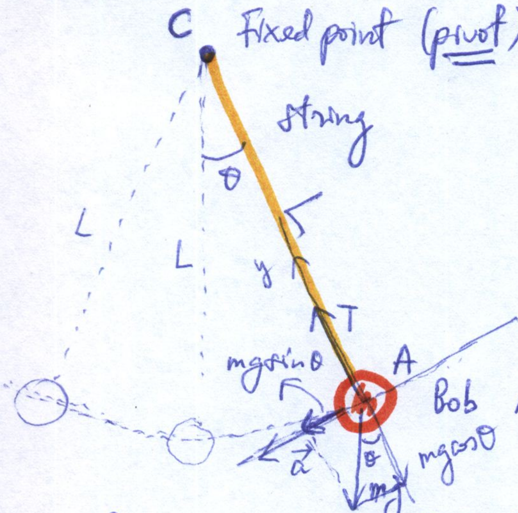
$$\mu_{s_{min}} = \frac{1}{2}\frac{\sin 60^\circ}{\sin 30^\circ} = 0.866$$

Ch 13 Oscillatory Motion

Another type of motion besides linear & rotational motion

1) Pendulum:

bob & string (negligible mass) with one end fixed (pivot)



bob rotate forward & backward around pivot. Length of string is L (CA)

→ Motion of pendulum ↔ motion of bob

{ always at separation L from C (tangential to a circular trajectory)
{ angle θ useful to describe this motion

↳ specify direction of string ↔ bob's location.

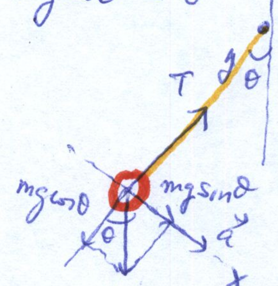
Derive equation of motion for pendulum:

$\boxed{1^{st}\text{ Method}}$: Using 2nd Newton's Law on bob: $\boxed{\vec{F}_{net} = m\vec{a}}$

↳ Convenient coord. system (motion is along an axis)

↳ $\vec{a}$ is tangential to circle centered @ C → x-axis

$$\begin{cases} x: -mg\sin\theta = +ma \Rightarrow \boxed{a = -g\sin\theta} \text{ @ position shown} \\ y: T - mg\cos\theta = 0 \end{cases}$$



$x: +mg\sin\theta = ma$

$\boxed{a = g\sin\theta}$

→ direction of $\vec{a}$ depends on bob's position

→ different than previous motions

{ bob left of vertical → tends to move right
{ bob right of vertical → tends to move left

↳ consequence: Oscillation

$a = -g \sin\theta$

$\alpha = \dfrac{a}{L}$ $\longrightarrow$ $\boxed{\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\sin\theta}$ 2nd order differential eq. non-linear.

rotational motion $\alpha = \dfrac{a}{R}$

Exact equation of motion for any pendulum!

→ Simple solution: one assumption: small angle approx. : θ is small

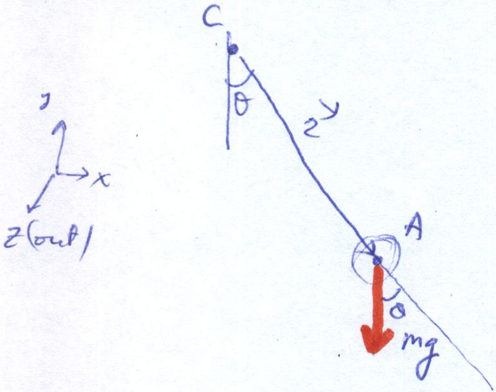
$\sin\theta \simeq \theta$ $\Rightarrow$ $\boxed{\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\theta}$ → General sol:

$\boxed{\theta(t) = \theta_M \cos(\omega t)}$

→ θ_M = amplitude of oscillation

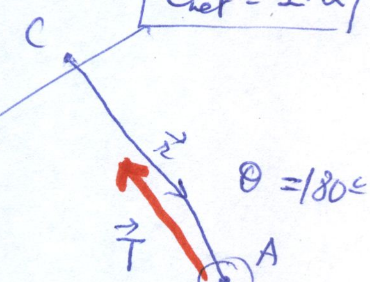
→ ω = angular frequency of osc. (# osc. per second) ↓ how fast pendulum swings.

$\boxed{\text{2nd Method:}}$ Using analog of 2nd Newton's Law for rotations: $\boxed{\vec{\tau}_{net} = I \cdot \vec{\alpha}}$



$\vec{\tau}_{mg} + \cancel{\vec{\tau}_T}^{\,0}$

$Lmg\sin\theta\,(-\hat{k})$



$I = mL^2$ $\Rightarrow$ $-\cancel{L}\cancel{m}g\sin\theta = \cancel{m}L^{\cancel{2}}\alpha \Rightarrow \alpha = -\dfrac{g}{L}\sin\theta$

mass m around C (a distance L)

$\Rightarrow \boxed{\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\sin\theta}$

Angular frequency: $\boxed{\omega = \sqrt{\dfrac{g}{L}}}$

Period of osc. = time for a full cycle

$\boxed{T = \dfrac{2\pi}{\omega}}$

$$a = -g\sin\theta$$

$$\alpha = \frac{a}{L} \longrightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

2nd order differential eq. non-linear.

rotational motion $\alpha = \frac{a}{R}$

Exact equation of motion for any pendulum!

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$$\boxed{\sin\theta \simeq \theta} \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta} \rightarrow \text{General sol.:}$$

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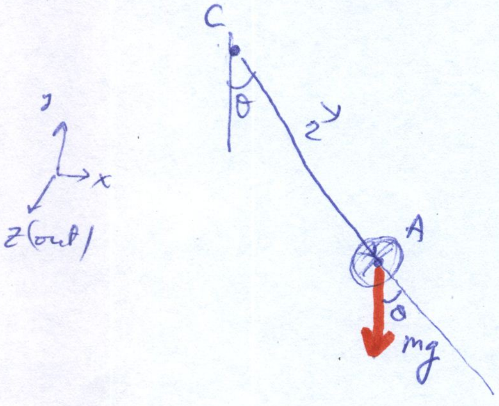
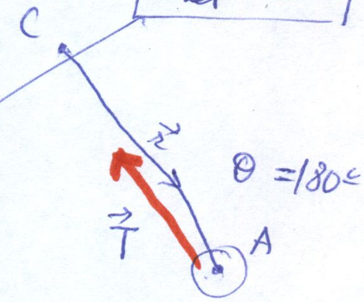


$$\vec{\tau}_{mg} + \cancel{\vec{\tau}_T} \rightarrow 0$$

$$L mg\sin\theta\,(-\hat{k})$$



$$I = mL^2 \Rightarrow -\cancel{L}mg\sin\theta = mL^{\cancel{2}}\alpha \Rightarrow \alpha = -\frac{g}{L}\sin\theta$$

mass m around C (a distance L)

$$\Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

Angular frequency: $\boxed{\omega = \sqrt{\frac{g}{L}}}$?

Period of osc. = time for a full cycle

$$\boxed{T = \frac{2\pi}{\omega}}$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta}$$ (small angle approximation)

General solution $\theta(t) = \theta_M \cos(\omega \cdot t)$ (position of bob at any time)

$$\underbrace{\frac{d^2}{dt^2}\left(\theta_M \cdot \cos(\omega t)\right)} = -\frac{g}{L}\left(\theta_M \cos(\omega t)\right)$$

θ_M time-independent $\rightarrow$ $\underbrace{\theta_M \frac{d}{dt}\left[(-\sin\omega t)\cdot\omega\right]} = -\frac{g}{L}\theta_M \cos\omega t$

ω time-independent

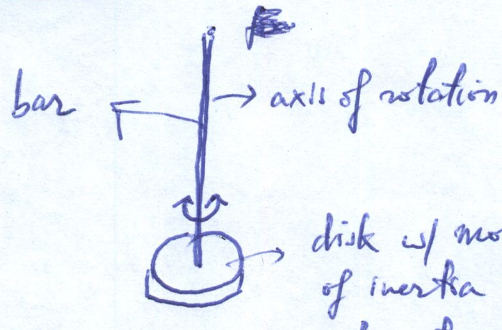
$$-\theta_M\omega \underbrace{\frac{d}{dt}\left[\sin\omega t\right]} = -\frac{g}{L}\theta_M \cos\omega t$$

$$\underbrace{-\theta_M\omega\left[(\cos\omega t)\cdot\omega\right]}$$

$$+\cancel{\theta_M}\omega^2 \cancel{\cos\omega t} = +\frac{g}{L}\cancel{\theta_M}\cancel{\cos\omega t} \Rightarrow \omega^2 = \frac{g}{L}$$

$$\boxed{\omega = \sqrt{\frac{g}{L}}}$$

2) Torsional pendulum: twisting / torsional motion:



similar to spring's law $F = -k\Delta x$ (k: spring constant)

$$\boxed{\tau = -\kappa \cdot \Delta\theta}$$

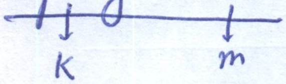
kappa: torsional constant (material & dimension of bar)

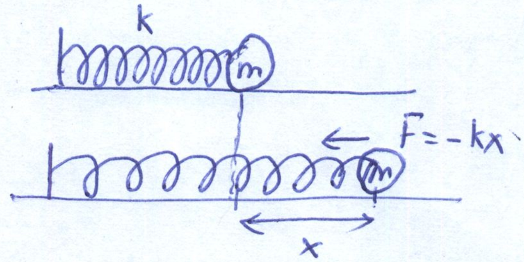
Equation of motion: $\tau = I \cdot \alpha$

$$-\kappa\theta = I \cdot \frac{d^2\theta}{dt^2} \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta}$$ 2nd order differential eq.

$$\Rightarrow \theta(t) = \underset{\text{max angle or amplitude}}{\theta_M} \cos(\omega t) \Rightarrow \boxed{\underset{\text{angular freq.}}{\omega} = \sqrt{\frac{\kappa}{I}}}$$

3) Spring & bob :





Equation of motion: $F = m \cdot a$

$$-kx = m \cdot \frac{d^2x}{dt^2} \Rightarrow \boxed{\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x}$$

2nd order differential eq. similar to those for pendulum & torsional pendulum

→ Solution (position of bob @ any given time): $x(t) = x_M \cdot \cos(\omega t)$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

angular frequency (can be defined even w/o a rotation)

Linear frequency $f = \frac{\omega}{2\pi}$ $\begin{cases} \text{unit for } \omega = \frac{rad}{s} = \frac{1}{s} \\ \text{unit for } f = \frac{1}{s} \text{ or Hz (Hertz)} \end{cases}$

Motion for $\left\{ \begin{array}{ll} \text{pendulum} & z = \theta \\ \text{torsional pendulum} & z = \theta \\ \text{spring \& bob} & z = x \end{array} \right\}$ $\frac{d^2z}{dt^2} = -\frac{a}{b}z \Leftrightarrow z(t) = z_M \cos(\omega t)$

$$\omega = \sqrt{\frac{a}{b}}$$

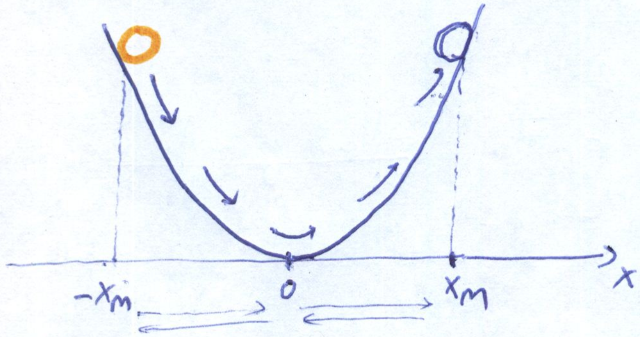
→ Simple Harmonic Motion (SHM).

In practice: z_M decays over time → Damped SHM :

$$\frac{d^2z}{dt^2} = -\frac{a}{b}z \underbrace{- \frac{c}{d}\frac{dz}{dt}}_{\text{damping term}} \Leftrightarrow z(t) = z_M e^{-\frac{c}{2d}t} \cos(\omega t + \phi)$$

phi

4) Particle trapped in a potential well:



w/o friction → SHM : $x(t) = x_M \cos(\omega t)$

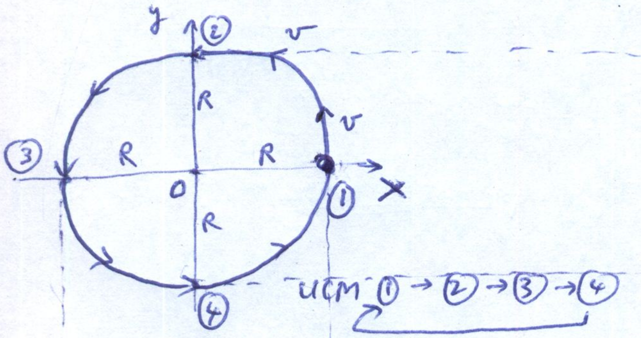
w/ friction → damped SHM:

$$x(t) = x_M e^{-\frac{b}{2m}t} \cos(\omega t + \phi)$$

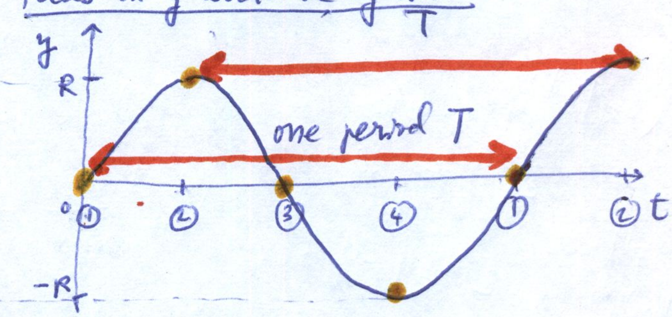
↓

exp. decay will bring the amplitude to 0 (due to friction particle will come to rest @ lowest point)

5) Coordinates x & y of an object in UCM follow SHM's:

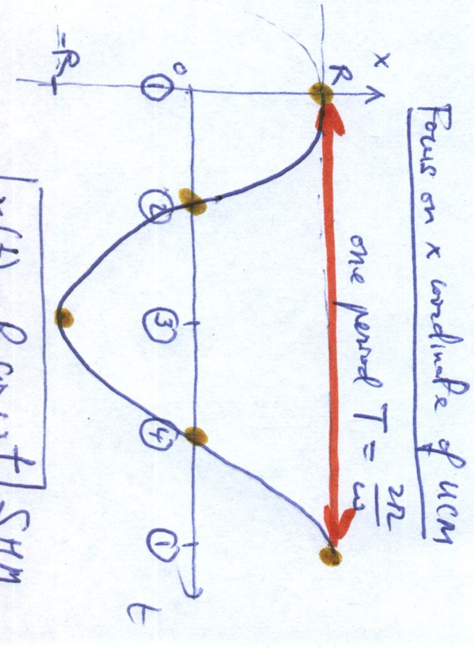


Focus on y coordinate of UCM



$\boxed{y(t) = R \sin \omega t}$ SHM

time to complete one circle or one cycle is $T = \frac{2\pi}{\omega}$



$\boxed{x(t) = R \cos \omega t}$ SHM

$$\left.\begin{aligned} x(t) &= R \sin \omega t \\ y(t) &= R \cos \omega t \end{aligned}\right\}$$ are SHM's shifted by $\frac{1}{4}$ circle or 90° or $\frac{\pi}{2}$

Total energy of a particle under SHM:

Spring & bob under SHM $\omega = \sqrt{\frac{k}{m}}$: $E = \overbrace{\frac{1}{2}kx^2}^{\text{spring}} + \overbrace{\frac{1}{2}mv^2}^{\text{bob}}$

x ↓ elongation or change of length of spring wrt its natural length

In principle:

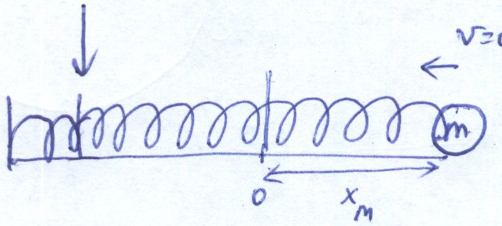
$$\left.\begin{aligned} x(t) &= x_M \cos\omega t \quad \text{SHM} \\ v(t) &= \frac{dx}{dt} = -x_M\omega \sin\omega t \quad \text{(SHM)} \end{aligned}\right\} E \text{ also in SHM?}$$

$$E = \frac{1}{2}k\,x_M^2 \cos^2\omega t + \frac{1}{2}m\,x_M^2\omega^2 \sin^2\omega t = \frac{1}{2}kx_M^2\underbrace{\left[\cos^2\omega t + \sin^2\omega t\right]}_{1}$$

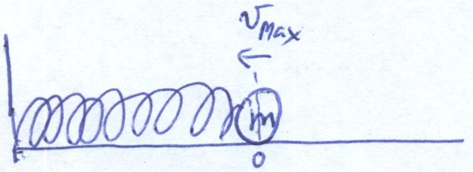
$$\omega^2 = \frac{k}{m} \rightarrow m x_M^2\omega^2 = \cancel{m}\, x_M^2\,\frac{k}{\cancel{m}}$$

→ Total energy of spring & bob is constant! $\boxed{E = \frac{1}{2}kx_M^2}$

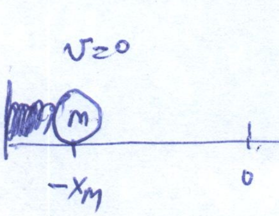
↓



→ spring & bob is released from position + $v_0 = 0$
(origin of coordinate x is @ natural length of spring)
$E = \frac{1}{2}kx_M^2$



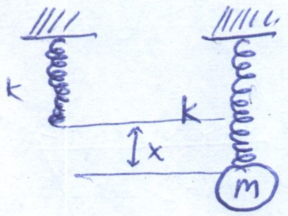
→ @ natural length: $E = \frac{1}{2}mv_{Max}^2$



@ max spring compression $E = \frac{1}{2}kx_M^2$

Total energy stays constant!

13.67 bob on vertical spring hangs from ceiling



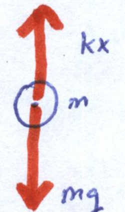
$k = 74 \frac{N}{m}$

$m = 0.49\,kg$

a) Amplitude of oscillation?

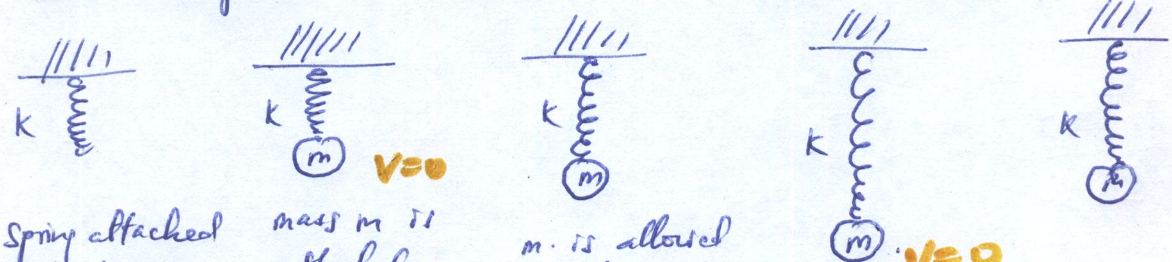
b) Period of motion?

Focus on bob:



when m is attached to spring it is stretch a distance x from natural length, it will pull in the opposite direction with a force equals kx

a) Different snapshots in time:



spring attached to ceiling

mass m is attached

m is allowed to dropped. V increasing ↓ $a \neq 0$

spring @ max. stretch → max amplitude: $a = 0$ ↓ Net force on bob: 0 ($F_{net} = m.a$)

SHM for bob
$$\begin{cases} x(t) = x_M \cos(\omega t) \\ v(t) = \frac{dx}{dt} = -x_M \omega \sin(\omega t) \\ a(t) = \frac{d^2x}{dt^2} = -x_M \omega^2 \cos(\omega t) \end{cases}$$

Max stretch
$$\begin{cases} x = x_M \Rightarrow \cos \omega t = 1 \Leftrightarrow \sin \omega t = 0 \\ \rightarrow v = 0 \rightarrow \boxed{a = -x_M \omega^2} \end{cases}$$

$F_{net} = kx - mg = ma$

$k x_M \cos\omega t - mg = -m x_M \omega^2 \cos \omega t$

$F_{net} = k x_M - mg = 0$

$$\boxed{x_M = \frac{mg}{k}}$$

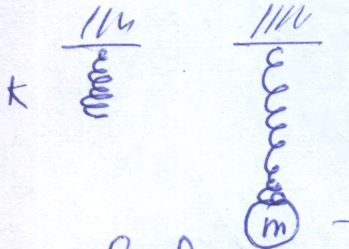
$$x_M = \frac{0.49 \times 9.81}{74} = 0.0649\,m$$

Max. stretch or amplitude of oscillation.

b) Period of oc. T: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{74}{0.49}}} = 0.511\,s.$

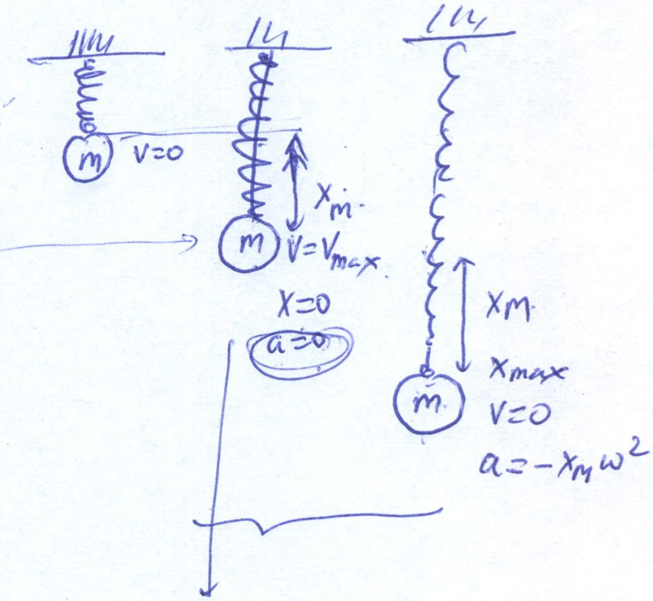
angular freq $\omega = \sqrt{\frac{k}{m}}$ (spring & bob system).

2 natural lengths: initial length w/o bob; new nat. length w/ bob — difference is x_M



original nat. length   new nat. length

→ our position of bob x is def. wrt. new natural length



$$F_{net} = 0 = kx_M - mg$$

$$\rightarrow \boxed{x_M = \frac{mg}{k}} \leftarrow$$