Ch 5 Applications of Newton's Equations

1) Static equilibrium
2) Multiple objects
3) Frictional forces
4) Circular motion

Strategies for solution: five-step process

1) Understand the problem: statements & sketch
2) Select a convenient coordinate system
   ↳ to simplify the analysis:
   - When most forces point along the axes of the coordinate system
   - When motion of interest points along an axis of the coordinate system

3) Make a free-body diagram of forces acting on each object so to facilitate the calculation of the net force on each object. Draw the x & y components of those forces not already aligned along the axes.

4) Write 2nd Newton's Law for each object ($\vec{F}_{net} = m\vec{a}$), for each component (x & y) as needed. Note: in each direction we can combine forces arithmetically (add & subtract)

5) Solve these equations for the requested information, obtaining numeric solutions with correct units. Check if these numbers make sense.

i) Static equilibrium:

5.36

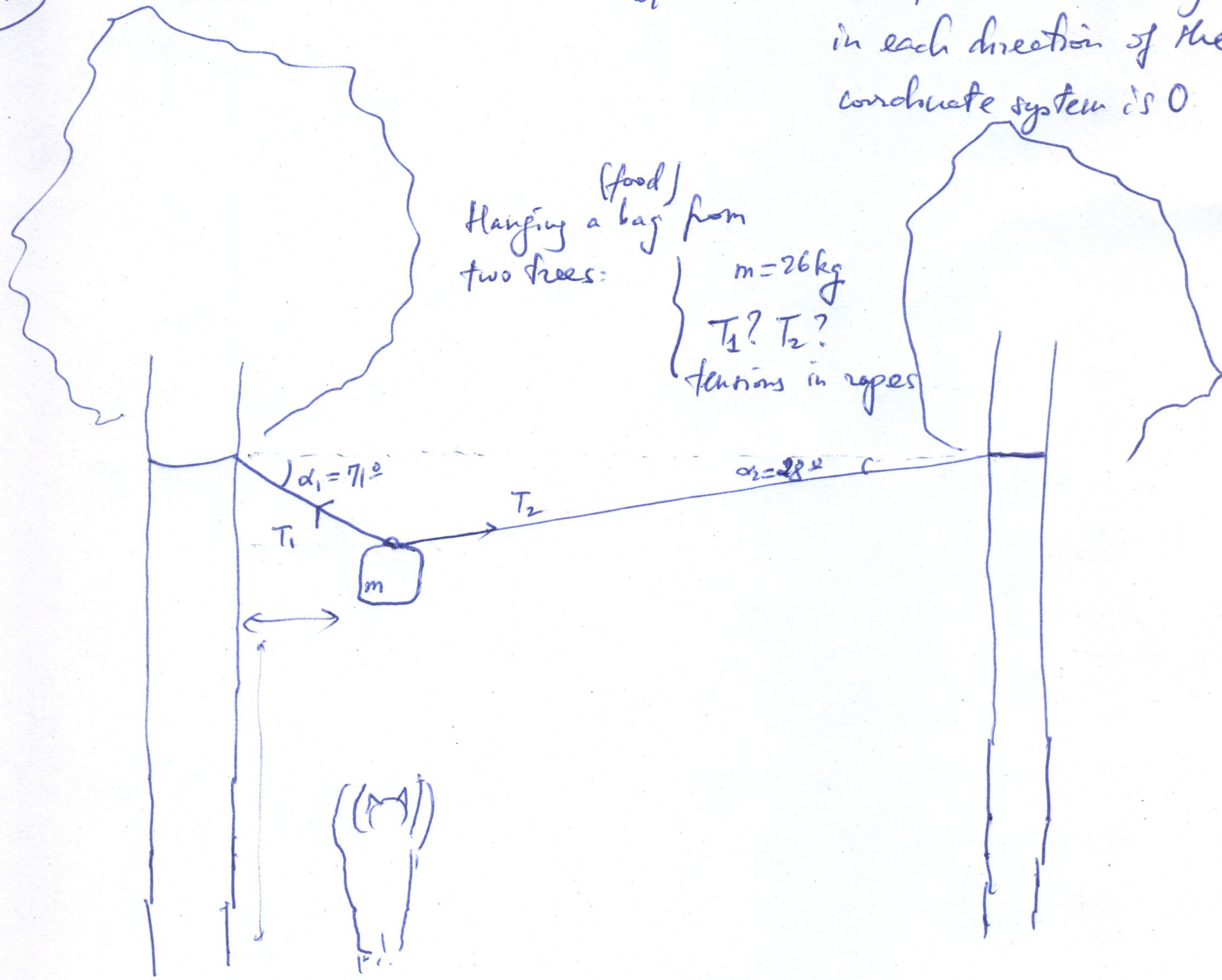
$\hookrightarrow \vec{a} = 0 \quad \longleftrightarrow \quad \vec{F}_{net} = 0$

Net force on each object in each direction of the coordinate system is 0

Hanging a (food) bag from two trees:

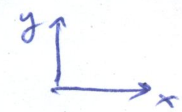
$m = 26\,kg$

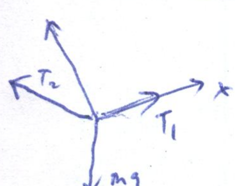
T_1? T_2? tensions in ropes



1) Statement: $\vec{F}_{net} = 0$ (tensions T_1 & T_2 cancel the weight mg)

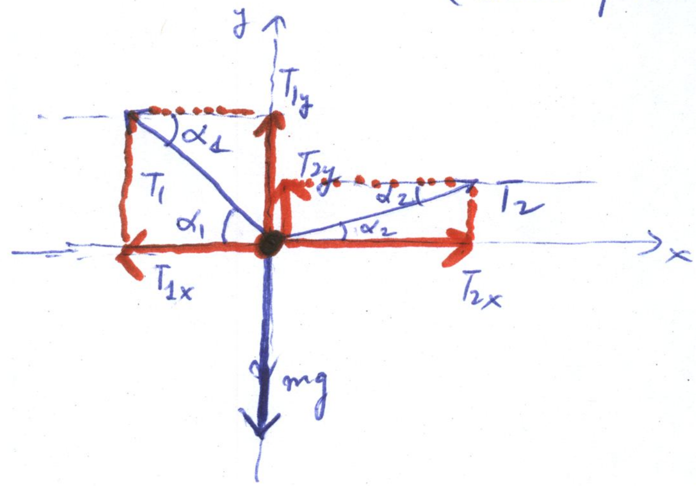
2) Most convenient coord. system: standard xy (mg along $-y$ direction).





does not offer any additional advantage since the forces involved: T_1 T_2 mg point in ~~directio~~ different directions not any two of them at 90° to each other.

3) F.B.D: free-body diagram: representing each object by a dot (where forces apply)



Notes: 1) T_{1x} & T_{2x} point in opposite directions ✓

2) T_{1y} & T_{2y} point in same direction ✓ They will cancel mg.

Components of tensions along axes of coord. system $\begin{cases}(T_{1x}, T_{1y}) \\ (T_{2x}, T_{2y})\end{cases}$

using unit vectors $\begin{cases}\vec{T}_1 = T_{1x}\hat{i} + T_{1y}\hat{j} = T_1\cos\alpha_1\hat{i} + T_1\sin\alpha_1\hat{j} \\ \vec{T}_2 = T_{2x}\hat{i} + T_{2y}\hat{j} = T_2\cos\alpha_2\hat{i} + T_2\sin\alpha_2\hat{j}\end{cases}$

$\Rightarrow$ Net forces in each direction for the object: $\begin{cases}F_{net_x} = T_{2x} - T_{1x} \\ F_{net_y} = T_{2y} + T_{1y} - mg\end{cases}$

4) Write 2nd Newton's law: $\vec{F}_{net} = m\vec{a} = 0$ (static equilibrium)

$$F_{net_x} = T_{2x} - T_{1x} = 0 \quad \Rightarrow T_2\cos\alpha_2 - T_1\cos\alpha_1 = 0$$

$$F_{net_y} = T_{2y} + T_{1y} - mg = 0 \Rightarrow T_2\sin\alpha_2 + T_1\sin\alpha_1 - mg = 0$$

$$\left.\begin{cases}a)\ T_2\cos 28^\circ - T_1\cos 71^\circ = 0 \\ b)\ T_2\sin 28^\circ + T_1\sin 71^\circ - 26\times 9.81 = 0\end{cases}\right\}$$ set of 2 equations & 2 unknowns T_1, T_2

5) Solve for T_1 & T_2: a) $T_1 = T_2\dfrac{\cos 28^\circ}{\cos 71^\circ} \rightarrow$ b) $T_2\sin 28^\circ + T_2\dfrac{\cos 28^\circ}{\cos 71^\circ}\sin 71^\circ = 26\times 9.81$

$$\Rightarrow T_2 = \frac{26\times 9.81}{\sin 28^\circ + \cos 28^\circ\tan 71^\circ} = 84\,N \Longrightarrow T_1 = 84\frac{\cos 28^\circ}{\cos 71^\circ} = 228\,N > T_2$$

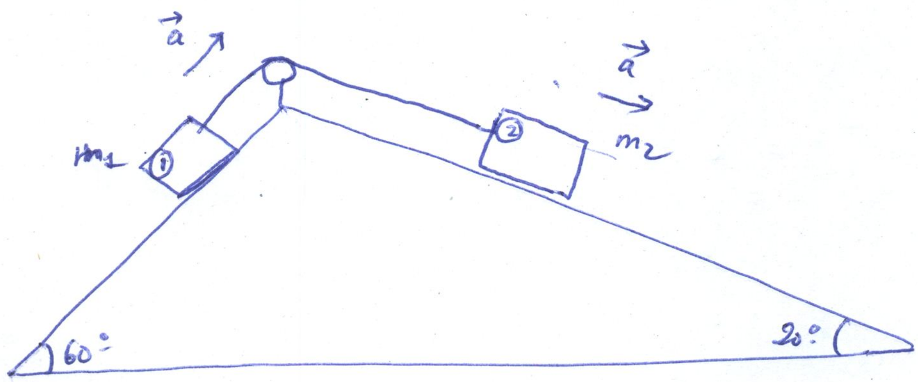
makes sense since $\alpha_1 = 71^\circ$ & $\alpha_2 = 28^\circ$

(ii) Multiple objects:
- → Five-step process, account for each object
- → If objects are connected: they will have the same acceleration although it could be in diferent directions (pulley is involved)

→ Two boxes m_1 & m_2 connected by a massless rope (same tension throughout)

$$a_1 = a_2 = a \text{ (magnitudes)}$$

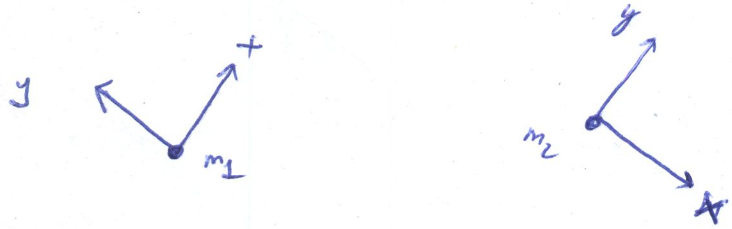
→ No friction b/w boxes & surface they are laying on



1) Statements:
a) same acceleration (in magnitude) for the boxes
$a_1 = a_2 = a$.
b) directions
- (A) box 1 moving up & box 2 moving down (CW around pulley)
- or (B) box 1 moving down & box 2 moving up. (CCW around pulley)

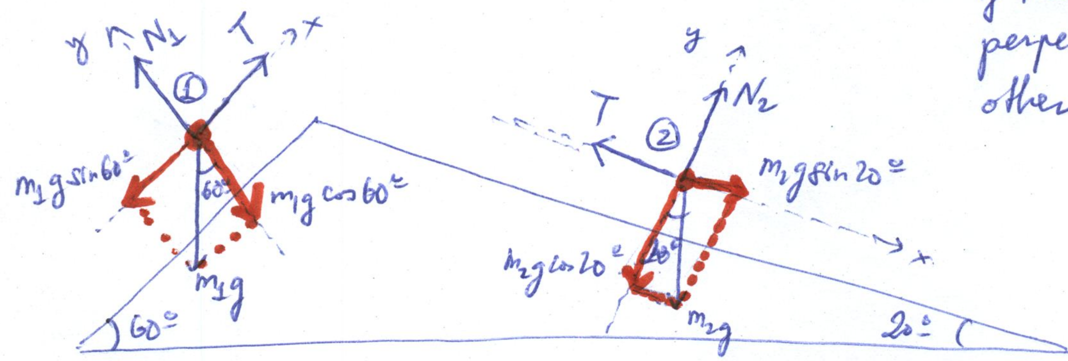
→ We will assume one scenario, for example (A), numbers will tell the actual directions
(If a comes out positive, → scenario (A)
if a comes out negative → opposite to what we assume → scenario (B)).

2) Most convenient coordinate systems
↓
direction of motion aligned along ~~axes~~ axis

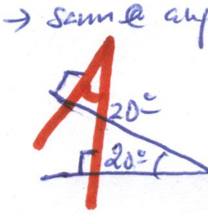


Also: forces on each box: tension & weight & normal
⇒ with these coordinate systems tension points along the x-axes, and normal forces along y-axes.

3) $\boxed{\text{FBD}}$ for each box



Angles: given 2 angles if their sides are perpendicular to each other → same angle



Notes: a) These coord. systems allow 2 ~~of~~ out of 3 forces on each object to be aligned along their axes

b) From F.B.D.

Box 1: net force $\begin{cases} F_{net_x} = T - m_1 g \sin 60° \\ F_{net_y} = N_1 - m_1 g \cos 60° \end{cases}$

Box 2: net force $\begin{cases} F_{net_x} = m_2 g \sin 20° - T \\ F_{net_y} = N_2 - m_2 g \cos 20° \end{cases}$

4) 2nd Newton's Law for each box: $\vec{F}_{net} = m\vec{a}$

Box 1: $\begin{cases} \text{(i) } T - m_1 g \sin 60° = m_1 a & \text{(acceleration in x is a)} \\ N_1 - m_1 g \cos 60° = 0 & \text{(boxes are not jumping on slope)} \end{cases}$

Box 2: $\begin{cases} \text{(ii) } m_2 g \sin 20° - T = m_2 a. & \text{(boxes are connected via rope \& pulley)} \\ N_2 - m_2 g \cos 20° = 0 \end{cases}$

5) Solve for unknown. For example assume m_1 & m_2 given, → solve for the acceleration of the system a.

Given m_1 & m_2: → 4 unknowns: T_1, N_1, N_2, a also 4 equations.

Using (i) & (ii) solve for T, then for a

$$\text{(i)} \Rightarrow T = m_1 g \sin 60^\circ + m_1 a$$

$$\Rightarrow \text{(ii)} \quad m_2 g \sin 20^\circ - m_1 g \sin 60^\circ - m_1 a = m_2 a$$

$$\boxed{a = \frac{m_2 g \sin 20^\circ - m_1 g \sin 60^\circ}{m_1 + m_2}}$$

This sign is very important! (if we had a +, we made a mistake!)

3 options for a:

- $a = 0 \iff m_2 \sin 20^\circ = m_1 \sin 60^\circ$ or $\dfrac{m_2}{m_1} = \dfrac{\sin 60^\circ}{\sin 20^\circ}$ (Static equilibrium)
- $a > 0$ (CW at pulley) $\iff \dfrac{m_2}{m_1} > \dfrac{\sin 60^\circ}{\sin 20^\circ}$
- $a < 0$ (CCW at pulley) $\iff \dfrac{m_2}{m_1} < \dfrac{\sin 60^\circ}{\sin 20^\circ}$

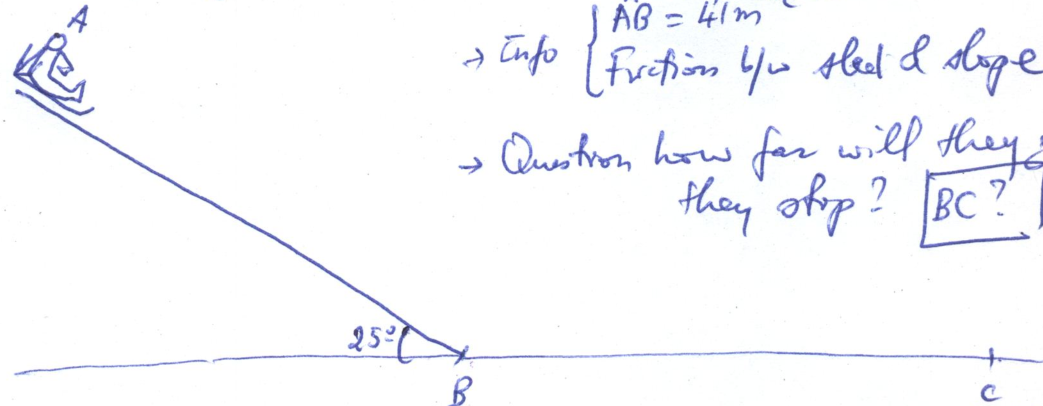
Note: w/o the − sign in the final equation for a we can't have these 3 options for any values of m_1 & m_2 ⇒ a contradiction.

(iii) Frictional Forces:

(5.50) Child sledding downhill & then along a flat bottom $A \to B \to C$

$\to$ Info $\begin{cases} v_A = 0 \,;\; v_C = 0 \\ AB = 41\,m \\ \text{Friction b/w sled \& slope } \mu_K = 0.12 \end{cases}$

$\to$ Question how far will they go until they stop? $\boxed{BC\,?}$



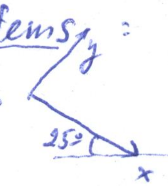
Child & sled stop ($v=0$)

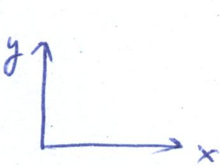
Five-step process for solution:

1) **Statements:**
   a) Child & sled stop @ C b/c of friction
   b) B/w A & B = constant acceleration (assuming they overcome friction)
   c) B/w B & C = constant deceleration b/c of friction

Sketch: see above

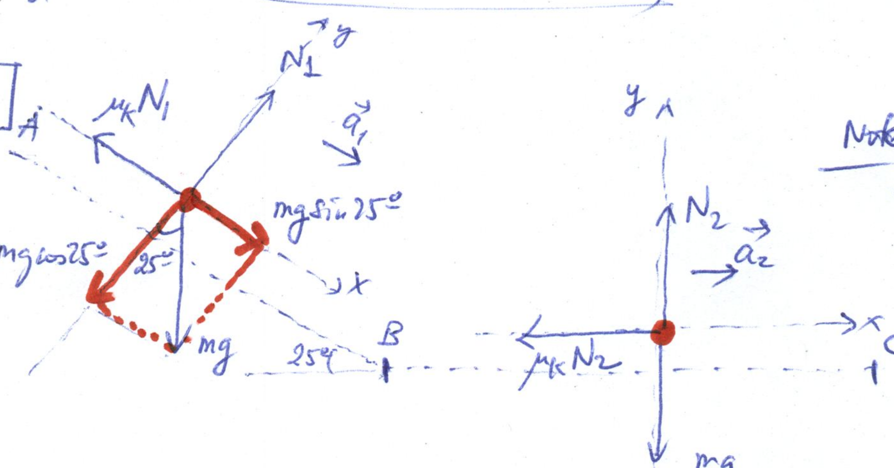
2) **Convenient coord systems:**

B/w A & B: $\to$ 

B/w B & C 

Directions of motion along x-axes

3) $\boxed{\text{FBD}}$



Note: Frictional forces always point in opposite direction as the motion

AB	BC
Net forces: $\begin{cases} F_{net_x} = mg\sin 25° - \mu_K N_1 \\ F_{net_y} = N_1 - mg\cos 25° \end{cases}$	$F_{net_x} = -\mu_K N_2$ $F_{net_y} = N_2 - mg$

4) Write 2nd Newton's Law: $\vec{F}_{net} = m \cdot \vec{a}$

AB	BC
$mg\sin 25° - \mu_K N_1 = m a_1$ $N_1 - mg\cos 25° = 0$	$-\mu_K N_2 = m a_2$ $N_2 - mg = 0$

→ This problem asks for distance until they stop or BC?

↳ need a_2 : $a_2 = -\frac{\mu_K N_2}{m} = -\mu_K \frac{\cancel{m}g}{\cancel{m}}$

$$\boxed{a_2 = -\mu_K g}$$

↳ $BC = x - x_0$: using 3rd kinematic equation in 1D

$$\boxed{\frac{v^2 - v_0^2}{x - x_0} = 2a}$$

↳ $\frac{\overset{0}{v_C^2} - v_B^2}{\boxed{x - x_0}} = 2a_2$ need ?

$$\boxed{(x - x_0)_{BC} = \frac{-v_B^2}{-2\mu_K g} = \frac{v_B^2}{2\mu_K g}}$$ → Now find v_B

To find v_B: final velocity v_B for 1st part of trajectory A to B (starts $v_A = 0$ @ top of hill, goes faster and faster till B)

↳ Need a_1:

↳ 2nd Newton's Law b/w A & B:

$$\boxed{a_1 = g \sin 25° - \mu_K g \cos 25°}$$

↳ 3rd kinematic eq. for constant acceleration

$$\frac{v_B^2 - \cancel{v_A^2}^{\,0}}{(x - x_0)_{AB}} = 2a_1$$

$$\rightarrow v_B = \sqrt{2g(\sin 25° - \mu_K \cos 25°)(x - x_0)_{AB}}$$

$$v_B = \sqrt{2 \times 9.81(\sin 25° - 0.12 \cos 25°)\, 41}$$

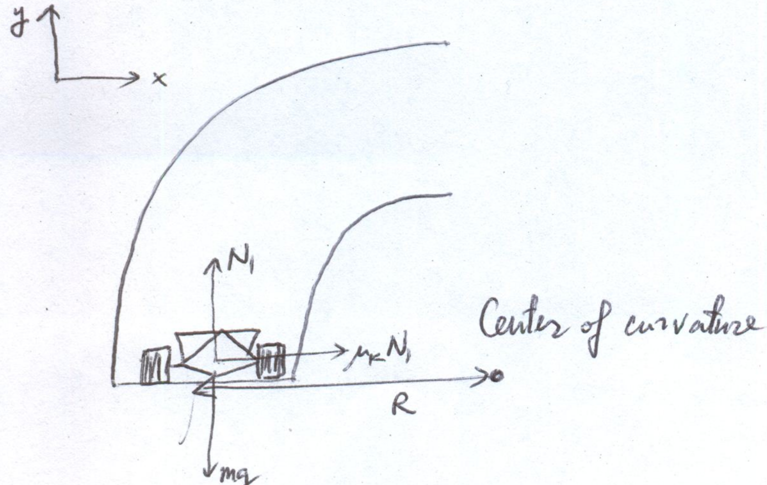
$$\boxed{v_B = 15.9\ m/s}$$

$$\Rightarrow (x - x_0)_{BC} = \frac{v_B^2}{2\mu_K g} = \frac{15.9^2}{2 \times 0.12 \times 9.81} = 107\ m$$

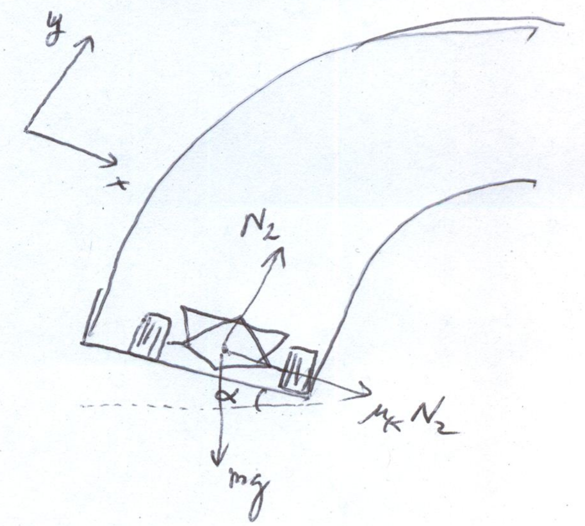
(IV) Circular motion: a) UCM object going at constant speed but not constant velocity ($a = \frac{v^2}{R}$) around a circular trajectory of radius R

b) Friction keeps car on track
(is the agent for $a = \frac{v^2}{R}$ toward center of curvature)

$\boxed{\text{Flat}}$ (Turn) race track

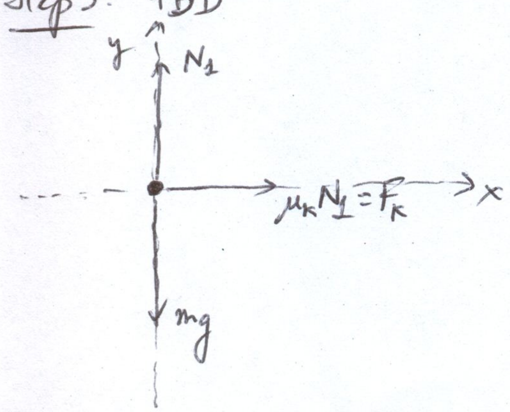


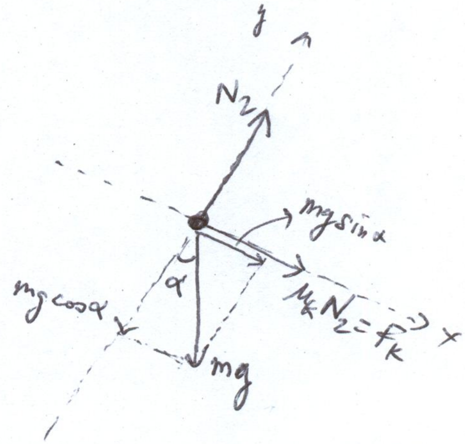
$\boxed{\text{Slanted}}$ (Turn) race track



Wide tires → more friction
(can go faster at turns)

Step 1: statements & sketches ✓
Step 2: convenient coord. systems ✓
Step 3: FBD





$F_{net_x} = \mu_k N_1$ (Flat turn)

$F_{net_y} = N_1 - mg$

$F_{net_x} = \mu_k N_2 \boxed{+ mg\sin\alpha}$ (Slanted turn)

$F_{net_y} = N_2 - mg\cos\alpha$

Step 4: 2nd Newton's Law $\vec{F}_{net} = m\vec{a}$

$\mu_k N_1 = m\frac{v^2}{R}$
$N_1 - mg = 0$

Step 5: $\mu_k g = \frac{v^2}{R}$

$\boxed{v_{Flat} = \sqrt{\mu_k g R}}$

Step 4: $\mu_k N_2 + mg\sin\alpha = m\frac{v^2}{R}$
$N_2 - mg\cos\alpha = 0$

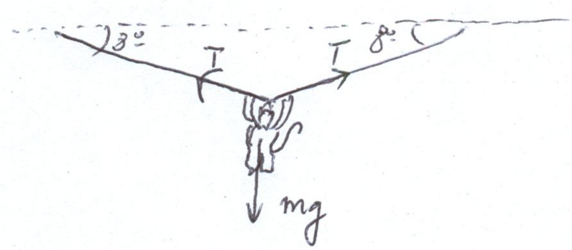
Step 5: $\boxed{v_{Slanted} = \sqrt{gR(\mu_k\cos\alpha + \sin\alpha)}}$

Let's compare:

$$\frac{v_{slanted}}{v_{flat}} = \sqrt{\frac{(\mu_K \cos\alpha + \sin\alpha)}{\mu_K}}$$

$\alpha = 20°$, $\mu_K = 0.2$ $\rightarrow$ $$\frac{v_{slanted}}{v_{flat}} = \sqrt{\frac{(0.2\cos 20° + \sin 20°)}{0.2}} = 1.63$$

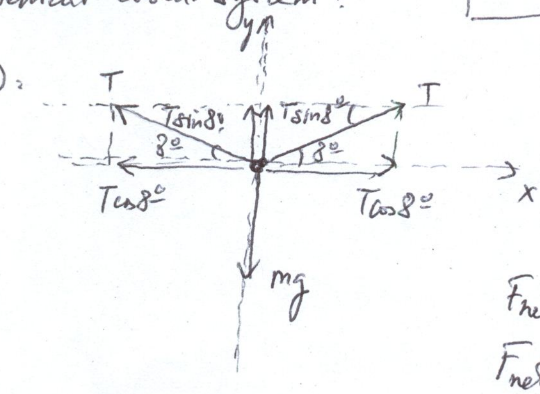
5.35 Monkey hangs in middle of massless rope
$m = 15\,kg$



Step 1: sketch ✓
statements: static equilibrium $\vec{F}_{net} = 0$ $\begin{cases} F_{net_x} = 0 \\ F_{net_y} = 0 \end{cases}$

Step 2: convenient coord. system: y x

step 3: FBD:



T's not aligned along axes
$\rightarrow$ find their Cartesian components.

$F_{net_x} = T\cos 8° - T\cos 8°$

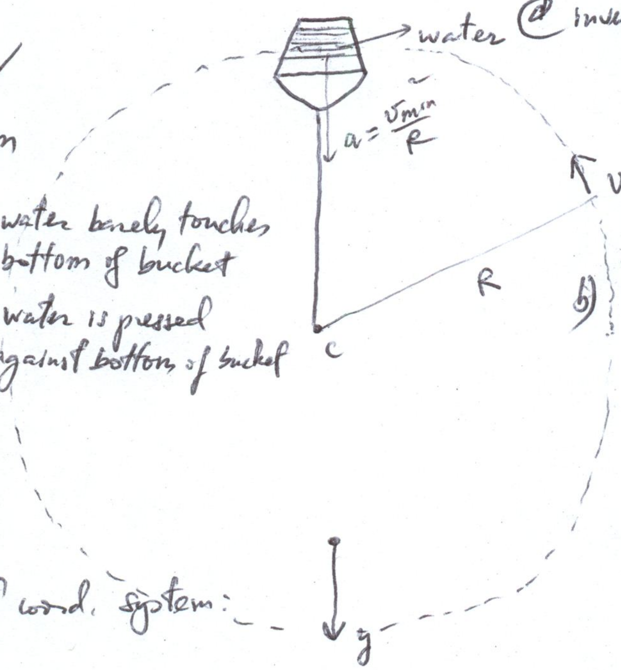
$F_{net_y} = 2T\sin 8° - mg$

step 4 $T\cos 8° - T\cos 8° = 0$

$$2T\sin 8° - mg = 0 \Rightarrow T = \frac{mg}{2\sin 8°} = \frac{15 \times 9.81}{2 \times \sin 8°}\,N = 528.7\,N$$

5.42

Water in bucket in vertical circular motion



Step 1: sketch ✓

$R = 0.85\,m$ ✓

Statement: a) { @ v_{min}: water barely touches bottom of bucket; > v_{min}: water is pressed against bottom of bucket }

⊥

if water can stay @ inverted position → it will make it through

b) $\boxed{\text{UCM}}$ for bucket & water

Step 2: convenient coord. system: ↓ y

Step 3: FBD: object is water @ inverted position & min speed.

↓ mg

Note: since water barely touches bottom of bucket (v_{min}) : N=0

$F_{net} = mg$

Step 4: 2nd Newton's law: $\vec{F}_{net} = m\vec{a}$

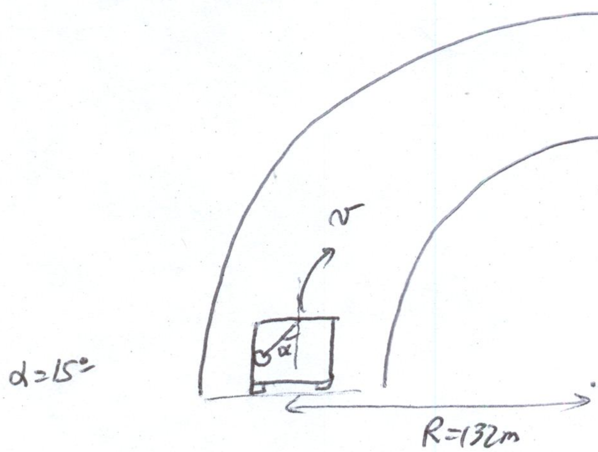
& Step 5.

$$\cancel{m}g = \cancel{m}\cdot\frac{v_{min}^2}{R} \rightarrow \boxed{v_{min} = \sqrt{gR}} = \sqrt{9.81 \times 0.85}$$

$$\boxed{v_{min} = 2.89\ m/s}$$

5.25

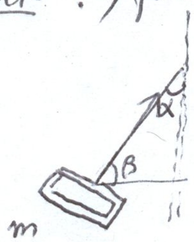
{ Train under UCM, flat track @ turn $R = 132m$
{ Hanging strap @ $15°$ to vertical



Step 1: sketch ✓

statements: a) from strap angle infer speed of train @ turn

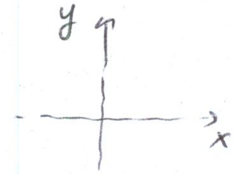
Why does it make an angle?



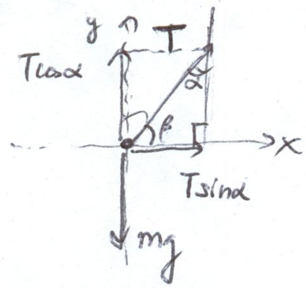
b) Strap also turns → it needs an acceleration toward center of curvature same as train

$a = \frac{v^2}{R}$ (UCM)

Step 2: conv. coord. system:



Step 3: FBD:



$\beta + \alpha + 90 = 180° \rightarrow \beta = 90 - \alpha$

β is the complement angle of α

$(\cos\beta = \sin\alpha \;\;\&\;\; \sin\beta = \cos\alpha)$

$F_{net_x} = T\sin\alpha$

$F_{net_y} = T\cos\alpha - mg$

Step 4: $\vec{F}_{net} = m\vec{a}$ (2^{nd} Newton's Law)

In x: $T\sin\alpha = m \cdot \frac{v^2}{R}$

In y: $T\cos\alpha - mg = 0$ (no motion in y direction)

Step 5: $T = \frac{mg}{\cos\alpha} \rightarrow \frac{\cancel{m}g}{\cos\alpha}\sin\alpha = \cancel{m}\frac{v^2}{R} \Rightarrow v = \sqrt{gR\tan\alpha} = \sqrt{9.81 \times 132 \times \tan 15°}$

$= 18.6\ m/s$ train went faster than speed limit.

Speed limit was $45\ \frac{km}{h} = \frac{45}{3.6}\ \frac{m}{s} = 12.5\ m/s$

Ch 6 Work, Energy, Power

→ Described motion ch 2 & 3
→ Included force ch 4 & 5

Force & Work: pushing a piano up a ramp

Stronger guy: more force
Weaker guy: less force

work: same ?
more ?
less ?

→ It is very important to know about work performed or energy spent

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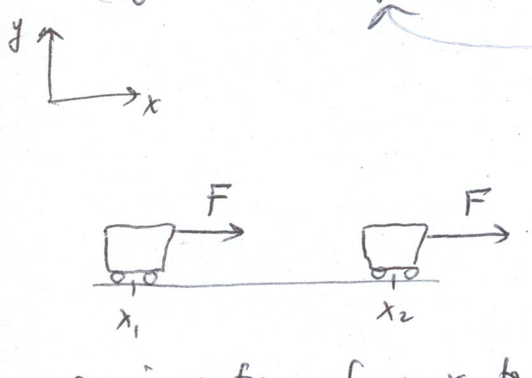
Work: $= \vec{F} \cdot \Delta\vec{r}$

$\vec{F}$: force applied (vector)
$\cdot$: scalar product (product of 2 vectors that is a number)
$\Delta\vec{r}$: displacement (vector)

↓
is independent of direction
a number or scalar
not a vector.

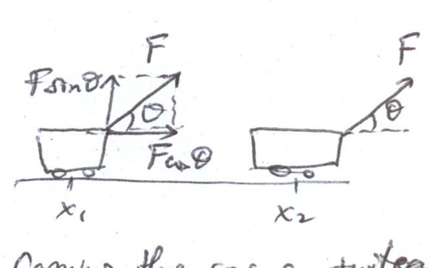
Why the scaler product? $\vec{A} \cdot \vec{B} = AB\cos(\theta)$ (θ angle b/w two vectors $\vec{A}$ & $\vec{B}$)



carrying suitcase from x_1 to x_2 applying a constant force F in x-direction ($x_2 - x_1 = \Delta x$)

Work done: $F \cdot \Delta x$

$\boxed{\text{Unit}}$: $N.m = J$ (Joule)

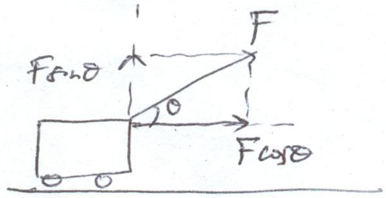


Carrying the same suitcase for same displacement Δx but F is @ angle θ

Work done: $\begin{cases} \text{in } x = F\cos\theta \cdot \Delta x = \vec{F} \cdot \Delta\vec{r} \\ \text{in } y = F\sin\theta \cdot 0 = 0 \end{cases}$

* Only the component of the force applied in the direction of motion performs work

Work & energy: differences:

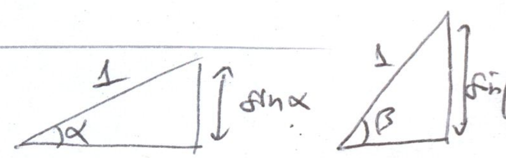


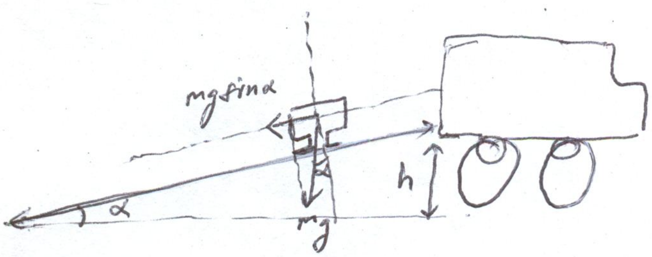
- → Only $F\cos\theta$ performs work
- → Some energy is spent to apply $F\sin\theta$

↳ More <u>efficient</u> is F is applied along direction of motion

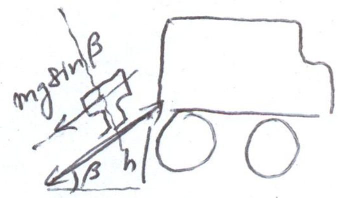
- Pushing a wall $\begin{cases} \Delta x = 0 \rightarrow \text{No work is perform} \\ \text{But energy is spent (calories)} \end{cases}$

<u>Pushing a piano up a ramp:</u>





Longer ramp.
smaller α



Shorter ramp
bigger angle $\beta > \alpha$

To push up: overcome $\boxed{mg\sin\alpha} < \boxed{mg\sin\beta}$

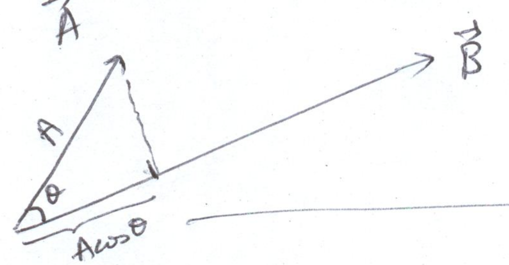
→ stronger guy can use a shorter ramp, weaker guy need long ramp.

Why would they ~~choose~~ a shorter ramp → overall work required is less!

(work is force × displacement)

Scalar product: math operation b/w 2 vectors that gives a number

Two vectors, with angle θ b/w them



Projection of $\vec{A}$ onto $\vec{B}$

Scalar product: $\vec{A}\cdot\vec{B} \equiv A.B.\cos\theta = \underbrace{A\cos\theta}_{\text{component of } \vec{A} \text{ along } \vec{B}} . B$

$\vec{A}\cdot\vec{B}$ ↓ scalar product; $A.B$ ↑↑ arithmetic product

Fits well on definition of work: $\text{Work} = \vec{F}\cdot\Delta\vec{r} = F\cos\theta . \Delta r$

Scalar product w/ unit vectors:

a) $\hat{i}\cdot\hat{i} = 1\cdot 1\cos 0 = 1$

$\hat{j}\cdot\hat{j} = 1 = \hat{k}\cdot\hat{k}$

b) $\hat{i}\cdot\hat{j} = 1\cdot 1\cos 90^\circ = 0$

$\hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = \hat{j}\cdot\hat{i} = \dots = 0$

2D: $\left.\begin{array}{l}\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}\\ \vec{F} = F_x\hat{i} + F_y\hat{j}\end{array}\right\}$ $W = \vec{F}\cdot\Delta\vec{r} = (F_x\hat{i} + F_y\hat{j})\cdot(\Delta x\,\hat{i} + \Delta y\,\hat{j})$

$= F_x\Delta x + F_y\Delta y$

$$\boxed{W = F\cos\theta\,\Delta x + F\sin\theta\,\Delta y}$$

Work performed by forces that change with position (e.g. spring force)

$$\text{Work} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot \underbrace{d\vec{r}}_{\text{infinitesimal displacement vector}}$$

Spring: $F = -kx \rightarrow \text{Work}_{\text{by spring}} = -k\int_0^x x . dx = -\frac{1}{2}kx^2$

(Negative as spring receives work when stretched (0 to x))

Power & Work: what is the difference

Two different cars going from rest to $60 \frac{mi}{h}$, assume same masses

Car #1

Work $= W_1$

Takes 10s

(Porsche)

$\downarrow$

Higher power

Car #2

Work $= W_2 = W_1 = W$

Takes 20s

(other)

Average power: $\overline{P} = \frac{\Delta Work}{\Delta t}$

Instantaneous power $P = \frac{d Work}{dt}$

Power & velocity: $P = \frac{dWork}{dt} = \frac{d(\vec{F}\cdot\Delta\vec{r})}{dt}$

$= \vec{F}\cdot\frac{d\Delta\vec{r}}{dt} = \vec{F}\cdot\vec{v}$

$\downarrow$ $\vec{F}$ constant

$\downarrow$ scalar product.

Work & Energy (Kinetic or energy in motion)

Ch4&5: Motion is changed when a force is applied.

Force & motion $\longrightarrow$ Work done

Work done to change a motion $\Longleftrightarrow$ Kinetic energy.

$$\hookrightarrow \int_{\vec r_1}^{\vec r_2} \vec F_{net}\cdot d\vec r \longrightarrow = m\int_{\vec r_1}^{\vec r_2} \frac{d\vec v}{dt}\cdot d\vec r$$

$\vec v$

2nd Newton's Law: $\vec F_{net} = m\cdot\frac{d\vec v}{dt}$

$\downarrow$

(constant m)

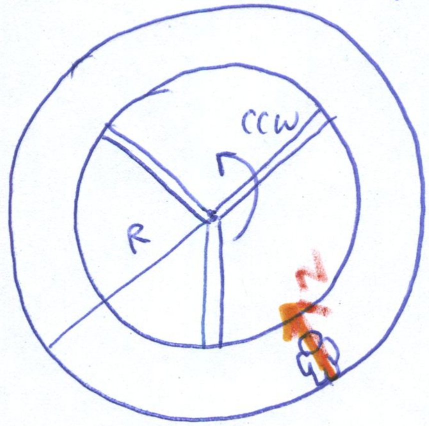
$$= m\int_{\vec r_1}^{\vec r_2} d\vec v\cdot\vec v = m\int_1^2 v\,dv = \left[\frac{1}{2}mv^2\right]_1^2$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = KE_2 - KE_1 = \Delta KE$$

$$\boxed{KE = \frac{1}{2}mv^2}$$

5.58 - Hollow ring space station $R = \frac{D}{2} = 225\,m$
- Rotates CCW to simulate effect of gravity $\Rightarrow$ RPM? (rev. per min.)

Step 1: sketch ✓



Statement:

a) Weightless in outer space (no mg!)

b) Astronaut standing on outer edge will rotate with space station under UCM $\Rightarrow$ $a = \frac{v^2}{R}$ (toward center of curvature or center of ring)

c) What force is giving the astronaut this acceleration?
Normal force by outer edge of space station N.

Step 2: ID

Step 3: FBD

Step 4: 2nd Newton's Law: $F_{net} = N = m \cdot a = m\frac{v^2}{R}$

To simulate weight as on Earth: $N = mg$ $\Big\}$ $g = \frac{v^2}{R}$

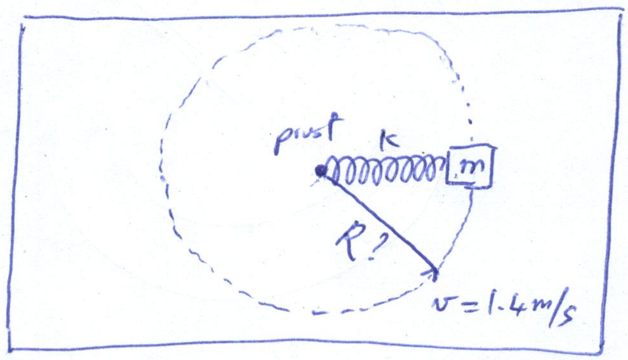
$v = \sqrt{gR}$

$v = \sqrt{9.81 \times 225} = 46.9\,m/s \xrightarrow{\text{to RPM}} 46.9\,\frac{m}{s} \cdot \frac{60\,s}{1\,min} \cdot \frac{1\,rev}{2\pi \times 225\,m} = 1.99\,RPM$

One rev → One circumference or one turn is $2\pi R$

5.65

Step 1: sketch ✓ system horizontal on a table.
view from above:



$m = 2.1 kg$
$k = 150 N/m$
$l_0 = 18 cm$
No friction
R?

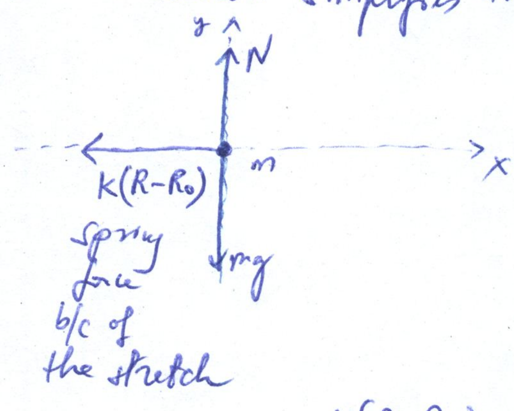
Statement: ✓ a) Mass m undergoes UCM @ constant speed $v = 1.4 \frac{m}{s}$
Velocity is changing → acceleration toward center of curvature $\underline{a} = \frac{v^2}{R}$.

b) Spring provides this acceleration, it will be stretched (to give a pull on m) $R > R_0 = l_0 = 0.18 m$

Step 2: convenient coord. system

(position of m as shown simplifies the analysis)

Step 3: FBD: of m



$F_{net_x} = -k(R - R_0)$
$F_{net_y} = 0$

Step 4: 2nd Newton's Law:
$$\begin{cases} -k(R-R_0) = m \cdot \left(-\frac{v^2}{R}\right) \\ 0 = m \cdot 0 \end{cases}$$

Step 5: solve for R $kR^2 - kR_0R = mv^2 \rightarrow kR^2 - kR_0R - mv^2 = 0$

$$150 R^2 - \underbrace{150 \cdot 0.18}_{27} R - \underbrace{2.1 \times 1.4^2}_{4.12} = 0 \rightarrow R = \frac{+27 \pm \sqrt{27^2 + 600 \times 4.12}}{300} \begin{cases} 0.279 m \\ - \text{ not relevant} \end{cases}$$

6.18

Work done by a force $\vec{F} = 1.8\hat{i} + 2.2\hat{j}$ N

$\Delta\vec{r} = \vec{r} - 0 = 56\hat{i} + 31\hat{j}$ m

$\text{Work} = \vec{F}\cdot\Delta\vec{r} = (1.8\hat{i} + 2.2\hat{j})\cdot(56\hat{i} + 31\hat{j})$

$= 1.8\times 56 + 2.2\times 31 \text{ J} = 169 \text{ J}.$

$$\begin{cases} \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1 \\ \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = 0 \end{cases}$$

6.72

How many lifts of a barbell to burn $\underbrace{230 \text{ kcal}}_{\text{unit of energy}}$

→ Conversion: $\frac{4.186 \text{ J}}{1 \text{ cal}} \times 230\times 10^3 \text{ cal} = 963\times 10^3 \text{ J}$

→ Work done per lift: 45 kg a height of 0.5 m

$\hookrightarrow \underbrace{mg}_{\text{Force applied}}\underbrace{\Delta h}_{\text{displacement}} = 45\times 9.81\times 0.5 = \frac{1100}{5} \text{ J}$

work = grav. potential energy

$\Rightarrow \#\text{ lifts} = \frac{963\times 10^3 \text{ J}}{\frac{1.1\times 10^3 \text{ J}}{5}} = 873 \text{ lifts.} \times 5 = 4359 \text{ lifts.}$

6-81 Related to egg dropping question in PP#1 { First part of a fall: free fall; Second part: crashing }

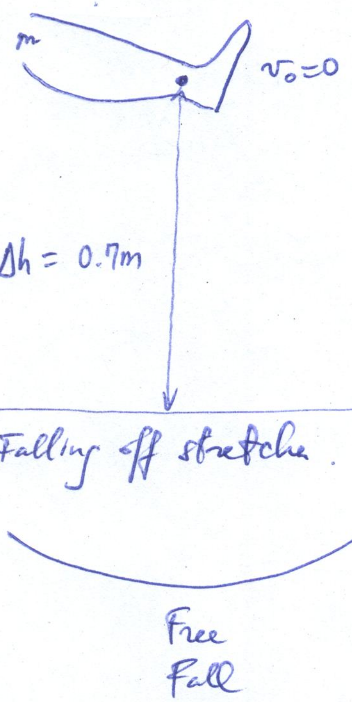
Statements: Leg falling off a stretcher { 1) Free fall: constant acceleration $a = g$ until it first touches ground (no hurt)

2) $\boxed{\text{Crashing:}}$ lasts from when it first touches ground until it completely stopped: Very short time & very large deceleration (to bring the final speed at end of free fall down to zero!) (it hurts) }

Leg → $m = 8kg$

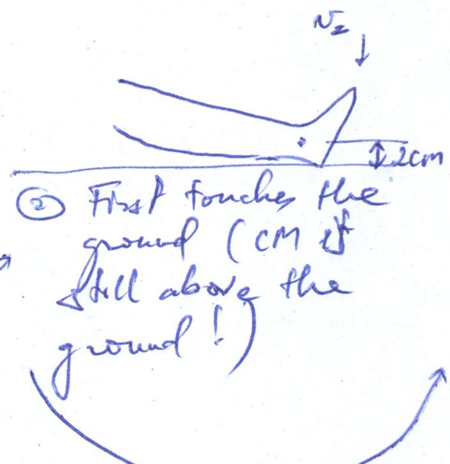
off → $v_0 = 0\frac{m}{s}$

Sketches:

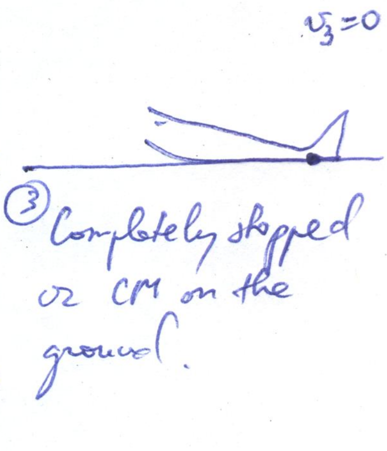


① Falling off stretcher

Free Fall



② First touches the ground (CM is still above the ground!)



③ Completely stopped or CM on the ground.

Constant deceleration

Stopping distance = 0.02m

$KE_1 = 0$ $\qquad KE_2 = \frac{1}{2} m v_2^2$ $\qquad KE_3 = 0$

KE_2 is gone in a very short period of time { some to the ground; mostly by bones & tissues! }

Hospital attorney argues stopping force = weight = 80N.
Let's calculate the actual stopping force:

Method #1: Work & energy: (Ch 6)

Energy acquired by leg 1 → 2: (Energy ⇔ Work: force × displ)

$$mg\Delta h = 8 \times 9.81 \times 0.7 \text{ J}$$

Energy absorbed by leg 2 → 3 = same as acquired 1 → 2

→ stopping force × stopping distance.

$$F_{stopping} = \frac{8 \times 9.81 \times 0.7}{0.02} \frac{N\cancel{m}}{\cancel{m}} = 2744\,N.$$

Method #2: Newton's Law & kinematic eqs (Ch 2, 3, 4)

$$F_{stopping} = m \cdot a_{23} \rightarrow \boxed{\text{Need } a_{23}} \text{ (deceleration)}$$

Kinematic eq #3: $\frac{v^2 - v_0^2}{\Delta x} = 2 \cdot a \xrightarrow{2 \to 3} \boxed{\frac{0 - v_2^2}{0.02} = 2a_{23}}$

→ $\boxed{\text{Need } v_2}$ final speed @ end of free fall ($\Delta h = 0.7m$)

Kinematic eq #3: $\frac{v_2^2 - 0}{0.7} = 2g \Rightarrow v_2 = \sqrt{2 \times 9.81 \times 0.7} = 3.7 \frac{m}{s}$

→ $a_{23} = -\frac{3.7^2}{0.02 \times 2} = -343.35 \frac{m}{s^2}$ (huge!)

→ $F_{stopp} = m \cdot a_{23} = 8 \times 343.35 = 2774\,N.$

(Shorter to use Work & energy)

6.36 $m = 75\,kg$ Long jumper

$$\underbrace{\overset{v_0}{0} \longrightarrow \overset{v}{10\,m/s}}_{\Delta t = 3.1\,s}$$

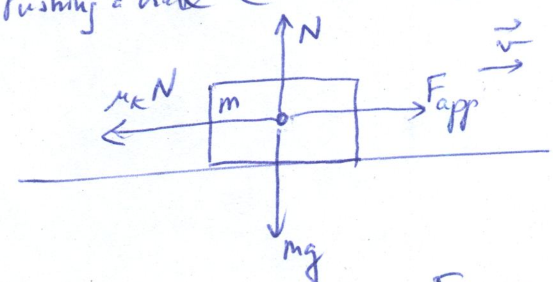
Power output?

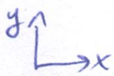
$$\bar{P} = \frac{Work}{\Delta t} = \frac{\Delta KE}{\Delta t} = \frac{\frac{1}{2} m v^2}{\Delta t} = \frac{\frac{1}{2} \cdot 75 \times 10^2\,J}{3.1\,s} = 1210\,W$$

6.67 Pushing a crate @ $v = 0.62\,m/s$ constant speed. Power? $\boxed{P = F_{app} \cdot v}$

$m = 95\,kg$

$\mu_K = 0.78$





a) Net forces $\left\{ \begin{array}{ll} x = & F_{app} - \mu_K N = m \cdot 0 \\ y = & N - mg = 0 \end{array} \right\} F_{app} = \mu_K mg$

$$P_{req} = F_{app} \cdot v = \mu_K mgv = 0.78 \times 95 \times 9.81 \times 0.62 = 450\,W$$

b) Work to push it 11 m:

$$Work = F_{app} \times d = 0.78 \times 95 \times 9.81 \times 11 = 8000\,J.$$