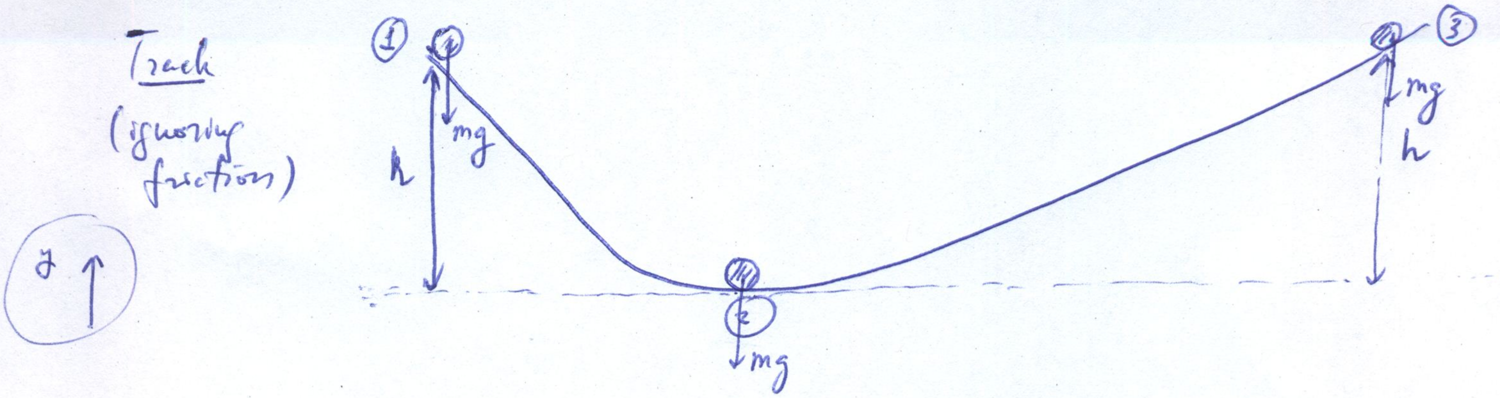
# Ch 7 Conservation of Energy

Motion → Forces → Work & Energy

Forces ; 2 types
- Conservative : gravity (lost & gain)
  (work is conserved)
- Non-conservative : friction (always a loss)
  (work is not conserved) ↓ against motion!




Work done by gravity: (if force was not changing over the displacement

Work done = $\vec{F}_{applied} \cdot \underbrace{\vec{\Delta r}}_{\text{displacement}} = (-\hat{j})mg \cdot \vec{\Delta h}$

↑ $F_{gravity}$ is vertical

1 → 2

$\vec{\Delta h} = (0-h)(+\hat{j}) = -h\hat{j}$

$Work_{12} = -\hat{j}mg \cdot (-h\hat{j}) = +mgh$

2 → 3

$\vec{\Delta h} = (h-0)\hat{j} = h\hat{j}$

$Work_{23} = -\hat{j}mg \cdot h\hat{j} = -mgh$

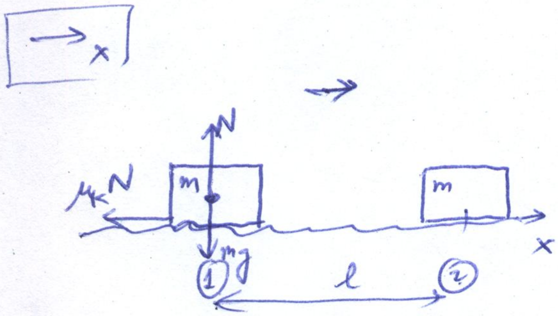
⇒ $Work_{12} + Work_{23} = +mgh - mgh = 0$

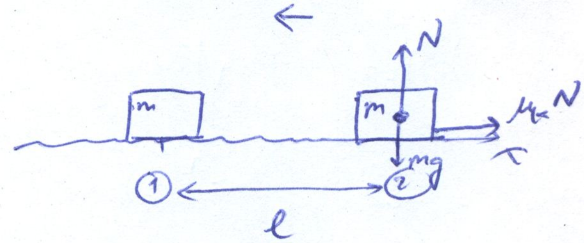
(Work done by a conservative force is conserved).

↳ Grav. potential energy is conserved.

## Work done by friction:

pushing a box of mass $m$ ① → ② → ①
on a rough surface.
* Friction is always against motion




## Work done by friction.

$$Work_{12} = \vec{F}_{f_{12}} \cdot \Delta\vec{r}_{12}$$
$$= -\mu_k N \hat{i} \cdot (l - 0)\hat{i}$$
$$= -\mu_k N l$$
$$\hat{i} \cdot \hat{i} = 1$$

$$Work_{21} = \vec{F}_{f_{21}} \cdot \Delta\vec{r}_{21}$$
$$= \mu_k N \hat{i} \cdot (0 - l)\hat{i}$$
$$= -\mu_k N l$$
$$\hat{j} \cdot \hat{j} = 1$$

Total work after ① → ② → ① : $W_{121} = -\mu_k N l - \mu_k N l = -2\mu_k N l$

↳ → Non-zero : work not conserved since friction is non-conservative
↳ → Negative : since friction did not perform any work it consumes work (work was performed by whatever force was applying on the box to move it ① → ② → ① )

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## Mechanical energy:

→ Gravitational potential energy $GPE = mgh$
($\Delta PE$ when there is a change in vertical position $h$)

→ Kinetic energy $KE = \frac{1}{2} m v^2$
($\Delta KE$ when there is a change in speed)

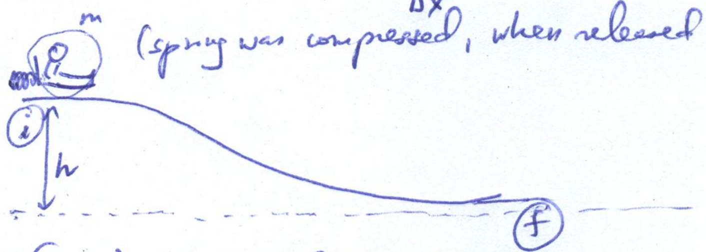
→ Elastic potential energy : $EPE = \frac{1}{2} k x^2$
($\Delta EPE$ when there is a change in length of spring)

Situations:

Only KE: horizontal motions

KE & GPE: vertical motions

KE & GPE & EPE:




(spring was compressed $\Delta x$, when released pushes sled & child)

Total mechanical energy (ME) = EPE + GPE + KE

→ If all forces involved are conservative: $\boxed{ME_{Total} = \text{conserved.}}$

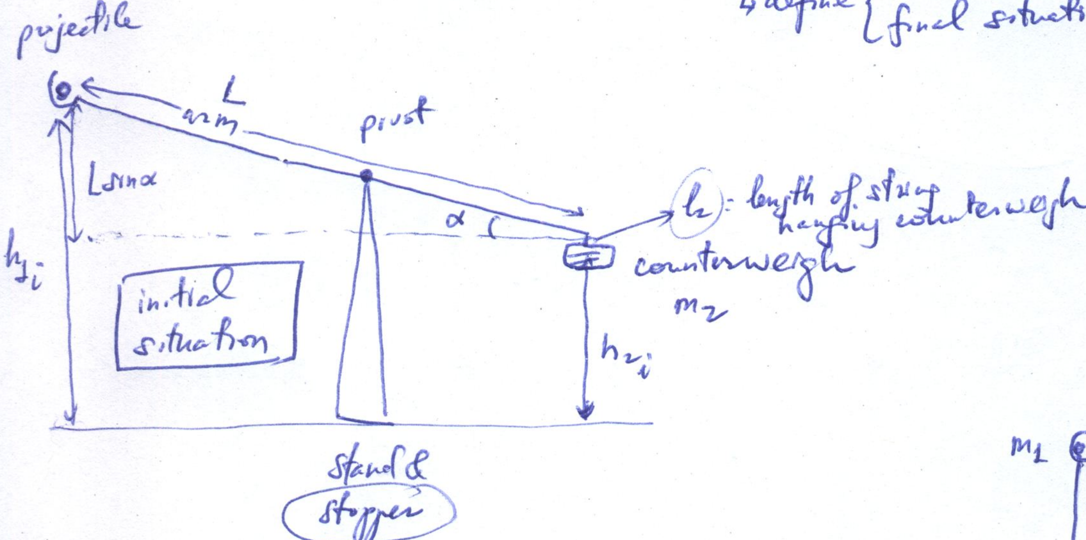
$$ME_i = ME_f$$

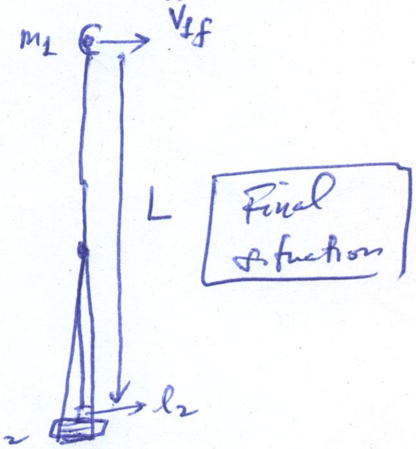
$$\frac{1}{2}k\Delta x^2 + mgh + 0 = 0 + 0 + \frac{1}{2}mv^2$$

speed @ bottom of hill.

**PP3 Q7.1** Catapult or trebuchet: uses a counterweigh to launch a projectile by conservation of mechanical energy

↳ define $\begin{cases} \text{initial} \\ \text{final situations: arm vertical, counterweigh @ lowest point, projectile leaving spoon with an initial velocity in the horizontal direction} \end{cases}$




→ Goal: what do we want to find:

$\boxed{\text{how far}}$ projectile will travel forward given ~~such~~ an initial height for counterweigh $h_{2i}$

$\boxed{\text{Need } v_{1f}} \rightarrow \boxed{\text{Conservation of ME}}$

$ME_i \begin{cases} GPE: m_1 \;\&\; m_2 \\ KE: 0 \; (v_{1i} = v_{2i} = 0) \end{cases}$

$ME_f \begin{cases} GPE: \begin{cases} m_1 g(L+l_2) \\ \text{none for counterweigh (approx.!)} \end{cases} \\ KE: \frac{1}{2} m_1 v_{1f}^2 \,; \\ \quad v_{2f} = 0 \text{ (supposedly!) with stopper.} \end{cases}$

Conserv. of ME:

$$m_1 g h_{1i} + m_2 g h_{2i} = m_1 g (L + l_2) + \frac{1}{2} m_1 v_{1f}^2$$

solve for this!

$$h_{1i} = L\sin\alpha + l_2 + h_{2i}$$

$$m_1 g (L\sin\alpha + l_2 + h_{2i}) + m_2 g h_{2i} = \frac{1}{2} m_1 v_{1f}^2 + m_1 g (L + l_2)$$

$$(m_1 + m_2) g h_{2i} = \frac{1}{2} m_1 v_{1f}^2 + m_1 g (L + \cancel{l_2} - L\sin\alpha - \cancel{l_2})$$

$$(m_1 + m_2) g h_{2i} = \frac{1}{2} m_1 v_{1f}^2 + m_1 g \, L(1 - \sin\alpha)$$

$$\Rightarrow \quad v_{1f} = \sqrt{\frac{2}{m_1}\left[(m_1+m_2)\,g\,h_{2i} - m_1 g L(1-\sin\alpha)\right]}$$

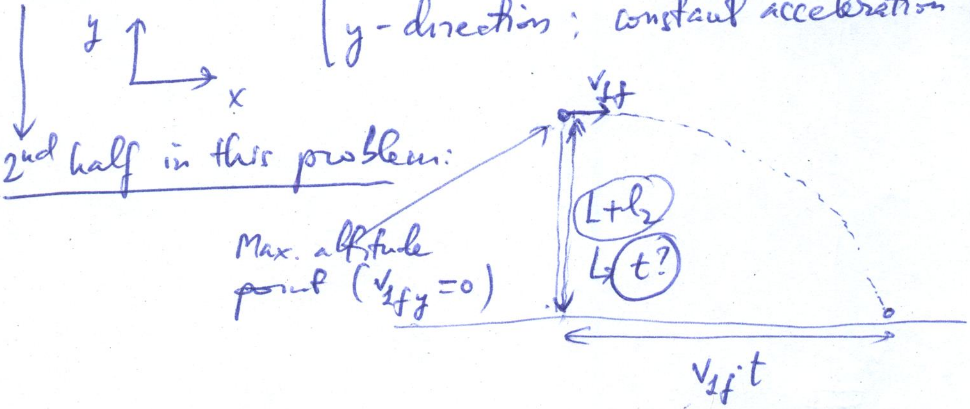
$h_{2i}$ → initial height of counterweigh

$\alpha$ → initial orientation of arm

Using $v_{1f}$ → calculate where projectile will land:

Projectile motion { x-direction: uniform motion; y-direction: constant acceleration ($-g$: upward — not here; $+g$: downward)

→ 2nd half in this problem:




→ Use motion in y (vertical) to find $t$: $L+l_2 = \frac{1}{2}gt^2$ ($v_{1fy}=0$)

$$t = \sqrt{\frac{2}{g}(L+l_2)}$$

→ Use motion in x (horizontal) to find $\boxed{x_{landing} = v_{1f}\cdot t = v_{1f}\sqrt{\frac{2}{g}(L+l_2)}}$

↓ initial height of counterweigh & how far projectile will land.

# Ch 8: Gravitation

So far: force of gravity $mg$; $GPE = mgh$

→ Universal law of gravitation

↓

Earth, Moon, other planets, galaxies, the universe

$$F = G\frac{m_1 \cdot m_2}{r^2}$$

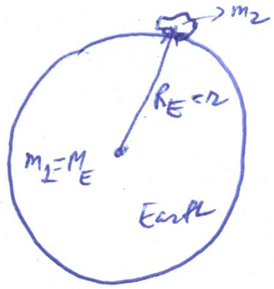
Force of gravitational attraction b/w two masses $m_1$ & $m_2$

G: Univ. Grav. Constant: $6.67\times10^{-11}\ \frac{Nm^2}{kg^2}$

r: separation (center to center) b/w $m_1$ & $m_2$ ("inverse square law") → doubled distance → one fourth of attraction)

Force is a vector: direction of grav. attraction is center to center towards the more massive mass.

→ Let's apply this law to an object on the surface of the Planet:




$$F = G\frac{M_E \cdot m_2}{R_E^2} = \underbrace{\left[\frac{6.67\times10^{-11} \times 5.97\times10^{24}}{(6.37\times10^{6})^2}\right]}_{9.81\,m/s^2 = g} \cdot m_2$$

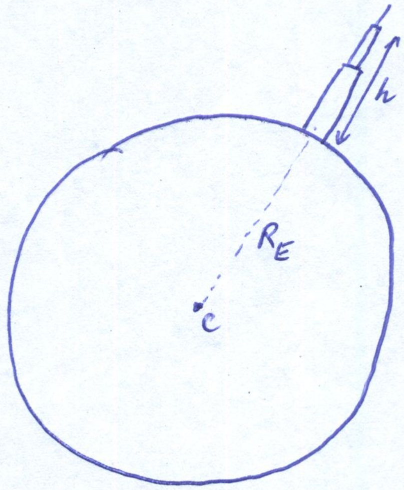
$$\boxed{\begin{aligned} M_E &= 5.97\times10^{24}\,kg \\ R_E &= 6.37\times10^{6}\,m \end{aligned}}$$

Objects on surface $h \ll R_E \Rightarrow F_{grav} = mg$

Objects on surface but $h$ can't be ignored (top of Sears tower)

$r = R_E + h \rightarrow g' < g \rightarrow$

8.17 **Sketch:**
(Not to scale)




Chicago's Willis Tower
(Formerly Sears Tower)
↳ height $h$ ↑

**Statement:** $h$ is small compared to $R_E = 6.37 \times 10^6\,m$

Order of magnitude for $h$: 10m, 100m ✓, 1000m, 10000m

↳ will calculate from $\Delta g = 1.36\ \frac{mm}{s^2}$ measured by gravimeter

↳ Use "Universal Law of Gravitation"

$$\Delta g = \underbrace{g_{street}}_{\text{street level}} - \underbrace{g_h}_{\text{top of Sears Tower } (<g)} = 1.36\ mm/s^2$$

$$F = G\frac{M_E m}{r^2} \quad (G\frac{M_E}{r^2} \equiv g) \Rightarrow \Delta g = \underbrace{G\frac{M_E}{R_E^2}}_{g_{street}} - G\frac{M_E}{(R_E+h)^2} = GM_E\left[\frac{1}{R_E^2} - \frac{1}{(R_E+h)^2}\right]$$

$$\Delta g = GM_E\left[\frac{(R_E+h)^2 - R_E^2}{R_E^2(R_E+h)^2}\right] = GM_E\,\frac{2R_Eh+h^2}{R_E^2(R_E+h)^2}$$

$$\Delta g = \underbrace{\frac{GM_E}{R_E^2}}_{g_{street}} \cdot \frac{2R_Eh+h^2}{(R_E+h)^2}$$

Simplify the 2nd fraction: use the approx: $h \ll R_E = 6{,}370{,}000\,m$

↳ To a very good approximation $\begin{cases} R_E + h \simeq R_E \\ 2R_E + h \simeq 2R_E \end{cases}$

$$\Delta g = g_{street} \cdot \frac{(2R_E+h)h}{(R_E+h)^2} \simeq g_{street} \cdot \frac{2\cancel{R_E}h}{R_E^{\cancel{2}}} = g_{street} \cdot \frac{2h}{R_E}$$

$$\Rightarrow h = \frac{\Delta g}{g_{street}} \frac{R_E}{2} = \frac{1.36\times10^{-3}}{9.81} \cdot \frac{6.37\times10^{6}}{2} = 442\,m$$

(We made use of the Universal Law of Grav.!)

→ <u>Circular Orbital Motion</u> (satellites in UCM)

↳ constant speed

varying $\vec{v}$ → $a = \frac{v^2}{R}$

→ Satellites going around the Earth in circular motion

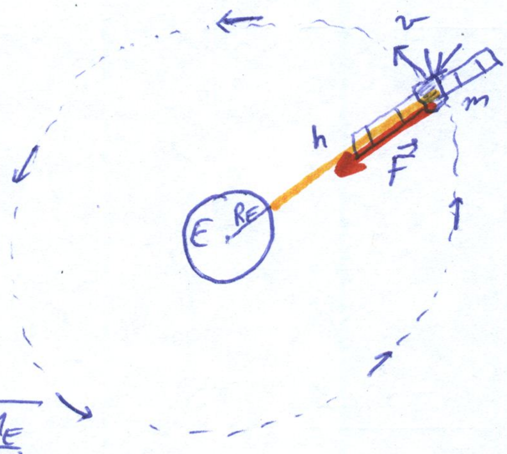
→ solar panels.

to provide energy for communications & operations <u>not for orbital motion</u>

→ Force that provides $a = \frac{v^2}{R}$ is the force of grav. attraction by the planet

Univ. Law of Grav. | 2nd Newton's Law

$$F = G\frac{M_E \cdot \cancel{m}}{(R_E+h)^{\cancel{2}}} = \cancel{m} \cdot \frac{v^2}{\cancel{(R_E+h)}}$$




<u>Orbital speed</u>: $v = \sqrt{\frac{GM_E}{R_E+h}}$

<u>Orbital period</u>: time to complete one orbit or one turn around the Earth

$$T = \frac{2\pi(R_E+h)}{\sqrt{\frac{GM_E}{R_E+h}}} = \frac{2\pi}{\sqrt{GM_E}}(R_E+h)^{3/2} \xrightarrow[\text{of } 2]{\text{to power}} T^2 = \frac{4\pi^2}{GM_E}(R_E+h)^3$$

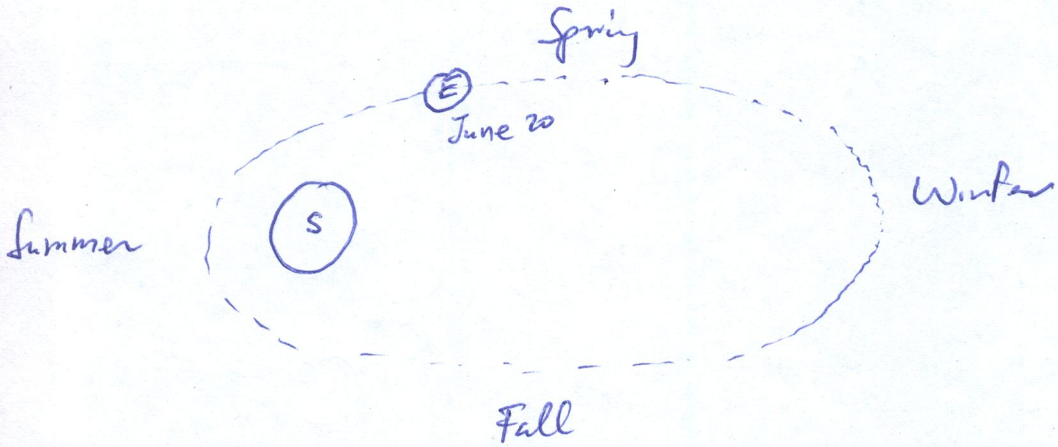
UCM or constant speed

$R \equiv R_E + h$ (radius of satellite wrt center of the Earth)

Orbital period : $T^2 \propto R^3$

Planetary Orbital Motion (Elliptical)

↳ Kepler's 3rd law: the period squared is proportional to the semi-major axis cubed



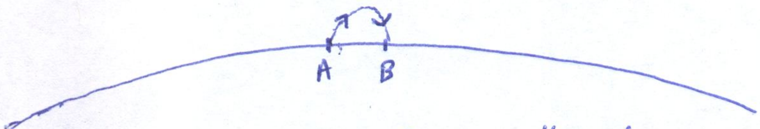

Cell phone satellites $h = 250\,km$

orbital period

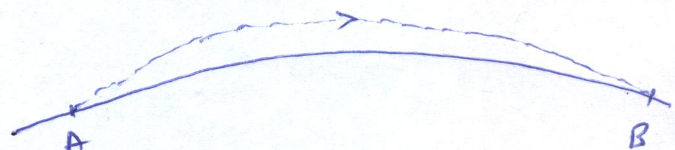
$$\hookrightarrow T = \frac{2\pi}{\sqrt{G M_E}} (R_E + h)^{3/2} = \frac{2\pi}{\sqrt{6.67\times10^{-11} \times 5.97\times10^{24}}} (6.37\times10^6 + 0.25\times10^6)^{3/2}$$

$$= 5400s \cdot \frac{1h}{3600s} = 1.5h$$

<u>Projectile motion</u> { horizontal direction : uniform motion
vertical direction : constant acceleration




Surface b/w A & B is "flat" to a very good approx.
→ Projectile trajectory is a parabola.




For longer range surface b/w A & B is not flat
→ Projectile trajectory is part of an elliptical orbit
→ Long-range missiles, etc..

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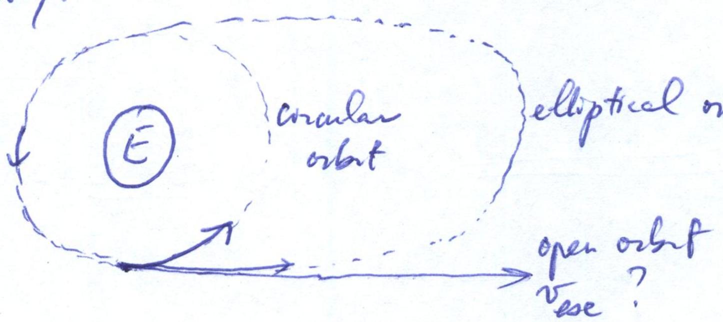
<u>Rocket escape speed</u> : 3 types of orbits for any object under the effect of gravity:




→ Total mechanical energy of an object trapped in a grav. field is negative

$$ME \underset{\text{=}}{\hookrightarrow} KE + GPE = \frac{1}{2}mv^2 - \boxed{\underbrace{\frac{GM_E\, m}{r}}_{\text{Generalization for } mgh \text{ (using Univ. Law of Grav.)}}} < 0$$

When KE is enough to ballance the negative GPE → $v_{esc}$

$$\frac{1}{2}\cancel{m} v_{esc}^2 - \frac{GM_E \cancel{m}}{r} = 0 \Rightarrow v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

$r \to R_E$ from surface

From surface

$$v_{esc} = \sqrt{\frac{2\times 6.67\times10^{-11} \times 5.97\times10^{24}}{6.37\times10^{6}}} = 11.2\ \frac{km}{s} = 40{,}320\ \frac{km}{h}$$

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## Gravitational Potential Energy : General expression (extension of mgh)

$\underbrace{\text{Potential Energy}}_{\text{Work (Ch 6)}}$

**Def. of work:**

$$\underbrace{\Delta U_{AB}}_{\substack{\text{Change of potential}\\ \text{energy from A to B}}} = +\int_A^B \vec{F}\cdot d\vec{r} = -\int_A^B \frac{GM_E m}{r^2}\,dr = -GM_E m \underbrace{\int_A^B \frac{dr}{r^2}}_{\left[-\frac{1}{r}\right]_A^B}$$

Univ Law of Grav. (negative sign for attraction)

$$= GM_E m \left[\frac{1}{r}\right]_A^B = GM_E m\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Def. $\boxed{U \equiv -\frac{GM_E m}{r}}$ $\longrightarrow$ $= U_A - U_B$ (makes sense!)

Gen. expression for GPE.

**Ref:** zero potential energy : $r = \infty \Rightarrow U_\infty = 0$

$$\Delta U_{A\infty} = U_A - U_\infty = U_A = -\frac{GM_E m}{r}$$

$r \to$ center to center separation (b/w $M_E$ & $m$)

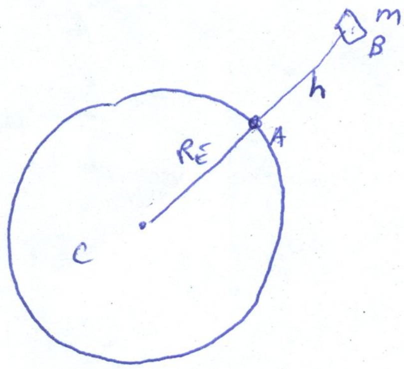
**Check:** if an object $m$ is on surface of Earth:

$$U = -\frac{GM_E m}{R_E} = -\frac{GM_E m}{R_E^2}\cdot R_E = -gm\cdot R_E$$

$$\boxed{U = -\frac{GM_E m}{r} \text{ is the extension of } mgh}$$

Proof:

Satellites:



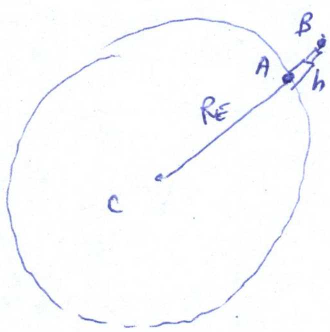

$$\Delta U_{AB} = -(GM_E m)\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$= -GM_E m\left(\frac{1}{R_E} - \frac{1}{R_E+h}\right)$$

$$= -GM_E m\left(\frac{\cancel{R_E}+h-\cancel{R_E}}{R_E(R_E+h)}\right)$$

$$= -GM_E m\frac{h}{R_E(R_E+h)}$$

Skyscrapers



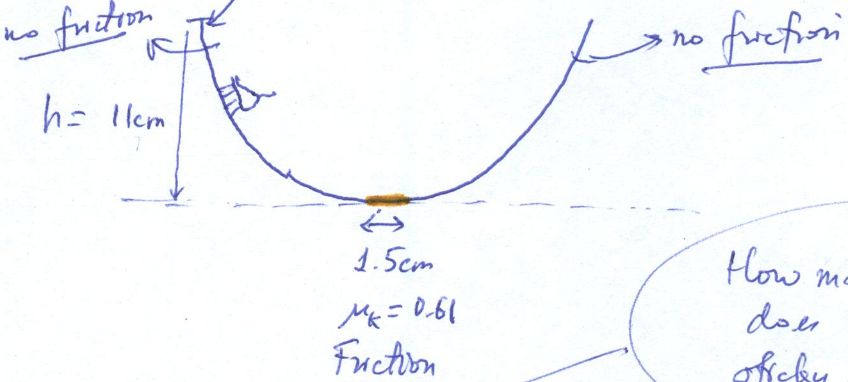

approx (very good) $\left\{ R_E + h \approx R_E \right.$

$$\Delta U_{AB} = -GM_E m\frac{h}{R_E \cdot R_E}$$

$$= -\frac{GM_E}{R_E^2}\cdot m \cdot h = -mgh$$

$g_{street}$

7.54 Sketch:




How many times does bug cross sticky region

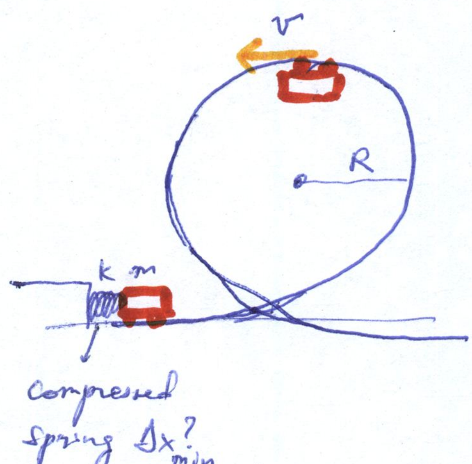
Statements: 1) Energy loss due to friction
↳ Work = force × displacement
↓ $\mu_K mg$ ↓ 1.5cm
↳ horizontal bottom $N = mg$

2) Bug started with GPE = mgh, it spends this on energy loss due to friction $\mu_K mg\, 0.015$ each time it crosses the sticky patch.

→ Answer: $\frac{\cancel{m}\cancel{g}h}{\mu_K \cancel{m}\cancel{g}\, 0.015} = \frac{\cancel{9.81} \times 0.11}{0.61 \times 0.015} = 12.02 \rightarrow 12$

7.57 Sketch:

$m = 840 kg$
$k = 31 \times 10^3 \frac{N}{m}$
No friction
$R = 6.2 m$




· Min $\Delta x$ so roller coaster car will make the loop?

Statements: 1) Car will make loop if it can go past the top point (inverted position) with a min. speed $v$ where car will barely touch the track! → $N = 0$
→ $F = mg$ is only force on car and it will provide $a = \frac{v^2}{R}$

to keep car under UCM

Conclusion: $\cancel{m}g = \cancel{m}\frac{v^2}{R} \Rightarrow v_{min} = \sqrt{gR}$

2) Make a connection b/w $v_{min}$ & $\Delta x_{min}$ :

Since there is no friction on track → energy provided by spring compression is conserved! or Total ME is conserved!

initial

car leaving spring @ (h=0) bottom of loop.

final

car @ top of loop $h = 2R$

Simple reference →

| KE | GPE | EPE | = | KE | GPE | EPE |
|---|---|---|---|---|---|---|
| 0 | 0 | $\frac{1}{2}k\Delta x_{min}^2$ | = | $\frac{1}{2}mv_{min}^2$ + | $mg2R$ + | 0 |

$$\frac{1}{\cancel{2}}k\Delta x_{min}^2 = \frac{1}{2}mgR + 2mgR = \frac{5}{\cancel{2}}mgR$$

$$\Delta x_{min} = \sqrt{\frac{5mgR}{k}} = \sqrt{\frac{5\times 840\times 9.81\times 6.2}{31000}}$$

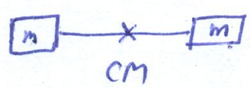
$$= 2.87m$$

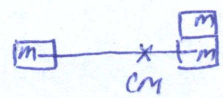
## ch9 System of Particles

So far we looked at objects (blocks, cars, ~~s~~leds, etc.) as point-like particles with a mass located at it's center of mass (FBD's). Now we will deal with systems of particles

**Definitions:**

**Center of Mass:** average position of all components of a system weighted by their masses:




Same 2 positions to average but the right position has a double weight!

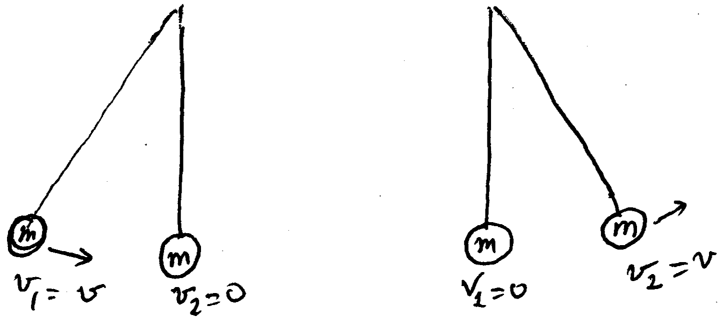
$\vec{R}$ CM position vector

- Discrete systems: $\vec{R} = \dfrac{\sum_i m_i \vec{r}_i}{M}$
  - $m_i$: mass of component $i$
  - $\vec{r}_i$: position vector for component $i$
  - $M = \sum_i m_i$ = total mass of system
- Continuous systems: $\vec{R} = \dfrac{\int \vec{r}\, dm}{M}$
  - $dm$: infinitesimal mass
  - $\vec{r}$: position vector for $dm$
  - $M = \int dm$ = total mass of system.

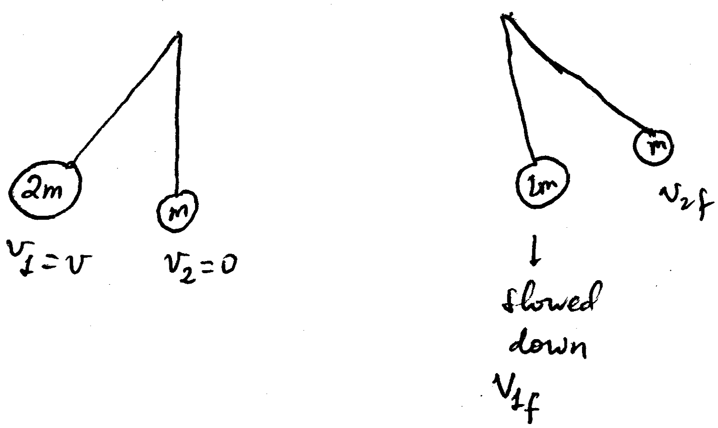
2nd Newton's Law for a system of particles:

$$\boxed{\underset{\text{on system}}{\vec{F}_{net}} = \underset{\text{Total mass}}{M} \cdot \frac{d^2 \vec{R}}{dt^2}}$$

$\vec{R}$ → CM position vector




~~change~~ trade of speed after the collision




linear momentum is conserved!

<u>Linear momentum of a system of particles:</u> $\vec{P}$ (vector!)

$$\vec{P} \equiv M.\vec{V} \quad \begin{cases} M: \text{total mass of system} \\ \vec{V}: \text{velocity vector of the CM of system} \end{cases}$$

$$= \sum_i \underbrace{\boxed{m_i \vec{v}_i}}_{\vec{P}_i} \quad \begin{cases} \text{sum of individual momenta of the} \\ \text{components of the system. } \vec{P}_i \equiv m_i \vec{v}_i \end{cases}$$

We mentioned linear momentum $\vec{p} = m\vec{v}$ when introducing 2nd Newton's law: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ (Ch 4)

For a system of particles 2nd Newton's law also applies:

$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}}$$

Net <u>external</u> force on the system equals the change of its total linear momentum over time

Very <u>important</u> consequence of 2nd Newton's Law on a system of particles:

$$\vec{F}_{net} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} \text{ is constant}$$

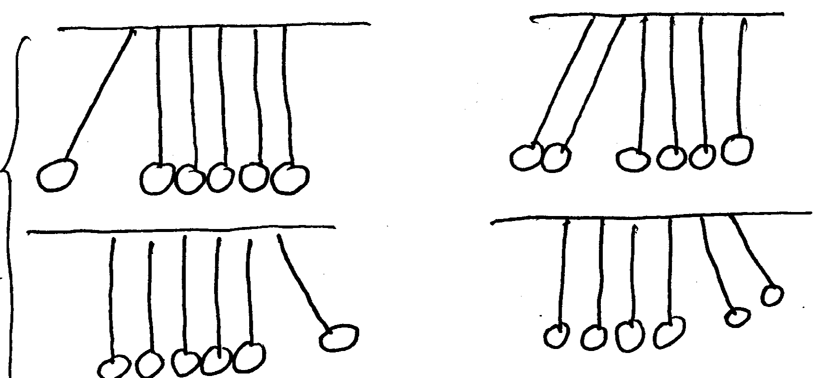
<u>Conservation of linear momentum</u>

(any collision: elastic & inelastic)

$\boxed{\vec{P}_i = \vec{P}_f}$ ⇔ define what are the initial & final situations

$\vec{F}_{net} = 0$

Net external force on system is 0



Clearly what is conserved is the linear momentum (product of mass & velocity)

So far: 2 conservation laws

Conservation of ME: $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$

Conservation of LM: $\vec{P}_i = \vec{P}_f$

\# of equations equals # of dimension

Collisions: net external force on system (minimum 2 components for a collision) is 0 $\Rightarrow$ $\vec{P}_i = \vec{P}_f$

**Inelastic:**

Colliding components stick together after collision:

$\rightarrow$ There is no conservation of ME: $KE_i > KE_f$

$KE_i = KE_f +$ energy lost into internal structure deformation or damage of colliding components

$\rightarrow$ Conservation of linear momentum: for a two component ~~collision:~~

$\boxed{\vec{V}_{1f} = \vec{V}_{2f} = \vec{V}_f}$

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f} = (m_1 + m_2)\vec{V}_f$$

(Vector equation: 1 eq in 1D, 2 eqs in 2D, etc)

**Elastic:** colliding components do not stick together after the collision. (no deformation, hard ball collisions)

For a 2 component collision:

$\rightarrow$ Conservation of KE: $\frac{1}{2}m_1 V_{1i}^2 + \frac{1}{2}m_2 V_{2i}^2 = \frac{1}{2}m_1 V_{1f}^2 + \frac{1}{2}m_2 V_{2f}^2$

(since final velocities are not the same $\rightarrow$ extra unknown, this additional eq. will help!)

$\rightarrow$ Conservation of LM: $m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f}$

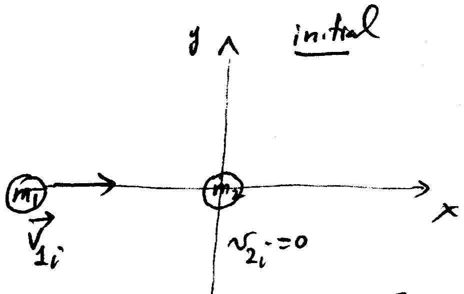
## Elastic Collisions: (hard ball collisions)

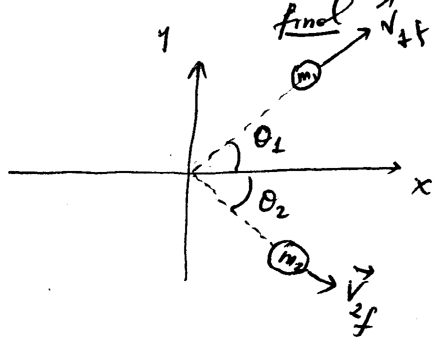
1) **1D elastic collision:** # of equations is 2 $\left\{ \begin{array}{l} KE_i = KE_f \\ P_i = P_f \end{array} \right\}$ can solve for 2 unknowns $\downarrow$ e.g. $V_{1f}$ & $V_{2f}$

**Can derive:**

2 component collision
$$\left\{ \begin{array}{ll} a) & V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i} \\ b) & V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} + \frac{m_2 - m_1}{m_1 + m_2} V_{2i} \\ c) & V_{1i} + V_{1f} = V_{2i} + V_{2f} \end{array} \right.$$

2) **2D elastic collisions** # of equations is 3 $\left\{ \begin{array}{l} KE_i = KE_f \\ P_{ix} = P_{fx} \\ P_{iy} = P_{fy} \end{array} \right.$

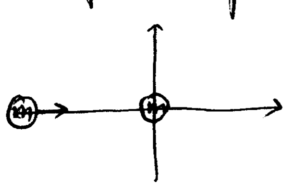


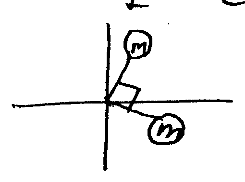




→ If we look for final velocities, for 2 component collisions: 4 unknowns (2 for each colliding component since we work in 2D)
One additional data will be provided
$V_{1f}$ or $\theta_1$ or $V_{2f}$ or $\theta_2$

Can derive:
$$\left\{ \begin{array}{ll} a) & V_{1i}^2 = V_{1f}^2 + \frac{m_2^2}{m_1^2} V_{2f}^2 + 2\frac{m_2}{m_1} V_{1f} V_{2f} \cos(\theta_2 - \theta_1) \\ b) & V_{1i}^2 = V_{1f}^2 + \frac{m_2}{m_1} V_{2f}^2 \\ c) & 0 = \left(\frac{m_2}{m_1} - 1\right) V_{2f} + 2V_{1f} \cos(\theta_1 - \theta_2) \end{array} \right.$$

Useful consequence when $m_1 = m_2$: eq c) $0 = 2V_{1f} \cos(\theta_1 - \theta_2)$

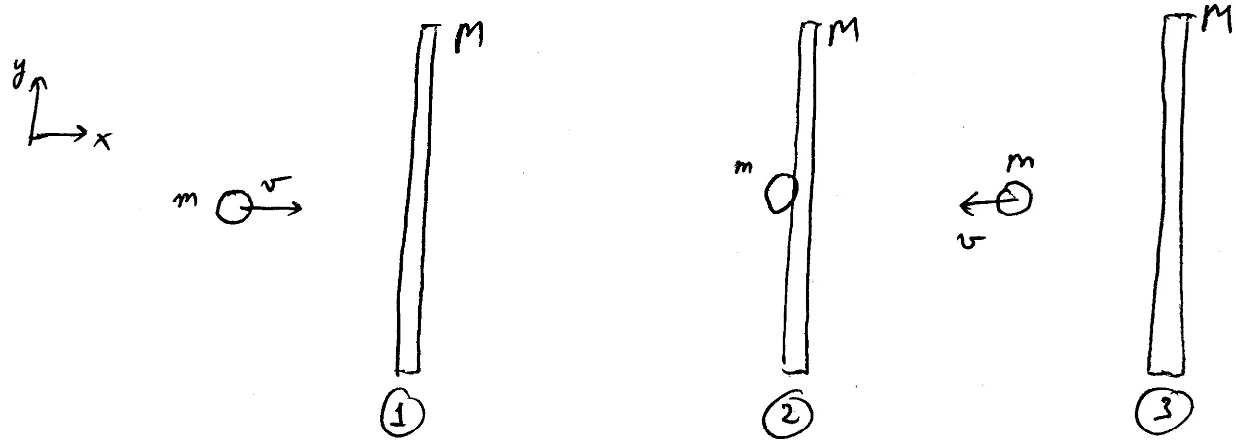



if $V_{1f} \neq 0 \Rightarrow \boxed{\theta_1 - \theta_2 = 90°}$

final directions of 1 & 2 form an angle of 90°

3) Collision of gas molecules (hard balls) with containing walls




Initial

final

No deformation neither for molecule nor the wall

$\downarrow$

Elastic collision ball/molecule comes back with same KE or same speed in the opposite direction!

In the x-direction: $P_{ix} = P_{fx}$ $(\vec{F}_{net} = 0)$

$\downarrow$

$$mv = m(-v) + 2mv$$

→ Mathematically

→ Physically:

$$2mv = MV.$$

$$V = \frac{2mv}{M}$$

(insignificant if $M >> m$)

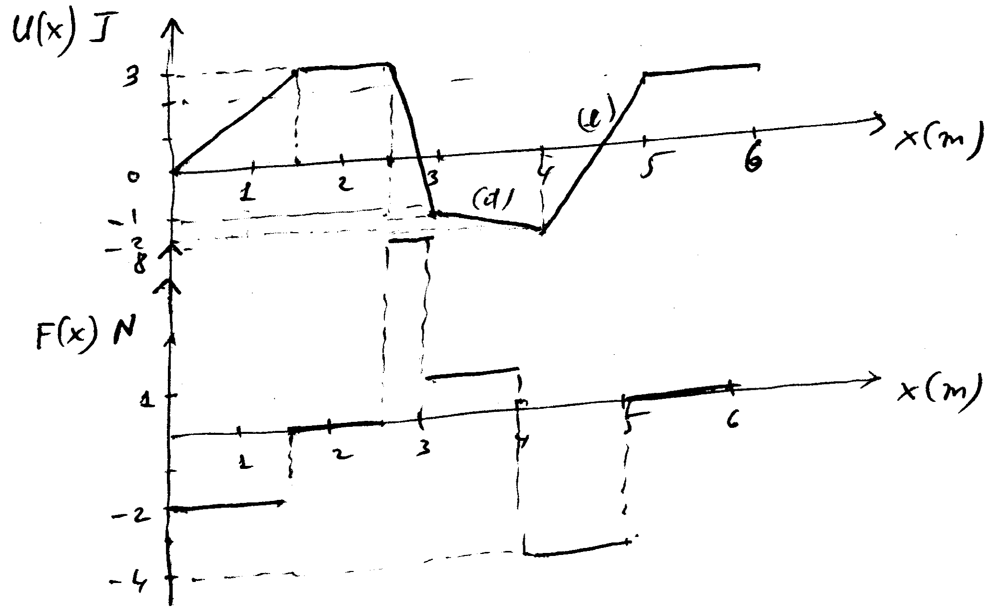
→ Momentum acquire by wall ⇒ air pressure on wall!

7.26 Find force from potential energy curve

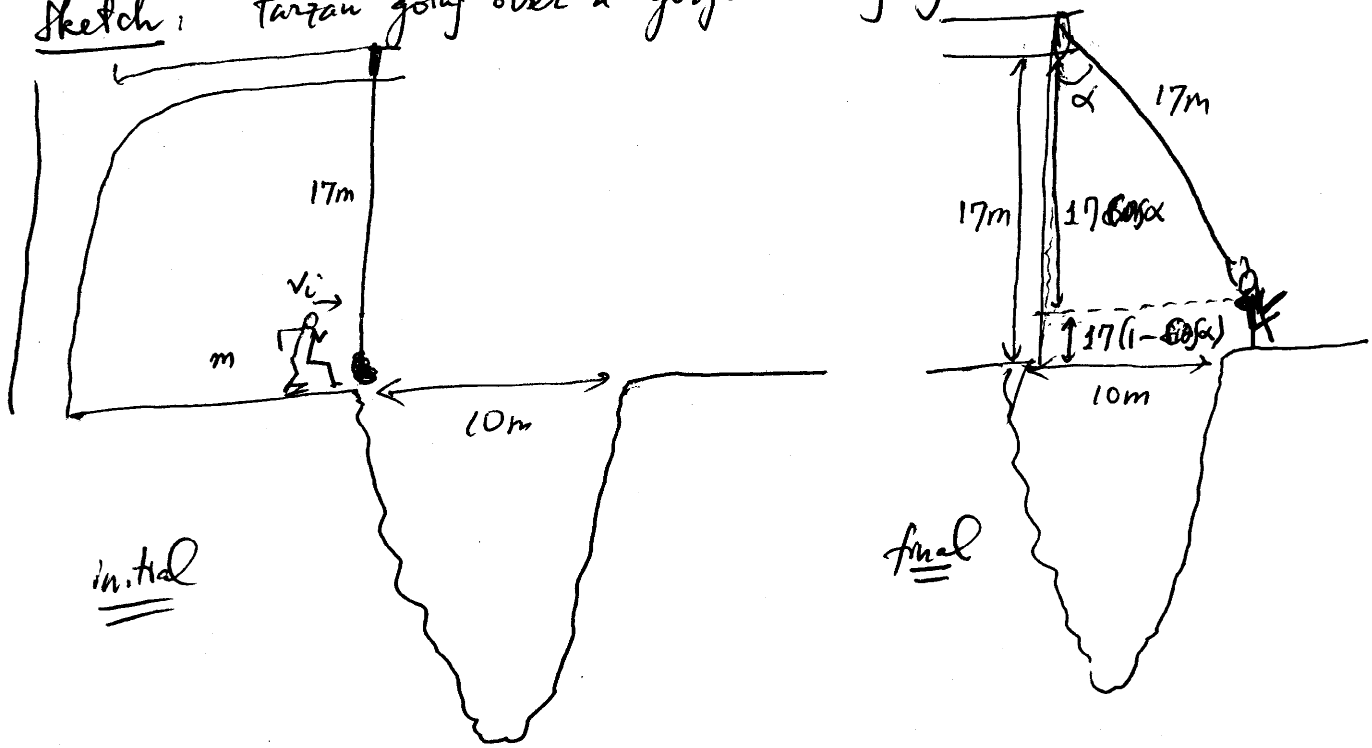
work

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r} \Rightarrow -\frac{d\,\Delta U_{AB}}{d\vec{r}} = \vec{F}$$

(Force equals minus the space derivative of potential energy)
or slope of potential energy curve



7.62 **Sketch:** tarzan going over a gorge swinging on a vine

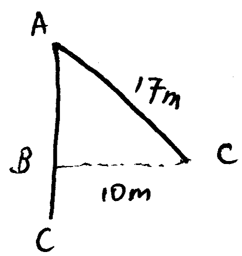



**Statements:** 1) When tarzan gets to the other side of gorge, he is @ $h = 17(1-\cos\alpha)$ above the ground!

$\sin\alpha = \frac{10}{17} \rightarrow \alpha = \sin^{-1}\frac{10}{17} \Rightarrow h = 17\left(1-\cos\left(\sin^{-1}\frac{10}{17}\right)\right)$

$\boxed{h = 3.25\,m}$

**alternative:**




$\Rightarrow AB^2 + 10^2 = 17^2$

$AB = \sqrt{17^2 - 10^2}$

$h = BC = 17 - AB = 17 - \sqrt{17^2-10^2}$

$= 3.25\,m$

→ the gained height by swinging from left side to the right side: he acquired a $GPE = mgh$

2) He got this energy from the $KE_i$ by running before grabbing the vine. Conservation of ME:

$\frac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh$

$v_{i\,min} = \sqrt{2gh} = \sqrt{2\times 9.81\times 3.25}$

$= 7.98\,m/s$

(For $v_{i\,min}$ he will just barely gets to the other side: $v_f = 0$ as he lands)

**8.32** For $v_{esc} = 30\frac{km}{s}$ what is $R_E$ for same $M_E$ ?

Any object under effect of gravity has a negative total ME:

$$\frac{1}{2}mv^2 - \frac{GM_E m}{r} < 0$$

At $v_{esc}$ → Total ME = 0 → open orbit

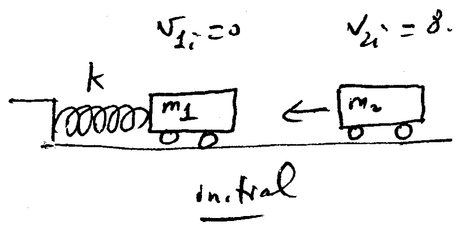
$$\frac{1}{2}\cancel{m}v_{esc}^2 = \frac{GM_E\cancel{m}}{r}$$

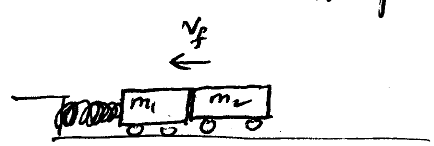
$$v_{esc}^2 = \frac{2GM_E}{r} \Rightarrow r = \frac{2GM_E}{v_{esc}^2} = \frac{2\times 6.67\times10^{-11}\times 5.97\times10^{24}}{30000^2}$$

**9.43** Statements 1) Two cars couple together: $\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f \Rightarrow$ inelastic collision.

2) Max compression for spring happens when the two cars going together @ speed $v_f$ come to stop: all of their KE is transferred to spring

y, x




final: two cars couple together compressing the spring

$k = 3.2\times10^5 \frac{N}{m}$

$m_1 = 11000\ kg,\ \vec{v}_{1i} = 0$

$m_2 = 9400\ kg,\ \vec{v}_{2i} = 8.5(-\hat{i})\ \frac{m}{s}$

$\vec{v}_f = v_f(-\hat{i})$

a) Max compression of spring: $\frac{1}{2}k\Delta x^2 = \frac{1}{2}(m_1+m_2)v_f^2 \Rightarrow \boxed{\Delta x = \sqrt{\frac{m_1+m_2}{k}}\, v_f}$

3) To find $v_f$: final speed after the inelastic collision b/w cars ① & ②.

↓

Conservation of LM $\boxed{\vec{P}_i = \vec{P}_f}$

$$\vec{P}_i = \vec{P}_f$$

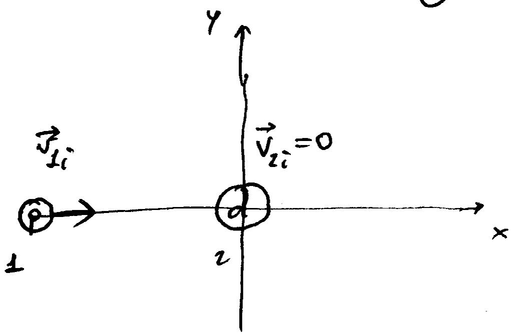
$$m_2\vec{v}_{2i} = (m_1+m_2)\vec{v}_f$$

$$-8.5m_2 = -v_f(m_1+m_2) \Rightarrow v_f = \frac{m_2}{m_1+m_2}8.5 = \frac{9400}{11000+9400}8.5 = 3.92\frac{m}{s}$$

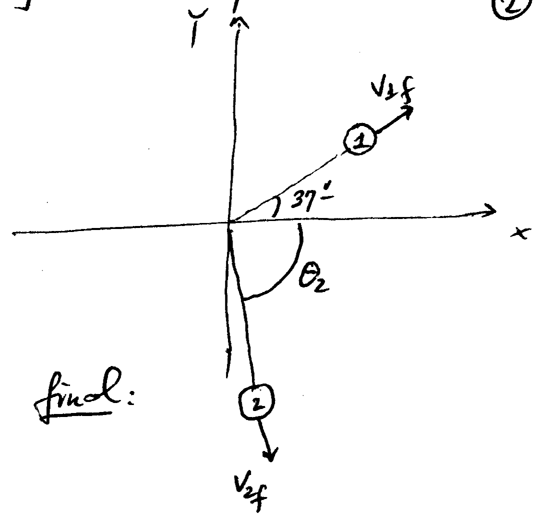
$$\boxed{\Delta x = \sqrt{\frac{11000+9400}{3.2\times10^5}}\ 3.92 = 0.989\,m}$$

b) Rebound speed of two cars : when spring returns all EPE back to KE $\vec{v}_f = -3.92\,\hat{i}$ m/s $\Rightarrow$ $\vec{v}_{rebound} = 3.92\,\hat{i}$ m/s

**9.71** Sketch: proton (1u) ① colliding elastically with a deuteron (2u) ②




initial




final:

Statements 1) 2D elastic collision

2) Note: final directions of ① & ② are not perpendicular to each other (since masses are not equal!)

3 unknows : $v_{1f}$, $v_{2f}$, $\theta_2$
3 equations : KE → 1 ; LM → 2

a) Fraction of KE transfered to deuteron: → proton slowed down after passing some KE to deuteron

$$\frac{KE_{2f}}{KE_{1i}} = \frac{KE_{1i} - KE_{1f}}{KE_{1i}} = 1 - \frac{KE_{1f}}{KE_{1i}} = 1 - \frac{\frac{1}{2}\cancel{m_1} v_{1f}^2}{\frac{1}{2}\cancel{m_1} v_{1i}^2}$$

$$\boxed{= 1 - \frac{v_{1f}^2}{v_{1i}^2}}$$

→ Use conservation of KE & LM to solve for $v_{1f}$ (eqs a) b) c))

$m_1 = 1u$ ; $m_2 = 2u$ → $\frac{m_2}{m_1} = 2$

a) $v_{1i}^2 = v_{1f}^2 + 4 v_{2f}^2 + 4 v_{1f} v_{2f} \cos(\theta_2 - 37°)$

b) $v_{1i}^2 = v_{1f}^2 + 2 v_{2f}^2$

c) $0 = v_{2f} + 2 v_{1f} \cos(\theta_2 - 37°)$ → $\cos(\theta_2 - 37°) = -\frac{v_{2f}}{2 v_{1f}}$

a) $V_{1i}^2 = V_{1f}^2 + 4V_{2f}^2 - 2V_{2f}^2 = V_{1f}^2 + 2V_{2f}^2$ or $1 = \frac{V_{1f}^2}{V_{1i}^2} + 2V_{2f}^2$

Fraction of KE transfered $= 1 - \frac{V_{1f}^2}{V_{1i}^2} = \boxed{2V_{2f}^2}$

$$V_{1f} = \frac{2V_{1i}\cos 37° \pm \sqrt{4V_{1i}^2\cos^2 37 + 12V_{1i}^2}}{6}$$

$$= V_{1i}\underbrace{\frac{2\cos 37° \pm 2\sqrt{\cos^2 37 + 3}}{6}}_{0.902}$$

$V_{1f} = 0.902V_{1i}$

$$1 - \frac{0.902^2 \cancel{V_{1i}^2}}{\cancel{V_{1i}^2}} = 1 - 0.902^2 = 0.186 \text{ or } 18.6\%$$

---

can also find $v_{2f}$ from eq b) $\quad V_{1i}^2 = V_{1f}^2 + 2V_{2f}^2$

$$\Rightarrow V_{2f}^2 = \frac{V_{1i}^2 - V_{1f}^2}{2}$$

$$= \frac{V_{1i}^2}{2}\underbrace{\left(1 - \frac{V_{1f}^2}{V_{1i}^2}\right)}_{0.186}$$

$$V_{2f} = V_{1i}\sqrt{0.93} = 0.305V_{1i}$$

---

can also find $\theta_2$ = from eq c):

$$\cos(\theta_2 - 37°) = -\frac{V_{2f}}{2V_{1f}} = -\frac{0.305\cancel{V_{1i}}}{2 \cdot 0.902\cancel{V_{1i}}} \Rightarrow \theta_2 - 37 = \cos^{-1}\left(\frac{-0.305}{2 \times 0.902}\right)$$

$$\boxed{\theta_2 = 62.75°}$$