9.28

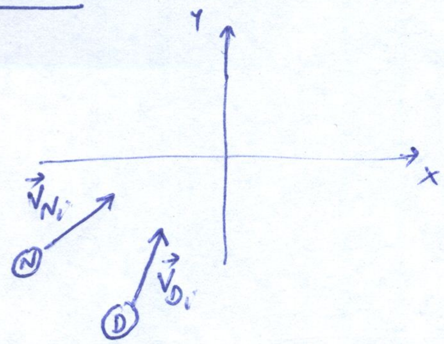
9.41

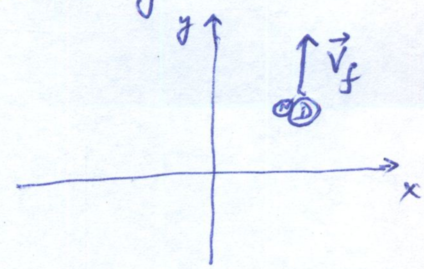
9.45, 9.57

9.63

9.28 Neutron (1u) striking a Deuteron (2u) and combine after collision!

Statement: inelastic collision : conservation of LM





$$\vec V_f = 12\hat i + 20\hat j \ \frac{Mm}{s}$$

$$\vec V_{N_i} = 28\hat i + 17\hat j \ \frac{Mm}{s}$$

$$\vec V_{D_i} = ?$$

No net external force $\vec F_{net} = 0 \Rightarrow \vec P_i = \vec P_f$

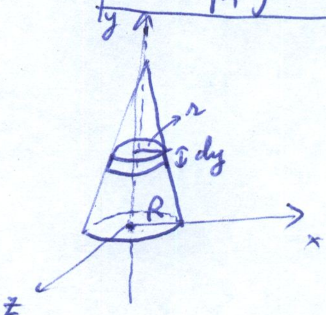
$$\cancel{m}\vec V_{N_i} + 2\cancel{m}\vec V_{D_i} = (\cancel{m} + 2\cancel{m})\vec V_f$$

$$\vec V_{D_i} = \frac{3\vec V_f - \vec V_{N_i}}{2}$$

$$\vec V_{D_i} = \frac{36\hat i + 60\hat j - 28\hat i - 17\hat j}{2} \ \frac{Mm}{s} = \frac{8\hat i - 43\hat j}{2} \ \frac{Mm}{s}$$

$$\boxed{\vec V_{D_i} = 4\hat i - 21.5\hat j \ \frac{Mm}{s}}$$

9.41 Find the CM of a continuous object $\leftrightarrow$ $\boxed{\vec{R} = \frac{1}{M}\int \vec{r}\, dm}$



$\vec{R}$: position vector of CM

Symmetry : CM on axis of cone

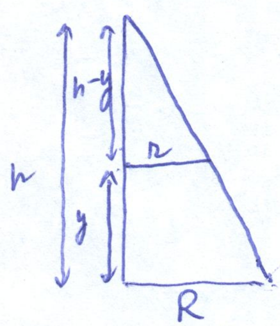
$\downarrow$

Coords. system centered @ center of base $\rightarrow$ only need the vertical position for CM !

$\downarrow$

$$y_{cm} = \frac{1}{M}\int y\, dm$$

$\Big\{$ y: vertical position of infinitesimal mass $dm \leftrightarrow$ disk of thickness dy and radius r



Similar triangles: $\frac{r}{h-y} = \frac{R}{h} \Rightarrow r = R\,\frac{h-y}{h}$

$$r = R\left(1 - \frac{y}{h}\right)$$

Also : $dm = \rho\, dV$ $\Big\{$ ρ = cone density ; dV = element of volume of disk $= \pi r^2 dy$

$$\rightarrow dm = \rho\, \pi r^2 dy$$

$$y_{cm} = \frac{1}{M}\rho\pi \int_0^h y \underbrace{R^2\left(1-\frac{y}{h}\right)^2}_{r^2} dy = \frac{\rho\pi R^2}{M}\int_0^h y\left(1 - \frac{2}{h}y + \frac{1}{h^2}y^2\right) dy$$

$$= \frac{\rho\pi R^2}{M}\left[\frac{y^2}{2} - \frac{2}{3h}y^3 + \frac{1}{4h^2}y^4\right]_0^h = \frac{3}{h}\left[\frac{h^2}{2} - \frac{2}{3h}h^3 + \frac{1}{4h^2}h^4\right]$$

$$= 3\left[\frac{h}{2} - \frac{2}{3}h + \frac{1}{4}h\right]$$

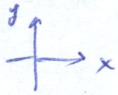
$$= 3h\underbrace{\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right]}_{\frac{1}{12}}$$

$$\rho_{cone} = \frac{M}{Vol_{cone}} = \frac{M}{\pi R^2 \frac{h}{3}} \quad \Bigg| \quad \frac{\rho \pi R^2}{M} = \frac{\cancel{M}}{\pi R^2 \frac{h}{3}}\,\frac{\cancel{\pi R^2}}{\cancel{M}} = \frac{3}{h}$$

$$\boxed{y_{cm} = \frac{h}{4}}$$

9.45 Car initially @ rest received a push by a jet of water hitting on its back window. No friction.

$\begin{cases} \text{water} = ① \\ \text{car} = ② \end{cases}$

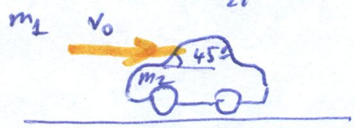


$\vec{v}_{1i} = v_0 \hat{i}$

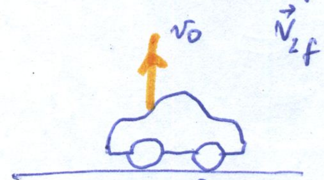
$\vec{v}_{2i} = 0$

$\vec{v}_{1f} = v_0 \hat{j}$

$\vec{v}_{2f} = ?$



initial



final

Car acquired a forward acceleration achieving some final velocity $\vec{v}_{2f}$

a) Car initial acceleration after receiving hit from waterjet?

Statements: 1) collision b/w jet of water & car

2) $\vec{F}_{net}$ on water & car $= 0 \Rightarrow \vec{P}_i = \vec{P}_f$

(Friction is ignored)

$$\underbrace{m_1 \vec{v}_{1i}}_{\vec{P}_i} = \underbrace{m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}}_{\vec{P}_f}$$

3) m_1 not given but we know the water flow rate is $\frac{dm_1}{dt}$

$$\begin{cases} \vec{a}_2 = \dfrac{d\vec{v}_{2f}}{dt} \end{cases}$$

$$\vec{v}_{2f} = \frac{m_1 (\vec{v}_{1i} - \vec{v}_{1f})}{m_2} = \underbrace{\frac{m_1}{m_2}}_{\text{constant}} \overbrace{(v_0 \hat{i} - v_0 \hat{j})}^{\text{constant}}$$

$$\vec{a}_2 = \frac{d\vec{v}_{2f}}{dt} = \frac{1}{m_2}(v_0 \hat{i} - v_0 \hat{j}) \frac{dm_1}{dt}$$

$$\begin{cases} \boxed{a_{2x} = \dfrac{1}{m_2} \dfrac{dm_1}{dt} v_0} \rightarrow \text{car accelerates forward.} \\ a_{2y} = -\dfrac{1}{m_2} \dfrac{dm_1}{dt} v_0 \end{cases}$$

(car feels a push downward)

b) Max speed car reaches:

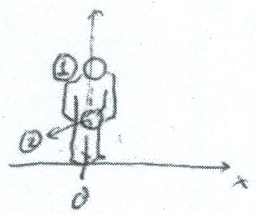
Why can't the jet of water accelerate the car to a very high speed? Relative motion: when car reaches v_0 (same as water speed) $\rightarrow$ no further push (no more momentum transfer) $\rightarrow$ max speed for car is v_0!

9.57 Tossing a rock standing on ice (no friction)

Conservation law: → Initial

↓

1) System: you + rock
2) $F_{net} = 0$
↳ $\vec{P}_i = \vec{P}_f$

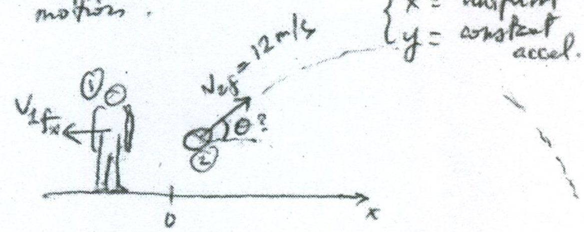


$\vec{P}_i = 0$

Final

① Uniform motion.

② → Projectile motion. $\begin{cases} x = \text{uniform} \\ y = \text{constant accel.} \end{cases}$

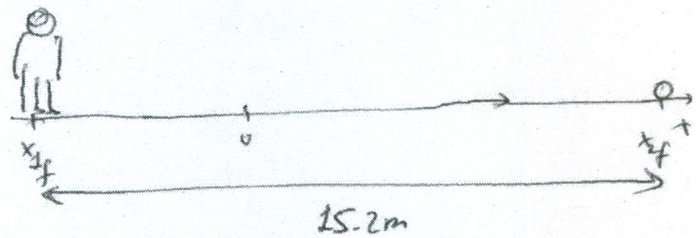


Find θ.

Kinematic equation:

$m_1 = 65\,kg$

$m_2 = 4.5\,kg$



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x-direction: $\begin{cases} \text{rock: } x_{2f} = v_{2f}\cos\theta \cdot 2t_{up} & \text{(a)} \\ \text{you: } x_{1f} = v_{1fx} \cdot 2t_{up} & \text{(b)} \end{cases}$

$\boxed{x_{2f} - x_{1f} = 15.2\,m}$

t_{up}: time for rock to go to max altitude point: final speed in y-direction of 0:

$$v_{2f_y} = v_{2fo_y} - g \cdot t$$

$$0 = v_{2f}\sin\theta - g \cdot t_{up} \rightarrow t_{up} = \frac{v_{2f}\sin\theta}{g}$$

→ Plug back into (a) $x_{2f} = v_{2f}\cos\theta \cdot 2 \cdot \dfrac{v_{2f}\sin\theta}{g}$

$$\left(x_{Range} = \frac{v_0^2 \sin 2\theta}{g}\right) \qquad = \frac{v_{2f}^2\, 2\cos\theta\sin\theta}{g} = \frac{v_{2f}^2 \sin(2\theta)}{g}$$

$$\boxed{x_{2f} = \frac{12^2 \sin(2\theta)}{9.81}} \quad (x_{2f} \text{ in term of } \theta)$$

(b) Writing $\underbrace{x_{1f}}_{\text{person}}$ in term of $\underbrace{\theta}_{\text{rock}}$: person & rock are related in conservation of Total momentum! $\vec{P}_i = \vec{P}_f$

$$\vec{P}_i = \vec{P}_f$$

$$0 = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Rock follows 2D motion
Person acquired $\vec{v}_{1f}$ but only v_{1fx} showed

x-direction: $\boxed{0 = m_1 v_{1fx} + m_2 v_{2fx}}$ (v_{1fy} just adds pressure on his feet!)

y-direction $0 = m_1 v_{1fy} + m_2 v_{2fy}$

$$0 = m_1 v_{1fx} + m_2 v_{2f} \cos\theta \rightarrow \boxed{v_{1fx} = -\frac{m_2}{m_1} v_{2f} \cos\theta}$$

$$x_{1f} = v_{1fx} \cdot 2 t_{up} = -\frac{m_2}{m_1} v_{2f} \cos\theta \cdot 2 \frac{v_{2f} \sin\theta}{g}$$

$$\boxed{x_{1f} = -\frac{m_2}{m_1} \frac{v_{2f}^2 \sin(2\theta)}{g}}$$

$$x_{2f} - x_{1f} = 15.2\,m$$

$$\frac{12^2 \sin(2\theta)}{9.81} + \frac{4.5}{65} \frac{12^2 \sin(2\theta)}{9.81} = 15.2m$$

$$\sin(2\theta) = \frac{15.2}{\left[\frac{144}{9.81} \left(1 + \frac{4.5}{65} \right) \right]}$$

$$\theta = \frac{1}{2} \sin^{-1} \left[0.974 \right]$$

$$\theta = 38.5°$$

9.63

$m_1 = 1200\,kg \quad m_2 = 2200\,kg$ — inelastic — then

a) System: Toyota + Buick ; lock together & skid 22m

colliding @ right angles. → 2D

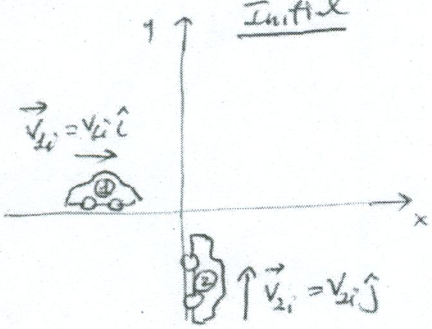
$\mu_K = 0.91$

Show at least one car exceeded $25\frac{km}{h}$ speed limit.

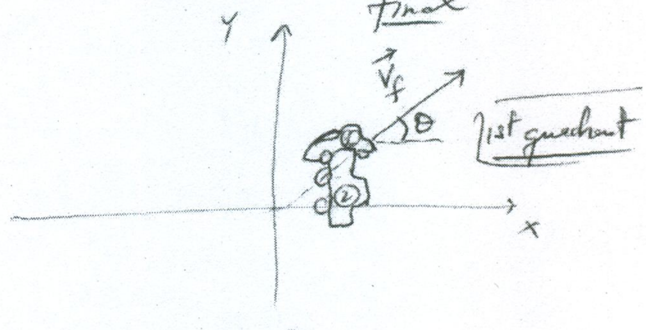
CLM → b) Define initial & final situations with a sketch:

$\vec{P}_i = \vec{P}_f$

Initial



final



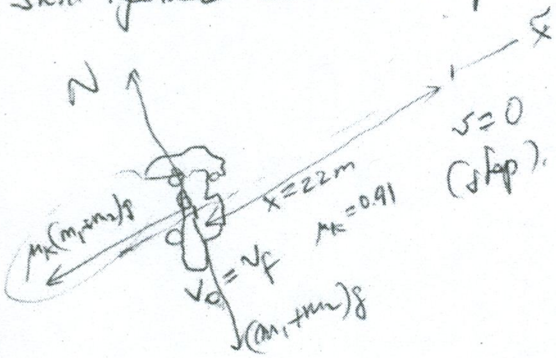
$\vec{P}_i$ = $\vec{P}_f$

1) $m_1 v_{1i}\hat{i} + m_2 v_{2i}\hat{j} = (m_1 + m_2)(v_f \cos\theta\,\hat{i} + v_f \sin\theta\,\hat{j})$

(I) $m_1 v_{1i} = (m_1 + m_2) v_f \cos\theta \rightarrow 1200 v_{1i} = 3400 v_f \cos\theta$

(II) $m_2 v_{2i} = (m_1 + m_2) v_f \sin\theta \rightarrow 2200 v_{2i} = 3400 v_f \sin\theta$

2) Skid together 22m to stop (b/c of friction $\mu_K = 0.91$)



$v=0$ (stop).

They will come to a stop when all kinetic energy has been used to overcome friction:

$\frac{1}{2}(m_1+m_2) v_f^2 = \mu_K (m_1+m_2) g \cdot x$

$v_f = \sqrt{2 \cdot \mu_K g x} =$

$= \sqrt{2 \cdot 0.91 \cdot 9.81 \cdot 22} =$

$\boxed{v_f = 19.82\,\frac{m}{s}}$

(I) $1200\, v_{1i} = 3400\, v_f \cos\theta \rightarrow \cancel{\cos\theta}$

(II) $2200\, v_{2i} = 3400\, v_f \sin\theta$

$$1200^2 v_{1i}^2 + 2200^2 v_{2i}^2 = (3400 \cdot 19.82)^2 \underbrace{(\cos^2\theta + \sin^2\theta)}_{1}$$

$$v_{1i}^2 + \underbrace{\left(\frac{2200}{1200}\right)^2}_{3.36} v_{2i}^2 = \underbrace{\left(\frac{3400 \times 19.82}{1200}\right)^2}_{3150}$$

$$v_{1i}^2 + 3.36\, v_{2i}^2 = 3150$$

Now: speed limit was $25 \frac{km}{h} = \frac{25}{3.6} \frac{m}{s} = 6.94 \frac{m}{s}$

<u>Hypothesis</u>: Each car was traveling @ $6.94 \frac{m}{s} \simeq 7 \frac{m}{s}$

$49 + 3.36 \cdot 49 \simeq 200 \rightarrow$ clearly at least one car was traveling well above $25 \frac{km}{h}$.

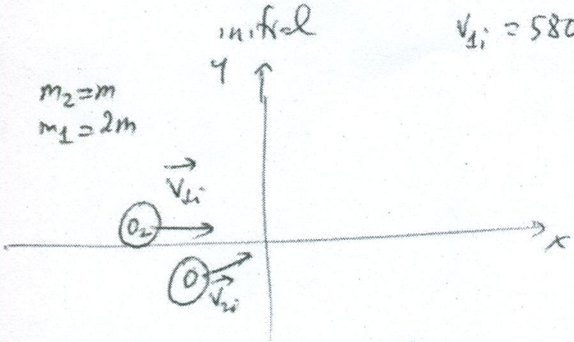
9.74 System: O_2 & O — inelastic collision. $\boxed{\text{stick}}$ together → Ozone, $\vec{v}_f$? 105

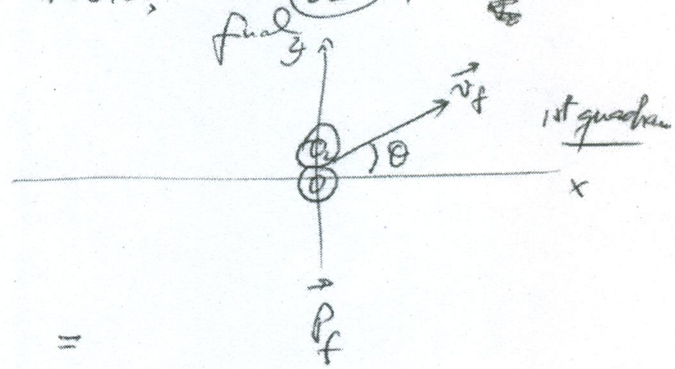
$(m_1 = 32u)$ $(m_2 = 16)$

$\vec{v}_{1i} = v_{1i}\,\hat{i}$ $\qquad \vec{v}_{2i} = \underbrace{v_{2i}\cos 27^\circ\,\hat{i} + v_{2i}\sin 27^\circ\,\hat{j}}_{2D}$

$v_{1i} = 580\,\frac{m}{s}$ $\qquad v_{2i} = 870\,\frac{m}{s}$

$m_2 = m$
$m_1 = 2m$





$$\vec{F}_{net} = 0 \rightarrow \vec{P}_i = \vec{P}_f$$

$$2\cancel{m}\,580\,\hat{i} + \cancel{m}\,870\left(\cos 27^\circ\,\hat{i} + \sin 27^\circ\,\hat{j}\right) = 3\cancel{m}\left(v_{fx}\,\hat{i} + v_{fy}\,\hat{j}\right)$$

$$1160 + 870\cos 27^\circ = 3\,v_{fx}$$
$$870\sin 27^\circ = 3\,v_{fy}$$

$$v_{fx} = \frac{1160 + 870\cos 27^\circ}{3} = 645.06\ \frac{m}{s}$$
$$v_{fy} = \frac{870\sin 27^\circ}{3} = 131.66\ m/s.$$

Cartesian → polar:

$$\boxed{\begin{aligned} v_f &= \sqrt{645.06^2 + 131.66^2} = 658.4\ m/s \\ &\text{(ozone)} \\ \theta &= \tan^{-1}\frac{131.66}{645.06} = 11.54^\circ \end{aligned}}$$

9.51

$m_1 = 60kg \quad m_2 = 14kg \quad m_3 = 5.8kg$

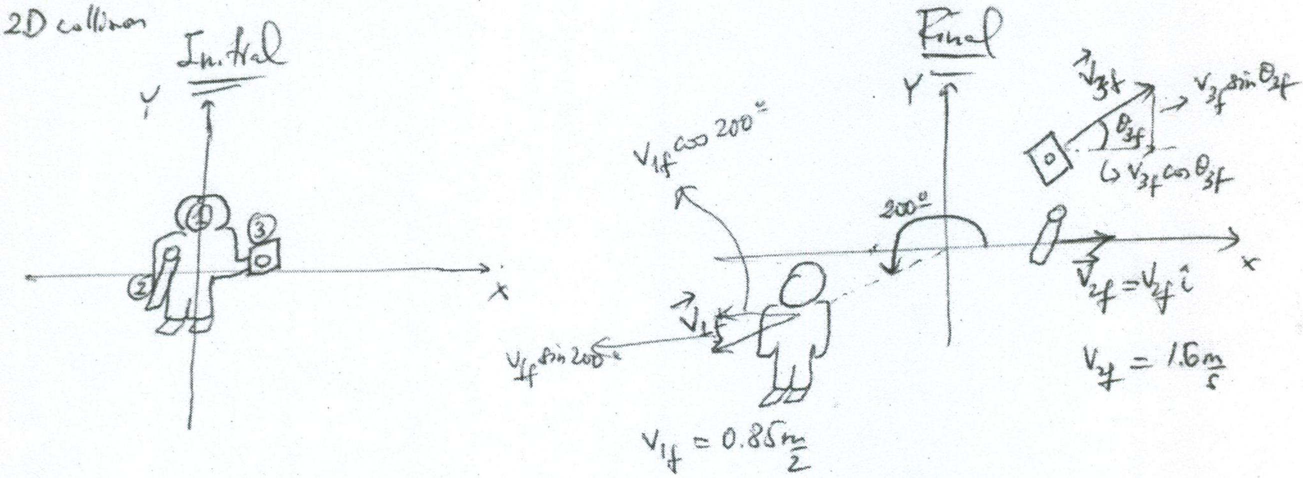
3 component system : astronaut + O_2 tank + camera

– In space

$\rightarrow \vec{F}_{net, external} = 0 \rightarrow \vec{P}_i = \vec{P}_f$

~~m1~~ recoils at 200° ccw from x-axis

$\rightarrow$ 2D collision



$$\vec{P}_i = 0 \quad = \quad \vec{P}_f = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} + m_3\vec{v}_{3f}$$

$$0 = (m_1 v_{1f}\cos 200° + m_2 v_{2f} + m_3 v_{3f}\cos\theta_{3f})\hat{i} + (m_1 v_{1f}\sin 200° + m_3 v_{3f}\sin\theta_{3f})\hat{j}$$

$0\hat{i} + 0\hat{j}$

x-direction : $0 = 60 \cdot 0.85\cos 200° + 14 \cdot 1.6 + 5.8\underbrace{\boxed{v_{3f}\cos\theta_{3f}}}_{\equiv v_{3fx}}$

y-direction : $0 = 60 \cdot 0.85\sin 200° + 5.8\underbrace{\boxed{v_{3f}\sin\theta_{3f}}}_{\equiv v_{3fy}}$

$$v_{3fx} = \frac{-60 \cdot 0.85\cos 200° - 14 \cdot 1.6}{5.8} = 4.4\frac{m}{s}$$

$$v_{3fy} = \frac{-60 \cdot 0.85\sin 200°}{5.8} = 3\frac{m}{s}$$

1st quadrant.

Cartesian $\rightarrow$ Polar $\rightarrow$

$$v_{3f} = \sqrt{4.4^2 + 3^2} = 5.33\frac{m}{s}$$

$$\theta_{3f} = \tan^{-1}\frac{3}{4.4} = 34.3°$$

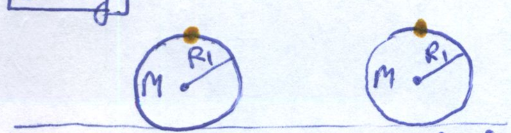
(ccw from axis-x 1st quadrant)

Ch.10 Rotational Motion

So far: linear motion, circular motion (object going around an <u>external</u> center of curvature)

<u>Translation</u> (linear motion)

$\boxed{\text{sliding}}$



bowling ball   icy, <u>no friction</u>

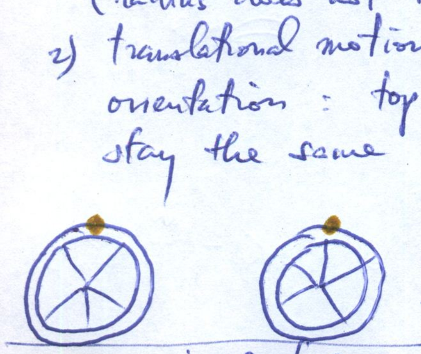
same position after same Δt

icy, <u>no friction</u>

smaller bowling ball
$R_2 < R_1$ same mass M

<u>Statements</u>: 1) sliding balls of equal masses but different radii $R_2 < R_1$ have <u>same</u> <u>translational motion</u> since they can be described as pointlike particles located at their centers of mass with mass equals to M (radius does not matter)

2) translational motion: same orientation: top dot always stay the same



icy surface

w/o friction here the same wheel translates but doesnt rotate!

<u>Rotation</u>

$\boxed{\text{Non sliding}}$: $\boxed{\text{Rolling}}$

bowling ball   regular, w/ friction

distance traveled after Δt is πR_1

smaller bowling ball   regular, w/ friction

distance traveled after Δt is πR_2

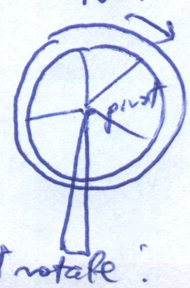
<u>Statements</u> = 1) In rotational motion radius does matter!

2) Rolling motion involves both translation (note the distance traveled) & rotation

↓

Is there any pure rotational motion? Is it always connected with a translation?

No! → flipped bike wheel: on a support rotates but does not translate.



1) Car wheels under normal road conditions: rolling motion
⇔ both translation & rotation

2) Car wheels stuck in sand, trying to get out: only rotation

3) ABS braking : anti-blocking system: when brake pedal is pressed, wheels are not instantly blocked (as in bike braking ⟶ although we learn to increase the pressure to avoid a dry stop @ high speed)
wheels makes a few rotations before stopping

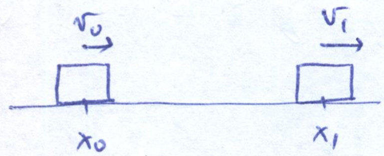
(i) Car wheels w/ brake applied w/o ABS : only translation
(ii) Car wheels w/ brake applied w/ ABS : both translation & rotation

To come to a full stop we need to lose all KE:
(i) Relying on friction only
(ii) In addition to friction some of the KE goes into the final few rotation of the wheels → shorter stopping distance (a little bit but can save lives)
→ better vehicle control when braking at curves.

Guide on previous knowledge: translation

Translational Motion

→ change of position



$$\bar{v} = \frac{v_0 + v_1}{2}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \text{(average velocity)}$$

$$v = \frac{dx}{dt} \quad \text{(instantaneous velocity)}$$

$$a = \frac{dv}{dt}$$

Equations of motion: (Ch 2 & 3)

1) $v = v_0 + a.t$

2) $x = x_0 + v_0 \cdot t + \frac{1}{2} a t^2$

3) $\dfrac{v^2 - v_0^2}{x - x_0} = 2a$

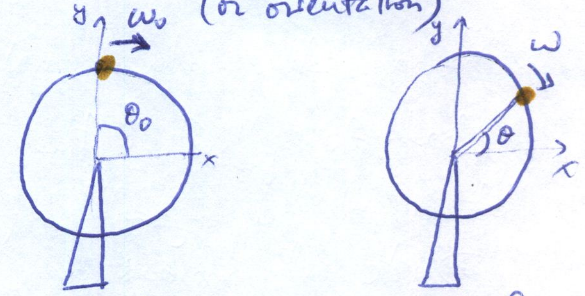
2nd Newton's Law (Ch 4)

$$F_{net} = m.a$$

(constant m)

Rotational motion

→ change of angle (or orientation)



ω_0: initial angular speed (omega sub zero)

ω: final angular speed. (omega)

$$\bar{\omega} = \frac{\omega_0 + \omega}{2}$$

$$\bar{\omega} = \frac{\Delta \theta}{\Delta t} \quad \text{(average angular velocity)}$$

$$\omega = \frac{d\theta}{dt} \quad \text{(instantaneous)}$$

SI units: $\frac{\text{radians}}{s}$ or $\frac{\text{rad}}{s}$

$\left(\frac{180°}{\pi}\right)$

$$\alpha = \frac{d\omega}{dt} \quad \text{(angular acceleration)}$$

SI units: $\frac{\text{rad}}{s^2}$

1) $\omega = \omega_0 + \alpha \cdot t$

2) $\theta = \theta_0 + \omega_0 \cdot t + \frac{1}{2} \alpha t^2$

3) $\dfrac{\omega^2 - \omega_0^2}{\theta - \theta_0} = 2\alpha$

Analog of 2nd Newton's Law

$$\tau_{net} = I \cdot \alpha$$

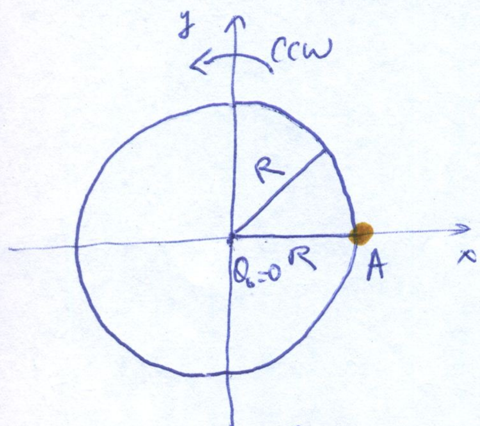
τ_{net}: net torque (radius does matter); I: moment of inertia (radius does matter)

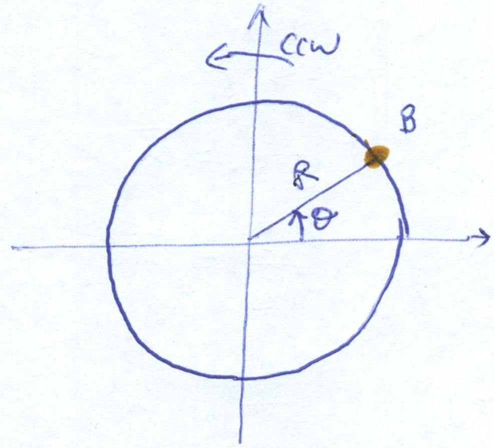
New items: torque (related to force but radius does matter)
moment of inertia (related to mass but radius does matter)

1) Rolling motion: $\overset{\text{quantitative}}{\text{connection}}$ b/w translation (linear vel.) & rotation (angular vel.)

$\downarrow$

Focus on a point on bowling ball & how its motion is related to the translation of center of bowling ball





center of ball is @ origin of coord.

a) In Δt : arc AB is s (while angle went from 0 to θ)

rotation $\theta = \dfrac{s}{R}$ translation

In rolling motion: displacement of center of mass equals the ~~chor~~ arc s

b) Connection b/w ω & v: $\dfrac{d}{dt}\left[\theta = \dfrac{s}{R}\right]$

$$\boxed{\omega = \frac{v}{R}} \longrightarrow \left(\frac{ds}{dt} = v\right)$$

R is constant

$\dfrac{\text{rad}}{s}$ $\qquad \dfrac{\frac{m}{s}}{m} = \dfrac{1}{s}$

angle is considered dimensionless $[\theta] = 1$

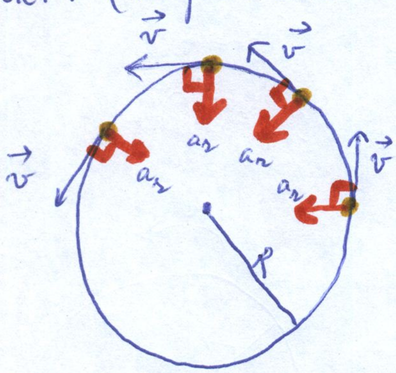
2) Angular acceleration α (alpha)

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} \quad ; \quad \alpha = \frac{d\omega}{dt} \qquad \left(\frac{rad}{s^2} = \frac{1}{s^2} = s^{-2} \right)$$

→ UCM (uniform circular motion) : $\alpha = \frac{d\omega}{dt} = \frac{1}{R}\frac{dv}{dt} = 0$

$\omega = \frac{v}{R}$

(v linear speed) or magnitude of velocity



$\vec{v}$: same length (uniform motion → constant v) but direction is always tangential to the circle → direction is always changing ⟹ we need the radial acceleration (pointing toward center of curvature) $a_r = \frac{v^2}{R}$

$$\text{UCM} \begin{cases} a_r = \frac{v^2}{R} & \text{(radial acceleration)} \\ a_t = \frac{dv}{dt} = 0 & \text{(tangential acceleration)} \\ \alpha = 0 \end{cases}$$

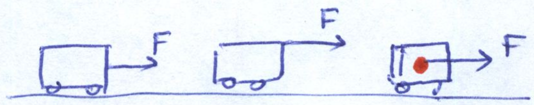
→ Non-UCM (non-uniform circular motion) : linear speed v along circle is not constant → $\alpha = \frac{d\omega}{dt} = \frac{1}{R}\frac{dv}{dt}$

$$\text{Non-UCM} \begin{cases} a_r = \frac{v^2}{R} & \text{(radial acceleration)} \\ \boxed{a_t = \frac{dv}{dt} \text{ (tangential acceleration)} = \frac{d(\omega R)}{dt} = R\frac{d\omega}{dt} = R\alpha} \\ \alpha \neq 0 \end{cases}$$

$\omega = \frac{v}{R} \Rightarrow \boxed{v = \omega R}$

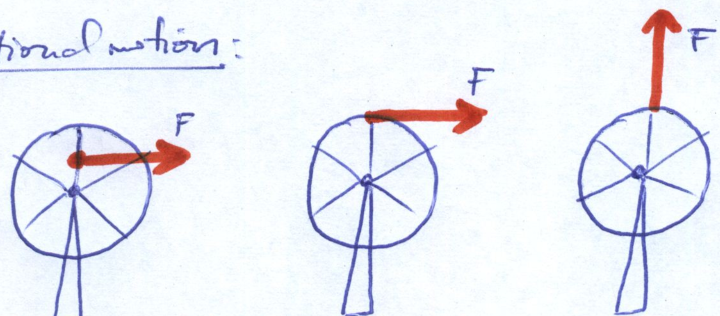
3) Torque: $\vec{\tau}$ (tau) (radius matters)

In linear motion:



(object can be described as a point @ its center of mass)

In rotational motion:



a) b) c)

✓

Good rotation:

Not c) $\vec{F}$ along radial direction does not help rotation (direction of F is important as it is vector)

b) Not just direction of $\vec{F}$ but where it is applied is important for rotation! (radius does matter!)

$$\vec{\tau} = \vec{F} \times \vec{r}$$

$\times$: "cross product" (different than the scaler product in work)

a product b/w 2 vectors that produces another vector that is perpendicular to both

$\vec{r}$ = position vector of the force application point. w.r.t. pivot or center of rotation

($\vec{r}$ goes from $\boxed{\text{pivot}}$ to the $\boxed{\text{force application point}}$)

$$= rF\sin\theta \ \hat{\tau}$$

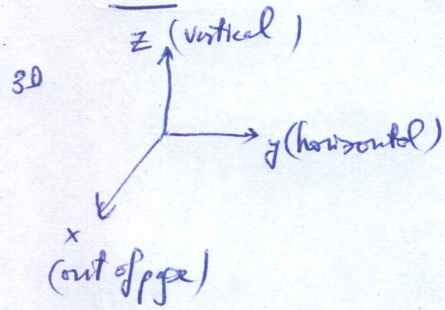
θ : angle b/w $\vec{r}$ & $\vec{F}$

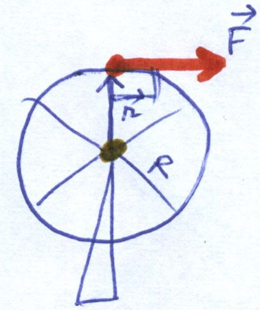
$\hat{\tau}$: unit vector in the direction of $\vec{\tau}$: perpendicular to plane formed by $\vec{r}$ & $\vec{F}$ direction given by the right hand rule (RHR)

Unit: Nm (same as work)

Torque is a vector whose magnitude is the product of the magnitude of the position vector $\vec{r}$ and the force applied $\vec{F}$ times the sine of the angle θ b/w those two vectors. Direction of torque is perpendicular to both $\vec{r}$ & $\vec{F}$ and given by RHR

RHR:





$\vec{r} = R\hat{k}$

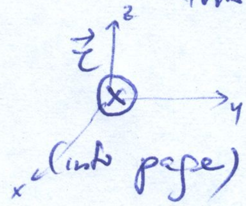
$\vec{F} = F\hat{j}$

what is $\vec{\tau}$? $\quad \vec{\tau} = RF\underbrace{\sin 90}_{1}\,\hat{k}\times\hat{j} = RF\,\hat{k}\times\hat{j}$

RHR on $\hat{k}\times\hat{j}$: align RH fingers along 1st vector $\hat{k}$ or $\vec{r}$ as you close your RH fingers toward the 2nd vector $\hat{j}$ or $\vec{F}$, RH thumb points in direction of $\hat{k}\times\hat{j}$ or $\vec{\tau}$

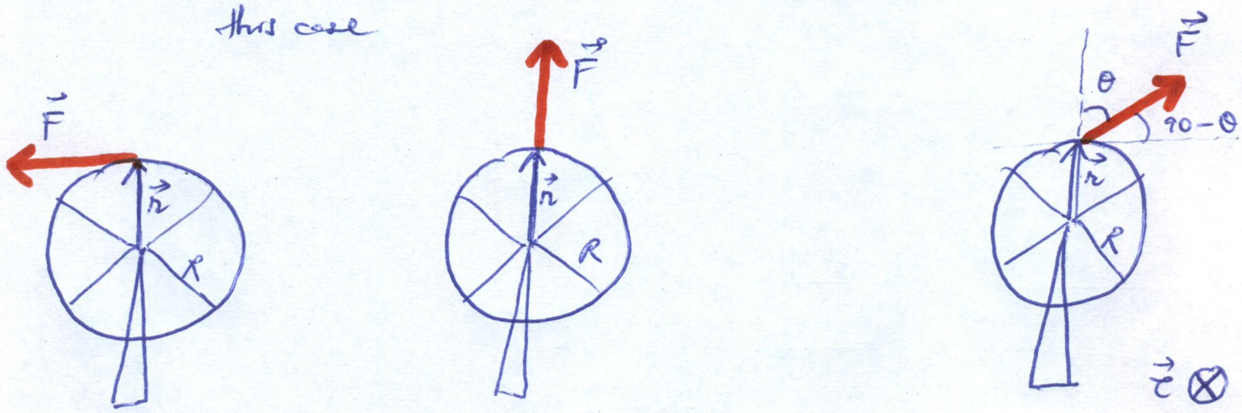
In this case: thumb points into the page or $(-\hat{i})$
this is the direction of $\hat{k}\times\hat{j}$ or $\vec{\tau}$

Convention:



$\vec{\tau}$ $\odot$ (out of page)

$$\boxed{\vec{\tau} = -RF\,\hat{i}}$$



$\vec{\tau} = RF\,\hat{i}$

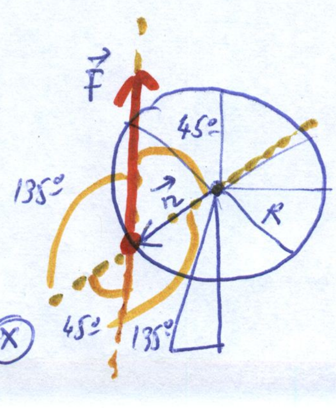
$\vec{\tau}$ $\odot$

$\vec{\tau} = RF\underbrace{\sin 0}_{0}\underbrace{(\hat{k}\times\hat{k})}_{0} = 0$

$\vec{\tau} = RF\sin\theta\,(-\hat{i})$

$= -\underbrace{RF\sin\theta}_{<RF}\,\hat{i}$

Angle θ:



Force is applied @ mid point of 3rd quadrant.

$\vec{\tau} = \underbrace{RF\sin\theta}_{\text{magnitude is always positive}}\,(-\hat{i}) = RF|\sin\theta|\,(-\hat{i})$

$\sin 45° = -\sin 135°$ ⇒ take absolute value on $\sin\theta$.

or use the smaller angle

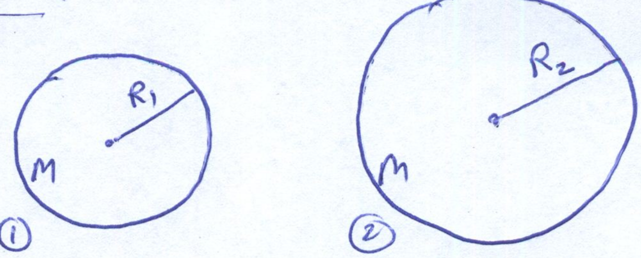
1) Rolling motion 2) Angular acceleration α 3) Torque $\vec{\tau}$ 4) Moment of inertia I

$$\underbrace{\underset{\vec{F}\ \&\ \vec{r}}{\downarrow} \qquad \underset{m\ \&\ r}{\downarrow}}_{\text{radius does matter in rotational motion.}}$$

$\vec{\tau}_{net} = \underset{\substack{\text{inertia for}\\ \text{rotational}\\ \text{motion.}}}{\underset{\downarrow}{I}}\alpha$ $\quad$ ($\vec{F}_{net} = ma$) inertia for linear motion m

4) Moment of Inertia I (

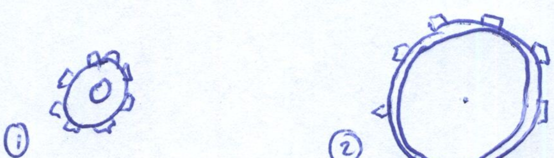
→ Size:



② offers more inertia to rotation although both would have same inertia to linear motion.

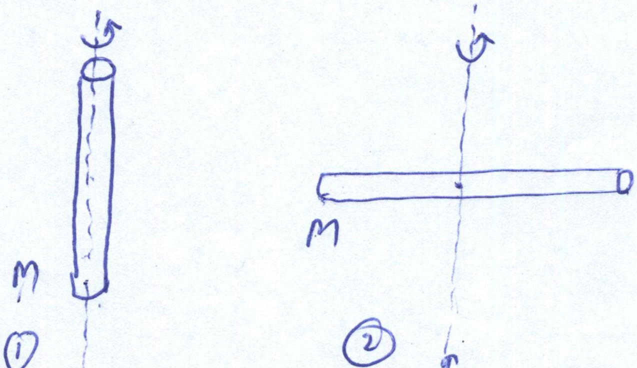
spheres with equal masses but different distribution or density $R_1 < R_2$

Bike gears



② is harder to pedal (more I)

→ Axis of rotation:



② offers more I

Rod wrt axis of symmetry

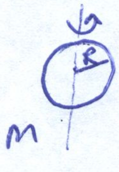
Axis of rotation = axis of symmetry

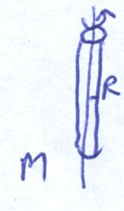
Axis of rotation

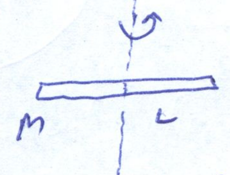
$$I = \begin{cases} \text{Discrete systems:} & I = \sum_i m_i r_i^2 \\ \text{Continuous systems:} & I = \int r^2\, dm \end{cases}$$

Simple geometrical objects: disk, rods, spheres, ...

$\hookrightarrow I = \underset{\text{constant}}{\underset{\downarrow}{\alpha}} MR^2$ $\begin{cases} M: \text{ total mass} \\ R: \text{ radius of the mass distribution wrt center / axis of rotation} \end{cases}$

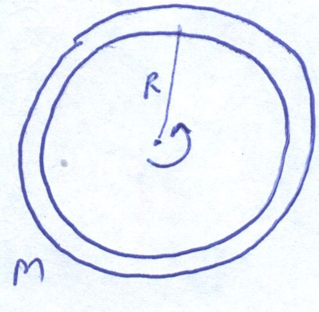
1) Sphere wrt center axis  $I = \frac{2}{5}MR^2$ $\left(\alpha = \frac{2}{5}\right)$

2) Cylinder " " "  $I = \frac{1}{2}MR^2$ $\left(\alpha = \frac{1}{2}\right)$

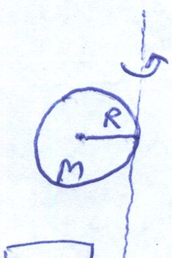
3) Thin rod of length L wrt middle axis  $I = \frac{1}{12}ML^2$

$\left(\alpha = \frac{1}{12}\right)$

4) Ring of mass M & radius R (bike wheel)



$(\alpha = 1) \Rightarrow I = MR^2$

5) Sphere wrt a tangential axis: 

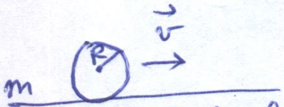
<u>Parallel axis theorem:</u> $I = I_{\substack{\text{center}\\ \text{axis}}} + \boxed{MR^2}$ (R is sep. b/w the tangential axis & the center axis)

$= \frac{2}{5}MR^2 + MR^2$

$I = \frac{7}{5}MR^2$

Kinetic Energy in rotational motion:

① Linear motion



frictionless surface

Disk skidding on frictionless surface with velocity $\vec{v}$

$$KE = \frac{1}{2}(m)v^2$$

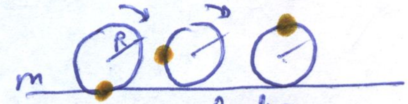
② Pure rotational motion

Disk rotating freely on a support (no translation)

$$KE = \frac{1}{2} I \omega^2$$

③ Rolling motion

$$\boxed{v_{cm} = \omega \cdot R}$$



friction. non-skidding

Disk rolling on surface: translation of cm + rotation w.r.t center axis

$$KE = \underbrace{\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2}_{\text{related!}}$$

$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right)\left(\frac{v_{cm}}{R}\right)^2$$

$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{4} m v_{cm}^2$$

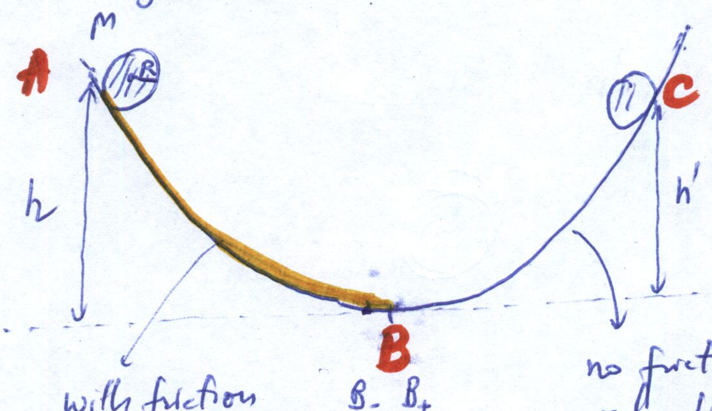
$$= \frac{1}{2}\left(\frac{3}{2} m\right) v_{cm}^2$$

→ Consequence of rolling vs skidding: 50% increase in mass (ABS braking system → can loose KE in shorter distance).

↳ Anti-blocking braking system
- w/o ABS: inertia mass is m
- w ABS: inertia mass is 1.5m

↳ Shorter stopping distance.

10.64 → Working with both rolling motion (left side) and linear motion (right side)



Max height h' of ball on right side

$$h' = \begin{cases} = h & \times \\ > h & \times \\ < h & \checkmark \text{ (friction!)} \end{cases}$$

with friction
no slipping or no skidding
↓
Rolling motion

no friction
only linear motion

Statement: 1) There is conservation of energy b/w B & C.
2) Friction allows rotation b/w A & B : with the additional rotational motion we can establish conservation of energy b/w A & B as well!

initial (A) = final (B_-)	initial (B_+) = final (C)
① $Mgh = \frac{1}{2}Mv_B^2 + \frac{1}{2}I\omega^2$	② $\frac{1}{2}Mv_B^2 = Mgh'$

→ Find v_B then h'

Rolling motion : $v_B = \omega \cdot R \Rightarrow \boxed{\omega = \frac{v_B}{R}}$

Sphere's moment of inertia (wrt center axis) : $\boxed{I = \frac{2}{5}MR^2}$

① → $Mgh = \frac{1}{2}Mv_B^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_B}{R}\right)^2$

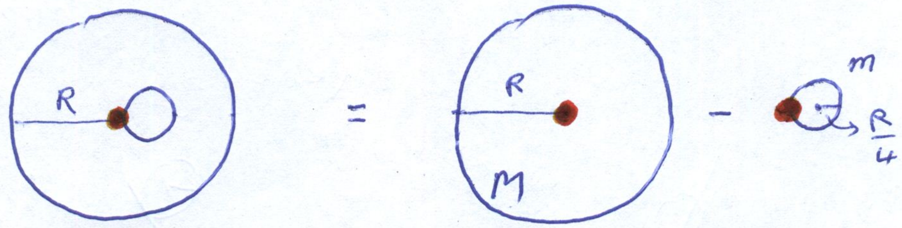
$= \frac{1}{2}Mv_B^2 + \frac{1}{2}\left(\frac{2}{5}M\right)v_B^2$

$= \frac{1}{2}\left(\frac{7}{5}M\right)v_B^2$

$\Rightarrow \frac{1}{2}Mv_B^2 = \frac{5}{7}Mgh$

② → $\frac{5}{7}Mgh = Mgh' \Rightarrow \boxed{h' = \frac{5}{7}h}$

10.65



Disk with hole of radius $\frac{R}{4}$	Whole disk	missing piece of radius $\frac{R}{4}$
Center axis of rotation	Center axis of rotation	Tangential axis of rotation

Parallel axis Theorem

$$\boxed{\overset{?}{I_{\text{with hole}}}} = \underbrace{I_{\text{without hole}}}_{\frac{1}{2}MR^2} - \underbrace{I_{\text{missing piece}}}_{\frac{1}{2}m\left(\frac{R}{4}\right)^2 + m\left(\frac{R}{4}\right)^2}$$

m is a fraction of M: $\frac{m}{M} = \frac{\pi\left(\frac{R}{4}\right)^2}{\pi R^2} = \frac{1}{16} \Rightarrow m = \frac{M}{16}$

$$I_{\text{with hole}} = \frac{1}{2}MR^2 - \frac{3}{2}m\left(\frac{R}{4}\right)^2 = \frac{1}{2}MR^2 - \frac{3}{2}\frac{M}{16}\frac{R^2}{16}$$

$$= \frac{1}{2}MR^2 \underbrace{\left(1 - \frac{3}{16^2}\right)}_{\frac{253}{256}}$$

Ch 11 Rotational Vectors & $\boxed{\text{Angular Momentum}}$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \leftrightarrow \vec{F}_{net} = 0 \text{ (collisions)} \leftrightarrow \vec{P}_i = \vec{P}_f$$ (linear momentum is conserved)

$$\vec{\tau}_{net} = I\alpha \leftrightarrow \frac{d\vec{L}}{dt} \leftrightarrow \vec{\tau}_{net} = 0 \leftrightarrow \vec{L}_i = \vec{L}_f$$ (angular momentum is conserved)

So far: r showed up in $\vec{\tau}$ & I

$\vec{L}$: angular momentum.
(related to $\vec{p}$ & radius does matter)

Linear

$\vec{p} = m\vec{v}$

linear momentum

Rotational

$\vec{L} = \vec{r} \times \vec{p}$

angular momentum of an object wrt an axis of rotation equals the cross product b/w its position vector $\vec{r}$ (wrt axis of rotation) and its linear momentum $\vec{p}$

($\vec{L}$ is perpendicular to both $\vec{r}$ & $\vec{p}$)

---

1) So far: $\begin{cases} \vec{\tau} = \vec{r} \times \vec{F} \\ \vec{L} = \vec{r} \times \vec{p} \end{cases}$

also $\vec{F} = \frac{d\vec{p}}{dt} \longleftrightarrow \boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

$\vec{L} = ?$

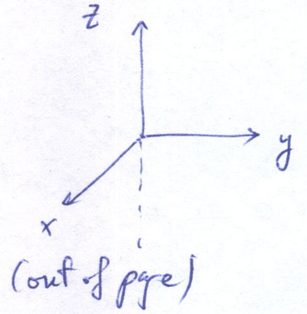
2) $\boxed{\vec{\tau} = I \cdot \vec{\alpha}}$ $\quad (\vec{F} = m\vec{a}) \longrightarrow \boxed{\vec{L} = I \cdot \vec{\omega}}$

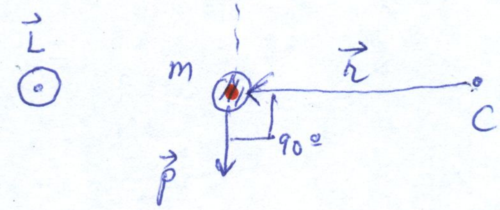
(recall $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$)

Angular momentum:

$$\begin{cases} \text{General motion: } & \vec{L} = \vec{r} \times \vec{p} \\ \text{Rotations: } & \vec{L} = I\vec{\omega} \end{cases}$$

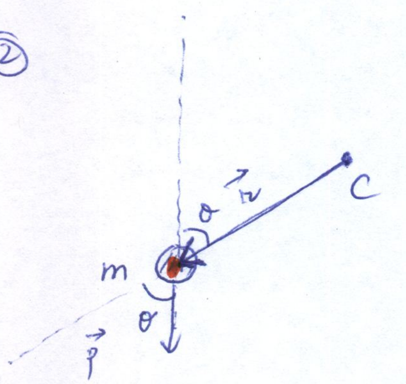
① → An object of mass m moving along $-z$ axis $(-\hat{k})$; our axis of "rotation" is @ C





Although there is no rotation we can calculate the angular momentum $\vec{L} = \vec{r} \times \vec{p} = rp\underbrace{\sin 90}_{1}\underbrace{\hat{i}}_{RHR}$ wrt to C

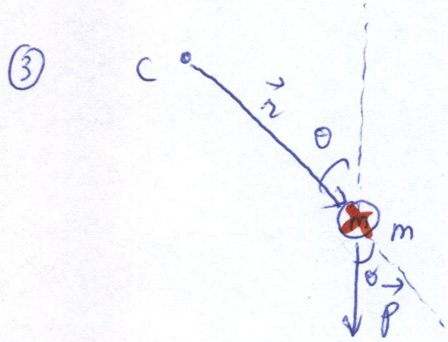
②



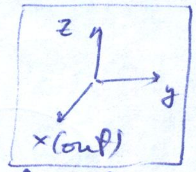
$\vec{L} = rp\sin\theta \underbrace{\hat{i}}_{RHR}$ (out of page)

RHR: to find direction of $\overset{1st}{\vec{r}} \times \overset{2nd}{\vec{p}}$: RH fingers along 1st vector, then close RH toward 2nd vector, thumb gives direction of $\vec{r} \times \vec{p}$

③



$\vec{L} = rp\sin\theta \underbrace{(-\hat{i})}_{RHR}$



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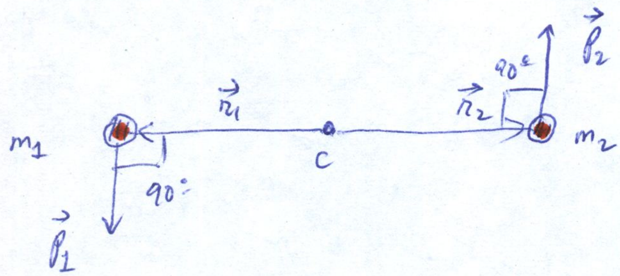


Angular momentum for this system?

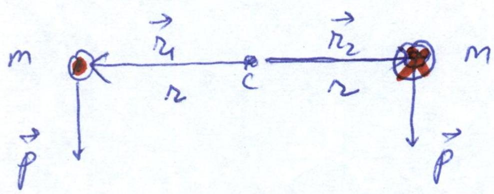
$$\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$= r_1 p_1 \hat{i} + r_2 p_2 \hat{i} = (r_1 p_1 + r_2 p_2)\hat{i} \quad \text{(out of page!)}$$

(equal masses $m_1 = m_2 = m$; equal radii : $|\vec{r}_1| = |\vec{r}_2| = r$; equal speeds $v_1 = v_2 = v$:

$$\vec{L} = 2rp\,\hat{i} \quad \odot$$

(5)



$\vec{L} = 0$!

<u>Conservation of angular momentum</u>:

<u>General motion:</u>

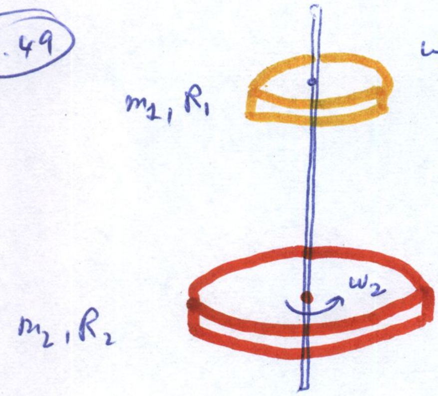
$$\vec{L}_i = \vec{L}_f$$

$$\vec{r}_i \times \vec{p}_i = \vec{r}_f \times \vec{p}_f$$

<u>Rotational motion</u>

$$I_i \omega_i = I_f \omega_f$$

11.49



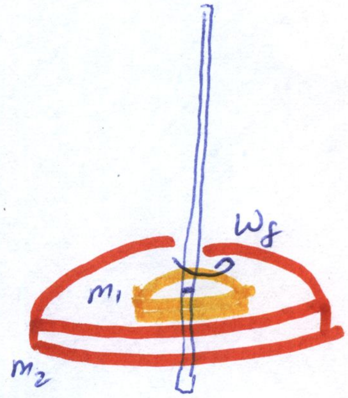
$m_1 = 0.27 kg$

$R_1 = 0.023 m$

$m_2 = 0.44 kg$

$R_2 = 0.035 m$

$\omega_2 = 180\ rpm$



Top @ rest (dropping freely)
Bottom @ $\omega_2 = 180\ rpm$.

Two disks rotating together @ ω_f $\begin{cases} \cancel{=180\ rpm} \\ \cancel{>180\ rpm} \\ <180\ rpm \checkmark \end{cases}$

Statement: $\vec{\tau}_{net} = 0 \quad \leftrightarrow \quad \vec{L}_i = \vec{L}_f$

(Nothing is applied on either disk!)

$$\boxed{I_i \omega_i = I_f \omega_f}$$

a) $\boxed{\omega_f ?}$

$$\left.\begin{aligned} I_i &= I_2 = \tfrac{1}{2} m_2 R_2^2 \\ \omega_i &= \omega_2 \\ I_f &= I_1 + I_2 = \tfrac{1}{2} m_1 R_1^2 + \tfrac{1}{2} m_2 R_2^2 \end{aligned}\right\} \Rightarrow I_2 \omega_2 = (I_1 + I_2)\,\omega_f$$

$$\omega_f = \frac{I_2}{I_1 + I_2}\,\omega_2$$

$$= \frac{m_2 R_2^2}{m_1 R_1^2 + m_2 R_2^2}\,\omega_2$$

$$= \frac{0.44 \times 0.035^2}{0.27 \times 0.023^2 + 0.44 \times 0.035^2}\,180\ rpm$$

$$\boxed{\omega_f = 142\ rpm}$$

b) Fraction of energy lost to friction (b/w top disk & bottom disk!)

$$\underbrace{\text{energy lost}}_{KE_i - KE_f}$$

$$\frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{\tfrac{1}{2}(I_1 + I_2)\,\omega_f^2}{\tfrac{1}{2} I_2 \omega_2^2} = 1 - \frac{0.27 \times 0.023^2 + 0.44 \times 0.035^2}{0.44 \times 0.035^2} \cdot \frac{142^2}{180^2}$$

$$= 0.21 \text{ or } 21\%$$

(note conversion from rpm to $\frac{rad}{s}$ would cancel out up & down)

10.36 **Rotational vs. translational kinetic energy**

Baseball $\leftrightarrow$ sphere (solid) $\begin{cases} m = 0.15\,kg \\ R = 0.037\,m \end{cases}$ $v_{cm} = 33\,m/s$, $\omega = 42\,rad/s$

$$\rightarrow KE = \frac{1}{2}mv_{cm}^2 + \underbrace{\frac{1}{2}I\omega^2}_{\text{Rotational}} \Rightarrow f = \frac{\overbrace{\frac{1}{2}I\omega^2}^{\text{Rotational}}}{\underbrace{\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2}_{\text{Total KE}}}$$

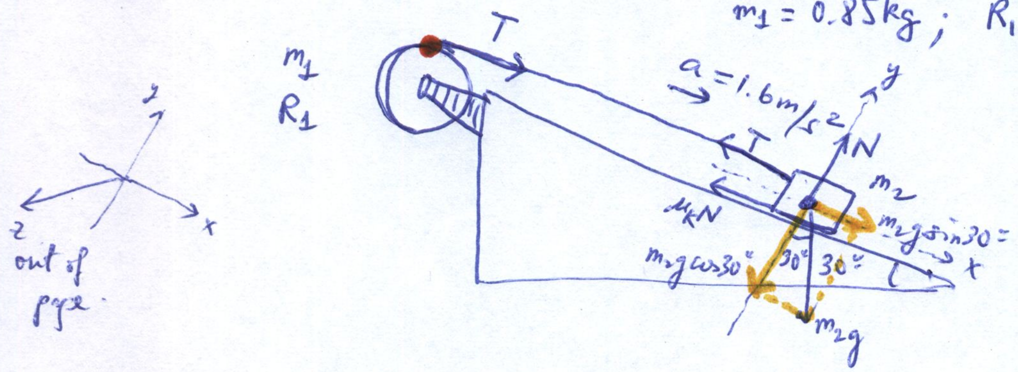
Solid sphere (2) $I = \frac{2}{5}mR^2 = \frac{2}{5}0.15\,0.037^2$

$$f = \frac{\frac{1}{2}\times\frac{2}{5}0.15\times 0.037^2\times 42^2}{\frac{1}{2}0.15\cdot 33^2 + \frac{1}{2}\frac{2}{5}0.15\times 0.037^2} = \frac{\frac{1}{2}8.2\times10^{-5}\times 42^2}{81 + \frac{1}{2}8.2\times10^{-5}\times42^2} = \cancel{10}$$

$$f = \frac{0.072}{81 + 0.072} = 8.88\times10^{-4}$$

10.57 Two objects $\begin{cases} \text{1) box under linear motion down a slope with friction} \\ \quad m_2 = 2.4\,kg \\ \text{2) drum under rotational motion} \\ \quad m_1 = 0.85\,kg;\ R_1 = 0.05\,m \end{cases}$

$\rightarrow$ String has negligible mass (same tension throughout)



Coefficient of kinetic friction μ_K?

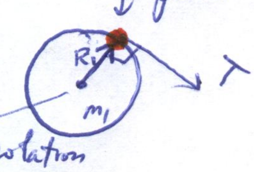
Box: $(F_{net} = ma)$

$$F_{net_x} = m_2 g\sin 30° - \mu_K N - T = m_2 a$$

$$F_{net_y} = N - m_2 g\cos 30° = 0$$

~~$T = m_2 g(\sin 30° - \mu_K$~~

$(\tau_{net} = I\alpha)$ Drum



$$\vec{\tau}_{net} = R_1 T\sin 90°(-\hat{k})\ \otimes$$

$$R_1 T = I\cdot\alpha = \frac{1}{2}m_1 R_1^{\cancel{2}}\cdot\frac{a}{\cancel{R_1}}$$

$$\cancel{R_1}T = \frac{1}{2}m_1\cancel{R_1}a \Rightarrow \boxed{T = \frac{1}{2}m_1 a}$$

<u>Statements</u> 1) Red dot has a linear acceleration same as that of the block (block + drum are connected by a rope)

$$\rightarrow \boxed{\alpha = \frac{a}{R_1}} \qquad \left(\text{recall also: } \omega = \frac{v}{R} \text{, since} \right.$$

$$\left. \theta = \frac{arc}{R} \rightarrow \frac{d\theta}{dt} = \frac{\frac{d(arc)}{dt}}{R} \text{ or } \omega = \frac{v}{R} \right)$$

Now using result from analysis for the drum rotational motion

$$T = \frac{1}{2} m_1 a$$

$$\rightarrow \quad m_2 g \sin 30^\circ - \mu_k m_2 g \cos 30^\circ - \frac{1}{2} m_1 a = m_2 a$$

$$\mu_k = \frac{m_2 g \sin 30^\circ - \left(\frac{1}{2} m_1 + m_2\right) a}{m_2 g \cos 30^\circ}$$

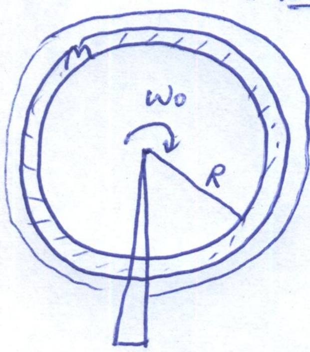
$$= \frac{2.4 \times 9.81 \times \sin 30^\circ - \left(\frac{0.85}{2} + 2.4\right) 1.6}{2.4 \times 9.81 \times \cos 30^\circ}$$

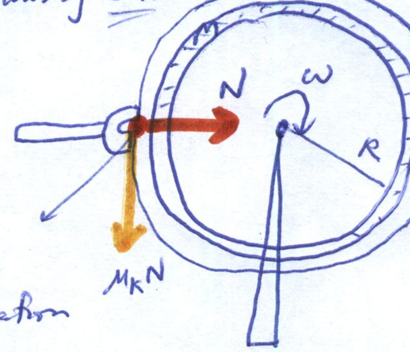
$$\mu_k = \cancel{0.5543} \quad \underline{0.3557}$$

10.58

Spinning inverted bike wheel, then a wrench was applied with a normal force 2.7N during 3.1s

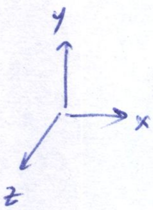
$R = 0.33\,m$, $m = 1.9\,kg$ | $\mu_k = 0.46$



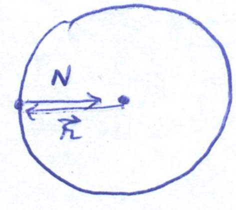


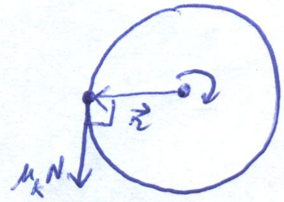
$w < w_0$

$w_0 = 230\,rpm$



$\tau_{\mu_k N}$ applies an angular deceleration α that will slow it down!





$\vec{\tau}_N = \vec{r} \times \vec{N} = rN\underbrace{\sin 180°}_{0} = 0$

Normal force itself applies no torque

$\vec{\tau}_{\mu_k N} = r\mu_k N \underbrace{\sin 90}_{1}\ \hat{k}$ $\odot$

Friction force applies torque $\tau_{\mu_k N} = R\mu_k N$ (out of page)

Analog of 2nd Newton's law: $\tau_{net} = I \cdot \alpha$

Bike wheel = a ring of mass m & radius R

$\tau_{\mu_k N} = mR^2 \cdot \alpha$

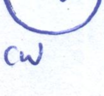
$\alpha = \dfrac{\mu_k N \cancel{R}}{mR^{\cancel{2}}} \Rightarrow \boxed{\alpha = \dfrac{\mu_k N}{mR}}$

Statement: How is it a deceleration in our equations?

Friction torque was out of page

$\vec{L}$	τ_{μ_k}	opposite
$\otimes$	$\odot$	→ deceleration → α is negative

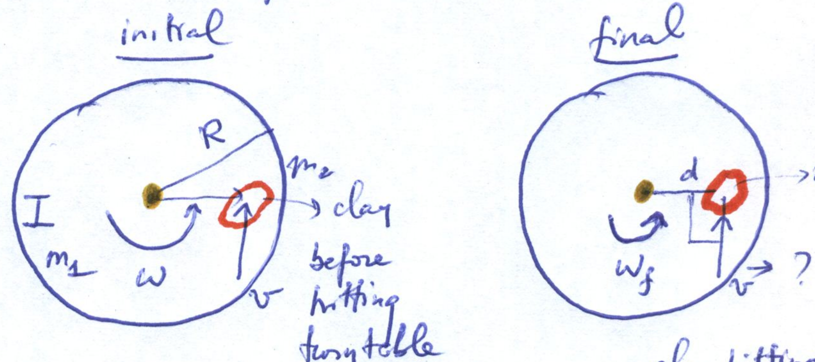
$\left\{ \begin{array}{l} \vec{\tau} = \vec{r} \times \vec{F} \\ \vec{L} = \vec{r} \times \vec{p} = I\vec{w} \text{ (rotation)} \end{array} \right\} \Rightarrow \vec{\tau} = \dfrac{d\vec{L}}{dt}$



CW $\vec{L}$ by RHR (RH fingers turning in CW, thumb into page)

11.45

Turn table ($\boxed{\text{disk}}$) rotating wrt its center axis, moment of inertia I. View from above:



→ clay hitting turn table horizontally & @ 90° to radius in same direction as rotation.
→ stays on with same final ω_f

Find v $\begin{cases} a) & \omega_f = \frac{\omega}{2} \\ b) & \omega_f = \omega \\ c) & \omega_f = 2\omega \end{cases}$

Statement: to relate ω_f to ω (initial):

$\tau_{net} = 0$ on turn table & clay $\rightarrow \frac{d\vec{L}}{dt} = 0 \rightarrow L_i = L_f$

System in rotational motion $\rightarrow L = I \cdot \omega$

$$L_i = L_f$$

$$\underbrace{\frac{1}{2} m_1 R^2 \cdot \omega}_{\text{turntable}} + \underbrace{d m_2 v}_{\text{clay}} = \left(\frac{1}{2} m_1 R^2 + m_2 d^2\right) \cdot \omega_f$$

(there is an angular momentum although no rotation as yet)

$L = \vec{r} \times \vec{p}$

($\vec{p} \perp \vec{r}$ as said $\theta = 90°$)

Solve for v: $$v = \frac{\left(\frac{1}{2} m_1 R^2 + m_2 d^2\right)\omega_f - \frac{1}{2} m_1 R^2 \omega}{m_2 d}$$

a) if $\omega_f = \frac{\omega}{2}$ $\rightarrow$ $v = \dfrac{\frac{1}{2} m_1 R^2 \frac{\omega}{2} - \frac{1}{2} m_1 R^2 \omega + m_2 d^2 \frac{\omega}{2}}{m_2 d}$

$$= \frac{\frac{1}{2} m_2 d^2 \omega - \frac{1}{4} m_1 R^2 \omega}{m_2 d} = \frac{1}{2} d\omega - \frac{1}{4}\frac{m_1 R^2}{m_2 d}\omega$$

$$= \left(\frac{1}{2} d - \frac{1}{4}\frac{m_1 R^2}{m_2 d}\right)\omega$$

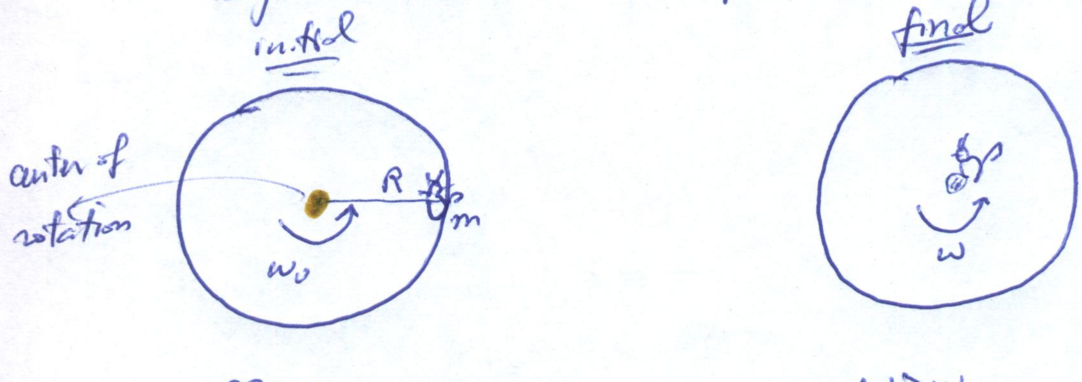
b) if $\omega_f = \omega$ $\quad v = \dfrac{\cancel{m_2} d^{\cancel{2}} \omega}{\cancel{m_2} \cancel{d}} = d\omega$

c) if $\omega_f = 2\omega$ $\quad v = \dfrac{m_1 R^2 \omega - \frac{1}{2} m_1 R^2 \omega + 2 m_2 d^2 \omega}{m_2 d}$

$$= \frac{\frac{1}{2} m_1 R^2 \omega + 2 m_2 d^2 \omega}{m_2 d}$$

$$= \left(\frac{1}{2}\frac{m_1 R^2}{m_2 d} + 2d\right)\omega$$

11.38 $\rightarrow$ Mouse walks on a turn table (I, R given) from outer edge to center. View from above:



$\omega_0 = 22$ rpm
freely (no friction)

$\omega > \omega_0$

a) $\tau_{net} = 0$ $\longrightarrow$ $L_i = L_f$ (rotations: $L = I \cdot \omega$)
on system of disk & mouse

$I \cdot \omega_0 + mR^2 \omega_0 = I\omega$ $\rightarrow$ solve for ω

1st kinematic eq for rotational motion: $\omega = \omega_0 - \underbrace{\alpha . t}_{\Delta\omega}$

ω_0 → 230 rpm non SI unit

<u>Note:</u> if we use SI units → α comes out as $\frac{rad}{s^2}$ or $\alpha . t$ as $\frac{rad}{s}$

→ We will calculate $\Delta\omega = \alpha . t$ in $\frac{rad}{s}$ then convert it to rpm before the subtraction

$$\Delta\omega = \frac{\mu_k N}{m R} . t = \frac{0.46 \times 2.7}{1.9 \times 0.33} \times 3.1 \cdot \frac{rad}{\cancel{s}} \cdot \frac{60 \cancel{s}}{min} \cdot \frac{1 \, rev}{2\pi \, \cancel{rad}} = 58.6 \frac{rev}{min}$$

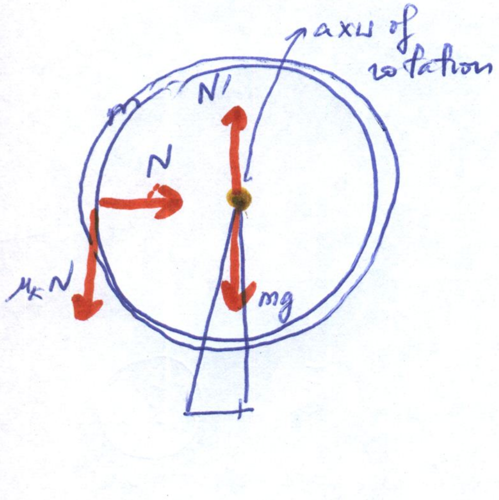
$$\Delta\omega = 58.6 \text{ rpm} \Rightarrow \omega = 230 - 58.6 = 171 \text{ rpm}$$

<u>Statements</u>:

What are the forces acting on the wheel? Is $\mu_k NR$ the net torque τ_{net} on wheel?

$\boxed{\text{Forces:}}$ N (by wrench), $\mu_k N$ (friction), mg (weight), N' (normal by support on wheel)

$\boxed{\text{Torques:}}$
- N → 0 ($\vec{N}$ & $\vec{r}$ forms angle 180°)
- $\mu_k N$ → $\mu_k NR$ ⊙
- mg → 0 ($\vec{r} = 0$ since force application point on axis of rotation)
- N' → 0 (same reason as no torque for mg)



$$\omega = \frac{(I + mR^2)}{I}\omega_0 = \left(1 + \frac{mR^2}{I}\right)\omega_0$$

$$= \left(1 + \frac{0.0195 \times 0.25^2}{0.0154}\right) 22\,\text{rpm} = \overset{23.7411}{\cancel{22.1714}}\ \text{rpm}$$

b) Work done by mouse : since system speeds up by 0.1714 rpm as it walks from outer edge to center of rotation

↳ Should equal $KE_f - KE_i = \frac{1}{2} I \omega_f^2 - \frac{1}{2}(I + mR^2)\omega_0^2$

$$= \frac{1}{2} \times 0.0154 \times 2.4862^2 - \frac{1}{2}(0.0154 + 0.0195 \times 0.25^2) 2.3038^2$$

$$= \boxed{0.0035\ \cancel{0.0026}\ J}$$

Need ω's in $\frac{rad}{s}$

$$\omega_0 = 22 \frac{rev}{min} \cdot \frac{1\,min}{60\,s} \frac{2\pi\,rad}{rev} = \frac{22 \times 2\pi}{60} \frac{rad}{s} = 2.3038 \frac{rad}{s}$$

$$\omega_f = 23.7411 \times \frac{2\pi}{60} \frac{rad}{s} = 2.4862 \frac{rad}{s}$$

11.35 Rotor:

$R = \frac{d}{2} = 150 \times 10^{-6}\,m$

$h = 2 \times 10^{-6}\,m$

$Vol = \pi R^2 h = \pi \times (150 \times 10^{-6})^2 \times 2 \times 10^{-6}\,m^3$

$m = \rho \cdot Vol$; $\omega = 800 \frac{rev}{min} \cdot \frac{1\,min}{60\,s} \frac{2\pi}{rev} = 800 \frac{2\pi}{60} \frac{rad}{s}$

↑ silicon

↓

$$\rho = 2.3296 \frac{g}{cm^3} \cdot \frac{1\,kg}{1000\,g} \frac{10^6\,cm^3}{1\,m^3} = \boxed{2329.6 \frac{kg}{m^3}}$$

$$L = I \cdot \omega$$

$$= \frac{1}{2} m R^2 \cdot \omega$$

$$= \frac{1}{2} \rho\, Vol\, R^2 \omega$$

$$= \frac{1}{2} 2329.6 \times \pi \times (1.5 \times 10^{-4})^2 \times 2 \times 10^{-6} \times (1.5 \times 10^{-4})^2 \times 800 \frac{2\pi}{60} = 3.1 \times 10^{-16}\ kg \frac{m^2}{s}$$